Lensing in cosmology: theory

A cosmologist turned strong lens modeller's review

News from the Dark, Marseille, November 2024

Natalie Hogg | Laboratoire univers et particules Montpellier

Contents

Cosmology and our current problems

- Why is the expansion rate of the Universe accelerating at late times?
- Why do different measurements of cosmological parameters disagree so severely?
- What is dark matter?

Gravitational lensing

Uniquely sensitive to cosmology and dark matter on a extremely wide range of scales.

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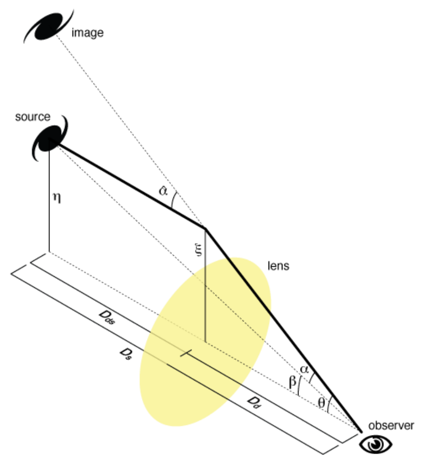
Massive objects distort local spacetime, curving the geodesics.

$$\beta = \theta - \alpha(\theta)$$

where β is the unlensed source position, θ is the lensed image position and α is called the deflection angle.



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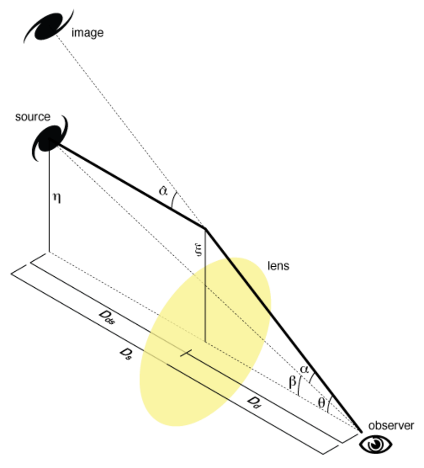
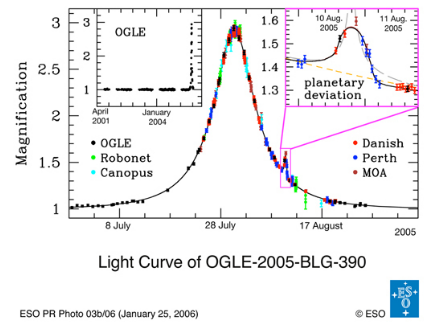
$$\alpha(\theta) = \nabla\psi(\theta)$$

where ψ is the gravitational potential of the lens.



Regimes of gravitational lensing

	Lens	Source	Images
Microlensing	Planet, star, PBH	Star	Single, highly magnified
Strong lensing	Galaxy, cluster	Galaxy	Multiple, magnified, strongly distorted
Weak lensing	Galaxies	Galaxies	Single, weakly distorted



ESO

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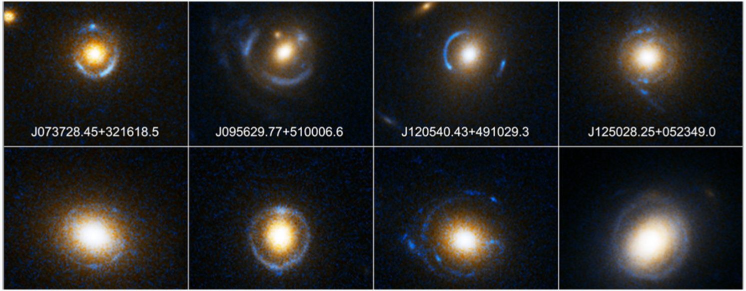

Bolton et al. (2006)

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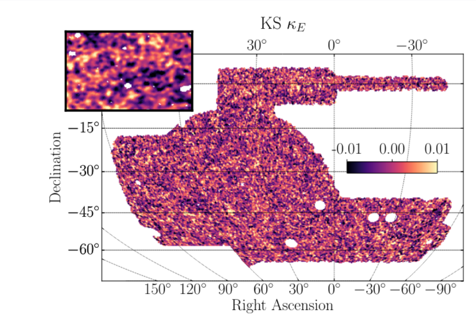

Jeffrey et al. (2021)

Cosmology with strong lensing

Strong lensing for cosmology: time delays

For a variable source, the delay between the arrival time of separate images is given by

$$t(\theta, \beta) = \frac{(1 + z_{\rm od})}{c} \frac{D_{\rm od} D_{\rm os}}{D_{\rm ds}} \left[\frac{(\theta - \beta)^2}{2} - \psi(\theta) \right].$$

Terms in yellow are dependent on the lens model and terms in red are dependent on the cosmology.

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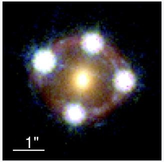
Simple picture:

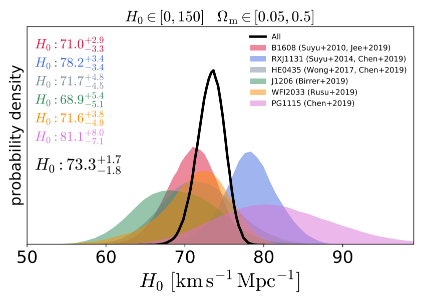
$$t \propto \frac{1}{H_0}.$$

Strong lensing for cosmology: time delays



plus $$\alpha(R,\varphi) = \frac{2b}{1+f}\left(\frac{b}{R}\right)^{t-1} e^{i\varphi}\, {}_2F_1\left(1, \frac{t}{2}; 2-\frac{t}{2}; -\frac{1-f}{1+f}e^{i2\varphi}\right)$$

Tessore & Metcalf (2015)



HoLiCOW collaboration, Wong et al. (2020); 2.4% precision measurement of H_0

Strong lensing for cosmology: time delays

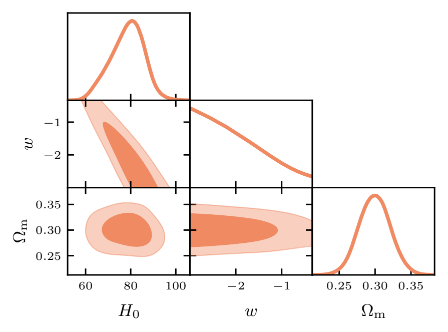
Assuming $\Omega_k = 0$,

$$\frac{H(z)}{H_0} = \left[\Omega_\mathrm{m}(1+z)^3 + \Omega_\mathrm{DE}(1+z)^{3(1+w)}\right]^{\frac{1}{2}}$$

where $w = -1$ for a cosmological constant.

Use time delays plus stellar kinematics combined using hierarchical Bayesian inference.

TDCOSMO collaboration, Birrer et al. (2020).



$w < -1.75$ from seven lenses + kinematics alone

$w = -1.025 \pm 0.029$ combined with other data

Hogg (2023) tdcosmo_ext

Strong lensing for cosmology: small-scale dark matter constraints

- Mass–concentration relation of lens galaxies.
- Halo and sub-halo mass functions.
- Inner density slope of lens galaxy mass profiles.
- Individual sub-halo detection via flux ratios.

Vegetti et al. (2023)

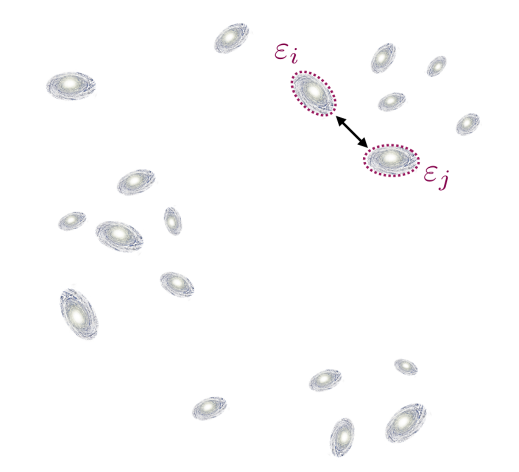
Cosmology with weak lensing

Weak lensing for cosmology: 3×2 point correlation functions

- Weak distortions mainly manifest as *shear*; squashing of circles into ellipses.
- Extremely noisy signal due to shape noise and intrinsic alignments.
- Noise beaten by statistics: 3×2 point correlation functions using millions of galaxy shape and position measurements.

Weak lensing for cosmology: 3×2 point correlation functions

$\langle \varepsilon_i \times \varepsilon_j \rangle$ 'cosmic shear'



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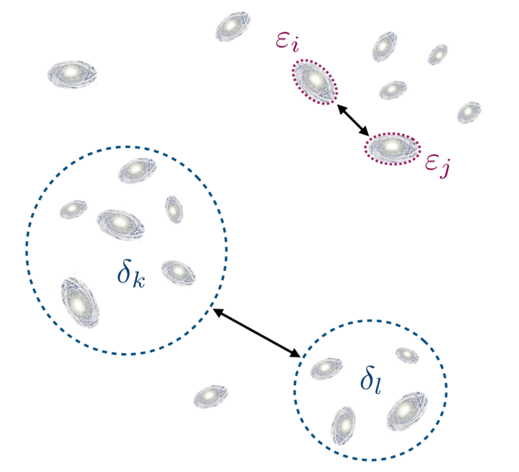

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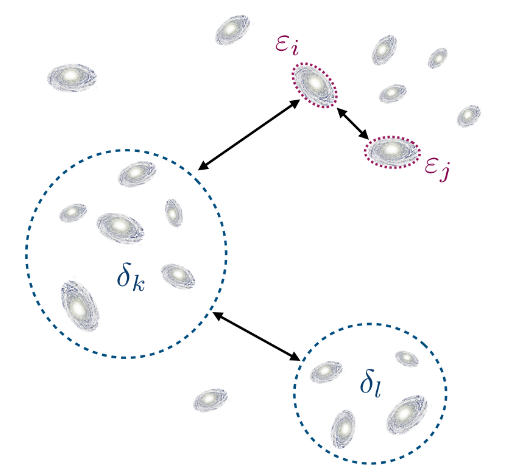
$\langle \varepsilon_i \times \delta_k \rangle$ 'galaxy–galaxy lensing'



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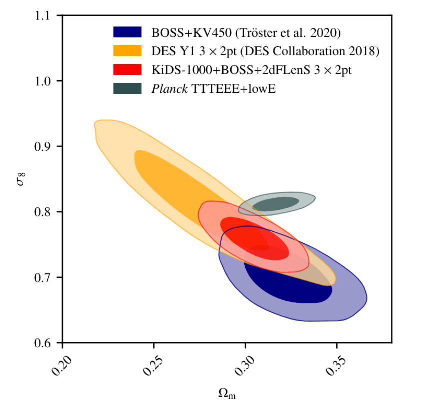

Heymans et al. (2020).

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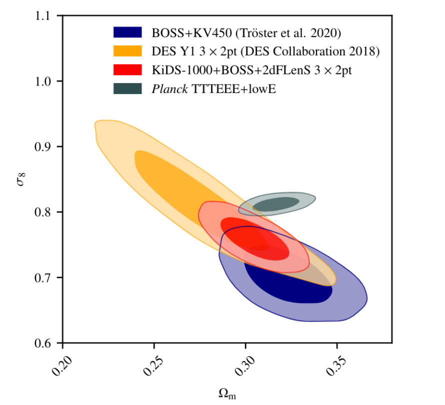

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How can the weak lensing constraints be improved?

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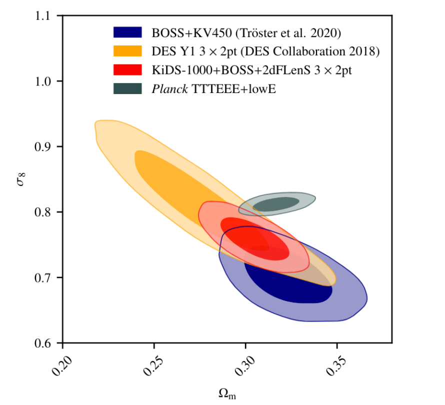

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How can the weak lensing constraints be improved?

How can uncertainties be reduced?

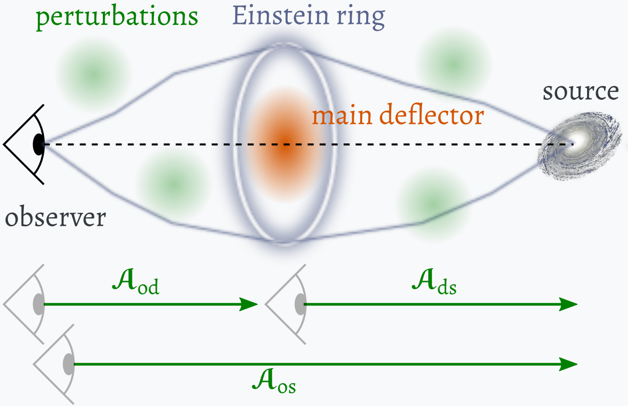
Is there more information to be found in lensing?

Cosmology with the weak lensing of strong lensing

Weak lensing of strong lensing for cosmology

- Strong lensing images also experience weak lensing distortions, called ‘line-of-sight effects’: if this ‘weak lensing of strong lensing’ can be measured it will provide additional cosmological information.
- Must be done statistically $\rightarrow 6 \times 2$ point correlation functions.
- How to model the line-of-sight effects on a strong lens image?

Modelling the line-of-sight effects



Modelling the line-of-sight effects

The amplification matrices are defined as

$$\mathcal{A}_{\mathrm{ab}} = \mathbf{1} - \begin{bmatrix} \kappa_{\mathrm{ab}} + \mathrm{Re}\left(\gamma_{\mathrm{ab}}\right) & \mathrm{Im}\left(\gamma_{\mathrm{ab}}\right) - \omega_{\mathrm{ab}} \\ \mathrm{Im}\left(\gamma_{\mathrm{ab}}\right) + \omega_{\mathrm{ab}} & \kappa_{\mathrm{ab}} - \mathrm{Re}\left(\gamma_{\mathrm{ab}}\right) \end{bmatrix}$$

where κ_{ab} is the convergence, γ_{ab} the shear and ω_{ab} the rotation; $\mathrm{ab} \in \mathrm{od}, \mathrm{ds}, \mathrm{os}$.

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The lens equation is thus modified,

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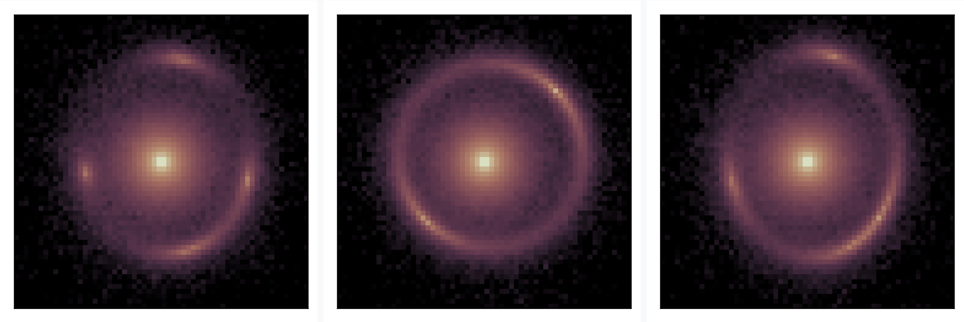
⚠ Valid in the tidal regime: perturbations are small.

For a treatment of beyond-tidal effects see Duboscq et al. (2024).

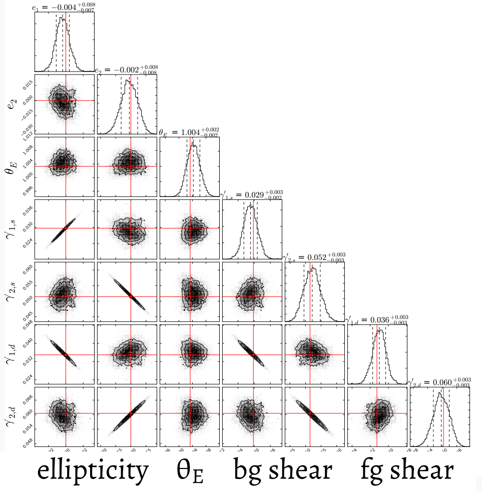
What makes an Einstein ring elliptical?

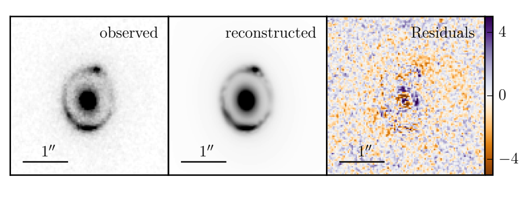


Shear or ellipticity?

Cosmic shear from Einstein rings

Birrer et al. (2016, 2017)





COSMOS 0038+4133

“fg shear” $= \gamma_{\rm od}$; “bg shear” $= \gamma_{\rm os} - \gamma_{\rm ds}$.

Conquering the shear–ellipticity degeneracy

Multiply the lens equation by the combination $\mathcal{A}_{\rm od}\mathcal{A}_{\rm ds}^{-1}$, creating the "minimal model",

$$\tilde{\boldsymbol{\beta}} = \mathcal{A}_{\rm LOS}\boldsymbol{\theta} - \mathcal{A}_{\rm od}\boldsymbol{\alpha}(\mathcal{A}_{\rm od}\boldsymbol{\theta}),$$

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Fleury et al. (2021)

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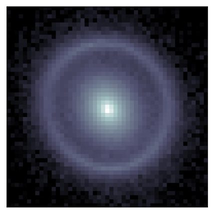
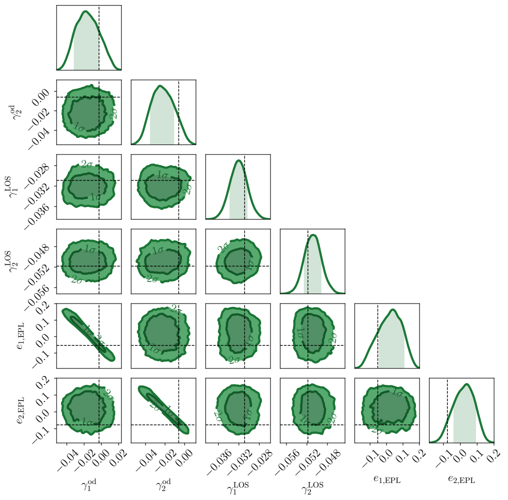
Such transformations are possible as we cannot access $\boldsymbol{\beta}$, the unlensed source position.

Fleury et al. (2021)

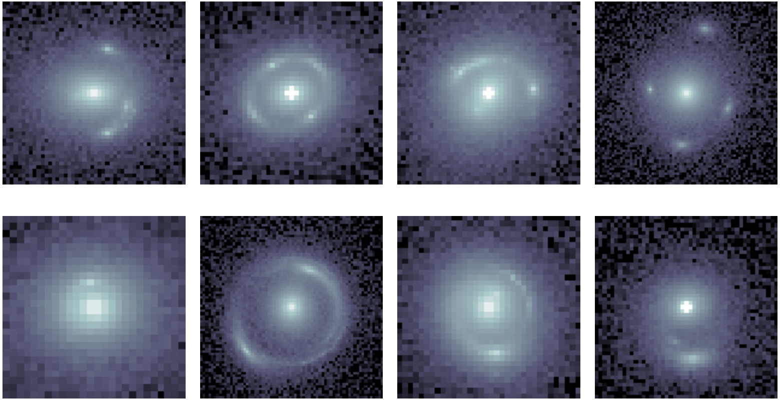
Demonstrating the efficacy of the minimal model





Hogg et al. (2023)

Measuring LOS shear: a proof of concept with 64 complex mocks



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Hogg et al. (2023)

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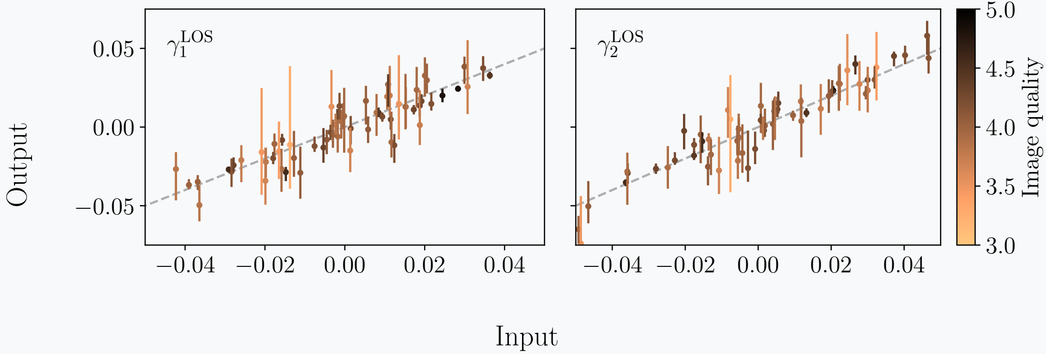
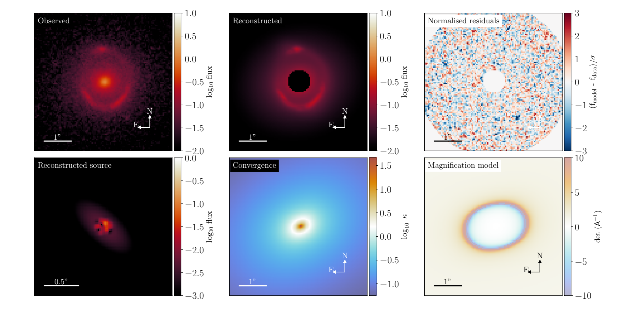
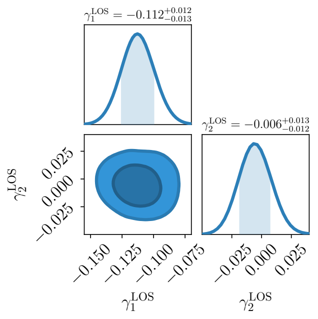


$\chi^2 = 1.0$; average precision of 1%; no outliers $> 2\sigma$

Hogg et al. (2023)

Measuring LOS shear in 50 SLACS lenses

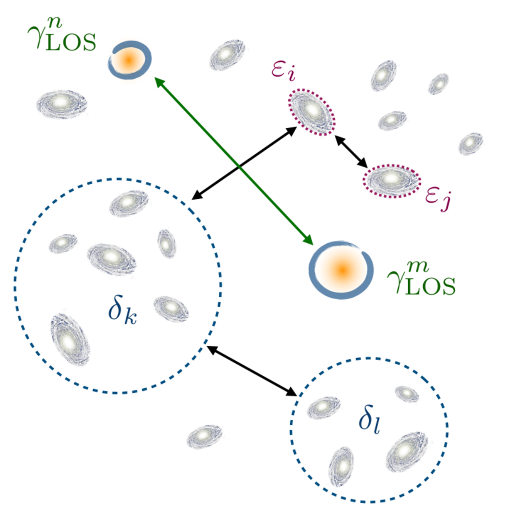




dolphin   Hogg et al. (in prep. 2024)

Weak lensing of strong lensing for cosmology: 6×2 point correlation functions

$\langle \gamma^m_{\rm LOS} \times \gamma^n_{\rm LOS} \rangle$ 'ring–ring'



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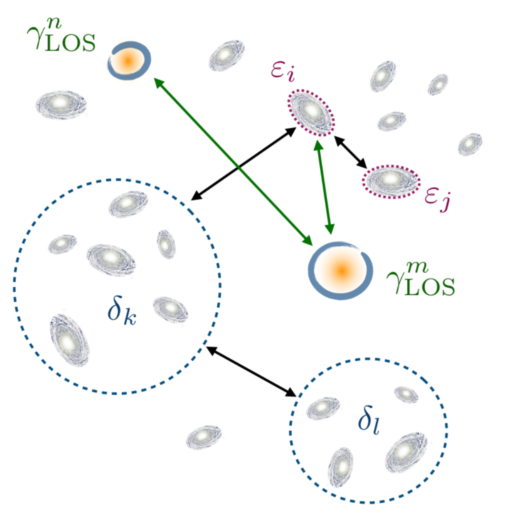

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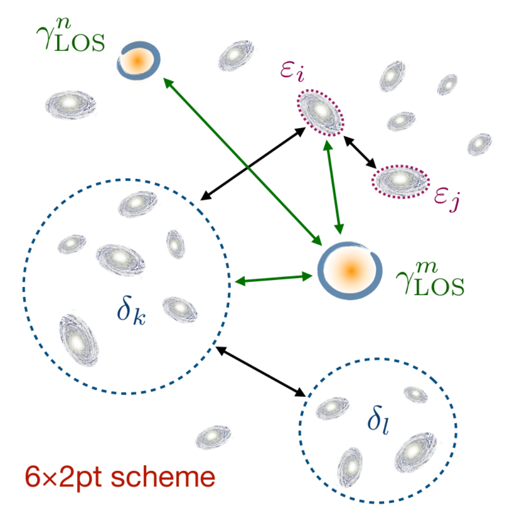

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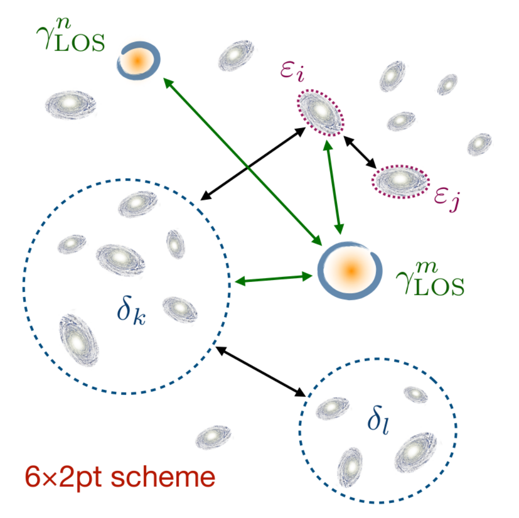
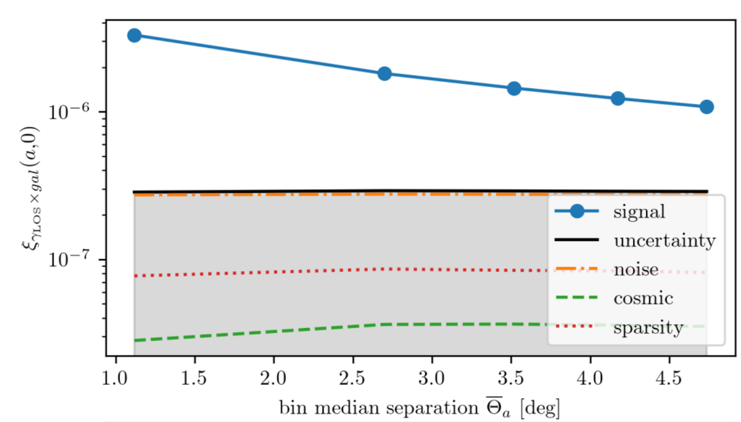
$\langle \varepsilon_i \times \varepsilon_j \rangle$; $\langle \delta_k \times \delta_l \rangle$; $\langle \varepsilon_i \times \delta_k \rangle$



Cosmology with the LOS shear: preliminary results

Example: cross-correlation of LOS shear with galaxy positions from a *Euclid*-like dataset.



LOS shear from 10^5 strong lenses with 5% precision 📷 Théo Duboscq.

What dust is under the carpet?

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Q: Multipolar distortions in lens mass; will 'boxy', 'disky', and 'twisty' features contaminate shear measurements?

Q: How prevalent are beyond-shear shape distortions (flexion) in real lines of sight?

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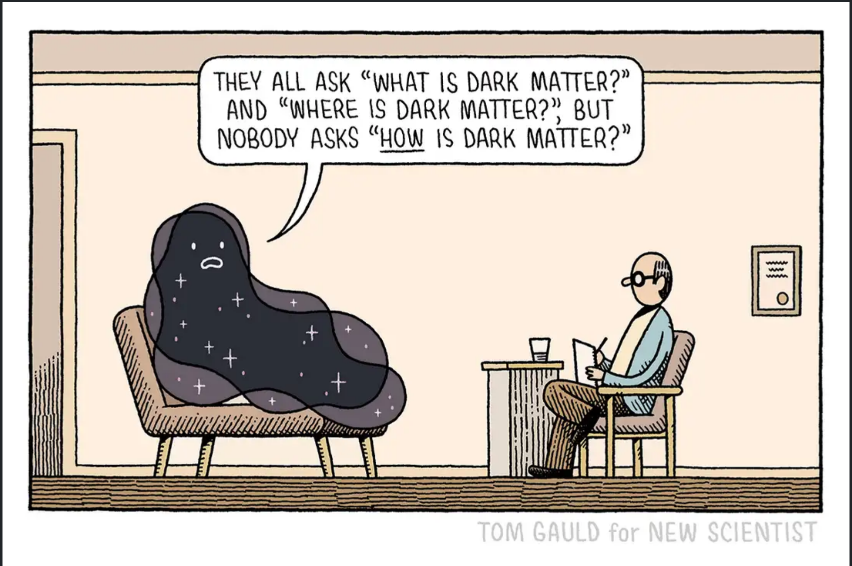
A: `JAX`-based codes or machine learning.

Summary

- Gravitational lensing is a unique probe of dark matter on a vast range of scales.
- A new probe, the weak lensing of strong lensing, has been proposed and can be accurately measured; preliminary results indicate that the cosmological signal will be detectable.
- *Euclid* and JWST (ask me about COSMOS-Web!) are ushering in a new era of lensing in cosmology.



natalie.hogg@lupm.in2p3.fr astronat nataliebhogg.com

Back-up slides

Weak and strong lensing in COSMOS-Web

Highest-ever resolution dark matter map from weak lensing

Scognamiglio et al. (2024)

Twenty spectacular strong lenses

Mahler et al. (2024)

A catalogue of 100 strong lenses

Nightingale et al. (2024)

Do strong lens forecasts match COSMOS-Web observations?

Hogg et al. (2024)

The mass-sheet degeneracy

Under multiplicative transformation of the lens equation,

$$\lambda\boldsymbol{\beta} = \boldsymbol{\theta} - \lambda\boldsymbol{\alpha}(\boldsymbol{\theta}) - (1-\lambda)\boldsymbol{\theta}, \tag{1}$$

where the source has been linearly displaced, $\boldsymbol{\beta} \rightarrow \lambda\boldsymbol{\beta}$, image positions are preserved.

Falco et al. (1985), Schneider and Sluse (2013, 2014)

Time delay constraints on H_0: using stellar kinematics

- Add mass-sheet degeneracy hyperparameters to the model.
- Constrain those parameters using stellar kinematics data from a separate strong lens catalogue.
- Resulting cosmological constraints will be the most precise possible whilst making minimal assumptions about the mass-sheet degeneracy.

TDCOSMO collaboration, Birrer et al. (2020)

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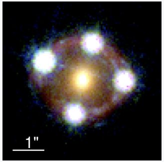

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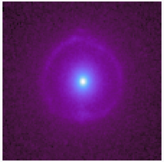

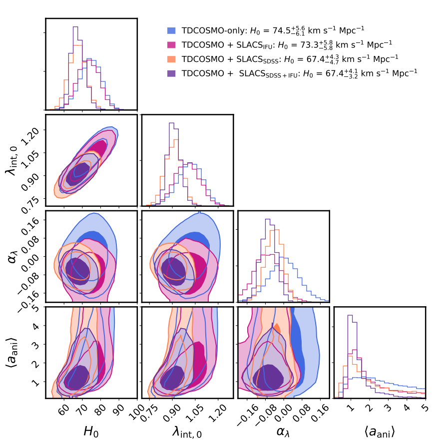

TDCOSMO collaboration, Birrer et al. (2020); 5% precision measurement of H_0

$$\gamma_{\mathrm{LOS}} = \gamma_{\mathrm{od}} + \gamma_{\mathrm{os}} - \gamma_{\mathrm{ds}}$$