Fission dynamics within state-of-the-art Langevin approach

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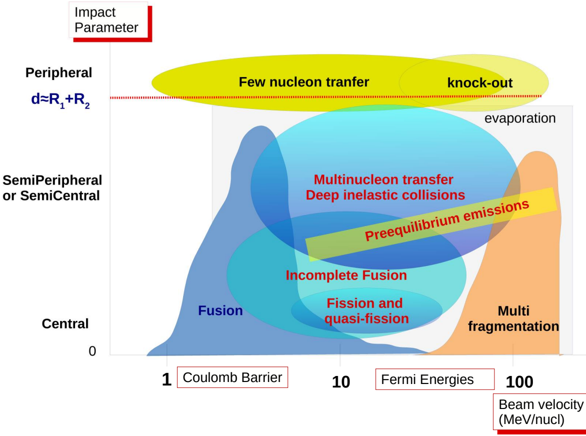
November 29, 2024

Outline

- Langevin equations and fission dynamics
- Review of the results
- Current study
- Plans



Thanks to A. Maj, G. Casini

Stochastic approach

Dynamical effect

- path from equilibrium to scission slowed-down by the nuclear viscosity
- description of the time evolution of the collective variables like the evolution of Brownian particle that interacts stochastically with a "heat bath".
- Monte Carlo method for choosing the shape, initial angular momentum, type and energy of emitted particles....

Coupling to the evaporation

Pre and post- scission emission of neutrons, protons, α and γ.

Ingredients

Inertia ($[M^{-1}(\vec{q})]_{ij}$); Friction ($\gamma_i(t)$) and fluctuation (g_{ik})

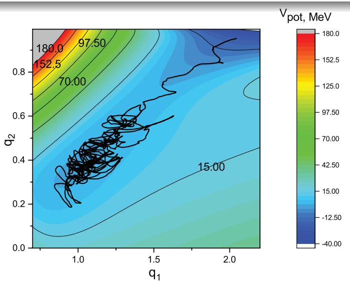
Macroscopic potential

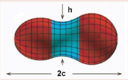
$(V(\vec{q}, K) \to F(\vec{q}, K) = V(\vec{q}, K) - \mathbf{a(\tilde{q})} T^2)$

Langevin equations

$$\frac{dq_i}{dt} = \sum_j [M^{-1}(\vec{q})]_{ij} p_j$$

$$\frac{dp_i}{dt} = -\frac{1}{2}\sum_{j,k} \frac{d[M^{-1}(\vec{q})]_{jk}}{dq_i} p_j p_k - \frac{dF(\vec{q}, K)}{dq_i} - \sum_{j,k} \gamma_{ij}(\vec{q})[M^{-1}(\vec{q})]_{jk} p_k + \sum_j g_{ij}(\vec{q})\Gamma_j(t)$$





- c - the elongation of the nucleus
- h - constriction coordinate
- α - mass-asymmetry parameter related to the ratio of the masses of nascent fragments

Model Ingredients

Energies

- Potential energy in deformation space e.g: FRLDM + Wigner or LSD + Congruence

$$\mathbf{E_{lsd}(q)} = \mathbf{b_{vol}\{1-\kappa_{vol} I^2\} A + b_{surf}\{1-\kappa_{surf} I^2\} A^{2/3} B_{surf}(q)}$$
$$+ \mathbf{b_{curv}\{1-\kappa_{curv} I^2\} A^{1/3} B_{curv}(q) + \frac{3}{5} e^2 \frac{Z^2}{r_0^{ch} A^{1/3}} B_{Coul}(q)}$$
$$\mathbf{I} = \mathbf{(N-Z)/A}$$

- Rotational energy:

$E_{rot}(q, I, K) = \frac{\hbar^2 I(I+1)}{2J_\perp(q)} + \frac{\hbar^2 K^2}{2J_{eff}(q)}$ where $J_{eff}^{-1} = J_{||}^{-1} - J_{\perp}^{-1}$ - the rigid-body moments of inertia.

- The friction parameter controlling coupling between K coordinate and the heat bath.

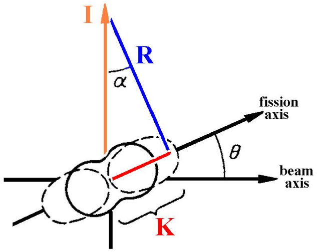
T. Dosing, J. Randrup, Nucl. Phys. A 433,215; J. Randrup, Nucl.Phys. A 383, 468

$$\gamma_K = \frac{1}{R_N R_{cm}\sqrt{2\pi^3 n_0}} \frac{J_{||} | J_{eff} | J_R}{J_\perp^3}$$

where R_N - neck radius, R_{cm}- distance between the center of the nascent fragments, n_0=0.0263 MeV zs fm^{-4} - the bulk flux in standard nuclear mass and $J_R = M_0 R_{cm}^2/4$ for reflection symmetric shape.

Collective coordinates(4D)

- Description of the nuclear shape by elongation, neck and asymmetry – 3 parameters.
- **K** – spin about the fission (symmetry) axis



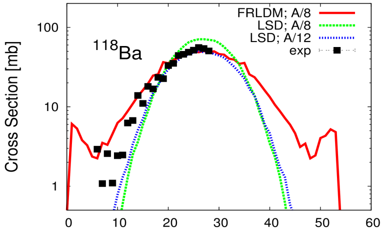
"Fission-fragment distributions within dynamical approach" K. M., P. N. Nadtochy, E. G. Ryabov, G. D. Adeev, Eur. Phys. J. A, 53 (2017) 79E

Energy Models study: **LSD or FRLDM, Congruence**

PHYSICAL REVIEW C **84**, 014610 (2011)

Critical insight into the influence of the potential energy surface on fission dynamics

K. Mazurek,[1,2,*] C. Schmitt,[2] J. P. Wieleczko,[2] P. N. Nadtochy,[3] and G. Ademard[2]



EPJ Web of Conferences **17**, 16006 (2011)
DOI: 10.1051/epjconf/20111716006

Influence of the potential energy landscape on the fission dynamics

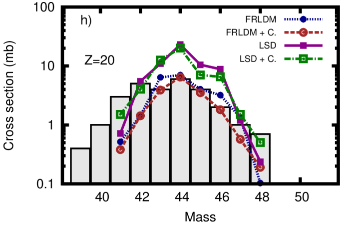
K. Mazurek[1,2,a], J.P. Wieleczko[1], C. Schmitt[1], and P. N. Nadtochy[3]

The change of the fission fragment mass distribution due to different PES taken to solve Langevin equations

PHYSICAL REVIEW C **88**, 054614 (2013)

Examining fine potential energy effects in high-energy fission dynamics

K. Mazurek,[1,*] C. Schmitt,[2] P. N. Nadtochy,[3] M. Kmiecik,[1] A. Maj,[1] P. Wasiak,[1] and J. P. Wieleczko[2]



Vol. 44 (2013) ACTA PHYSICA POLONICA B

FISSION FRAGMENT MASS DISTRIBUTION AS A PROBE OF THE SHAPE-DEPENDENT CONGRUENCE ENERGY TERM IN THE MACROSCOPIC MODELS* **

K. Mazurek[a], C. Schmitt[b], P.N. Nadtochy[c], A. Maj[a]
P. Wasiak[a], M. Kmiecik[a], B. Wasilewska[a]

Exp: Y. Futami, et al., Nucl. Phys. A607, 85 (1996)

Other works

EPJ Web of Conferences **62**, 02002 (2013)
DOI: 10.1051/epjconf/20136202002

Pre- and post- scission particle emission in 3D Langevin calculations with various macroscopic potentials

K. Mazurek[1,a], P.N. Nadtochy[2], C. Schmitt[3], P. Wasiak[1], M. Kmiecik[1], A. Maj[1], E. Bonnet[3], A. J. Frankland[3], D. Gruyer[3], and J.-P. Wieleczko[3]

PHYSICAL REVIEW C **100**, 064606 (2019)

New procedure to determine the mass-angle correlation of quasifission

C. Schmitt,[1,*] K. Mazurek,[2] and P. N. Nadtochy[3]

Physics Letters B 737 (2014) 289–292

Contents lists available at ScienceDirect

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Physics Letters B

www.elsevier.com/locate/physletb

On current ambiguity in the interpretation of fission at intermediate excitation energy

CrossMark

C. Schmitt[a,*], K. Mazurek[b], P.N. Nadtochy[c]

PHYSICAL REVIEW C **94**, 064602 (2016)

Going beyond statistical models for fission in the Businaro-Gallone region

K. Mazurek,[1] C. Schmitt,[2,*] P. N. Nadtochy,[3] and A. V. Cheredov[4]

RAPID COMMUNICATIONS

PHYSICAL REVIEW C **91**, 041603(R) (2015)

Description of isotopic fission-fragment distributions within the Langevin approach

K. Mazurek,[1] C. Schmitt,[2,*] and P. N. Nadtochy[3]

Nuclear level density

Importance

- Statistical models of decay
- Dynamical macroscopic models
- Astrophysics

Level Density Formulas

- Mass dependent LD $a(A) = \alpha A$
- Deformation depedent (Ignatuk) $a(A) = \alpha A + \beta A^{2/3}$
- (N-Z) dependent $a(A, Z) = \alpha A / exp[\beta(N - Z)^2]$
- $(Z - Z_0)$ dependent
  $a(A, Z) = \alpha A / exp[\gamma(Z - Z_0)^2]$
  $Z_0 = 0.5042A/(1 + 0.0073A^{2/3})$
- Energy dependent (Reisdorf)
  $a(U, A, Z) = a(A) * [1 + dWg(Z, N) * f(U_{cor})/(U_{cor})]$
  $f(x) = 1 - exp(-\gamma * x)$

NUCLEAR LEVEL DENSITIES[1] 5531

J. R. Huizenga
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AND
L. G. Moretto
Lawrence Berkeley Laboratory, University of California, Berkeley, California

CONTENTS

INTRODUCTION

Nuclear level density

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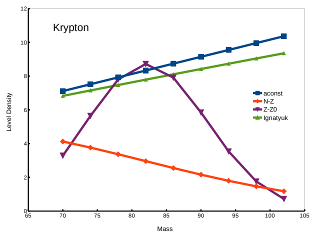
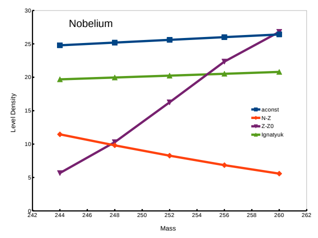
Light system



Heavy system



Nuclear level density

Previous works

- *Are the level densities for r- and rp-process nuclei different from nearby nuclei in the valley of stability?* S.I. Al-Quraishi et al. PRC 63,065803
- *Level densities for 20 $\leq A \leq$ 110* S.I. Al-Quraishi et al. PRC 67,015803
- *Continuum corrections to the level density and its dependence on excitation energy, n-p asymmetry, and deformation* R. J. Charity and L. G. Sobotka, PRC 71, 024310
- *Compound nucleus evaporative decay as a probe for the isospin dependence of the level density* R. Moro et al. EPJA 48,159

	Partial multiplicity		*Total* multiplicity		
	Protons	Alphas	Protons	Alphas	Neutrons
Exp	1.21 ± 0.18	0.40 ± 0.06	1.79 ± 0.36	0.76 ± 0.15	–
NoIso	1.47	0.46	2.17	0.88	2.31
$N-Z$	1.24	0.47	1.89	0.90	2.57
$Z-Z_0$	2.17	0.61	3.08	1.17	0.68

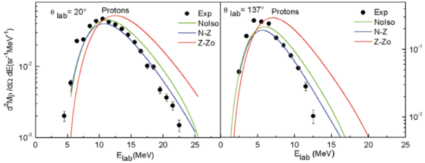


Fig. 4. Experimental and calculated proton spectra in coincidence with the PPACdown. The laboratory angles of the detectors with respect to the beam direction are indicated in the figure.

Previous work

- *Isospin dependence of nuclear level density at A = 120 mass region* R. Shil et al., PLB 831, 137145.

R. Shil, K. Banerjee, P. Roy et al.

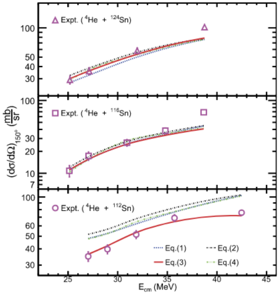
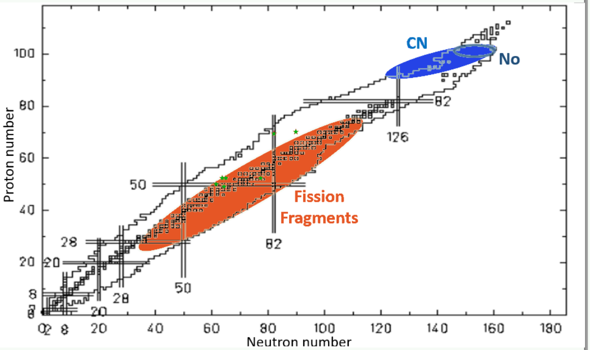


Fig. 4. Measured $d\sigma/d\Omega$ at 150° plotted as a function of centre-of-mass energy E_{cm} are shown in symbols. Lines represent calculated differential cross sections considering all neutron evaporation channels. Calculations are done using different formulae of NLD parameter described in Eqs. (1)-(4). Calculated cross sections are scaled down by the same factors as used in Fig. 3(a)-(c).

- G.K. Prajapi et al. PRC 102, 054605
- Pratap Roy et al. PRC 102, 061601(R)

Nuclear level density

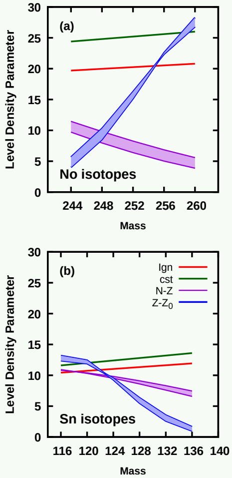
Fusion-Fision



While previous studies rely on the fusion- evaporation approach, and are restricted to the mass range used to fit the functional forms of the (N, Z) dependent $\tilde{a}$, fission permits to test the predictive capability of the proposed formulae outside the range of their adjustment.

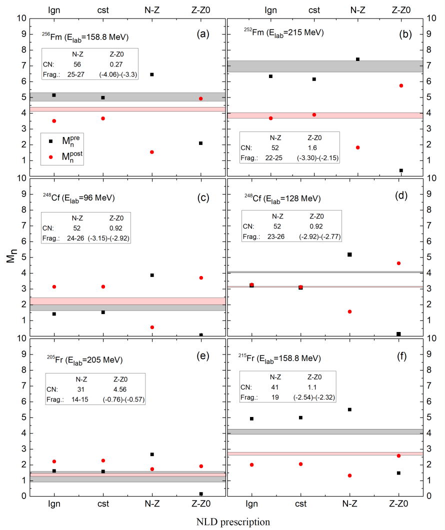
C. Schmitt, P.N. Nadtochy, K.Mazurek Phys. Lett. B, 840 (2023) 137873

Fusion-fission



Nuclear level density

Fusion-Fision - Validation of the method



Fusion-Fision - Verified reactions

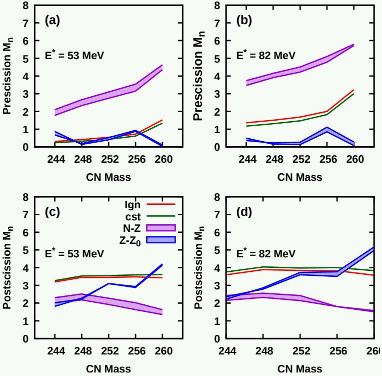
	CN	Sym. Fiss.
^{256}Fm, E^*=108 MeV	^{256}Fm	^{126}Sn
a_{N-Z}	56	26
a_{Z-Z_0}	0.27	-3.68
^{252}Fm, E^*=150 MeV	^{252}Fm	^{122}Sn
a_{N-Z}	52	22
a_{Z-Z_0}	1.6	-2.15
^{205}Fr, E^*=79 MeV	^{205}Fr	^{102}Tc
a_{N-Z}	31	16
a_{Z-Z_0}	4.56	-1.36
^{215}Fr, E^*=113 MeV	^{215}Fr	^{105}Tc
a_{N-Z}	41	19
a_{Z-Z_0}	1.1	-2.54
^{248}Cf, E^*=55 MeV	^{248}Cf	^{123}In
a_{N-Z}	52	25
a_{Z-Z_0}	0.92	-3.53
^{248}Cf, E^*=83 MeV	^{248}Cf	^{122}In
a_{N-Z}	52	24
a_{Z-Z_0}	0.92	-3.15

Nuclear level density - Testing both side of stability line

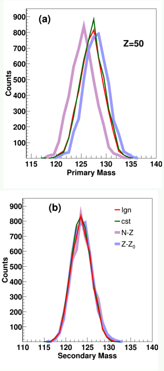
Fusion-fission ($^{40,48}Ca + ^{204,208}Pb$, $^{22}Ne + ^{238}U$)

Pre and post scission neutron multiplicity as a LD probe

Heavy/superheavy compound nucleus - neutron deficient (Mnpre)



Mass distribution



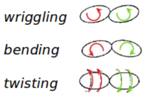
Fission fragments - neutron rich (Mnpost)

The $^{x}Ca + ^{y}Pb$ collisions which populate the No chain by fusion is very good candidates, since several projectile-target combinations can be readily accessible at current accelerator facilities. The further enhancement of this set of reactions with $^{22}Ne + ^{238}U$ involving a radioactive beam that is currently available. The PARIS+VAMOS detection system ca be used

a) Angular momentum of fission fragments (with J. Randrup)

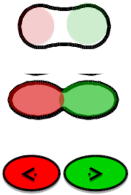
Useful framework for discussion of fission fragment angular momenta:

Normal modes:



Fission populates several distinct *rotational modes* whose presences reflect the physical *mechanisms* generating the angular momenta of the fragments

Generation of fission fragment angular momenta by nucleon exchange:



$t_{\mathrm{wrig}} < t_{\mathrm{fiss}} \approx t_{\mathrm{bend}} < t_{\mathrm{twst}} << t_{\mathrm{tilt}}$

Wriggling will reach equilibrium

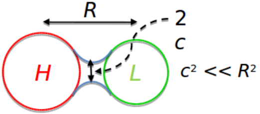
Bending probably has some presence; it increases with the mass asymmetry

Twisting is unlikely to play a major role; it grows more prominent with excitation

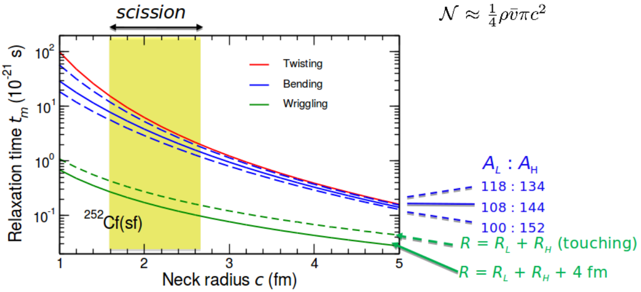
a) Angular momentum of fission fragments

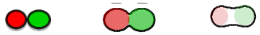
Relaxation times
of dinuclear rotational modes caused by *nucleon exchange*

$$t_m = I_m / M_m \qquad \begin{cases} M_{\rm wrig} = m\mathcal{N}R^2 \\ M_{\rm bend} = m\mathcal{N}\left[\left(\frac{\mathcal{I}_H R_L - \mathcal{I}_L R_H}{\mathcal{I}_L + \mathcal{I}_H}\right)^2 + c_{\rm ave}^2\right] \\ M_{\rm twst} = m\mathcal{N}c_{\rm ave}^2 \end{cases}$$



Transfer rate:
$\mathcal{N} \approx \frac{1}{4}\rho\bar{v}\pi c^2$

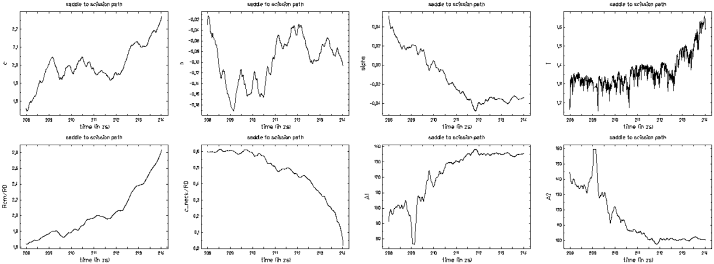


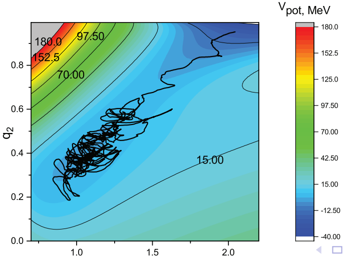


J. Randrup, T. Døssing, R. Vogt,
Phy.s Rev. C **106,** 014609 (2022)

a) Angular momentum of fission fragments

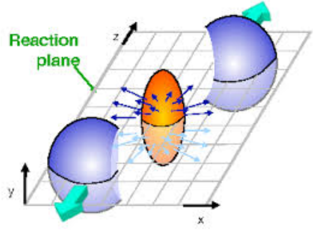
Preliminary results of Langevin trajectory from saddle to scission

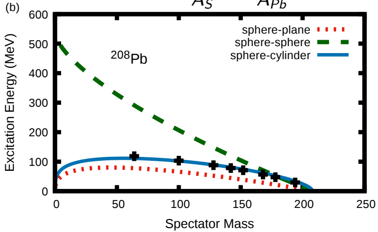




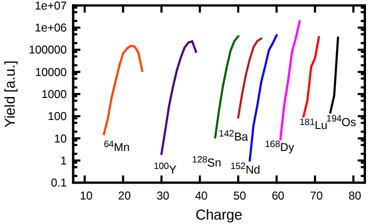
Pb + Pb Collision - Dynamic Evolution of Spectator

The excited Compound Nuclei (eight) have been evaluated in 4D Langevin code to estimate the evaporation and fission channels. We assume $\frac{Z_S}{A_S} = \frac{Z_{Pb}}{A_{Pb}}$

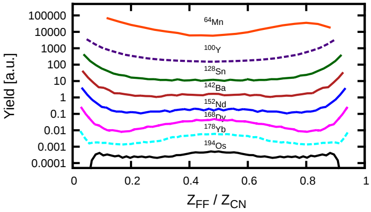




Evaporation Residue charge distribution



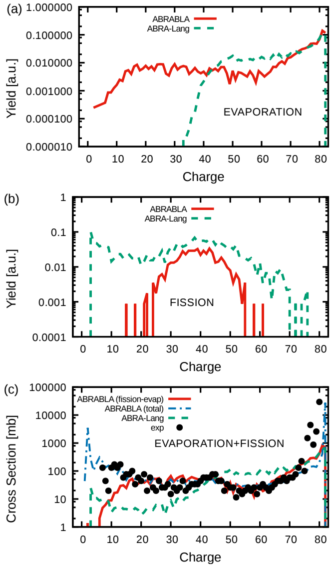
Fission Fragment charge distribution



K. M., A. Szczurek, C. Schmitt, P.N. Nadtochy, Phys. Rev. C, 97 (2018) 024604

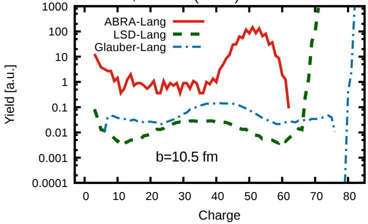
Pb + Pb Collision - Dynamic Evolution of Spectator

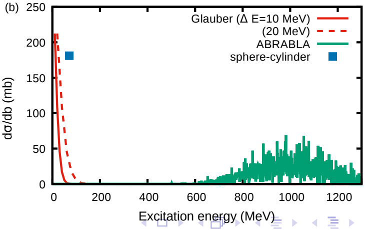
ABRA+Langevin



Impact parameter b=10.5 fm

following predictions of A. Rybicki and A. Szczurek, PR**C75** (2007) 054903;PR**C87** (2013) 054909.





b) Fission in ultrarelativistic collisions

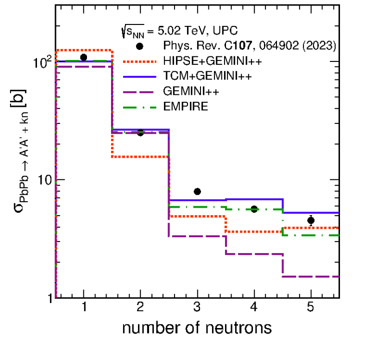
Nuclear processes in ultrarelativistic, ultraperipheral Pb+Pb collisions

Multiplicity of emitted particle

Results of our calculations using photon flux, photoabsorption cross section, and de-excitation models (EMPIRE and HIPSE) compared to values obtained from ALICE.

σ [b]	EMPIRE	HIPSE+GEMINI++	ALICE data
$1n$	100.8	124.1	108.4 $\pm$3.9
$2n$	25.3	15.6	25.0$\pm$1.3
$3n$	5.9	4.9	7.95 $\pm$0.25
$4n$	5.6	3.6	5.65 $\pm$0.33
$5n$	3.4	3.9	4.54 $\pm$0.44
n_{tot}	152.21	174.79	151.5 $\pm$4.7
1p	7.1	28.5	40.4
1α	64.0	42.4	-
1d	2.65	5.24	-
fission	-	18.3	-

ALICE data



P. Jucha, M. Klusek-Gawenda, A. Szczurek, M. Ciemala, and K. Mazurek, arXiv:2411.17865

Summary

- The fission description with Langevin type transport equation solving gives good estimation of various observables: mass, charge, TKE, angular distributions, emitted particles multiplicities and spectra and so on.
- The study of the correlation between mass and angular distributions allows to distinguish between fission and quasifission reactions.
- Four different prescriptions of $\tilde{a}$ have been implemented in the code, and their predictions for a number of reactions and the pre- and post-scission neutron multiplicity observables were analyzed.
- A single fission reaction makes available simultaneously two observables, viz. M_{pre} and M_{post}, which scan different regions of the (N , Z) phase space with the CN and fragments, respectively.
- Such analytical parameterizations remain of valuable guidance for fundamental theory, in particular for probing the relative evolution with a specific quantity, like e.g. here the isospin.
- The angular momentum of the fission fragments will be estimated, including the effects of exotic rotation on the path from the saddle to the scission point.
- The new field of interest for fission fragment calculations is the deexcitation of spectators in ultrarelativistic ultraperipheral Pb+Pb collisions.