NEWS FROM THE DARK, OM VS. DM, MARSEILLE, FRANCE - 15/11/2024

Deciphering the dynamics of the Milky Way bar and spiral arms with Gaia

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cnrs
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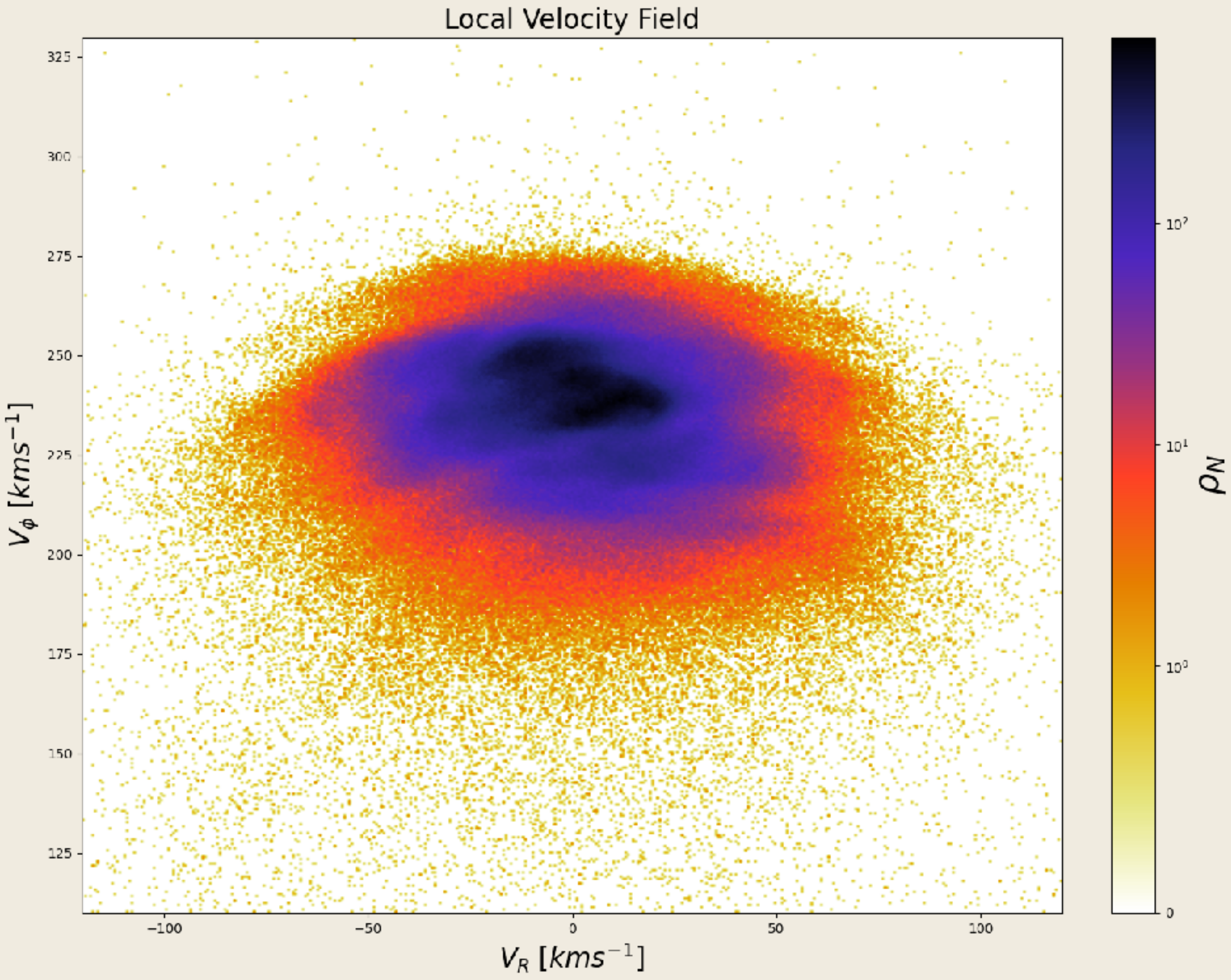
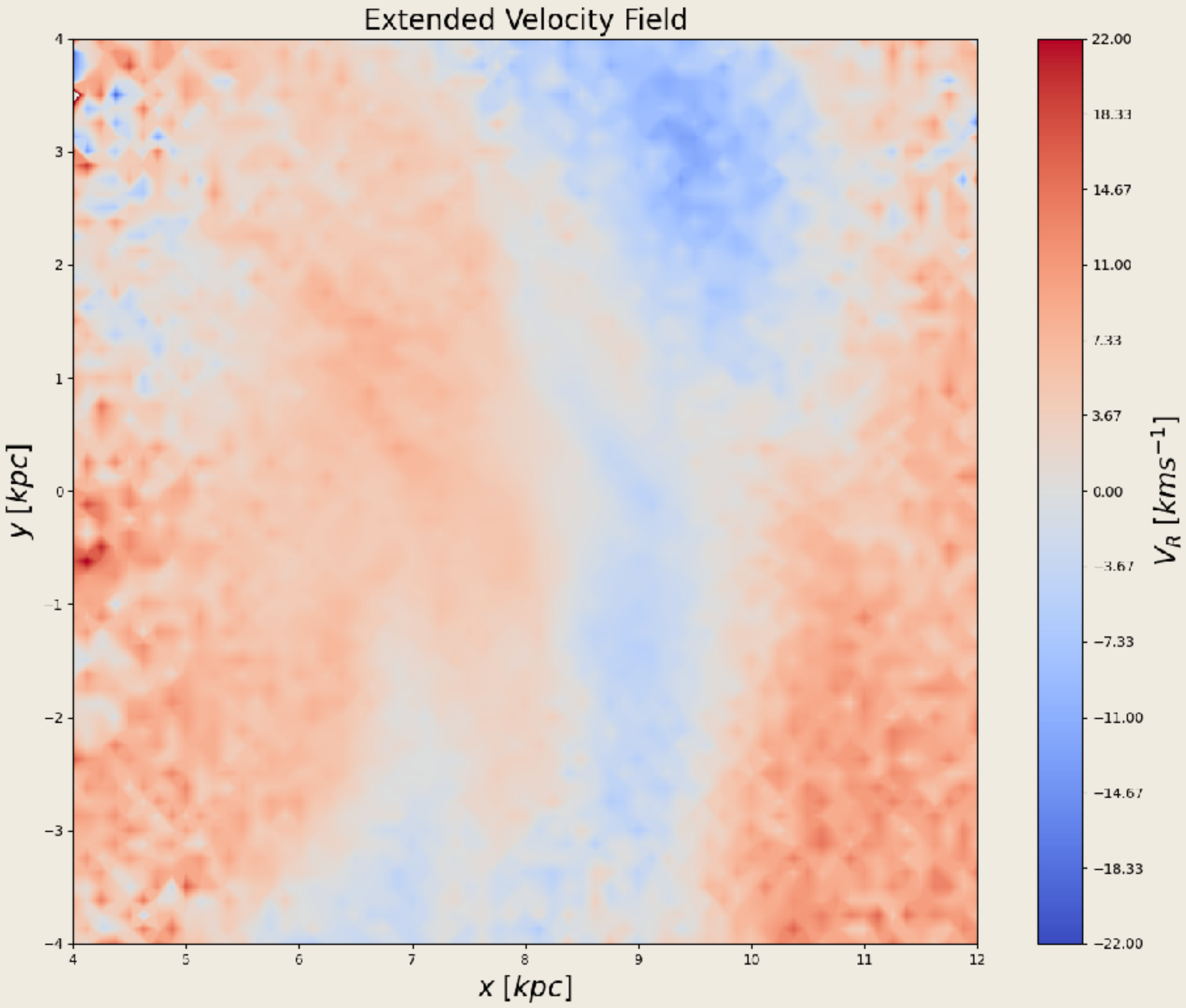
Université de Strasbourg
Observatoire astronomique de Strasbourg | ObAS
Research in Mathematics, Interactions & Applications | IRMIA++
The Interdisciplinary thematic institutes of the University of Strasbourg & cnrs & Inserm funded under the Excellence Initiative program

Gaia mission sample: Velocity fields





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Local velocity field: Gaia DR3

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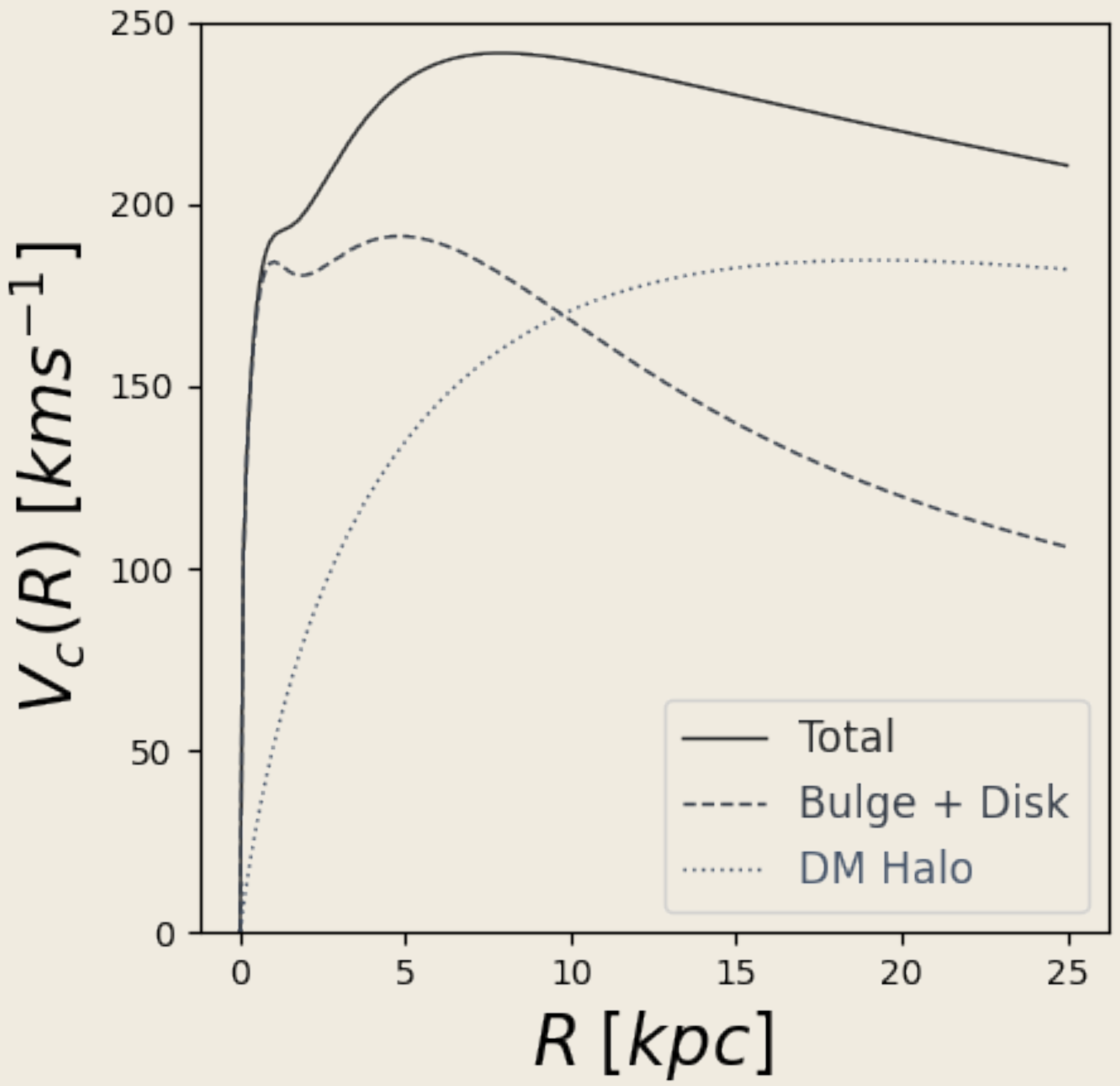
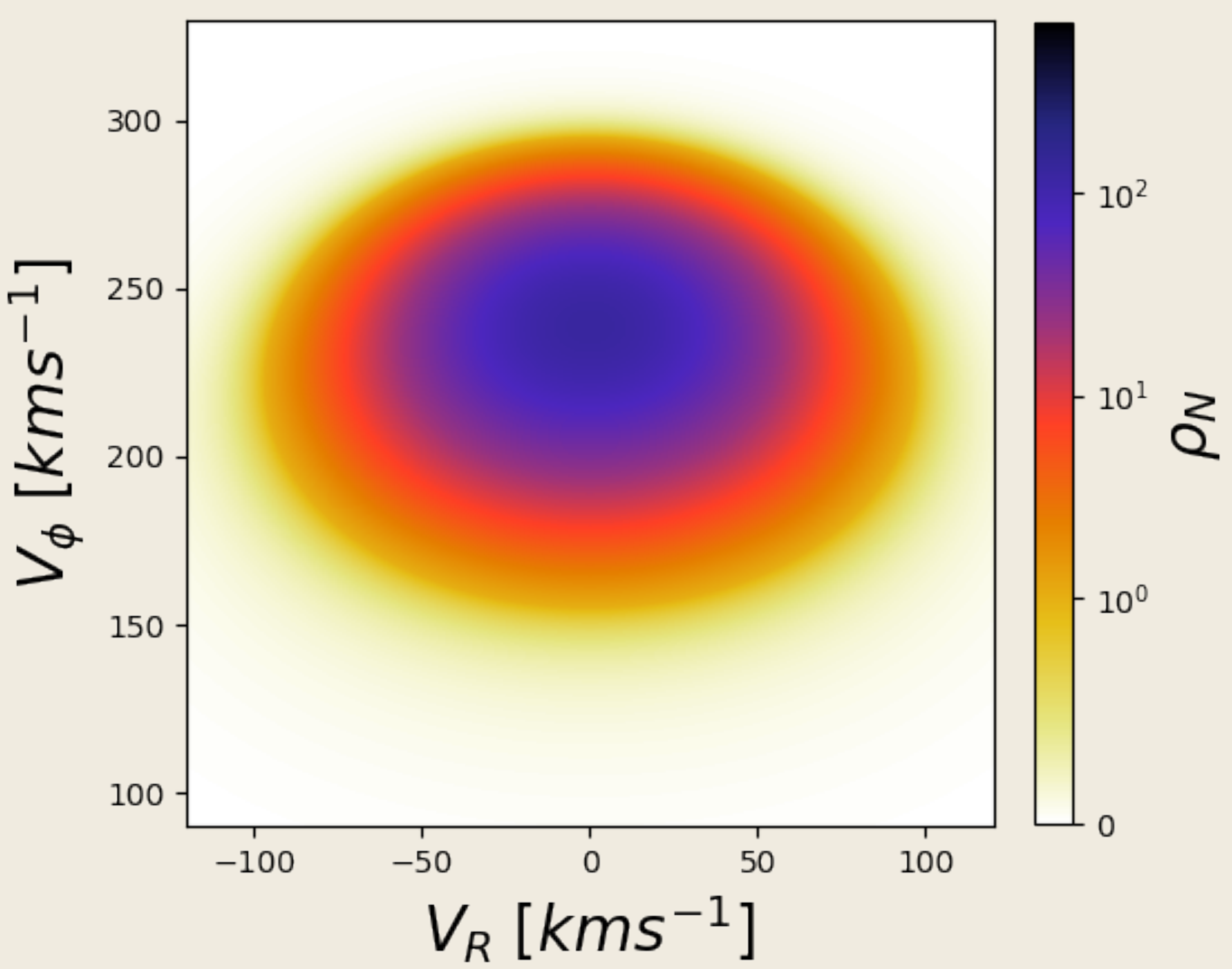
Axisymmetric model





- Thin disk
- Thick disk
- Dark Matter
- Bulge

$$f = \eta \frac{\Omega}{2^{\frac{5}{2}}\pi^{\frac{3}{2}}\kappa\tilde{\sigma}_R^2} exp\left(-\frac{R_g}{h}\right) exp\left(-\frac{J_r\kappa}{\tilde{\sigma}_R^2}\right)$$

Properties

DM halo mass: $3.1 \times 10^{11}\, M_\odot$

Baryonic mass: $6 \times 10^{10}\, M_\odot$

Mass inside $20\, kpc$: $2.2 \times 10^{11}\, M_\odot$

$\Sigma_{DM,\odot} = 1.3 \times 10^{-2}\, M_\odot pc^{-3}$

$s(0) = 0;\ s(1) = -0.6,\ s(3) = -1$

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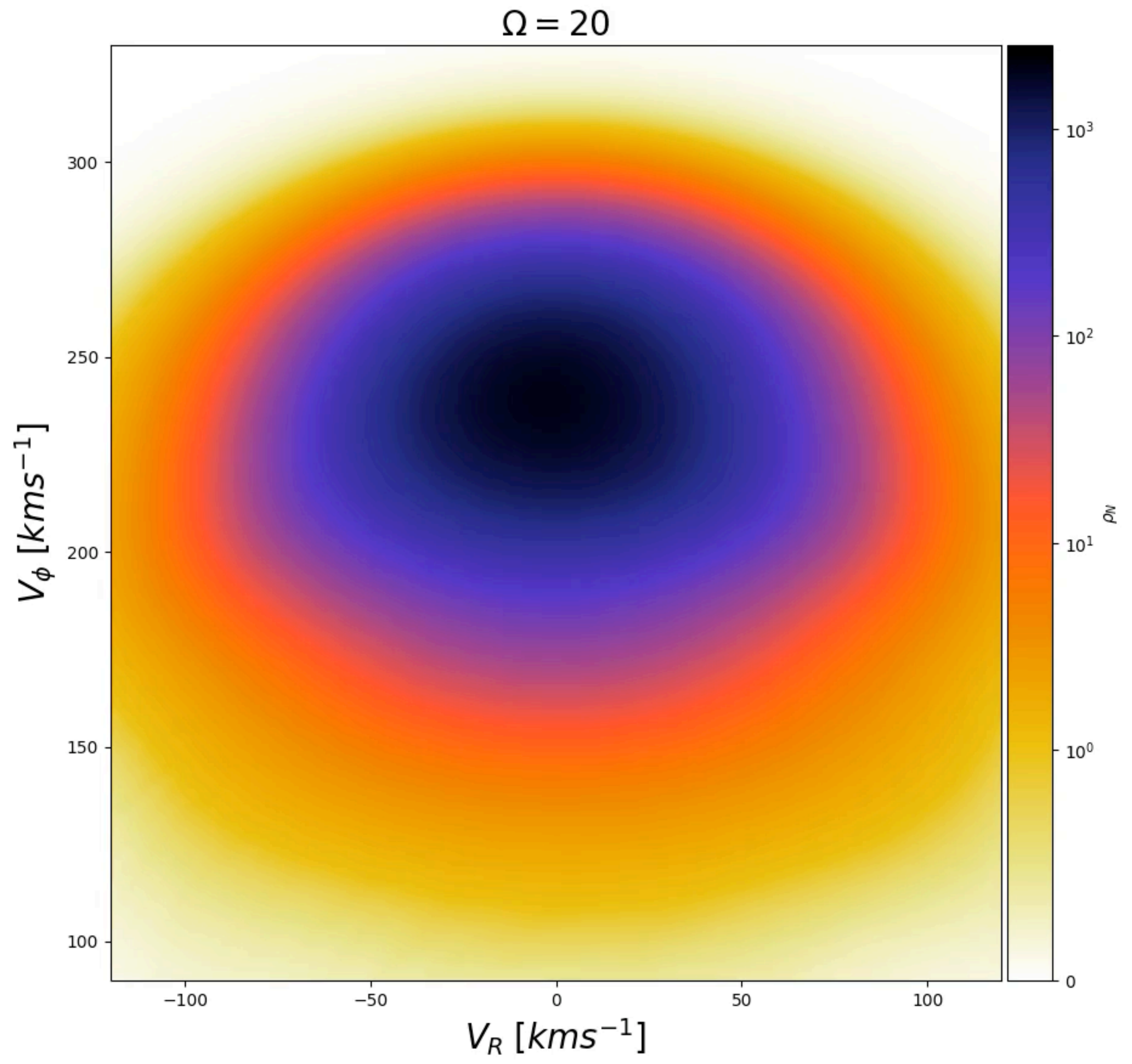
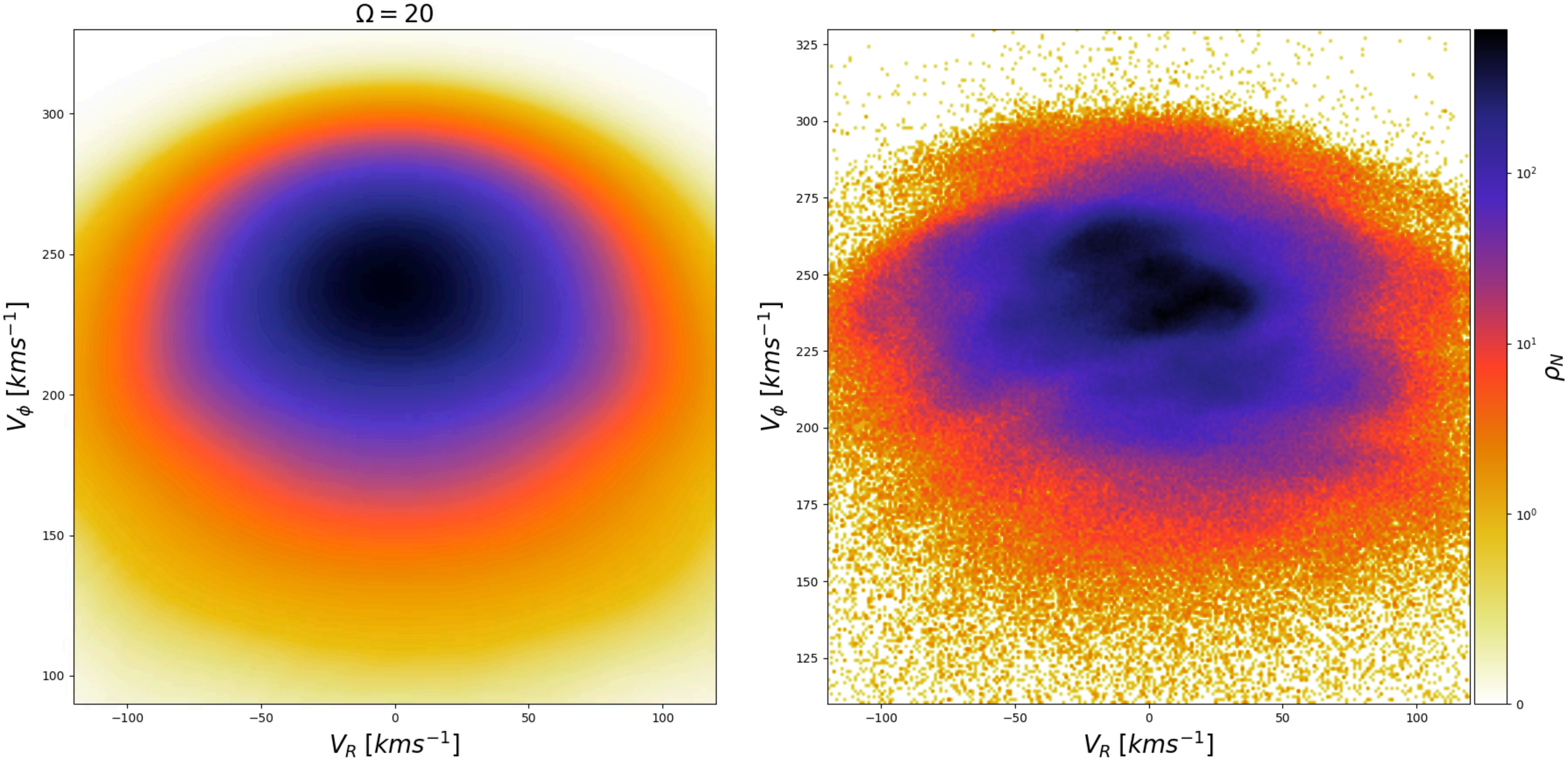
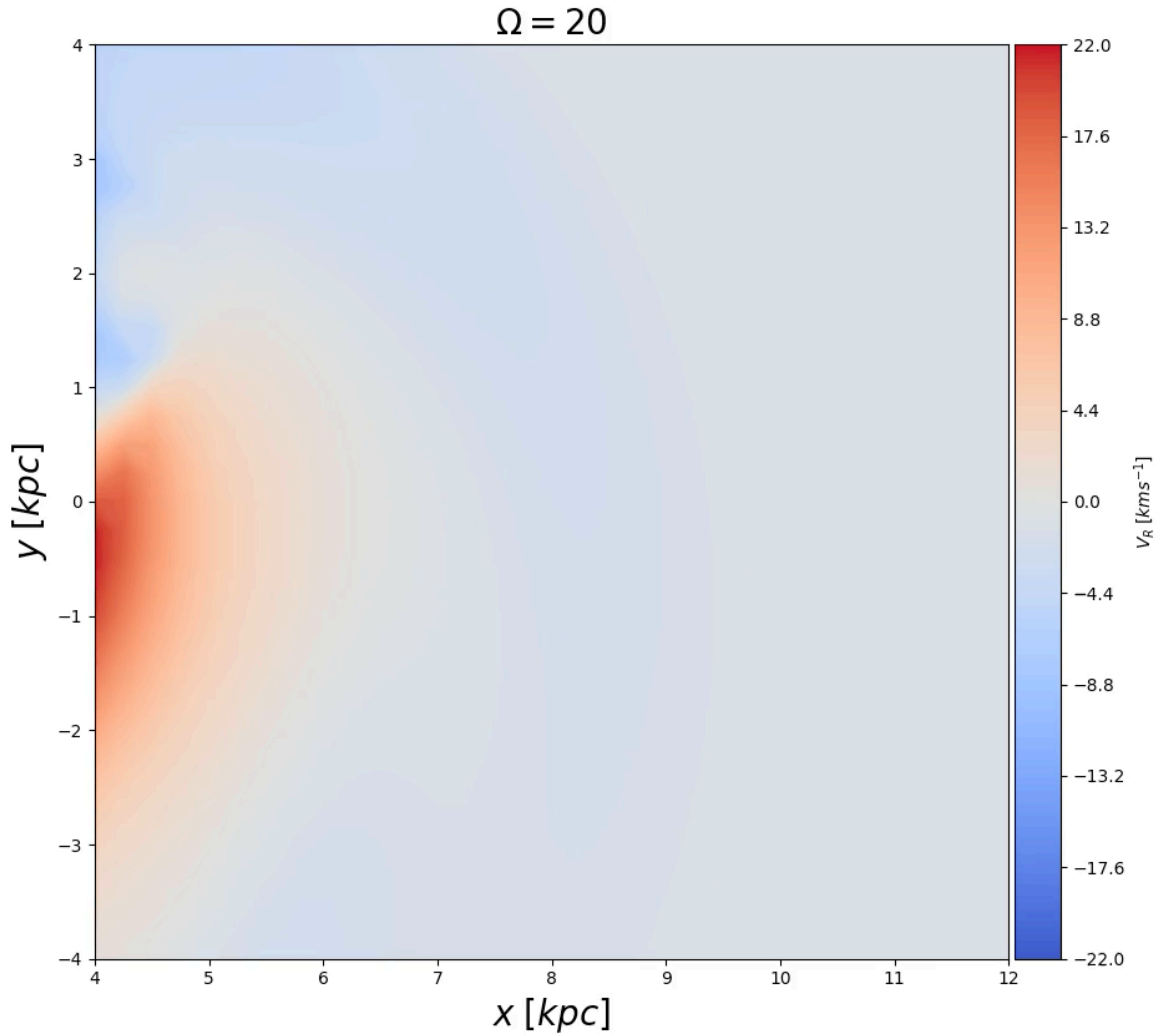


Bar model

- Model from Thomas et al., 2023
- 3 superposed modes $m = 2{,}4{,}6$
- Angle: 28°
- Length $\approx\ 5kpc$
- Visible perturbations and resonances
  - $l\kappa + m(\Omega_{bar} - \Omega) = 0$
- Co-rotation and Hat are very constraining

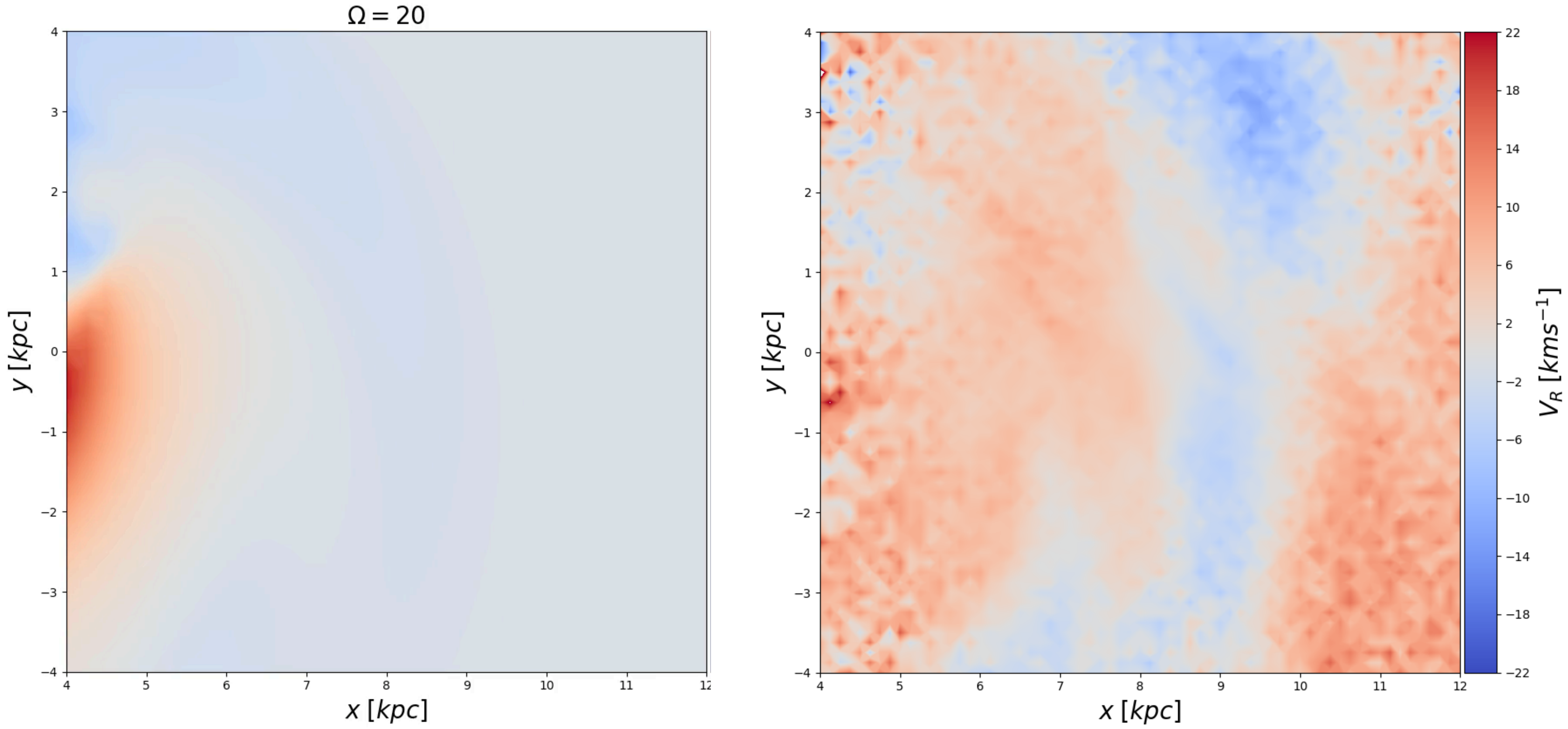


$\Omega = 20$
$V_\phi\ [kms^{-1}]$
$V_R\ [kms^{-1}]$
ρ_N



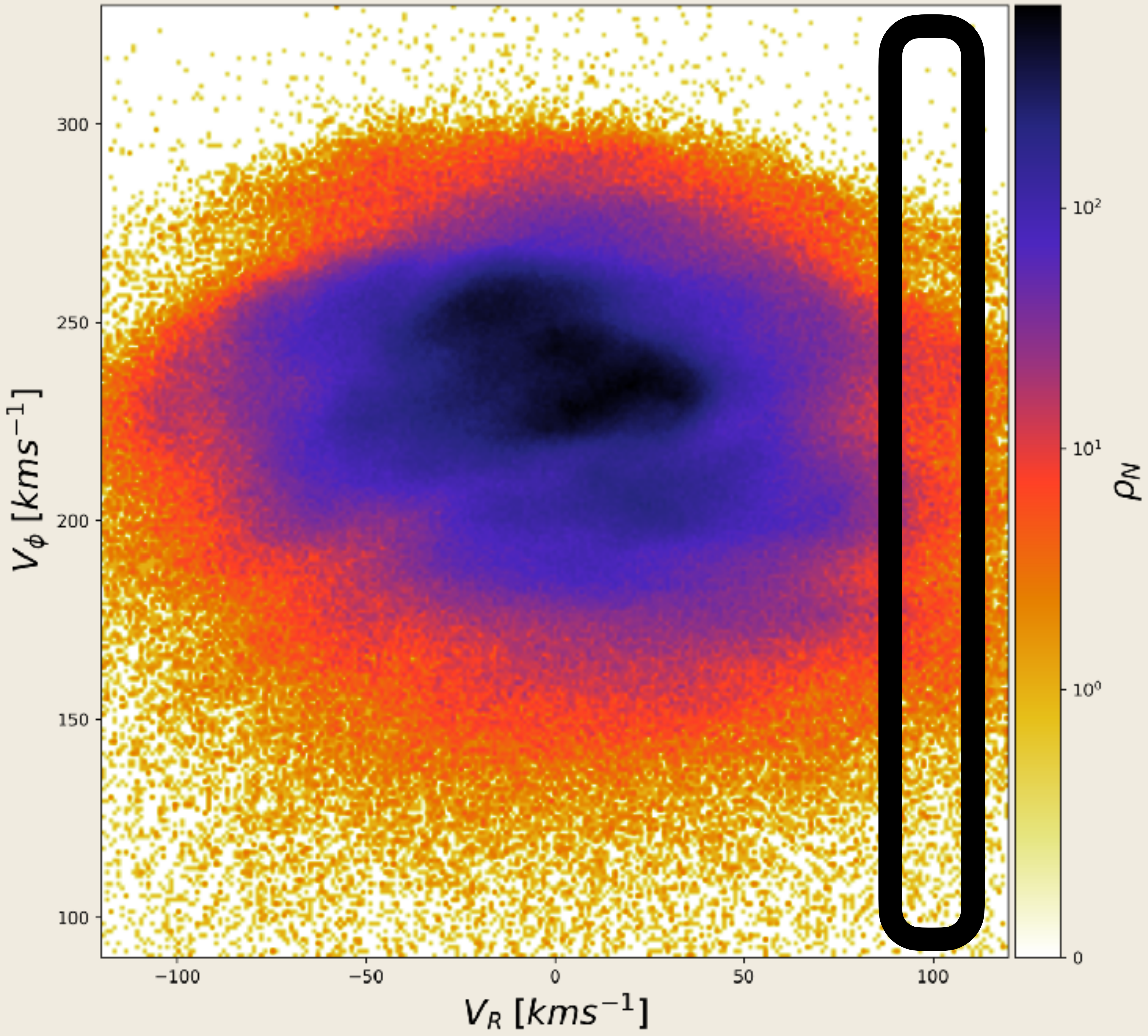
Bar model

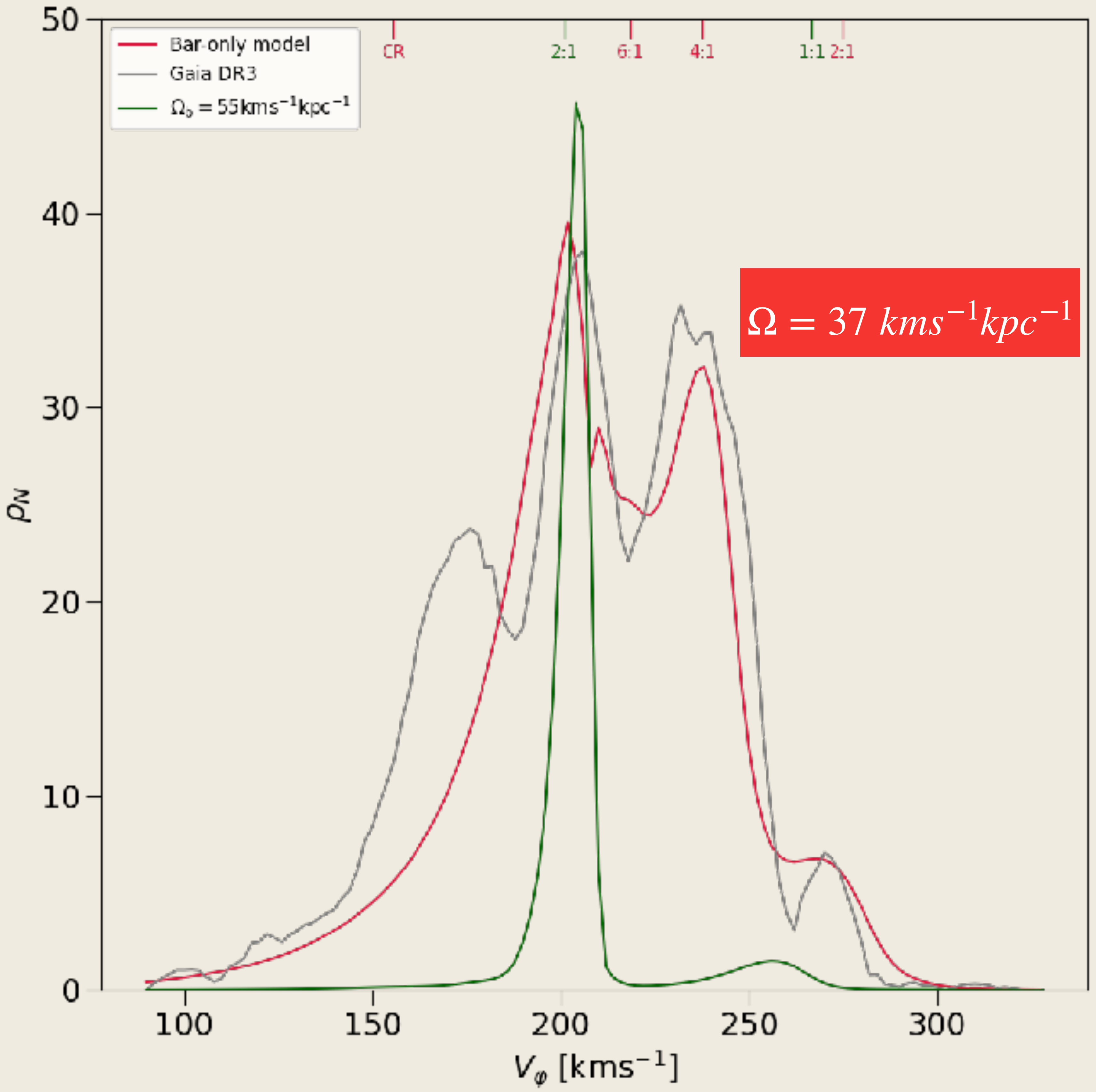
- Don’t reproduce data fully
- Good at inner regions when $40 \gtrsim \Omega \gtrsim 35$
- Too strong features at $\Omega > 40$
- Can be fixed when exploring the spiral arms (as it affect mostly the inner parts)



$\Omega = 20$
$y\ [kpc]$
$x\ [kpc]$
$y\ [kpc]$
$x\ [kpc]$
$V_R\ [km s^{-1}]$

Bar model





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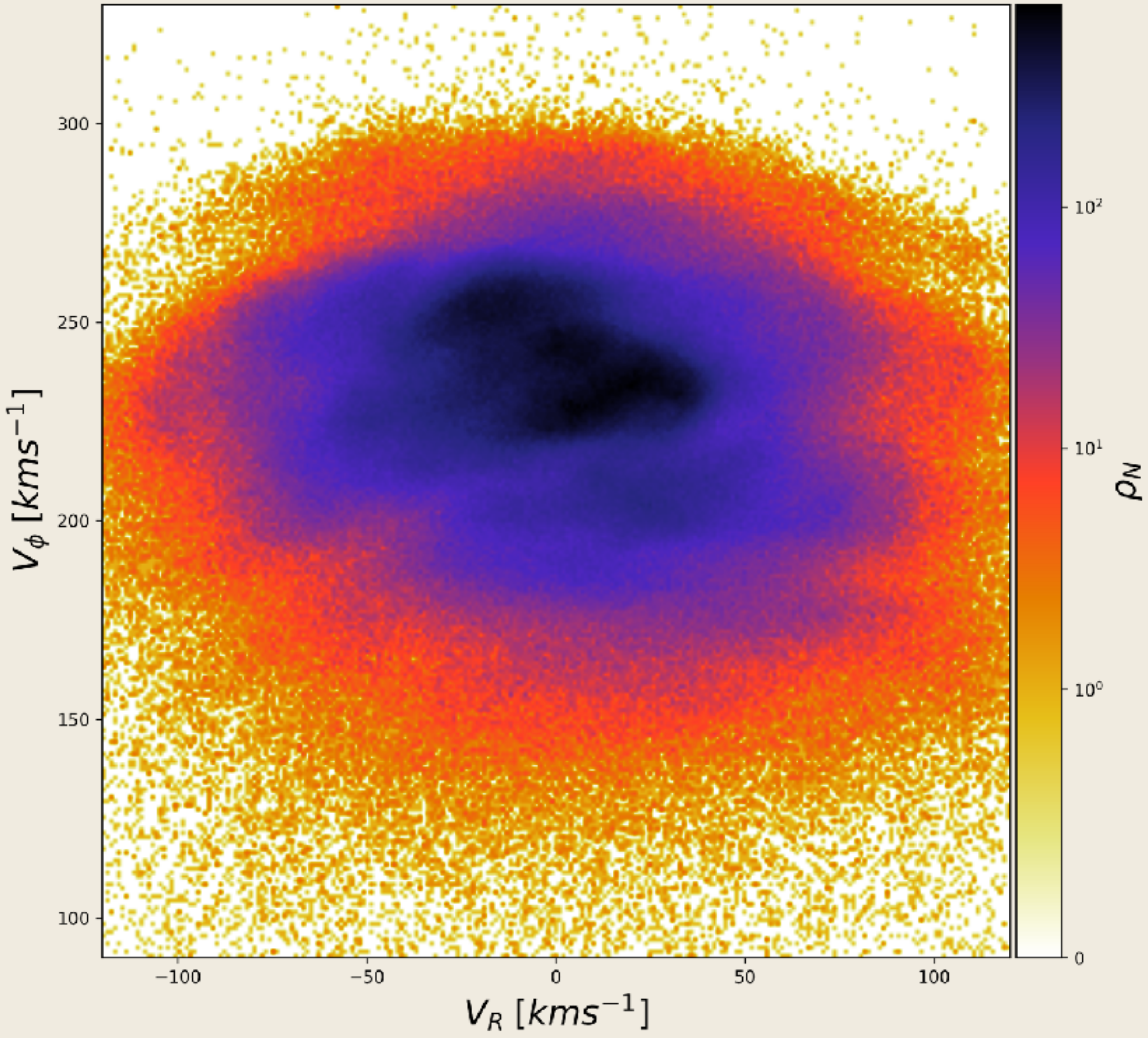
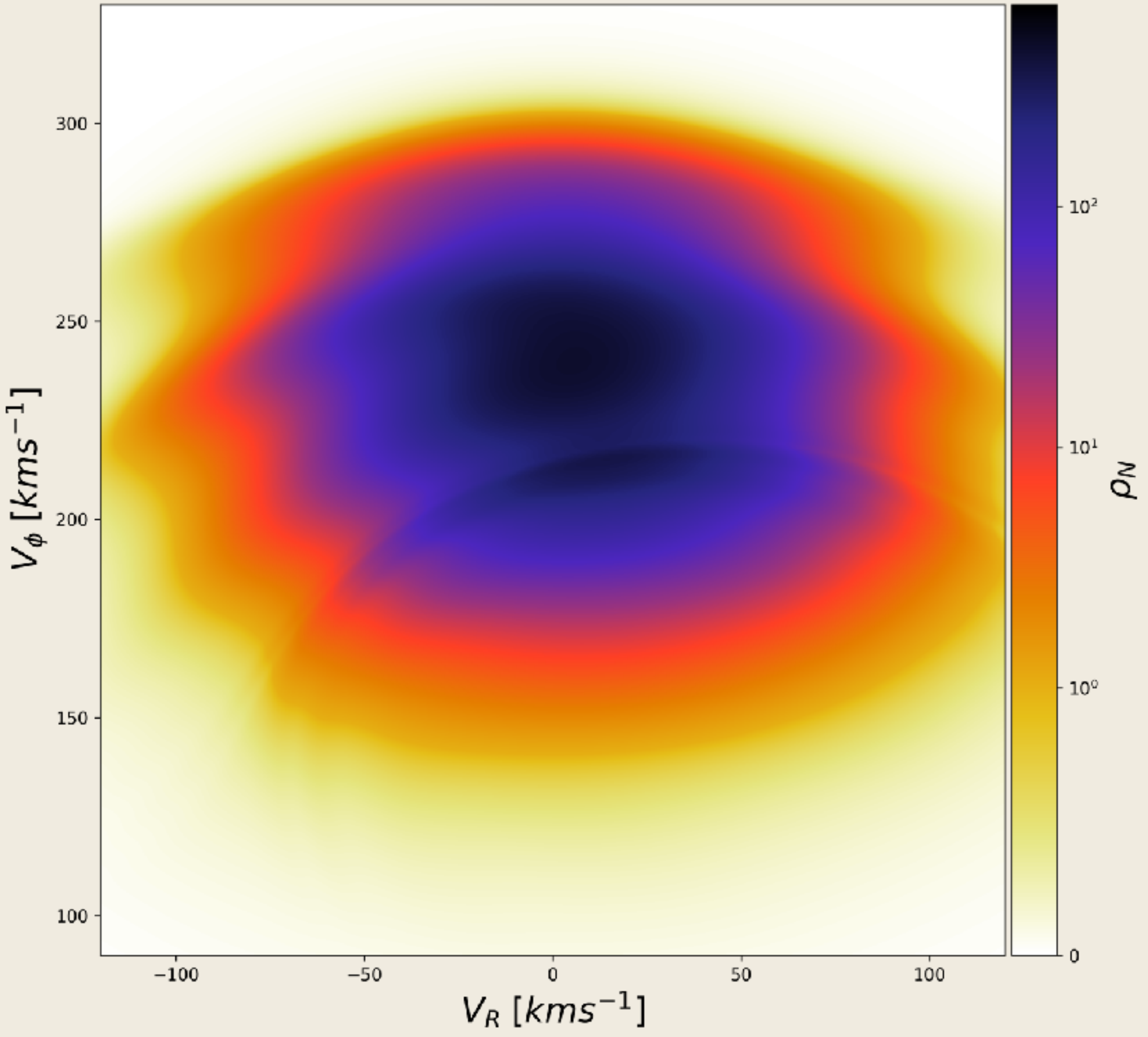
Bar model





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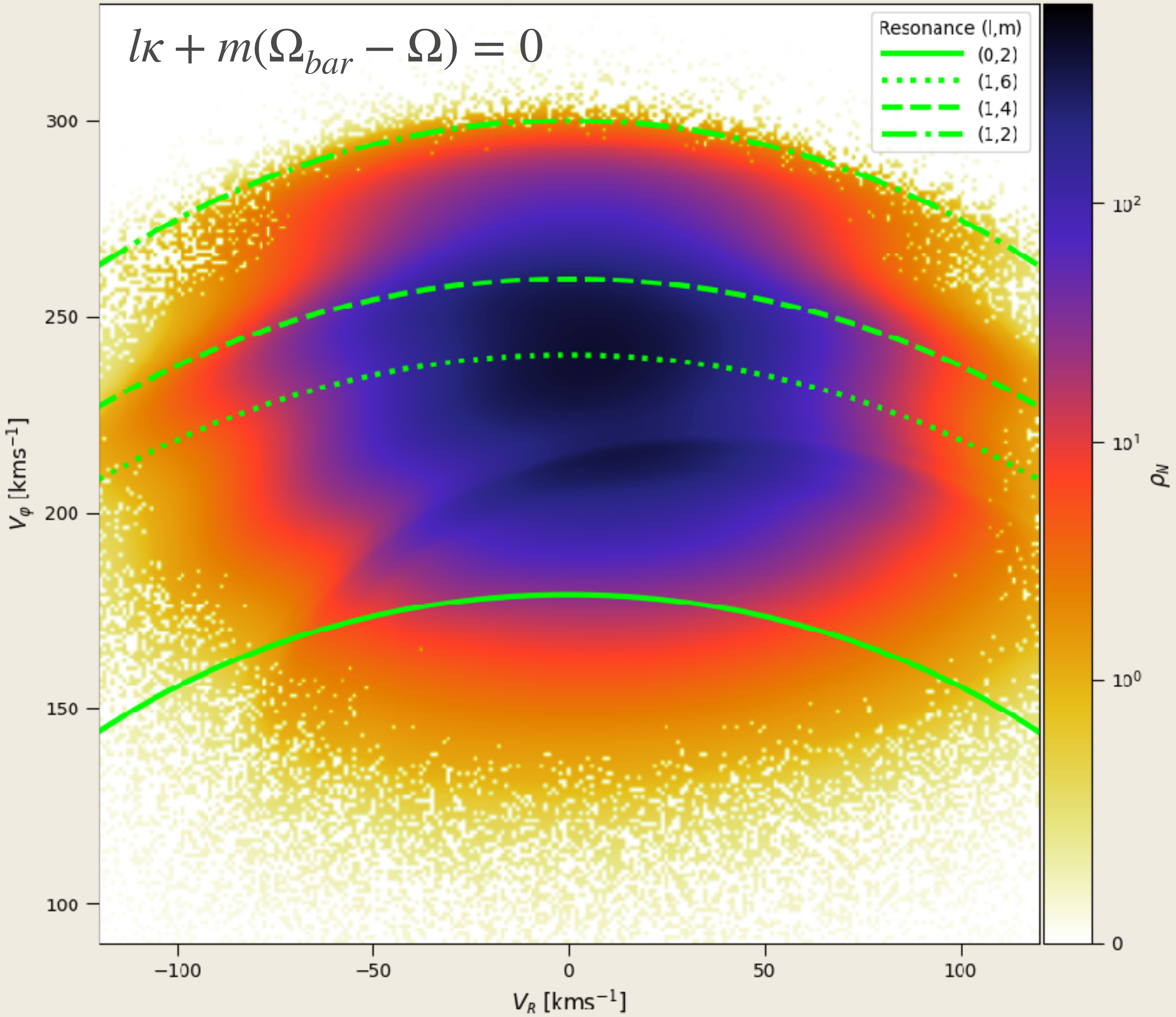
Bar model



Using Pau Ramos approximation.

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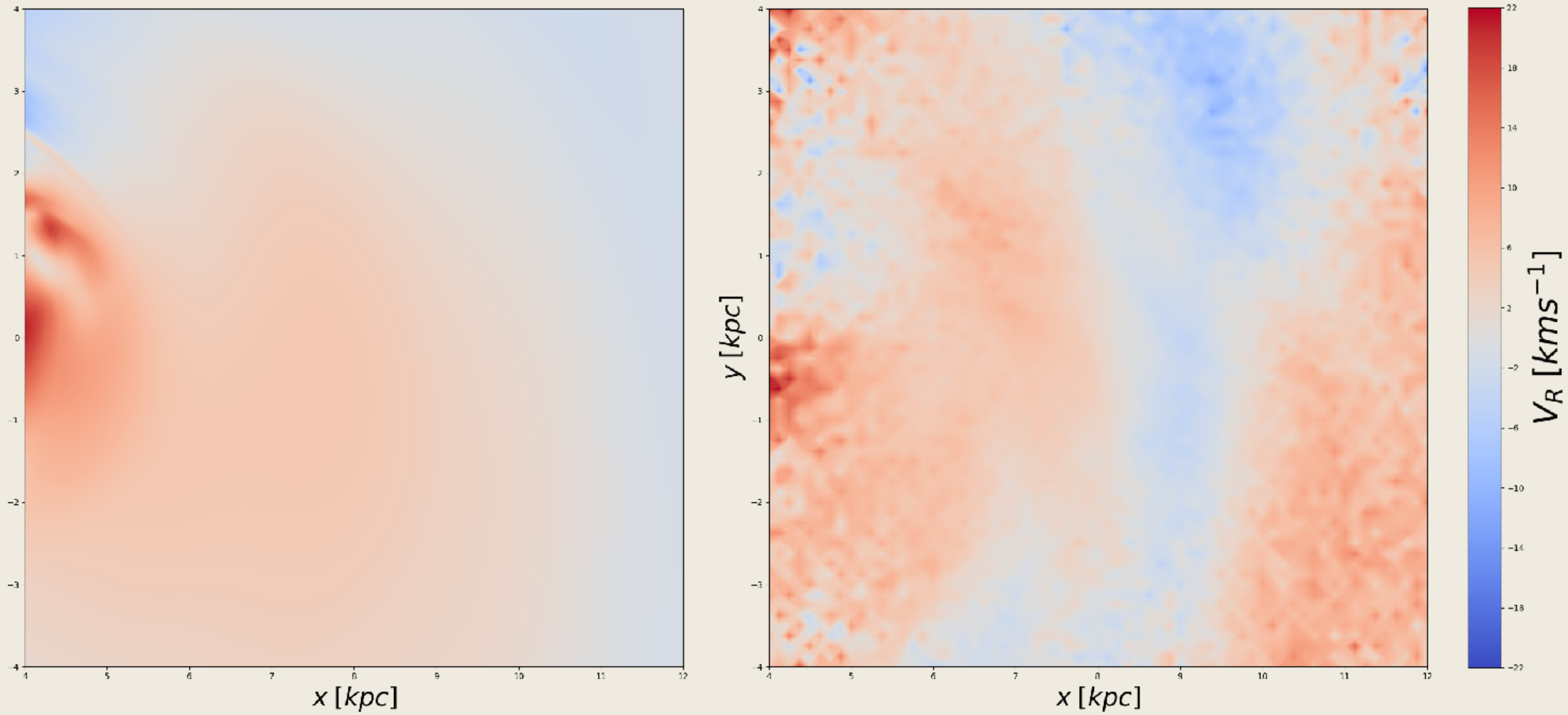
Bar model



Probing the parameter space

- Differential evolution (DE) to search parameters candidates for axisymmetric+bar+spiral arms models

- Constraints on velocity sign for some points
    - Constrained DE (J. Lampinen 2002 in Scipy)
- Constraints on Sirius at Solar Neighbourhood
- Constraints on V_R for 16 points on the disk
- Constraint on DF possible

- Pitch angle: $6^\circ < i < 30^\circ$
- Phase: $0^\circ < \phi_0 < 360^\circ$
- Density contrast: $0\,\% < \delta < 35\,\%$
- Pattern speed $10 < \Omega < 37\ (kms^{-1}kpc^{-1})$

- Best candidate
    - Mode m=2
        - Start growing 60 Myr after the Bar
        - Contrast of density of about 25%
    - Mode m=3
        - Start growing 160 Myr after the Bar
        - Contrast of density of about 10%

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Extended velocity field

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Local velocity field

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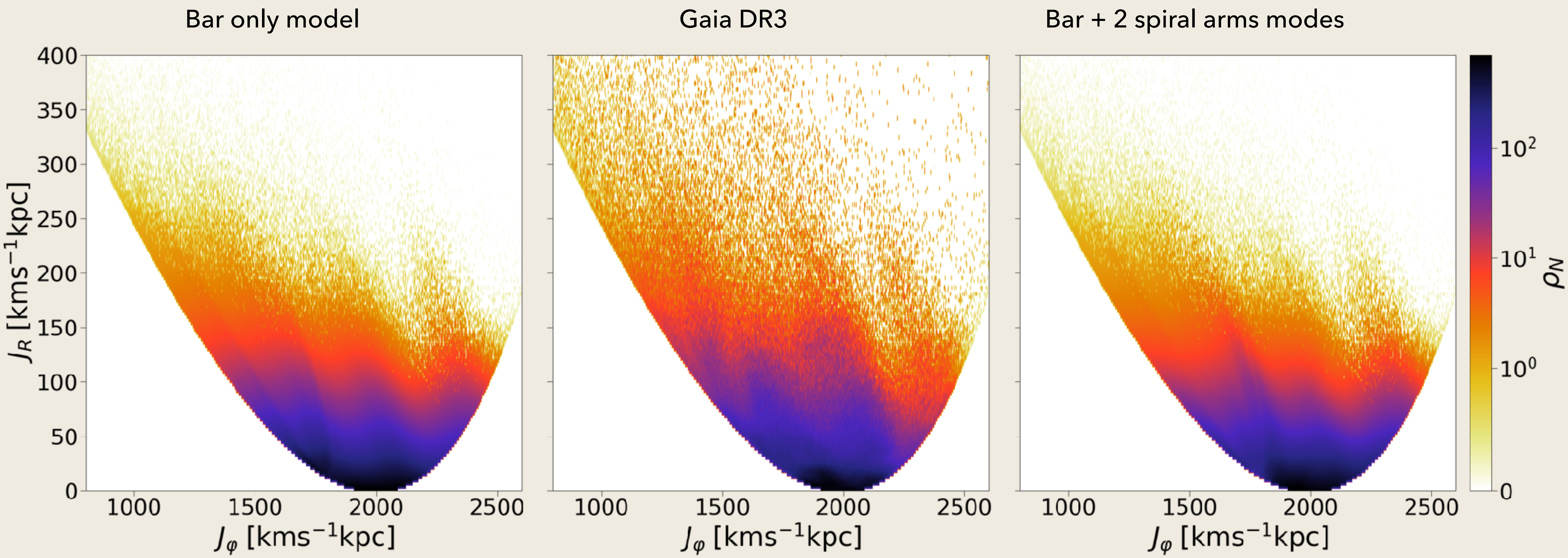

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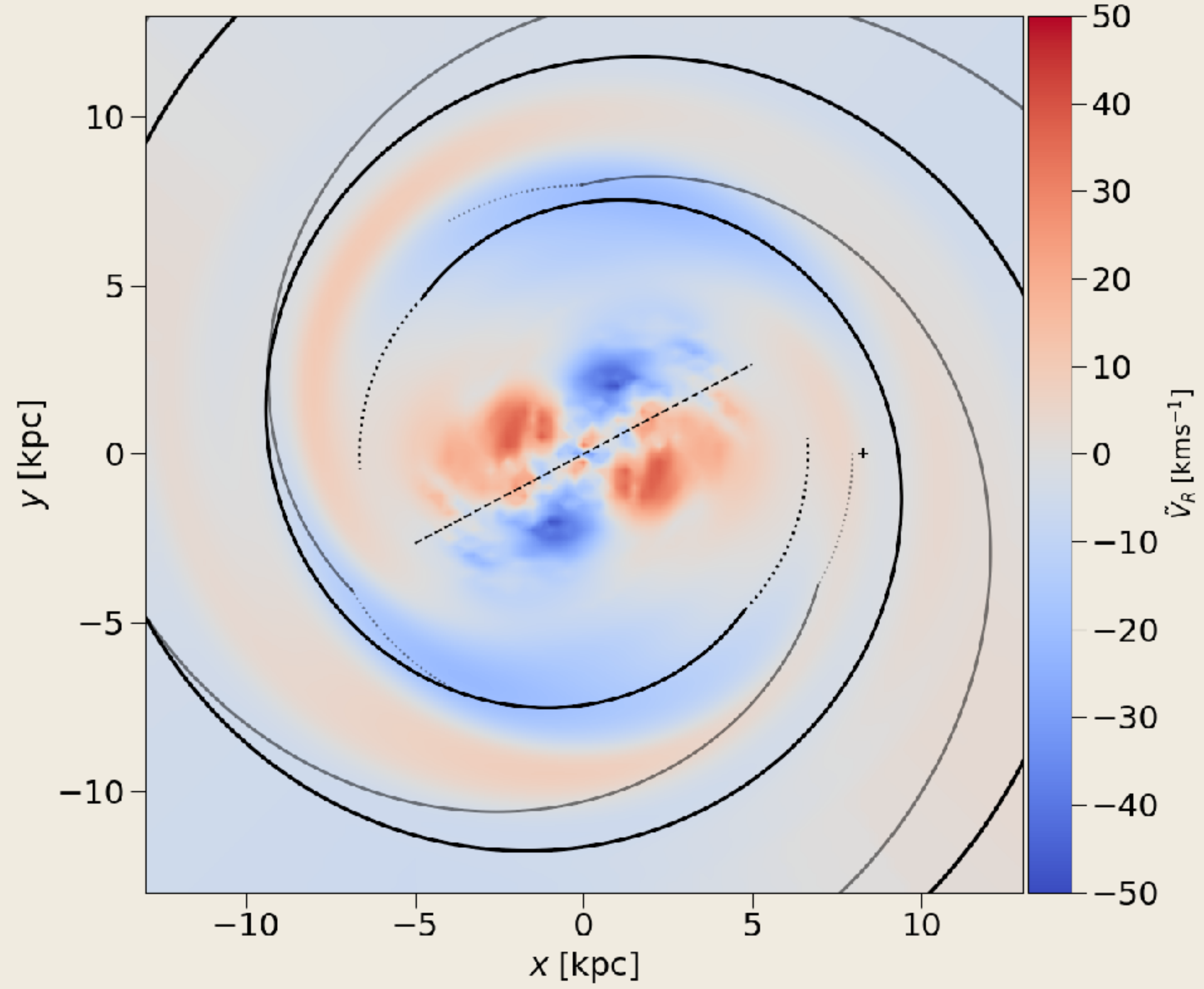
Milky Way disk: Median radial velocity



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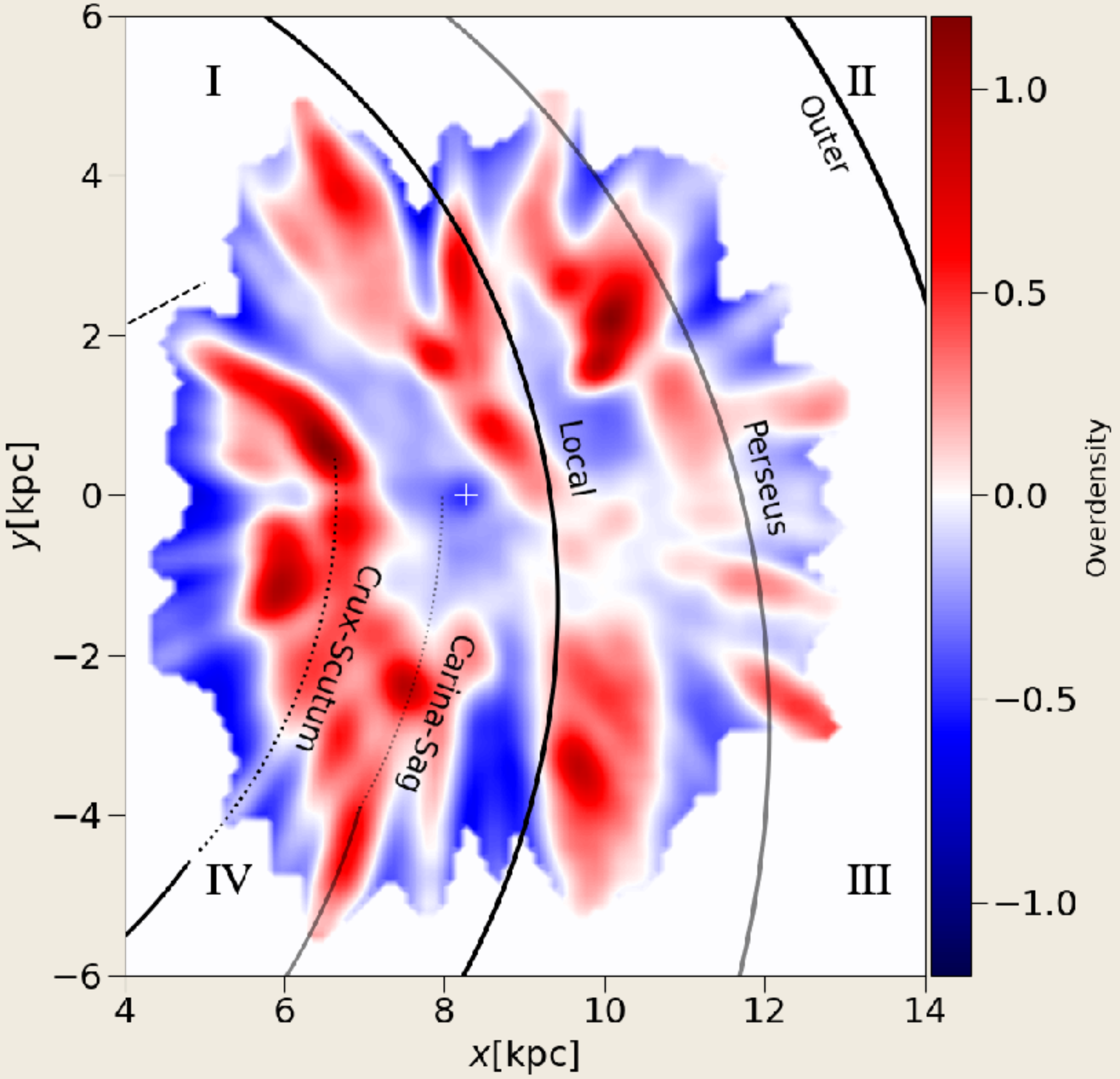
Position of the spiral arms



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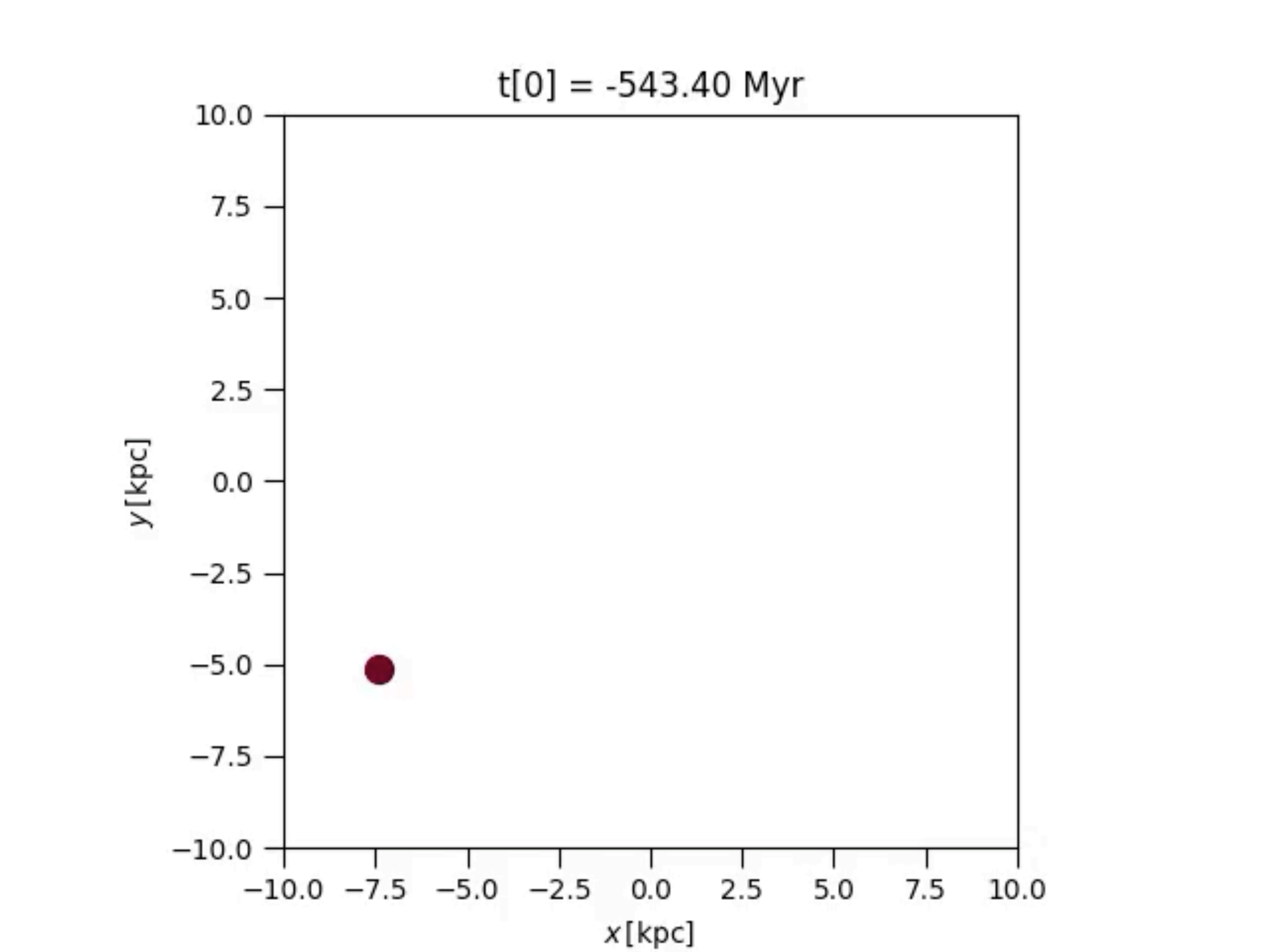
Implications on the Solar orbit



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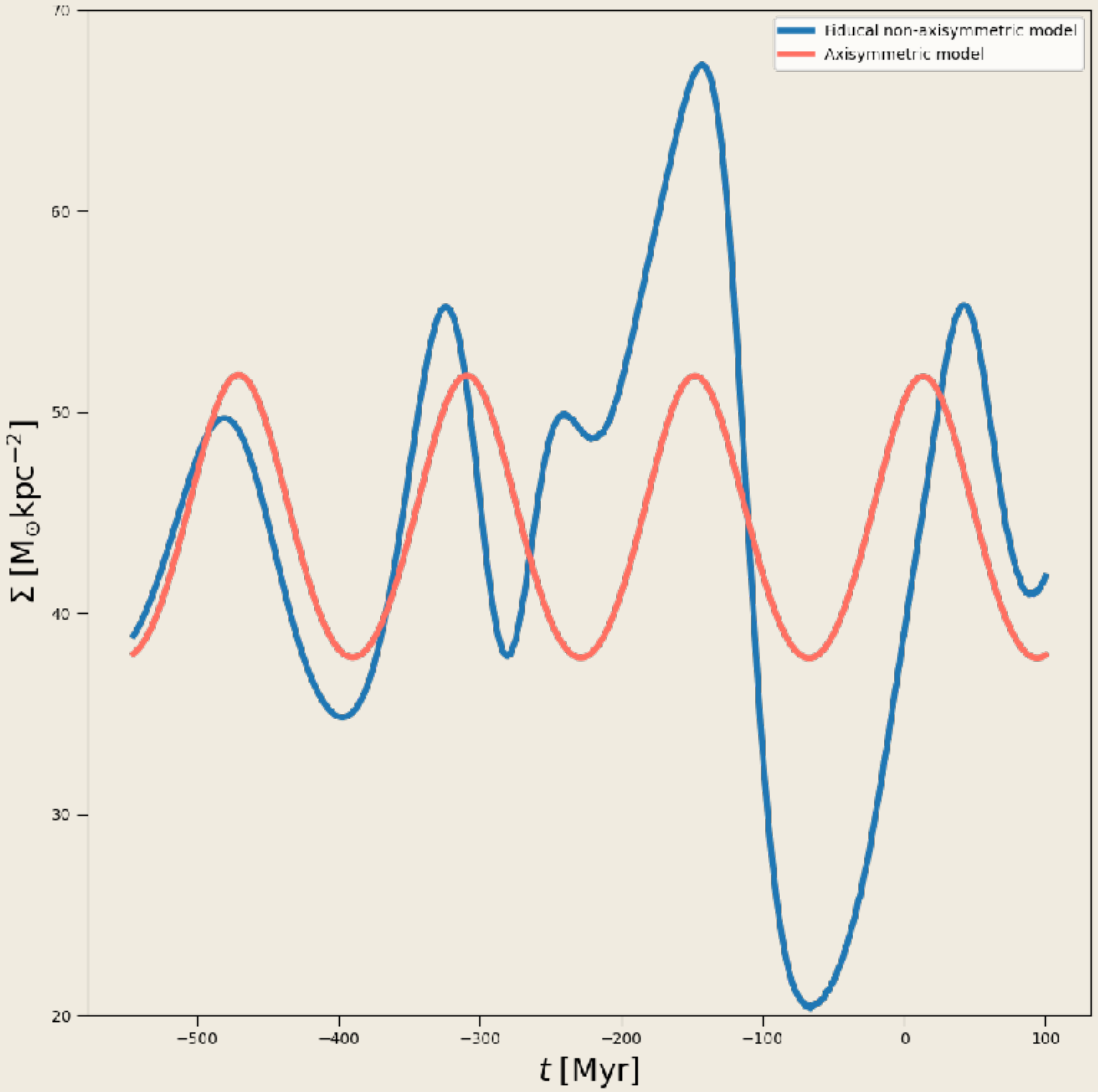
Implications on the Solar orbit



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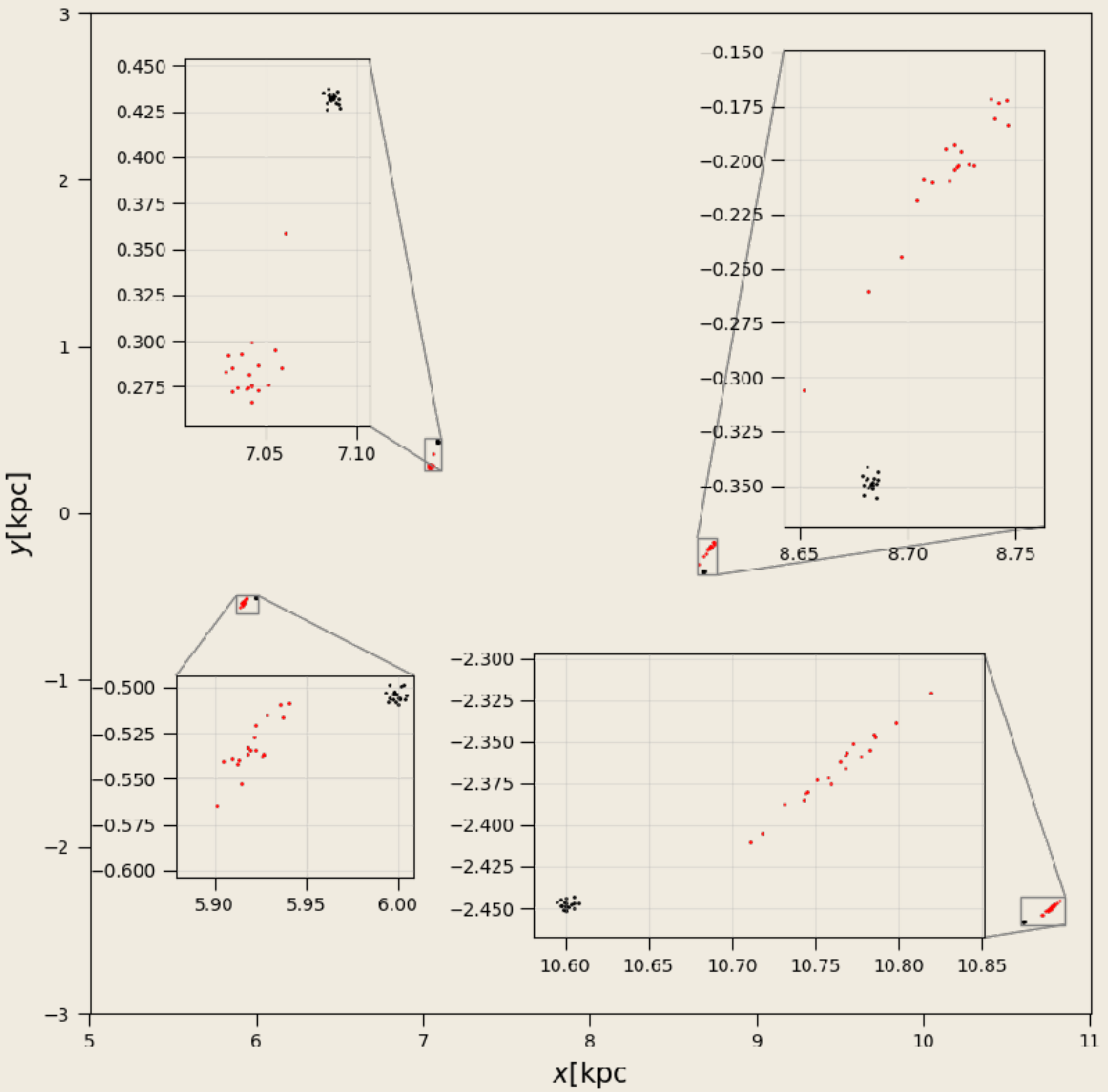
Implications on Young Associations



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Conclusion & Perspectives

- Possibly the most realistic non-axisymmetric dynamical model for the Milky Way disk
- It can be extended to 3 dimensions
- It is possible to improve the approach to constrain at once the non-axisymmetric and axisymmetric structures
- Other configurations can be explored as evolving pattern speed for the bar and/or for the spiral arms
- The established model can be used to improve direct measurements of spiral arms pattern speed with young associations

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Thank you !

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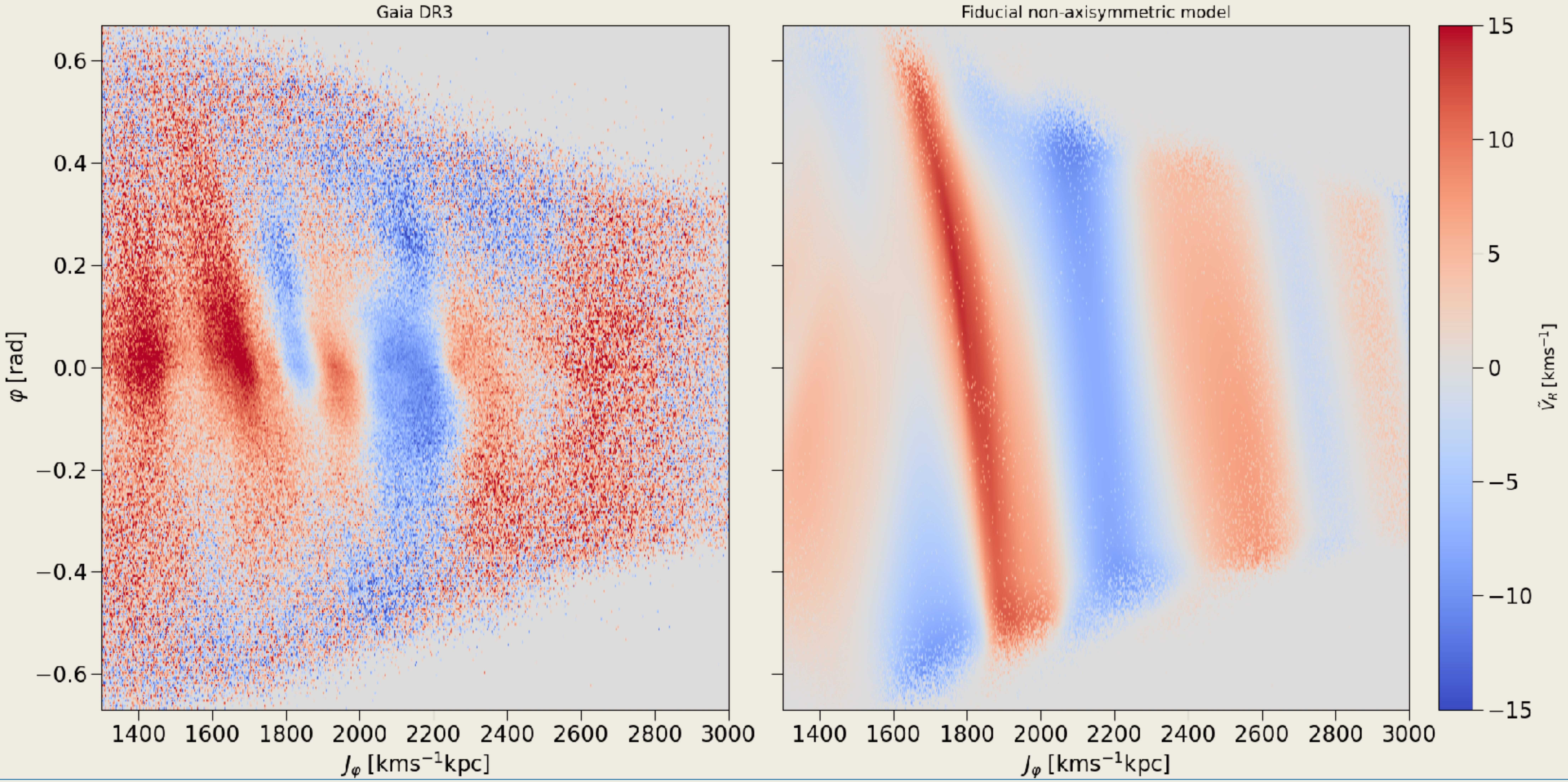
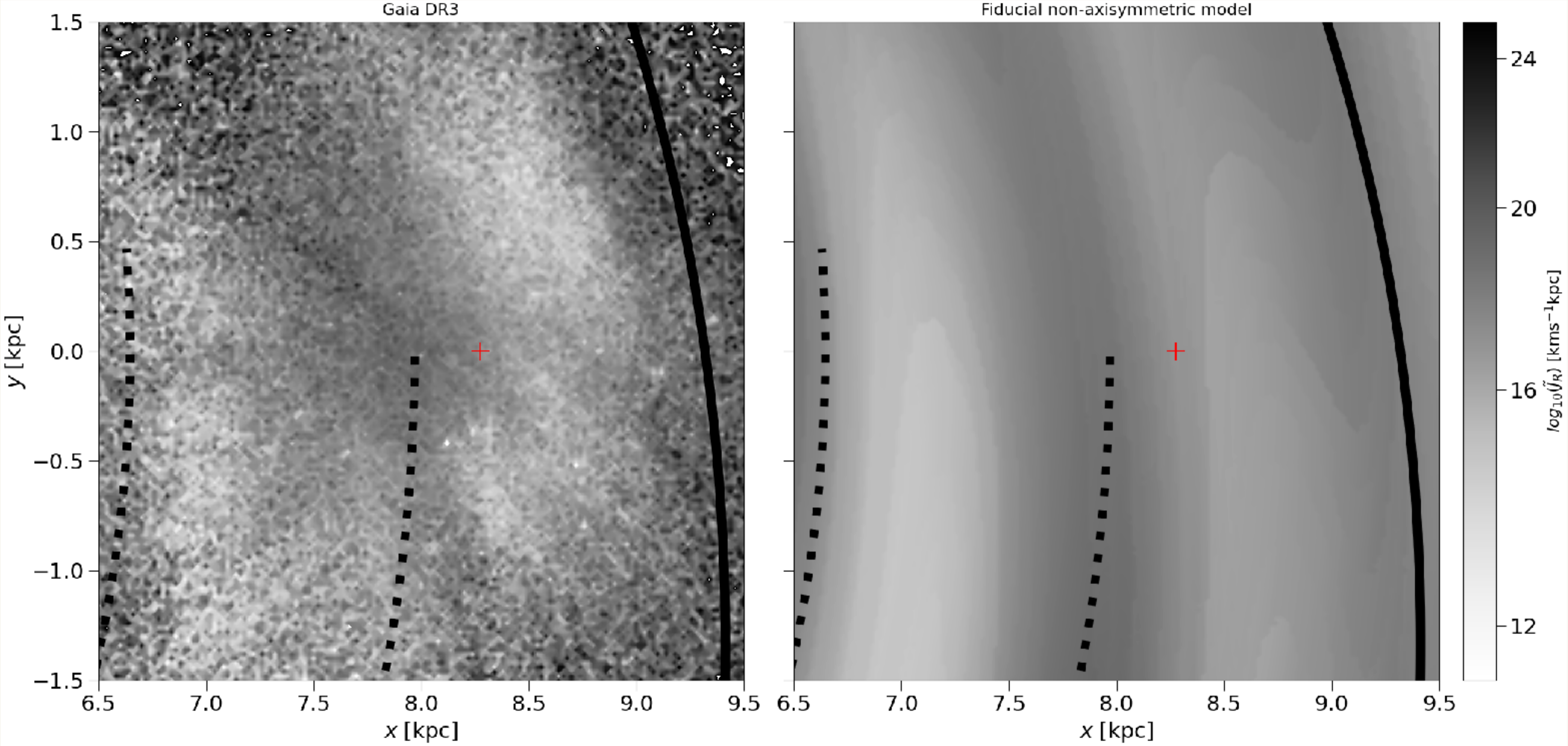
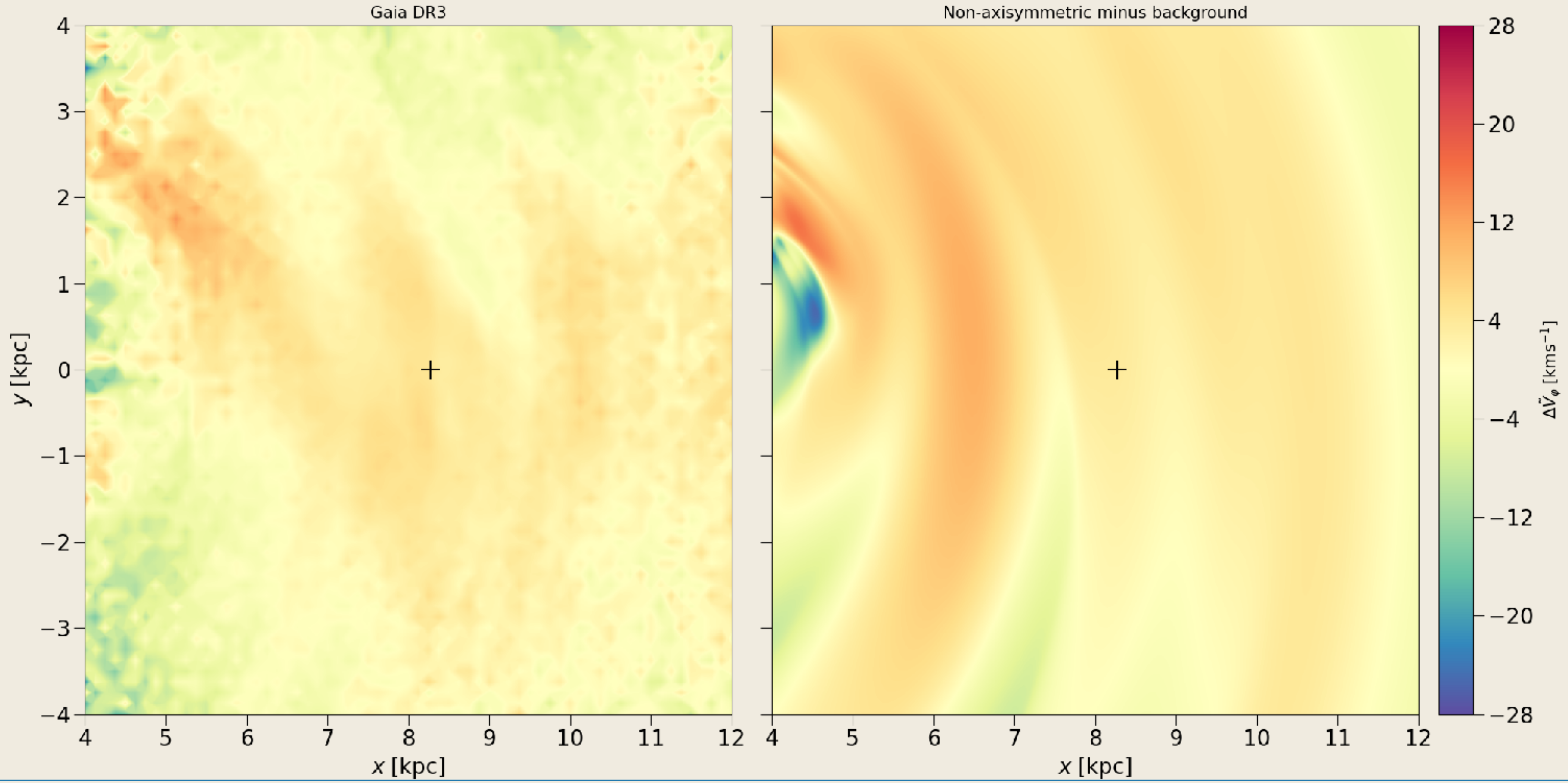
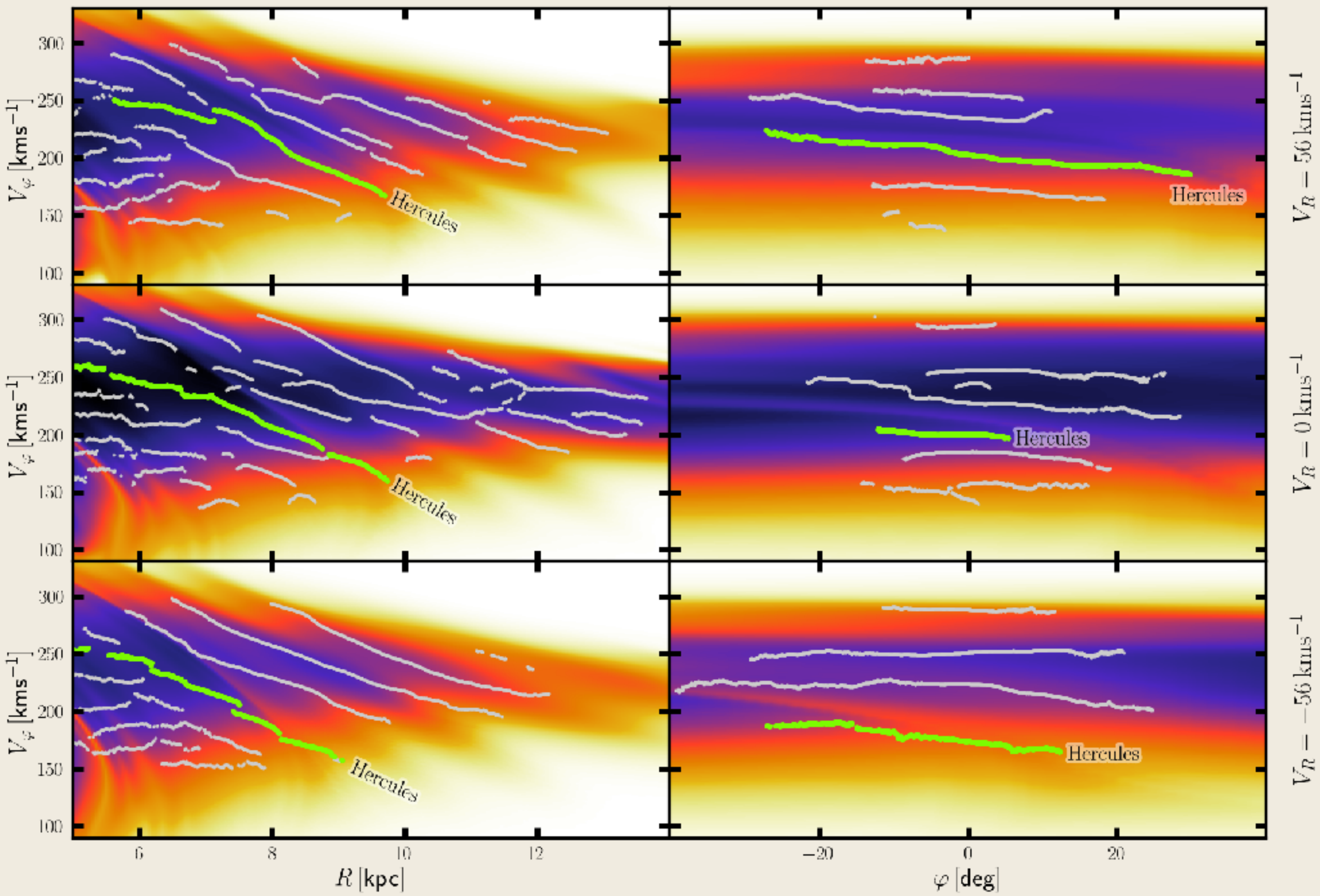

Gaia DR3
Fiducial non-axisymmetric model
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
φ [rad]
1400 1600 1800 2000 2200 2400 2600 2800 3000
J_φ [kms^{-1}kpc]
15
10
5
0
−5
−10
−15
$\tilde{V}_R$ [kms^{-1}]


Gaia DR3
Fiducial non-axisymmetric model
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
y [kpc]
6.5
7.0
7.5
8.0
8.5
9.0
9.5
x [kpc]
24
20
16
12
$log_{10}(\tilde{J}_R)$ [kms^{-1}kpc]



Gaia DR3
Non-axisymmetric minus background
y [kpc]
x [kpc]
x [kpc]
4
3
2
1
0
−1
−2
−3
−4
4
5
6
7
8
9
10
11
12
28
20
12
4
−4
−12
−20
−28
$\Delta\tilde{V}_\phi$ [kms^{-1}]



Hercules
V_φ [kms^{-1}]
R [kpc]
φ [deg]
$V_R = 56$ kms^{-1}
$V_R = 0$ kms^{-1}
$V_R = -56$ kms^{-1}

Distribution functions

- **Stellar systems:** systems of stars bounded by gravitational long-range force.
  - We assume a system of N stars of same mass m.
  - Each star has a position $\mathbf{x} = (x, y, z)$ and velocity $\mathbf{v} = (v_x, v_y, v_z)$

- **Distribution function**
  - Gives the probability $f(\mathbf{x}, \mathbf{v}, t)d^3\mathbf{x}d^3\mathbf{v}$ to find a star in the volume $d^3\mathbf{x}d^3\mathbf{v}$ centred on $(\mathbf{x}, \mathbf{v})$ at time t
  - **Density at position $\mathbf{x}$:** $\rho(\mathbf{x}, t) = \int f(\mathbf{x}, \mathbf{v}, t)d^3\mathbf{v}$
  - **Number density at $\mathbf{x}$:** $N\rho(\mathbf{x}, t)$**,** with N the number of stars
  - **Average velocity at position $\mathbf{x}$:** $\overline{\mathbf{v}} = \rho^{-1}(\mathbf{x}, t)\int \mathbf{v}f(\mathbf{x}, \mathbf{v}, t)d^3\mathbf{v}$

Vlasov-Poisson Equation

- **Vlasov-Poisson Equation**

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{x}} - \nabla \Phi \cdot \frac{\partial f}{\partial \mathbf{v}} = 0$$

$$\nabla^2 \Phi = 4\pi G \int d^3\mathbf{v} f$$

- **Asymptotic limit** of an infinite particle stellar system for the first equation (with negligible interaction term) of **BBGKY hierarchy** formulation of the **Liouville equation**

- **Collision-less dynamics**
  - **Relaxation time** τ_{relax}**:** time to a star's velocity change by its order thorough stellar encounters
    - Typically $\tau_{relax} > \tau_{Hubble}$. Increases with number of stars and crossing time $\left(\frac{R}{v}\right)$.

Vlasov-Poisson Equation

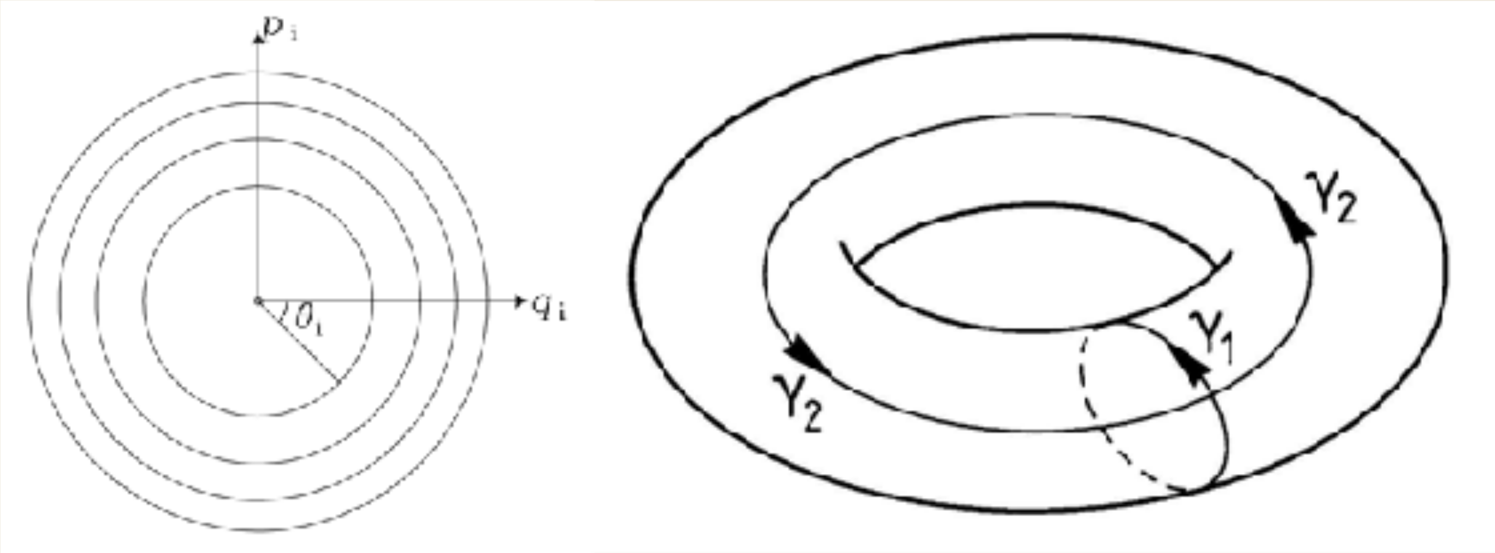
- Key results encoded in the Vlasov equation:
  - The distribution function in an infinitesimal Lagrangian volume is conserved. $$\frac{df}{dt} = 0$$
  - The distribution function is conserved along the orbits

- Open the possibility to the backwards integration method (Vauterin & Dejonghe 1997; Dehnen 1999)
  - Why this is important ?
    - It is very hard to compute distribution functions for the galaxy with both the bar and spiral arms
      - Multiple pattern speed are concerned
        - Resonant effects and overlap of resonances can be very hard to characterise
      - Usually made with perturbation theory for one structure alone

Actions-Angles variables

- Jeans theorems help us solve for the Vlasov equation for equilibrium:
  - If integrable system: $f = f(I_1, I_2, I_3)$ and if in axisymmetry and equilibrium: $f = f(E, L_z, I_3)$.

- But, how to choose the integrals I? Actions $\mathbf{J}$ and Angles $\boldsymbol{\theta}$
  - Canonical variables: $H = H(\mathbf{J})$ and $\mathbf{J} = const$ as well as $\dot{\boldsymbol{\theta}} = const$
  - Natural phase-space coordinates for regular orbits in (quasi)-integrable systems.
  - Transforming $(\mathbf{x}, \mathbf{v})$ to $(\mathbf{J}, \boldsymbol{\theta})$ is volume-conserving (appropriate for DFs).



Angle-Action variables as polar coordinates. Binney & Tremaine 2008.

Actions-Angles variables

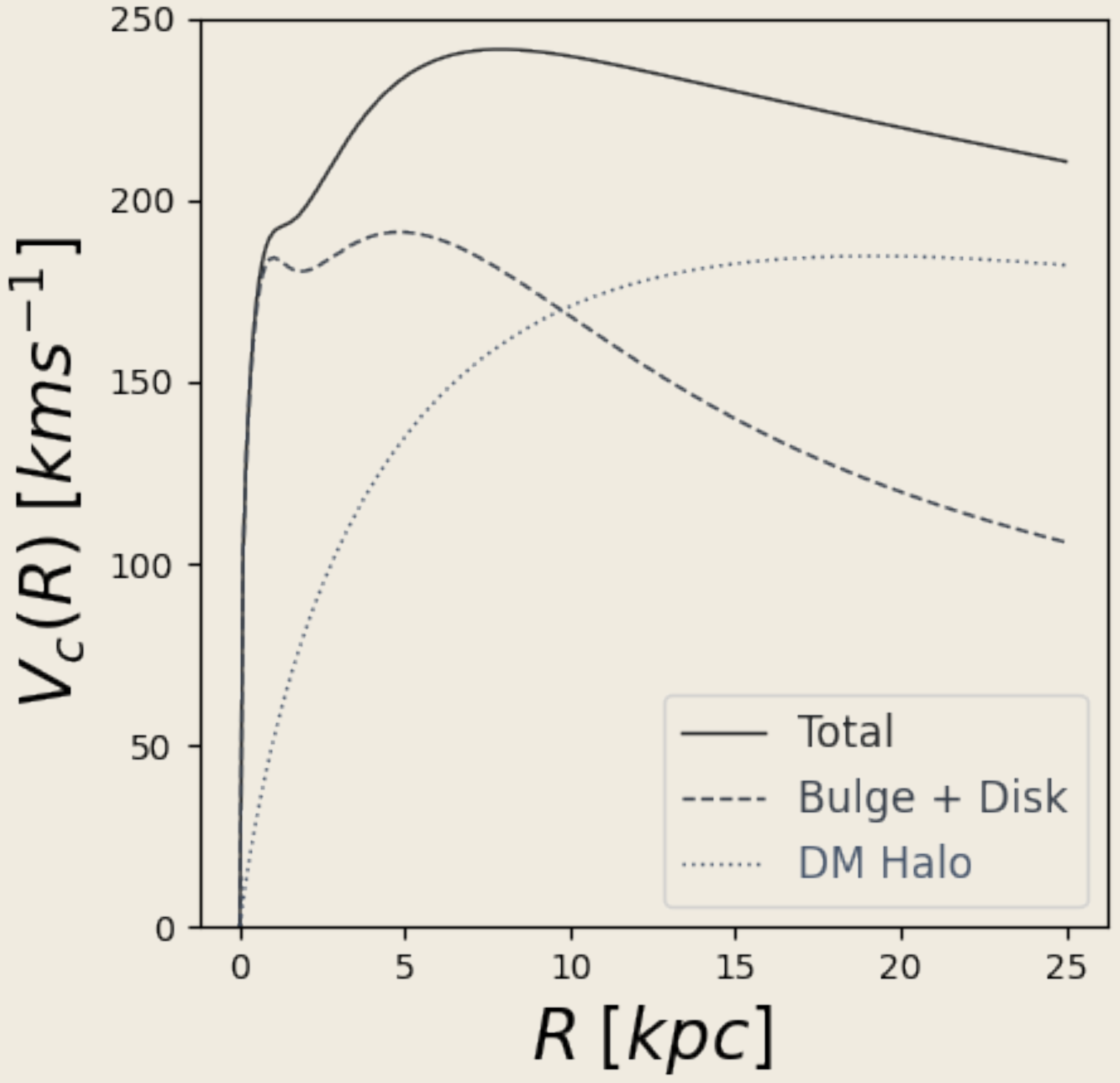
- How we compute it ?
  - Transformations of $(\mathbf{x}, \mathbf{v})$ to $(\mathbf{J}, \boldsymbol{\theta})$ is exactly know for some separable potentials like the Stäckel potential
  - In the Stäckel Fudge we use the real potential locally as if it were a Stäckel potential:

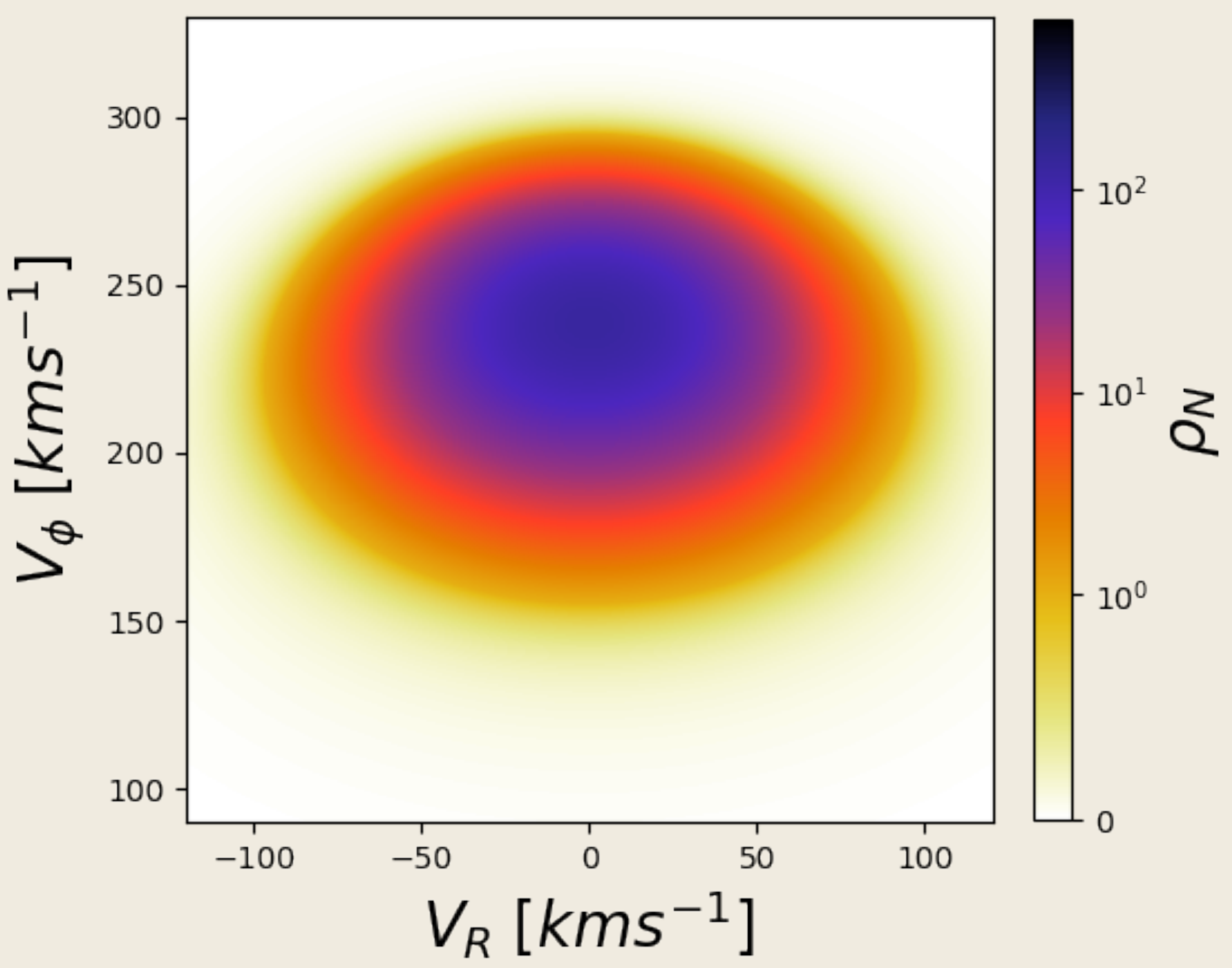
$$\Delta_2 = z^2 - R^2 + 3\left[3z\frac{\partial\Phi}{\partial R} - 3R\frac{\partial\Phi}{\partial z} + Rz\left(\frac{\partial^2\Phi}{\partial R^2} - \frac{\partial^2\Phi}{\partial z^2}\right)\right]\left(\frac{\partial^2\Phi}{\partial R\partial z}\right)^{-1}$$

(Sanders 2012)

- In practice, accurate and efficient computations it with AGAMA (Vasiliev 2019) library

Axisymmetric model





Khalil et al., in preparation.

- Thin disk
- Thick disk
- Dark Matter
- Bulge

$$f = \eta \frac{\Omega}{2^{\frac{5}{2}} \pi^{\frac{3}{2}} \kappa \tilde{\sigma}_R^2} exp\left(-\frac{R_g}{h_R}\right) exp\left(-\frac{J_r \kappa}{\tilde{\sigma}_R^2}\right)$$

$$\tilde{\sigma}_R(R_g) = \tilde{\sigma}_R(R_0) exp\left(-\frac{R_g - R_0}{h_{\sigma,R}}\right)$$

$$f(\mathbf{J}) = f_{thin}(\mathbf{J}) + \beta f_{thick}(\mathbf{J})$$

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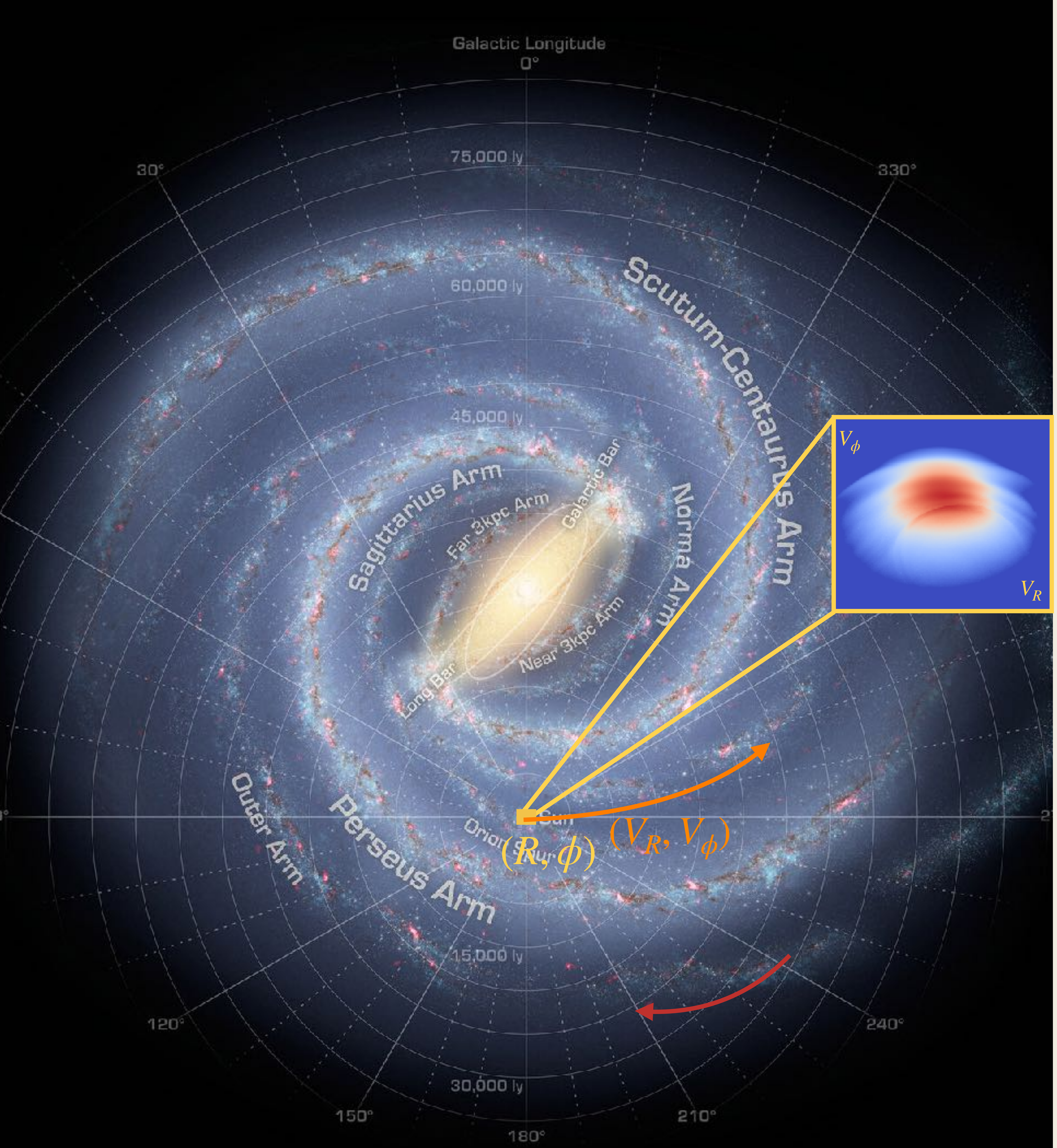


Credits: NASA/JPL-Caltech/R. Hurt (SSC/Caltech) + Adaptions (Yassin Rany Khalil)

Backwards Integrations
(Vauterin & Dejonghe 1997; Dehnen 1999)

1. Integrate orbits starting from different « local » positions in the configuration space
2. Integrate back in time to a time where bar and spiral arms were not present.
3. Compute the orbits actions at this time.
4. Compute the axisymmetric DF at this original time :
   - By conservation we have the DF at $t = now$ for joint bar and spiral arms

Liouville equation

- Idealised stellar system of N identical stars of mass μ.

- Position $\mathbf{x} = (x_0, \ldots, x_N)$ and velocities $\mathbf{v} = (v_0, \ldots, v_N)$.
- Phase-space probability distribution function $P_N(\mathbf{x}, \mathbf{v})$.

- Temporal evolution of $P_N(\mathbf{x}, \mathbf{v})$
  - Liouville equation :

$$\frac{\partial P_N}{\partial t} + \sum_{i=1}^{N} \left[\mathbf{v_i} \cdot \frac{\partial P_N}{\partial \mathbf{x_i}} + \mu \mathcal{F}_i^{tot} \cdot \frac{\partial P_N}{\partial \mathbf{v_i}} \right] = 0.$$

BBGKY Hierarchy

- The reduced distribution functions are defined as*: $f_n(\Gamma_1, \ldots, \Gamma_n, t) \equiv \mu^n \frac{N!}{(N-n)!} P_n(\Gamma_1, \ldots, \Gamma_n, t)$

- So that the BBGKY hierarchy is given by:

$$\frac{\partial f_n}{\partial t} + \sum_{i=1}^{n} \mathbf{v_i} \cdot \frac{\partial f_n}{\partial \mathbf{x_i}} + \sum_{i=1}^{n} \sum_{k=1, k \neq i}^{n} \mu \mathscr{F}_{\mathbf{ik}} \cdot \frac{\partial f_n}{\partial \mathbf{v_i}} + \sum_{i=1}^{n} \int d\Gamma_{n+1} \mathscr{F}_{\mathbf{i,n+1}} \cdot \frac{\partial f_{n+1}}{\partial \mathbf{v_i}} = 0.$$

*With $P_n(\Gamma_1, \ldots, \Gamma_n, t) \equiv \int d\Gamma_{n+1} \ldots d\Gamma_N P_N(\Gamma_1, \ldots, \Gamma_N, t)$ with $\Gamma_m = (\mathbf{x}_m, \mathbf{v}_m)$.

BBGKY Hierarchy for $n = 1$

- $f_1(\Gamma_1, t)$ is the one-particle phase-space density in terms of mass.

- For the two-particle reduced distribution function, let's define $g_2(\Gamma_1, \Gamma_2)$ such that:

$$f_2(\Gamma_1, \Gamma_2, t) = f_1(\Gamma_1, t) f_1(\Gamma_2, t) + g_2(\Gamma_1, \Gamma_2).$$

- The BBGKY hierarchy for $n = 1$:

$$\frac{\partial f_1}{\partial t} + \mathbf{v_1} \cdot \frac{\partial f_1}{\partial \mathbf{x_1}} + \left[\int d\Gamma_2 f_1(\Gamma_2, t) \mathscr{F}_{\mathbf{12}}\right] \cdot \frac{\partial f_1}{\partial \mathbf{v_1}} + \int d\Gamma_2 \mathscr{F}_{\mathbf{12}} \cdot \frac{\partial g_2(\Gamma_1, \Gamma_2)}{\partial \mathbf{v_1}} = 0$$

- In the limit $n \to N$, $\int d\Gamma_2 \mathscr{F}_{\mathbf{12}} \cdot \dfrac{\partial g_2(\Gamma_1, \Gamma_2)}{\partial \mathbf{v_1}} \to 0$

Collionsless Boltzmann Equation (= Vlasov Equation)

- BBGKY hierarchy for $n = 1$ in the limit $n \to N$: $\frac{\partial f_1}{\partial t} + \mathbf{v_1} \cdot \frac{\partial f_1}{\partial \mathbf{x_1}} + \left[\int d\Gamma_2 f_1(\Gamma_2, t) \mathscr{F}_{\mathbf{12}} \right] \cdot \frac{\partial f_1}{\partial \mathbf{v_1}} = 0\,.$
    - Notice that: $-\nabla\Phi = \int d\Gamma_2 f_1(\Gamma_2, t)\mathscr{F}_{\mathbf{12}}.$

THE PHASE-SPACE DENSITY OF STARS
IN AN INFINITESIMAL LAGRANGIAN VOLUME IS CONSERVED.

$$\frac{\partial f_1}{\partial t} + \mathbf{v_1} \cdot \frac{\partial f_1}{\partial \mathbf{x_1}} - \nabla\Phi \cdot \frac{\partial f_1}{\partial \mathbf{v_1}} = 0 \Longleftrightarrow \frac{df_1}{dt} = 0.$$

Background potential

- 2 disk density profiles: Stellar thin disk and Interstellar medium thick disk

  $\rho = \Sigma_0 \exp[-(R/R_d)^{1/n} - R_0/R + \epsilon \cos(R/R_d)]$

  3 parameters each

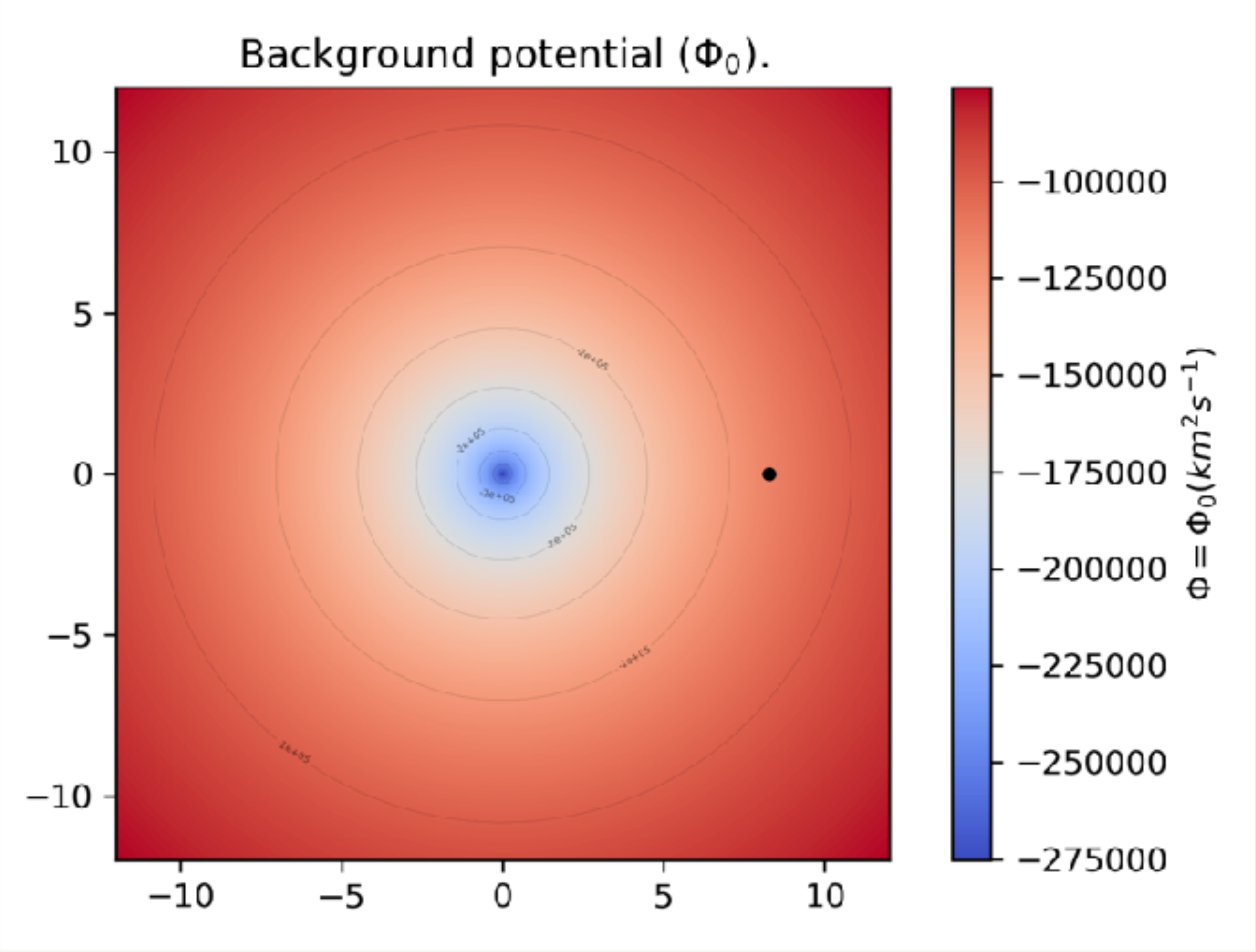
- 2 spheroidal density profiles: Dark Matter and Bulge.

  $\rho = \rho_0 (r/r_0)^{-\gamma} (1 + (r/r_0)^{\alpha})^{(\gamma-\beta)/\alpha} \exp[-(r/r_{cut})^{\xi}]$

  5 parameters each

- So the background model has $2 \cdot (3 + 5) = 16$ parameters

- Sun's velocity and position counts for more 6 parameters



Bar potential

- Bar potential:

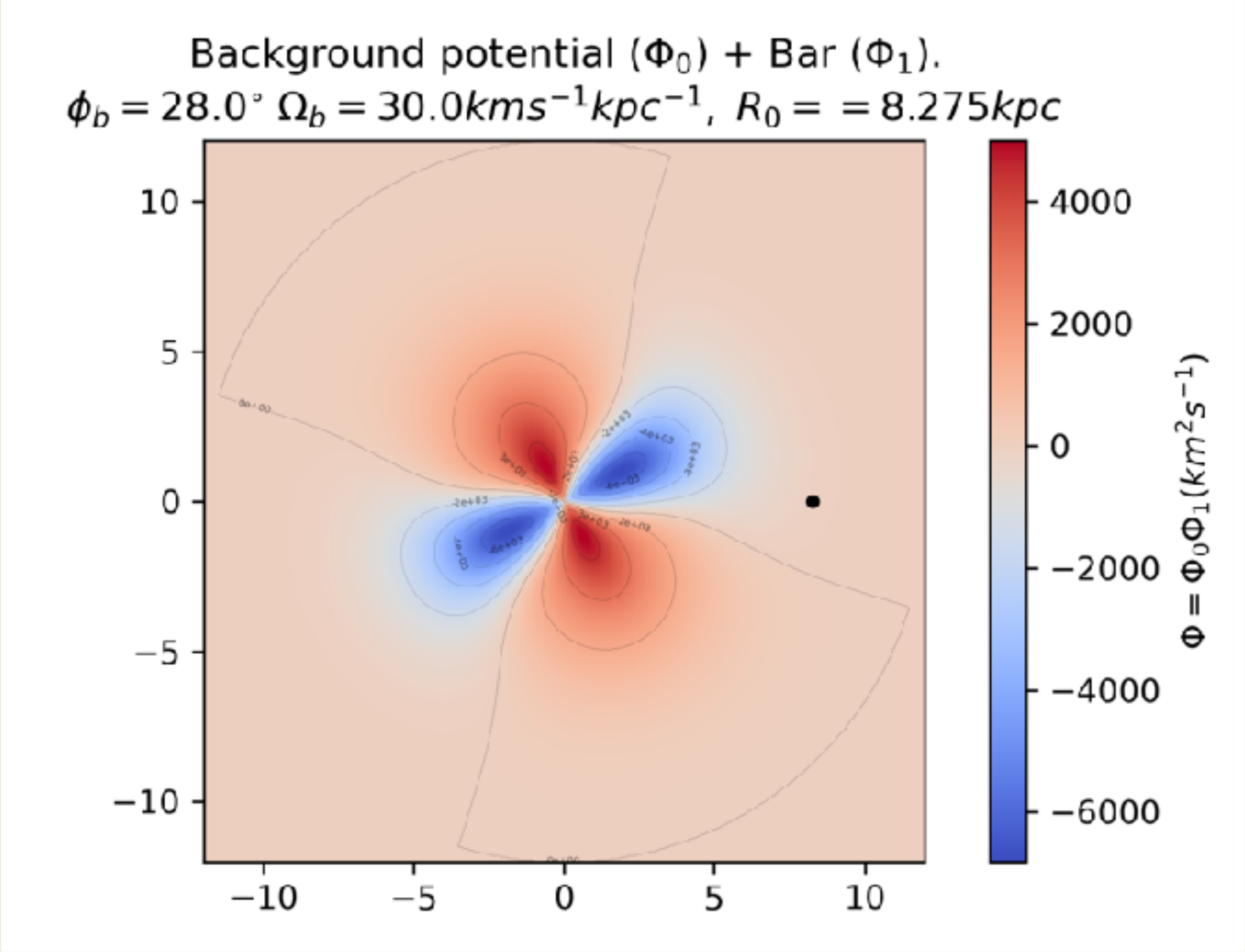
$$\Phi_1(r, \phi, z, m, R_{max}, A, a, b, \phi_b, \Omega_{bar}, t) = A\bar{r}^{a-1}(1-\bar{r})^{b-1}cos\left(m(\phi - \phi_b - \Omega_{bar}t)\right)$$

  - $\bar{r} = \frac{r}{R_{max}}$. Cutoff at $R_{max} = 12kpc$.

- Bar angle ϕ_b
- Bar pattern speed Ω_{bar}
- Amplitude A
- Radial profile parameters a, b

- 3 superposed modes $m = 2,4,6$
  - Each mode has 3 free parameters: A, a, b

- So the Bar model has $2 + 3 \cdot 2 = 11$ parameters



Spiral arms potential

- Spiral Arms potential:

$$\Phi_2 = A cos\left(m\left((\phi - \phi_0) - \Omega_s t + ln\left(\frac{R}{R_0}\right) tan(i)^{-1}\right)\right)$$

- Amplitude A
- Pit angle i
- Spiral arms pattern speed Ω_s
- Phase ϕ_0
- Arms number m

- So the Spiral Arms model has 5 parameters