

Inferring astrophysics and cosmology with individual compact binary coalescences and their gravitational-wave stochastic background

Sarah Ferraiuolo, GdR Ondes Gravitationnelles, 16/10/2024



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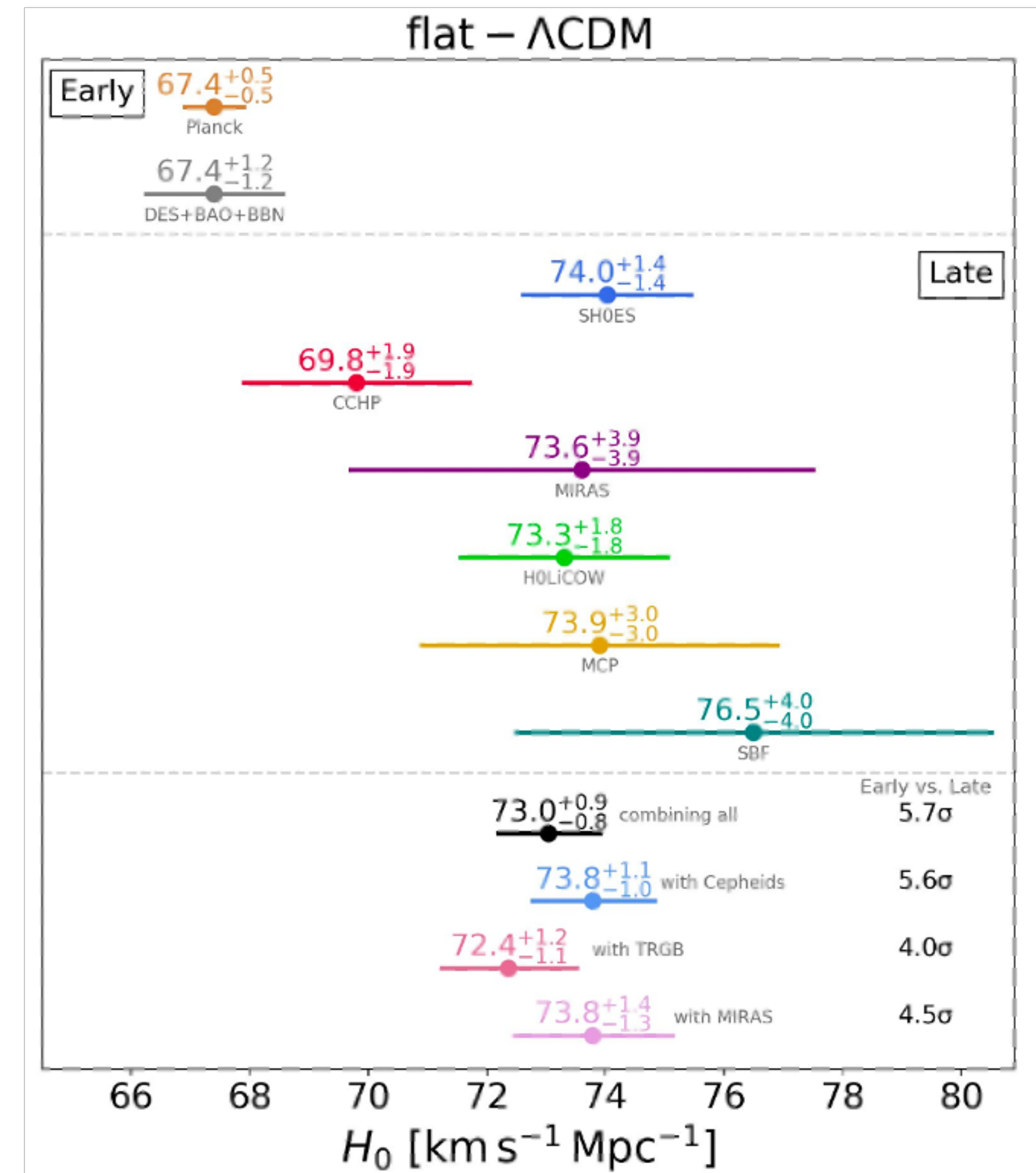


Outline

- Introduction: gravitational waves as standard sirens and stochastic gravitational wave background
- Analysis method
- LVK at A+ design sensitivity simulated data
- Preliminary Results

The Hubble tension

- There is a tension between direct and indirect measurements of the Hubble constant
- Although in-depth studies for hidden systematics the tension has not been yet alleviated
- We are required to directly measure the Universe expansion in all the observable Universe



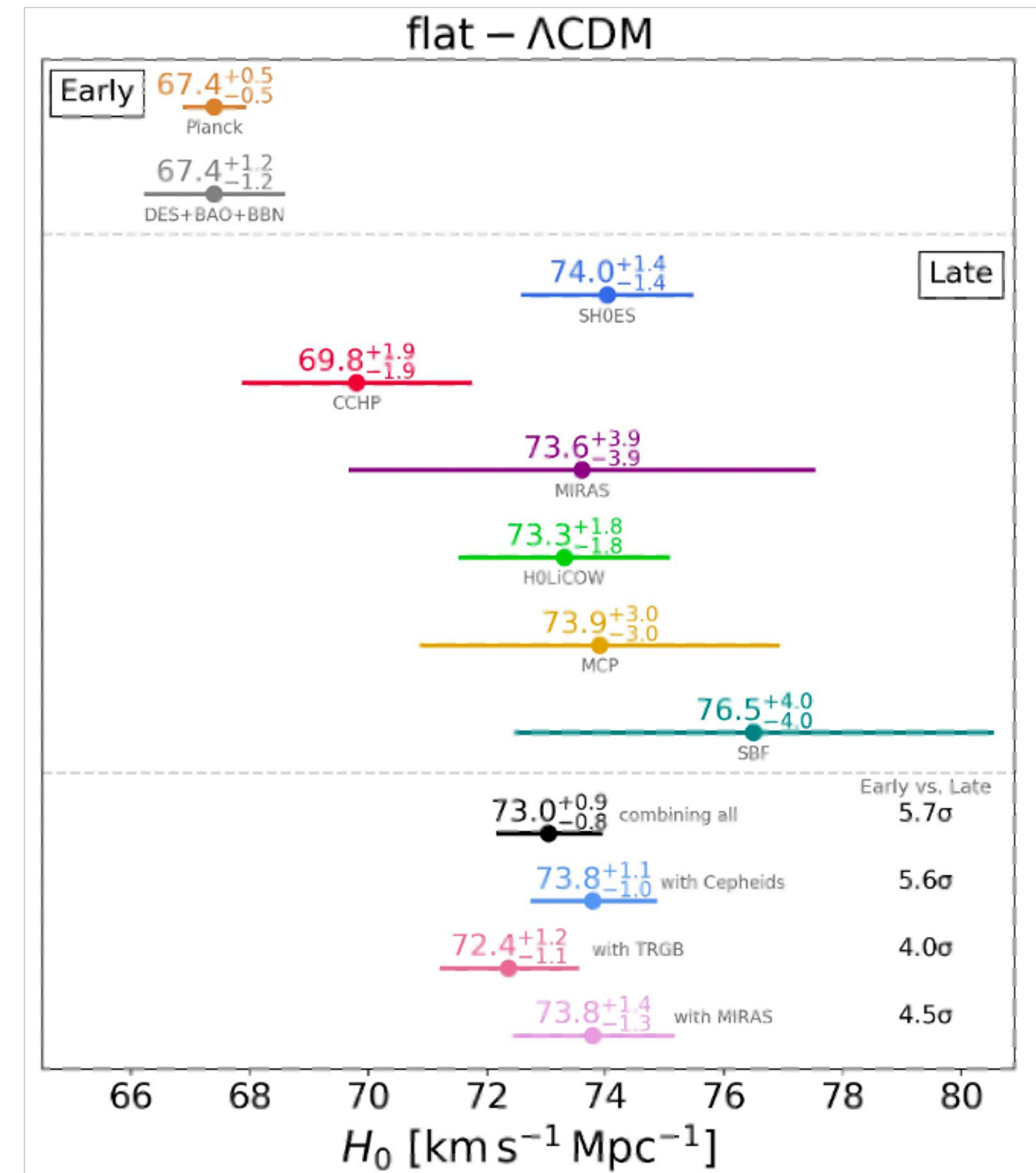
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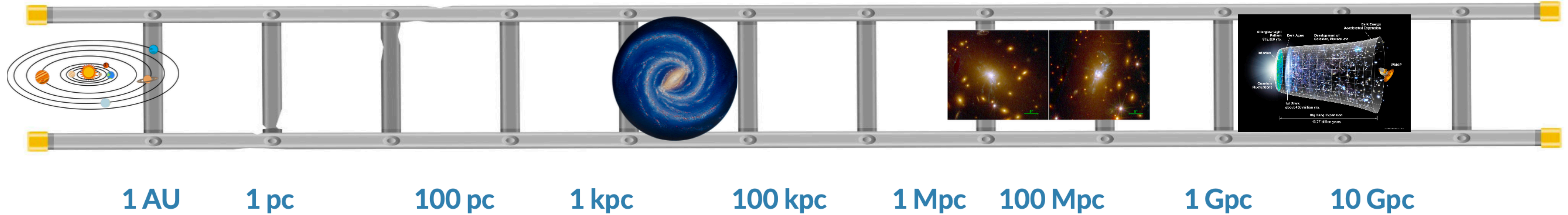
Gravitational Waves!



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The Hubble tension

Why use gravitational waves for cosmology



Distances with Electromagnetic observations



Distances with GW observations



Gravitational Waves

They exist in multiple forms:

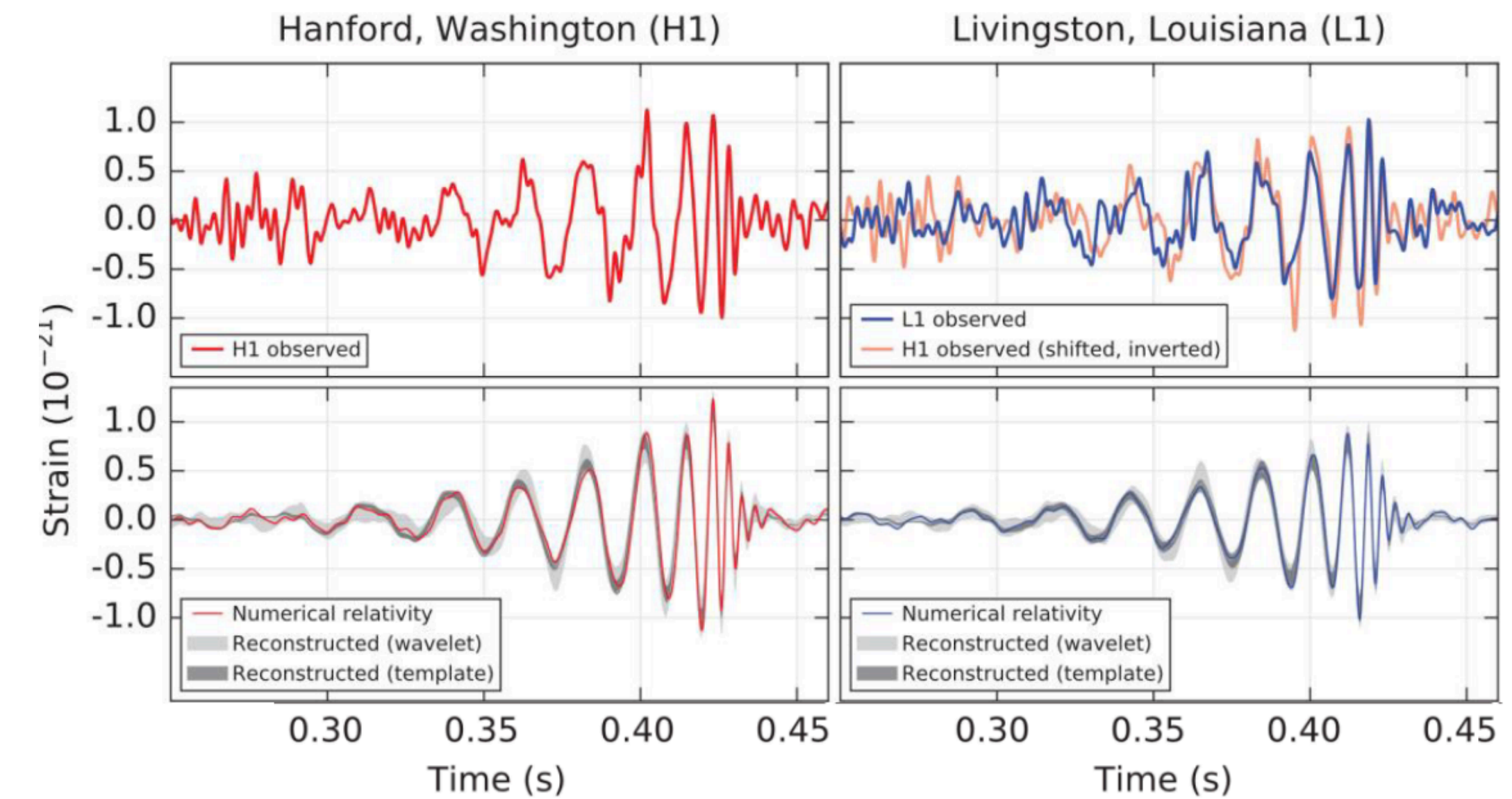
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- Bursts
- Continuous waves
- Stochastic gravitational-wave background (SGWB)

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Phys. Rev. Lett. 116, 061102

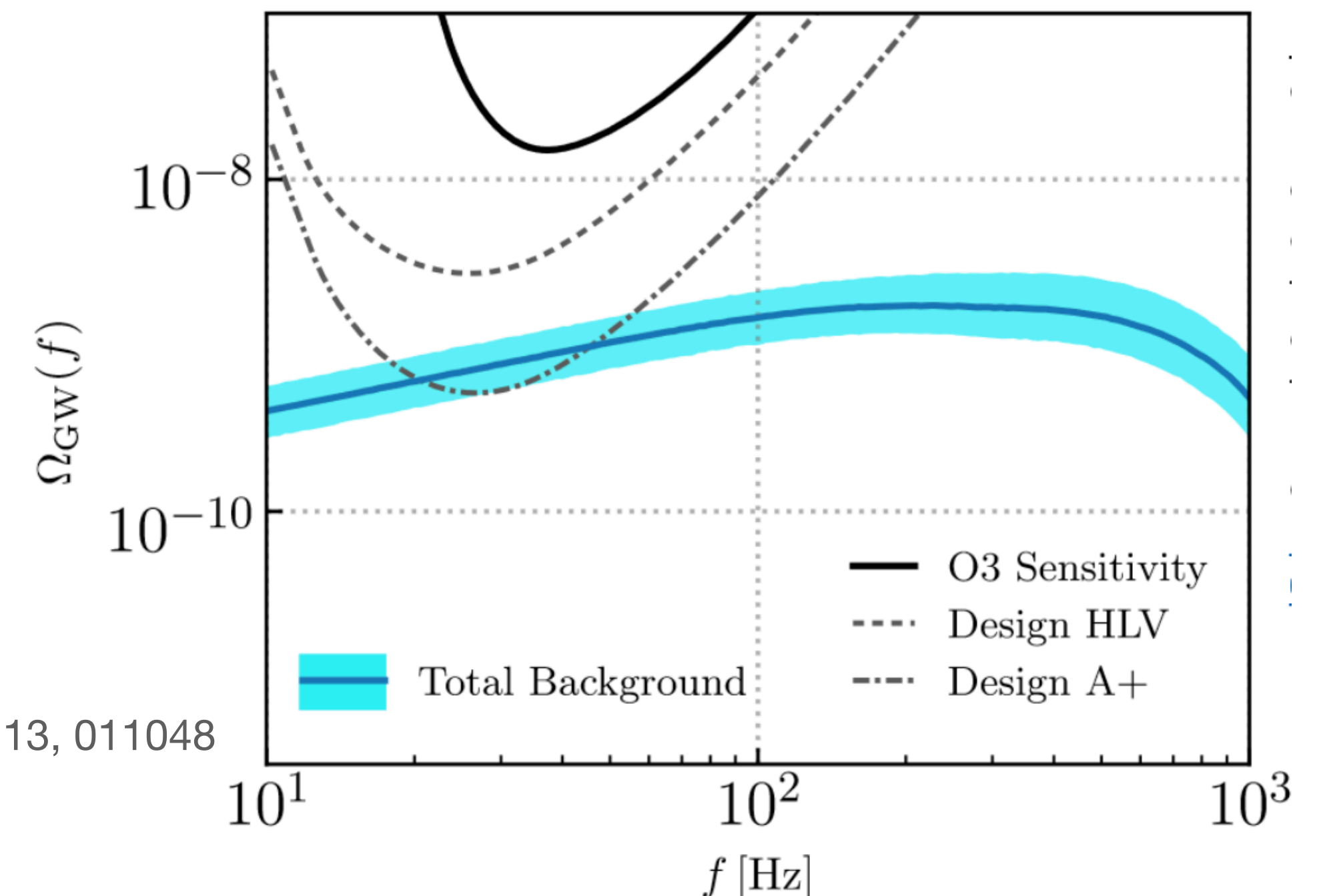
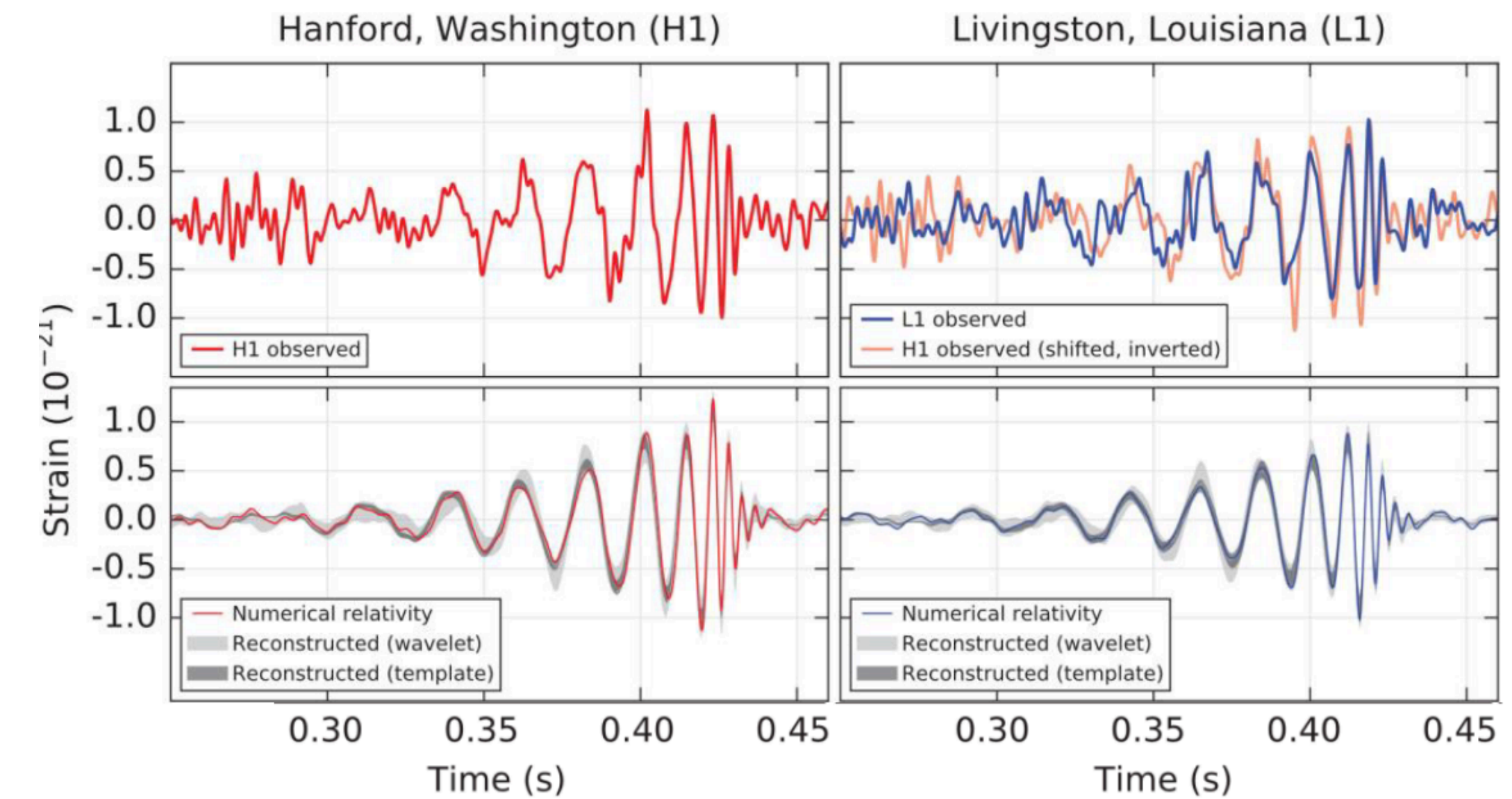


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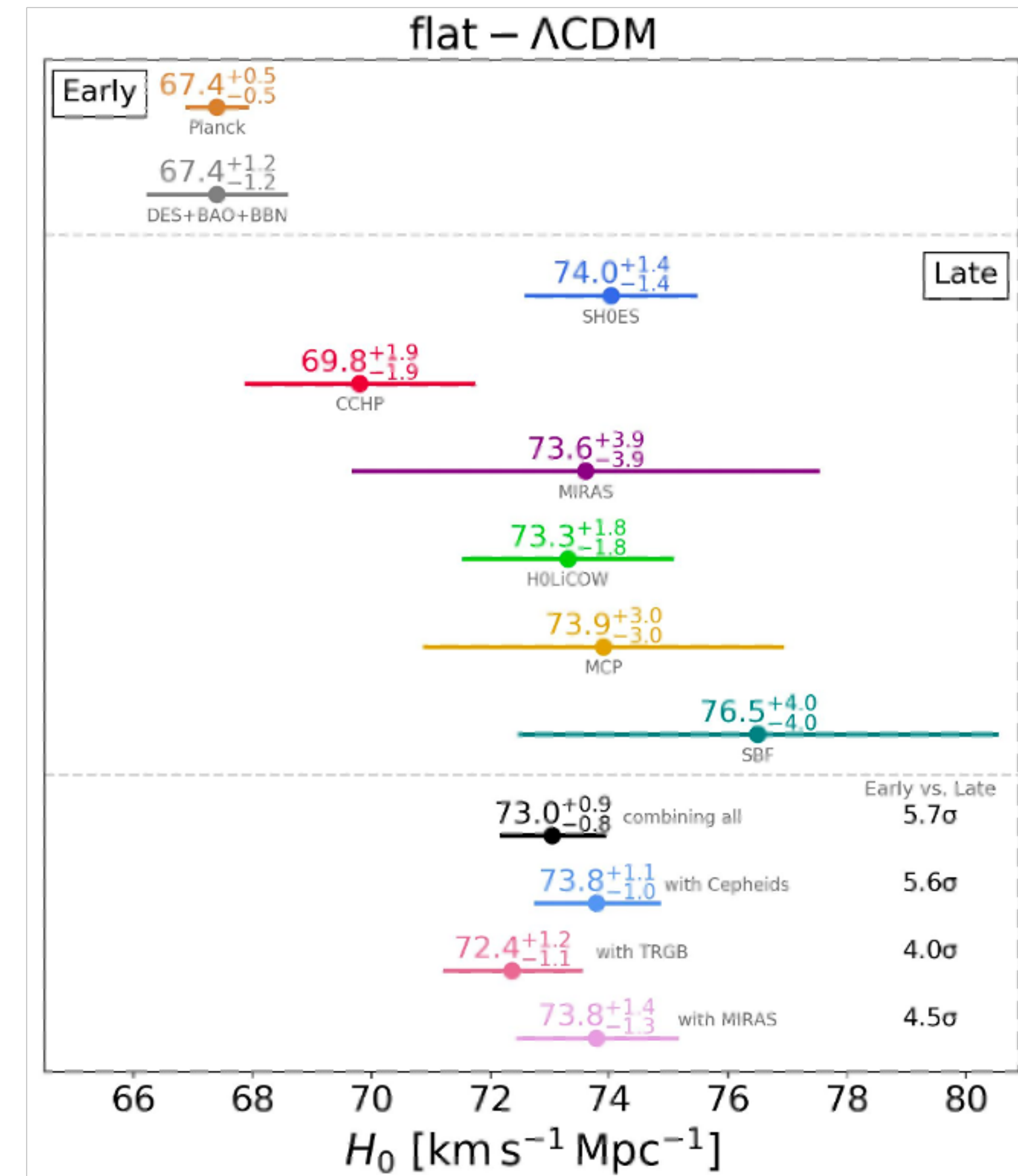
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Phys. Rev. X 13, 011048

Standard Sirens with CBCs

- $cz \approx H_0 d_L$



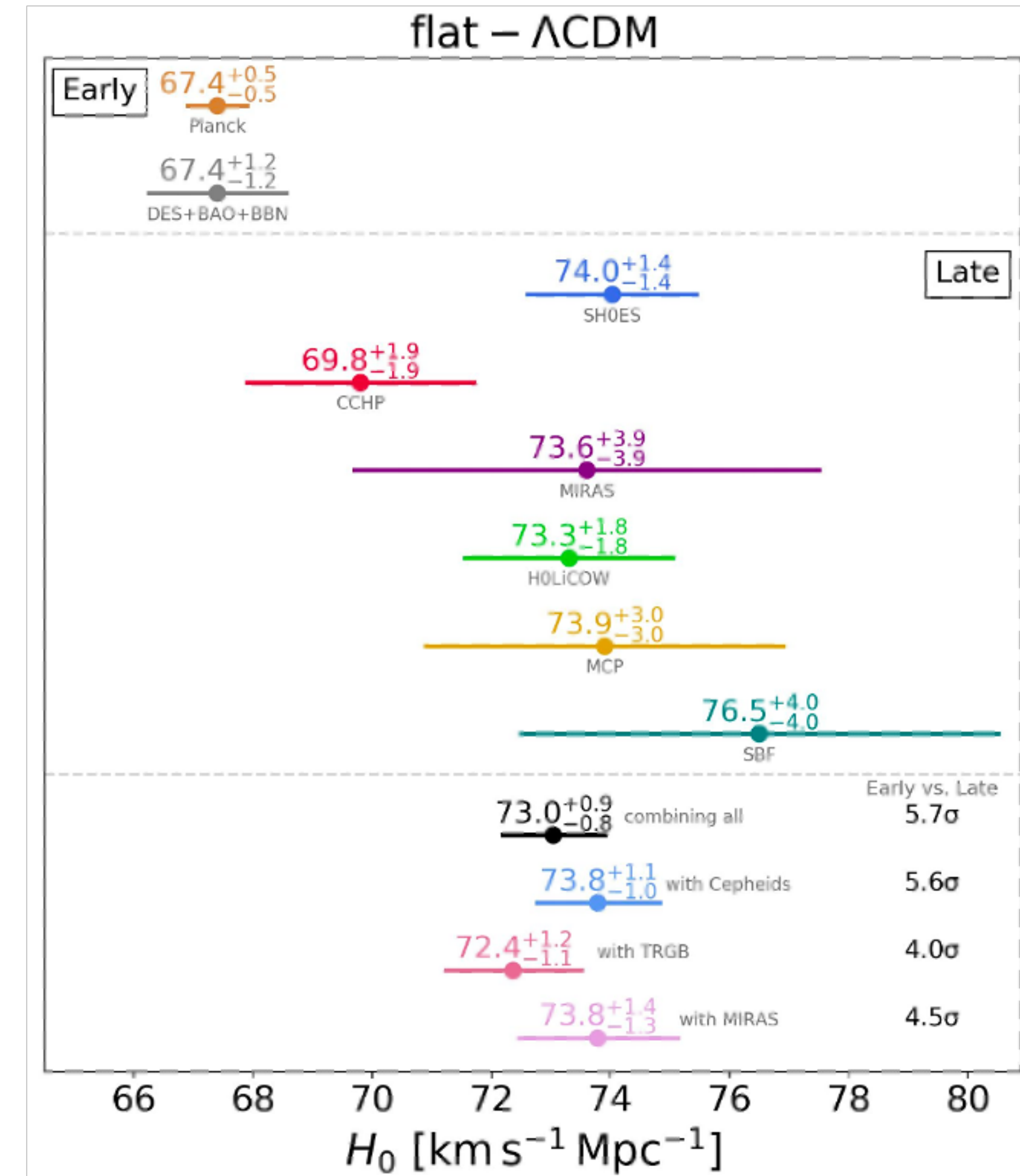
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Luminosity distance: directly measurable from the amplitude of the gravitational wave signal

- $h(t) = A(M_z, d_L, t) e^{i\Phi(M_z, t)}$



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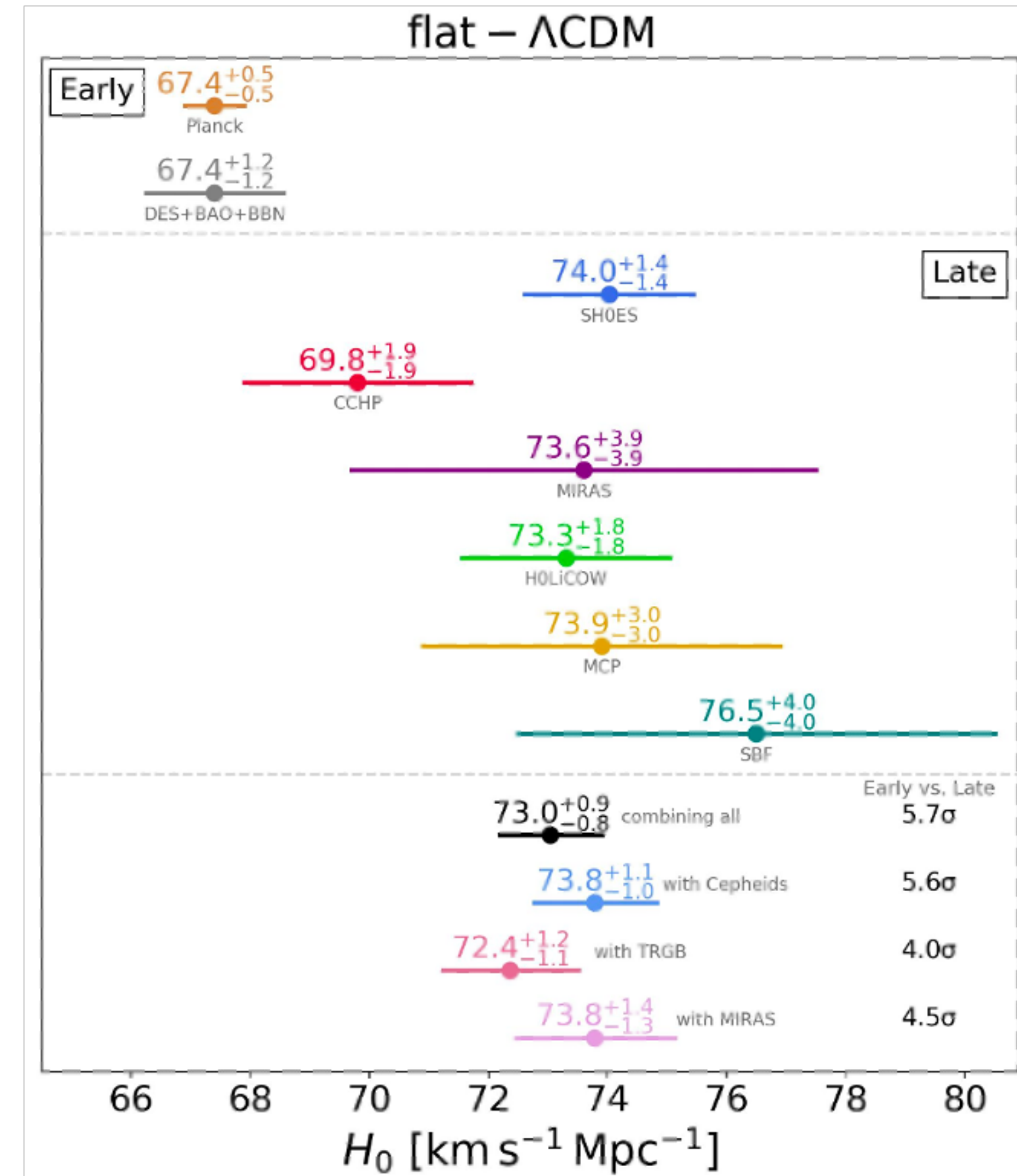
Standard Sirens with CBCs

Redshift: degenerate with mass in the GW signal. Must be found through other methods

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Detect GW from CBC  Detect EM counterpart?

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Bright Sirens

- An EM counterpart is observed and used to obtain the host galaxy's redshift
- E.g. GW170817 and NGC4993

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Spectral Sirens

- Features in the mass distribution of the GW population break the mass-redshift degeneracy \longrightarrow

$$m_{det} = m_{source}(1 + z)$$

Dark Sirens

- Galaxy surveys are used to provide redshift estimates for potential host galaxies \longrightarrow **the galaxy catalog method**

Standard Sirens with CBCs

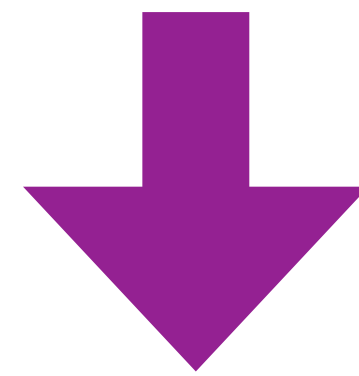
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Measure H_0

Spectral Sirens

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$$m_{det} = m_{source}(1 + z)$$

Dark Sirens

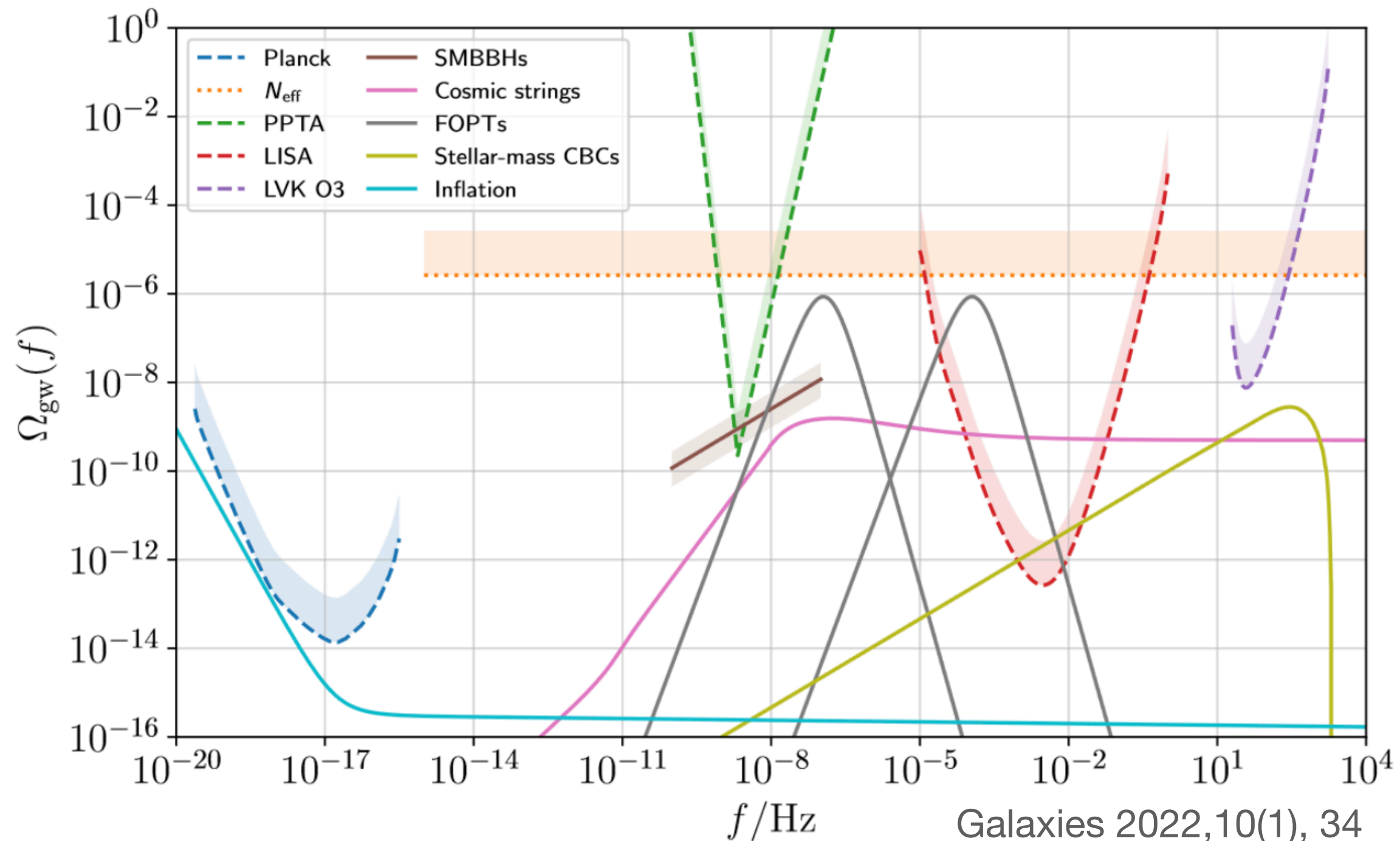
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Stochastic Gravitational Wave Background

The SGWB is a superposition of weak GW sources.

It can exist in many forms from two categories:

- Astrophysical background
- Cosmological background

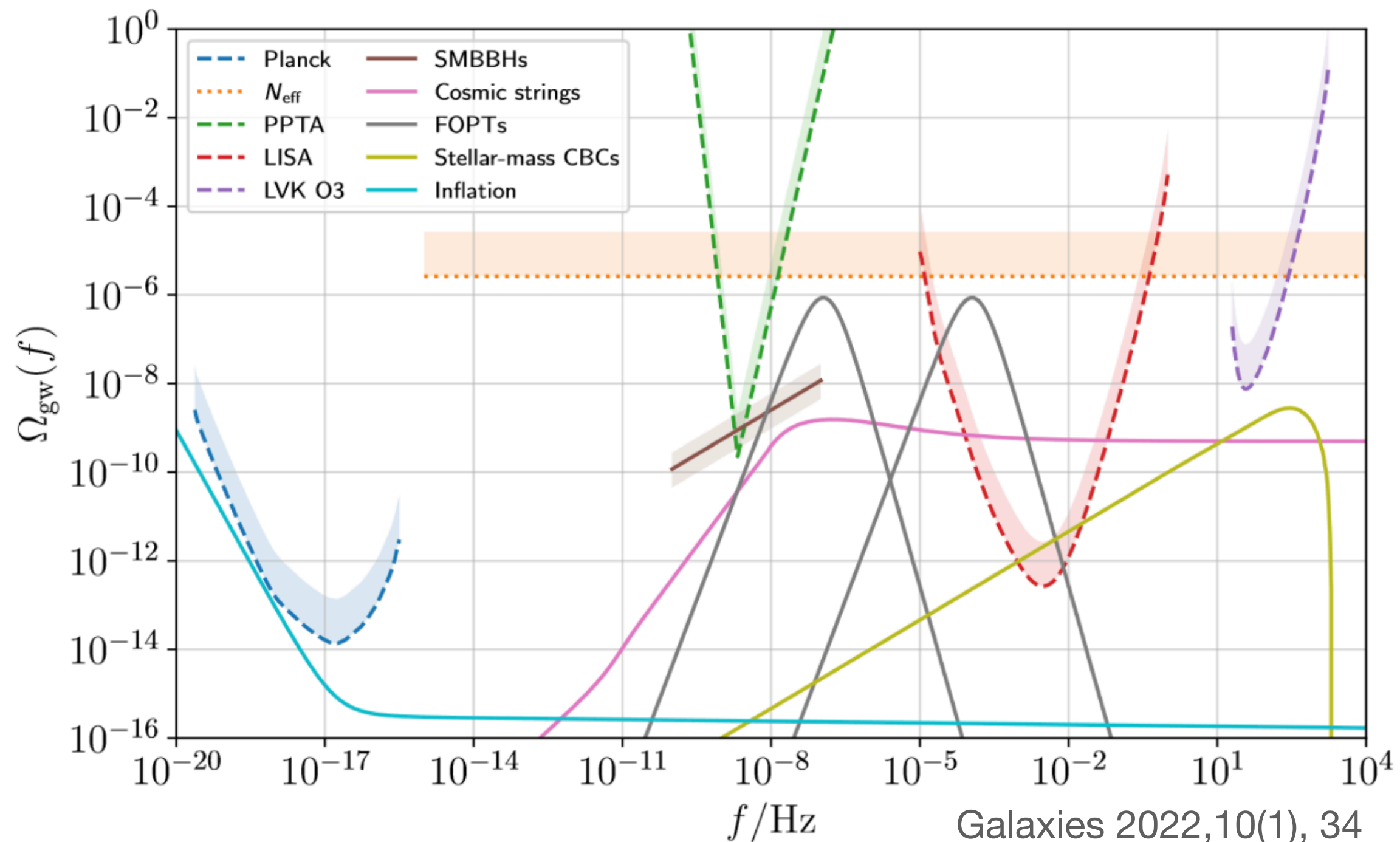


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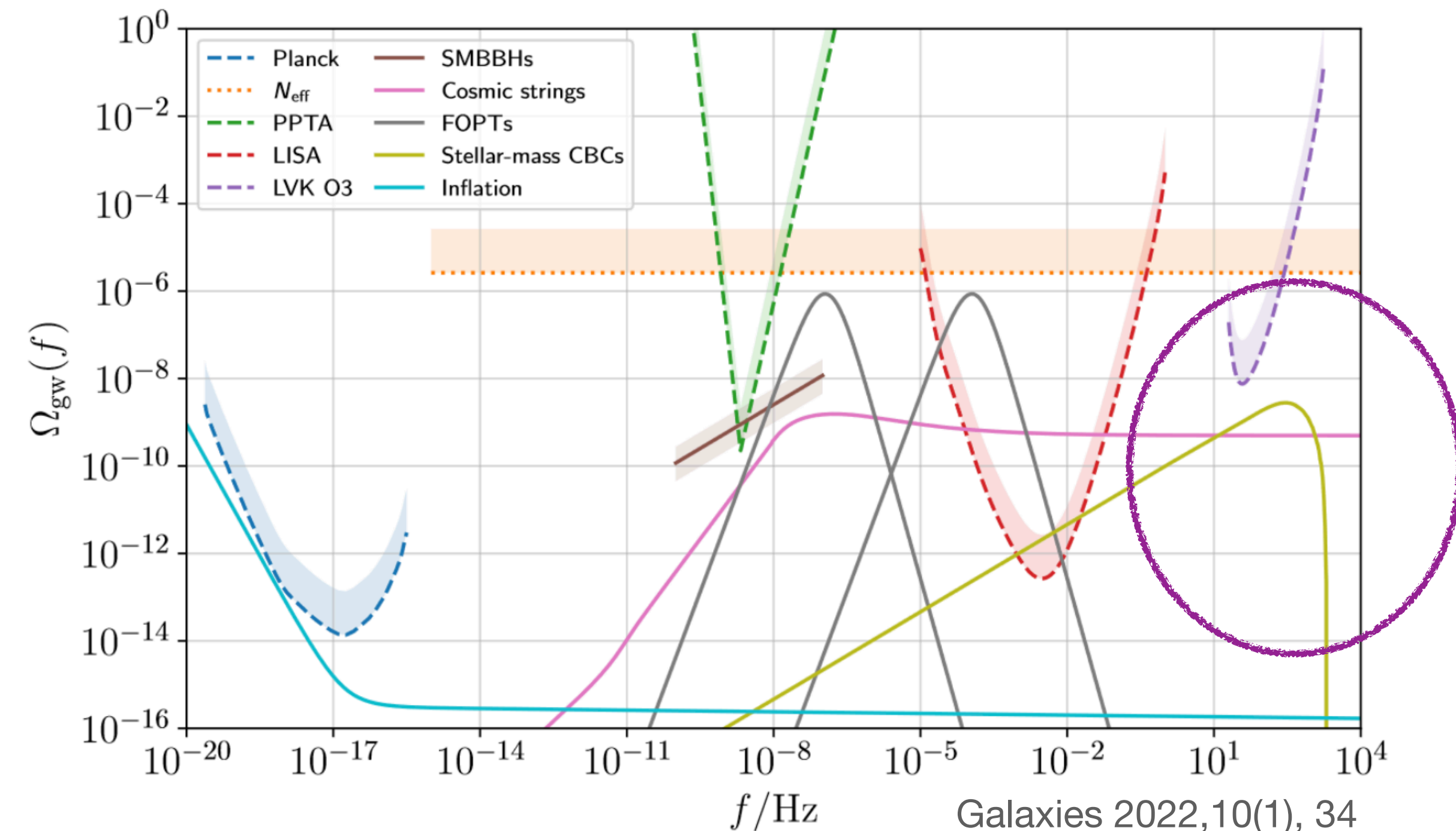


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The SGWB is characterized by dimensionless energy density

$$\Omega_{\text{GW}}(f) = \frac{1}{\rho_c} \frac{d\rho_{\text{GW}}(f)}{d \ln f}$$

Since we are only interested in BBH events, we only look at the astrophysical background from BBHs.

Stochastic Gravitational Wave Background

For a BBH population

$$\Omega_{\text{GW}} = \frac{f}{\rho_c} \int_0^{z_{\text{max}}} dz \frac{R(z)}{(1+z)H(z)} \left\langle \frac{dE_s}{df_s} \Big|_{f(1+z)} \right\rangle$$

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Analysis Method

Turbang et al. 2020, Callister et al. 2023

Individual BBH detections

BBH events



Likelihood on BBH and cosmology parameters (e.g., masses and H_0)

Stochastic Backgrounds

GW data from multiple detectors



Optimal estimator for \hat{C}
GWB spectrum $\Omega_{\text{GW}}(f)$

$$\mathcal{L}(d_i, \hat{C} | \Lambda) = \mathcal{L}_{\text{BBH}}(d_i | \Lambda) \mathcal{L}_{\text{GWB}}(\hat{C} | \Lambda)$$

We want to infer hyperparameters Λ consistent with the individual BBH events and the measurements on the GWB

Analysis Method

Likelihood

- Hierarchical Likelihood for a GW dataset x

$$\mathcal{L}_{BBH}(\{x\} | \Lambda) \propto e^{-N_{exp}(\Lambda)} \sum_i \int d\theta dz \mathcal{L}_{BBH}(x_i | \theta, z, \Lambda) \frac{1}{(1+z)} \frac{N_{CBC}}{(d\theta dz dt_s)}$$

- Gaussian Likelihood

$$\mathcal{L}_{GWB}(\hat{C} | \Lambda) \propto \exp \left[-\frac{1}{2} \sum_k \left(\frac{\hat{C}(f_k) - \Omega(f_k, \Lambda)}{\sigma(f_k)} \right)^2 \right]$$

LVK at A+ design sensitivity simulated data

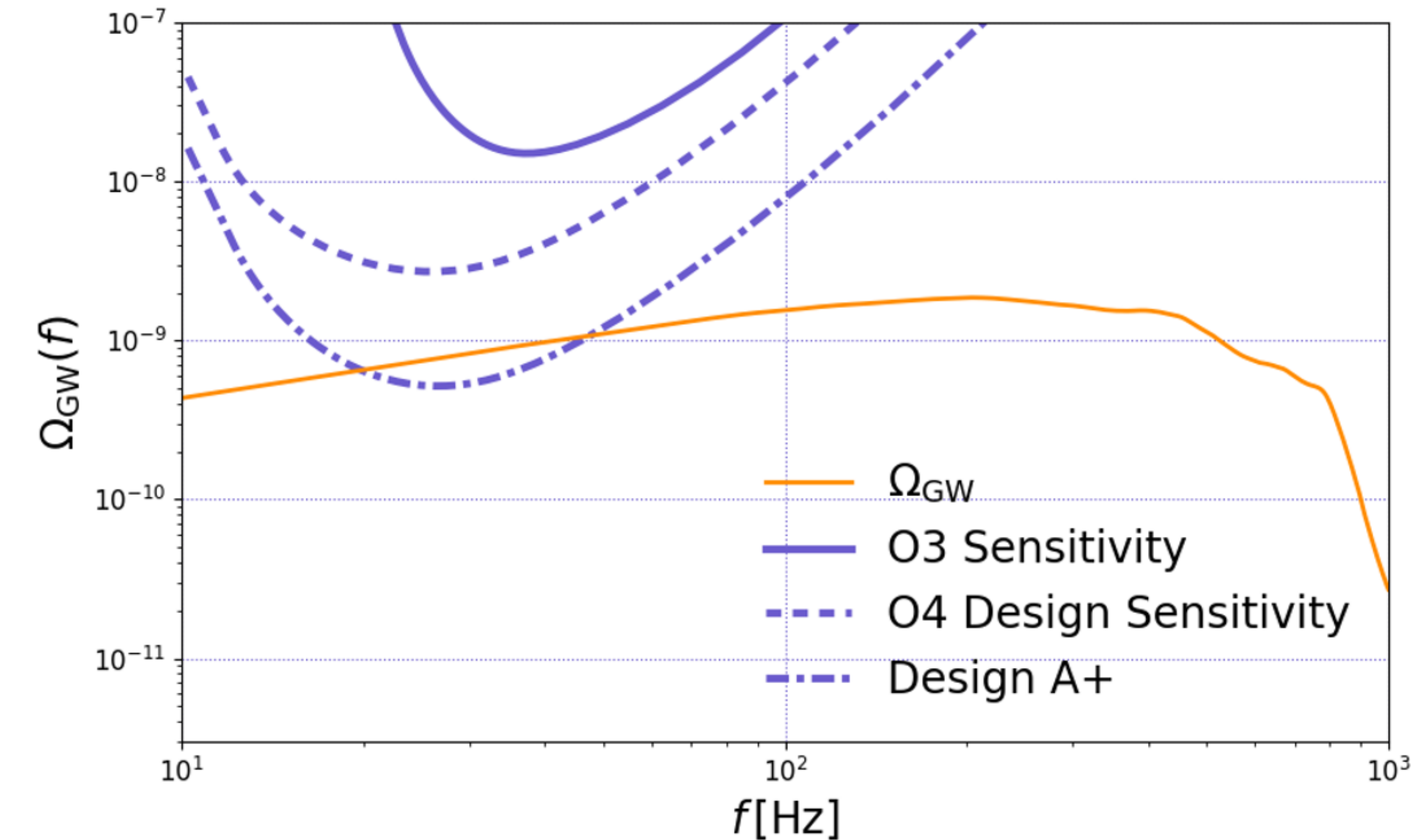
SGWB Simulation

- We simulated \hat{C} and σ based on O5 sensitivity
- The simulations rely on a population model of gravitational wave event rates and masses

$$p(m_1 | m_{min}, m_{max}, \alpha) = (1 - \lambda) \times \\ \times P(m_1 | m_{min}, m_{max}, -\alpha) + \lambda G(m_1 | \mu_g, \sigma)$$

$$p(m_2 | m_{min}, m_1, \beta) = P(m_2 | m_{min}, m_1, \beta)$$

$$\psi(z; \Lambda_p) = \psi_{MD}(z; \gamma, k, z_p)$$



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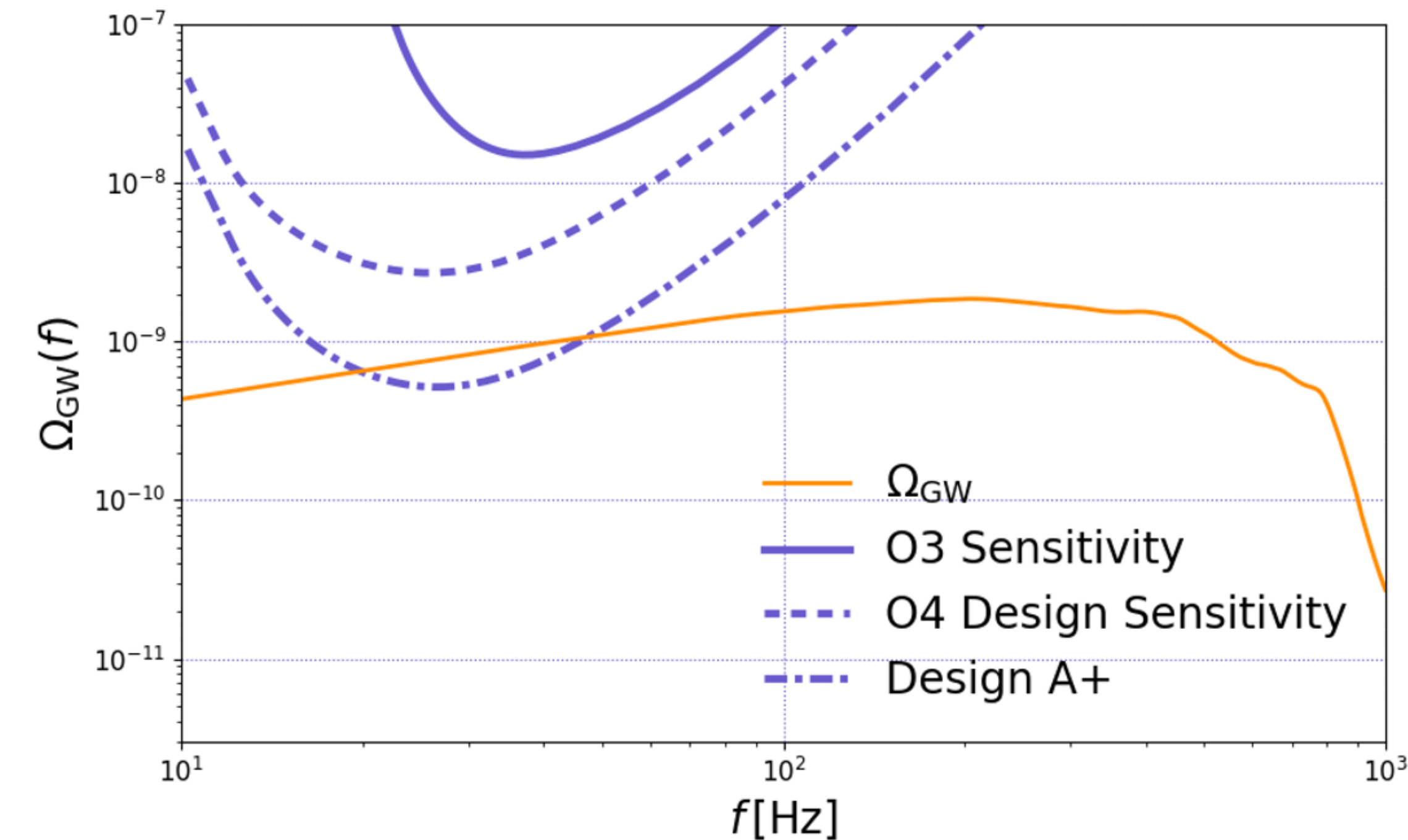
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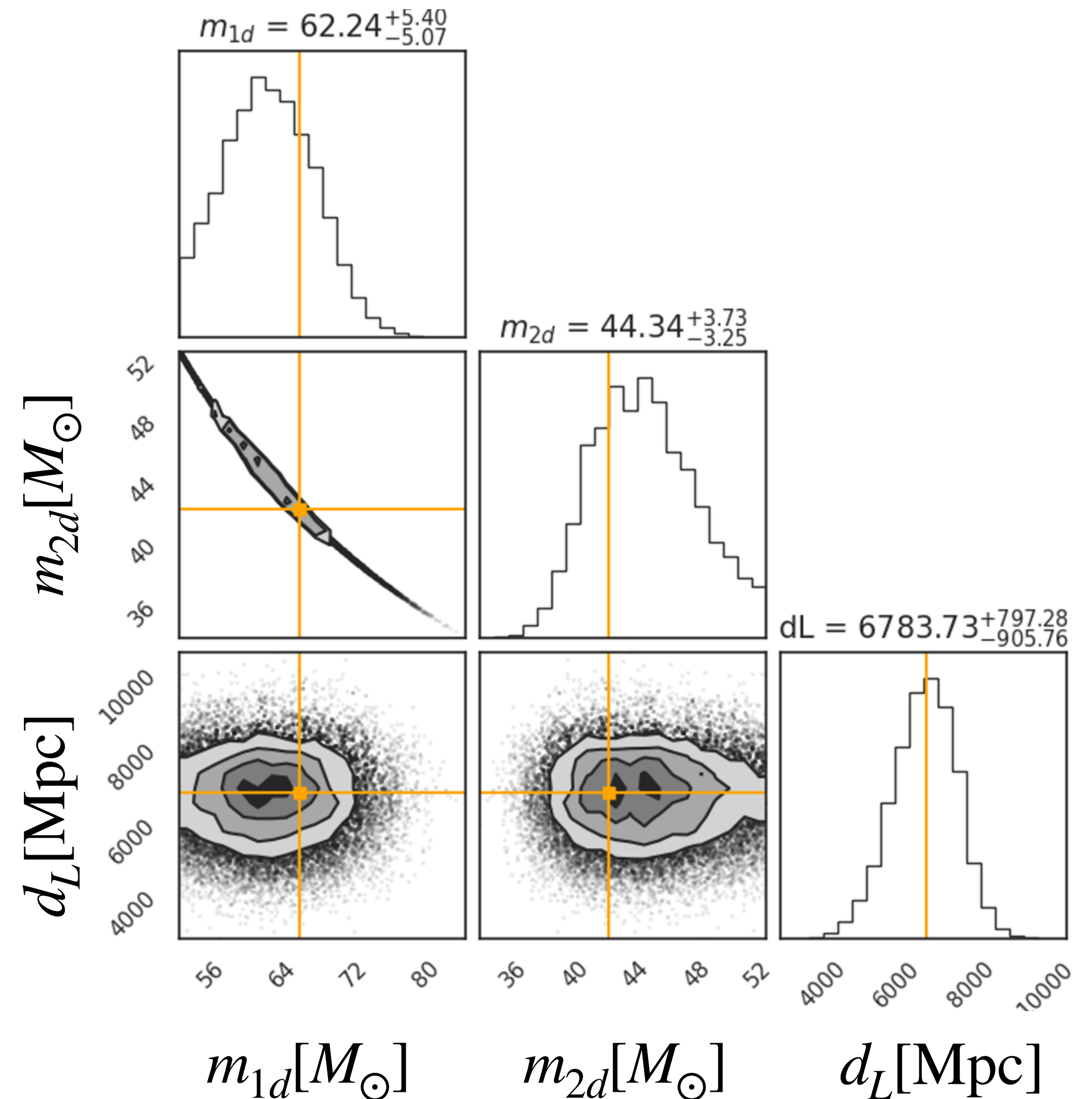


Power Law+Peak model

LVK at design sensitivity simulated data

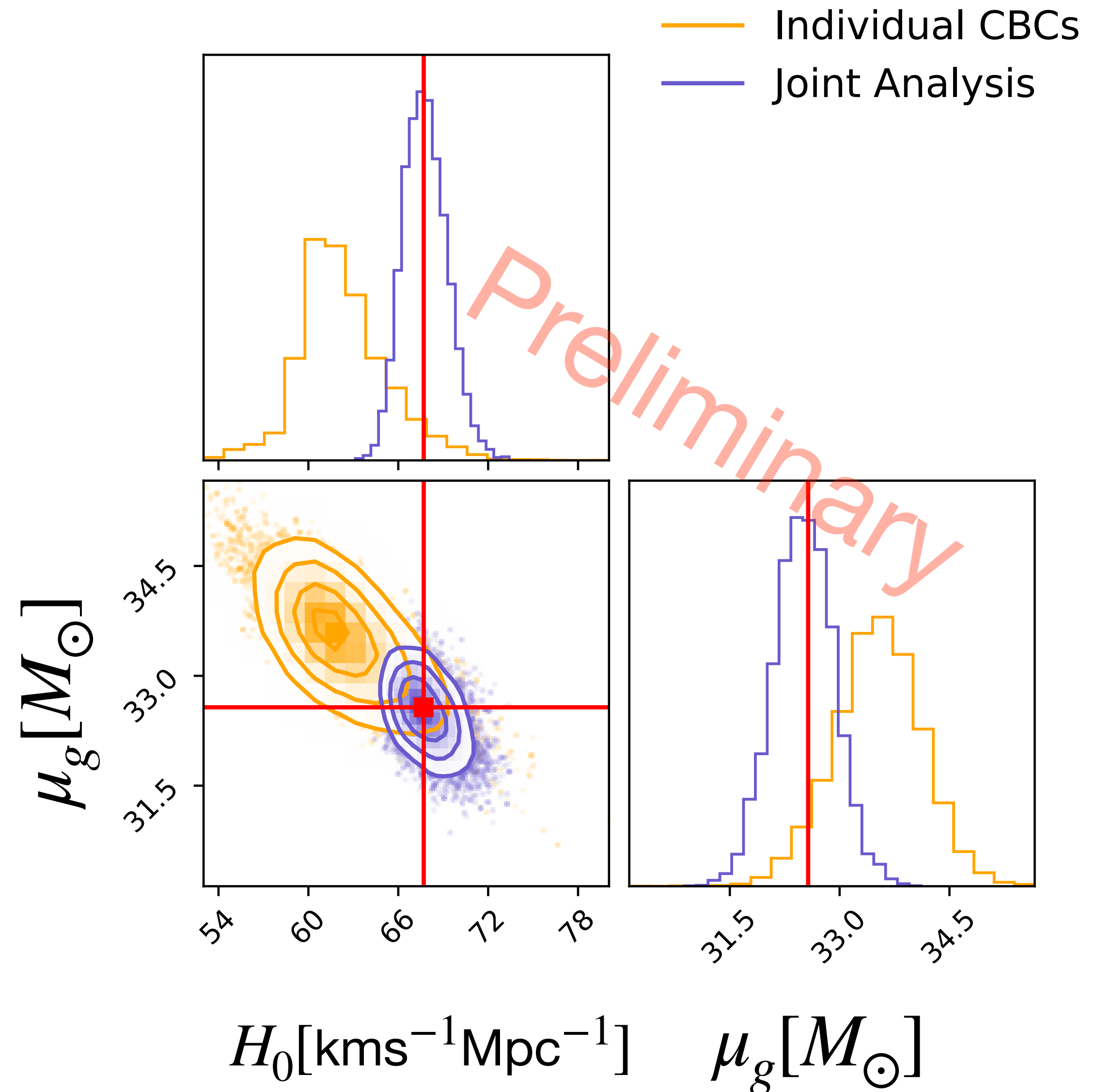
GW Simulation

- We assumed the same population model and merger rate
- A toy model simulates parameter estimation posteriors for 500 detected BBH
- The corner plot show posterior distributions for detector-frame masses and luminosity distance



Preliminary Results

- The prior range for the Hubble constant is set to $H_0[\text{kms}^{-1}\text{Mpc}^{-1}] \in [10,200]$ and for the mean $\mu_g[M_\odot] \in [20,50]$
- The values for H_0 and μ_g are better constrained using the joint analysis
- It assumes the mass spectrum is perfectly known



Conclusions and future steps

- Gravitational Wave sources are rapidly becoming important cosmic tracers
- Adding SGWB data can improve knowledge of H_0 and of the population parameters
- Joint inference can extend to other populations and key cosmological parameters (e.g., Ω_m)
- This approach applies to the Einstein Telescope (ET) for further analysis

Thank you for your attention!

