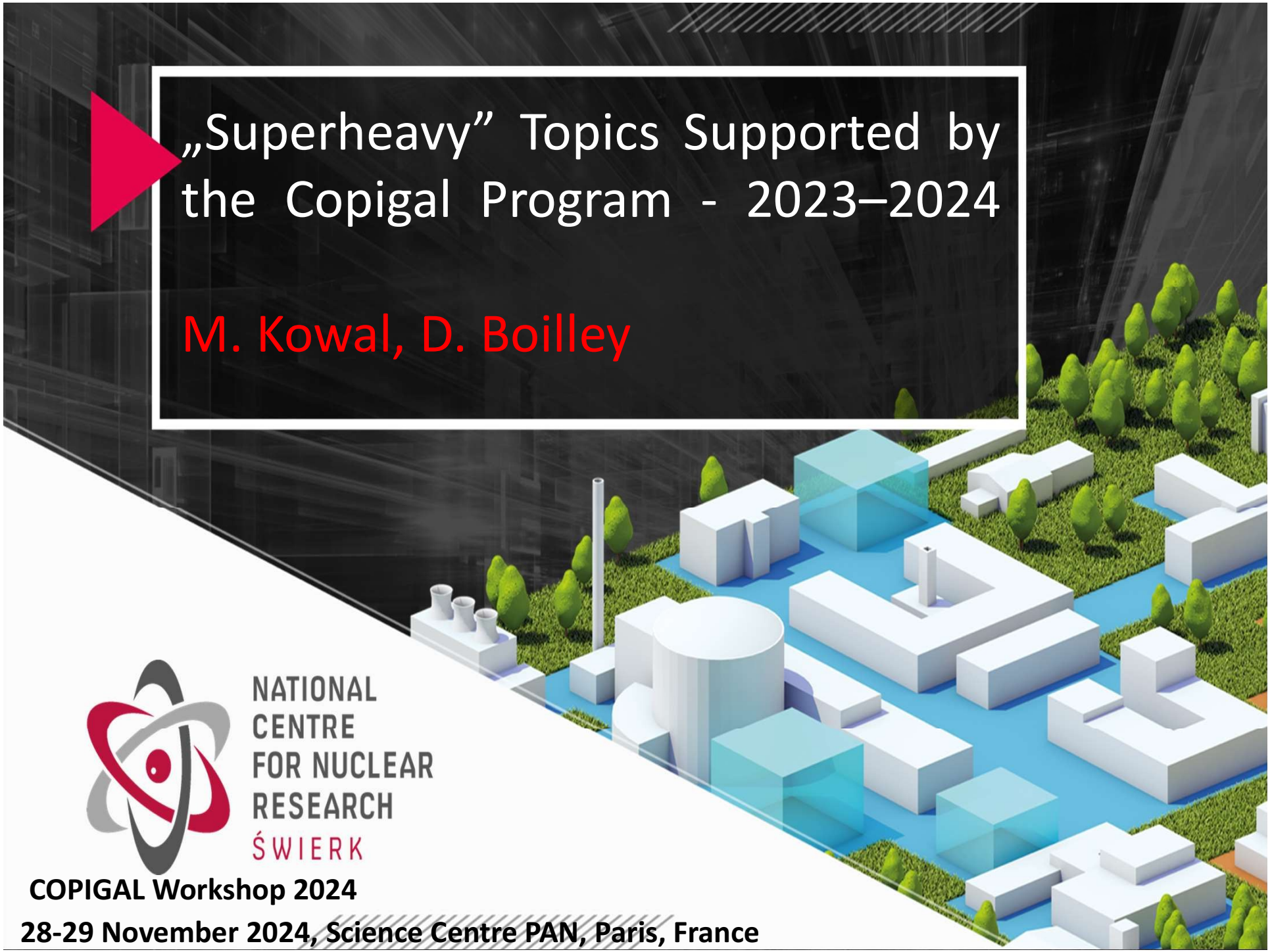
„Superheavy” Topics Supported by the Copigal Program - 2023–2024

M. Kowal, D. Boilley



NATIONAL CENTRE FOR NUCLEAR RESEARCH ŚWIERK

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Entropies, level-density parameters and fission probabilities along the triaxially- and axially-symmetric fission paths in 296Lv
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Multi-step particle emission probabilities in superheavy nuclei at moderate excitation energies
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Journal: Physics Letters B, 2023, Volume 844

Candidates for three-quasiparticle K isomers in odd-even Md–Rg nuclei
Authors: P. Jachimowicz, M. Kowal, J. Skalski
Journal: Physical Review C, 2023, Volume 108, 064309

Cassinian oval parametrization within the macroscopic–microscopic approach
Authors: P. Jachimowicz, R. Capote, M. Kowal
Journal: European Physical Journal A, 2024, Volume 60, Article 160

Dipole-driven multidimensional fusion: An insightful approach to the formation of superheavy nuclei
Authors: T. Cap, A. Augustyn, K. Siwek-Wilczyńska., M. Kowal
Journal: Physical Review C, 2024, Volume 109, L061603

Predict:

- **The probability of production of superheavy nuclei**
- **The optimal bombardment energy**
- **The best combination of projectile and target for successful element creation**

$$\sigma(\text{synthesis}) = \pi\lambda^2 \sum_{l=0}^{l\max} (2l+1) T_l \times P_l(\text{fusion}) \times P_{xn}^{\ell}(\text{survive})$$

Basic assumptions:

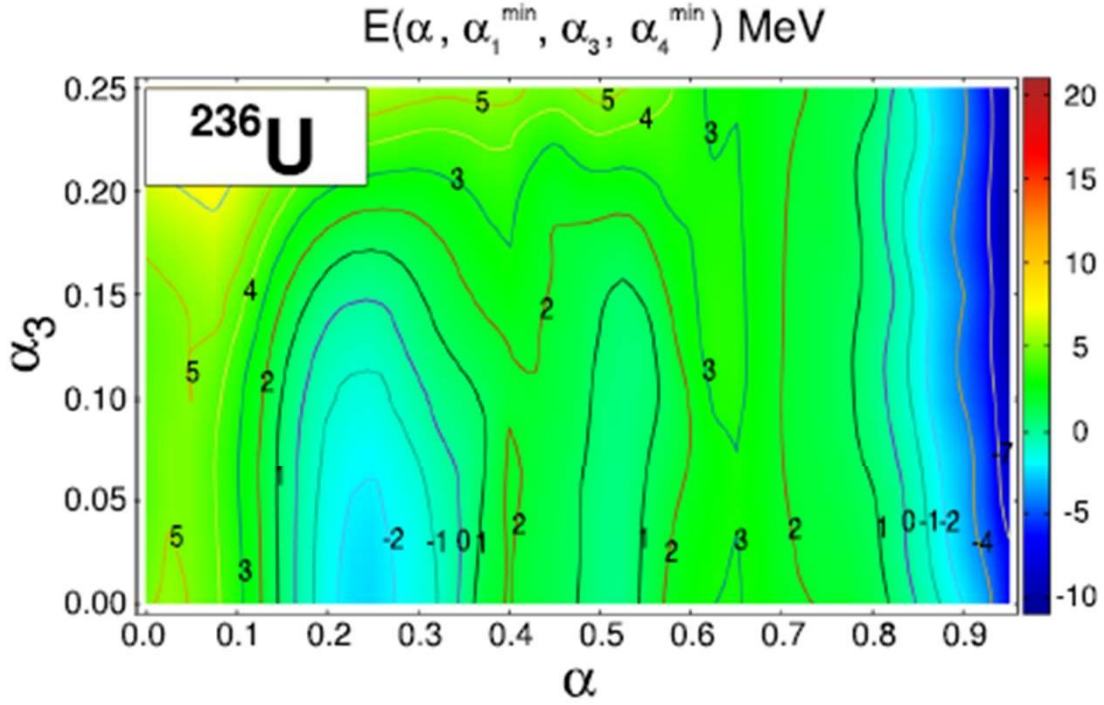
- After the capture stage, the internal mass asymmetrical conditional saddle point - the fusion barrier - must be overcome by the system.
- The stochastic nature of the fusion process is accompanied by dissipation of energy and angular momentum.
- All nucleons are transformed from projectile to the target by a diffusion process.

$$\sigma(\text{synthesis}) = \pi\lambda\!\!\!^{-2} \sum_{l=0}^{l\max} (2l+1) T_l \times P_l(\text{fusion}) \times \boxed{P_{xn}^{\ell}(\text{survive})}$$

$$\frac{\Gamma_n}{\Gamma_f} = \frac{gA^{2/3}}{K_0} \frac{\int_0^{U-B_n} \varepsilon \rho_{\mathrm{GS}}(U - B_n - \varepsilon) d\varepsilon}{\int_0^{U-B_f} \rho_{\mathrm{SP}}(U - B_f - \varepsilon) d\varepsilon},$$

U>Bn>Bf>T

$$\frac{\Gamma_n}{\Gamma_f} = \frac{4mR_0^2}{\hbar^2} \exp\left(-\frac{B_n - B_f}{T}\right).$$



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Regular Article-Theoretical Physics

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THE EUROPEAN PHYSICAL JOURNAL A

Regular Article - Theoretical Physics

Entropies, level-density parameters and fission probabilities along the triaxially- and axially-symmetric fission paths in ^{296}Lv

A. Rahmatinejad[1], T. M. Shneidman[1], G. G. Adamian[1], N. V. Antonenko[1], P. Jachimowicz[2], M. Kowal[3,a]

METHOD

PES: take into account all the collective variables that we know play an important role in the process

NLD: determine the density of states at each point of a multidimensional network in a fundamental way so that the available phase space determines the transition

ME; take into account the stochastical nature (Master Equation) of the process and give the system as much freedom as possible to choose the path

RATES

- **Initial Setup:**
  - Thermodynamic quantities (entropy, intrinsic level density, shell corrections) are calculated as functions of excitation energy for each deformation β.
  - Kinetic energy for motion in deformation space is neglected.
- **Free Energy and Excitation Energy:**
  - The total energy is expressed as $U(\beta, T) + F(\beta) = E_0$.
  - $F(\beta)$ is the free energy, consisting of macroscopic (E_{mac}) and microscopic (δF_{mic}) terms.
  - $U(\beta, T)$ represents the excitation energy.
- **Damping Effects:**
  - With damping of microscopic corrections, the total energy becomes $U^*(\beta, T) + F(\beta, U^*) = E_0$.
  - $F(\beta, U^*)$ includes damped microscopic corrections.
- **Local Temperature:**
  - Equations define local temperatures at deformation sets account for changes in excitation energy and the damping of microscopic corrections in a self-consistent manner.

Superfluid Formalism and Nuclear Excitations

Based on the superfluid formalism and an assumption of thermal equilibrium between neutron and proton subsystems, the excitation energies ($U = U_Z + U_N$), entropies ($S = S_Z + S_N$), and intrinsic level densities ρ are calculated at each temperature T as:

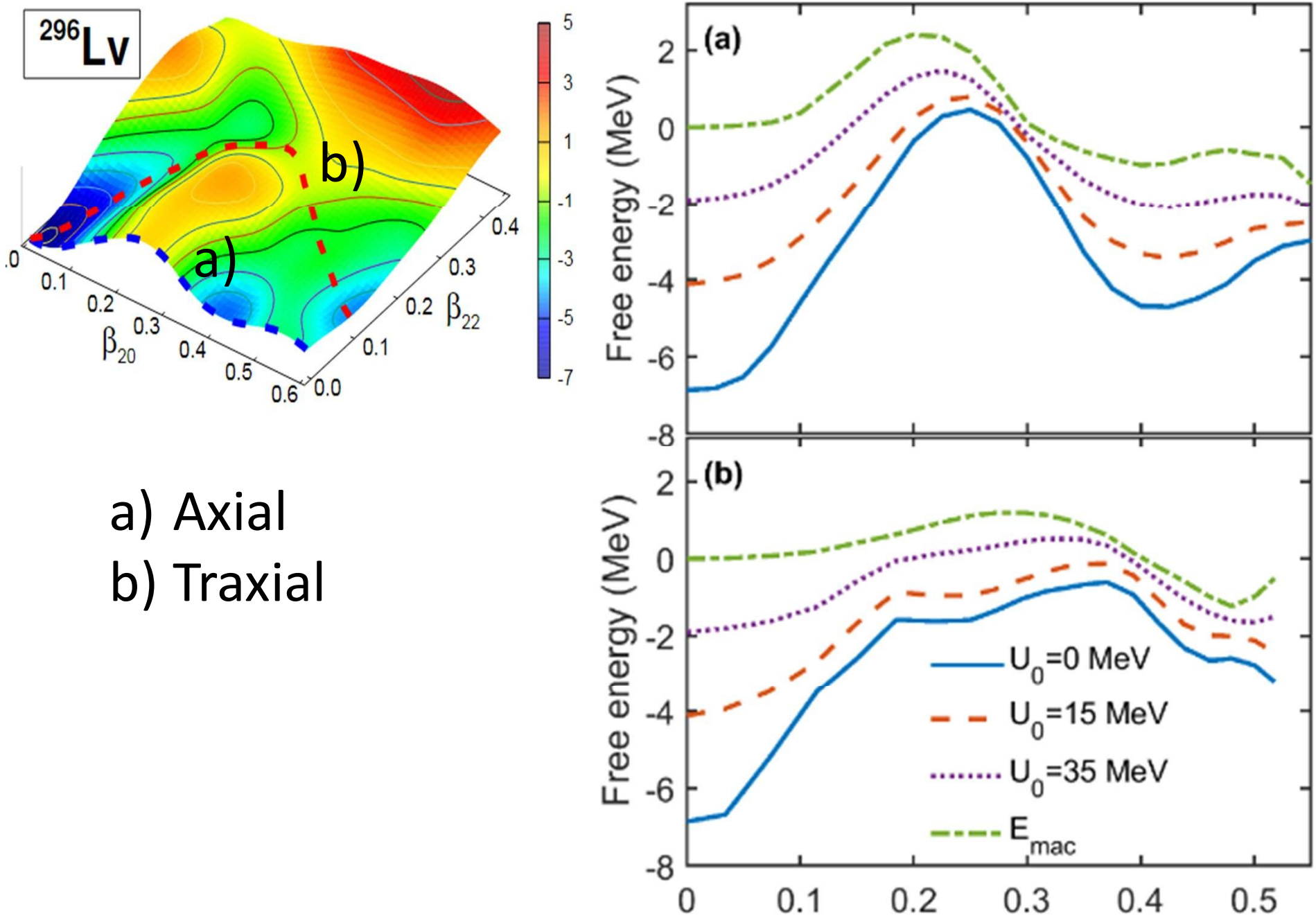
$$E_{N(Z)}(T) = 2\sum_k \varepsilon_k n_{\Delta,k}^{N(Z)} - \frac{\Delta_{N(Z)}^2}{G_{N(Z)}},$$

$$U_{N(Z)}(T) = E_{N(Z)}(T) - E_{N(Z)}(0),$$

$$S_{N(Z)}(T) = \sum_k \left\{ \ln\left[1 + \exp\left(-\frac{E_k^{N(Z)}}{T}\right)\right] + \frac{E_k^{N(Z)}}{T\left[1 + \exp\left(\frac{E_k^{N(Z)}}{T}\right)\right]} \right\},$$

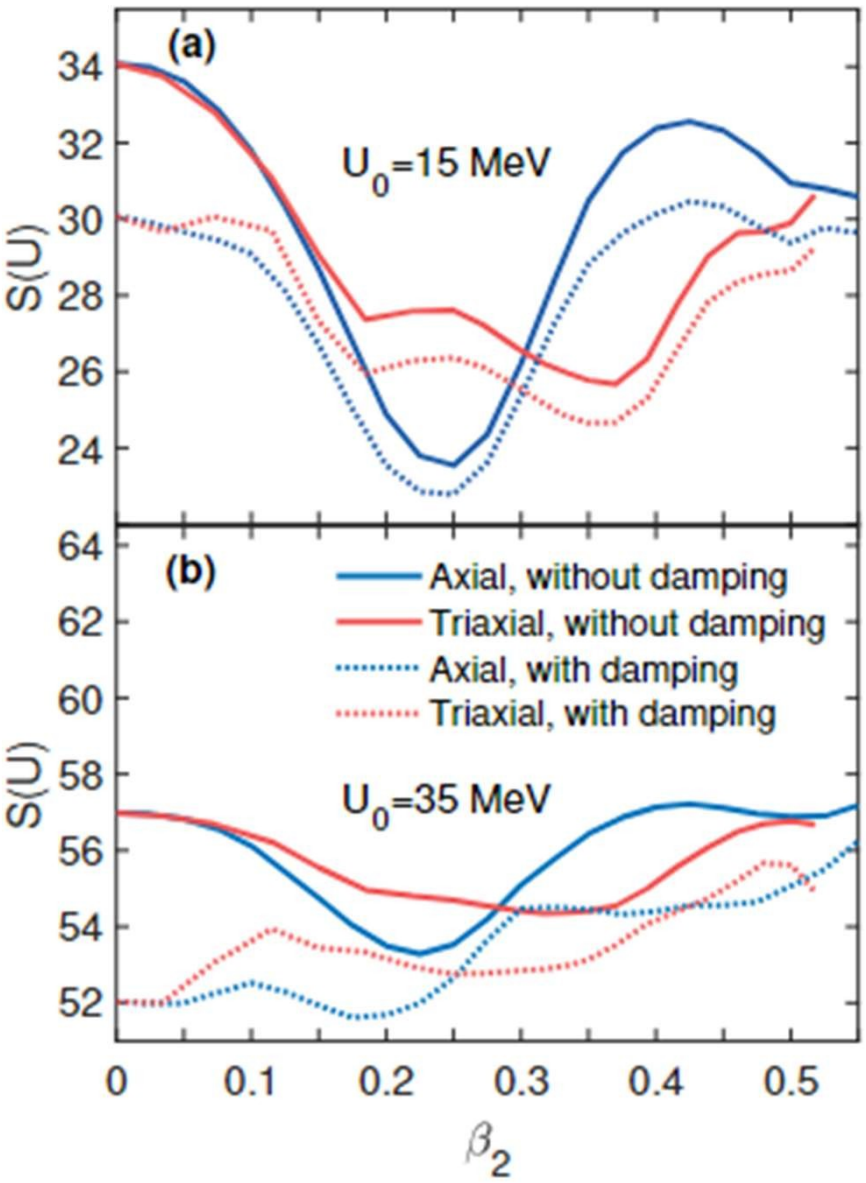
$$\rho = \frac{\exp(S)}{(2\pi)^{\frac{3}{2}}\sqrt{D}},$$

where the determinant D is the second derivative of entropy with respect to $1/T$ and $\mu = \lambda/T$.



a) Axial
b) Traxial

Entropy



1. Damping Effect - Damped microscopic corrections lower entropy along both axial and triaxial paths, especially at higher energies.

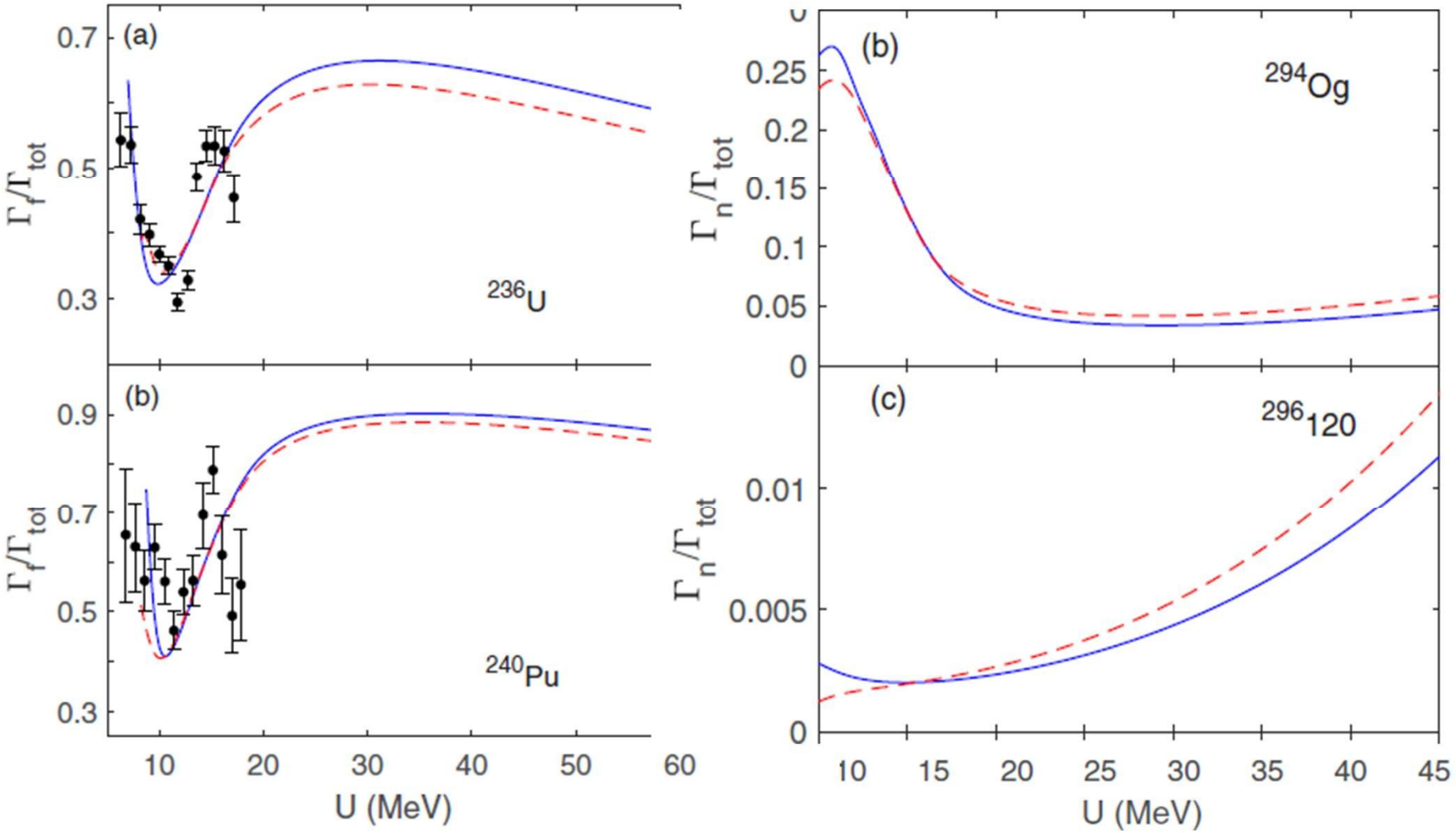
2. Low Energy - At low energies, the GS has higher entropy due to excitation energy, but entropy decreases with increasing deformation towards the SP.

3. High Energy - At higher energies, the entropy at the GS becomes lower than at the SP, with damping further reducing GS entropy due to reduced excitation energy.

4. Axial vs. Triaxial - The entropy difference between axial and triaxial paths decreases with energy

$$\frac{\Gamma_n}{\Gamma_f} = \frac{4A^{2/3}a_f(U - B_n - \Delta_n)}{K_0 a_n\left[2a_f^{1/2}(U - B_f - \Delta_f)^{1/2} - 1\right]} \times \exp\left[2a_n^{1/2}(U - B_n - \Delta_n)^{1/2} - 2a_f^{1/2}(U - B_f - \Delta_f)^{1/2}\right],$$

$$\frac{\Gamma_n}{\Gamma_f} = \frac{gA^{2/3}}{K_0} \frac{\int_0^{U-B_n} \varepsilon \rho_{GS}(U - B_n - \varepsilon)d\varepsilon}{\int_0^{U-B_f} \rho_{SP}(U - B_f - \varepsilon)d\varepsilon},$$



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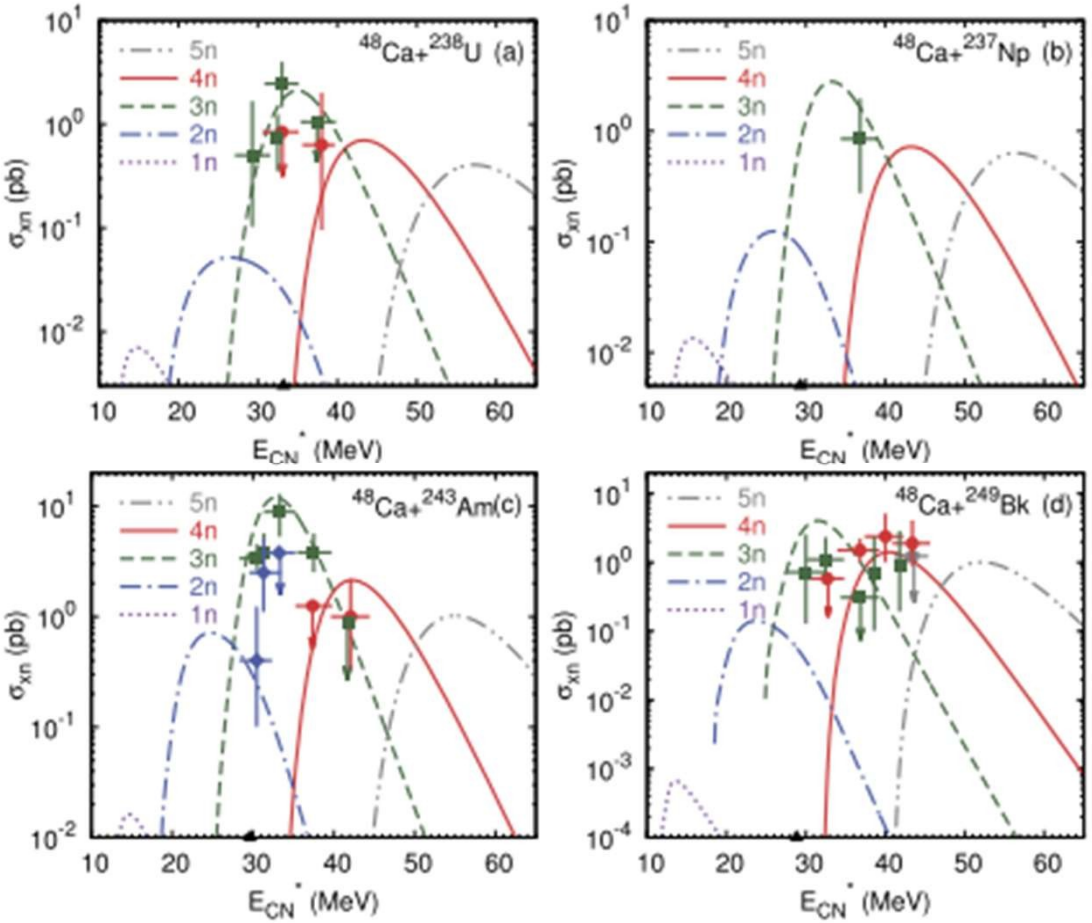

PHYSICAL REVIEW C **109**, L061603 (2024)

Letter

Dipole-driven multidimensional fusion: An insightful approach to the formation of superheavy nuclei

T. Cap,[1,*] A. Augustyn,[1] M. Kowal,[1] and K. Siwek-Wilczyńska[2]

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We present an approach to describe the fusion of two heavy ions in which the colliding system has access to a wide spectrum of shapes through the utilization of an auxiliary reference frame and the employment of a multipole expansion of the nuclear radius with the dipole term treated as an actual and leading shape variable. Access to fusion shapes that would otherwise be unattainable is possible by initially placing the origin of the auxiliary reference frame in the neck region between the colliding nuclei. The fusion process is modeled as an unconstrained biased random walk in a four-dimensional deformation space with step probabilities correlated to the density of available states. Deformation energy is calculated using the macroscopic-microscopic method, incorporating rotational energy. The presented approach successfully describes fusion probabilities for reactions involving ^{48}Ca, ^{50}Ti, and ^{54}Cr projectiles with a ^{208}Pb target.

DOI: 10.1103/PhysRevC.109.L061603

Features of the new model

- Using multidimensional deformation space, including the dipole
- Adopting an auxiliary reference frame giving access to otherwise unattainable shapes, specifically the starting configuration
- Adding the shell effect and rotational energy energy to the whole deformation space
- Replacing the Smoluchowski diffusion equation with a biased, unconstrained random walk

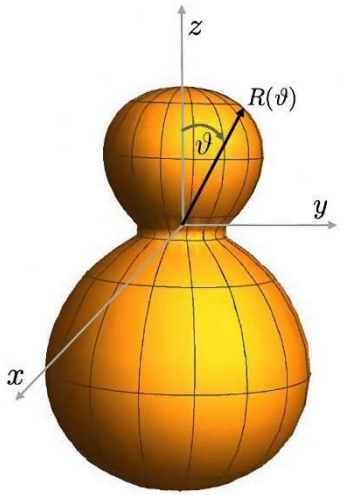
Goals of the new model

- Comparison with fragment mass distributions (fission, fusion-fission, quasi-fission) and TKE (total kinetic energy) distributions from experiments
- Study of the competition between fusion-fission and quasi-fission
- Study of the shape evolution during fusion and fission
- Modeling the effect of angular momentum on fusion, fission and quasi-fission
- Prediction of fusion probabilities for new SHE synthesis reactions

T. Cap, A. Augustyn, M. Kowal, K. Siwek-Wilczyńska, "Dipole-Driven Multidimensional Fusion: An Insightful Approach to the Formation of Superheavy Nuclei", Phys. Rev. C 2024

Starting point parametrization

- After overcoming the entrance channel barrier, the projectile and the target are assumed to be spherical and in a touching configuration
- The spherical harmonic parametrization is fitted, with the origin situated in the neck, giving the β parameters for the starting point configuration
- For now calculations are limited to 4 dimensions (β_{10} - β_{40})



$\beta_{10} = +0.917$
$\beta_{20} = +1.757$
$\beta_{30} = +0.001$
$\beta_{40} = -0.292$
$\beta_{50} = -0.001$
$\beta_{60} = +0.078$
$\beta_{70} = +0.001$
$\beta_{80} = -0.018$

T. Cap, A. Augustyn, M. Kowal, K. Siwek-Wilczyńska, "Dipole-Driven Multidimensional Fusion: An Insightful Approach to the Formation of Superheavy Nuclei", Phys. Rev. C 2024

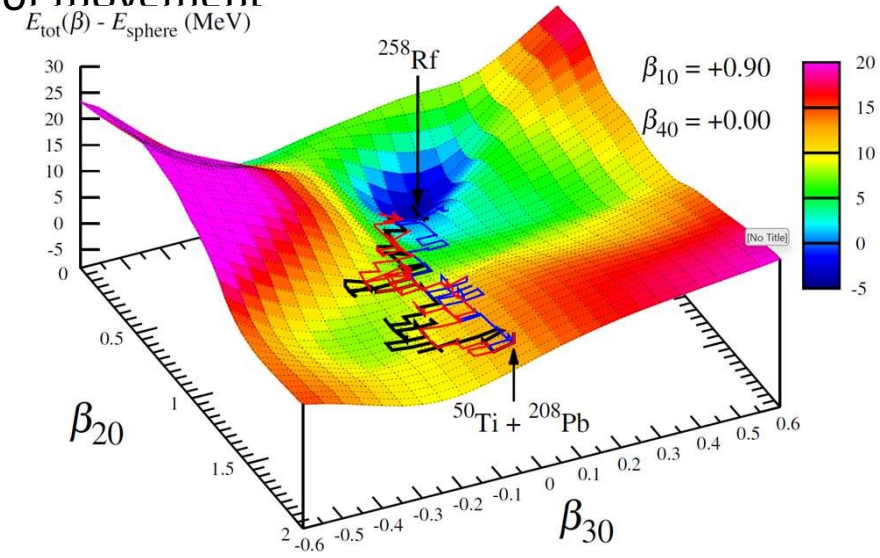
Biased, unconstrained random walk method

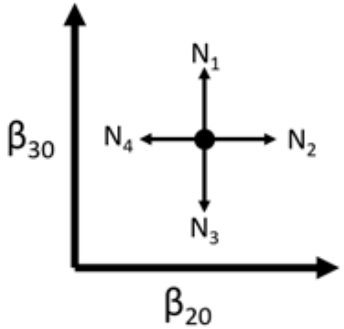
- The probability of transitioning from one shape to another is determined by the number of available energy levels for a given shape β -> biased

$$N_i(\beta_i, \ell) \propto \exp\left(2\sqrt{a\left(E^*_{\max}(\beta_i) - E_{\mathrm{rot}}(\beta_i, \ell)\right)}\right)$$

a – constant density parameter

- Only one β parameter changes at a time, by a step of 0.05, giving 8 possible directions of movement

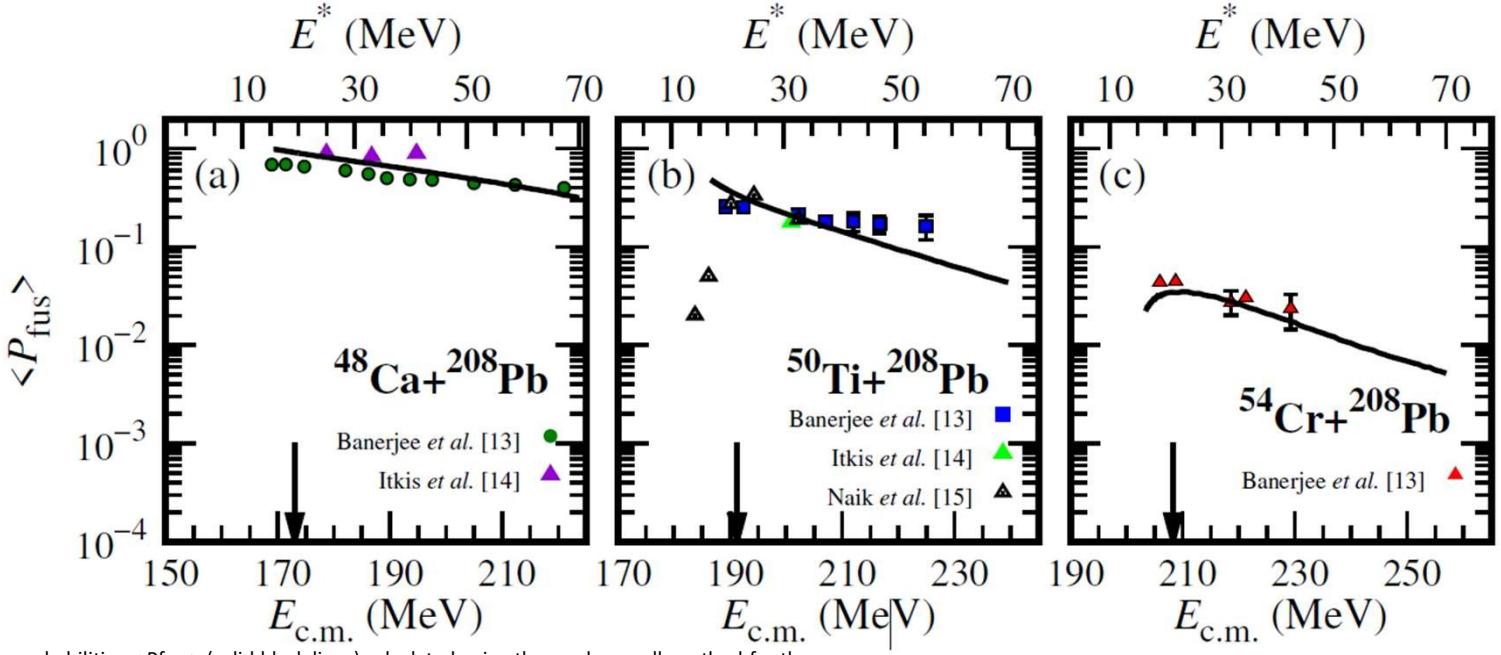




$$P_{i\to j}(\ell) = \frac{N_j(\beta_j, \ell)}{\sum_{k=1}^{8} N_k(\beta_k, \ell)}$$

Fusion probability from the random walk

Average over *l* to get P_{fus} dependant on E_{cm}: $P_{fus}(E_{cm}) = \frac{\sum_{l=0}^{lmax}(2l+1)P_{fus}(l)}{(2l_{max}+1)^2}$ fusion probability averaged over l



The averaged fusion probabilities < Pfus > (solid black lines) calculated using the random walk method for the ^{48}Ca+^{208}Pb, ^{50}Ti+^{208}Pb, and ^{54}Cr+^{208}Pb reactions. Experimental data are taken from [K. Banerjee et al., PRL 122, 232503 (2019)], [M. Itkis et al., EPJ 58, 178 (2022] and [R. S. Naik et al., PRC 76, 054604 (2007)]
The arrows represent the locations of the mean entrance channel barrier B_0 for each reaction.