Angular integrals for NNLO corrections to Drell-Yan production with a jet in the nested soft-collinear subtraction scheme

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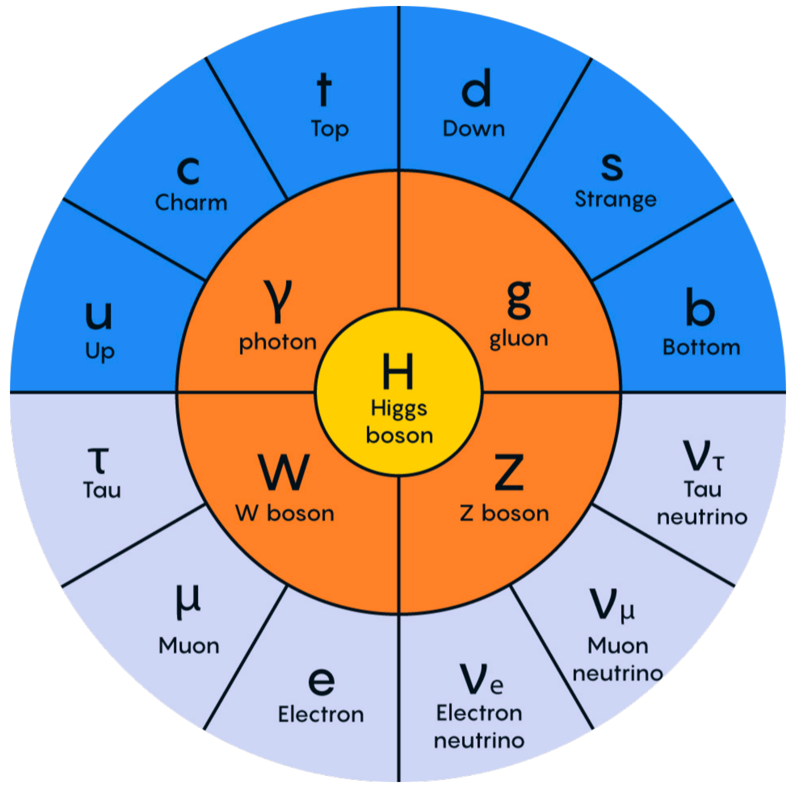
Particle physics needs precision

- 2012: **Higgs boson** at LHC



SM ✓

- Higgs couplings to heavy particles

  → Up to $5\,\%$ - $10\,\%$ accuracy SM ✓

- Work underway for couplings
  - ✦ Higgs to lighter particles
  - ✦ Higgs self-couplings
- Evidence from **non-collider** data:
  - ✦ Dark matter
  - ✦ Neutrino oscillations

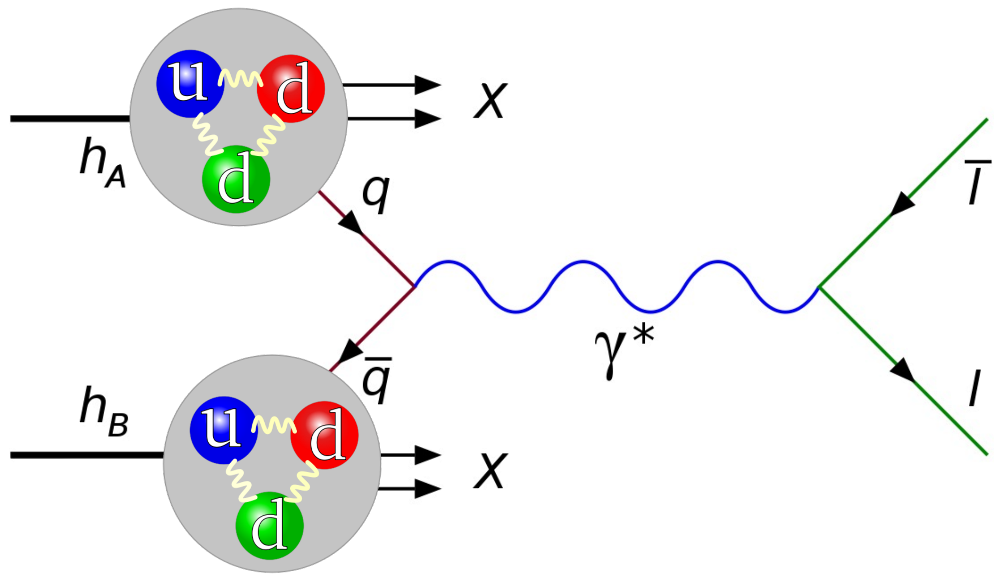
⟶ Possible **New Physics** at LHC? Any deviations from SM?

Experimental
Theoretical

Advance in the **precision** program at LHC → $\mathcal{O}(1\%)$ precision goal

Particle physics needs precision

QCD $\alpha_s \sim 0.1$
(Quantum ChromoDynamics) → Hadronic collisions →



$$d\sigma^H = f_i \otimes f_j \otimes d\hat{\sigma}_{ij}$$

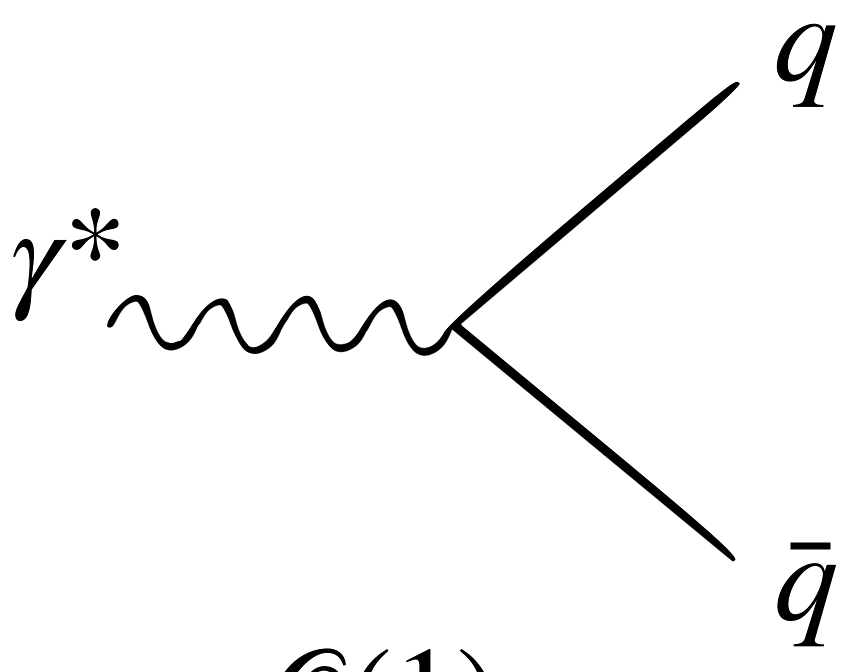
- **Perturbative** calculations at **partonic** level:

  More orders $\Longrightarrow$ More precision

- $\mathcal{O}(1\%)$ precision: **next-to-next-to-leading order (NNLO)** calculations needed

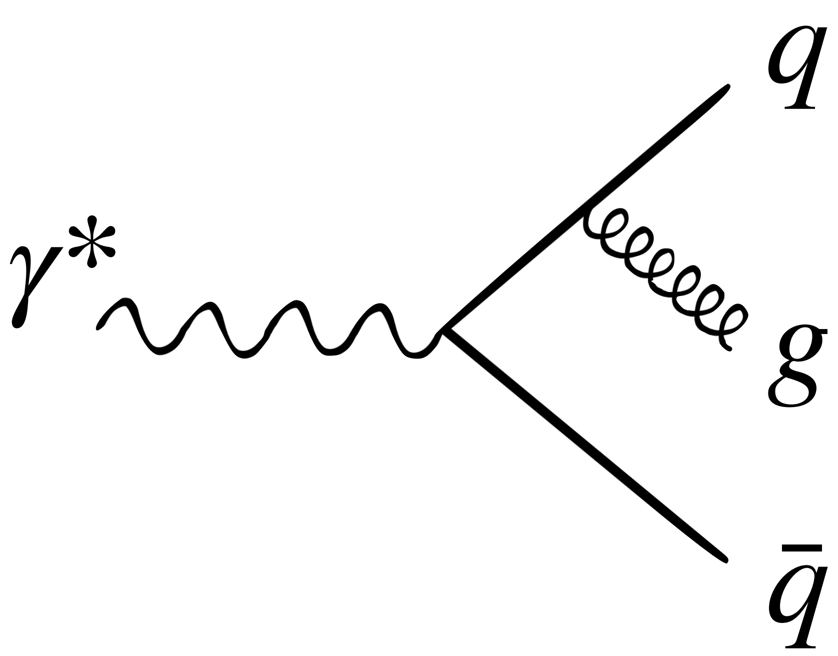
$\alpha_s^2 \sim 0.01$

LO



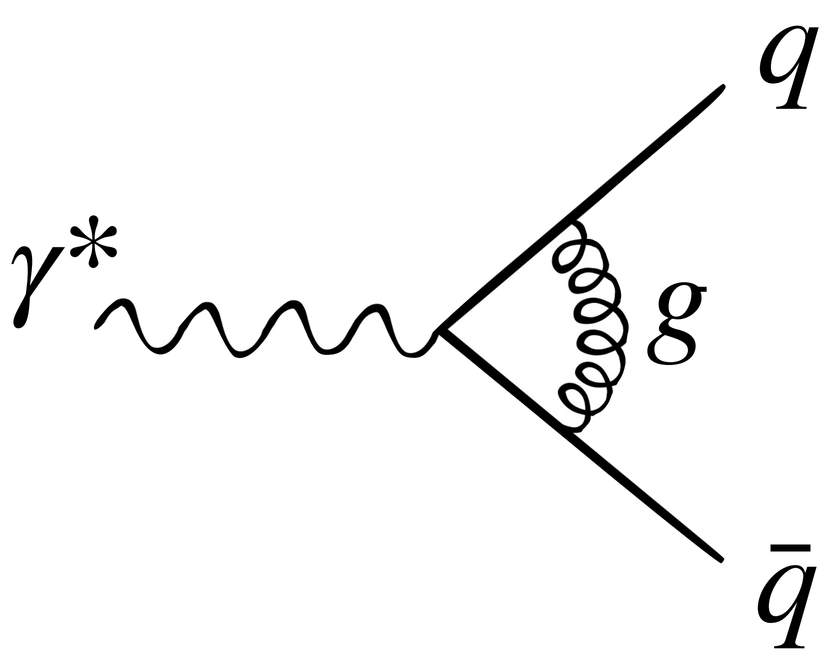
$\mathcal{O}(1)$

NLO



Real

$\mathcal{O}(\alpha_s)$



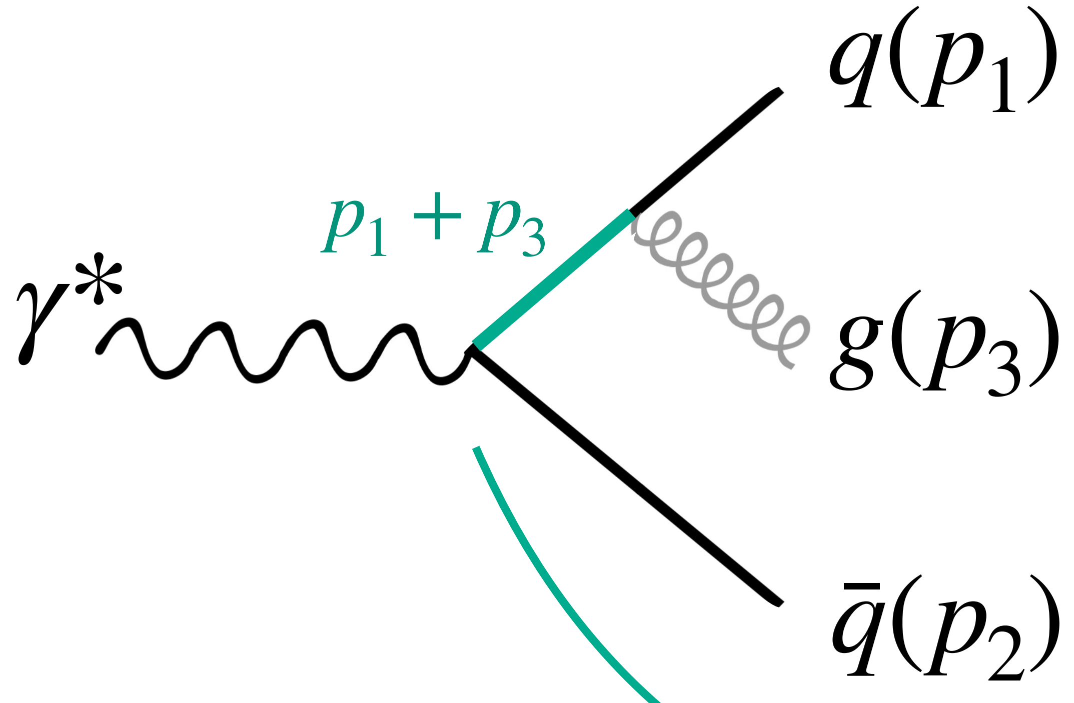
Virtual

Infrared (IR) singularities

IR singularities

$$p_i = (E_i, \vec{p}_i)$$
$$\rho_{ij} = 1 - \cos\theta_{ij} \propto p_i \cdot p_j$$

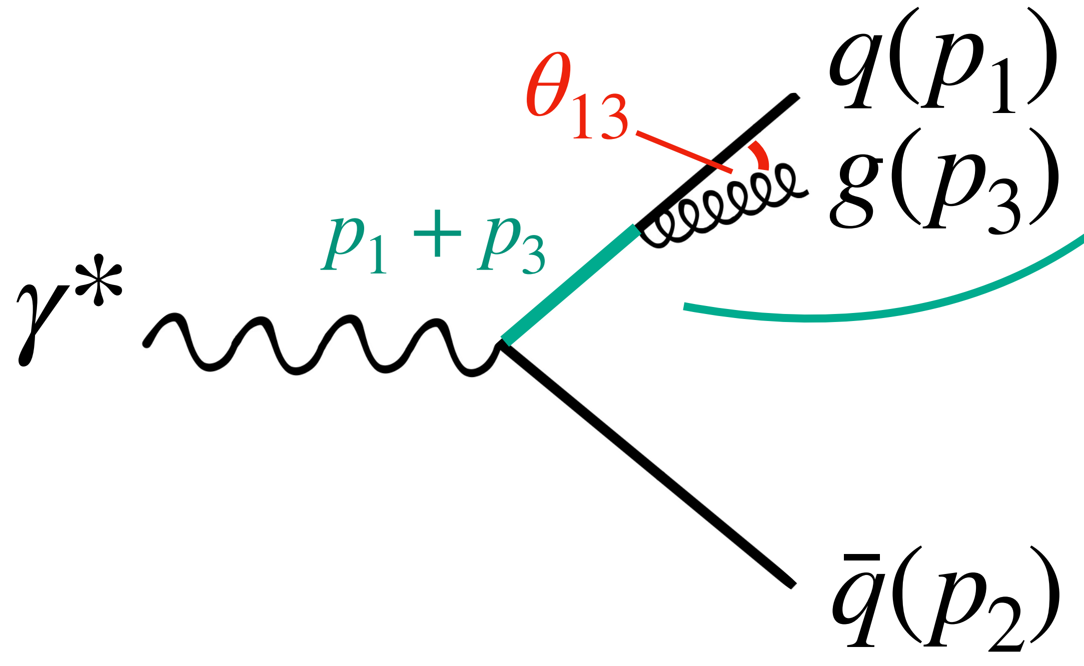
- **Soft:**



$$E_3 \to 0$$

$$\propto \frac{1}{(p_1+p_3)^2} = \frac{1}{E_1 E_3 (1 - \cos\theta_{13})}$$

- **Collinear:**



$$\rho_{13} = 1 - \cos\theta_{13} \to 0$$

- Unphysical individually
- Extract the singularities

↓

Subtraction scheme

General subtraction*

$d = 4 - 2\epsilon,\ \ \epsilon \to 0$ (Dimensional Regularization)

$$\int d\Phi^{(d)} F = \boxed{\int d\Phi^{(4)} \left[F - \mathcal{S}\right]} + \boxed{\int d\Phi^{(d)} \mathcal{S}}$$

Converges → MC

Analytically computable → Extract the poles $1/\epsilon^n$

- Sum **real** and **virtual** contributions
- **Cancellation** of the poles

↓

Finite parts

***local subtraction schemes**

Inside the $\mathcal{S}$

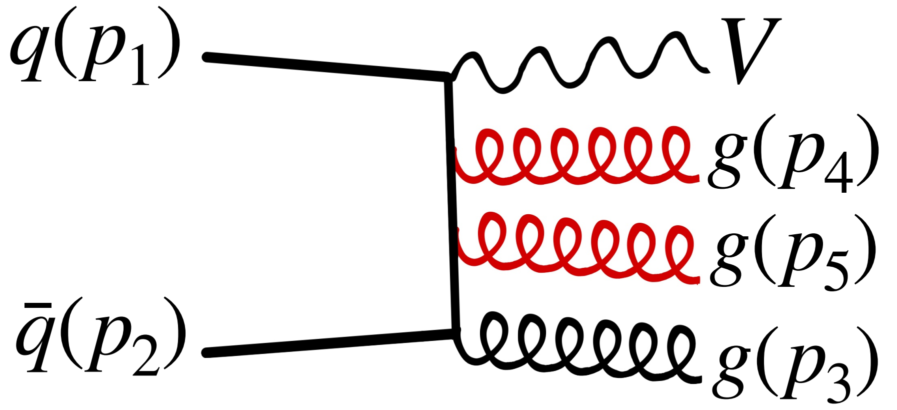
The nested soft-collinear subtraction scheme [Caola, Melnikov, Röntsch, 2017]

- **Soft** and **collinear** operators:

$$S_5 \equiv \lim_{E_5 \to 0} \qquad \mathbb{S} \equiv \lim_{E_4, E_5 \to 0}$$

$$C_{ij} \equiv \lim_{\rho_{ij} \to 0} \qquad \mathbb{C}_i \equiv \lim_{\rho_{4i}, \rho_{5i} \to 0}$$

NNLO



e.g. Drell-Yan + jet process

- **Soft** regulation:

$$F = (1 - \mathbb{S})F + \mathbb{S}F$$

$$= (1 - S_5)(1 - \mathbb{S})F + S_5(1 - \mathbb{S})F + \mathbb{S}F$$

Soft regulated

Analytically computable → Poles

- **Collinear** regulation:

$(1 - S_5)(1 - \mathbb{S})F$ → Partitioning → Sectoring

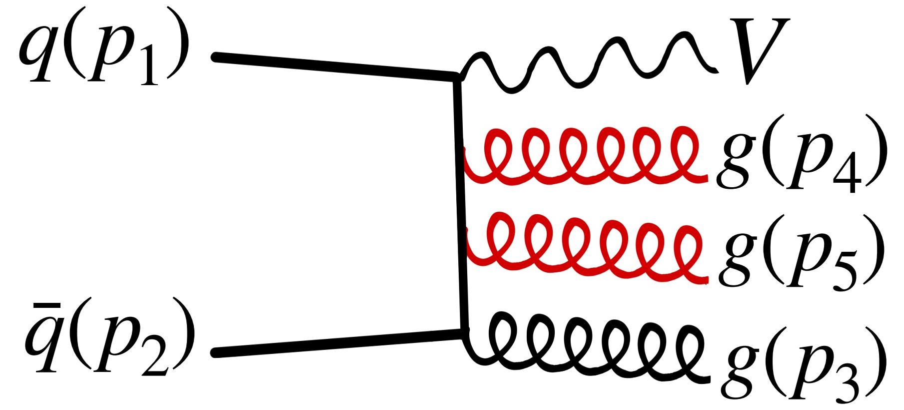
Subtracting collinear limits

Overlapping collinear singularities

Triple-collinear limits

Inside the $\mathcal{S}$

- **Nested soft-collinear** subtraction scheme
- **Drell-Yan + jet** process: $P + P \to V + j$
- NNLO: **two** radiated partons



Double real

Intricate IR structure

→ **overlapping** collinear singularities

Soft limit

↓

Eikonal functions

Finite part

subtraction term

↑

Collinear limit

↓

Partition functions

Sectoring

$$I = \int d\Omega_4^{(d-1)} \; \frac{\rho_{12}}{\rho_{14}\rho_{24}} \; \tilde{w}_{5\|1}^{41,51} \left(\frac{\rho_{14}}{4}\right)^{-\epsilon}$$

$$\rho_{ij} = 1 - \cos\theta_{ij} \propto p_i \cdot p_j$$

$$\tilde{w}^{41,51}_{5||1} = \frac{\rho_{24}\rho_{34}}{\rho_{12}+\rho_{13}}\Bigg[\frac{\rho_{12}}{(\rho_{14}+\rho_{24})(\rho_{14}+\rho_{24}+\rho_{34})} + \frac{\rho_{12}\rho_{14}}{(\rho_{14}+\rho_{24})^2(\rho_{14}+\rho_{24}+\rho_{34})}$$
$$+\frac{\rho_{12}}{(\rho_{14}+\rho_{34})(\rho_{14}+\rho_{24}+\rho_{34})} + \frac{\rho_{12}\rho_{14}}{(\rho_{14}+\rho_{34})^2(\rho_{14}+\rho_{24}+\rho_{34})}$$
$$+\frac{\rho_{12}\rho_{14}}{(\rho_{14}+\rho_{24})(\rho_{24}+\rho_{34})(\rho_{14}+\rho_{24}+\rho_{34})} + \frac{\rho_{12}\rho_{14}}{(\rho_{14}+\rho_{34})(\rho_{24}+\rho_{34})(\rho_{14}+\rho_{24}+\rho_{34})}$$
$$+\frac{\rho_{13}}{(\rho_{14}+\rho_{24})(\rho_{14}+\rho_{24}+\rho_{34})} + \frac{\rho_{13}\rho_{14}}{(\rho_{14}+\rho_{24})^2(\rho_{14}+\rho_{24}+\rho_{34})}$$
$$+\frac{\rho_{13}}{(\rho_{14}+\rho_{34})(\rho_{14}+\rho_{24}+\rho_{34})} + \frac{\rho_{13}\rho_{14}}{(\rho_{14}+\rho_{34})^2(\rho_{14}+\rho_{24}+\rho_{34})}$$
$$+\frac{\rho_{13}\rho_{14}}{(\rho_{14}+\rho_{24})(\rho_{24}+\rho_{34})(\rho_{14}+\rho_{24}+\rho_{34})} + \frac{\rho_{13}\rho_{14}}{(\rho_{14}+\rho_{34})(\rho_{24}+\rho_{34})(\rho_{14}+\rho_{24}+\rho_{34})}\Bigg]$$

- Not so few terms
- Analytical structure

structure

ing collinear
es

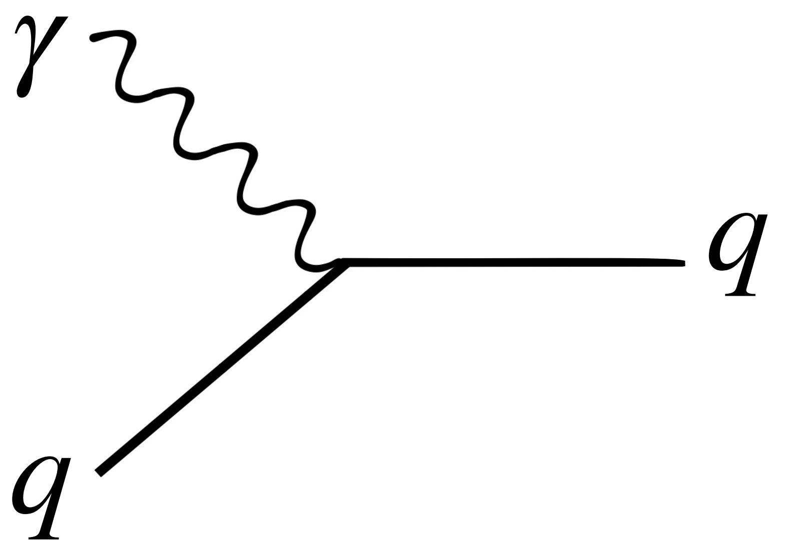
ring

$$I = \int d\Omega_4^{(d-1)} \frac{\rho_{12}}{\rho_{14}\rho_{24}} \boxed{\tilde{w}^{41,51}_{5||1}} \left(\frac{\rho_{14}}{4}\right)^{-\epsilon}$$

$$\rho_{ij} = 1 - \cos\theta_{ij} \propto p_i \cdot p_j$$

Feynman parameter method

From Deep Inelastic Scattering NNLO QCD



- Simpler **partition-dependent** integrals in DIS [Asteriadis, Caola, Melnikov, Röntsch, 2019]

$d = 4 - 2\epsilon$
$\epsilon \to 0$

$$\int d\Omega_4^{(d-1)} \xrightarrow[\mathcal{O}(\epsilon)]{\epsilon\text{-expansion}} \int d\Omega_4^{(3)} = \int_0^{\pi} d\theta \int_0^{2\pi} d\phi$$

Well-known formulas

$$\int_0^{2\pi} d\phi \frac{1}{(a - b\cos\phi)^n} = 2\pi \times \begin{cases} 1 & \text{if} \quad n = 0 \\ (a^2 - b^2)^{-1/2} & \text{if} \quad n = 1 \\ a(a^2 - b^2)^{-3/2} & \text{if} \quad n = 2 \\ (a^2 + b^2)(a^2 - b^2)^{-5/2} & \text{if} \quad n = 3 \\ & \vdots \end{cases}$$

Rationalization
- Change of variables

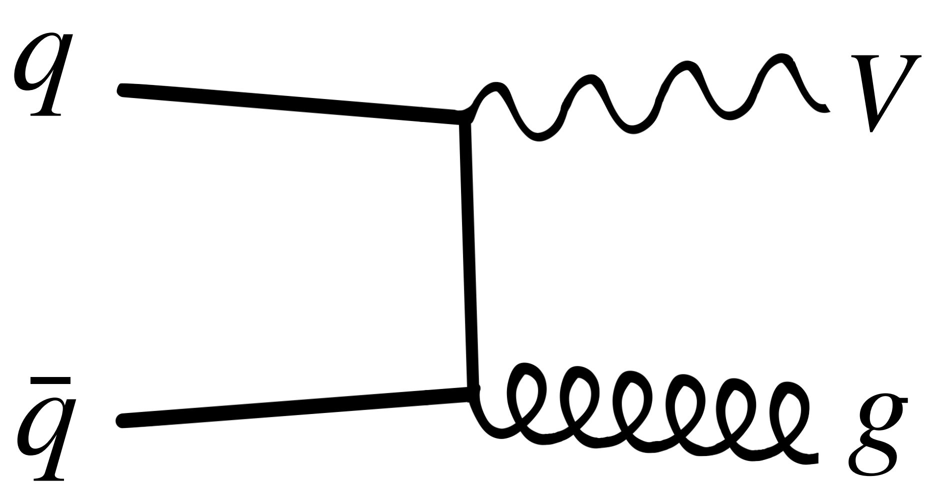
$$\theta \to \cdots \to t$$

→ Domain of integration →

Integration over t:
- Indefinite integral
- Evaluation at extremes

Feynman parameter method

To Drell-Yan + jet NNLO QCD



- $I_1^{(n_1,n_2,n_3)} = \int d\Omega_4^{(3)} \frac{\rho_{14}^{n_1}}{(\rho_{14}+\rho_{24})^{n_2}} \log^{n_3}\left(\frac{\rho_{14}}{4}\right)$ [Present in DIS]
- $I_4^{(n_1,n_2,n_3)} = \int d\Omega_4^{(3)} \frac{\rho_{14}^{n_1}}{(\rho_{14}+\rho_{24}+\rho_{34})^{n_2}} \log^{n_3}\left(\frac{\rho_{14}}{4}\right)$ $\longrightarrow$
  - Integration over t: evaluation at extremes $\longrightarrow$ spurious singularities
  - Computed

- $I_5 = \int d\Omega_4^{(3)} \frac{\log\left(\rho_{14}/4\right)}{\left(a_1\rho_{14}+a_2\rho_{24}+a_3\rho_{34}\right)\left(b_1\rho_{14}+b_2\rho_{24}+b_3\rho_{34}\right)}$ $\xrightarrow{\text{Feynman parameter}}$ $I_5 = \int_0^\infty dx \int d\Omega_4^{(3)} \frac{\log\left(\rho_{14}/4\right)}{L_x^2\left(c_x+\rho_{4x}\right)^2}$

Computed

$$\int_0^\infty dx \frac{\pi(2x+3)}{2\left(\eta_{12}(x+1)^2+\eta_{13}(x+1)+\eta_{23}(x+1)\right)} \cdot \frac{\log\left(\frac{3+2x+\sqrt{(2x+3)^2-4\left(\eta_{12}(x+1)^2+\eta_{13}(x+1)+\eta_{23}(x+1)\right)}}{3+2x-\sqrt{(2x+3)^2-4\left(\eta_{12}(x+1)^2+\eta_{13}(x+1)+\eta_{23}(x+1)\right)}}\right)}{\sqrt{(2x+3)^2-4\left(\eta_{12}(x+1)^2+\eta_{13}(x+1)+\eta_{23}(x+1)\right)}}$$

Mellin-Barnes method

$\Gamma(n) = (n-1)!$

Angular integrals

$$\int d\Omega_q^{(d-1)} \frac{1}{(p_1 \cdot q)^{j_1} \cdots (p_n \cdot q)^{j_n}}$$

[Somogyi, 2011] →

Contour integrals

Mellin-Barnes representation

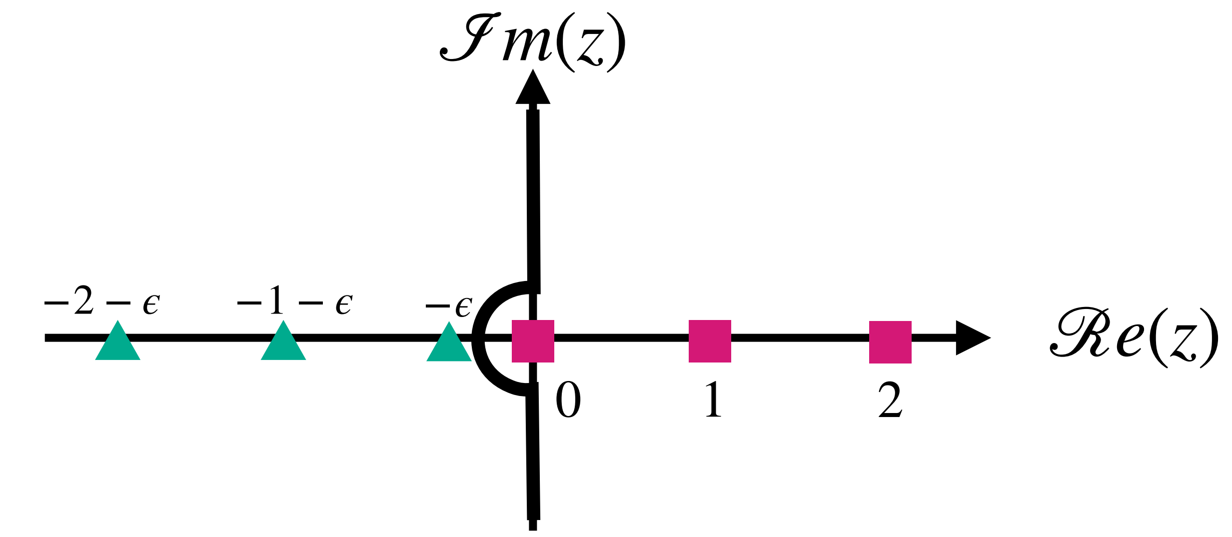
$$\int_{-i\infty}^{+i\infty} dz_1 \ldots dz_k \;\; \Gamma(z_1 + \ldots) \cdots \Gamma(z_k + \ldots)$$

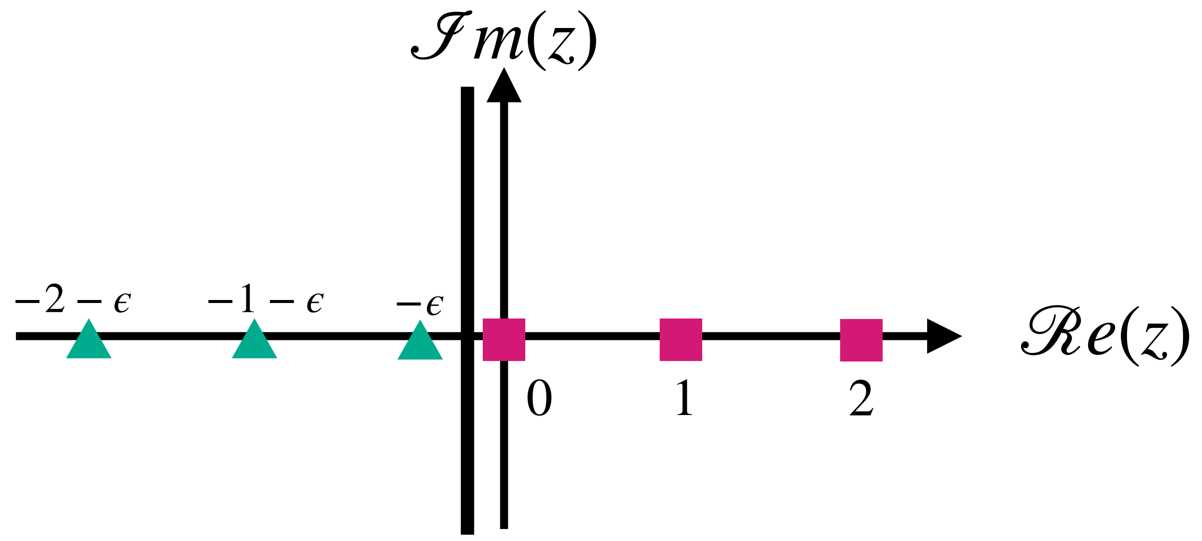
RESIDUE THEOREM

- Soft-limit integrals in DY + N jets [Devoto, Melnikov, Röntsch, Signorile-Signorile, Tagliabue, 2023]
- Gamma functions $\Gamma(z)$ have poles at $z = -k$, with $k = 0,1,2,\ldots$
- <u>*Caveat*</u>: contour separates *left* poles $\Gamma(\ldots + z)$ from *right* poles $\Gamma(\ldots - z)$

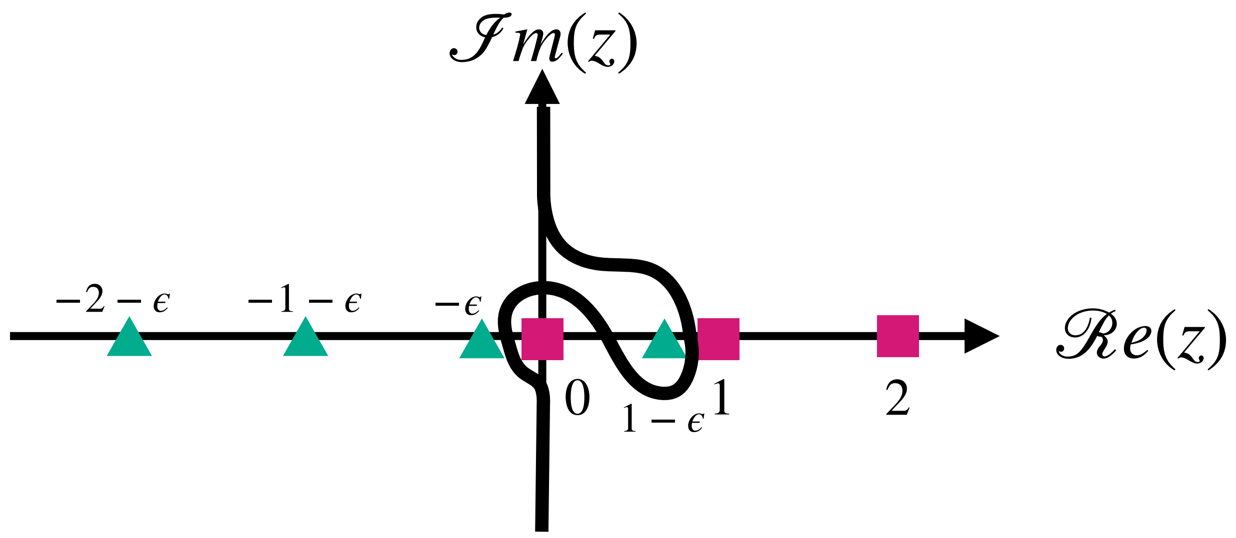
e.g.

$$\int_{-i\infty}^{+i\infty} dz \; \Gamma(z+\epsilon) \; \Gamma(-z)$$



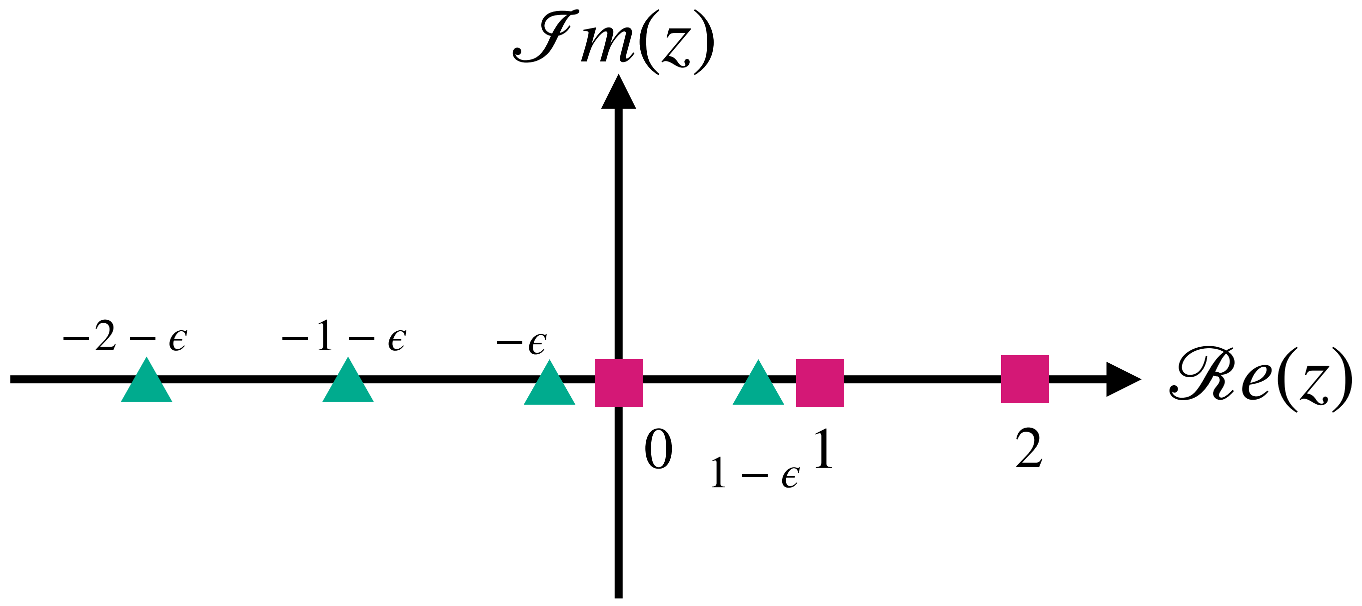


$$\int_{-i\infty}^{+i\infty} dz \; \Gamma(z-1+\epsilon) \; \Gamma(-z)$$

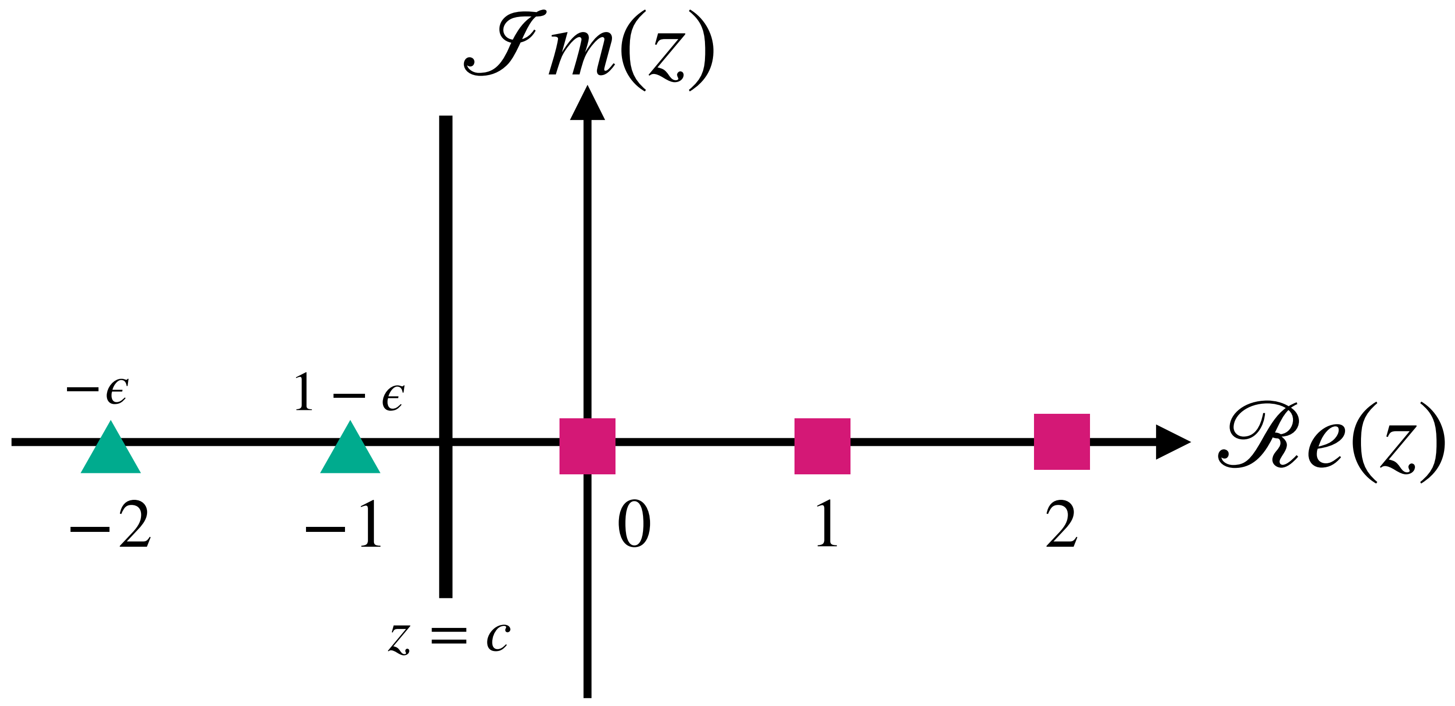


- Involved contours
- Dependence on ϵ
  → series expansion?
- No straight lines

Mellin-Barnes method



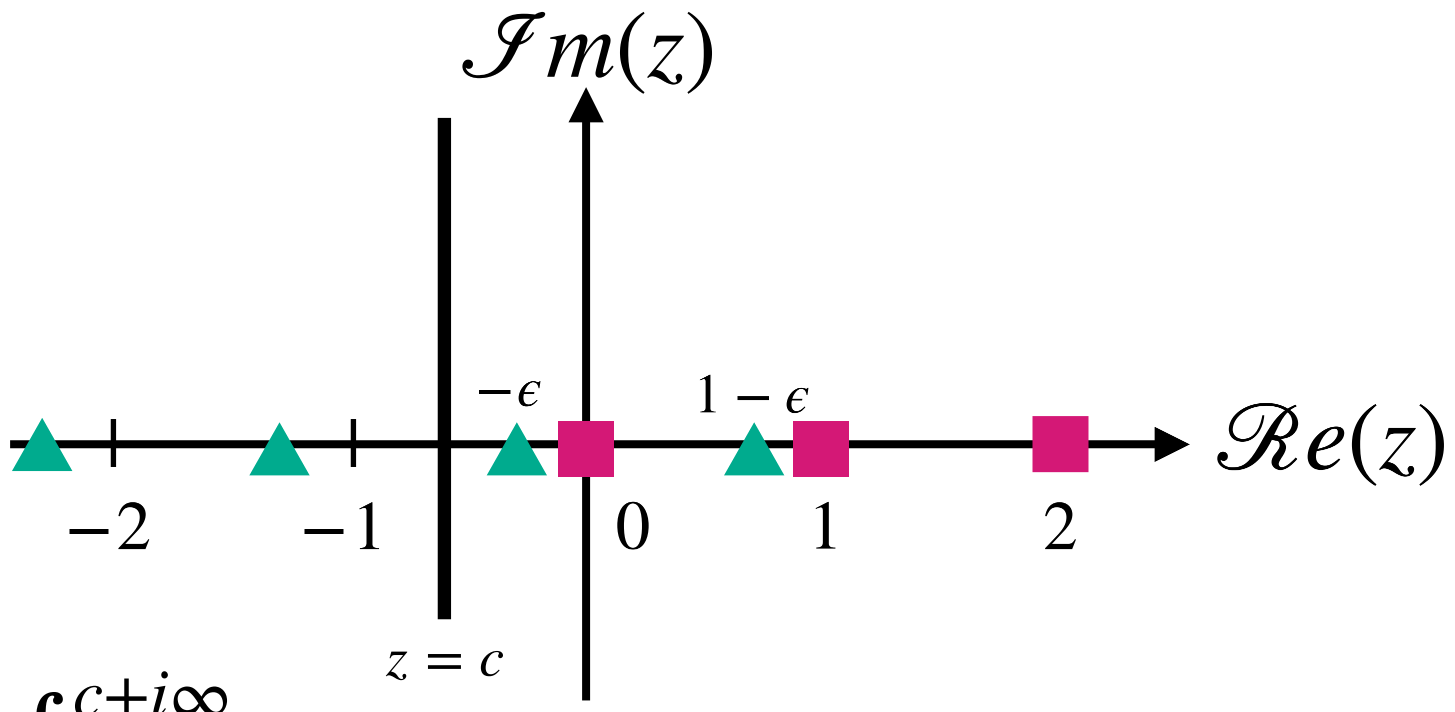
Shift $\epsilon \approx 2$



$$\int_{c-i\infty}^{c+i\infty} dz \;\; \Gamma(z-1+\epsilon) \;\; \Gamma(-z)$$

- Well-defined MB representation ($\epsilon \approx 2$)
- Straight line contour

Analytic continuation $\epsilon \to 0$



$$\int_{c-i\infty}^{c+i\infty} dz \;\; \Gamma(z-1+\epsilon) \;\; \Gamma(-z) \quad (\epsilon \to 0)$$

$+$ Res$[z \to -\epsilon]$

$+$ Res$[z \to 1-\epsilon]$

- Several gamma functions
- Multi-dimensional integrals

MBresolve.m
MB.m

[Smirnov, ’09]
[Czakon, ’06]

- List of integrals
- Straight contour positions
- Fully automated

Mellin-Barnes method

Integral families

One-den. massless

Two-denominator one-massive

$$\int d\Omega_4^{(d-1)}\Big[-\frac{1}{\rho_{12}\rho_{14}^{1+\epsilon}}+\frac{1}{\rho_{14}^{\epsilon}\left(\rho_{14}+\rho_{24}\right)^2}+\frac{1}{\rho_{14}^{1+\epsilon}\left(\rho_{14}+\rho_{24}\right)}$$

$$+\frac{1}{\rho_{14}^{\epsilon}\rho_{24}\left(\rho_{14}+\rho_{34}\right)}-\frac{1}{\rho_{14}^{\epsilon}\rho_{24}\left(\rho_{14}+\rho_{24}+\rho_{34}\right)}$$

$$+\frac{1}{\rho_{14}^{\epsilon}\rho_{34}\left(\rho_{14}+\rho_{24}+\rho_{34}\right)}-\frac{1}{\rho_{14}^{\epsilon}\rho_{34}\left(\rho_{14}+\rho_{24}\right)}$$

$$+\frac{1}{\rho_{14}^{-1+\epsilon}\rho_{24}^2\left(\rho_{14}+\rho_{34}\right)}-\frac{1}{\rho_{14}^{-1+\epsilon}\rho_{24}^2\left(\rho_{14}+\rho_{24}+\rho_{34}\right)}-\frac{1}{\rho_{14}^{-1+\epsilon}\rho_{24}\left(\rho_{14}+\rho_{34}\right)^2}$$

$$+\frac{1}{\rho_{14}^{\epsilon}\left(\rho_{14}+\rho_{24}\right)\left(\rho_{24}+\rho_{34}\right)}-\frac{1}{\rho_{14}^{\epsilon}\left(\rho_{14}+\rho_{34}\right)\left(\rho_{24}+\rho_{34}\right)}\Big]4^{\epsilon}\rho_{12}$$

$\epsilon,\ 1,\ 1$

$-1+\epsilon,\ 1,\ 2$

$-1+\epsilon,\ 2,\ 1$

$$\rho_{ij} = 1-\cos\theta_{ij} \propto p_i \cdot p_j$$

Notation:

$\dfrac{1}{\rho_{i4}^a}$ Massless

$\dfrac{1}{(\rho_{i4}+\rho_{i4}+\ldots)^a}$ Massive

Three-denominator one-massive terms

Mellin-Barnes method

Three-denominator one-massive: $-1+\epsilon,\ 1,\ 2$

$$\int d\Omega_4^{(d-1)} \frac{1}{\rho_{14}^{-1+\epsilon}\rho_{24}\left(\rho_{14}+\rho_{34}\right)^2}$$

- Mellin-Barnes representation:

$$\Omega_{-1+\epsilon,1,2} = k(\epsilon)\int_{-i\infty}^{+i\infty} dz_1 dz_2 dz_3 dz_4\ f(p_1,p_2,p_3,\epsilon,\{z_i\})\,\Gamma(-z_1)\Gamma(-z_2)\Gamma(-z_3)\Gamma(-z_4)\times$$
$$\times\Gamma(z_1+z_3+1)\Gamma(\epsilon+z_1+z_2-1)\Gamma(z_2+z_3+2z_4+2)\Gamma(-2\epsilon-z_1-z_2-z_3-z_4-1)$$

↓

`MBresolve.m` —(ϵ-expansion, $\mathcal{O}(\epsilon)$)→ 6 Mellin-Barnes integrals:

- 1 trivial
- 2 one variable
- 3 **two variables**

→ equipped with relative **contours**

Mellin-Barnes method

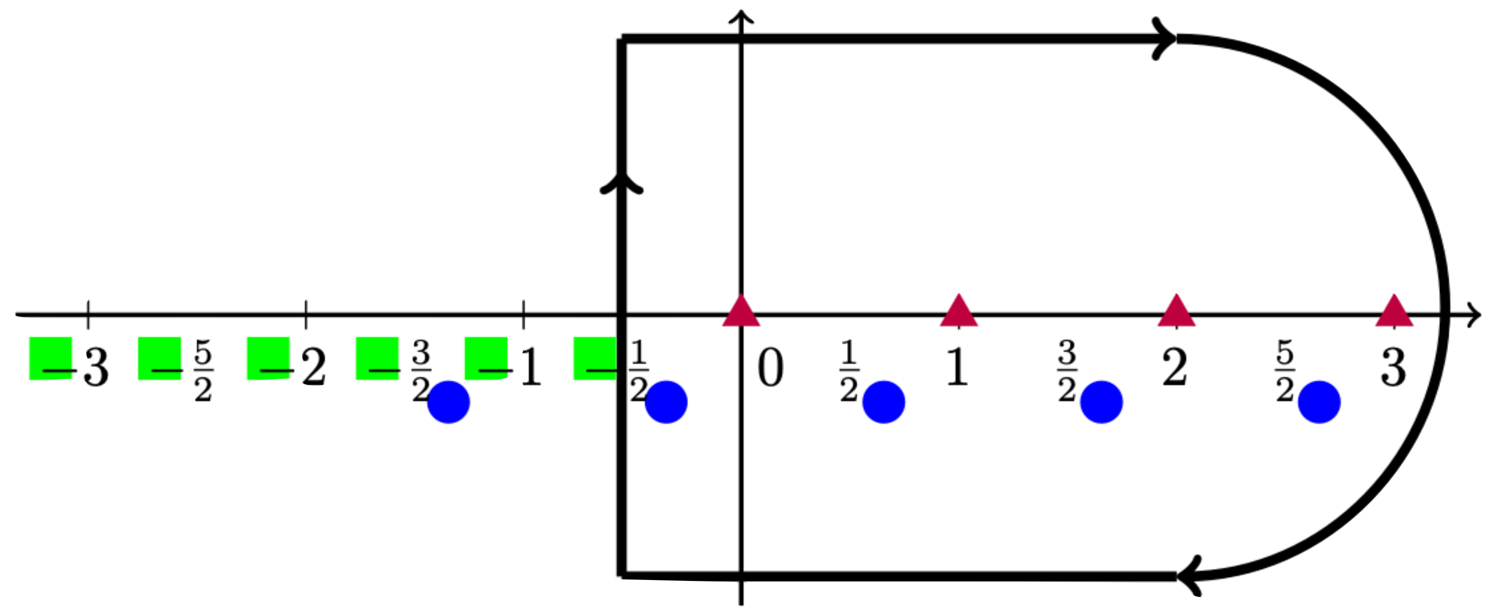
Three-denominator one-massive:

$-1+\epsilon,\ 1,\ 2$

$$\int d\Omega_4^{(d-1)} \frac{1}{\rho_{14}^{-1+\epsilon}\rho_{24}\left(\rho_{14}+\rho_{34}\right)^2}$$

Two variable integrals

$$\int_{-i\infty}^{+i\infty} dz_1 \int_{-i\infty}^{+i\infty} dz_2\ \Gamma(z_1+z_2+\ldots)\cdots\Gamma(-z_1-z_2+\ldots)$$



- Gamma poles at **non-integer** values

    - Very general functions: $_2F_1(a,b;c;z)$ Hypergeometric functions

        → Still one integration missing

Particular values → Elementary functions

Integral representation → much harder

- Contour integration over second variable

Mellin-Barr

Three-denomina

Two variable integrals

$$\int_{-i\infty}^{+i\infty} dz_1 \int_{-i\infty}^{+i\infty} dz_2\, \Gamma($$

- Gamma poles at
  - Very general … netric functions
    - → Still one inte
  - Partic … presentation
    - → Elen … rder
- Contour integrati

$$
\begin{aligned}
\Omega_{-1+\epsilon;1;2} = & -\frac{v_{12}}{v_{23}^2}\frac{1}{\epsilon} + \frac{\log v_{12} v_{12}}{v_{23}^2} + \frac{2\log(v_{23}) v_{12}}{v_{23}^2} - \frac{\log v_{33} v_{12}}{v_{23}^2} + \frac{v_{13}v_{23} - 2v_{12}v_{33}}{v_{23}^2 v_{33}} \\
& + \epsilon\left\{ \left[\frac{\log v_{33} v_{12}}{v_{23}^2} + \frac{\log\left(1-\frac{v_{13}v_{23}}{v_{12}v_{33}}\right) v_{12}}{v_{23}^2} - \frac{2\log(v_{23})v_{12}}{v_{23}^2} + \frac{2v_{12}}{v_{23}^2} \right] \log v_{12} \right. \\
& - \frac{v_{12}\log^2 v_{12}}{2v_{23}^2} + \frac{3v_{12}\log^2\left(1-\frac{v_{23}}{v_{33}}\right)}{2v_{23}^2} + \frac{v_{12}\log^2\left(1-\frac{v_{13}v_{23}}{v_{12}v_{33}}\right)}{2v_{23}^2} - \frac{2v_{12}\log^2 v_{23}}{v_{23}^2} \\
& - \frac{3v_{12}\log\left(1-\frac{(r+1)v_{23}}{2v_{33}}\right)\log v_{33}}{v_{23}^2} + \frac{3v_{12}v_{33}\log(v_{33}-v_{23})}{v_{23}^2(v_{33}-v_{23})} - \frac{v_{12}\log^2 v_{33}}{2v_{23}^2} - \frac{5\pi^2 v_{12}}{6v_{23}^2} \\
& + \left[\frac{4v_{12}v_{33} - v_{23}(2v_{23}v_{12}+v_{12}+3v_{13}-3v_{23})}{v_{23}^2((v_{23}-1)v_{23}+v_{33})} + \frac{2v_{12}\log v_{33}}{v_{23}^2} + \frac{3v_{12}\log\left(\frac{(r-1)v_{23}}{2v_{33}}+1\right)}{v_{23}^2} \right. \\
& \left. + \frac{3v_{12}\log\left(1-\frac{(r+1)v_{23}}{2v_{33}}\right)}{v_{23}^2} - \frac{v_{12}\log\left(1-\frac{v_{13}v_{23}}{v_{12}v_{33}}\right)}{v_{23}^2} \right] \log v_{23} - (\log 2 - \log(r+1))\cdot \\
& \times \left[\frac{3(r+1)\left(v_{13}v_{23}(v_{23}+2v_{33}-1)+v_{33}\left(-2v_{23}^2+v_{23}+v_{12}-4v_{12}v_{33}\right)\right)}{v_{23}(r-4v_{33}+1)v_{33}((v_{23}-1)v_{23}+v_{33})} \right. \\
& \left. + \frac{3v_{12}\log\left(1-\frac{(r+1)v_{23}}{2v_{33}}\right)}{v_{23}^2} - \frac{3v_{12}\log\left(\frac{(r-1)v_{23}}{2v_{33}}+1\right)}{v_{23}^2} \right] + \left[-\frac{v_{13}}{v_{23}v_{33}} \right. \\
& \left. - \frac{v_{12}\log\left(1-\frac{v_{13}v_{23}}{v_{12}v_{33}}\right)}{v_{23}^2} \right] \log v_{13} + \left[\frac{3v_{12}v_{33}}{v_{23}^2(v_{23}-v_{33})} - \frac{3v_{12}\log\left(-\frac{v_{23}}{v_{33}}\right)}{v_{23}^2} \right] \log\left(1-\frac{v_{23}}{v_{33}}\right) \\
& + \left[v_{33}\left((r+1)(v_{12}+3v_{23})v_{23}^2 + (v_{12}(6r-4v_{23}+2) - 3(r+1)v_{23})v_{33}v_{23} \right.\right. \\
& \left.\left. -2v_{12}(5r+4v_{23})v_{33}^2\right) - v_{13}v_{23}(v_{23}-v_{33})(4rv_{33}+v_{23}(r+2v_{33}+1))\right] \cdot \\
& \times \frac{\log v_{33}}{rv_{23}^2(rv_{23}+v_{23}-2v_{33})(v_{23}-v_{33})v_{33}} \\
& + \left[\frac{v_{12}\log v_{33}}{v_{23}^2} + \frac{v_{12}\log\left(\frac{v_{13}v_{23}}{v_{13}v_{23}-v_{12}v_{33}}\right)}{v_{23}^2} \right] \log\left(1-\frac{v_{13}v_{23}}{v_{12}v_{33}}\right) + \frac{3v_{12}\mathrm{Li}_2\left(\frac{2v_{23}}{r+1}\right)}{v_{23}^2} \\
& \left. + \frac{3v_{12}\mathrm{Li}_2\left(-\frac{v_{33}}{v_{23}-v_{33}}\right)}{v_{23}^2} + \frac{3v_{12}\mathrm{Li}_2\left(\frac{(r+1)v_{23}}{2v_{33}}\right)}{v_{23}^2} - \frac{3v_{12}\mathrm{Li}_2\left(\frac{v_{23}}{v_{33}}\right)}{v_{23}^2} - \frac{v_{12}\mathrm{Li}_2\left(\frac{v_{12}v_{33}}{v_{12}v_{33}-v_{13}v_{23}}\right)}{v_{23}^2} \right\},
\end{aligned}
\tag{B.4}
$$

v_{ij}

- contain kinematics
- obtain integral terms by substitution

Mellin-Barnes method

Integral families

$$\int d\Omega_4^{(d-1)}\Bigg[-\frac{1}{\rho_{12}\rho_{14}^{1+\epsilon}}+\frac{1}{\rho_{14}^{\epsilon}\left(\rho_{14}+\rho_{24}\right)^{2}}+\frac{1}{\rho_{14}^{1+\epsilon}\left(\rho_{14}+\rho_{24}\right)}$$

$$+\frac{1}{\rho_{14}^{\epsilon}\rho_{24}\left(\rho_{14}+\rho_{34}\right)}-\frac{1}{\rho_{14}^{\epsilon}\rho_{24}\left(\rho_{14}+\rho_{24}+\rho_{34}\right)}$$

$$+\frac{1}{\rho_{14}^{\epsilon}\rho_{34}\left(\rho_{14}+\rho_{24}+\rho_{34}\right)}-\frac{1}{\rho_{14}^{\epsilon}\rho_{34}\left(\rho_{14}+\rho_{24}\right)}$$

$$+\frac{1}{\rho_{14}^{-1+\epsilon}\rho_{24}^{2}\left(\rho_{14}+\rho_{24}\right)}-\frac{1}{\rho_{14}^{-1+\epsilon}\rho_{24}^{2}\left(\rho_{14}+\rho_{24}+\rho_{34}\right)}-\frac{1}{\rho_{14}^{-1+\epsilon}\rho_{24}\left(\rho_{14}+\rho_{34}\right)^{2}}$$

$$+\frac{1}{\rho_{14}^{\epsilon}\left(\rho_{14}+\rho_{24}\right)\left(\rho_{24}+\rho_{34}\right)}-\frac{1}{\rho_{14}^{\epsilon}\left(\rho_{14}+\rho_{34}\right)\left(\rho_{24}+\rho_{34}\right)}\Bigg]4^{\epsilon}\rho_{12}$$

$$\rho_{ij}=1-\cos\theta_{ij}\propto p_i\cdot p_j$$

- Single-denominator massless
- Two-denominator one-massive
- Three-denominator one-massive
- Three-denominator two-massive
  - Summation of digamma products
  - Generalised hypergeometric functions

Conclusions

- Addressed the problem of computing **angular integrals** which contribute to **NNLO** QCD corrections to **Drell-Yan+jet** cross section

- It is very demanding due to the presence of involved **partition functions** for regulating the overlapping collinear limits

- Two approaches:

  ✦ **Feynman parameter method:**

    - Computed simpler terms

    - Small margin of improvement in generalizing to more complicated terms

  ✦ **Mellin-Barnes method:**

    - Applied to non-trivial families with success

    - Computed terms **not** feasible with **Feynman parameter method**

    - Last missing family

Outlooks

- Finishing the computation of **three-denominators two-massive** family:
  - ✦ Expected to be more involved but MB integration can be **improved**
  - ✦ Key result for completing the NSC scheme **finite part of subtraction terms** for DY+jet at NNLO
    - ➡ Also for processes with same QCD structure: $H+j$, $VBF+j$, $e^+e^- \to 3j, \ldots$
- Looking further into the future:
  - ✦ NSC subtraction scheme is focussing on n**-parton** processes [Devoto, Melnikov, Röntsch, Signorile-Signorile, Tagliabue, 2023]
  - ✦ This project would be the **starting point** for developing a general tool for computing similar IR finite integrals appearing in subtraction counterterms

Thank you for your attention

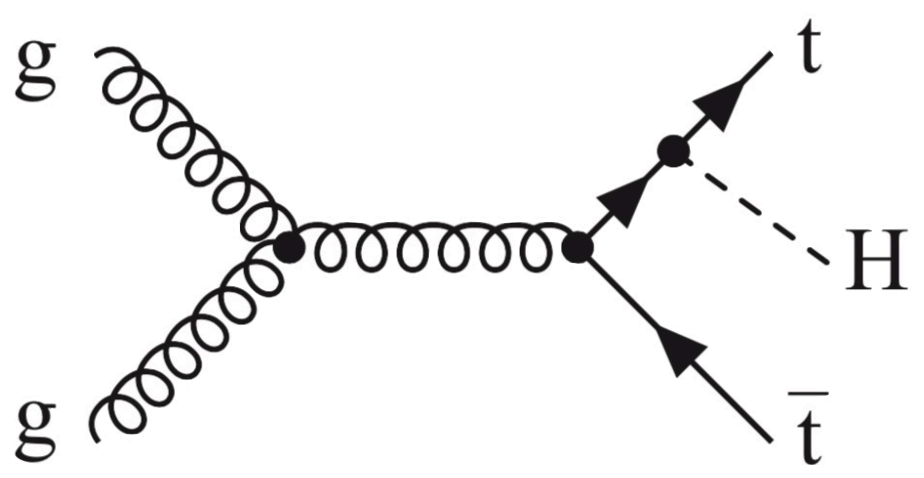
Backups

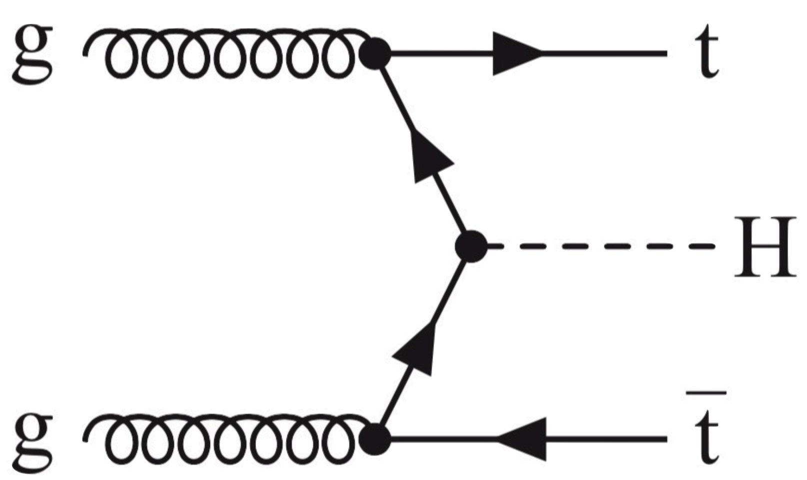
Particle physics needs precision

Experimental
Theoretical

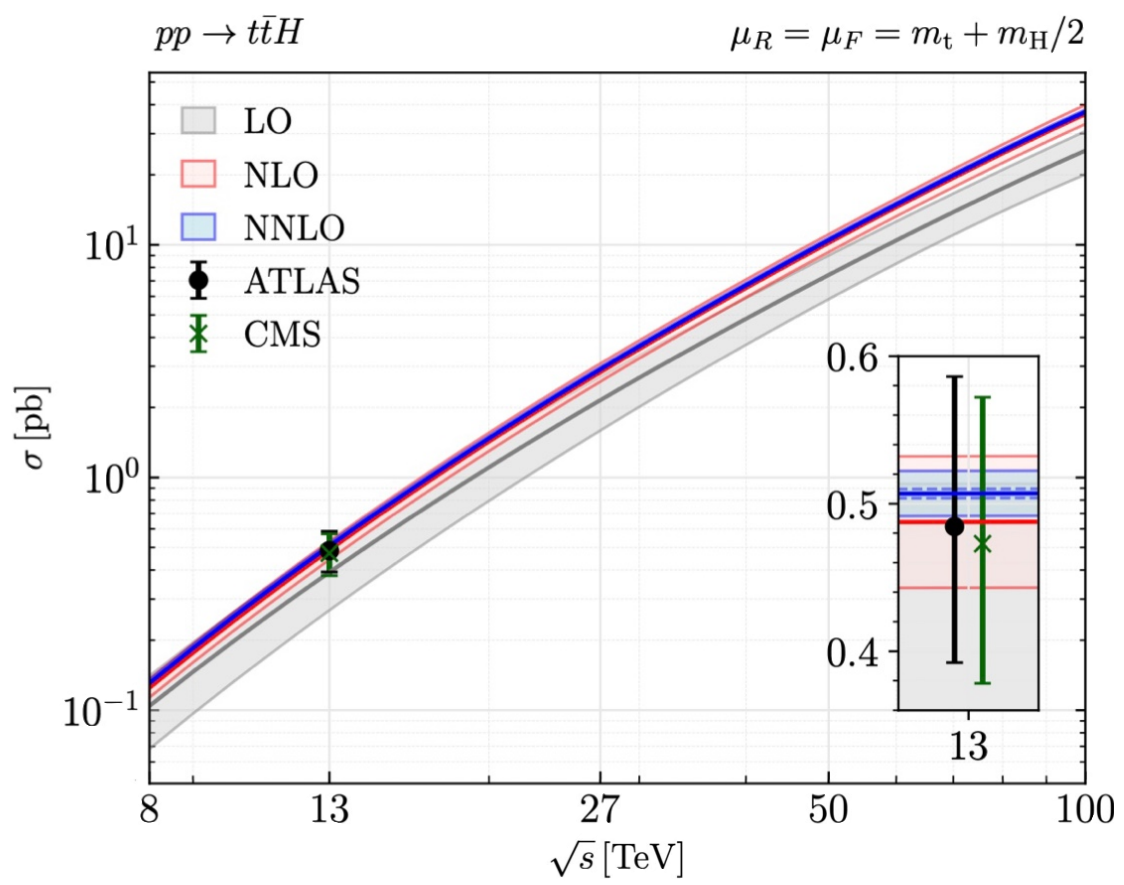
Advance in the **precision** program at LHC → $\mathcal{O}(1\%)$ precision goal

- Higgs couplings to heavy particles: e.g. $pp \to t\bar{t}H$

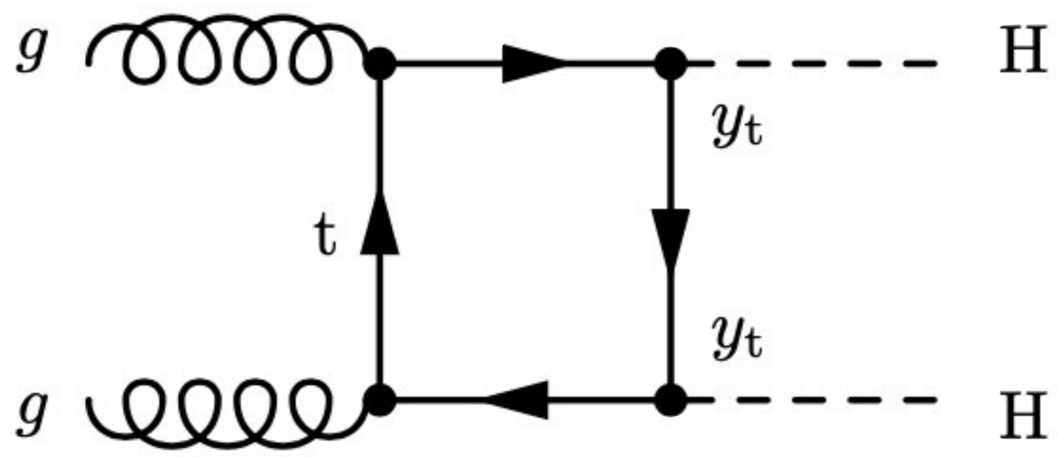


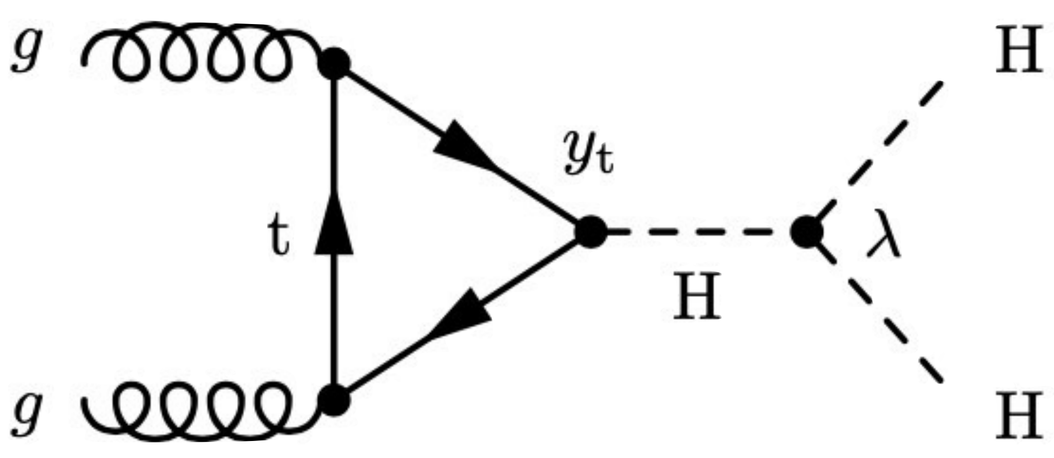


[Catani, S. Devoto, Grazzini, Kallweit, Mazzitelli, Savoini, 2023]

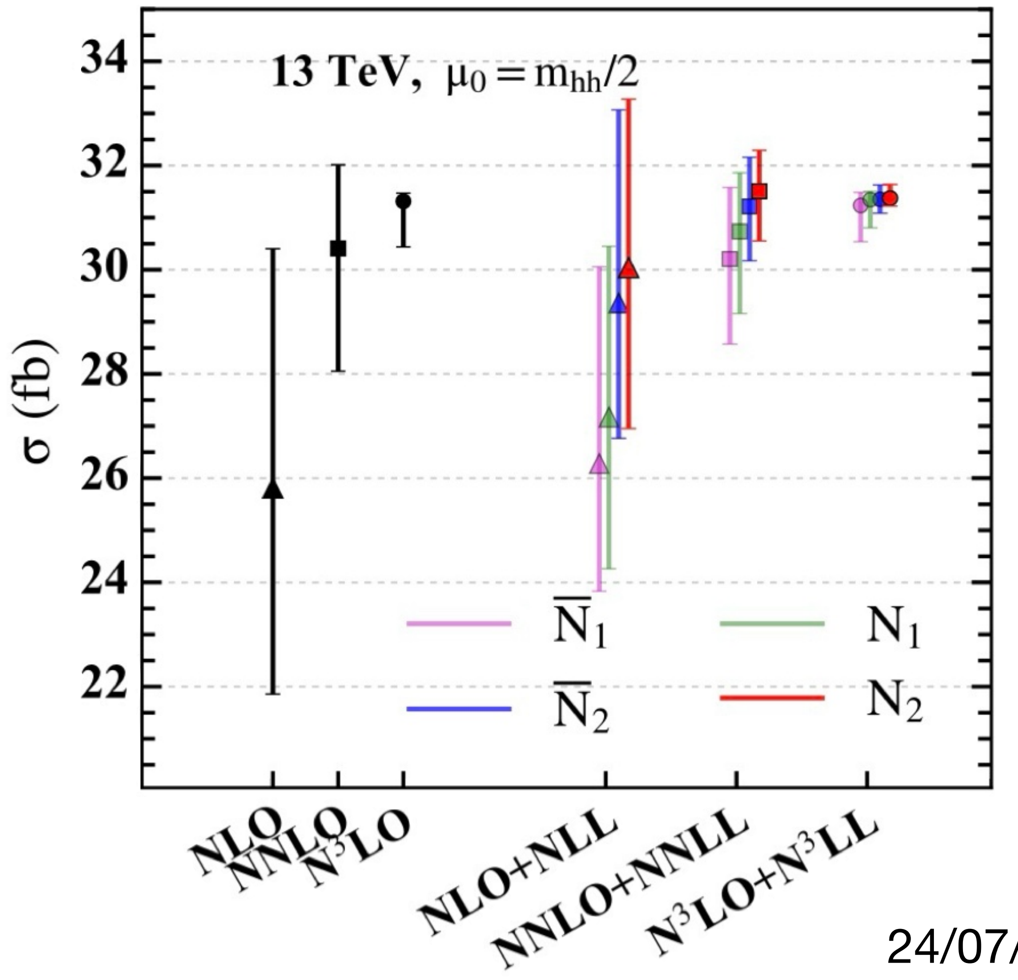


- Higgs self-couplings: e.g. $pp \to HH$





[Ajjath, Shao, 2023]



Analytic computation of subtraction terms

Motivation

- Analytic integration of **singular part**:
  - ⋆ Exact **subtraction** of IR poles
- Analytic integration of **finite part**:
  - ⋆ Their expression can be directly inserted in codes for cross section computation
  - ⋆ Reduces the dimensionality of numerical Monte Carlo integrals, improving **efficiency** and **stability**

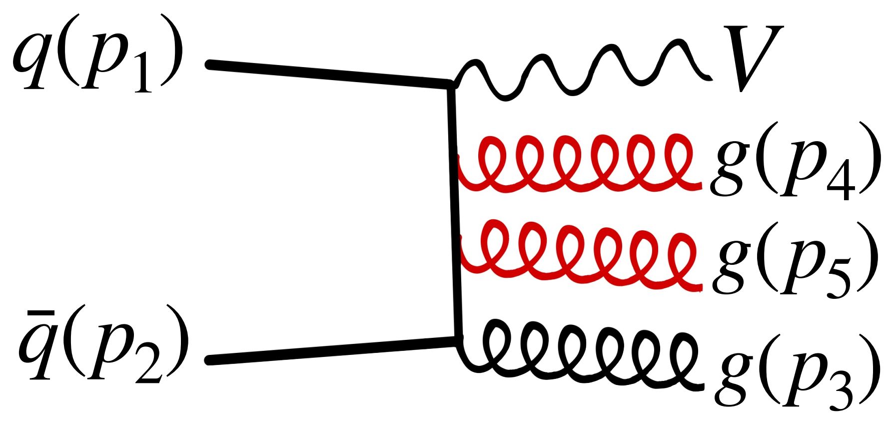
Partition functions

Properties

$$C_{ij} \equiv \lim_{\rho_{ij}\to 0}$$

Arbitrary expression, respecting:

- **Unitarity:**



$$1 = \sum_k w^k$$

$k \in \{(41,52); (42,51); (41,53); (43,51); (42,53); (43,52); (41,51); (42,52); (43,53)\}$

- **Selection of collinear singularities:** $C_{mn} w^{ij,kl} = 0$ unless $(m, n) = (i, j), (k, l)$

  $C_{mn} w^{ij,kj} = 0$ unless $(m, n) = (i, j), (k, l)$, or (i, k)

$$\begin{cases} w^{ij,kl} |\mathcal{M}|^2 & \text{singular in the limits } C_{ij}, C_{kl}, \text{ if } j \neq l \\ w^{ij,kj} |\mathcal{M}|^2 & \text{singular in the limits } C_{ij}, C_{kj}, \text{ and } C_{ik} \end{cases}$$

Partition functions

Expressions

DIS: $q(p_1) + e^-(p_2) \to e^-(p_2) + q(p_4) + g(p_5) + g(p_6)$

$$w^{51,61} = \frac{\rho_{54}\rho_{64}}{d_5 d_6}\left(1 + \frac{\rho_{15}}{d_{5641}} + \frac{\rho_{16}}{d_{5614}}\right)$$

$$w^{54,64} = \frac{\rho_{15}\rho_{16}}{d_5 d_6}\left(1 + \frac{\rho_{46}}{d_{5641}} + \frac{\rho_{45}}{d_{5614}}\right)$$

$$w^{51,64} = \frac{\rho_{45}\rho_{16}\rho_{56}}{d_5 d_6 d_{5614}}$$

$$w^{54,61} = \frac{\rho_{15}\rho_{46}\rho_{56}}{d_5 d_6 d_{5641}}$$

$$d_{i=5,6} \equiv \rho_{1i} + \rho_{4i}, \quad d_{5614} \equiv \rho_{56} + \rho_{15} + \rho_{46}, \quad d_{5641} \equiv \rho_{56} + \rho_{45} + \rho_{16}$$

[Asteriadis, Caola, Melnikov, Röntsch, 2019]

DY+j:

$$w^{4i,5j}\Big|_{i=j} = \frac{1}{2}\frac{\rho_{4k}\rho_{4n}\rho_{5k}\rho_{5n}}{d_4 d_5}\left[\left(\frac{1}{d_{4k}} + \frac{1}{d_{4n}}\right)\left(\frac{1}{d_{5k}} + \frac{1}{d_{5n}}\right) + \left(\frac{1}{d_{4i}} + \frac{1}{d_{4k}}\right)\left(\frac{1}{d_{5k}} + \frac{1}{d_{5n}}\right)\frac{\rho_{4i}}{d_{45ni}} + \left(\frac{1}{d_{4i}} + \frac{1}{d_{4n}}\right)\left(\frac{1}{d_{5k}} + \frac{1}{d_{5n}}\right)\frac{\rho_{4i}}{d_{45ki}} + \left(\frac{1}{d_{4k}} + \frac{1}{d_{4n}}\right)\left(\frac{1}{d_{5i}} + \frac{1}{d_{5k}}\right)\frac{\rho_{5i}}{d_{45in}} + \left(\frac{1}{d_{4k}} + \frac{1}{d_{4n}}\right)\left(\frac{1}{d_{5i}} + \frac{1}{d_{5n}}\right)\frac{\rho_{5i}}{d_{45ik}}\right]$$

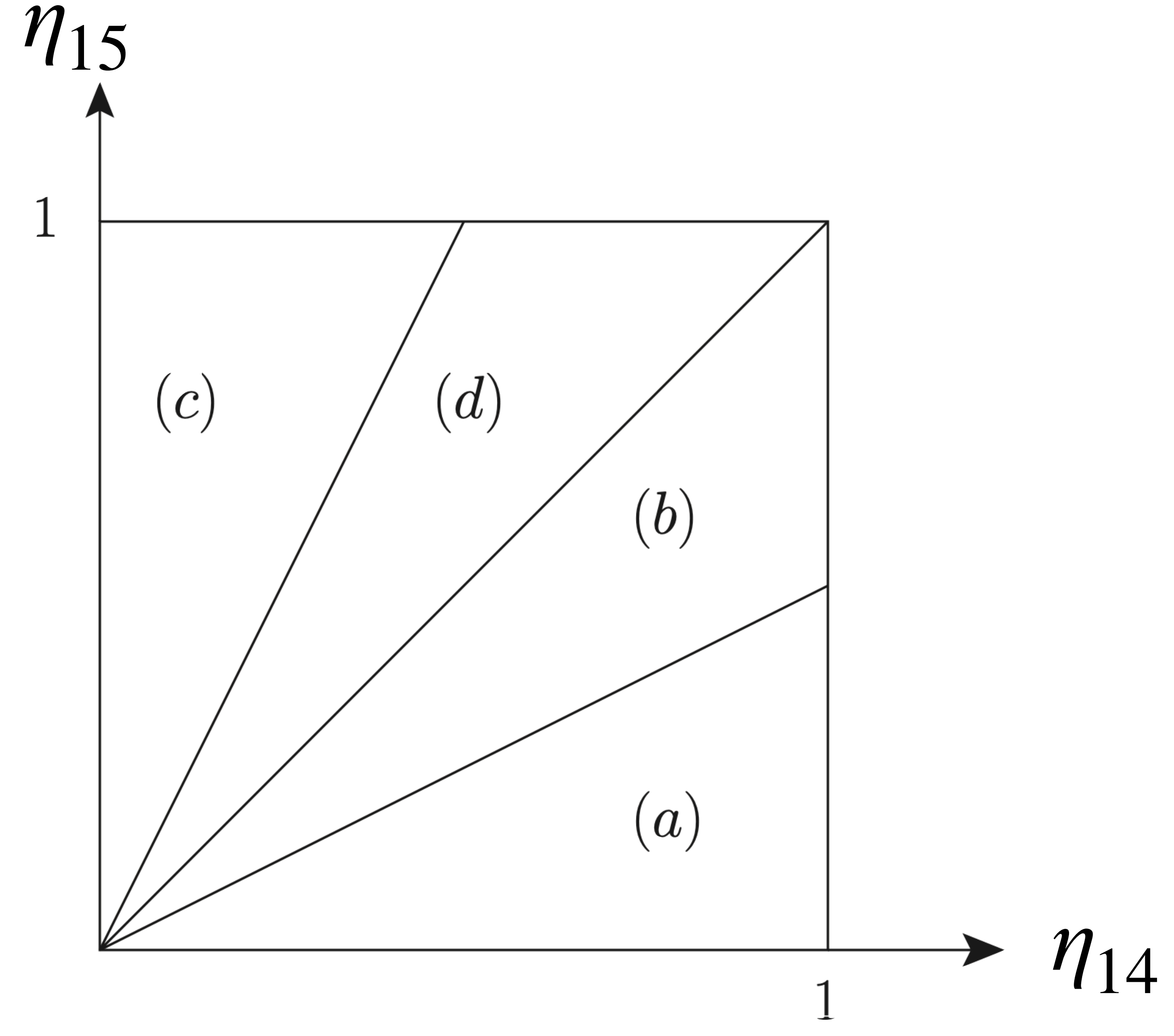
$$w^{4i,5j}\Big|_{i\neq j} = \frac{1}{4}\frac{\rho_{4k}\rho_{4n}\rho_{5l}\rho_{5m}}{d_4 d_5}\left(\frac{1}{d_{4k}} + \frac{1}{d_{4n}}\right)\left(\frac{1}{d_{5l}} + \frac{1}{d_{5m}}\right)\frac{\rho_{45}}{d_{45ij}}$$

$$d_{i\in[4,5]} = \sum_{j=1}^{3}\rho_{ij}, \quad d_{i\in[4,5]k} = \sum_{j=1,j\neq k}^{3}\rho_{ij}, \quad d_{45ij} = \rho_{45} + \rho_{4i} + \rho_{5j}$$

[Boughezal, Caola, Melnikov, Petriello, and Schulze, 2015]

Sectoring

$$1 = \theta\left(\eta_{51} < \frac{\eta_{41}}{2}\right) + \theta\left(\frac{\eta_{41}}{2} < \eta_{51} < \eta_{41}\right) + \theta\left(\eta_{41} < \frac{\eta_{51}}{2}\right) + \theta\left(\frac{\eta_{51}}{2} < \eta_{41} < \eta_{51}\right)$$
$$\equiv \theta^{(a)} + \theta^{(b)} + \theta^{(c)} + \theta^{(d)},$$



- The **triple-collinear** limits will be separated:

$$1 = \sum_k w^k = \sum_{(ij)\in dc} w^{4i,5j} + \sum_{i\in tc} w^{4i,5i}\left[\theta^{(a)} + \theta^{(b)} + \theta^{(c)} + \theta^{(d)}\right],$$

Feynman parameter method

Integral expression

$$\int \frac{\left[d\Omega_4^{(d-1)}\right]}{\left[\Omega^{(d-2)}\right]} \frac{1}{\rho_{14}} \left[\frac{\rho_{12}}{\rho_{24}} \tilde{w}_{5||1}^{41,51} - 1\right] \left[\left(\frac{\rho_{14}}{4}\right)^{-\epsilon} - 1\right] =$$

$$= -\frac{\epsilon}{2\pi} \int d\Omega_4^{(3)} \frac{1}{\rho_{14}} \left[\frac{\rho_{12}}{\rho_{24}} \tilde{w}_{5||1}^{41,51} - 1\right] \log\left(\frac{\rho_{14}}{4}\right) + \mathcal{O}(\epsilon^2)$$

Feynman parameter method

Rationalization of the denominator

Completing the square $\longrightarrow$ **Shift**

- Denominators with $\cos\theta,\ \cos^2\theta$ written as $(\cos\theta + shift)^2 + \cdots$

$\cos\theta \to \tilde{x} - shift$ $\quad \sqrt{\alpha + \beta\tilde{x}^2}$

Rationalization

- Change of variables:

$$\tilde{x} \longrightarrow k\frac{2t}{t^2-1}, \quad k = \sqrt{\frac{\alpha}{\beta}} \quad \longrightarrow \quad \alpha + \beta\tilde{x}^2 \longrightarrow \left(\sqrt{\alpha}\cdot\frac{t^2+1}{t^2-1}\right)^2$$

Notation for Mellin-Barnes method

- **# denominators**

$$\rho_{ij} = 1 - \cos\theta_{ij} \propto p_i \cdot p_j$$

- **# masses:**

$p_1^2 = p_2^2 = p_3^2 = 0$ *massless*

e.g.

$$\rho_{14} + \rho_{24} + \rho_{34} \propto \underbrace{(p_1 + p_2 + p_3)}_{k} \cdot p_4$$

⟶ Sum no longer *massless*:

$k^2 \neq 0$ *massive*

Notation:

$\dfrac{1}{\rho_{i4}^a}$	Massless
$\dfrac{1}{(\rho_{i4} + \rho_{i4} + \ldots)^a}$	Massive