EFT-based methods for classical gravity

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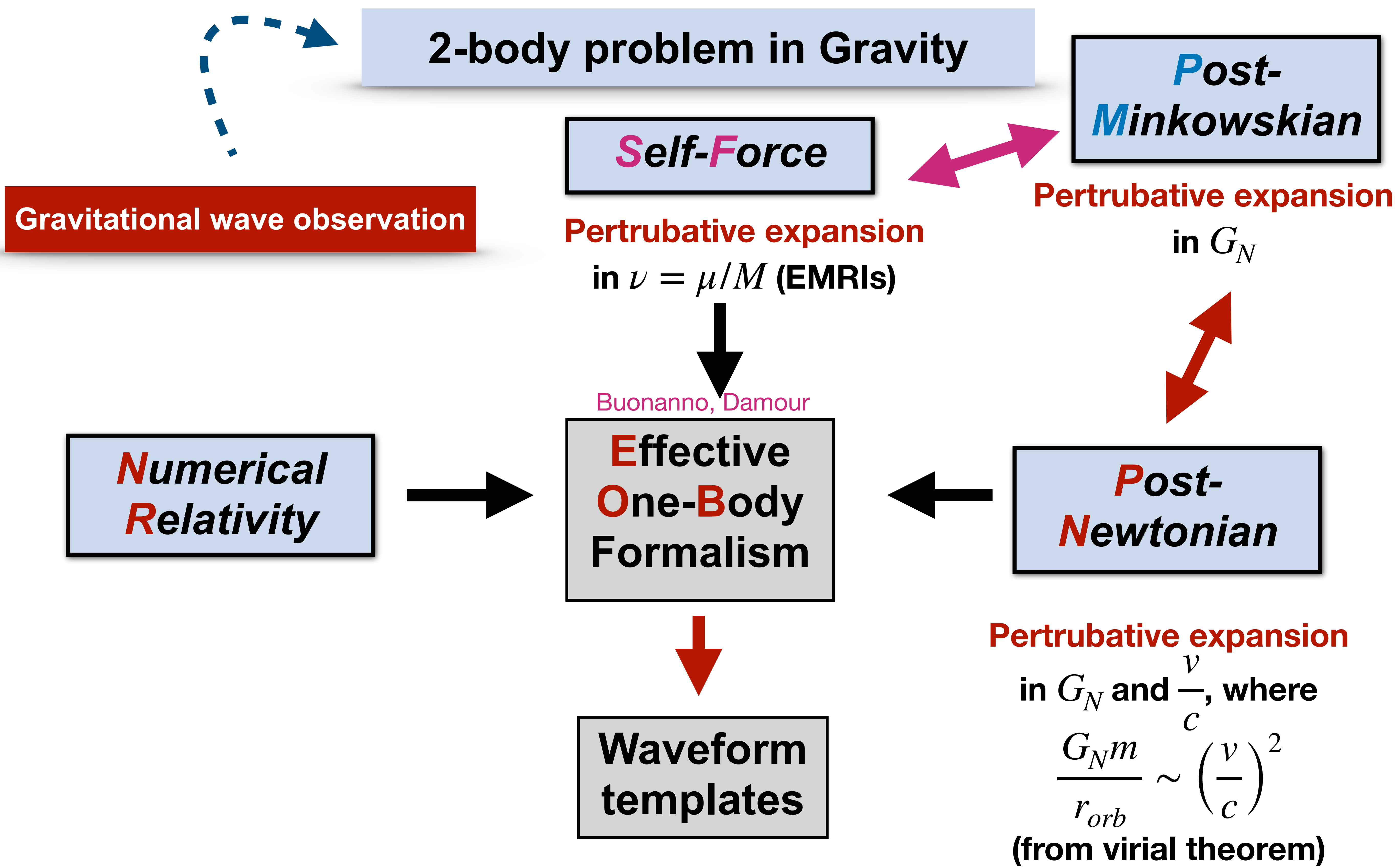
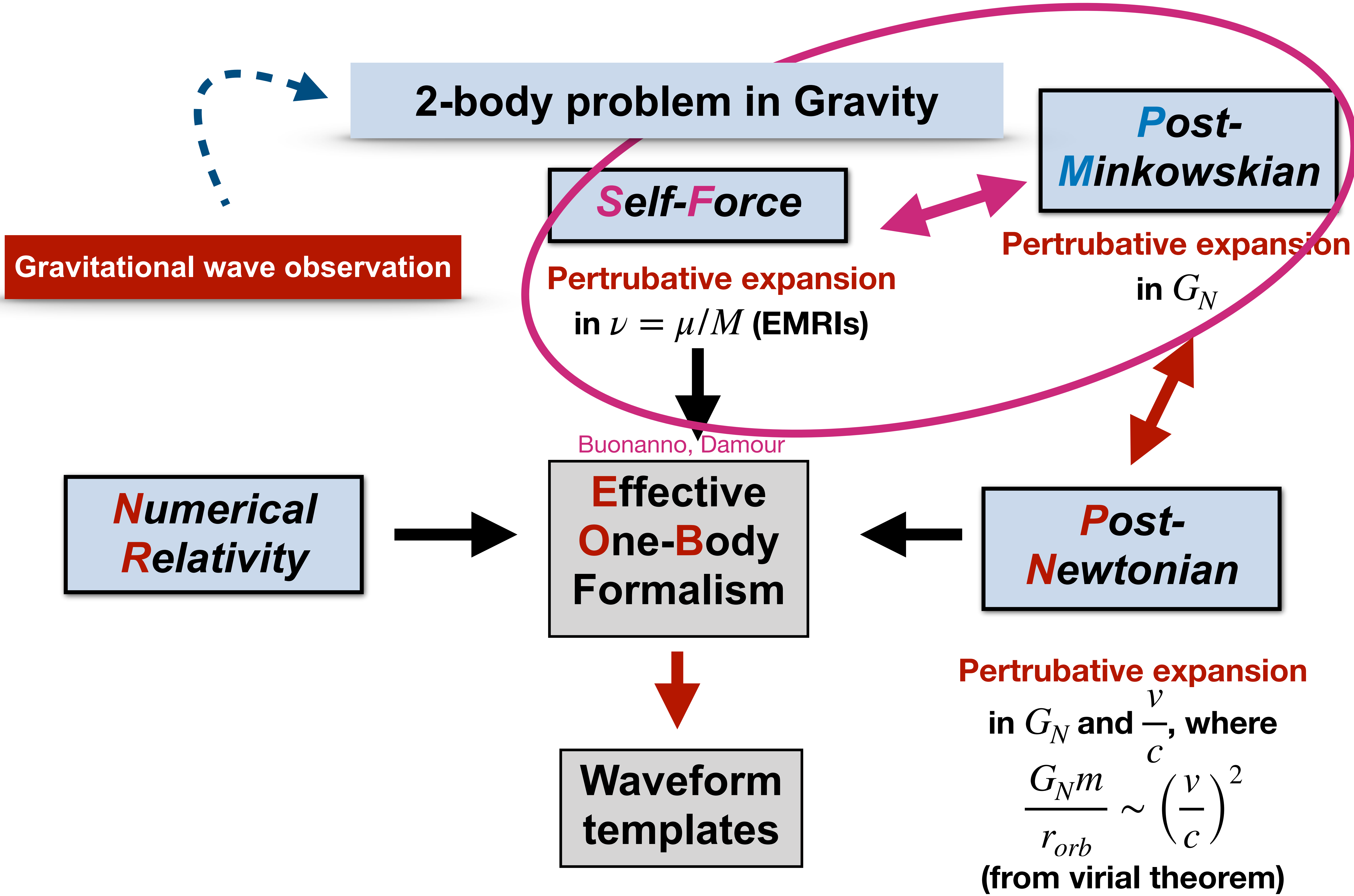
Atelier API “Ondes gravitationnelles et objets compacts”, Meudon 08/10/2024

Based on works with: P.Vanhove **[2407.09448],[2405.14421]**



2-body problem in Gravity
Gravitational wave observation
Self-Force
Pertrubative expansion
in $\nu = \mu/M$ (EMRIs)
Post-Minkowskian
Pertrubative expansion
in G_N
Buonanno, Damour
Effective One-Body Formalism
Numerical Relativity
Post-Newtonian
Pertrubative expansion
in G_N and $\frac{v}{c}$, where
$\frac{G_N m}{r_{orb}} \sim \left(\frac{v}{c}\right)^2$
(from virial theorem)
Waveform templates



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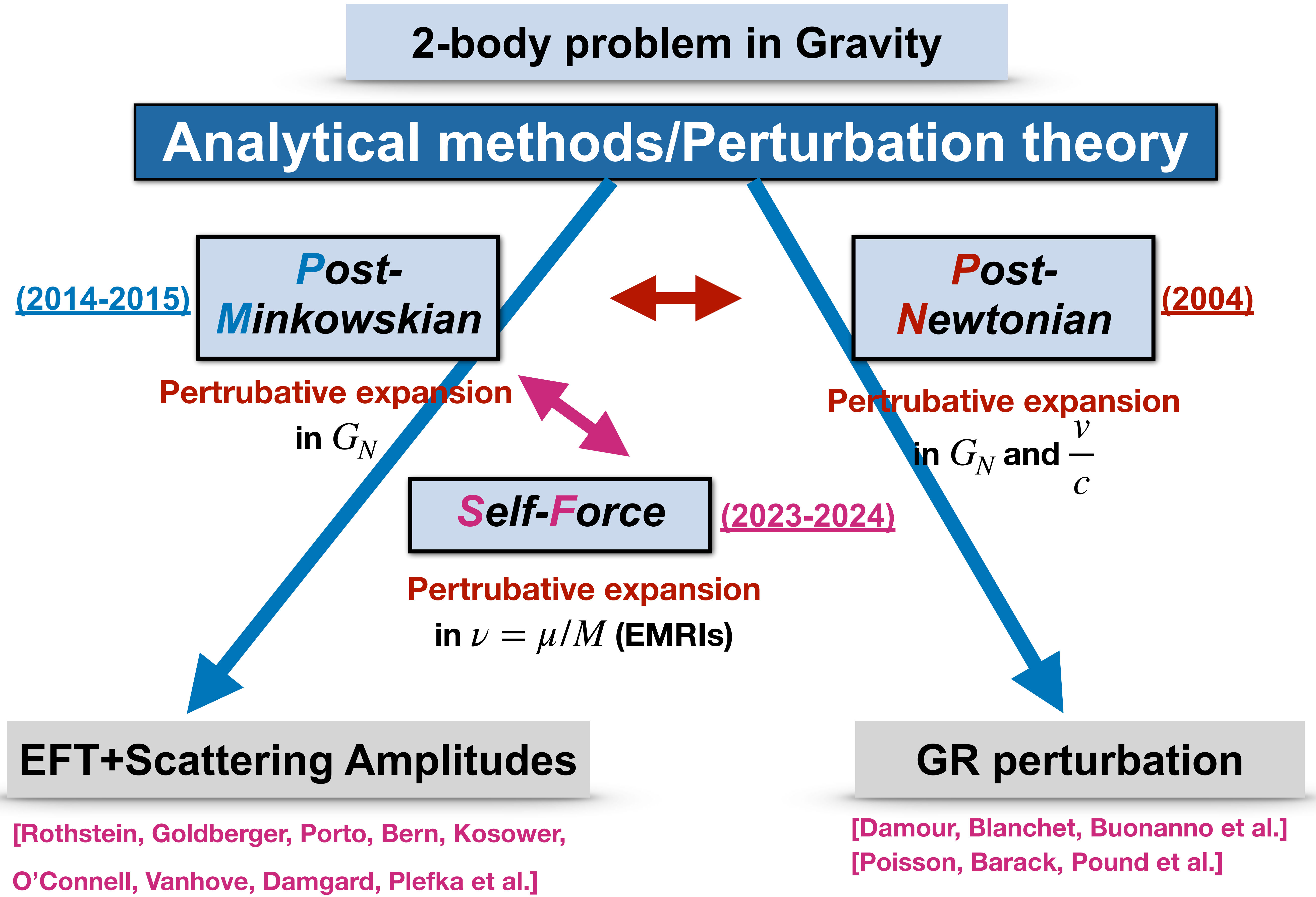
2-body problem in Gravity

Post-Minkowskian vs **Self-Force**

	1PM	2PM	3PM	4PM	5PM	...	
0SF	$\Big[(G\,m_1)$	$+(G\,m_1)^2$	$+(G\,m_1)^3$	$+(G\,m_1)^4$	$+(G\,m_1)^5$	$+\ldots\Big]$	$\times\, m_2$
1SF			$\Big[(G\,m_1)^2$	$+(G\,m_1)^3$	$+(G\,m_1)^4$	$+\ldots\Big]$	$\times\, G\,m_2^2$
2SF					$\Big[(G\,m_1)^3$	$+\ldots\Big]$	$\times\, G^2\,m_2^3$


2-body problem in Gravity
Analytical methods/Perturbation theory
(2014-2015)
Post-Minkowskian
Post-Newtonian
(2004)
Pertrubative expansion
in G_N
Pertrubative expansion
in G_N and $\frac{v}{c}$
Self-Force
(2023-2024)
Pertrubative expansion
in $\nu = \mu/M$ (EMRIs)
EFT+Scattering Amplitudes
GR perturbation
[Rothstein, Goldberger, Porto, Bern, Kosower,
O'Connell, Vanhove, Damgard, Plefka et al.]
[Damour, Blanchet, Buonanno et al.]
[Poisson, Barack, Pound et al.]

2-body problem in Gravity

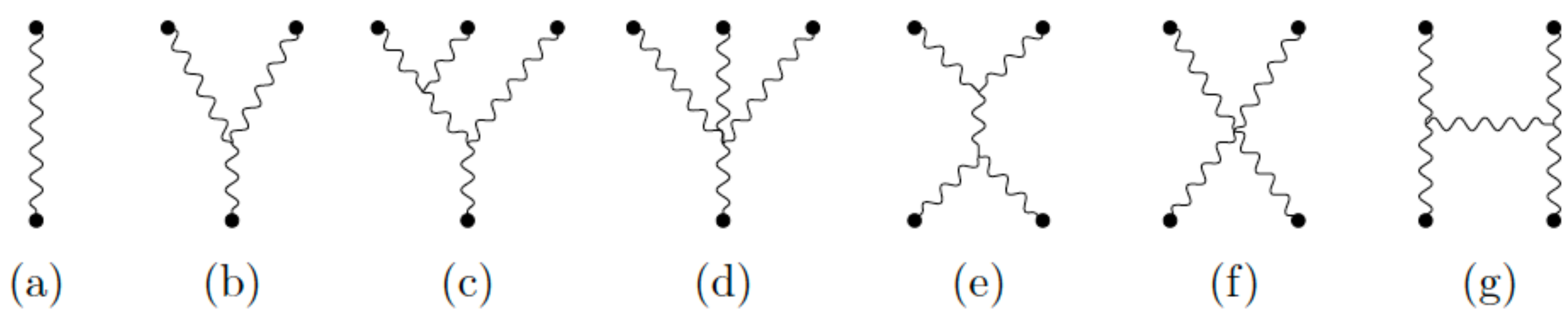
EFT+Scattering Amplitudes (in a nutshell)

$$e^{iS_{\rm eff}[x_a]} = \int \mathcal{D}h_{\mu\nu}\, e^{iS_{\rm EH}[h]+iS_{\rm GF}[h]+iS_{\rm pp}[x_a,h]}$$

→ **Point particle approximation +weak metric perturbations**

Feynman rules

integrate-out graviton (classical=NO grav. loops)



Schwarzschild from Amplitudes to all orders in G

[2407.09448],[2405.14421] **S.M.**, P. Vanhove

Towards SF-EFT, it is necessary to have a self-consistent framework to all orders in G

Simplest case: *Schwarzschild metric*

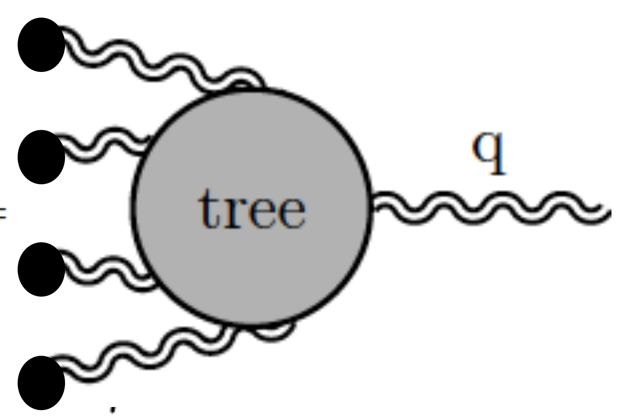
Schwarzschild from Amplitudes to all orders in G

[2407.09448],[2405.14421] S.M., P. Vanhove

GREFT ⟷ **General Relativity**

Equivalence up to:

1. **Choice of DOFs** → $g_{\mu\nu} = \eta_{\mu\nu} + \sum_{n=1}^{\infty} h_{\mu\nu}^{(n)}$
2. **Choice of Gauge** → **harmonic gauge**

$\mathcal{M}_3(p_1, q) =$ 

$$i\mathcal{M}_3^{(l)}(p_1, q) = -\frac{i\sqrt{32\pi G_N}}{2}\langle T^{(l)\,\mu\nu}(q^2)\rangle\epsilon_{\mu\nu}$$

Duff (1974)

$$h_{\mu\nu}^{(l+1)}(\vec{x}) = -16\pi G_N \int \frac{d^d\vec{q}}{(2\pi)^d} e^{i\vec{q}\cdot\vec{x}} \frac{1}{\vec{q}^2}\left(\langle T_{\mu\nu}^{(l)}\rangle^{\text{class.}}(q^2) - \frac{1}{d-1}\eta_{\mu\nu}\langle T^{(l)}\rangle^{\text{class.}}(q^2)\right)$$

Expectation: n**-loop diagrams generate** G_N^{n+1} **terms of the metric**

Schwarzschild from Amplitudes

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Too complicated!!

Main problems from previous attempts:

[2010.08882] **S.M**, Vanhove

1. **Infinite tower of *non-minimal couplings* (due to intermediate UV-divs)**
2. **No algorithm for higher loops (3-loops was already complicated)**

Schwarzschild from Amplitudes

[2407.09448],[2405.14421] **S.M.**, P. Vanhove

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Take a step back

Schwarzschild from Amplitudes

[2407.09448],[2405.14421] **S.M.**, P. Vanhove

Cubic formulation of GREFT

[1705.00626] Cheung, Remmen

Choice of DOFs: **1)Gothic metric:** $\mathfrak{g}^{ab} = \sqrt{-g}g^{ab} = \eta^{ab} - \sqrt{32\pi G_N}h^{ab}$

(GR people do it, we should pay more attention)

2) Extra Auxiliary field : $A^a_{bc} = \Gamma^a_{bc} - \frac{1}{2}\delta^a_{(b}\Gamma^d_{c)d}.$

(unorthodox but necessary to constrain to 3pt vertices)

Choice of Gauge: **harmonic gauge*(non unique)**

+ couple to worldline $\mathcal{L}_{p.p.} = -\frac{m}{2}\int d\tau\ \left(e^{-1}\, g^{\mu\nu}v_\mu v_\nu + e\right) = -\frac{m}{2}\int d\tau\ \left(\frac{\mathfrak{g}^{\mu\nu}v_\mu v_\nu}{(\sqrt{-\mathfrak{g}})^{\frac{2}{D-2}}} + 1\right)$

Schwarzschild from Amplitudes

[2407.09448],[2405.14421] **S.M.**, P. Vanhove

Cubic formulation of GREFT

Complete ansatz:

Single multi-loop Master Integral

$$\sqrt{32\pi G_N}\, h_{\mu\nu}^{(n)}(\mathbf{x}) = \int d^{D-1}x\; e^{i\mathbf{k}\mathbf{x}}\; J_{\mu\nu}^{(n)}(\mathbf{k}),$$

$$\sqrt{32\pi G_N}\, A^{a\;(n)}_{bc}(\mathbf{x}) = \int d^{D-1}x\; e^{i\mathbf{k}\mathbf{x}}\; Y^{a\;(n)}_{bc}(\mathbf{k}),$$

$$J_{\mu\nu}^{(n)}(\mathbf{k}) = \rho(|\mathbf{k}|, D, n)\left(\chi_1^{(n)}\delta_\mu^0\delta_\nu^0 + \chi_2^{(n)}\eta_{\mu\nu} + \chi_3^{(n)}\frac{k_\mu k_\nu}{\mathbf{k}^2}\right)$$

$$\rho(|\mathbf{k}|, D, n) = \frac{\Gamma\left(\frac{2-(D-3)(n-1)}{2}\right)}{\Gamma\left(\frac{n(D-3)}{2}\right)} \frac{\left(\Gamma\left(\frac{D-3}{2}\right)G_N m\right)^n}{(|\mathbf{k}|/(2\sqrt{\pi}))^{2-(D-3)(n-1)}}$$

$$Y^{a\;(n)}_{bc}(\mathbf{k}) = -i\rho(|\mathbf{k}|, D, n)\Big(k_{(b}\left(\chi_7^{(n)}\delta^0_{c)}\delta^a_0 + \chi_8^{(n)}\delta^a_{c)}\right) + k^a\left(\chi_4^{(n)}\delta^0_b\delta^0_c + \chi_5^{(n)}\eta_{bc} + \chi_6^{(n)}\frac{k_b k_c}{\mathbf{k}^2}\right)\Big).$$

form factors

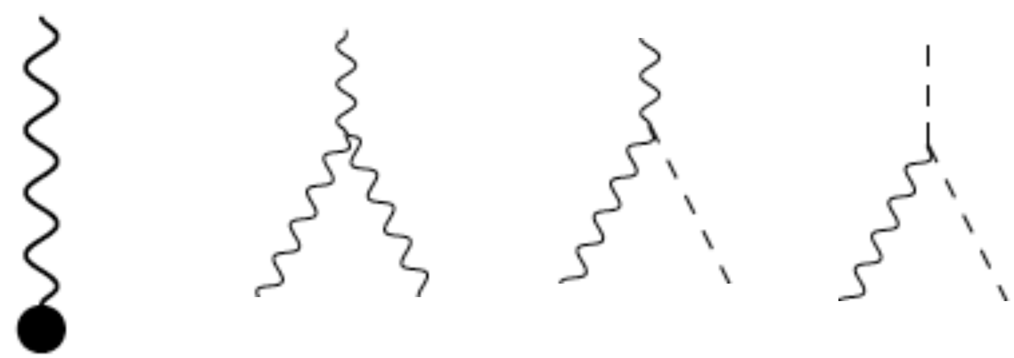
$$\boldsymbol{\chi}^{(n)}(D) = (\chi_1^{(n)}, \ldots, \chi_8^{(n)})$$

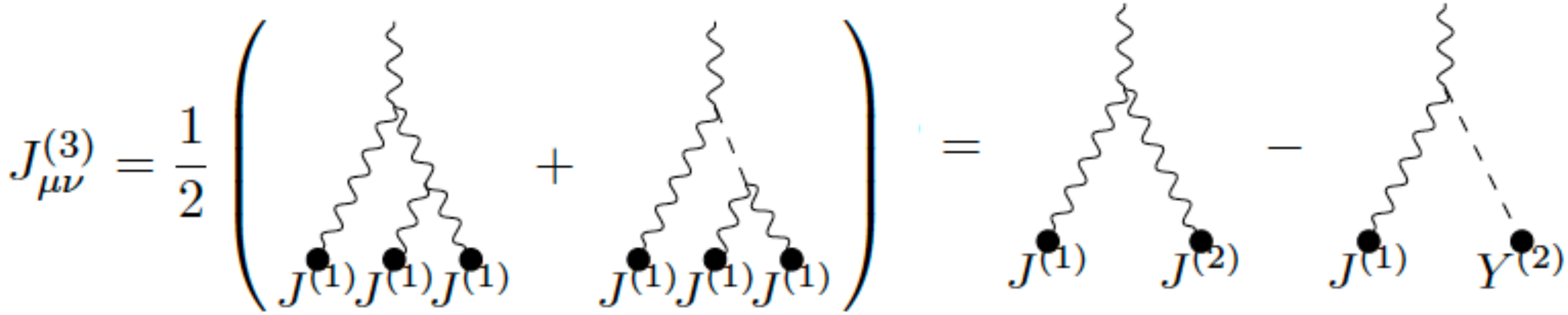
Schwarzschild from Amplitudes

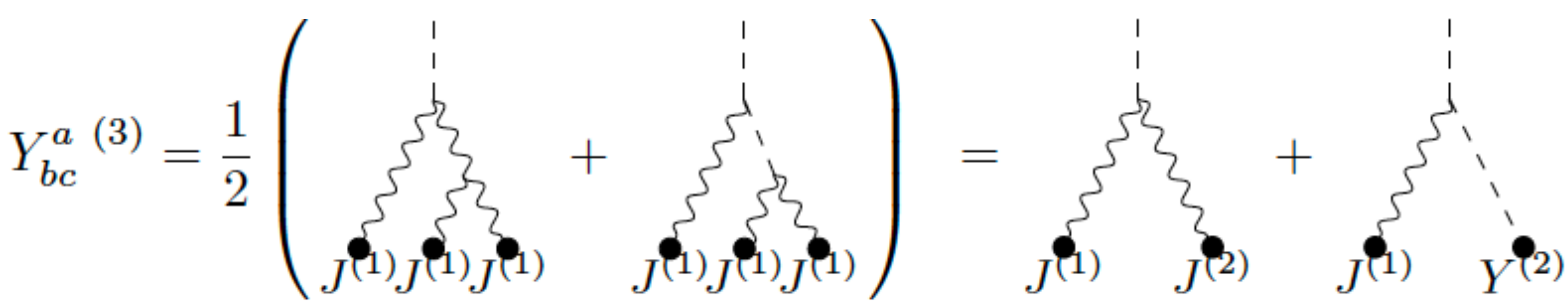
[2407.09448],[2405.14421] S.M., P. Vanhove

Cubic formulation of GREFT

Feynman rules:







Schwarzschild from Amplitudes

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Cubic formulation of GREFT

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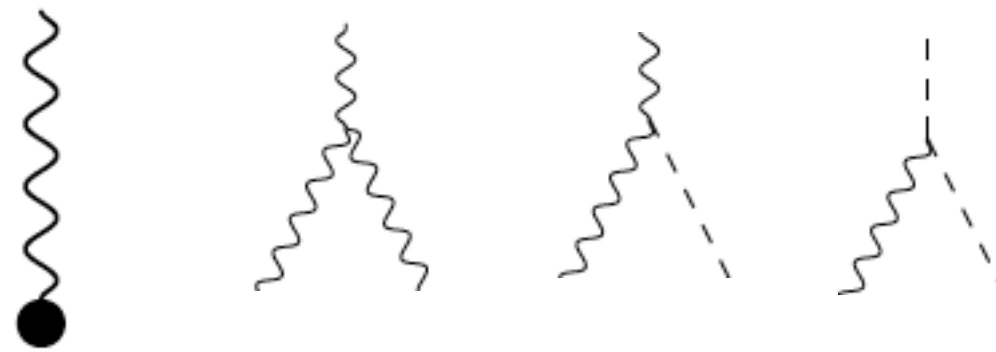

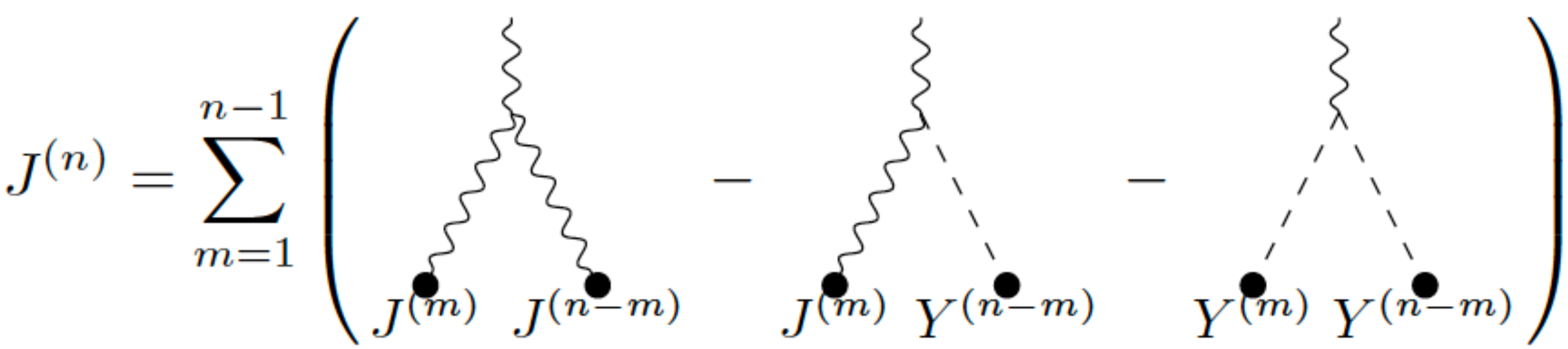

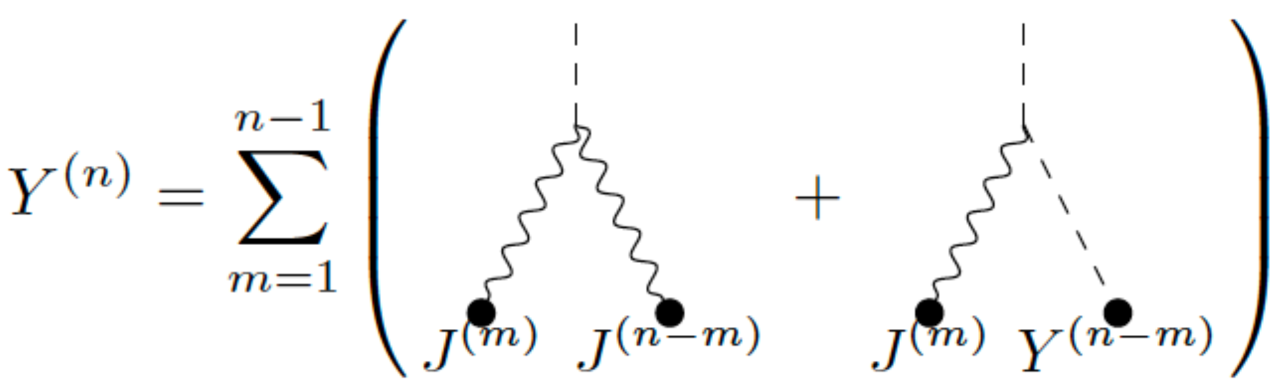

Iterative structure to all orders!!!
due to 3pt interactions

Schwarzschild from Amplitudes

[2407.09448],[2405.14421] S.M., P. Vanhove

Metric to all orders in G

Recursion relations

$$\chi_k^{(n)}(D) = \sum_{i,j=1}^{8} \sum_{m=1}^{n-1} \chi_i^{(m)}(D)\chi_j^{(n-m)}(D)M_k^{ij}(D)$$

Solvable at D=4 →

$$\chi^{(n)}(4) = \Big(8, 0, 0, 4 + n(-1)^n + \frac{1+3(-1)^n}{2n(n+2)}, 2 + \frac{1+3(-1)^n}{2n(n+2)}, \frac{1+3(-1)^n}{2n(n+2)}(n-3), \frac{1}{n} - 4 - \frac{1+3(-1)^n}{2n(n+2)}(n+1), \frac{1+3(-1)^n}{2n(n+2)}\Big)$$

no UV-divs!!!

→ **Resums to GR solution** (**GREFT computation "picks" the simplest harmonic gauge**)

Schwarzschild from Amplitudes

[2407.09448],[2405.14421] S.M., P. Vanhove

Geodesic motion (0SF)

SF expansion: $\mathcal{S} = \mathcal{S}_{EH} + \mathcal{S}_l + \mathcal{S}_H$; $\mathcal{S}_H = -\frac{M}{2}\int d\tau_H \left(\frac{\mathfrak{g}^{\mu\nu} v_{H\mu} v_{H\nu}}{(\sqrt{-\mathfrak{g}})^{\frac{2}{D-2}}} + 1 \right)$ *n-point graviton vertices* contrary to PM-expansion

$$e^{i\mathcal{S}_{\text{eff}}[x_l, x_H]} = \int \mathcal{D}h\, \mathcal{D}A\; e^{i\mathcal{S}_{EH}[h,A] + i\mathcal{S}_{GF}[h] + i\mathcal{S}_l[x_l,h] + i\mathcal{S}_H[x_H,h]}$$

integrate-out via diagrams →

$$\mathcal{S}_{\text{eff}} = -\frac{M}{2}\int d\tau_H\, \eta^{\mu\nu} v_{H\mu} v_{H\nu} + M \sum_{n=0}^{\infty} \int d\tau_l \left(\frac{m}{M}\right)^{n+1} \mathcal{L}_n[x_l(\tau_l), x_H(\tau_H)]$$

+SF expand trajectories $x_l^\mu(\tau_l) \equiv x^\mu(\tau) = \sum_{n=0}^{\infty} \left(\frac{m}{M}\right)^n \delta x^{(n)\,\mu}(\tau),\; x_H^\mu(\tau_H) = u_H^\mu \tau_H + \sum_{n=1}^{\infty} \left(\frac{m}{M}\right)^n \delta x_H^{(n)\,\mu}(\tau_H)$

Schwarzschild from Amplitudes

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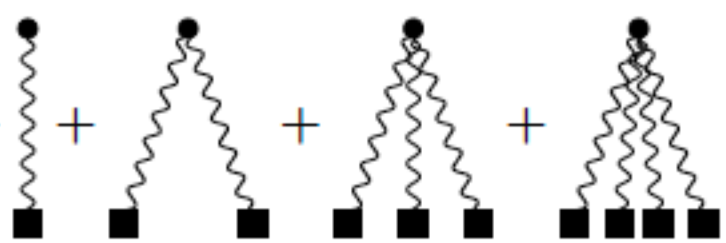
$$\mathcal{L}_0[x^\mu(\tau), u^\mu_H \tau_H] = \bullet + \cdots + \cdots + \cdots + \cdots + \cdots$$



1) Infinite tower of n-graviton worldlines vertices

where $$J_{\mu\nu}(\mathbf{k}) = \cdots = \sum_{n=1}^{\infty} J^{(n)}_{\mu\nu}(\mathbf{k})$$

2) Effective 1pt contains an infinite series -*dressed graviton emission*-, already known

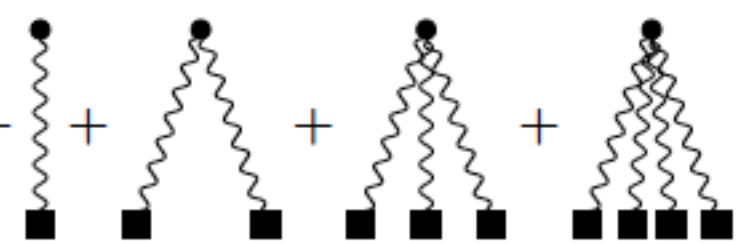
Schwarzschild from Amplitudes

[2407.09448],[2405.14421] **S.M.**, P. Vanhove

Geodesic motion (0SF)

We managed to perform explicitly the resummation of the above infinite diagrams where each one contains an infinite sum-*double resummation*!

$$\mathcal{L}_0[x^\mu(\tau), u_H^\mu \tau_H] = \bullet +$$



$$+ \cdots = -\frac{1}{2} v_\mu(\tau) v_\nu(\tau) g^{\mu\nu}(|\mathbf{x}|(\tau))$$

trivially gives geodesic eq.

Rediscovered America!

1)Trivial from GR perspective but highly non-trivial from EFT
2)Crucial stepping stone to go beyond leading SF order

Conclusion

Classical GR through the lens of QFT approach

1. Efficiently exploits previous knowledge from particle physics
2. Possibly overcomes shortcomings of other methods (*bremmstrahlung example*)
3. Easily applicable to GR modifications
4. Done well in PN, PM. Needs to be extended for SF
5. Non-perturbative, EFT-based approach can be used for questions regarding BHs (Love numbers, quantum effects etc.)