Demystifying the fusion mechanism in heavy ion collisions within full Langevin dissipative dynamics

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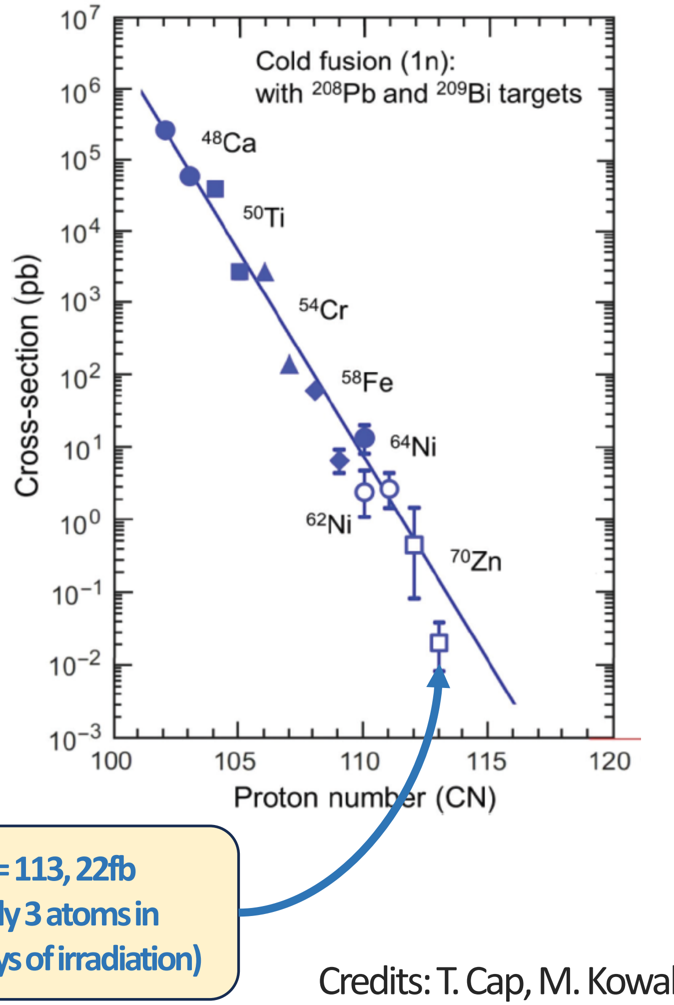
NARODOWE CENTRUM BADAŃ JĄDROWYCH ŚWIERK

Description of the fusion mechanism within Langevin dynamics

Super-heavy elements with Z > 103 do not occur in nature.
They can only be produced in the laboratory by fusing two lighter nuclei.

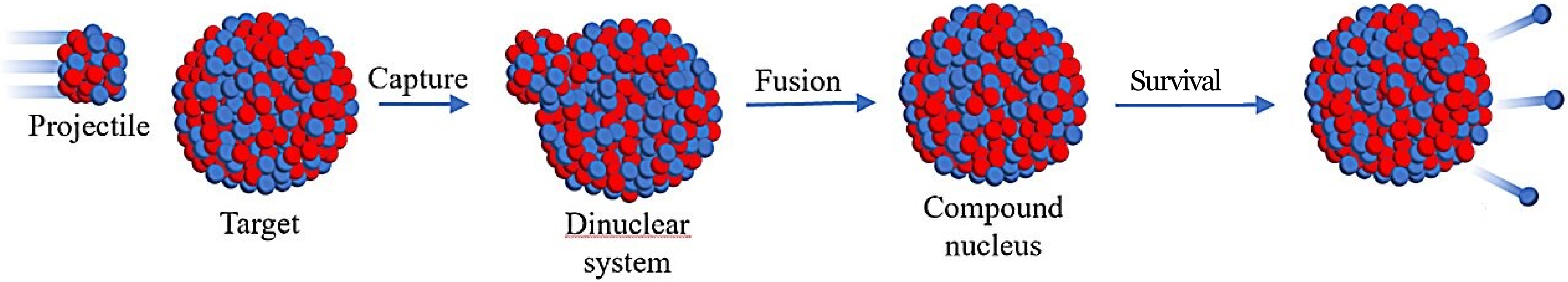
Our primary goal is to gain insights into the **fusion reaction mechanisms in the domain of cold synthesis reactions** ($Z < 113$, $E^* \approx 10 - 20$ MeV), in particular on the **understanding of the hindrance mechanism which prevents the formation of super-heavy nuclei.**

We propose a comprehensive dissipative dynamics **Langevin**-based formalism to describe the unrestricted motion of the systems in terms of **elongation**, **neck** and **asymmetry** variables.



Credits: T. Cap, M. Kowal

The fusion process (Schematic view)



Micro-macroscopic description of fusion:

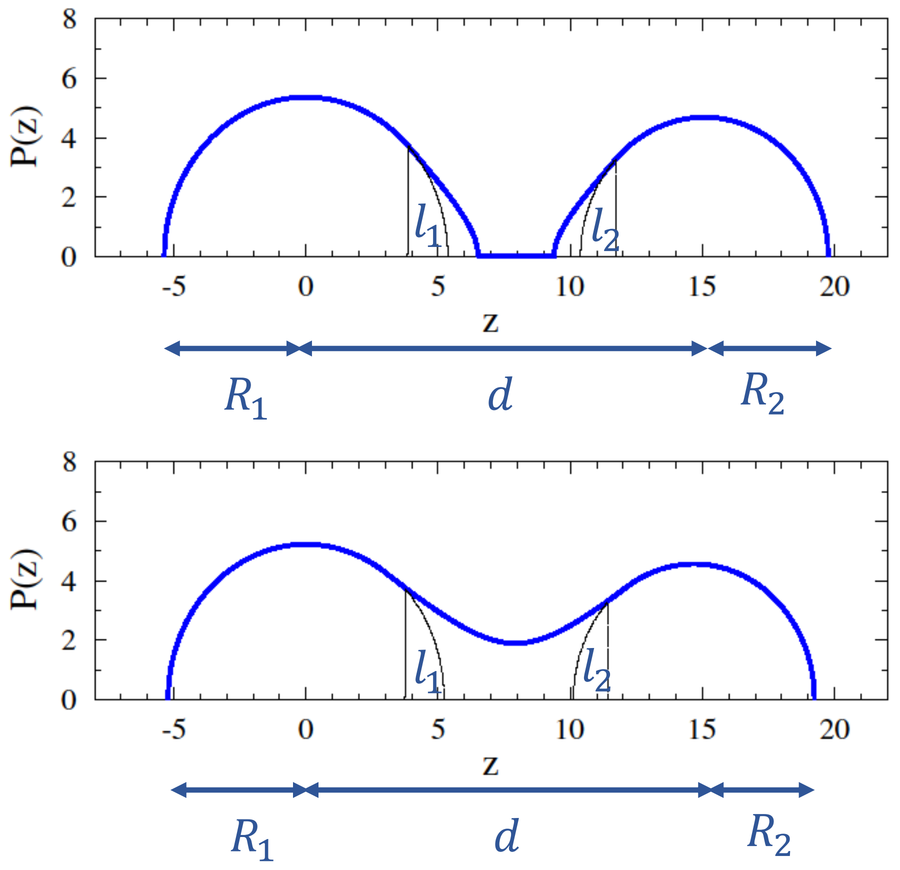
- Complexity/impossibility of tracking all internal degrees of freedom
- Identification of **slow collective degrees of freedom** immersed in a **bath** of faster dynamics
- Emergence of the mechanisms of **friction** and **random forces**

Collective variables adapted to fusion/fission – Shape variables

$^{92}Zr + ^{64}Ni$

- Axially symmetric shapes
- **Spherical** cups connected by quadratic surfaces[1]

- **Shape collective/slow variables**:
  - **Distance/elongation** : $\rho = \frac{d}{R_1+R_2}$
  - **Neck/deformation** : $\lambda = \frac{l_1+l_2}{R_1+R_2}$
  - **Asymmetry**: $\Delta = \frac{R_1-R_2}{R_1+R_2}$



Bipartite
$\rho = 1.5$
$\lambda = 0.3$
$\Delta = \Delta_0$

Monopartite
$\rho = 1.5$
$\lambda = 0.4$
$\Delta = \Delta_0$

[1] J. Błocki, H. Feldmeier and W. J. Świątecki, Nucl. Phys. A 459 (1986) 145

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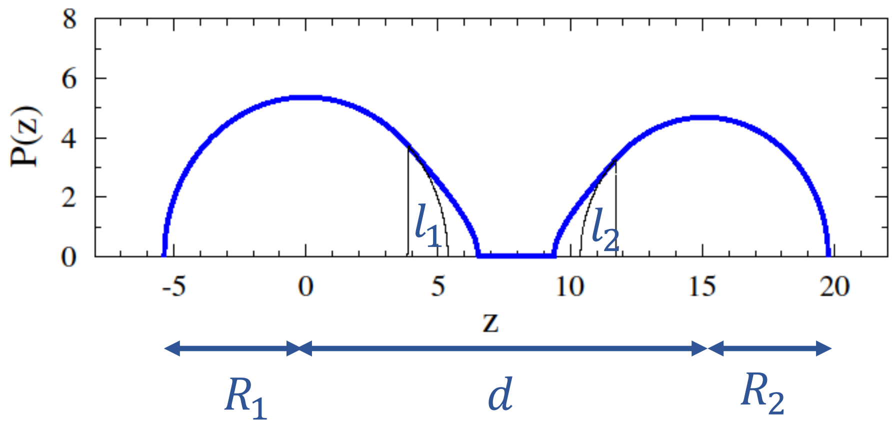
  - **Asymmetry**: $\Delta = \frac{R_1-R_2}{R_1+R_2}$
- **Scission is well-defined**: $\lambda_{\text{scission}} = 1 - \frac{1}{\rho_{\text{scission}}}$

  → **Suited to describe fusion/fission** (vs. multipole moments)

$^{92}Zr + ^{64}Ni$



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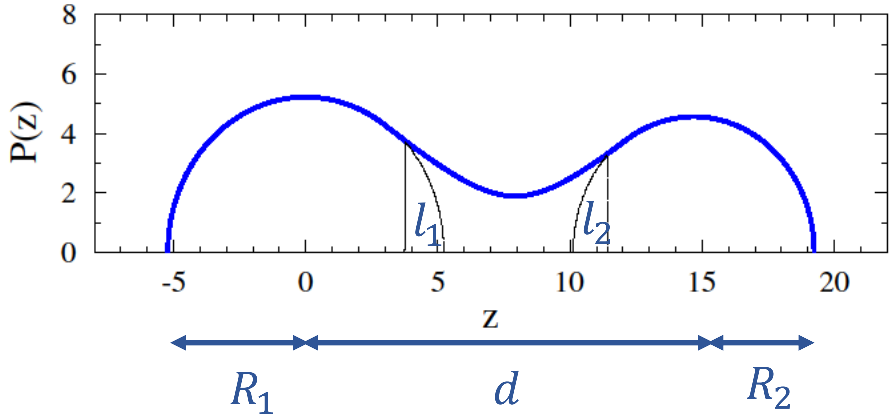

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Collective variables adapted to fusion/fission – Angle variables

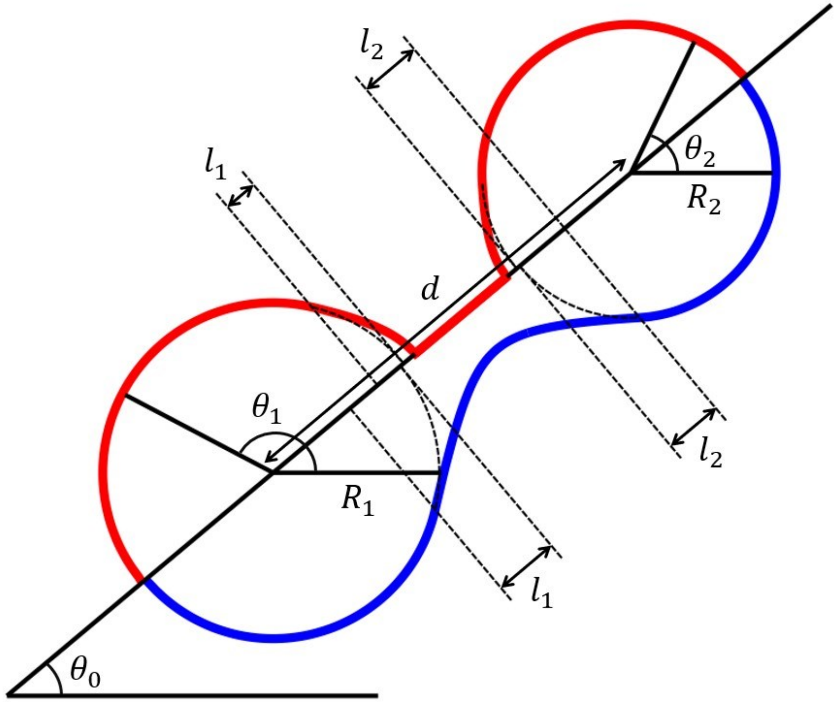
- **Collective angle variables**
  - Angle of the whole system θ_0
  - Angle of the first sphere θ_1
  - Angle of the second sphere θ_2

- Variations linked to angular momentum, in particular:

$$p_{\theta_0} + p_{\theta_1} + p_{\theta_2} = L_{init}$$

- **Exact treatment of angular momentum**

→ **Full Langevin 6-dimensional dissipative dynamics**



The Langevin system of equations

- Denoting **collective/slow variables** $q_i(t)$ and their **associated moments** $p_i(t)$, the **Langevin equations** read:

$$\dot{q}_i(t) = \sum_k (\mathcal{M}^{-1})_{ik}\, p_k \qquad \leftrightarrow (P = MV)$$

$$\dot{p}_i(t) = -\frac{\partial H}{\partial q_i} - \sum_k \gamma_{ik}\, \dot{q}_k + \sum_k g_{ik}\, \xi_k(t) \qquad \leftrightarrow \left(\frac{dP}{dt} = \sum F\right)$$

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Mass tensor

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Conservative forces $(\boldsymbol{H} = \boldsymbol{T} + \boldsymbol{V})$

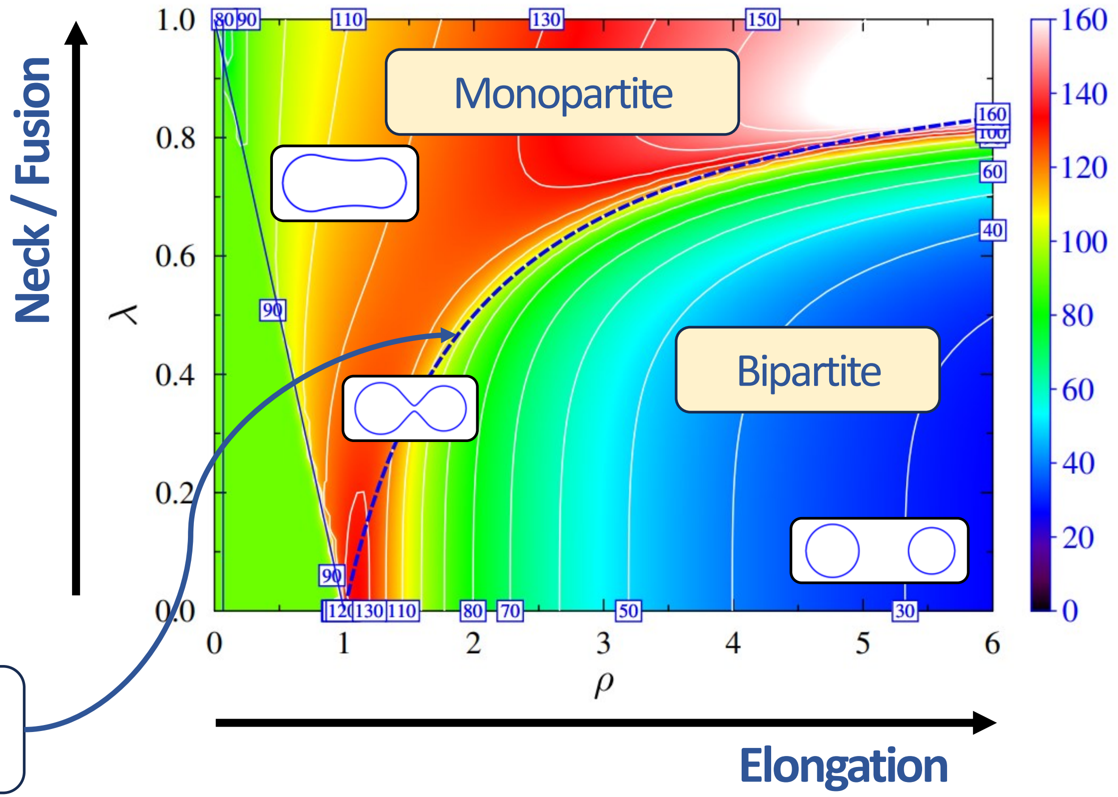
Friction forces

Langevin/random forces

$\longrightarrow$ A **comprehensive understanding of the dynamics process** (in comparison to the random walk f. eg.).

Potential Energy

- **Yukawa-plus-exponential folding potential + Coulomb**
- Parameters taken from a previous fit to experimental masses and fusion barrier heights [1]
- No shell effects at the moment.



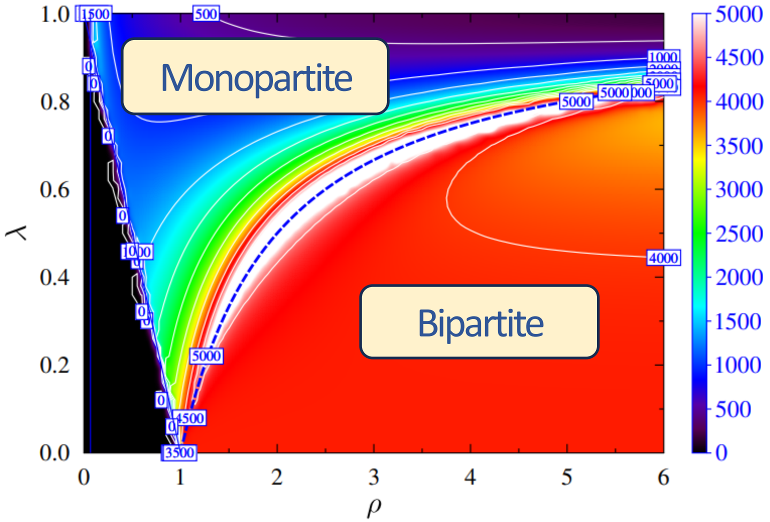
Scission line $\lambda = 1 - \frac{1}{\rho}$

Deformation potential of $^{92}Zr + ^{64}Ni$ in MeV

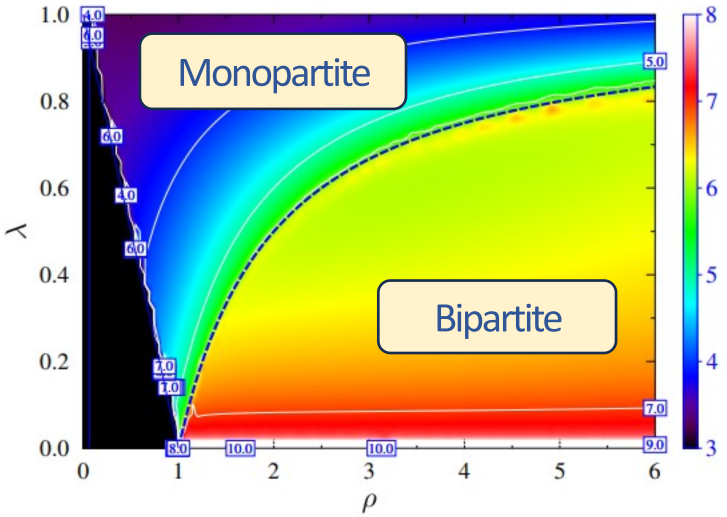
[1] H. J. Krappe *et al.*, Phys. Rev. C 20 (1979) 992–1013

Mass tensor / Kinetic Energy

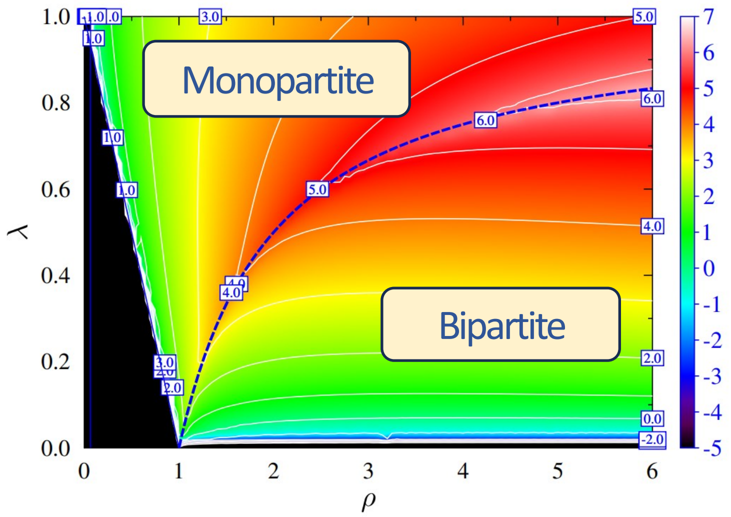
- **Werner-Wheeler flow approximation**:
  - **Incompressibility** (matter density is uniformly distributed)
  - The flow is **irrotational** (the moving planes remain plane)



$\mathcal{M}_{\rho\rho}$



$\log(\mathcal{M}_{\Delta\Delta})$

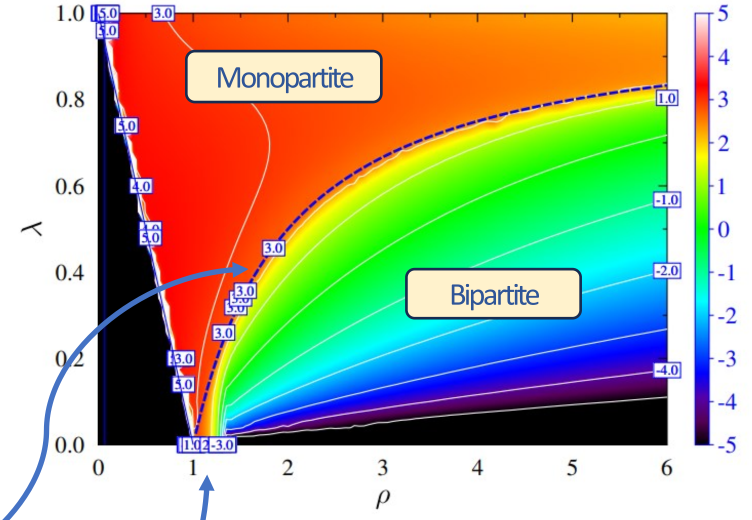


$\log(\mathcal{M}_{\lambda\lambda})$

Friction forces

- **Proximity formalism** (to account for some quantum effects):
  Possible matter flow/friction before contact (d = 3.2 fm)

- **Shape friction**:
  - **Wall friction** (collisions nucleons ↔ nuclear surface)
  - + **Wall-plus-window friction** (between the two fragments)



$\log(\gamma_{\rho\rho})$

Scission line $\lambda = 1 - \frac{1}{\rho}$

Friction **even when the system is separated** (proximity)

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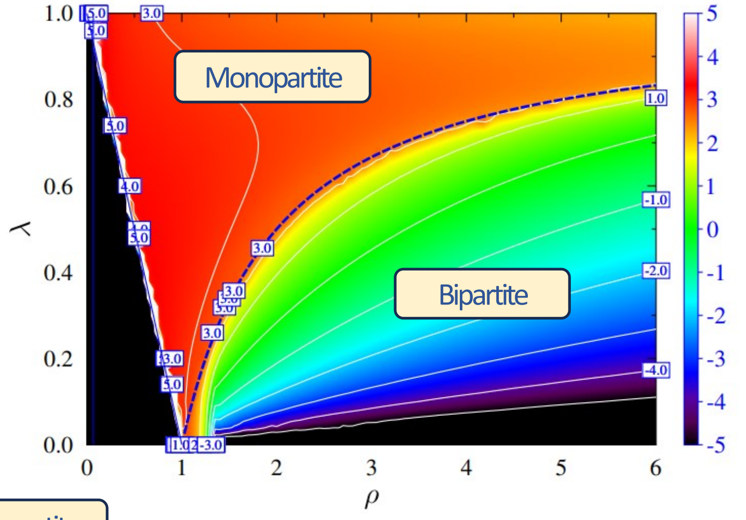
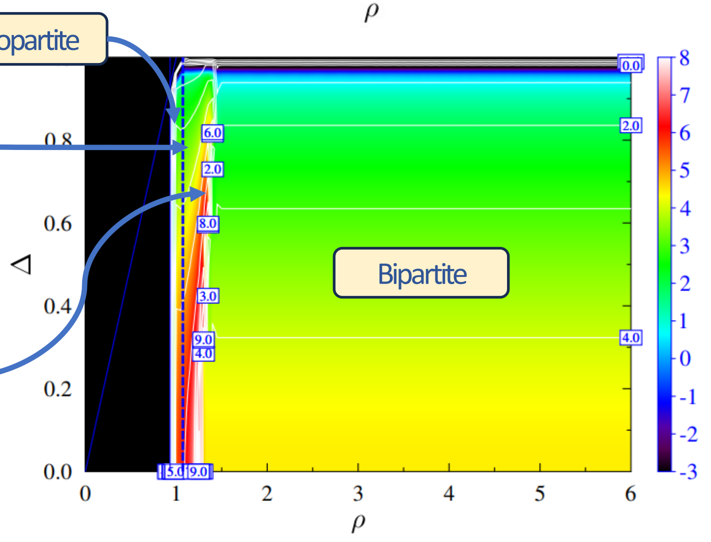


$\log(\gamma_{\rho\rho})$

Scission line $\lambda = 1 - \frac{1}{\rho}$



$\log(\gamma_{\Delta\Delta})$, $\lambda = 0.1$

Friction **even when the system is separated** (proximity)

Friction forces

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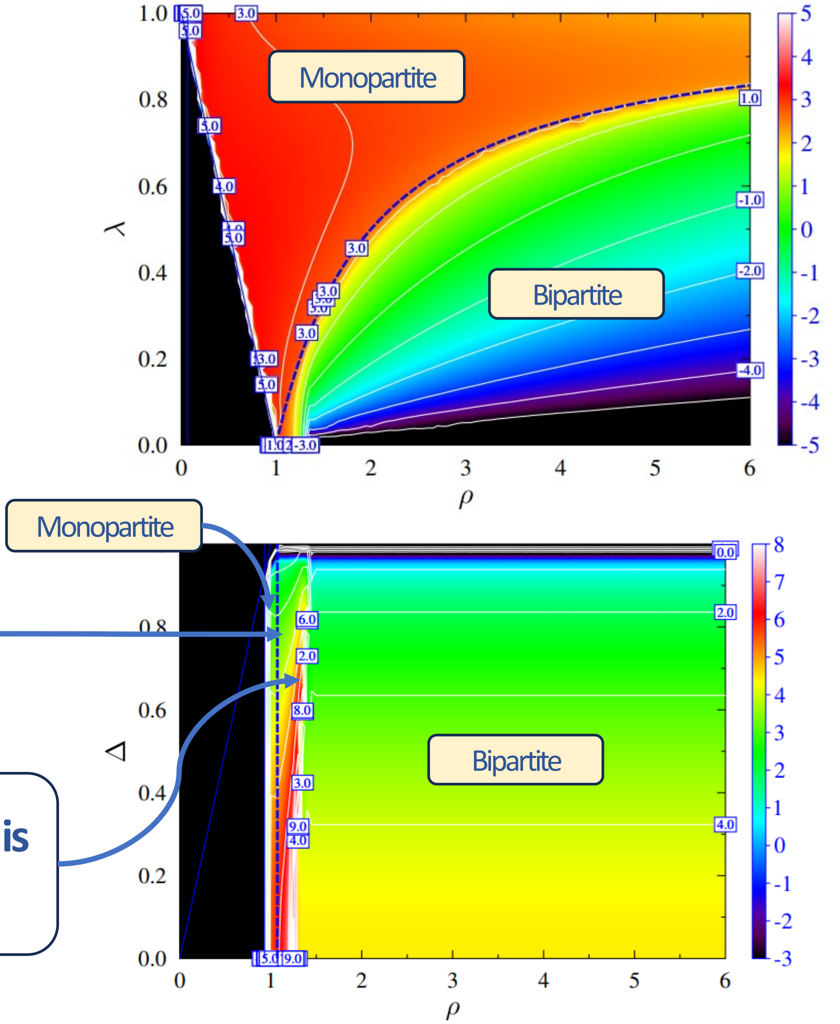
  Possible matter flow/friction before contact (d = 3.2 fm)

- **Shape friction**:
  - **Wall friction** (collisions nucleons ↔ nuclear surface)
  - + **Wall-plus-window friction** (between the two fragments)

- **Angular friction**:
  - **Sliding friction**
  - **No rolling friction**



$\log(\gamma_{\rho\rho})$

$\log(\gamma_{\Delta\Delta})$, $\lambda = 0.1$

Scission line $\lambda = 1 - \frac{1}{\rho}$

Friction **even when the system is separated** (proximity)

Langevin/random forces

- We assume a simple memoryless Langevin force (white noise):

$$F_i = \sum_k g_{ik} \xi_k(t)$$

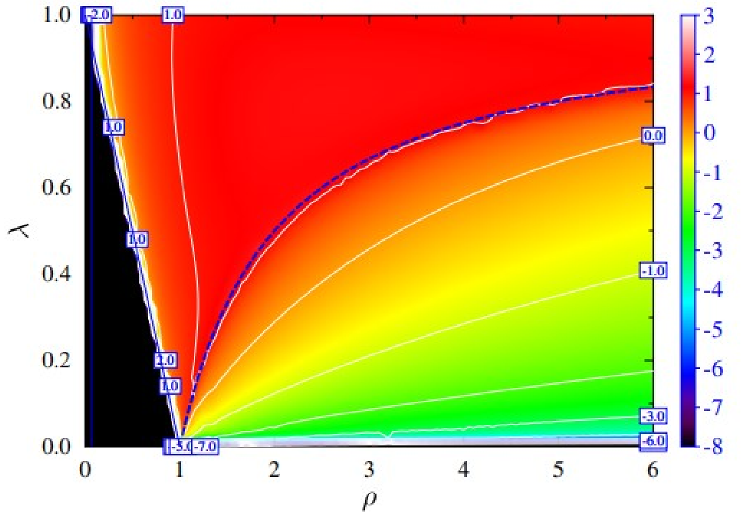
$\xi_k(t)$ are **time-dependent Gaussian random variables:**

$$\langle \xi_k(t) \rangle = 0$$
$$\langle \xi_k(t), \xi_{k'}(t') \rangle = 2\delta_{kk'} \delta(t - t')$$

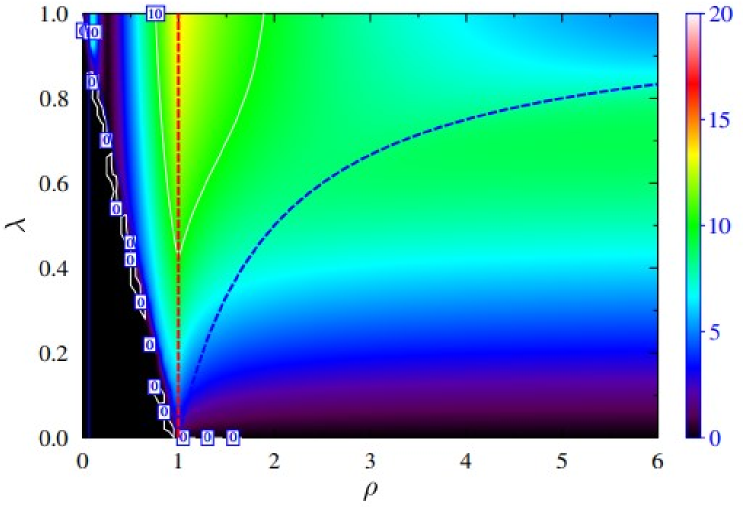
- The diffusion tensor is given by the **Einstein relation**:

$$\sum_k g_{ik} g_{kj} = D_{ij} = k_B T^* \gamma_{ij}, \qquad T^* = \frac{E_0}{\tanh(E_0/T)}, \qquad T = \sqrt{E^*/a}$$

$E_0 = 2$ MeV is the zero-point collective energy of the heat bath oscillators
E^* is the **dissipated energy**, $a = A/8$ MeV is the **level density parameter**.



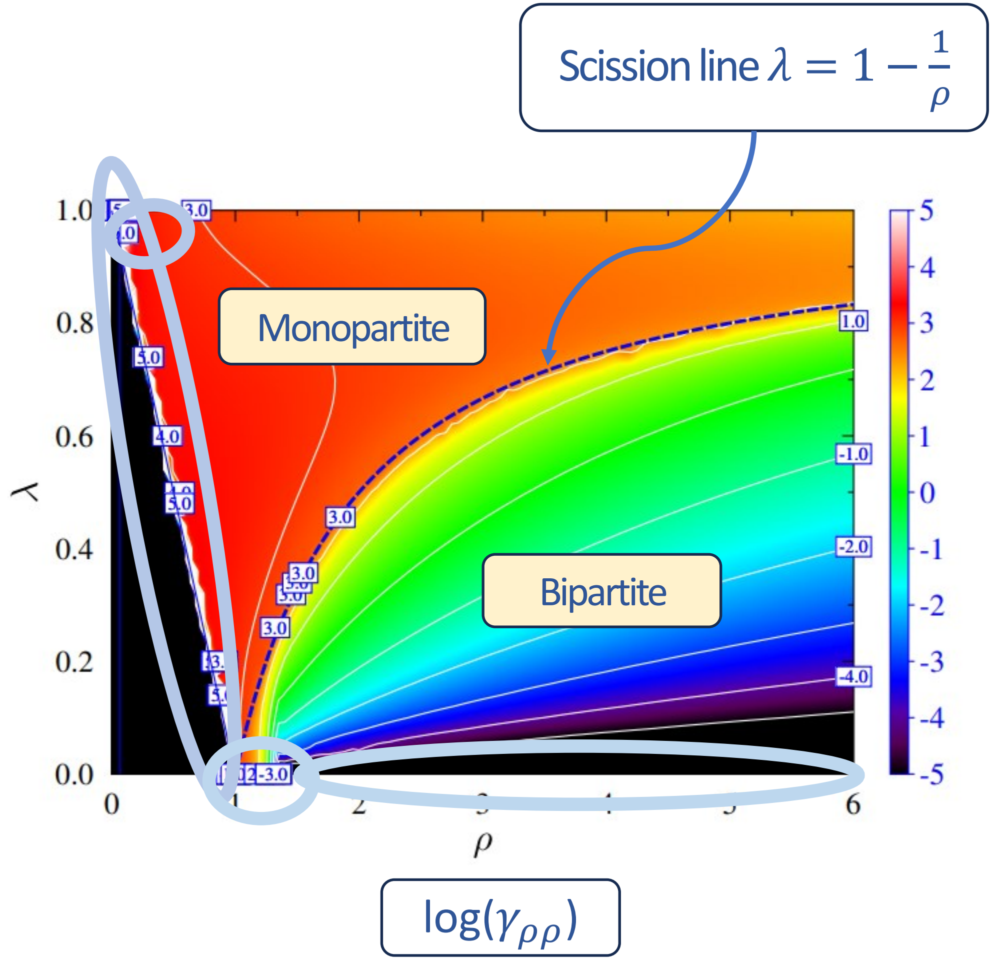
$\log(g_{\rho\lambda})$



$g_{\theta_2\theta_2}$

Physical vs. practical collective variables

- Unlike the multipole moments, the (ρ, λ, σ) variables are **extremely irregular**:
  - **Many borders,**
  - **Small proximity region,**
  - **Regime change.**
- **A correct treatment of numerical precision is needed.**
- **Extrapolation** is needed for the calculations of the physical quantities and their derivatives.
  The regions of extrapolation should **avoid the borders.**
- The process of fusion itself is not fully tractable numerically.



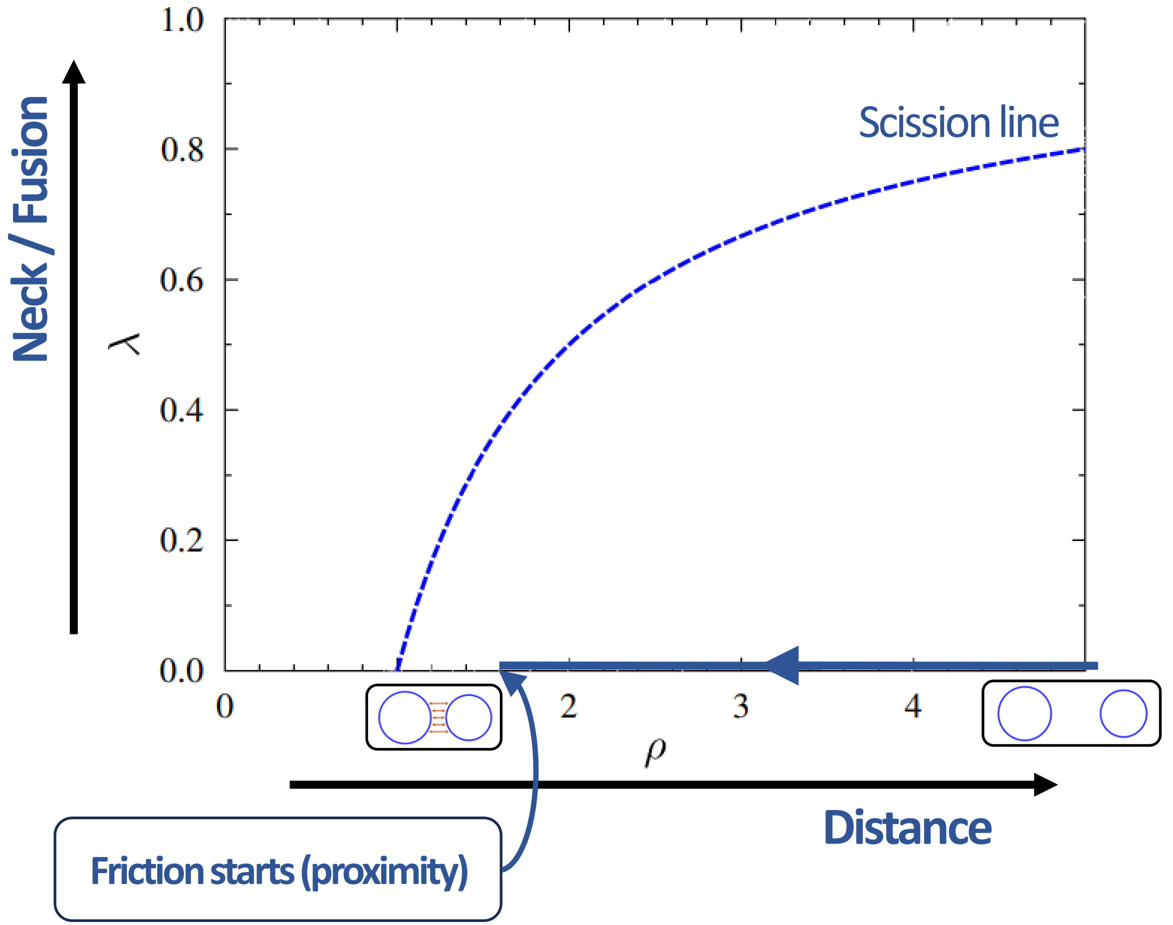
The fusion process

The three main stages of the collision:

1. **A first violent deceleration** during which:
   - The **system loses most of its kinetic energy**
   - There is **almost no deformation of the nuclei** as **no interaction has happened yet**
   $(\dot{\lambda} = (\mathcal{M}_{\rho\lambda})^{-1} p_\rho + (\mathcal{M}_{\lambda\lambda})^{-1} p_\lambda$
   $= -\infty + \infty)$

   - **Unstable balance close to the lambda border**

   - **Treated exactly** (conservative forces).

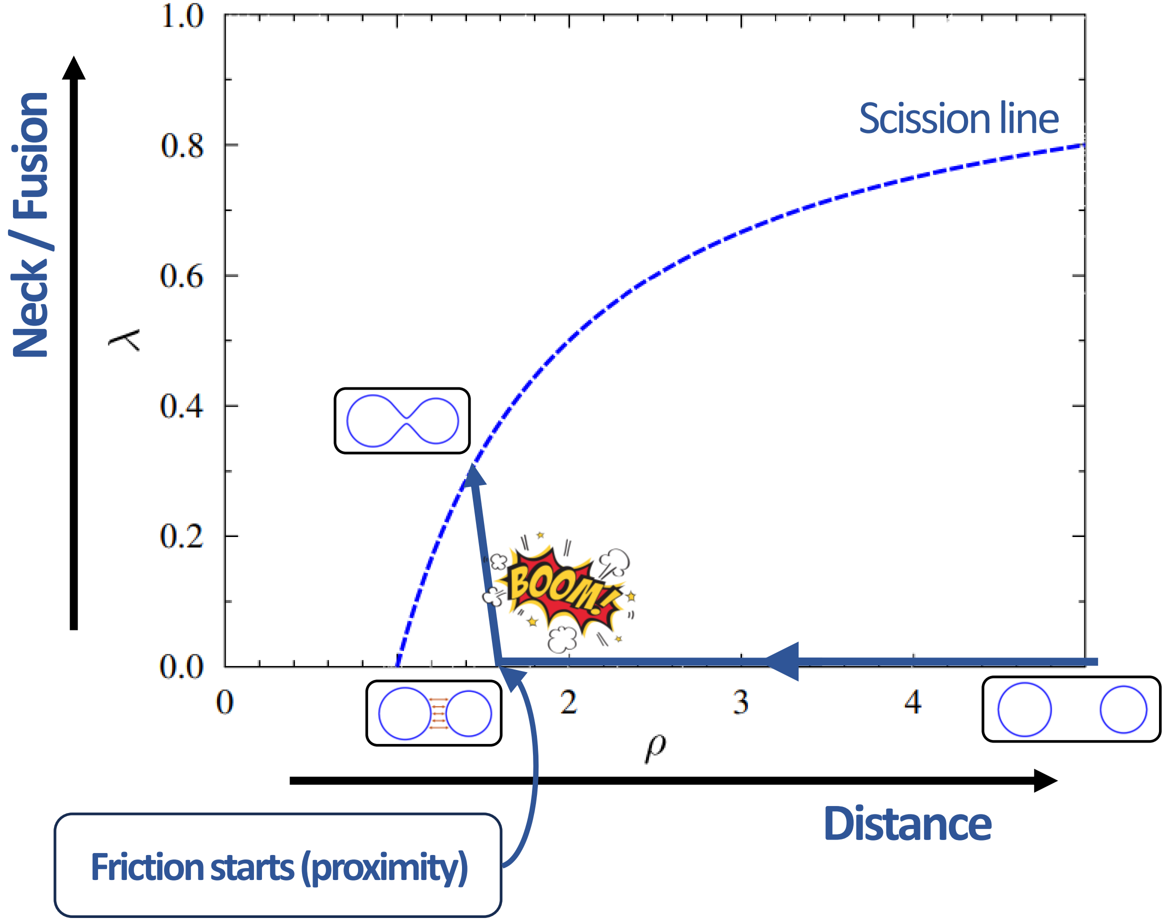


The fusion process

2. The **"Kiss of death"** when friction starts

- **A little sudden change in** p_ρ → **infinite push to the scission line** $(\dot{\lambda} = (\mathcal{M}_{\rho\lambda})^{-1} p_\rho + (\mathcal{M}_{\lambda\lambda})^{-1} p_\lambda = -\infty + \infty)$

- **Deformation starts and remains.**



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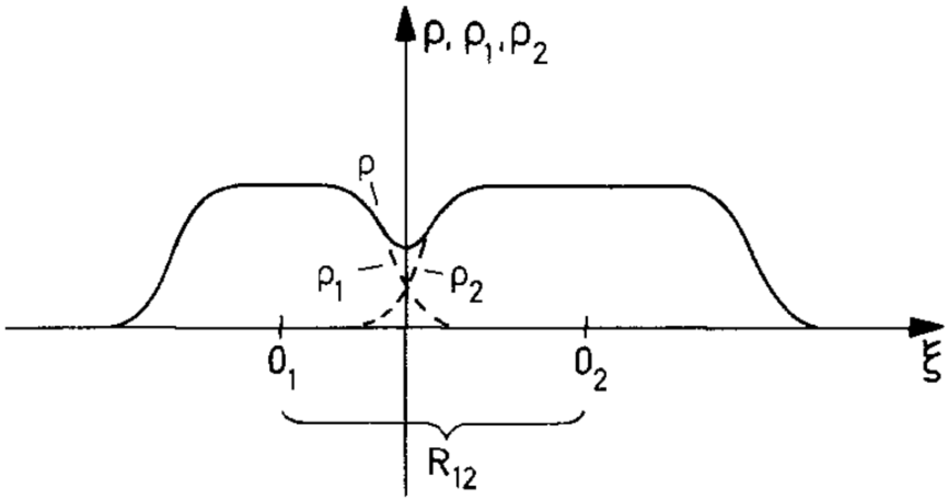
  $= -\infty + \infty)$

- **Deformation starts and remains.**

- **Inherent instability of non-saturated nucleonic densities**
  (**Sudden approximation** in HF when the nuclear tails touch)

- **This step is NOT treated numerically**:
  We start the calculation from the touching point

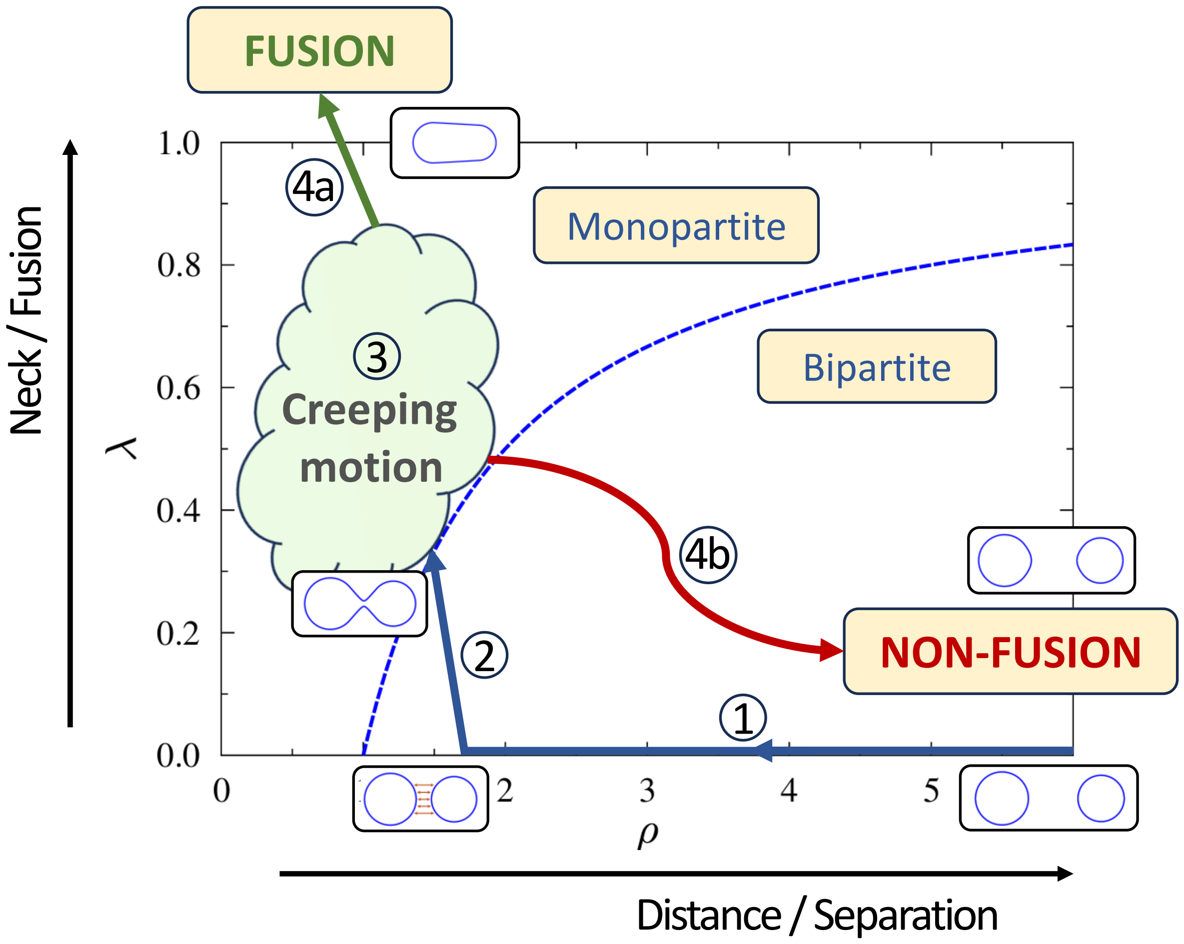


"Sudden approximation" – K. Pomorski, K. Dietrich
Z. Phys. A, 295 (1980) 335

The fusion process

The three main stages of the collision:

3. A **long creeping motion**
   - which leads to fusion or separation
   - **Solved numerically**



The observables

- The resolution of the Langevin equations generates **a distribution of trajectories due to the fluctuation force.**
- We use **500,000 - 1,000,000 trajectories.**
- **Asymmetry is free to change**.
- Calculations performed on the **CiŚ cluster** (Świerk/Warsaw).

- The **spin distribution** is calculated as a Monte-Carlo integral on a given bin $i \equiv \ell_i$:

$$\sigma_\ell = \left(\frac{d\sigma_{fus}}{d\ell}\right)_{\ell_i} = \frac{2\pi}{k^2}\ell_i^2 \frac{N_i^{fus}}{N_i^{tot}}$$

where $\ell_{init} = \ell_{max}\sqrt{x}, x$ a random number in [0,1] (for easy derived formulas).

- From the spin distribution, one can calculate:
  - The **total cross section / probability for the formation of the compound nucleus**
  - $\langle \ell \rangle , \langle \ell^2 \rangle$
  - **Excitation functions.**

$^{64}Ni + ^{92,96}Zr \longrightarrow ^{156,160}Er$ – Effect of the Asymmetry

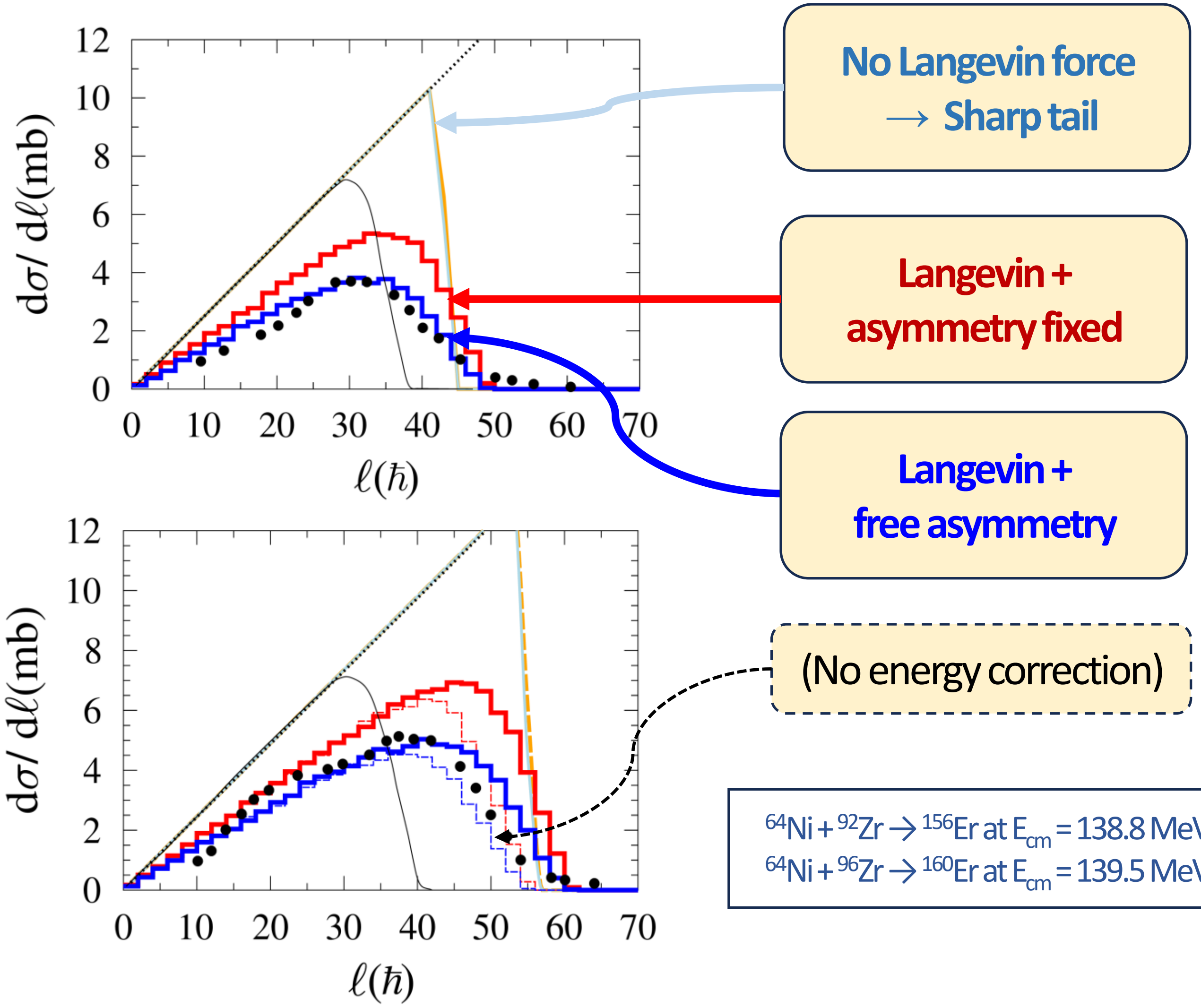
- ^{64}Ni + ^{92}Zr: $Q_{fus,calc} = Q_{fus,exp} \longrightarrow$ no correction needed
- ^{64}Ni + ^{96}Zr: Difference of 3.5 MeV $\longrightarrow$ **correction needed**

- **Spin distributions are more natural with the Langevin force** (vs. Ref .[1])

- **Great agreement to the experimental data[2,3] .**

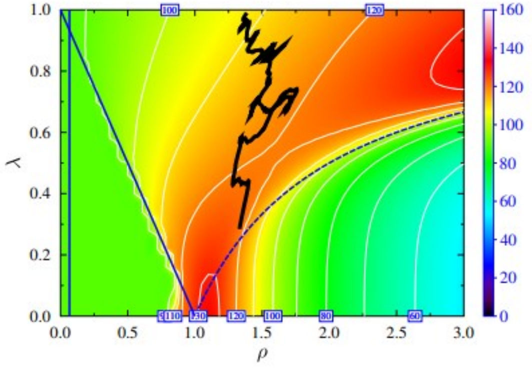
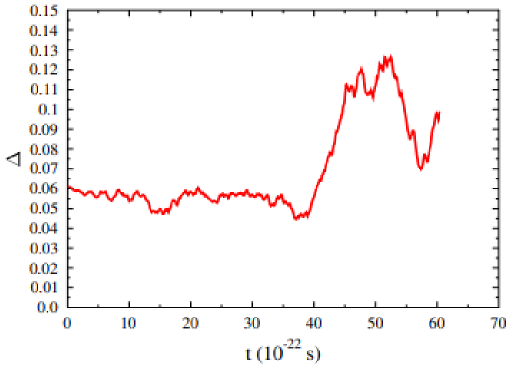
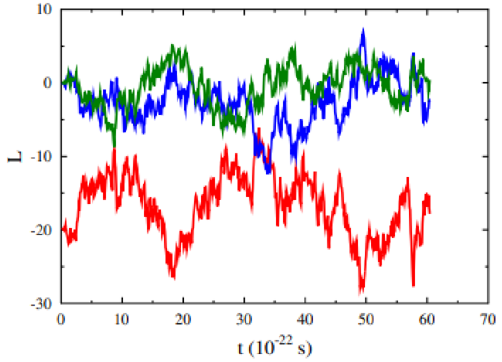
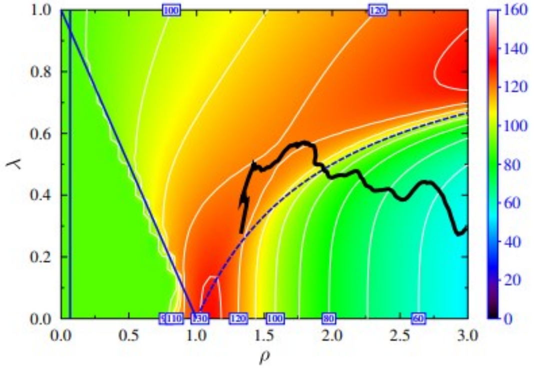
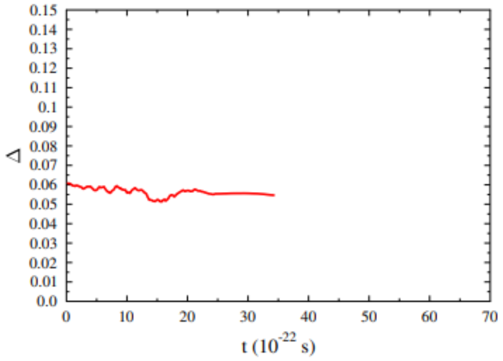
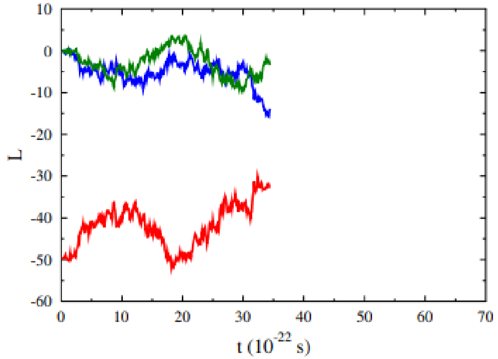
- In the ^{64}Ni + ^{96}Zr case, little discreprancies in the spin distributions (deformation?)
- **The total cross section (160 mb) is well reproduced.**

[1] W. Przystupa, K. Pomorski, Nucl. Phys. A 572(1) (1994) 153
[2] W. Kuhn *et al.*, Phy. Rev. Lett. 62 (1989) 1103
[3] A. M. Stefanini *et al.*, Phys. Lett. B 252 (1990) 43



$^{64}Ni + ^{92}Zr \rightarrow ^{156}Er$ at $E_{cm} = 138.8$ MeV
$^{64}Ni + ^{96}Zr \rightarrow ^{160}Er$ at $E_{cm} = 139.5$ MeV

A deep understanding of the fusion process (Preliminary examples)

	Paths	Asymmetry vs. time	Angular momenta
▸ $\ell = 20\hbar$, Fusion case:			
▸ $\ell = 50\hbar$, No-fusion case:			

- And various other observables: **diffusion tensor**, the **number of rotations the system udergoes before fusion** etc.

Summary and Perspectives

- We have derived a fully **6-dimensional dissipative dynamics Langevin**-based formalism to describe the **unrestricted motion of the systems** in terms of **elongation**, **neck** and **asymmetry** variables.

- Thanks to a **correct treatment of the different stages of fusion**, the spin distributions are now **in great agreement with experimental data**.
- The Langevin formalism allows for a deep understanding of the fusion process: evolution of the asymmetry, angular momentum/rotations of the fragments etc.

- In the very near future:
  - We will study the effect of the **asymmetry of entrance channel** on the formation of the compound system.
  - We will tackle the **hindrance problem** by comparing the **$^{48}Ca/^{50}Ti/^{54}Cr + ^{208}Pb$** systems.

- We are planning to make the following improvements of the formalism:
  - The addition of **shell effects** for a **fully microscopic-macroscopic picture**
  - The testing of different forms of stochastic noises (**color noises**), which will allow us to explore **memory effects** as the process evolves.

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Thank you for your attention!