October 16th 2024

GdR Ondes Gravitationnelles

Rencontre du groupe de travail - “Méthodes d’analyse des données”

Pulsar Timing Array Data Analysis

Université Paris Cité

EPTA

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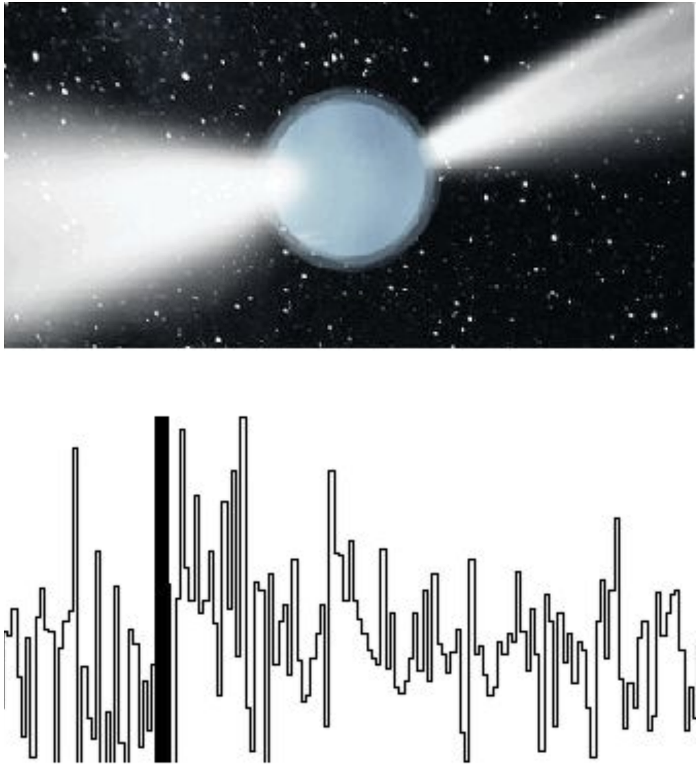
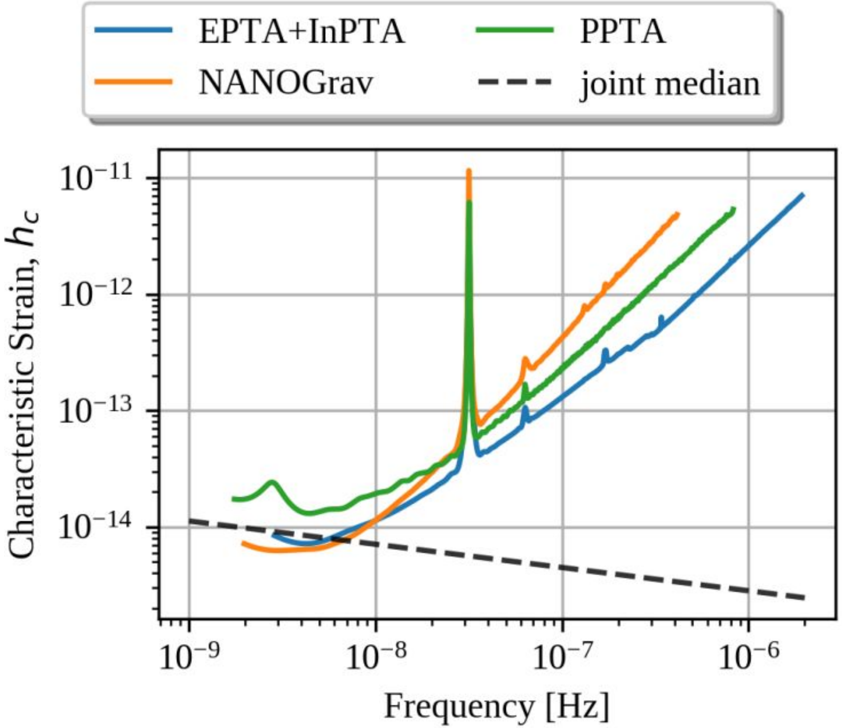
Principle of PTA



credits: Mikel Falxa



[arXiv:2309.00693]

Main expected sources of GW for PTA

SGWB

Population of SMBHBs

Cosmological origin

(Phase transition, Cosmic strings, Inflation…)

Continuous wave

Individual SMBHB

Inspiraling phase
Circular or Eccentric

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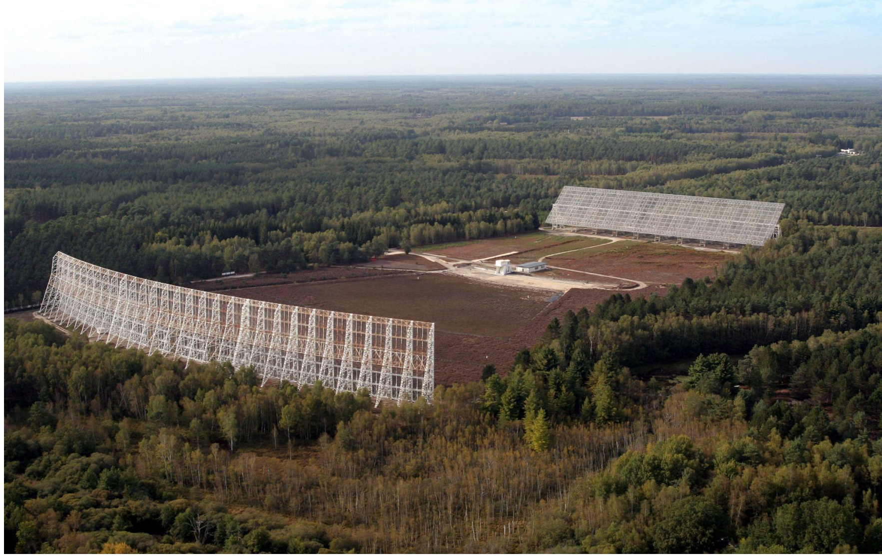
The PTA data

→ Radio telescopes measure time of arrival (ToA) of radio pulses from most stable MSP (~twice/month)

→ **Unevenly sampled** data

→ Each ToA is associated with a radio frequency of observation (mostly in L-band: $1 < \boldsymbol{\nu} < 2$ GHz)

→ Observations span over **years**



Credits: CNRS

The PTA data

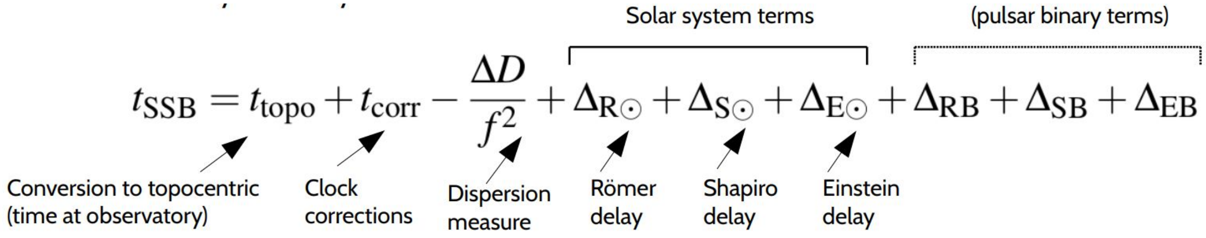
The data used in PTA are **residuals**

$$\delta t^i_\alpha = t^i_{\text{obs}} - t^i_{\text{TM}}$$

ToA index

Pulsar index

Where the Timing Model (TM) is a complex deterministic function



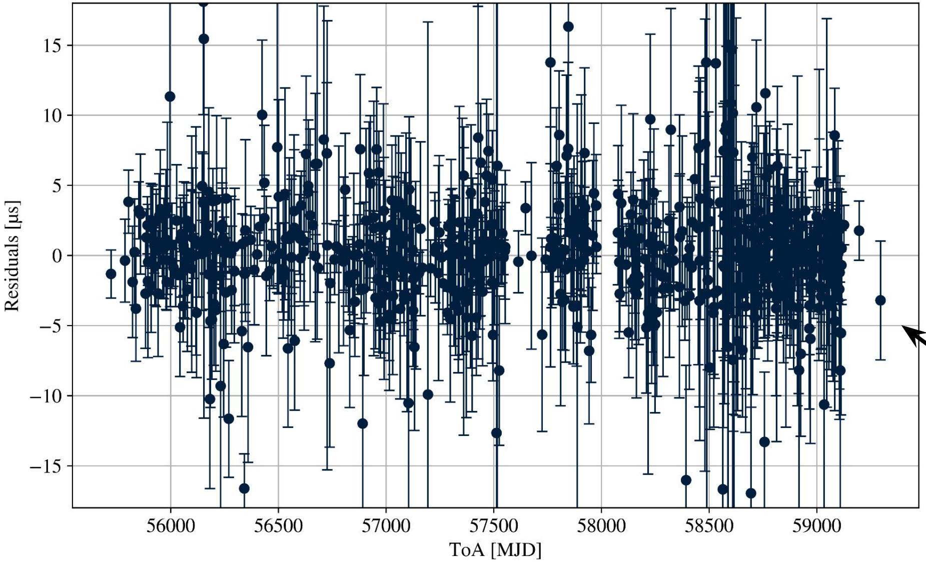
$$t_{\text{SSB}} = t_{\text{topo}} + t_{\text{corr}} - \frac{\Delta D}{f^2} + \Delta_{\text{R}\odot} + \Delta_{\text{S}\odot} + \Delta_{\text{E}\odot} + \Delta_{\text{RB}} + \Delta_{\text{SB}} + \Delta_{\text{EB}}$$

This function is parameterised by tens of parameters determined by a least-squares fit

Spin period and derivative(s), sky position, mass of the MSP, dispersion measure…

[arXiv:astro-ph/0607664]

The PTA data



× 25 MSPs for the EPTA Data Release 2

Each ToA is associated
with an error (radiometer)

~ 10 years of data

Single pulsar noise analysis

What are those residuals made of? And how to model it?

ToA index

$$\delta t_{\alpha}^{i} = t_{\text{obs}}^{i} - t_{\text{TM}}^{i}$$

Pulsar index

Bayesian framework

Assumptions: noise is Gaussian and stationary

$$\mathcal{L} = \frac{1}{\sqrt{|2\pi C|}} e^{-\frac{1}{2}(\delta t - h)^\top C^{-1}(\delta t - h)}$$

Stochastic signals

Deterministic signals

(DM variation event, Individual SMBHB,
GW burst, ...)

Both deterministic and stochastic signals are parameterised by several parameters, whose posterior distributions are computed using MCMC

White Noise

Origin

- → Radiometer noise
- → Pulse jitter

ToA uncertainty

$$N_{ij} = \langle n_{i,\mu}^{\mathrm{WN}} n_{j,\nu}^{\mathrm{WN}} \rangle = \left(\mathrm{F}_{\mu}^{2}\sigma_{i}^{2} + \mathrm{Q}_{\mu}^{2}\right)\delta_{ij}\delta_{\mu\nu}$$

ToA index

Instrument system index

White noise is parameterised by two parameters per instrument system (receiver + backend)

[arXiv:1407.1838]

Modeling of stochastic processes (and more)

Marginalization over Gaussian Process realisations

The idea is to decompose the noise residuals over a basis of functions (usually Fourier basis)

$$\delta t = \mathbf{F} \cdot \mathbf{c}$$

where $\mathbf{c}$ follow a multivariate Gaussian distribution

For the stochastic processes we use a Fourier basis $\delta t_i^{(\alpha)} \propto \sum_{n=1}^{N_f^{(\alpha)}} \left[a_n \cos\left(2\pi f_n t_i\right) + b_n \sin\left(2\pi f_n t_i\right)\right]$

Such that

$$f_n = n/T$$

$$B_{kl}^{(\alpha)} = \langle c_k^{(\alpha)} c_l^{(\alpha)} \rangle = \frac{1}{T} S^{(\alpha)}(f_k; \theta^{(\alpha)}) \delta_{kl}$$

→ The idea is that we do not sample over the $\mathbf{c}$ coefficients but over the hyper parameters $\theta^{(\alpha)}$ that parameterise the statistical properties of the noise

Marginalization over Gaussian Process realisations

$$
\begin{aligned}
p(\delta t \mid \theta) &= \int p\left(\delta t \mid c\right) p\left(c \mid \theta\right) dc \\
&= \int \frac{1}{\sqrt{|2\pi N|}} e^{-\frac{1}{2}(\delta t - Fc)^\top N^{-1}(\delta t - Fc)} \times \frac{e^{-\frac{1}{2}c^\top B^{-1}(\theta) c}}{\sqrt{|2\pi B|}} dc \\
&= \frac{e^{-\frac{1}{2}\delta t^\top N^{-1}\delta t}}{\sqrt{|2\pi N||2\pi B|}} \int e^{-\frac{1}{2}c^\top \left(F^\top N^{-1} F + B^{-1}\right) c + \delta t^\top N^{-1} F c} dc \\
&= \frac{1}{\sqrt{|2\pi N||B||\Sigma|}} e^{-\frac{1}{2}\delta t^\top \left(N^{-1} - N^{-1} F \Sigma^{-1} F^\top N^{-1}\right)\delta t}
\end{aligned}
$$

where $\Sigma = B^{-1} + F^\top N^{-1} F$

Advantages of the method

→ Huge reduction of computational cost as the computational bottleneck matrix inversion is now in frequency domain ($N_f \ll N_{\mathrm{ToA}}$)

→ Can also be useful for deterministic signals (see next slide)

→ Addition of several processes is easy (concatenation of the F and c)

$$\delta t^i_\alpha = t^i_{\mathrm{obs}} - t^i_{\mathrm{TM}}$$

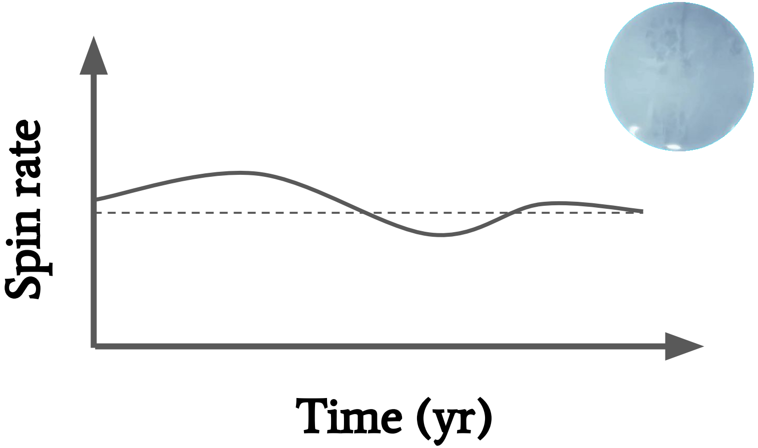
Timing model errors

Before conducting a noise analysis, an initial **timing model solution** β_0 is obtained using a least-squares fitting method. Errors on this timing solution can then be modeled as

$$\delta t^{\mathrm{TM}} = \nabla_{\vec{\beta}} f_{\mathrm{TM}}(\vec{\beta}_0) \cdot (\vec{\beta} - \vec{\beta}_0)$$
$$= M \cdot \epsilon$$

→ We can marginalize over the TM errors for each pulsar using infinitely wide Gaussian prior

Achromatic red noise (spin noise)



Origin

→ Long-term variability of pulsar spin rate

Characteristics

→ Power law expected [Shannon & Cordes, 2010]

Modeling

→ Fourier basis up to $N_f^{(\mathrm{RN})}$ components (free parameter) with a power law PSD

Parameters (per pulsar)

$$A_{\mathrm{yr}}\,,\,\gamma$$

Dispersion measure variations

Origin

→ Stochastic variations of the DM along the pulsar line of sight: delay is **chromatic**

Characteristics

→ If originate from turbulent Kolmogorov spectrum: Power law PSD [Keith et al, 2012]

Modeling

→ Fourier basis up to $N_f^{(\mathrm{DM})}$ components scaled with observing radio frequency $\left(\frac{\nu_{\mathrm{ref}}}{\nu_i}\right)^2$

→ Power law PSD

$$\delta t_i^{(\alpha)} \propto \sum_{n=1}^{N_f^{(\alpha)}} \left[a_n \cos\left(2\pi f_n t_i\right) + b_n \sin\left(2\pi f_n t_i\right)\right]$$

Parameters (per pulsar)

$$A_{\mathrm{yr}}, \gamma$$

Building custom noise models for individual pulsars

→ Not every pulsar will exhibit each noise component

→ Bayesian model comparison is used to select the most preferred model

WN only, WN + RN, WN + DMv, WN + RN + DMv, ...

→ Once the model is picked we can also select the preferred number of Fourier components to model each noise component $N_f^{(\alpha)}$

→ The resulting single pulsar noise models are then used in the GW search

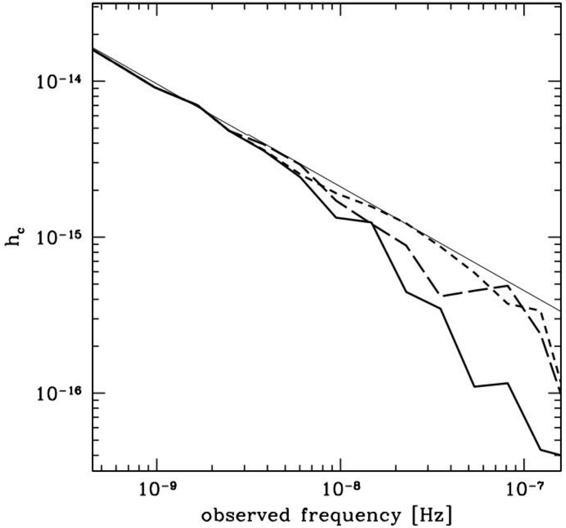
The search for GW background(s)

The isotropic and stationary GWB model

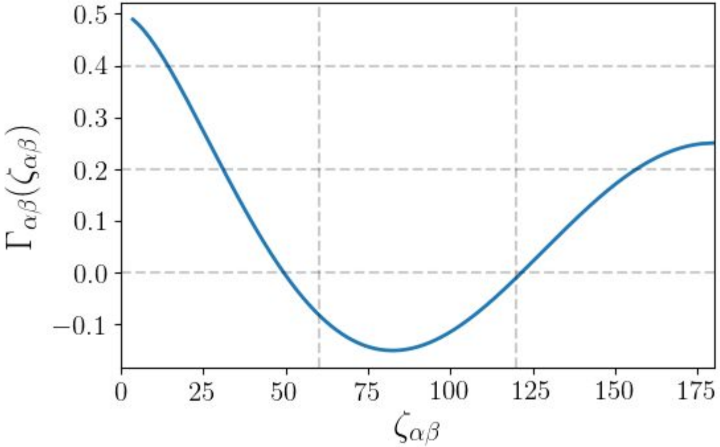
For a Gaussian, isotropic and stationary background $\langle \delta t_\mu(f) \delta t_\nu(f') \rangle = \frac{1}{2} \Gamma_{\mu\nu} S_{\mathrm{GWB}}(f) \delta_{ff'}$

$\Gamma_{\mu\nu}$ is the overlap reduction function, quantifying the expected average correlation

S_{GWB} is the Power Spectral Density of the GWB



[Sesana, 2008]



Hellings-Downs (HD) correlation pattern

[Hellings & Downs, 1983]

$$f_k = k/T_{\rm PTA}$$

GWB modeling

→ As for individual pulsar noise, we use Gaussian process to model the GWB

$$\delta t^{\mu}_{\rm GWB} = F^{\mu} \cdot c^{\mu}_{\rm GWB}$$

→ But here, coefficients are correlated amongst pulsars

$$\langle c^{\mu}_{{\rm GWB},k} c^{\nu}_{{\rm GWB},l} \rangle = \Gamma^{\mu\nu} \frac{1}{T_{\rm PTA}} S_{\rm GWB}(f_k) \delta_{kl}$$

→ Computationally very heavy as cannot be inverted block by block as in the case of uncorrelated red noise (some fast methods have been developed: resampling, …)

→ The PSD model can either be a power law or a free spectrum

GWB search - Summary of the analysis

Deterministic signals

$$\ln \mathcal{L} = -\frac{1}{2} \left(\delta t - h\right)_{\mu}^{\top} C_{\mu\nu}^{-1} \left(\delta t - h\right)_{\nu} + \dots$$

Pulsar index

The covariance matrix (mainly) contains

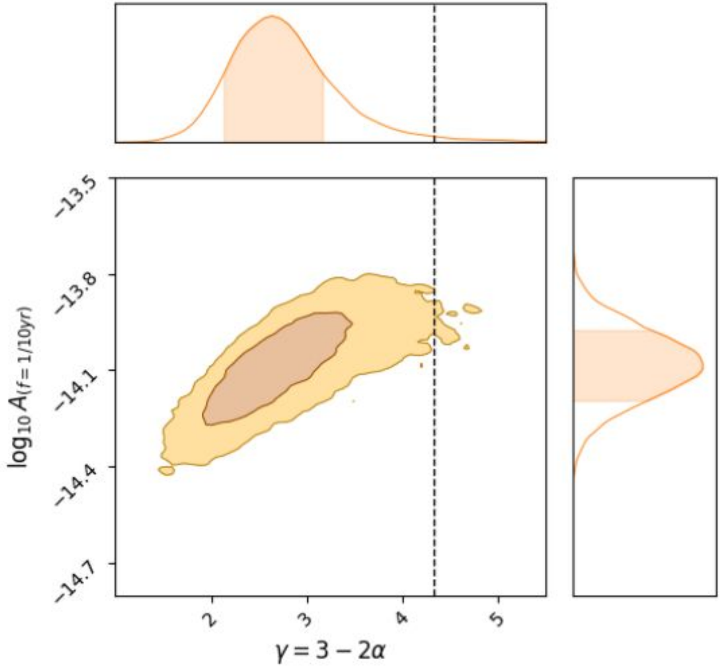
- TM error marginalization of each pulsar
- Achromatic red noise (2 parameters for each pulsar that show aRN)
- DM variation noise (2 parameters for each pulsar that show DMv)
- GWB noise (2 parameters) → correlations given by the HD curve

→ In total: **hundreds** of parameters to sample

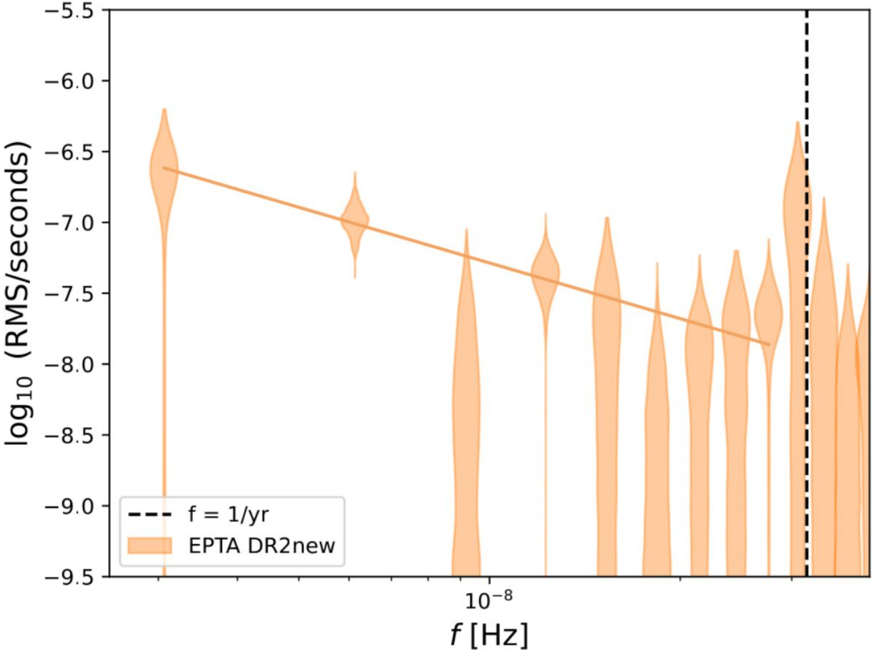
[EPTA Paper IV - 2023]

Inferring the properties of the GWB

Power law PSD model



Free Spectrum PSD model



[EPTA Paper III - 2023]

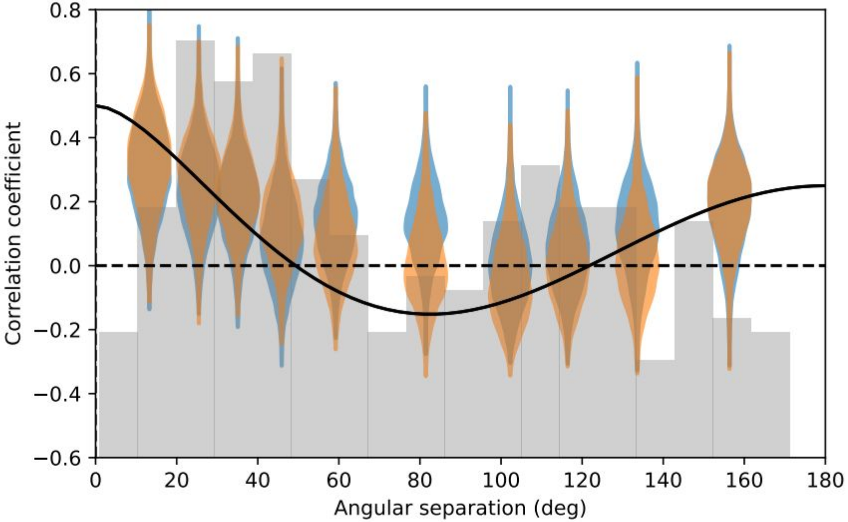
Evaluation of the significance of the correlations

1. Model comparison via Bayes Factor (BF) evaluation of

Common Uncorrelated RN vs GWB

$$\Gamma_{\mu\nu} = \delta_{\mu\nu} \qquad \Gamma_{\mu\nu} = \Gamma_{\mu\nu}^{\mathrm{HD}}$$

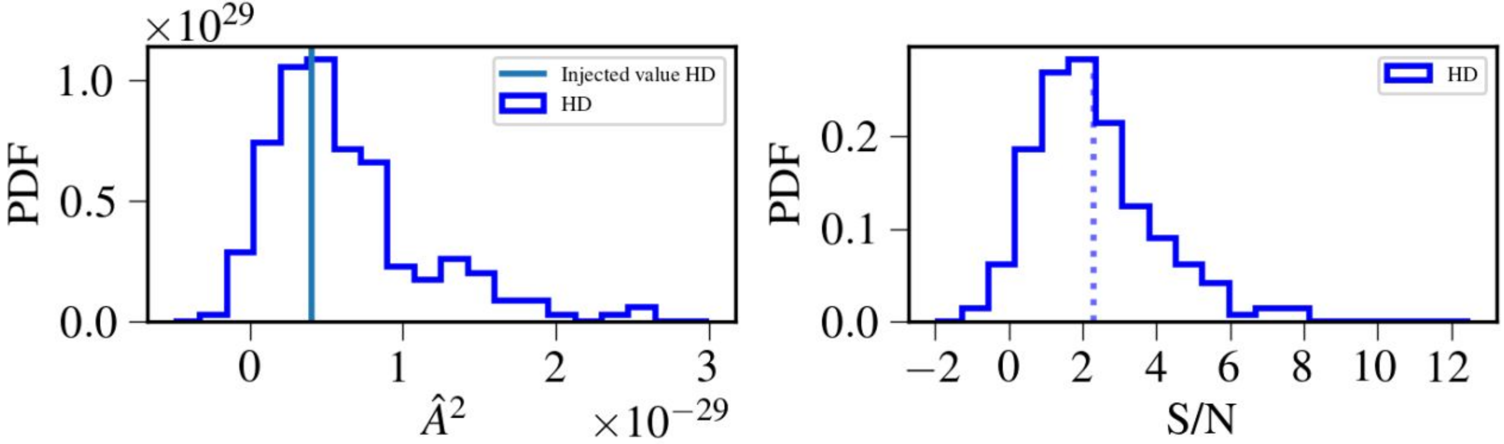
2. Bayesian inference of the ORF (assuming it depends only on the pulsar pair angular separation)



Evaluation of the significance of the correlations

3. Frequentist approach (F-statistic)

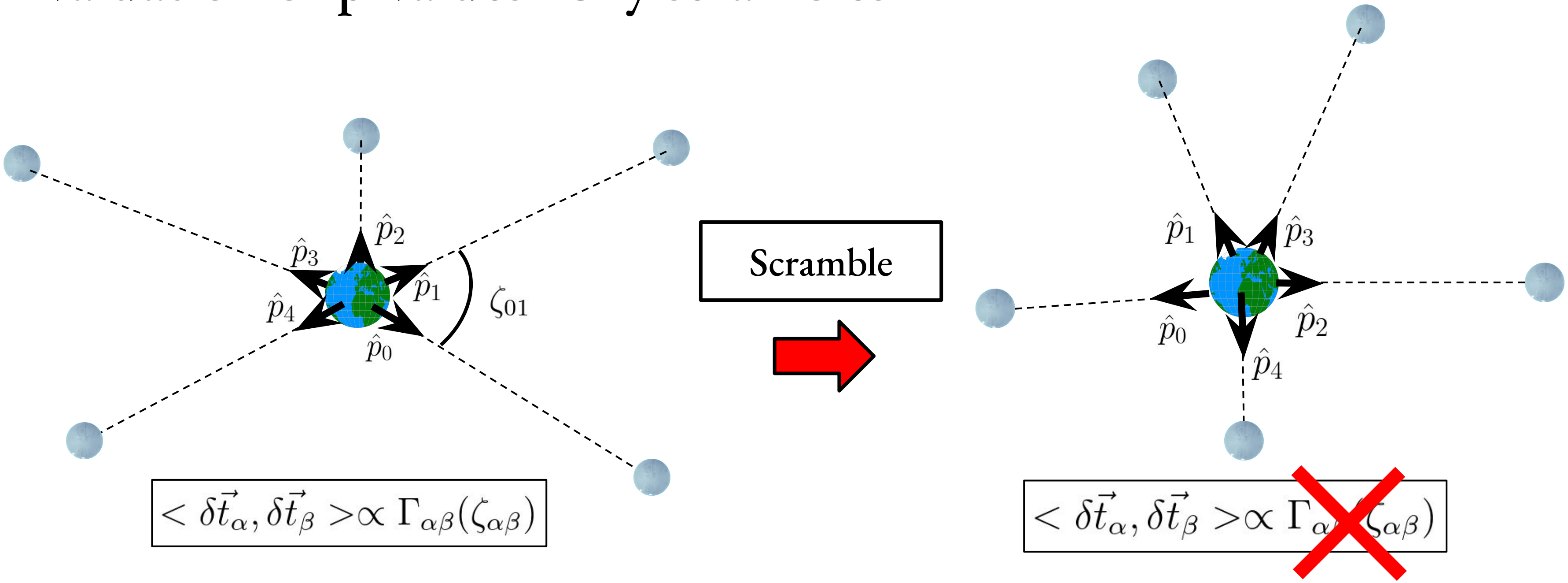
→ Maximization of likelihood ratio (GWB/CURN) $\Leftrightarrow \chi^2 = \sum_{ab, a<b} \left(\frac{\rho_{ab} - A^2 \Gamma_{ab}}{\sigma_{ab}} \right)^2$



[Sardesai et al, 2023]

[Taylor et al, 2017]

Evaluation of p-values - Sky scrambles



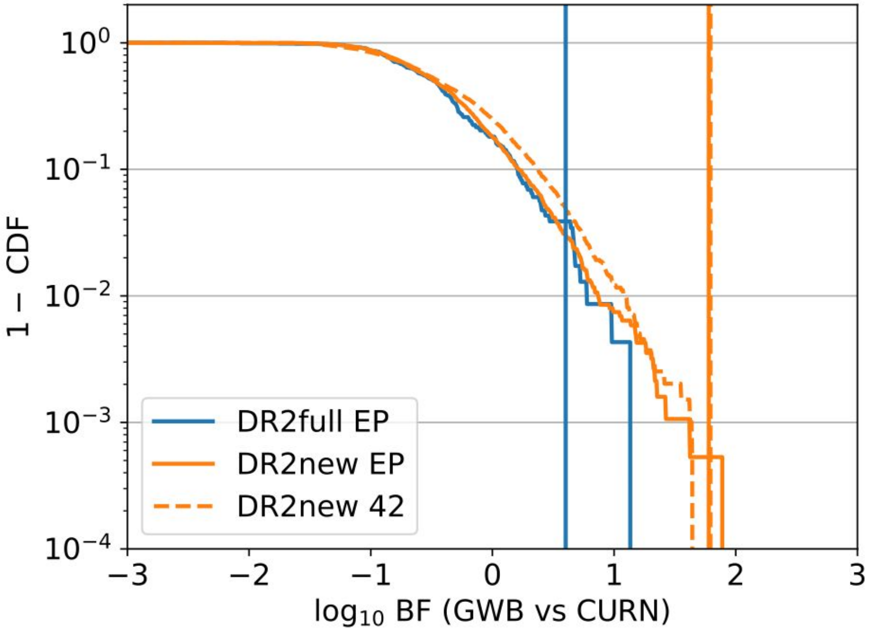
$$< \delta\vec{t}_\alpha, \delta\vec{t}_\beta > \propto \Gamma_{\alpha\beta}(\zeta_{\alpha\beta})$$

$$< \delta\vec{t}_\alpha, \delta\vec{t}_\beta > \propto \cancel{\Gamma_{\alpha\beta}(\zeta_{\alpha\beta})}$$

1. Conservation of noise properties
2. Destruction of the pulsar cross-correlations
   → construction of the statistics null distribution

[EPTA Paper III - 2023]

Evaluation of p-values - Sky scrambles



→ We construct the null distribution for our statistic

$$BF^{\mathrm{HD}}_{\mathrm{CURN}} \text{ (in this case)}$$

using thousands of sky scrambles realisations

→ We estimate a p-value looking at the fraction of scrambles that give a BF higher than our measurement

Recent developments

- Non-stationarity modeling [Falxa - arXiv:2405.03295]

- Use of hierarchical models [van Haasteren - arXiv:2406.05081]

- Including Low Frequency data from other telescopes (LOFAR, CHIME…)
  - Help disentangling achromatic and chromatic noise

Packages

Computation of the likelihood

- enterprise
  - [github.com/nanograv/enterprise/]
- enterprise_extensions
  - [github.com/nanograv/enterprise_extensions/]

Sampler

- PTMCMC
  - [github.com/nanograv/PTMCMCSampler]

Tutorials of NANOGrav

- [github.com/nanograv/15yr_stochastic_analysis]

Useful references

The Nanohertz Gravitational Wave Astronomer

S. Taylor - arXiv.2105.13270

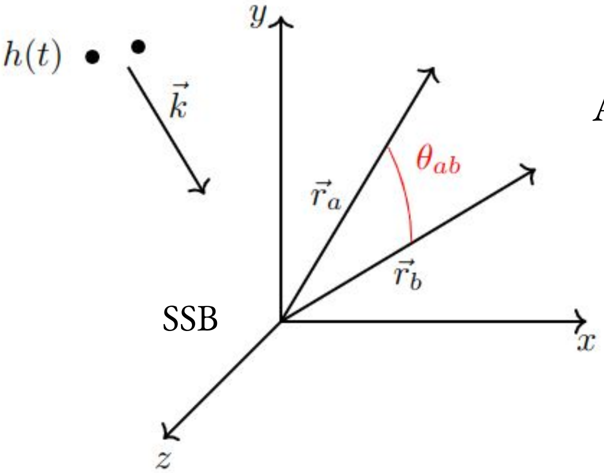
The NANOGrav 15-year Gravitational-Wave Background Analysis Pipeline

Johnson et al. - arXiv:2306.16223

New advances in the Gaussian-process approach to pulsar-timing data analysis

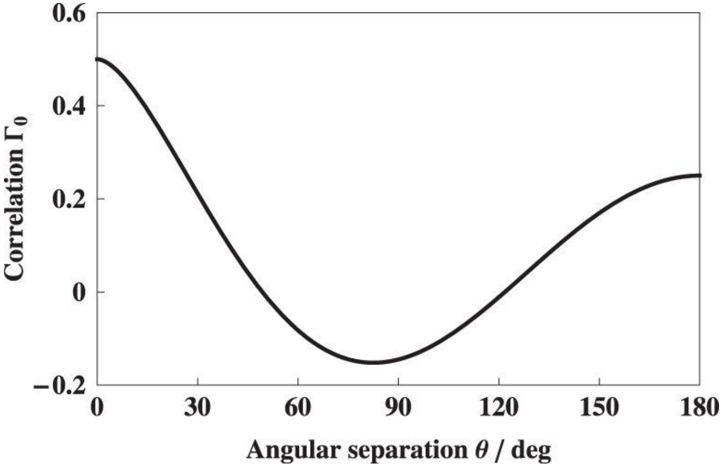
M. Vallisneri, R. van Haasteren - arXiv:1407.1838

Backup slides



Averaging over source localisation

Averaging over pulsar pairs



Allen, Romano (2023)