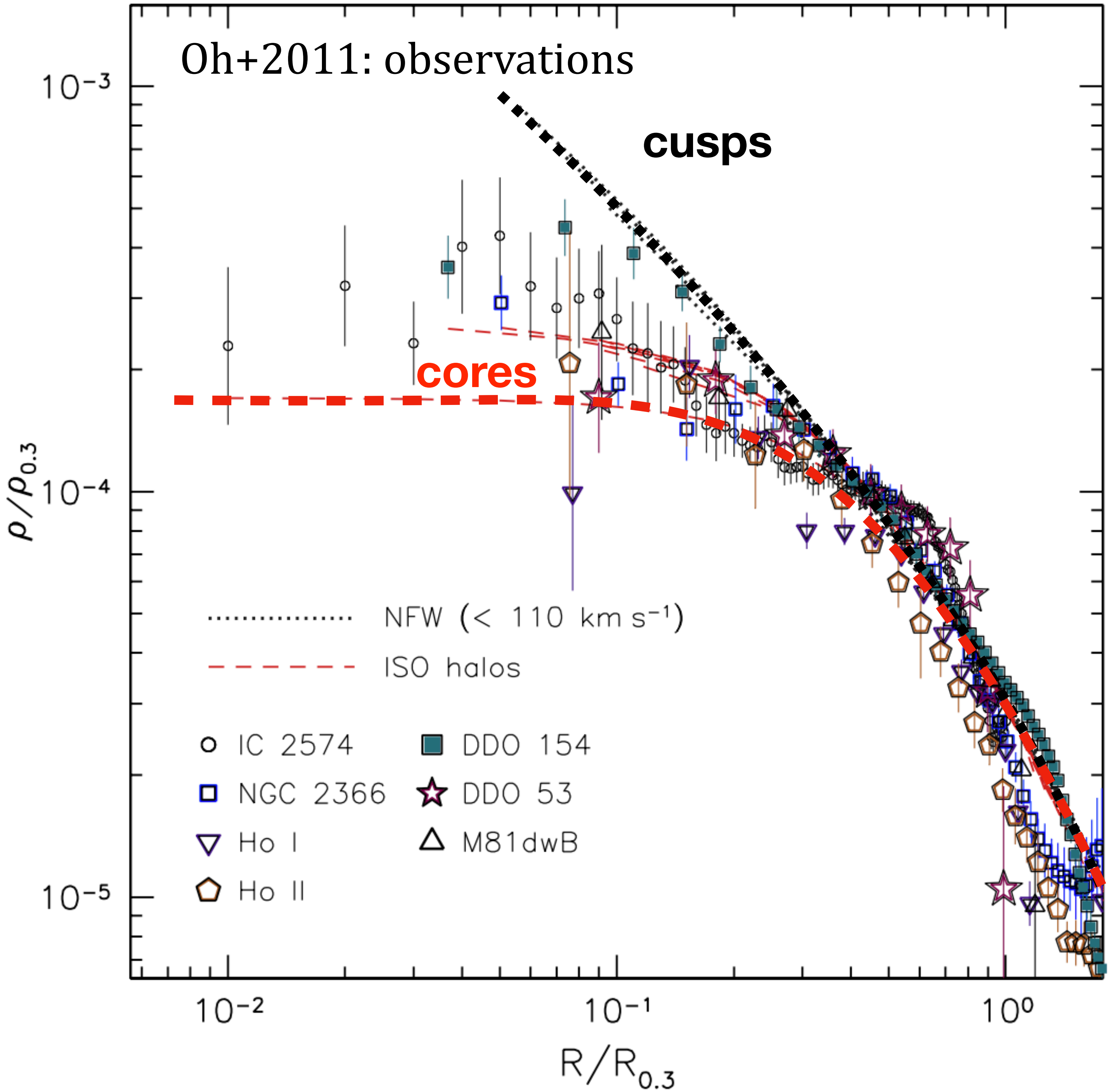
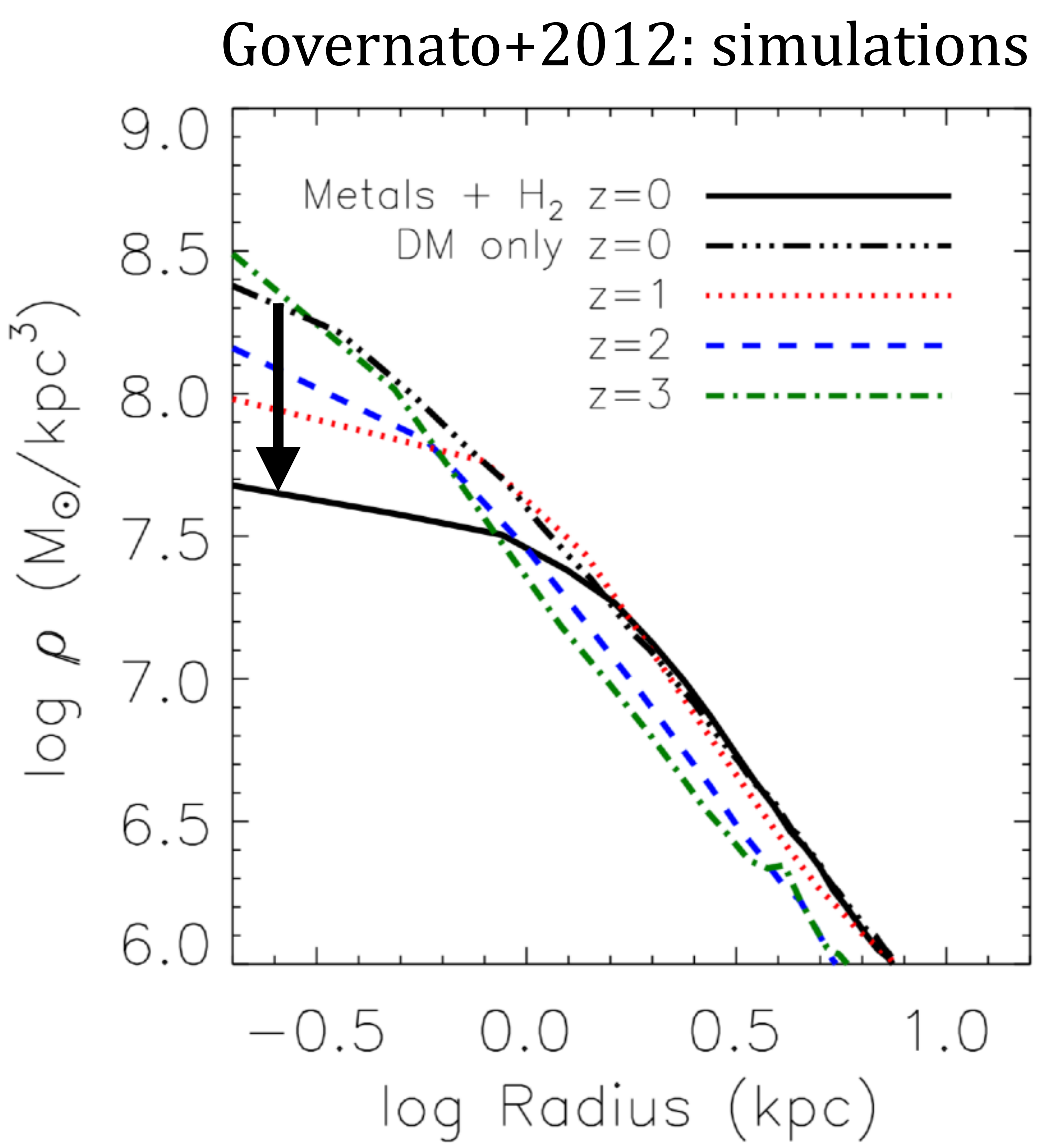
Observatoire astronomique de Strasbourg | ObAS

On the effect of stochastic density fluctuations on collisionless systems

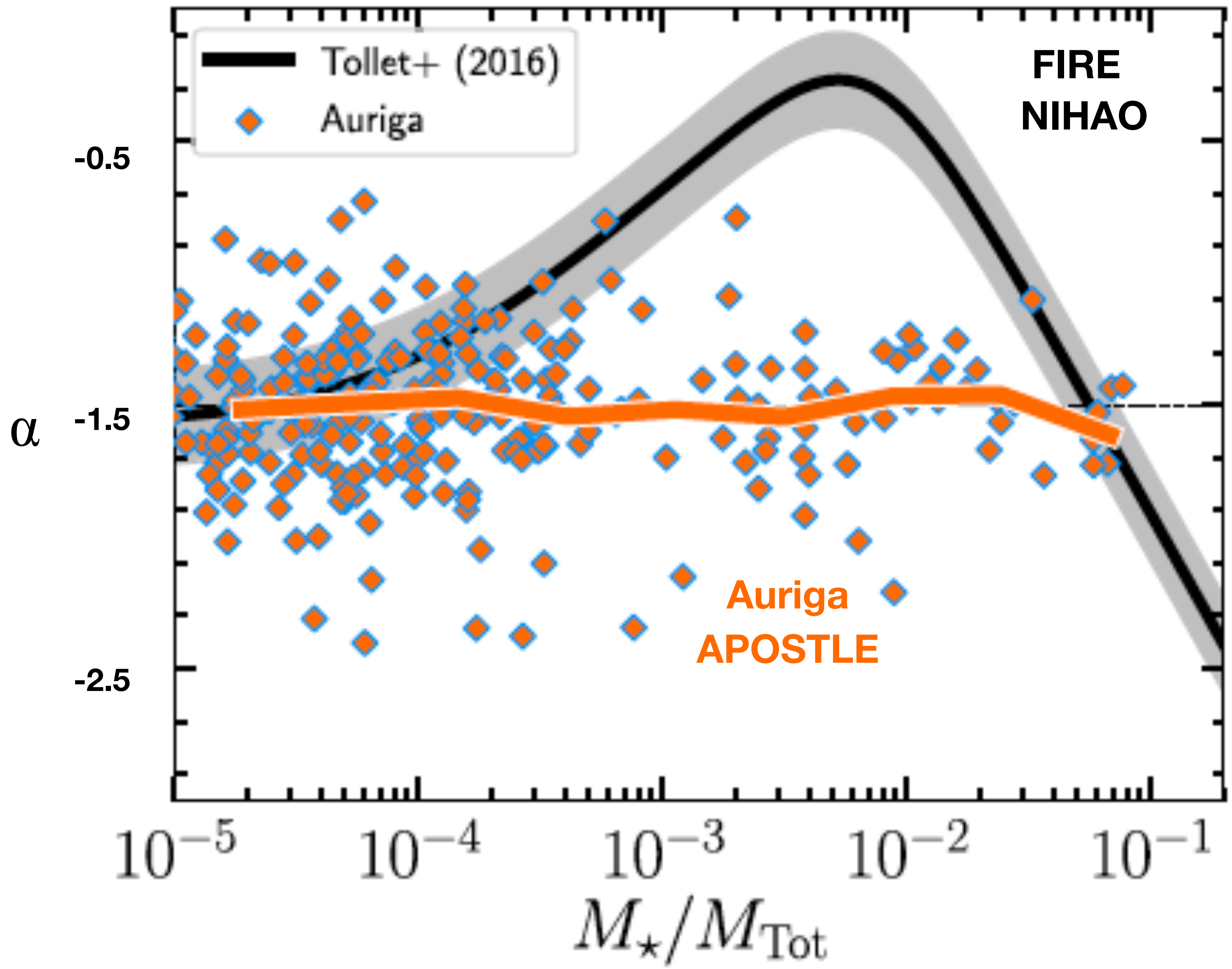
Jonathan Freundlich

The cusp-core discrepancy





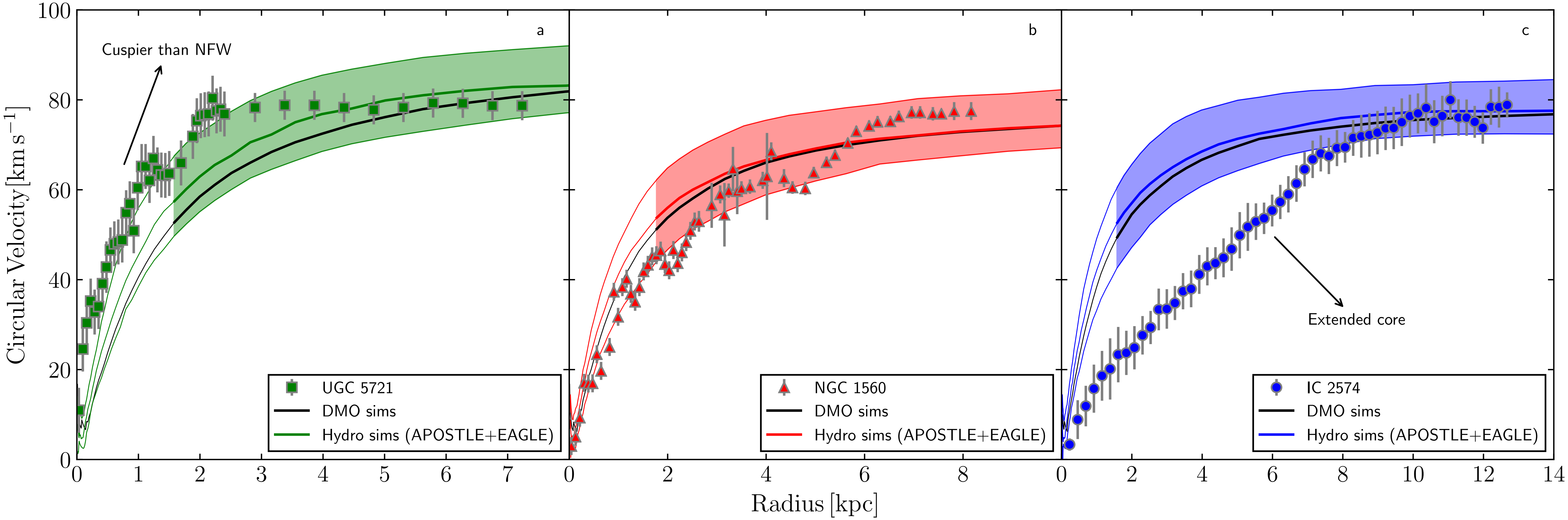
Different predictions for the halo response



Bose et al. 2019

A diversity of rotation curves

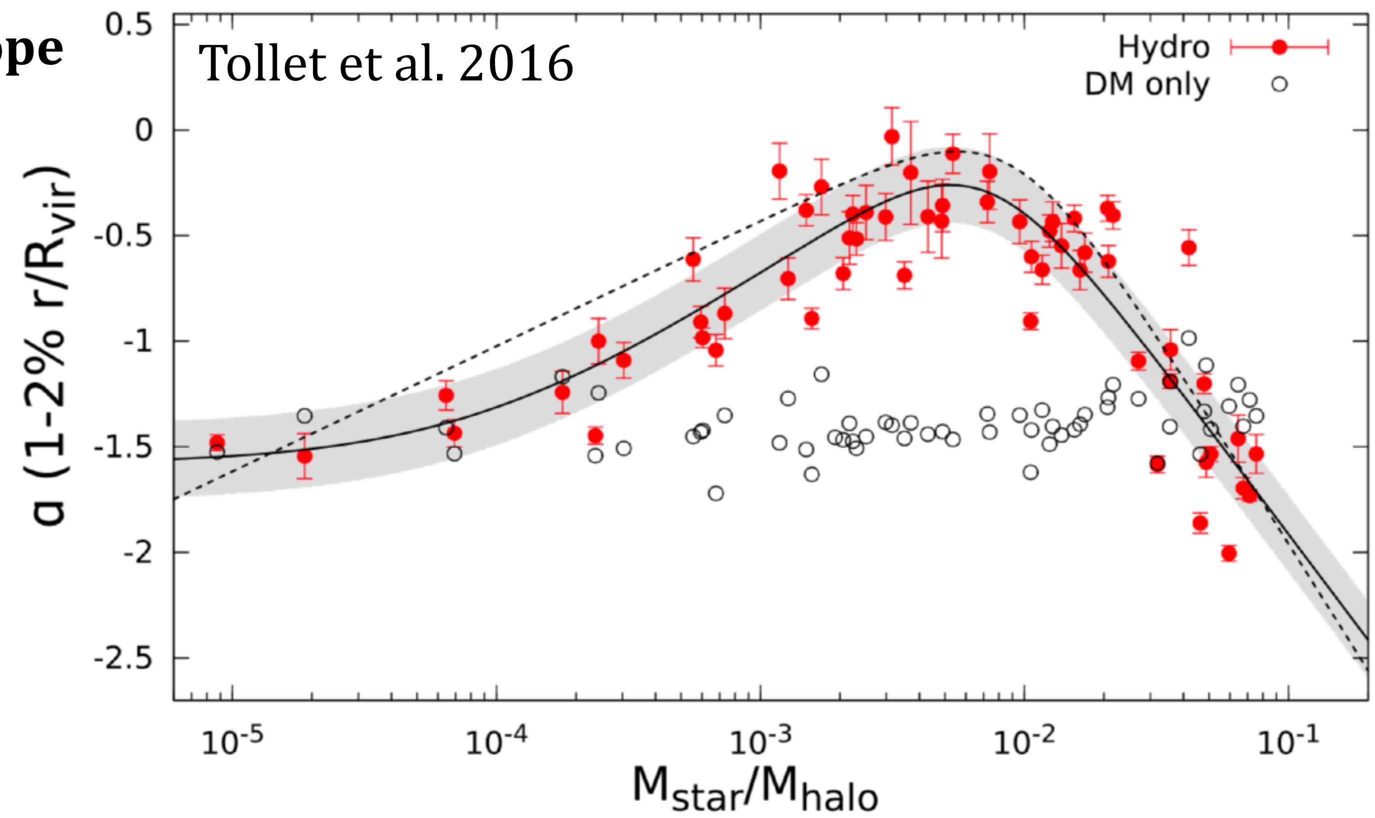
Three dwarf galaxies with similar outer rotation velocity but distinct inner behavior:



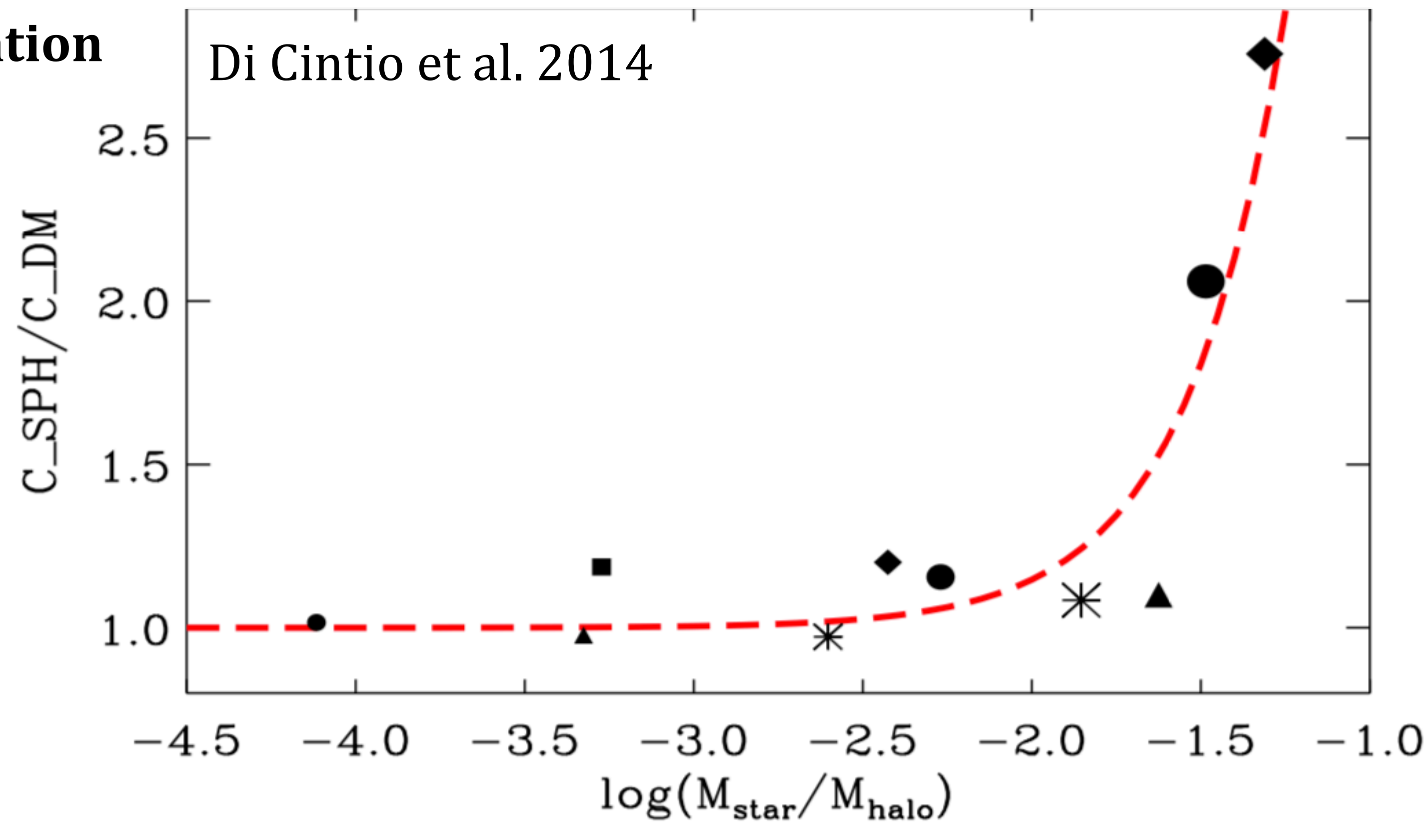
Oman et al. 2015, adapted by Sales et al. 2022

Dark matter halo response to baryons

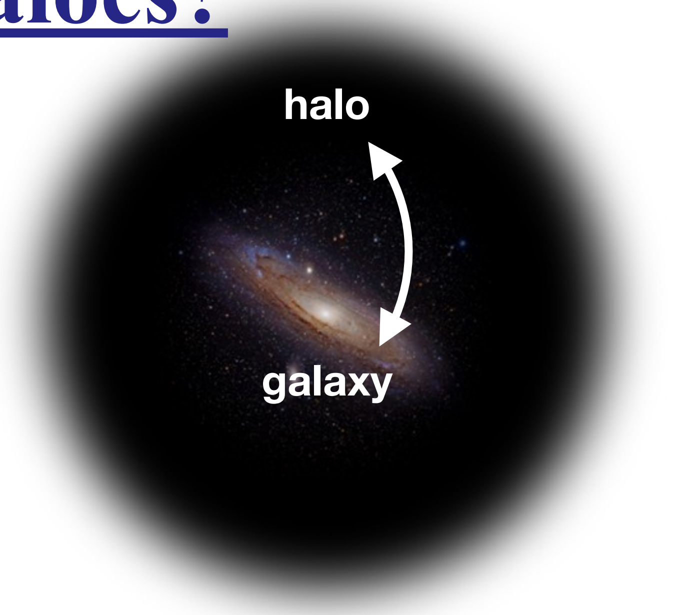
Inner slope



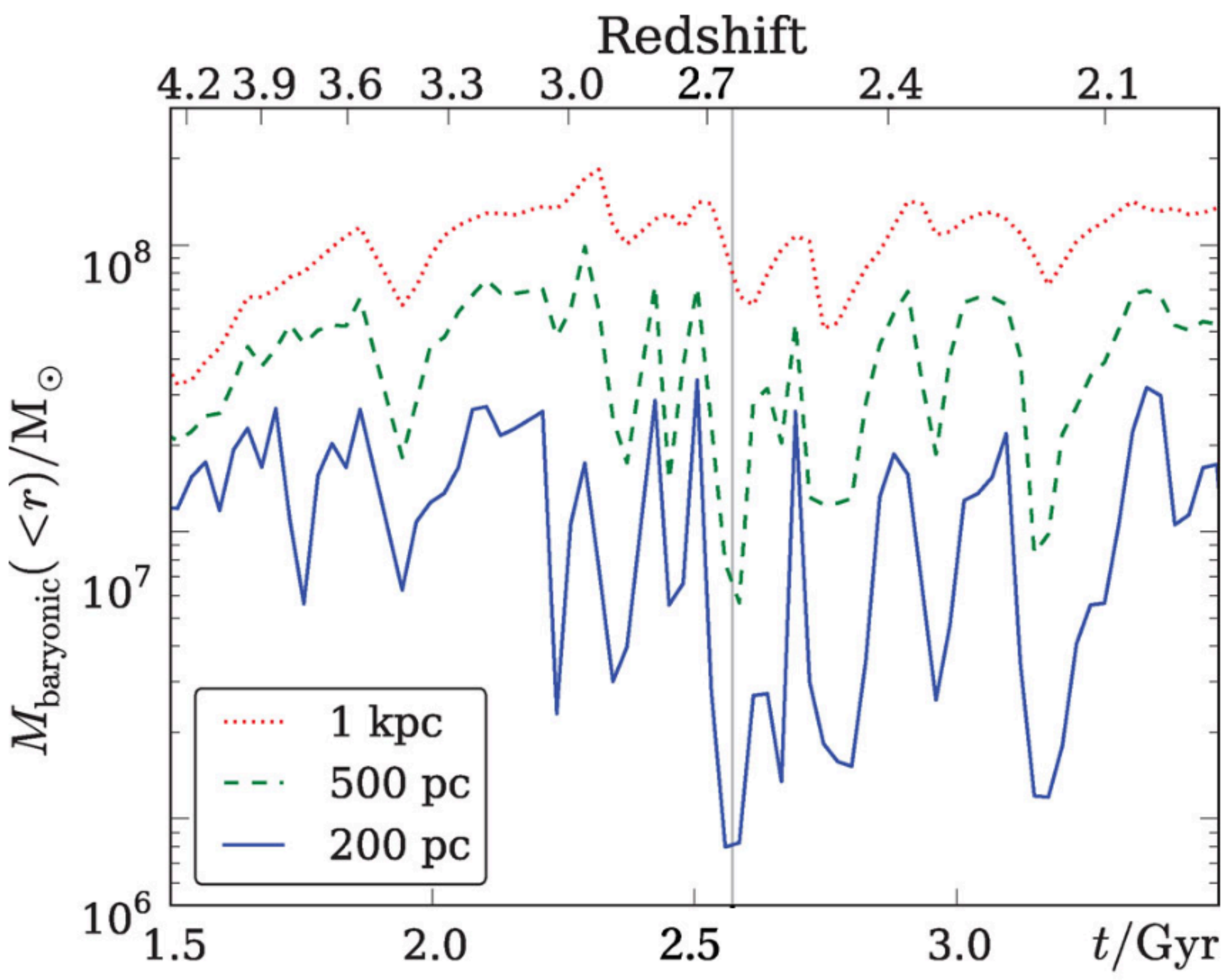
Concentration

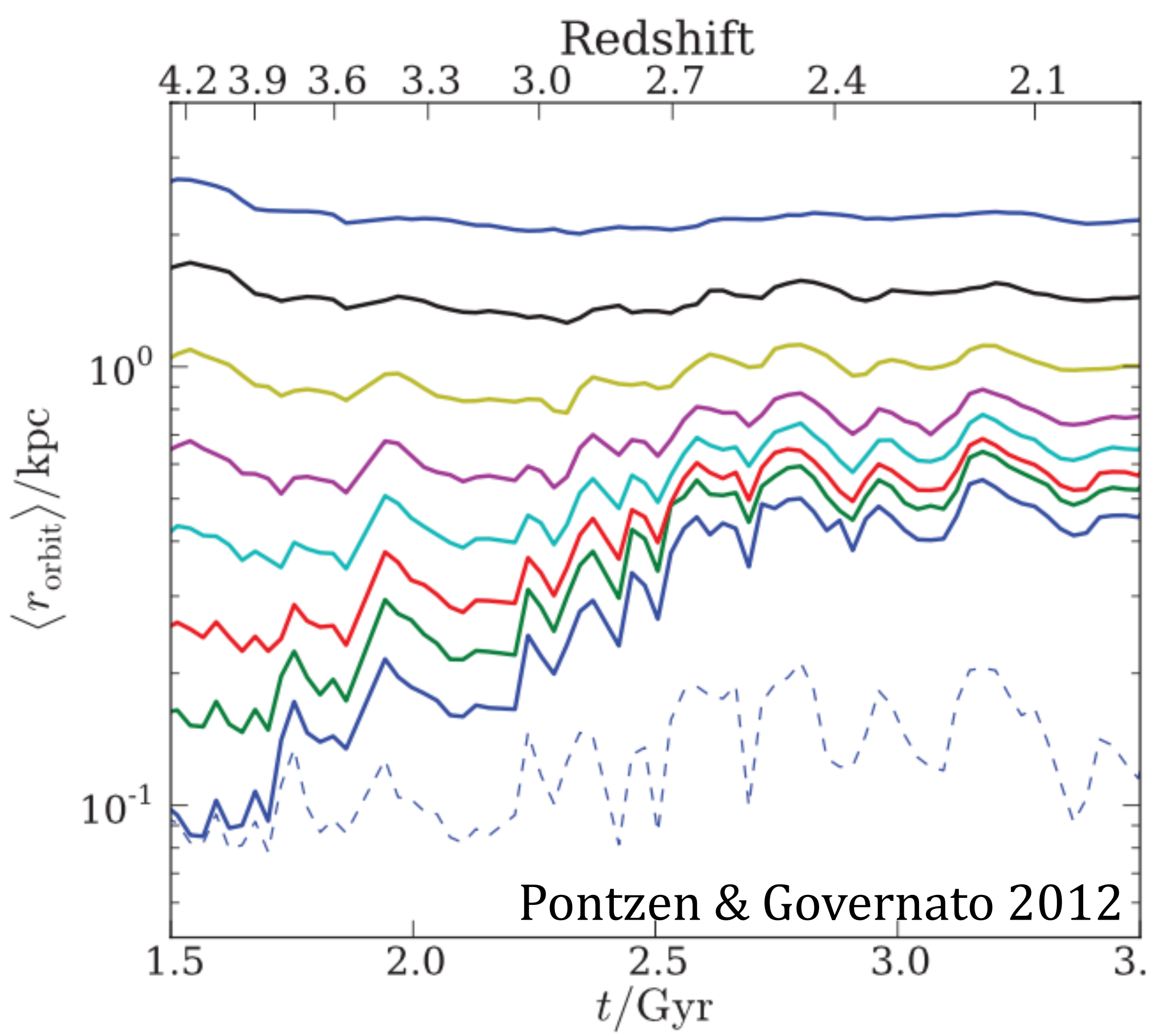


How can baryons affect dark matter haloes?

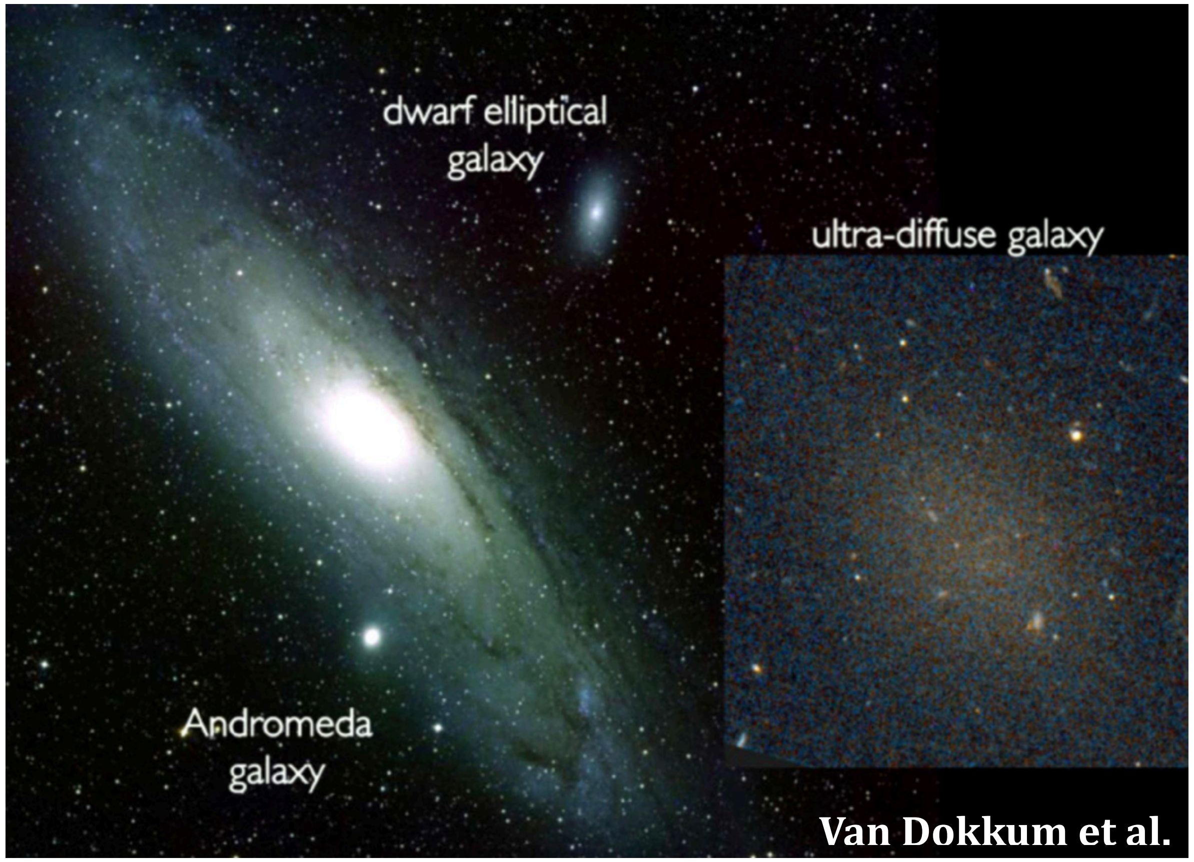


- ✦ **Adiabatic contraction** (Blumenthal+1986)
- ✦ **Dynamical friction** (El-Zant+2001, 2004)
- ✦ **Repeated potential fluctuations from feedback processes** (Pontzen & Governato 2012)





The same process at stake in ultra-diffuse galaxies?

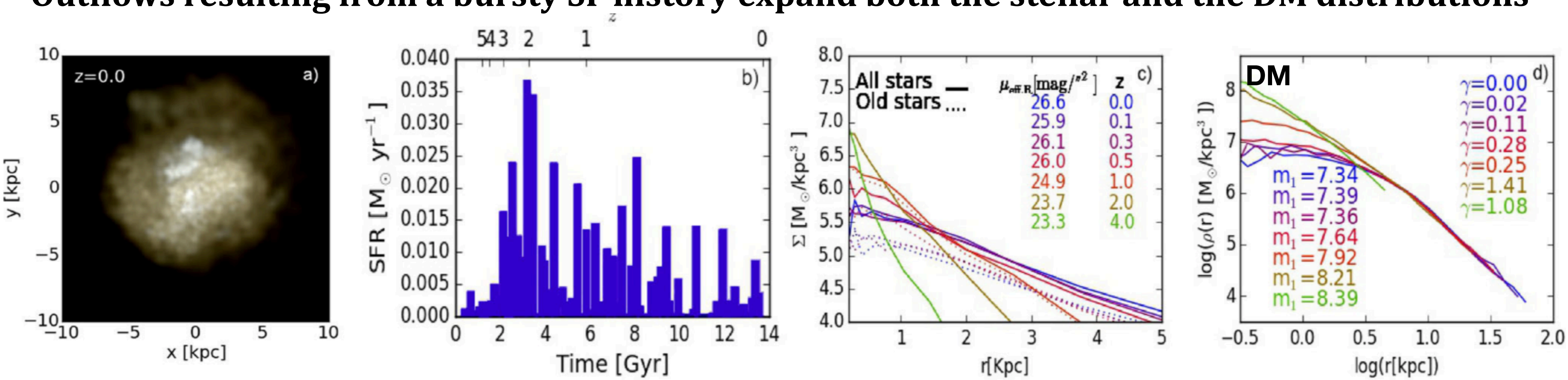


- ✦ **Stellar masses of dwarf galaxies**
  $7 < \log(M_{\rm star}/M_{\odot}) < 9$
- ✦ **Effective radii of MW-sized objects**
  $1 < r_{\rm eff}/{\rm kpc} < 5$

Possible formation scenarii:

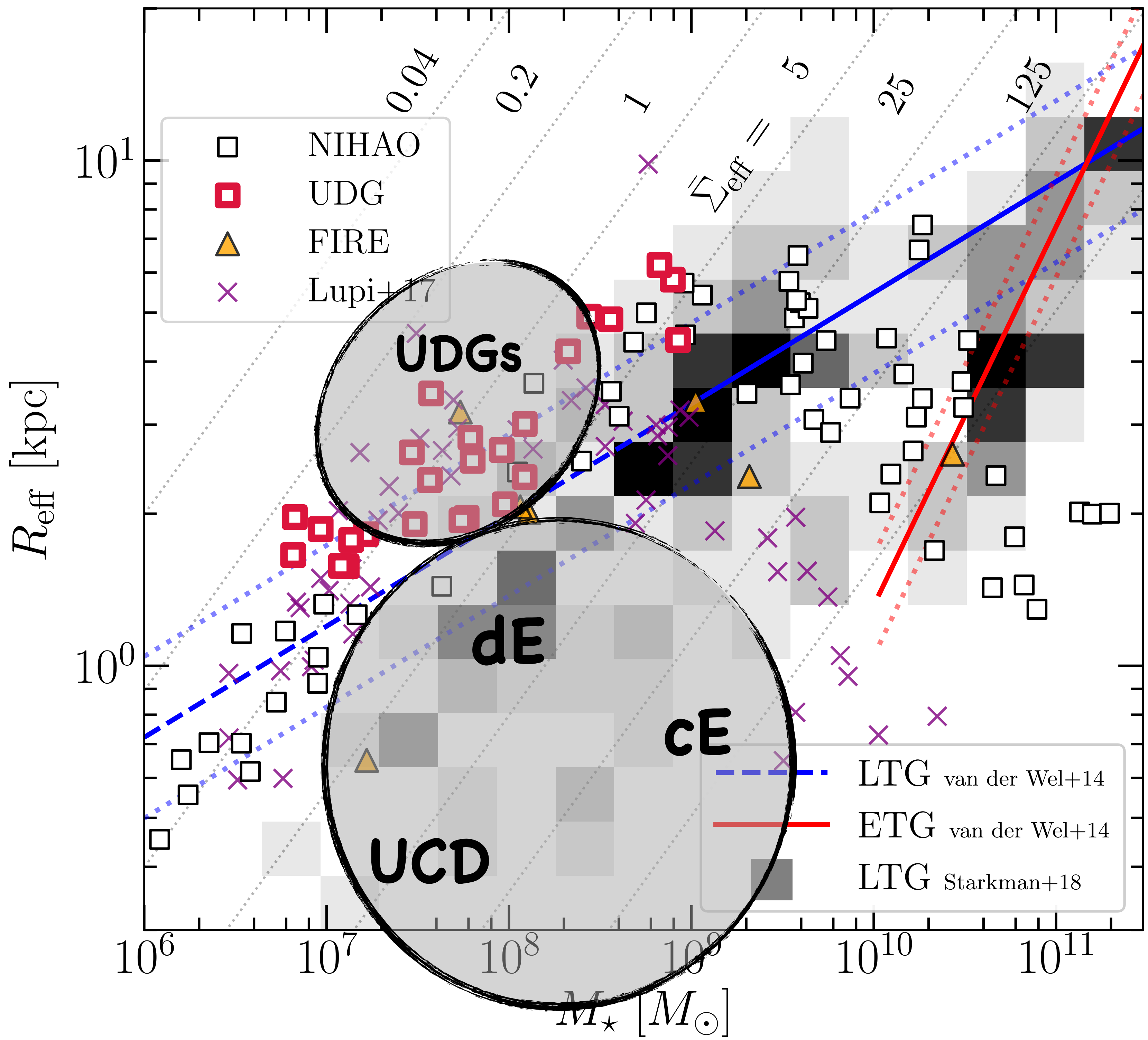
- ✦ **Failed MW-like galaxies** (Van Dokkum+2015)
- ✦ **High-spin tail** (Amorisco & Loeb 2016)
- ✦ **Tidal debris** (Greco+2017)
- ✦ **Collisions** (Van Dokkum+2022)
- ✦ **Stellar feedback outflows** (Di Cintio+2017)

Outflows resulting from a bursty SF history expand both the stellar and the DM distributions



Di Cintio et al. 2017

A dwarf-galaxy diversity problem in simulations



Jiang, Dekel, Freundlich et al. 2019

1/ Core formation from bulk outflows

Freundlich et al. (2020a), Dekel et al. (2021), Li et al. (2022)

CuspCore: Core formation from bulk outflows

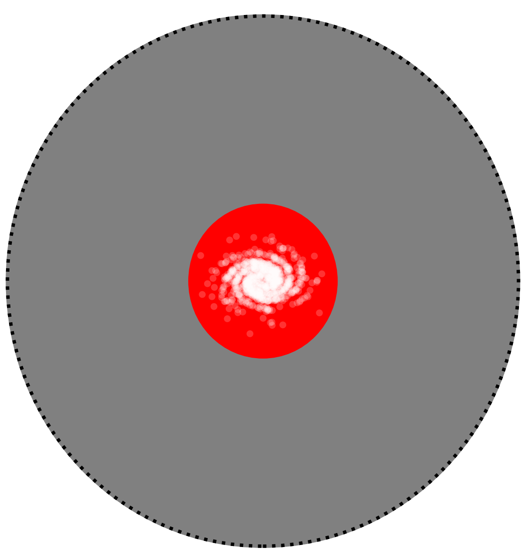
Evolution of a spherical shell encompassing a collisionless mass M when a baryonic mass m is removed (or added) at the center

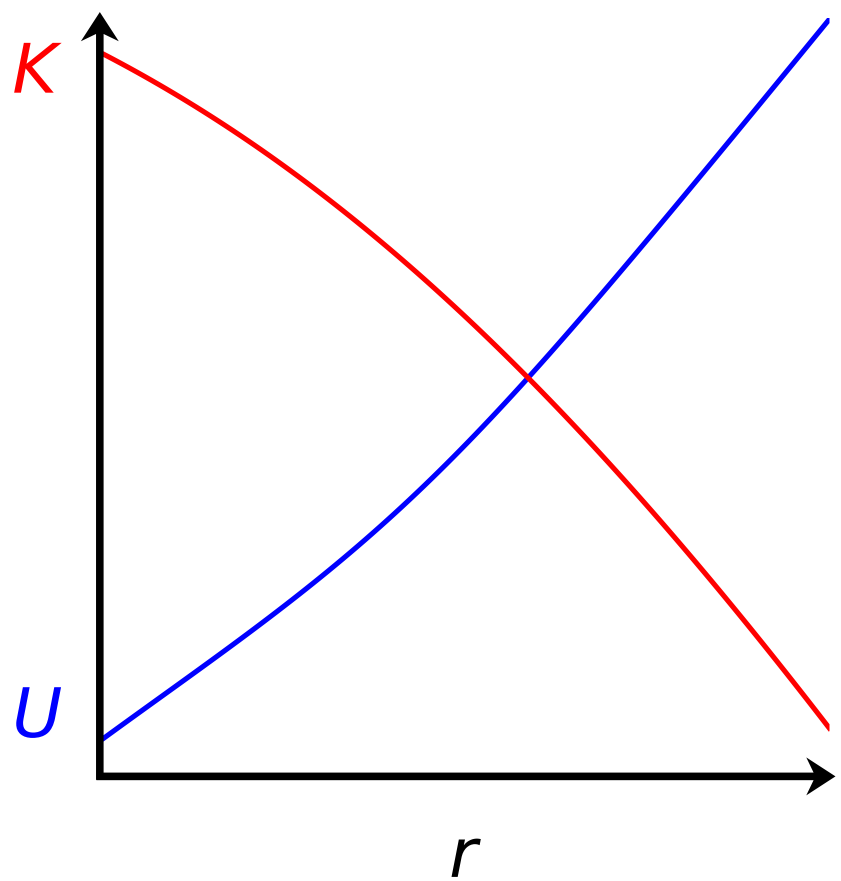
✦ **Slow mass change**

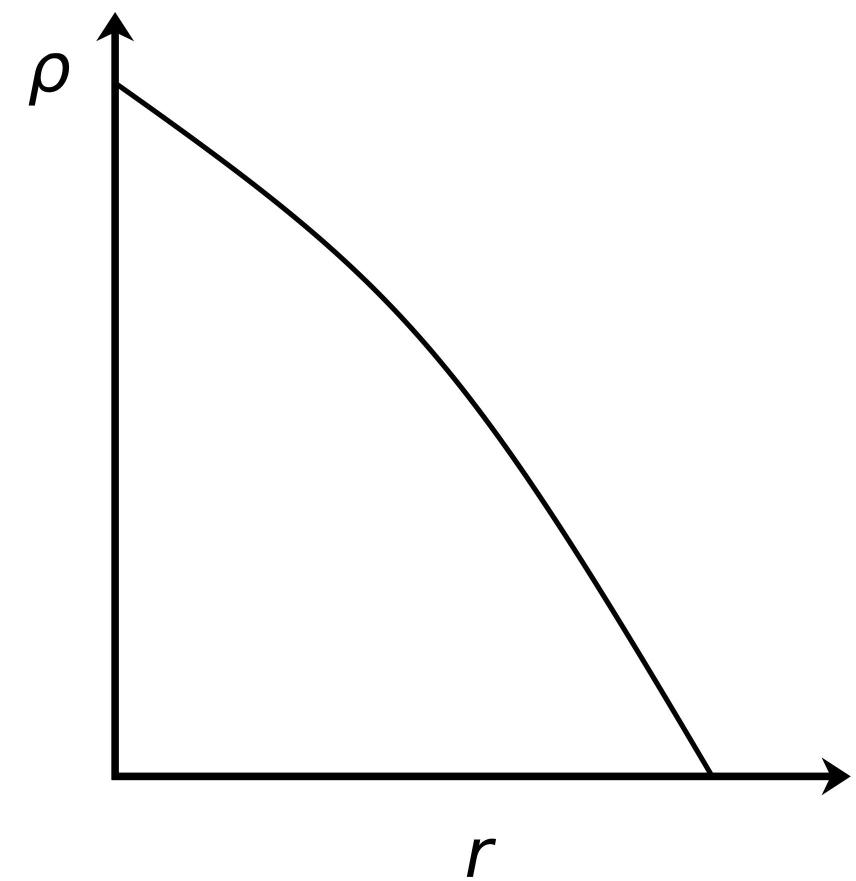
Angular momentum conservation on circular orbits: $\frac{r_f}{r_i} = \frac{M}{M+m} = \frac{1}{1+f}$ with $f = \frac{m}{M}$

✦ **Instant mass change**

① Initial equilibrium $E_i(r_i) = U_i(r_i) + K_i(r_i)$







Freundlich et al. (2020a)

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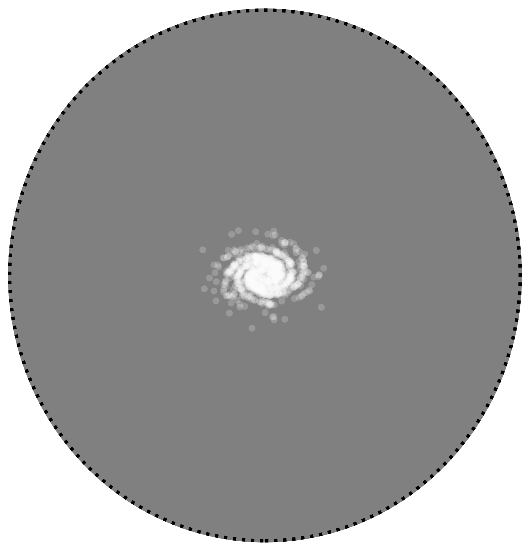
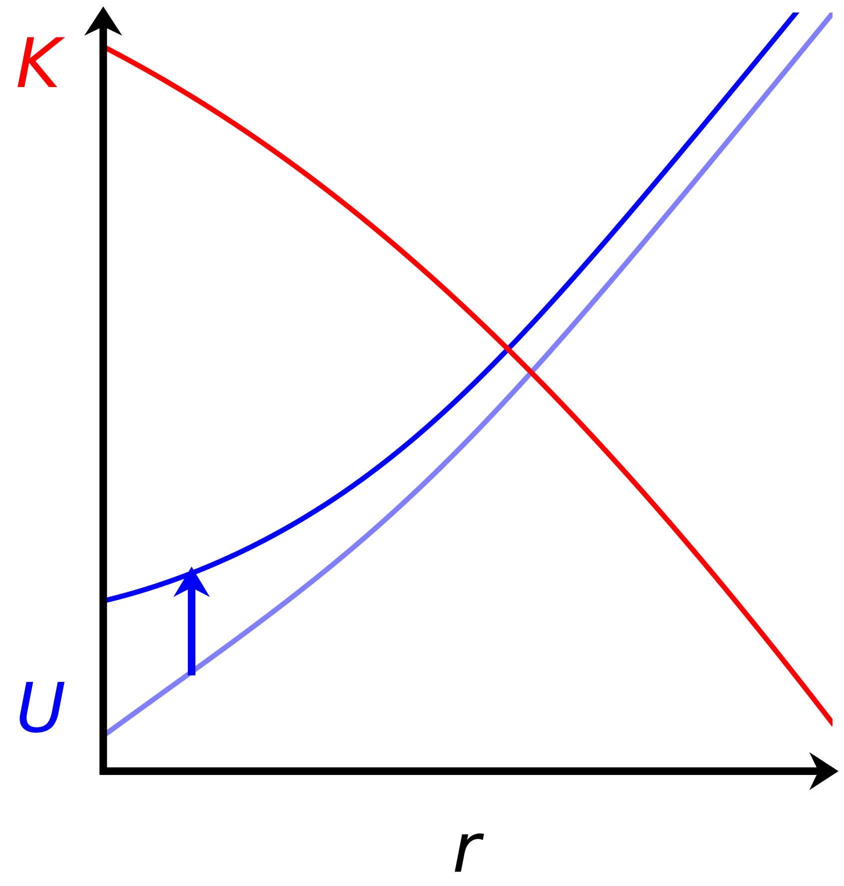
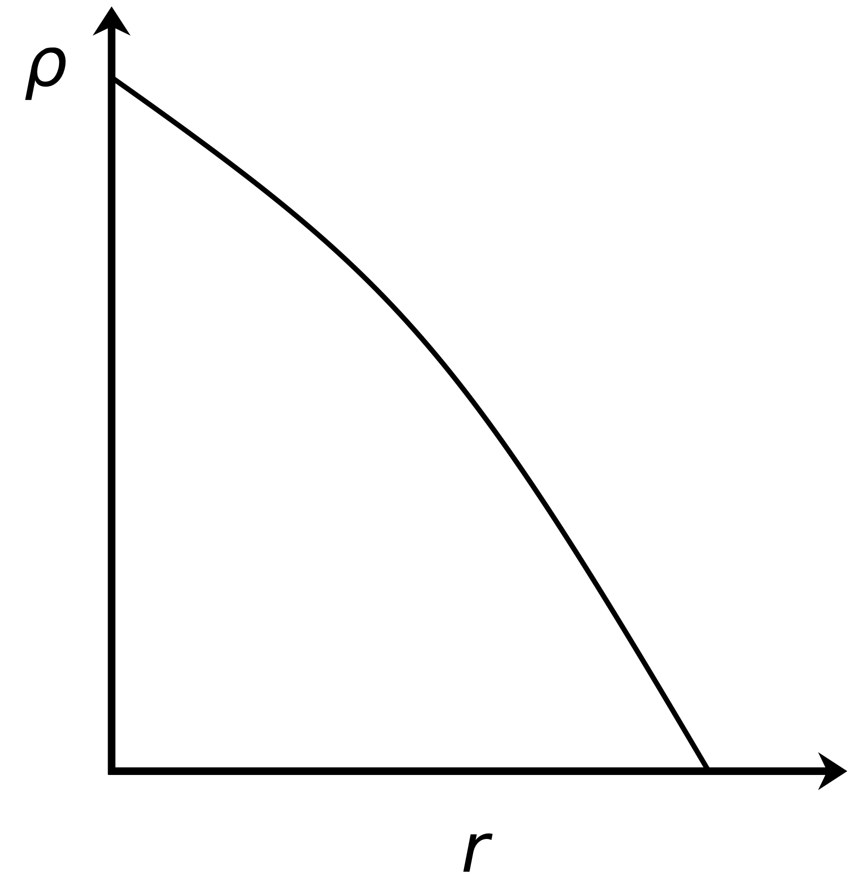
✦ **Instant mass change**

② Sudden gas removal $E_t(r_i) = U_i(r_i) - Gm/r_i + K_i(r_i)$







Freundlich et al. (2020a)

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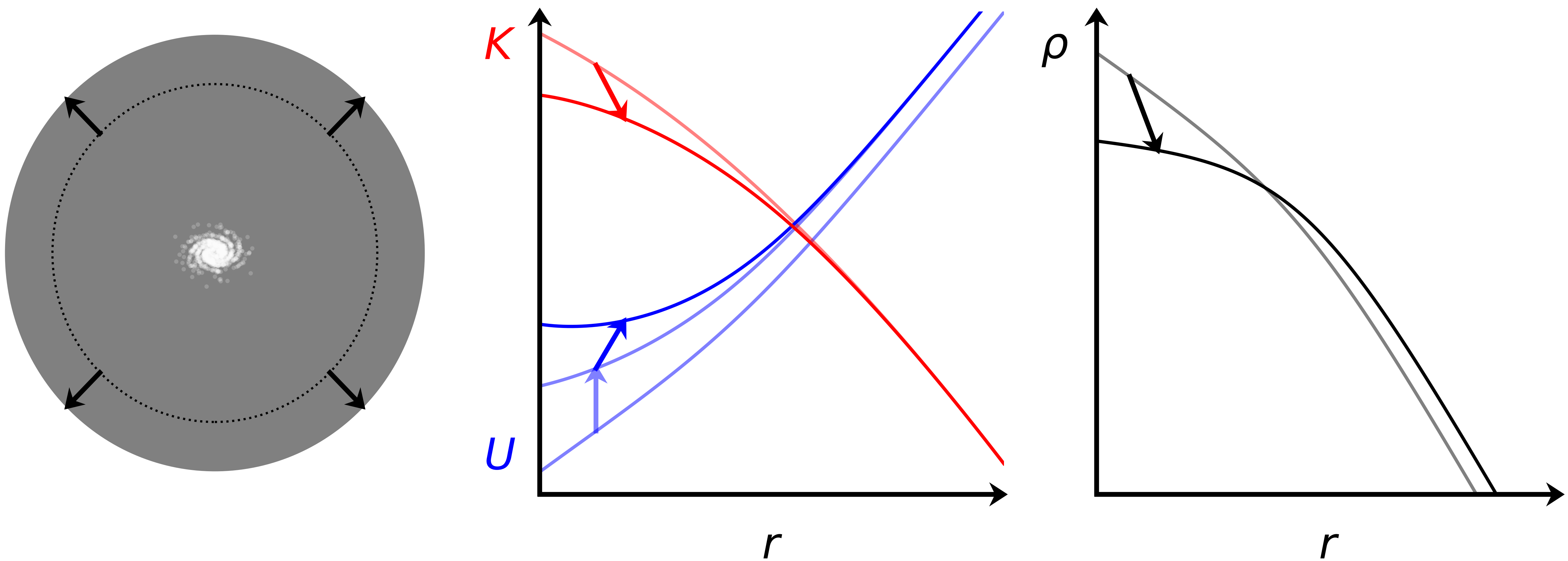
  Angular momentum conservation on circular orbits: $\frac{r_f}{r_i} = \frac{M}{M+m} = \frac{1}{1+f}$ with $f = \frac{m}{M}$
- ✦ **Instant mass change**

③ Relaxation

$E_f(r_f) = U_f(r_f) - Gm/r_f + K_f(r_f)$



Given functional forms ***U(r;p,m)*** and ***K(r;p,m)***, **energy conservation** $E_f(r_f) = E_t(r_i)$ during relaxation yields the final state (*CuspCore I*)

Freundlich et al. (2020a)

Dark matter halo parameterizations

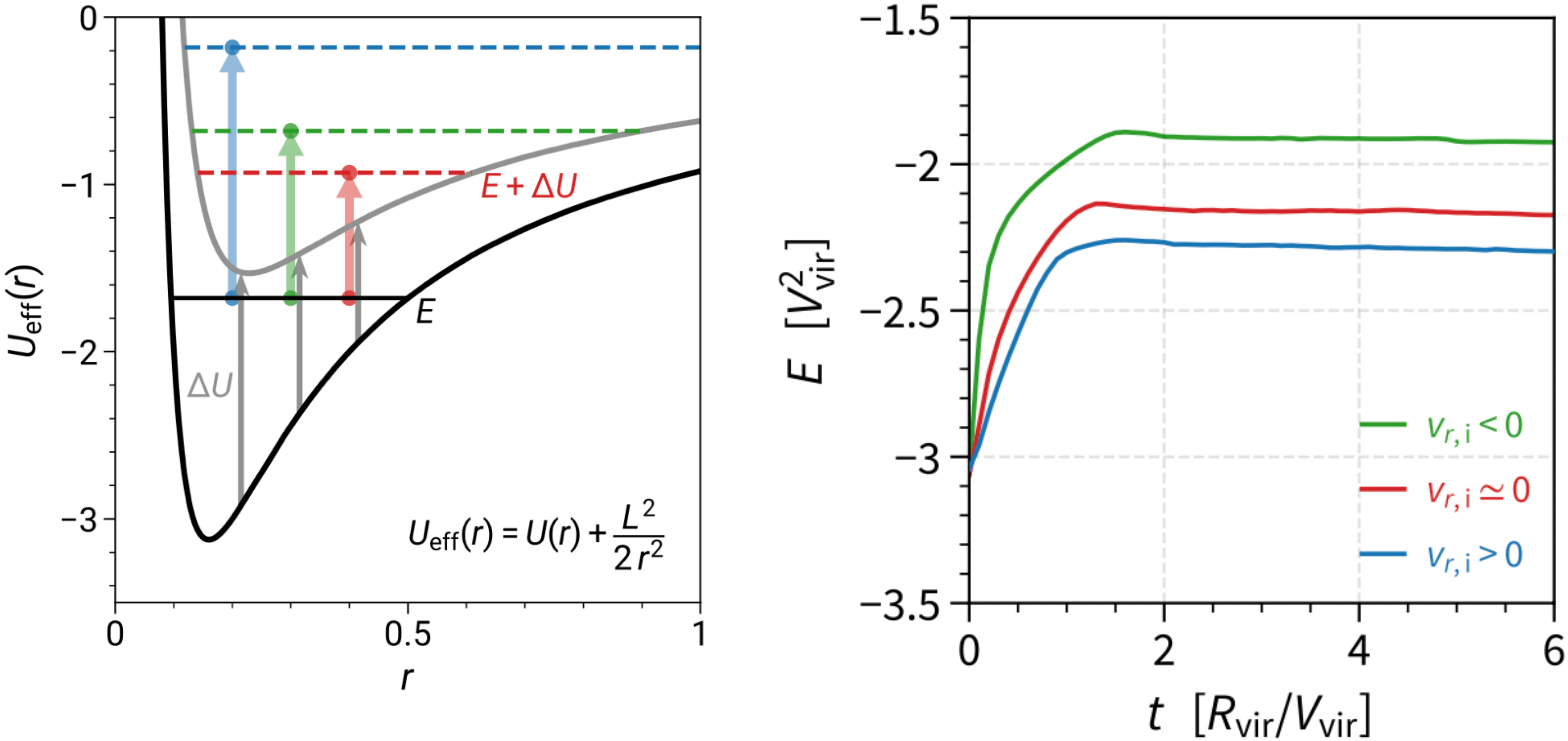
	Profile	Expression	Shape parameters	c_2	$M(r)$	$V(r)$	$\sigma_r(r)$	$\Phi(r)$	$\Sigma(r)$	$\overline{\Sigma}(r)$	$f(\mathscr{E})$	Mass-dependence
cusps	NFW NFW 1996 An & Zhao 2013	$\rho = \frac{\rho_c}{x(1+x^2)}$	c	✓	✓	✓	~	✓	✓	✓	~	$c(M_{\rm halo})$
cusps	superNFW Lilley+2018	$\rho = \frac{\rho_c}{x(1+x)^{5/2}}$	c	✓	✓	✓	✓	✓	~	✗	~	✗
cores	pISO	$\rho = \frac{\rho_c}{1+x^2}$	c	✓	✓	✓	✓	✓	✓	✓	~	✗
cores	Burkert Burkert 1995	$\rho = \frac{\rho_c}{(1+x)(1+x^2)}$	c	✓	✓	✓	~	✓	✗	✗	✗	✗
cores	Lucky13 Li+2020	$\rho = \frac{\rho_c}{(1+x)^3}$	c	✓	✓	✓	~	✓	~	~	~	✗
variable inner slope	Einasto Einasto 1965 An & Zhao 2013	$\rho = \rho_c \exp\left[-\frac{2}{\alpha}\left(x^\alpha - 1\right)\right]$	c, α	✓	~	~	~	~	~	~	✗	✗
variable inner slope	coreEinasto Lazar+2020	$\rho = \rho_c \exp\left[\frac{-2}{\alpha}\left(\left(x+x_c\right)^\alpha - 1\right)\right]$	c, x_c, α	✗	~	~	✗	~	✗	✗	✗	$r_c(x_M), c_2(x_M)$ with $\alpha=0.16$
variable inner slope	$\alpha\beta\gamma$/Di Cintio Zhao 1996 Di Cintio+2014	$\rho = \frac{\rho_c}{x^a(1+x^{1/b})^{b(g-a)}}$	c, a, b, g	✓	~	~	◐	◐	◐	◐	✗	$a(x_M), b(x_M), g(x_M), c_2(x_M)$
variable inner slope	gNFW	$\rho = \frac{\rho_c}{x^a(1+x)^{3-a}}$	c, a	✓	~	~	~	~	◐	◐	✗	✗
variable inner slope	coreNFW Read+2016	$M = f^n M_{\rm NFW}, f = \tanh\left(r/r_c\right)$	c, r_c, n	✗	✓	✓	✗	✗	✗	✗	✗	$c(M_{\rm halo})$
	Dekel-Zhao Zhao 1996 Dekel+2017 Freundlich+2020b	$\rho = \frac{\rho_c}{x^a(1+x^{1/2})^{2(3.5-a)}}$	c, a (or c_2, s_1)	✓	✓	✓	✓	✓	~	~	✗	$c_2(x_M), s_1(x_M)$

$x = r/r_s$ $c = R_{\rm vir}/r_s$ $x_M = M_{\rm star}/M_{\rm halo}$ $c_2 = R_{\rm vir}/r_{-2}$ ✓ available ~ non-elementary functions ✗ not available ◐ only certain cases

*for the $\alpha\beta\gamma$ profile, $M(r)$, $V(r)$, $\sigma_r(r)$, and $\Phi(r)$ can be expressed using elementary functions in certain cases (in particular when $\alpha = n, \beta = 3 + k/n$ with $k, n \in N$)

CuspCore: shortcomings

- ✦ **Energy diffusion:** particles on the same orbit experience different energy gains depending on their orbital phase
- ✦ **Violent relaxation** followed by **phase mixing**



Li et al. 2022 (incl. Freundlich)

CuspCore II: iteratively updating the distribution function

① Initial equilibrium

$$U_0 = U_{\rm g,i} + U_{\rm dm,i}$$

$$f_0(E) = \frac{1}{\sqrt{8}\pi^2} \int_E^0 \frac{d^2\rho_{\rm dm,i}}{dU_0^2} \frac{dU_0}{\sqrt{U_0 - E}}$$

② Sudden gas removal/addition

$$U_1 = U_{\rm g,f} + U_{\rm dm,i}$$

$$\Delta U(r) = U_1(r) - U_0(r)$$

③ Relaxation to new equilibrium

New distribution function for $E \to E + \Delta U(r)$

$$f_k(E) = \frac{\int_0^{r_E} f_{k-1}(E - \Delta U(r))\sqrt{E - U_k(r)}r^2\,dr}{\int_0^{r_E} \sqrt{E - U_k(r)}r^2\,dr}$$

Equilibrium density in *static* U_k

$$\rho'_{{\rm dm},k}(r) = 4\sqrt{2}\pi \int_{U_k(r)}^0 f_k(E)\sqrt{E - U_k(r)}dE$$

Evolve $\rho_{\rm dm}$ towards $\rho'_{{\rm dm},k}$ with a finite step

$$\rho_{{\rm dm},k}(r) = \mu\rho'_{{\rm dm},k}(r) + (1-\mu)\rho_{{\rm dm},k-1}(r)$$

Update potential

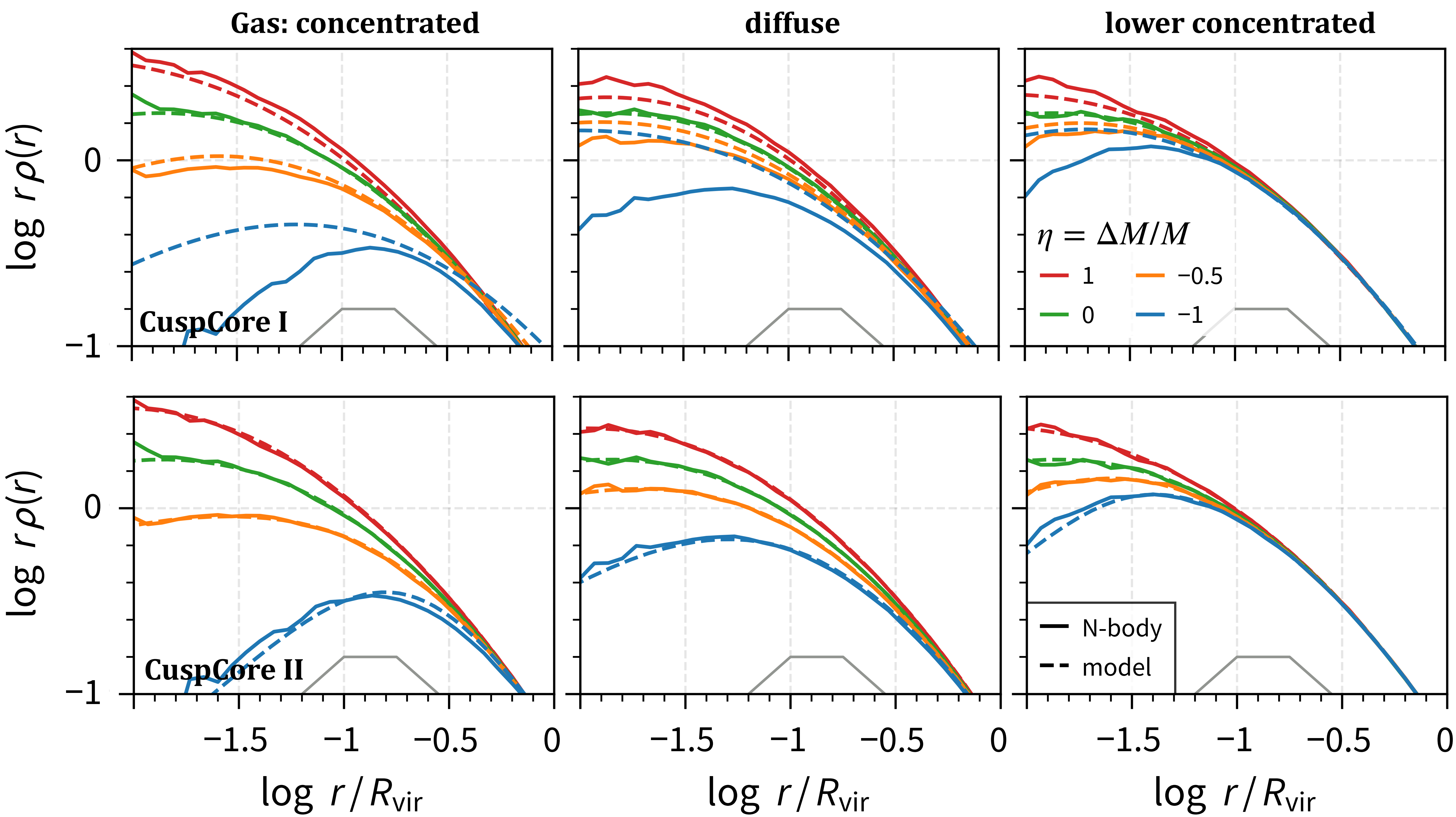
$$U_{k+1}(r) = U_{\rm g,f}(r) - 4\pi G \int_r^\infty \frac{dy}{y^2} \int_0^y \rho_{{\rm dm},k}(x)x^2\,dx$$

$k \mathrel{+}= 1$

$\Delta U(r) = U_k(r) - U_{k-1}(r)$

Li et al. 2022 (incl. Freundlich)

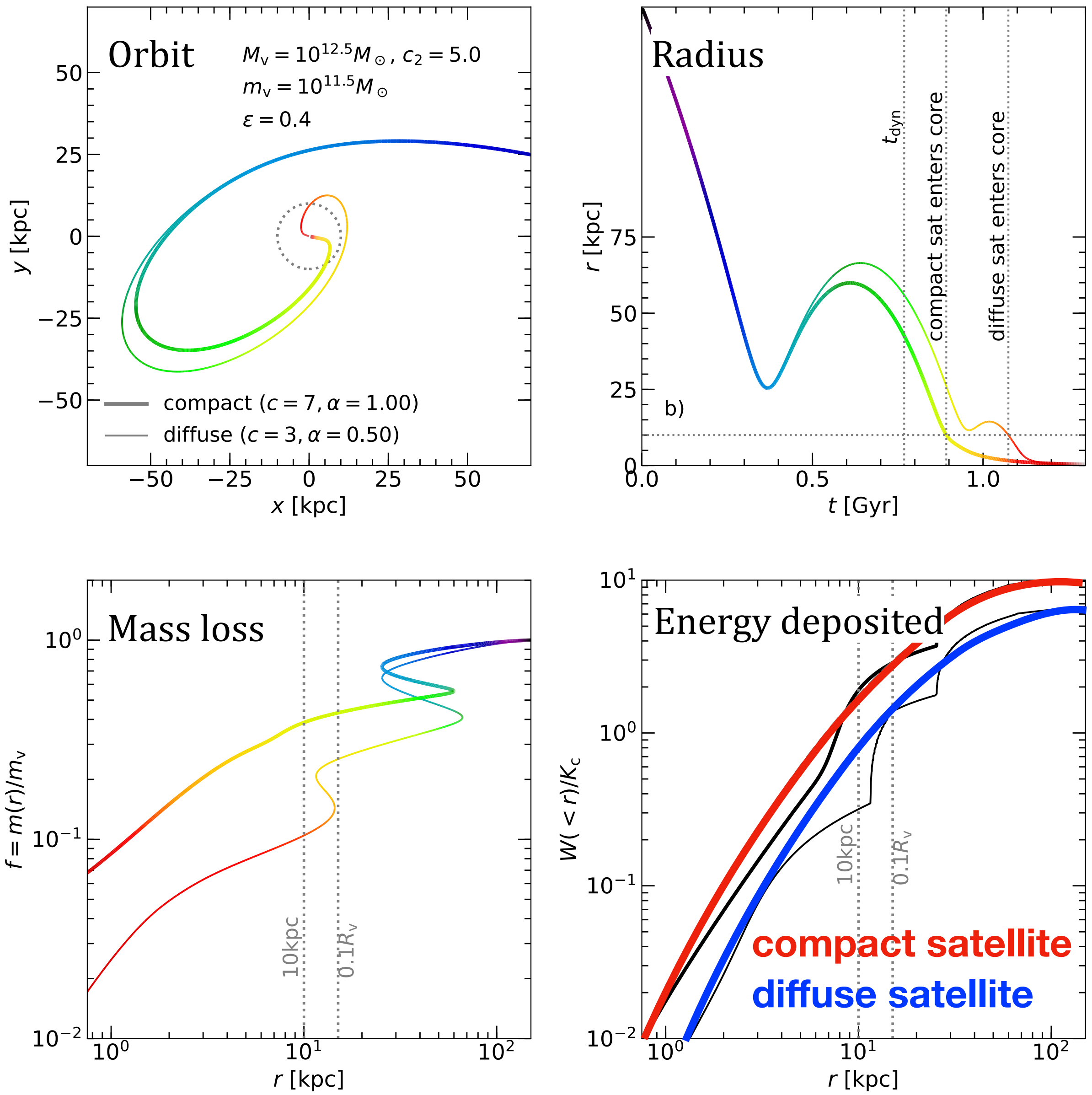
CuspCore II: numerical test



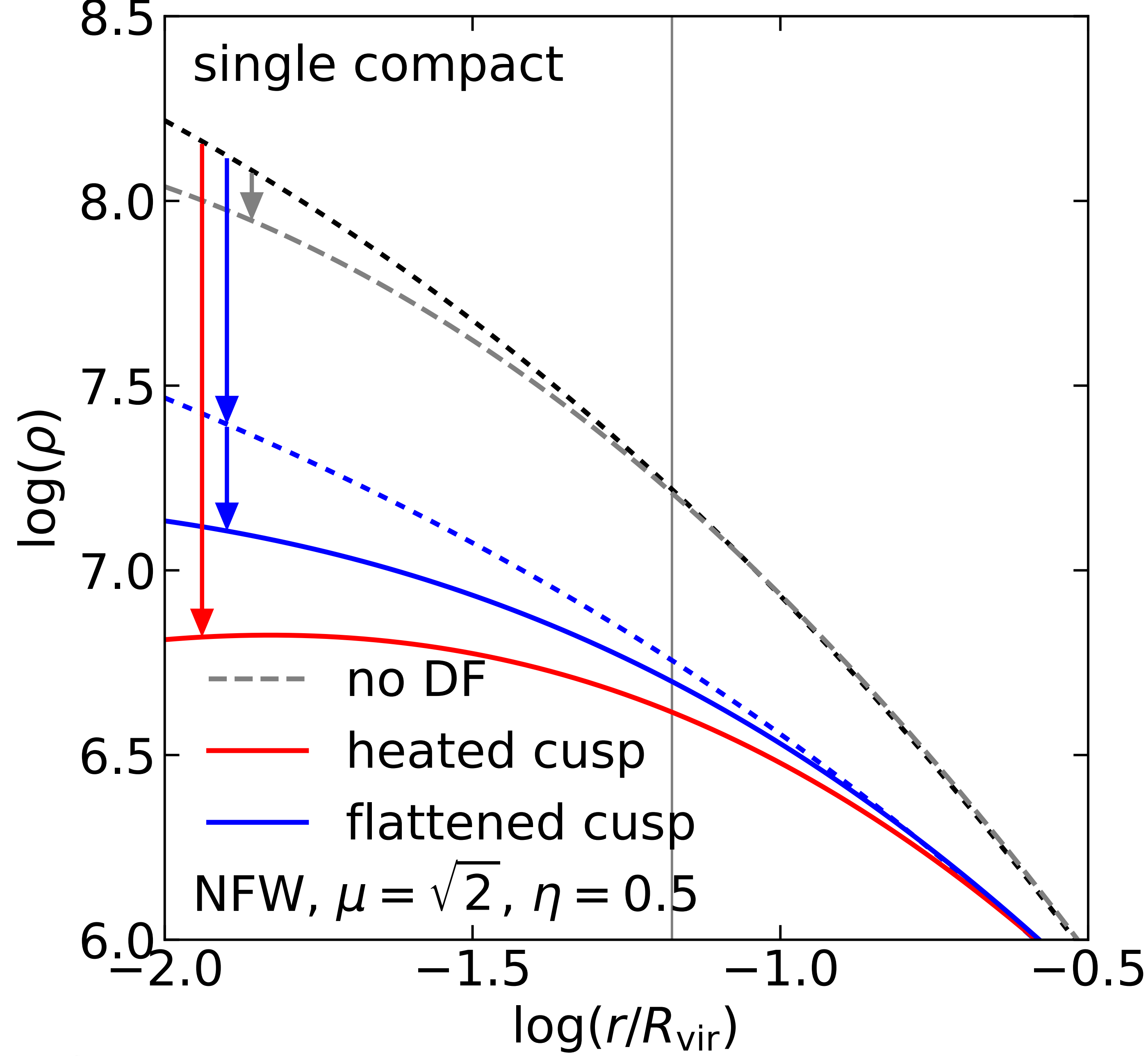
Li et al. 2022 (incl. Freundlich)

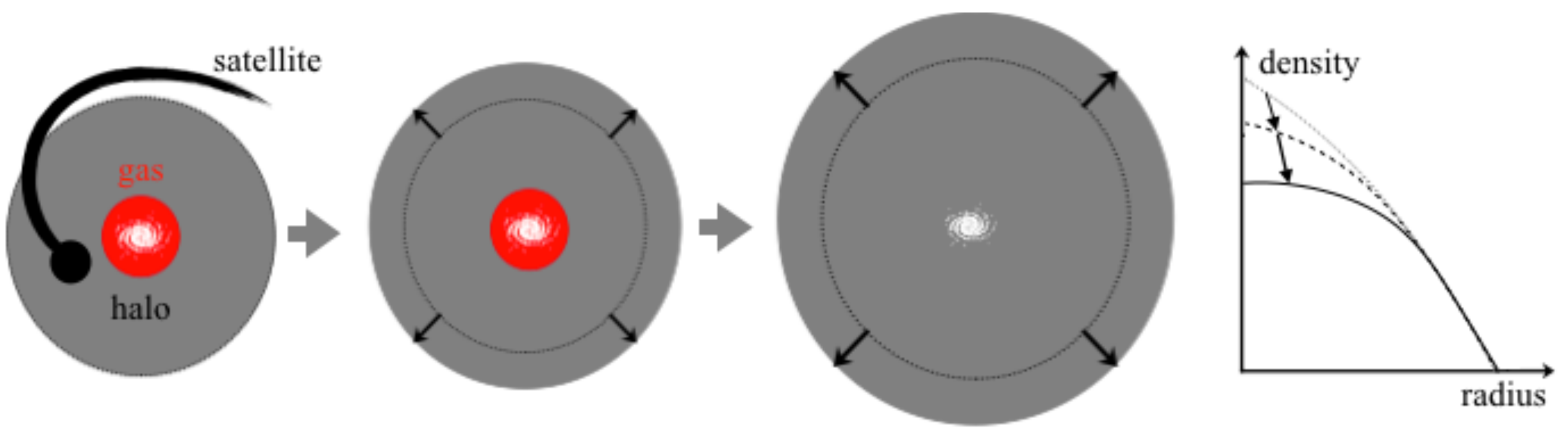
Enhanced core formation with dynamical heating and outflows

Energy deposited through dynamical friction…



…followed by a sudden AGN outflow





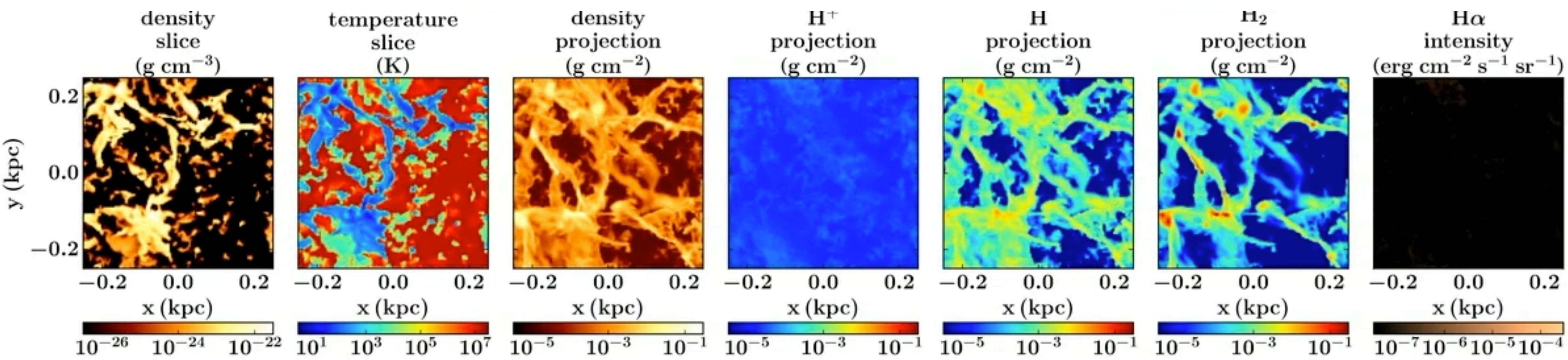
Dekel, Freundlich, Jiang et al. 2021

2/ Core formation from stochastic density fluctuations

El-Zant, Freundlich & Combes (2016), Hashim et al. (2023)

Core formation from stochastic density fluctuations

✦ **Effects of radiation, stellar winds and supernovae on the interstellar medium (e.g., SILCC Peters+17)**



✦ **Stochastic gas density fluctuations in an unperturbed homogeneous medium**

— Density contrast

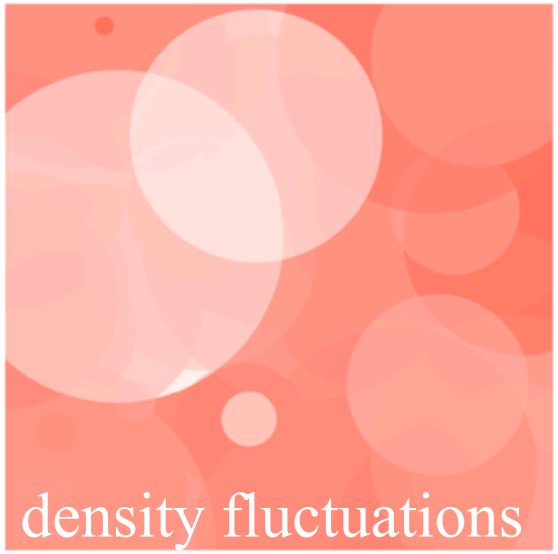
$$\delta(\mathbf{r}) = \frac{V}{(2\pi)^3} \int \delta_{\mathbf{k}}\, e^{i\mathbf{k}.\mathbf{r}}\, d^3\mathbf{k}$$

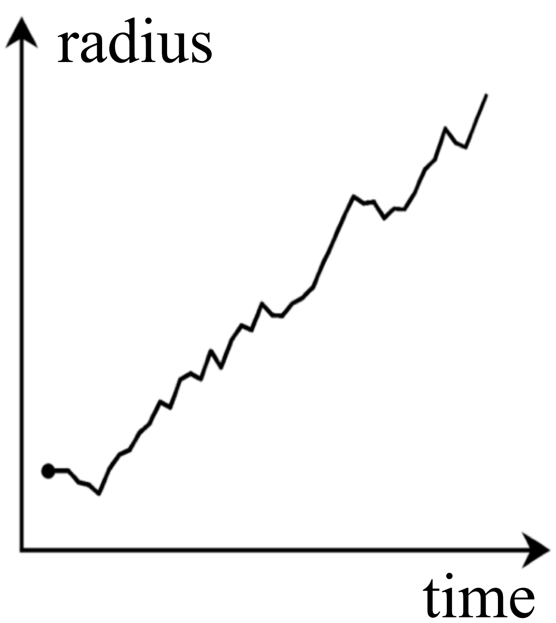
— Each mode induces a ‘kick’

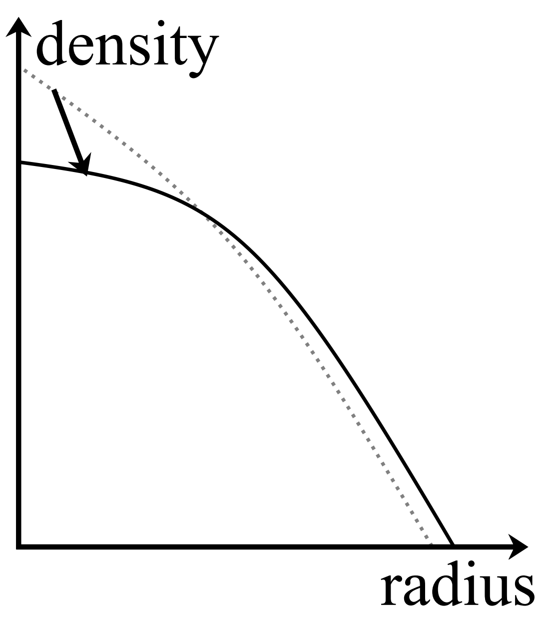
$$\mathbf{F}_{\mathbf{k}} = 4\pi i\, G\rho_0\, \mathbf{k}\, k^{-2}\, \delta_{\mathbf{k}}$$

— Which cumulatively induces the dark matter particles to deviate from their trajectories by

$$\langle \Delta v^2 \rangle = 2 \int_0^T (T-t)\langle F(0)F(t)\rangle\, dt$$



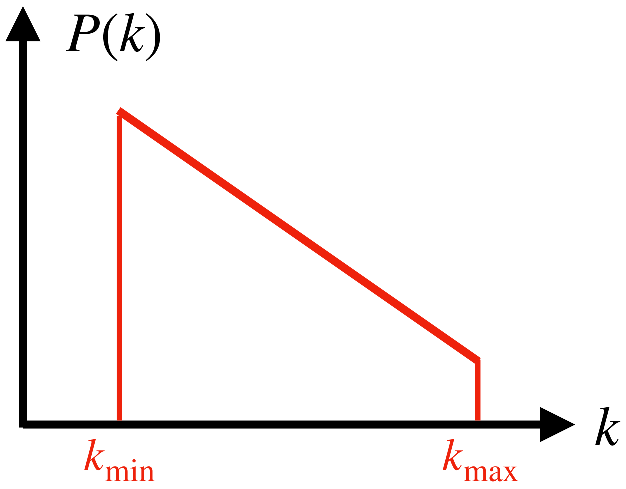




El-Zant, Freundlich & Combes (2016)

Main assumptions of the model

✦ **Isotropic, stationary fluctuations described by a power-law power spectrum with min/max cutoff scales**



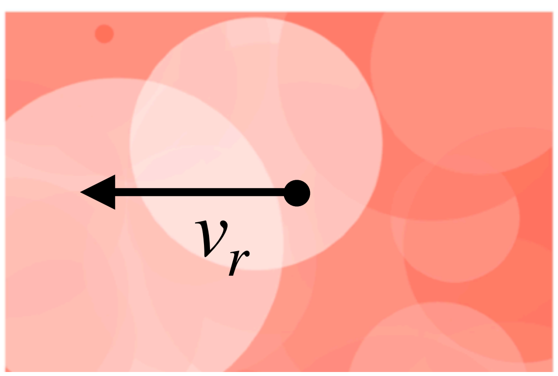
$$P(\mathbf{k}) = V\langle|\delta_{\mathbf{k}}|^2\rangle \propto k^{-n}$$

$$\phi_{\mathbf{k}} = -4\pi G\rho_0\delta_{\mathbf{k}}k^{-2}$$

$$P_F(k) = Vk^2\langle|\phi_k|^2\rangle$$

$$\langle\mathbf{F}(0)\,.\,\mathbf{F}(r)\rangle = \frac{1}{(2\pi)^3}\int_{k_{\rm min}}^{k_{\rm max}} P_F(k)\frac{\sin(kr)}{kr}4\pi k^2 dk$$

✦ **Spatial statistical properties transported (swept) into the temporal domain**



$$\langle\Delta v^2\rangle = 2\int_0^T (T-t)\langle F(0)F(t)\rangle\ dt = \frac{2}{v_r^2}\int_0^{R=v_rT}(R-r)\langle F(0)F(r)\rangle\ dr$$

✦ **Diffusion limit where the density fluctuations are small compared to the distance *R* travelled**

$2\pi/k_{\rm max} \ll 2\pi/k_{\rm min} \ll R$

$$t_{\rm relax} = \frac{nv_r\langle v\rangle^2}{8\pi(G\rho_0)^2V\langle|\delta_{k_{\rm min}}|^2\rangle}$$

El-Zant, Freundlich & Combes (2016)

Numerical implementation and test

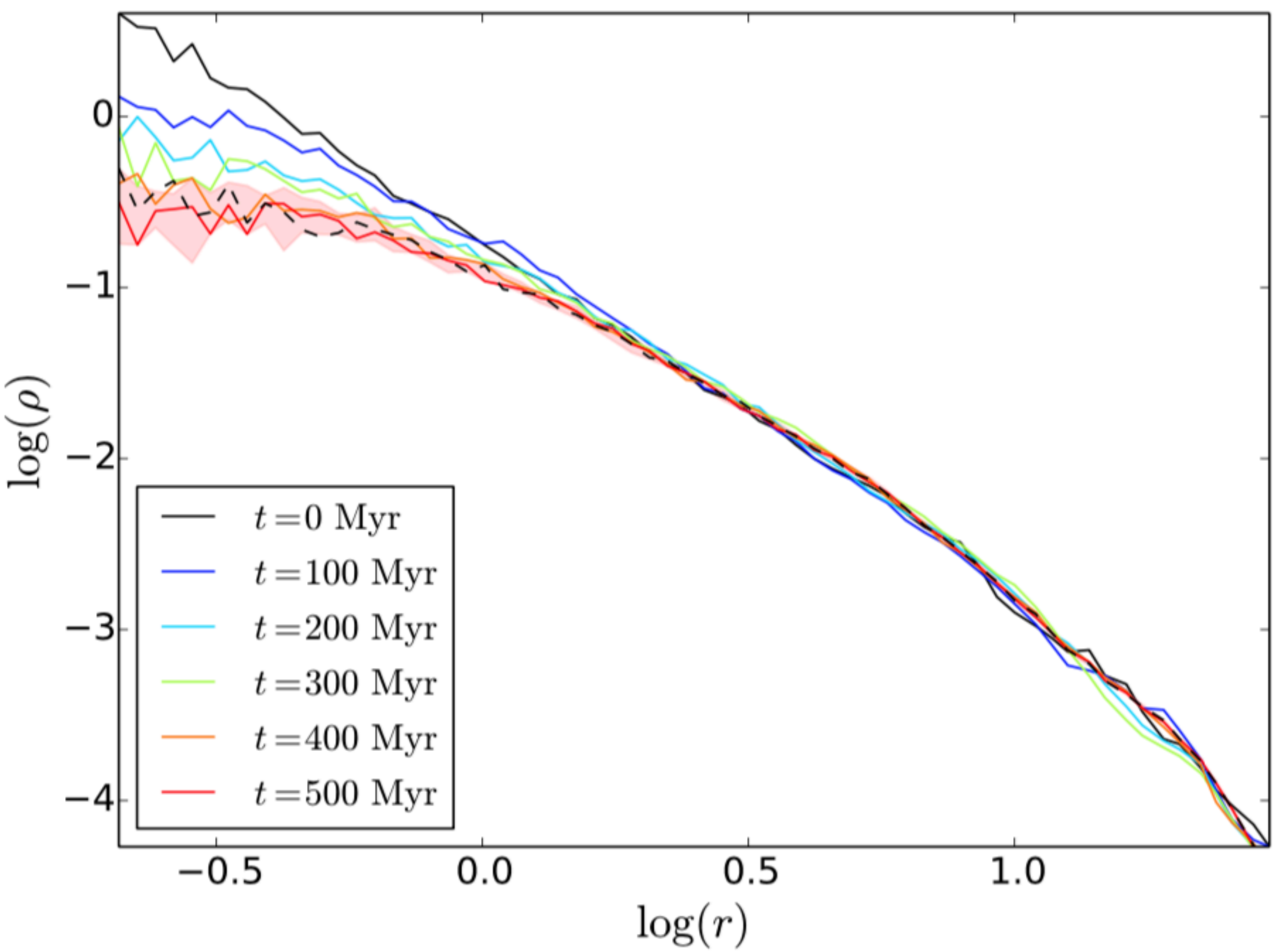
✦ **Self Consistent Field (SCF) method (Hernquist & Ostriker 1992)**

✦ **Fiducial dwarf NFW halo + force resulting from the stochastic density fluctuations**

— for each k, force kick according to the power spectrum in a random direction

— force normalization $\langle F(0)^2 \rangle = \frac{8(G\rho_0)^2 k_{\rm min} P(k_{\rm min})}{n-1} \left(1 - \left(\frac{k_{\rm min}}{k_{\rm max}} \right)^{n-1} \right)$

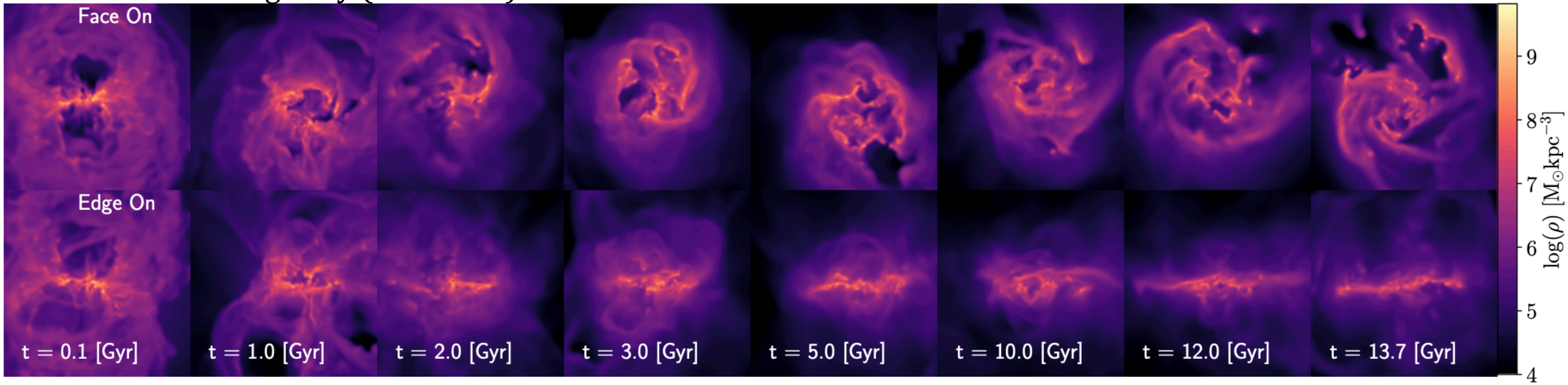
✦ **Core formation within a timescale comparable to the relaxation time**



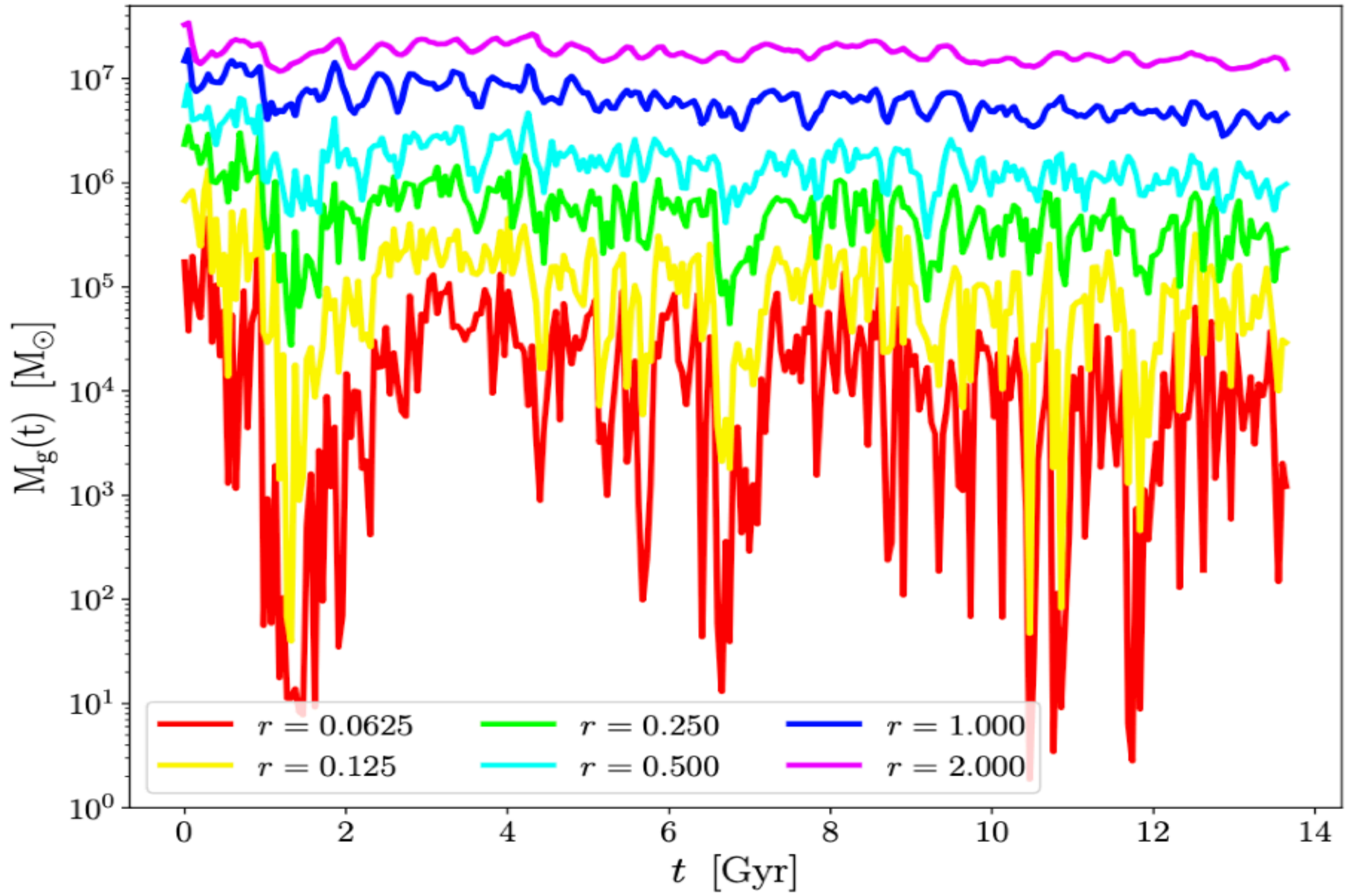
El-Zant, Freundlich & Combes (2016)

Hydrodynamical test

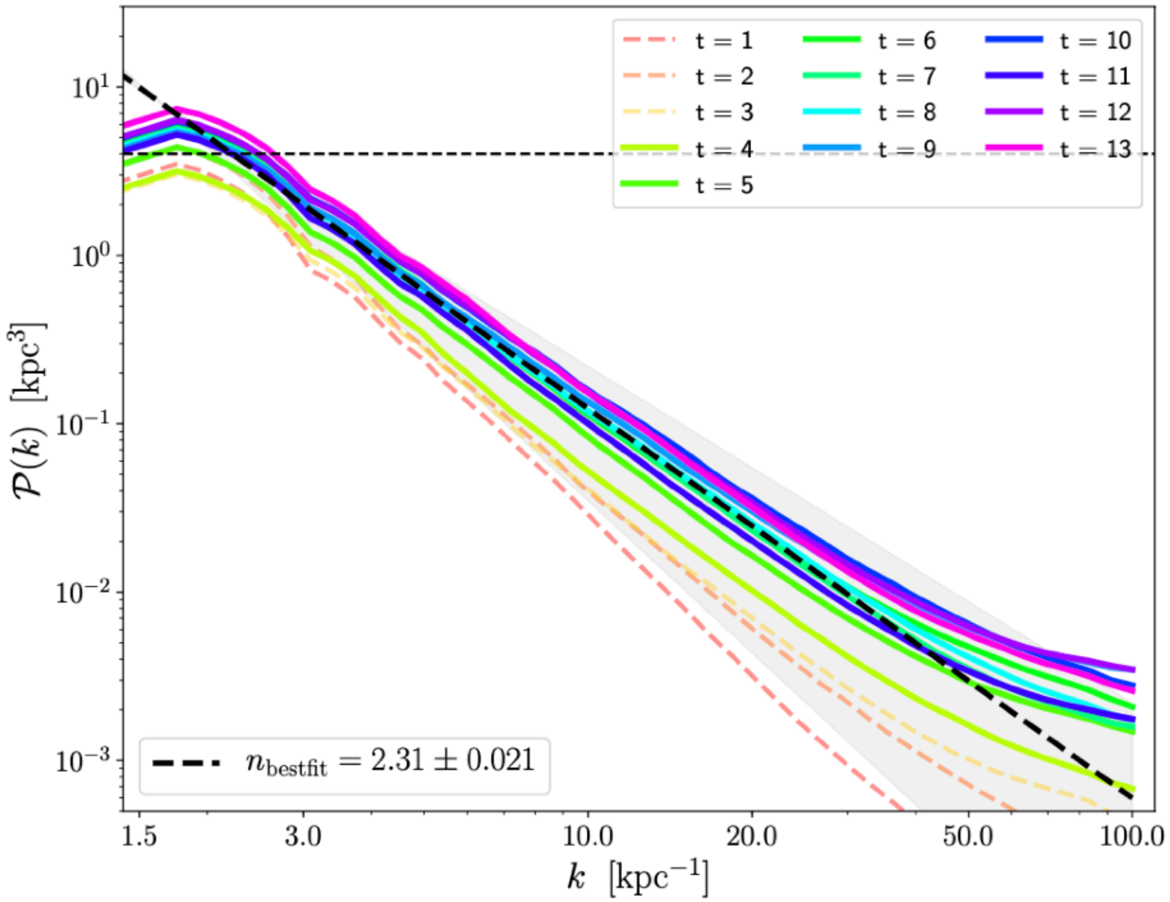
A simulated dwarf galaxy (10^9 Msun)



Mass fluctuations at different scales



A power-law power spectrum



Hashim et al. 2023 (incl. Freundlich)

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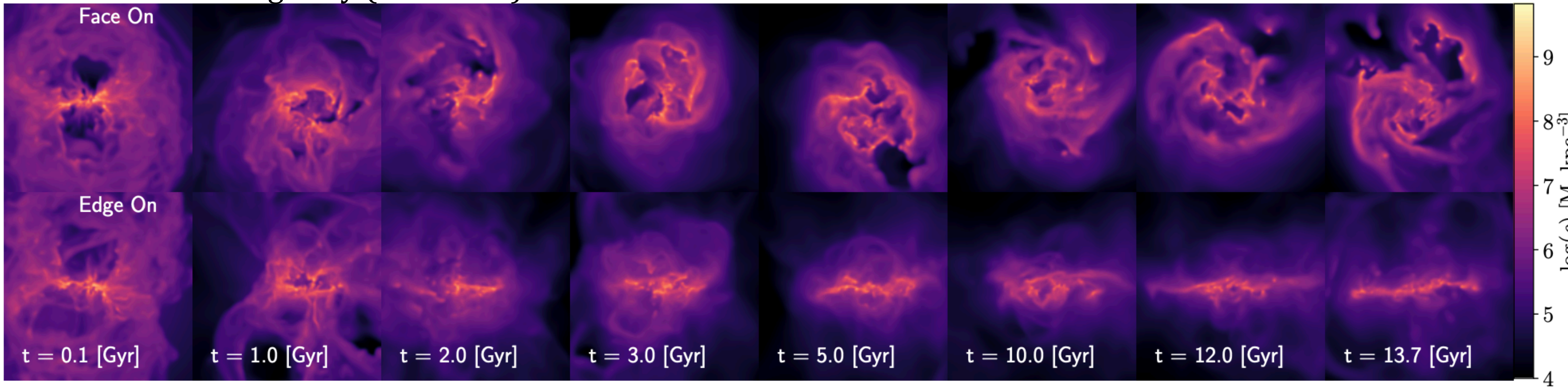
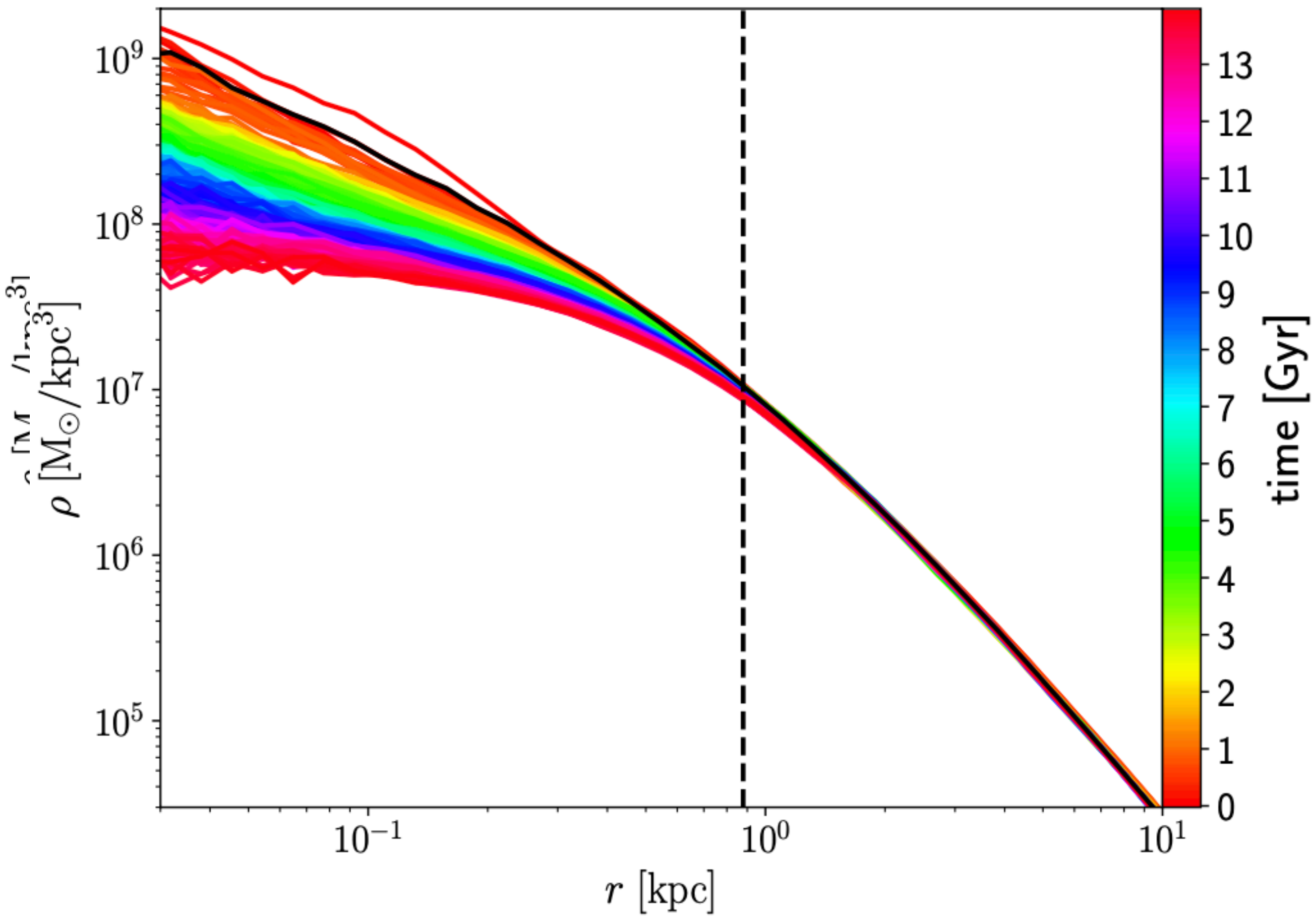
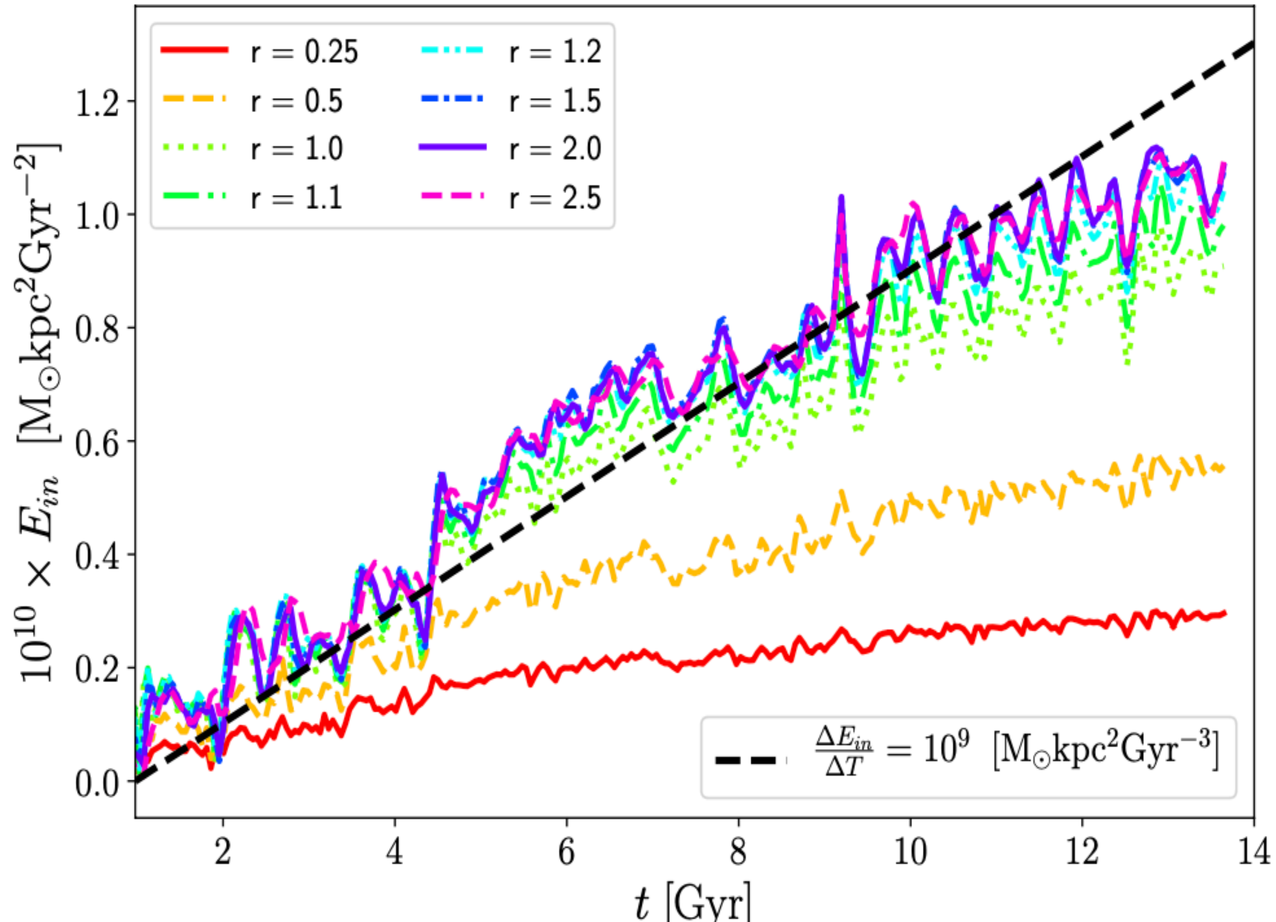


Density profile: core formation



Energy input: a diffusion process

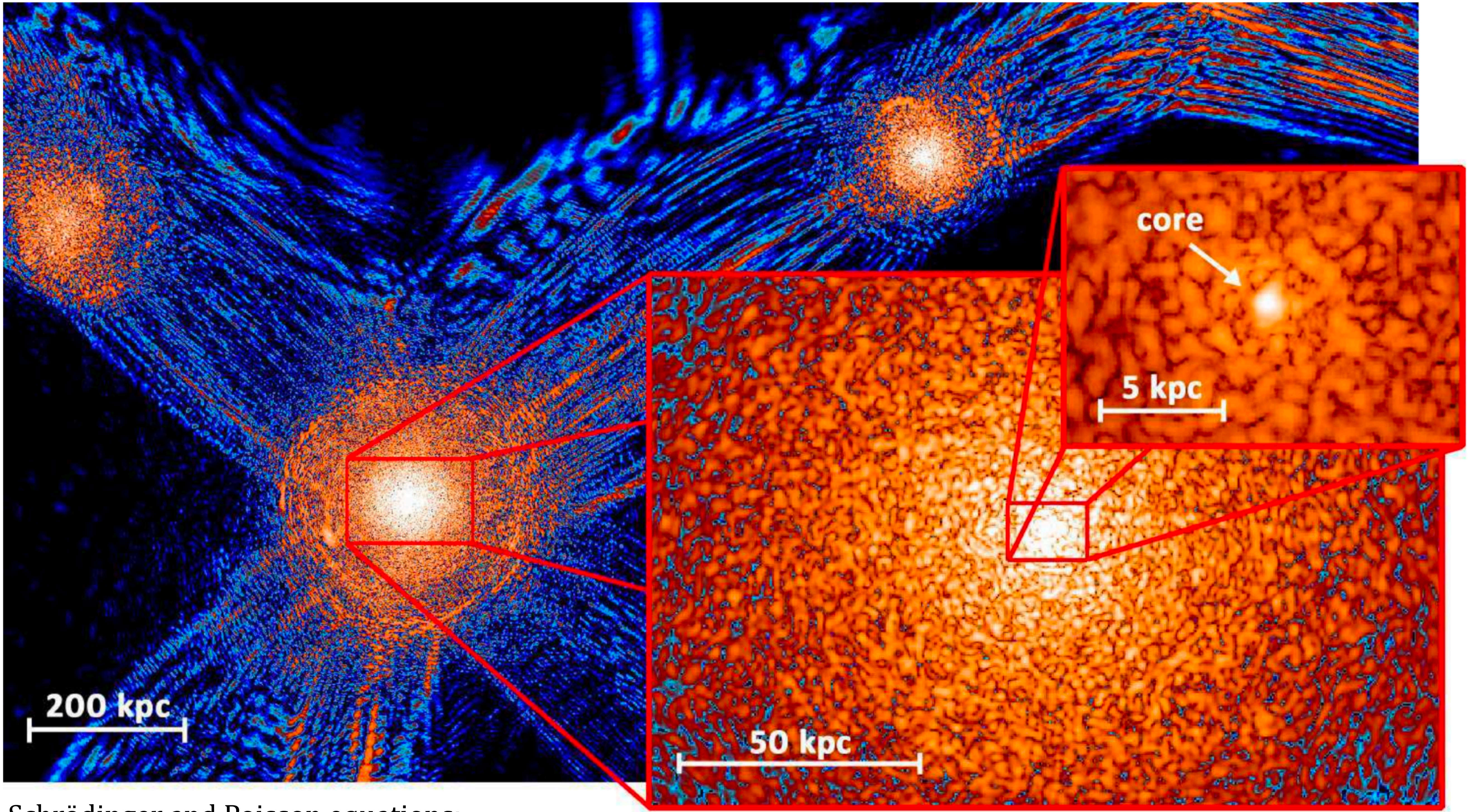


Hashim et al. 2023 (incl. Freundlich)

3/ Heating stellar systems with fuzzy dark matter?

Marsh & Niemeyer (2019), **El-Zant, Freundlich, Combes & Hallé (2020), Hallé et al. (in prep.)**

Fuzzy dark matter or ultra-light axions ($m \sim 10^{-22}$ eV)



Schrödinger and Poisson equations:

$$i\hbar \frac{\partial}{\partial t}\psi(\mathbf{r},t) = -\frac{\hbar^2}{2m}\nabla^2\psi(\mathbf{r},t) + m\ \Phi_s(\mathbf{r},t)\psi(\mathbf{r},t)$$

$$\nabla^2\Phi_s(\mathbf{r},t) = 4\pi G\ |\psi(\mathbf{r},t)|^2$$

Interferences, granules, core

Schive et al. (2014)

Constraining fuzzy dark matter: dynamical heating

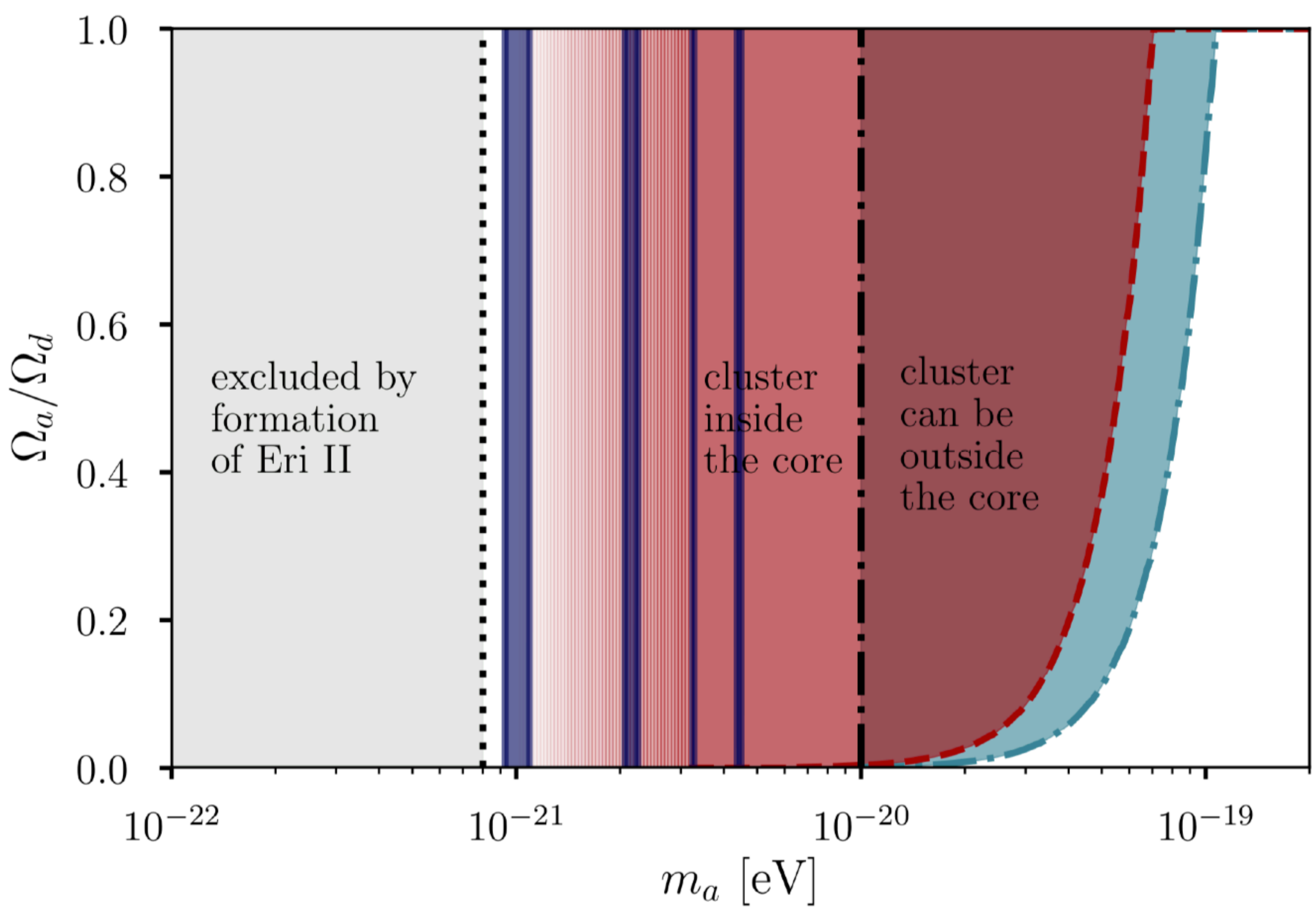
Marsh & Niemeyer 2019: Fuzzy dark matter (FDM) halo density fluctuations should heat up stellar structures, such as the old stellar cluster at the center of Eridanus II dwarf galaxy

— using the model of **El-Zant et al. (2019)**

— the central cluster is assumed to expand in virial equilibrium as it stars heat up

$$\frac{dr_h}{dt} = \frac{D}{G}\left(\frac{\alpha M_\star}{r_h^2} + 2\beta\rho_0 r_h\right)^{-1}$$

r_h half-mass radius, α=0.4, β=10 for a cored Sersic profile (cf. Brandt 2016)
D diffusion coefficient stemming from El-Zant et al. (2019)



Constraining fuzzy dark matter: dynamical heating

El-Zant, Freundlich, Combes & Hallé (2020): Adapting the formalism of El-Zant, Freundlich & Combes (2016), we derive the effect FDM halo fluctuations on test particles.

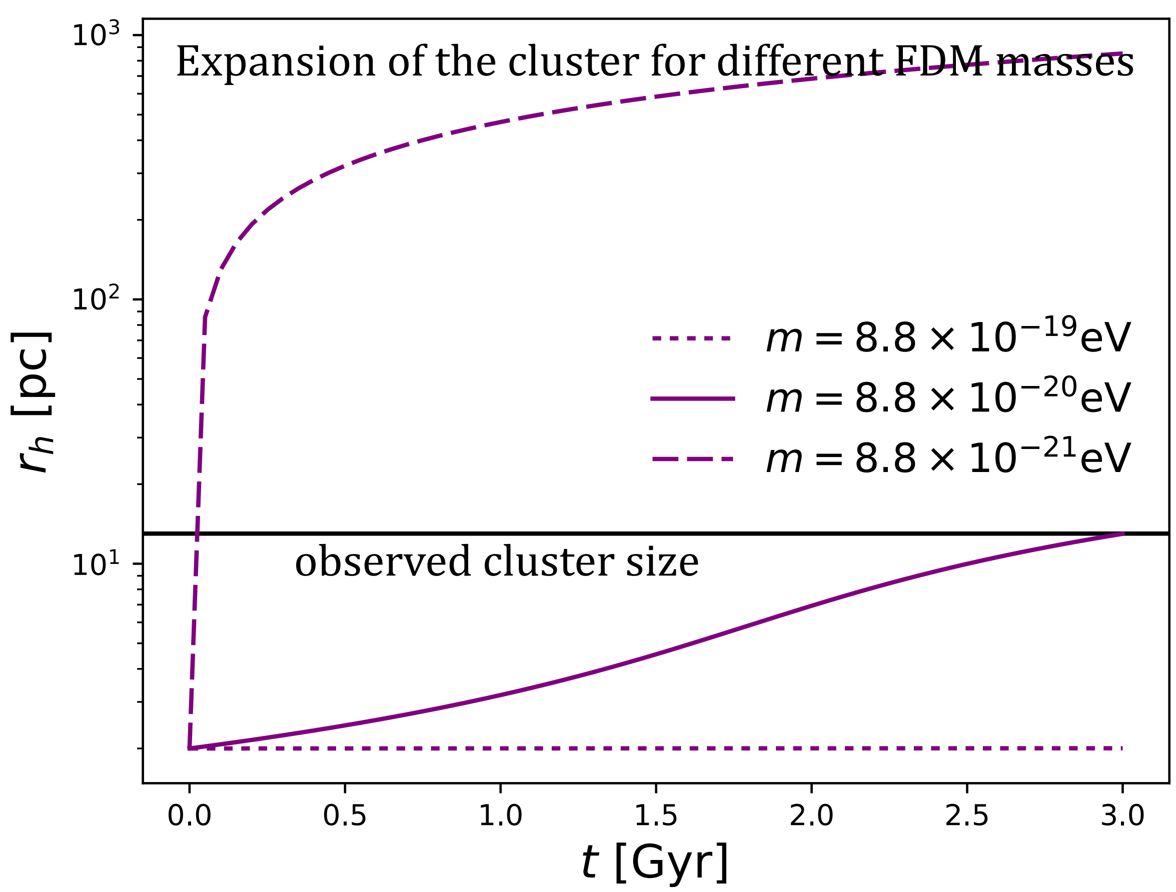
➡ Density power spectrum in line with FDM simulations

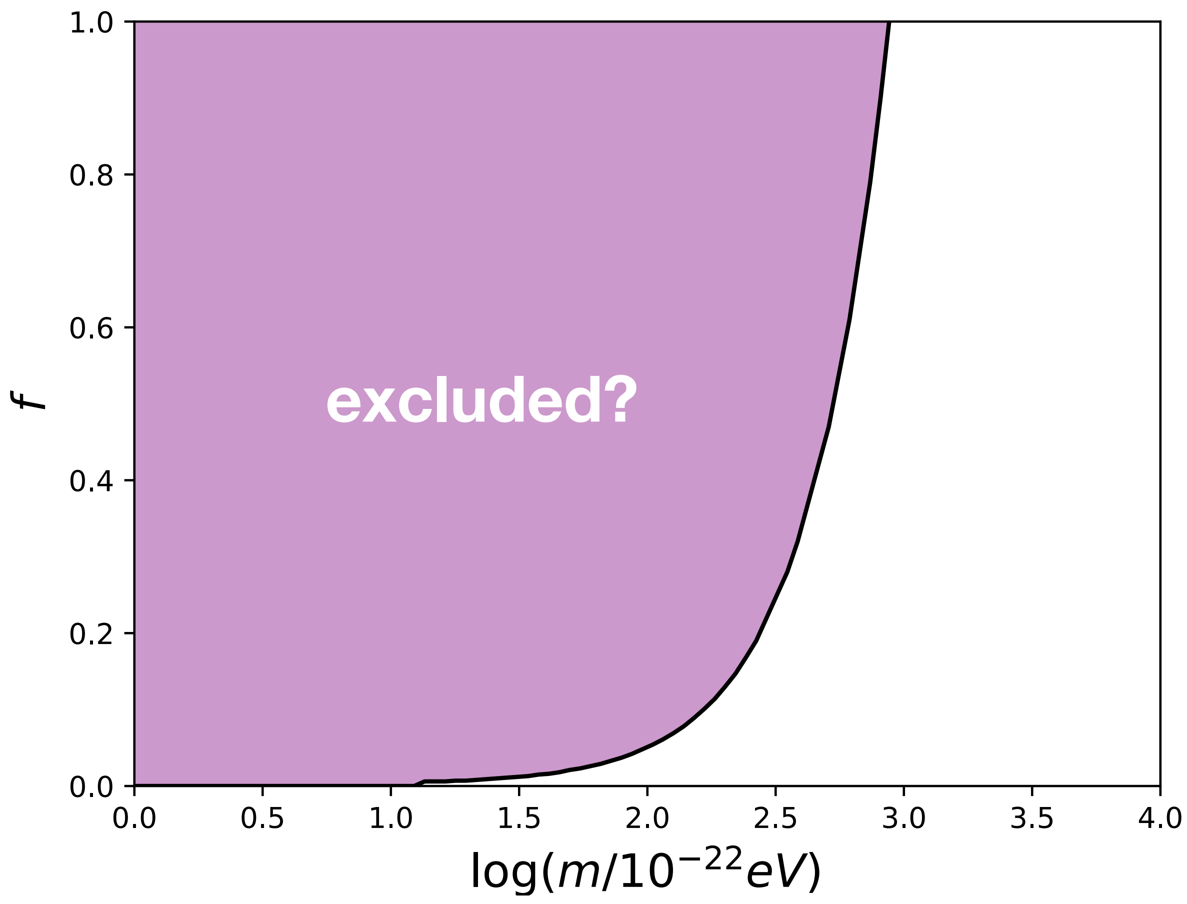
➡ Diffusion coefficient for a Maxwellian velocity distribution

➡ FDM particle mass m>2x10^{-22} eV from the local velocity dispersion in the Milky Way

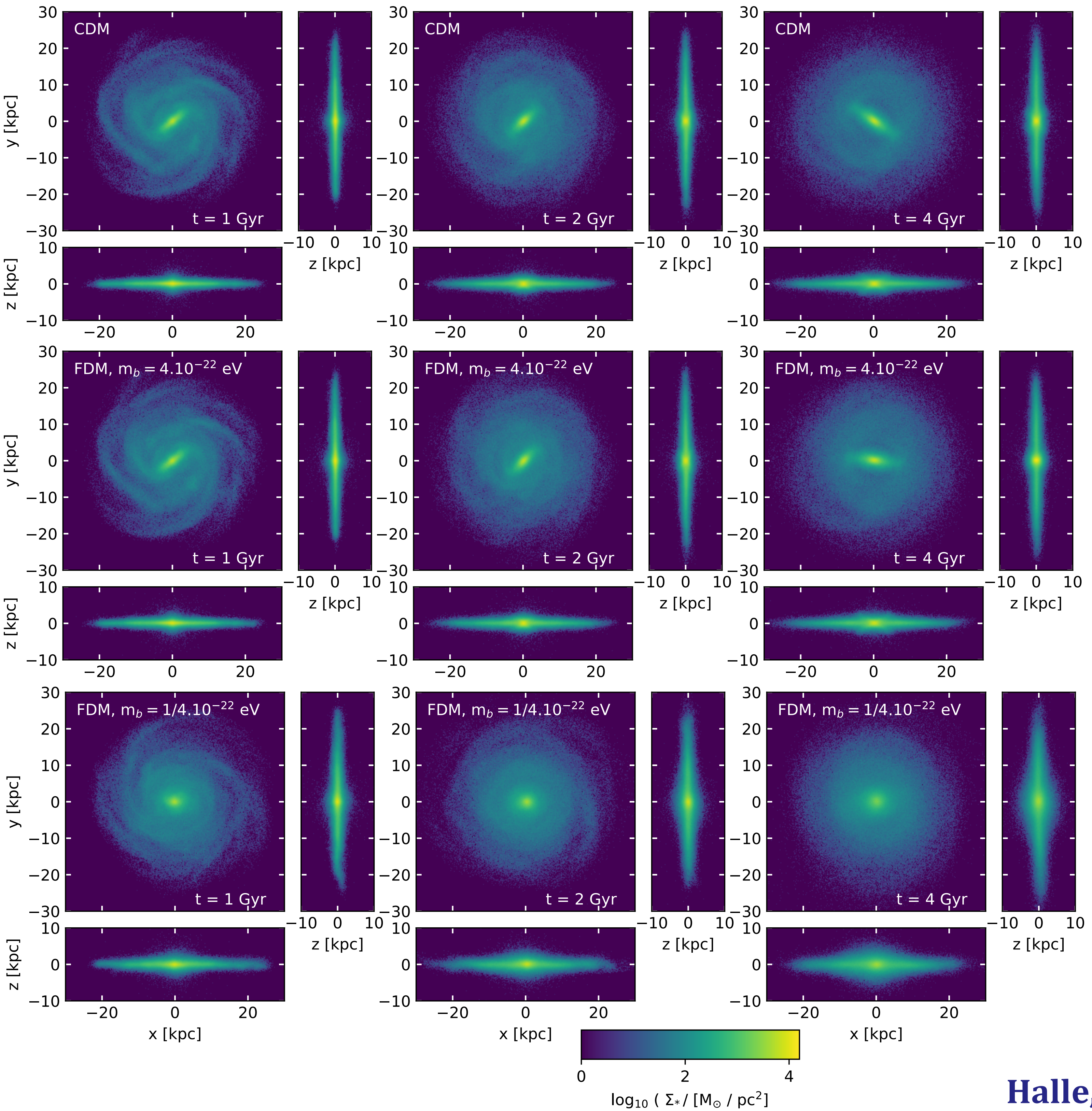
➡ The existence of the central stellar cluster of Eridanus II could in principle yield m ≳ 8.8 10^{-20} eV if all dark matter were fuzzy (cf. Marsh & Niemeyer 2019)

➡ **But:**
- The cluster lies inside the core for m ≲ 8.8 10^{-20} eV so the formalism does not rigorously apply
- For m ≳ 10^{-19} eV the granule size is bigger than the initial cluster size assumed: the fluctuations should not just heat up the cluster but affect it as a whole (e.g. displace it from the center)





Effect of FDM fluctuations on galactic disks



- ➡Thicker disk with increasing FDM mass
- ➡Weaker bar
- ➡Variation in the pattern speed?

Isolated disk + halo simulation
- Gadget-2 N-body code
- 80 pc softening length
- Additional force from FDM fluctuations (from El-Zant, Freundlich, et al. 2020)

Halle, El-Zant, Freundlich & Combes in prep.

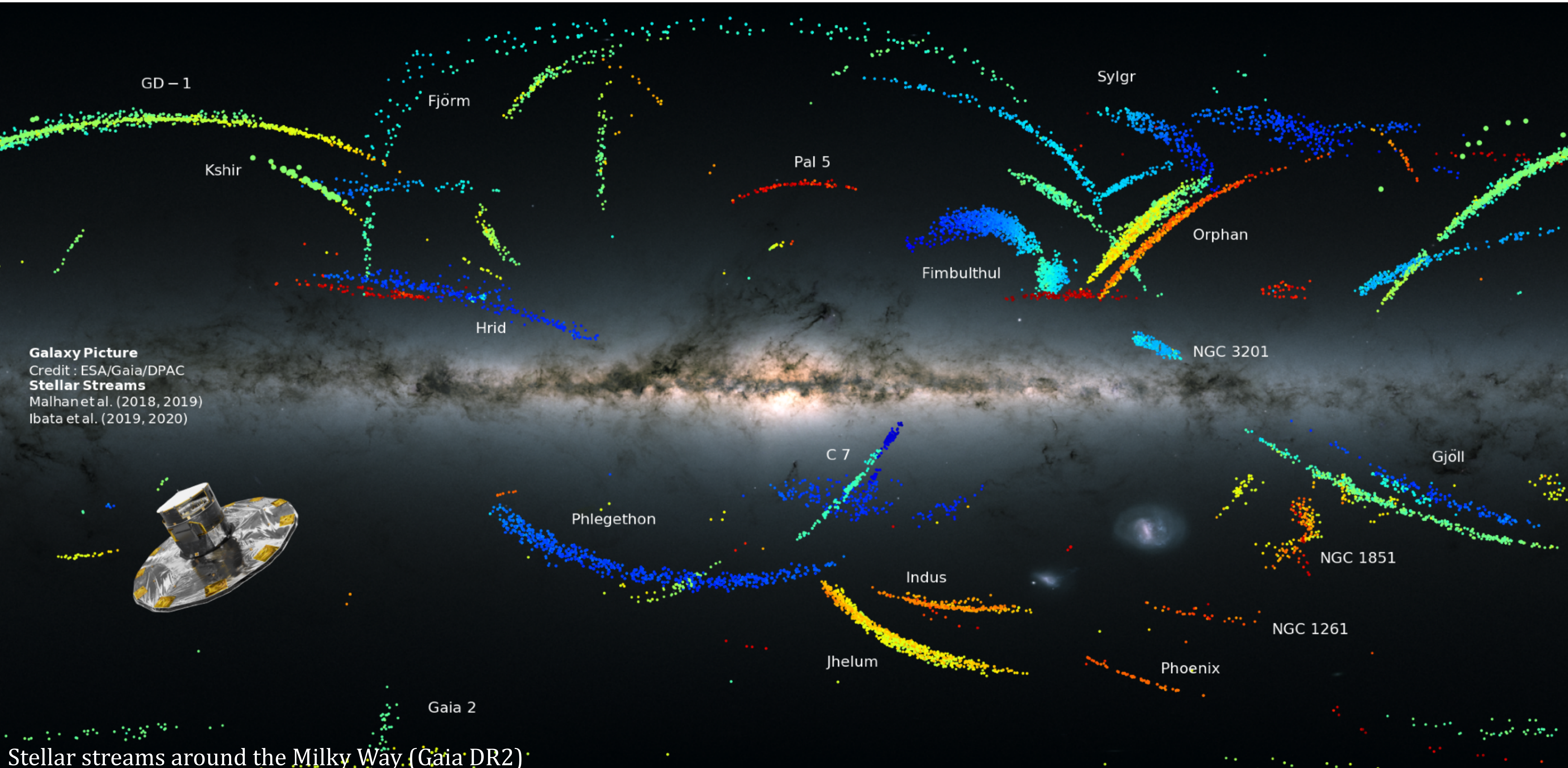
Constraining fuzzy dark matter with stellar streams?

Fuzzy dark matter halo fluctuations should dynamically heat or deform stellar streams…

Fokker-Planck equation $\partial_t f = -\nabla_{\mathbf{x}} \cdot (\mathbf{v}f) + \frac{1}{2}\nabla_{\mathbf{x}} \cdot \left(D_x \nabla_{\mathbf{x}} f\right) + \frac{1}{2}\nabla_{\mathbf{v}} \cdot \left(D_v \nabla_{\mathbf{v}} f\right)$

$$D\left[(\Delta v)^2\right] = \frac{4\sqrt{2}\pi G^2 \rho_1 m_{\mathrm{eff}}}{\sigma_{\mathrm{eff}}} \ln \Lambda \left[\frac{\mathrm{erf}(X_{\mathrm{eff}})}{X_{\mathrm{eff}}}\right]$$

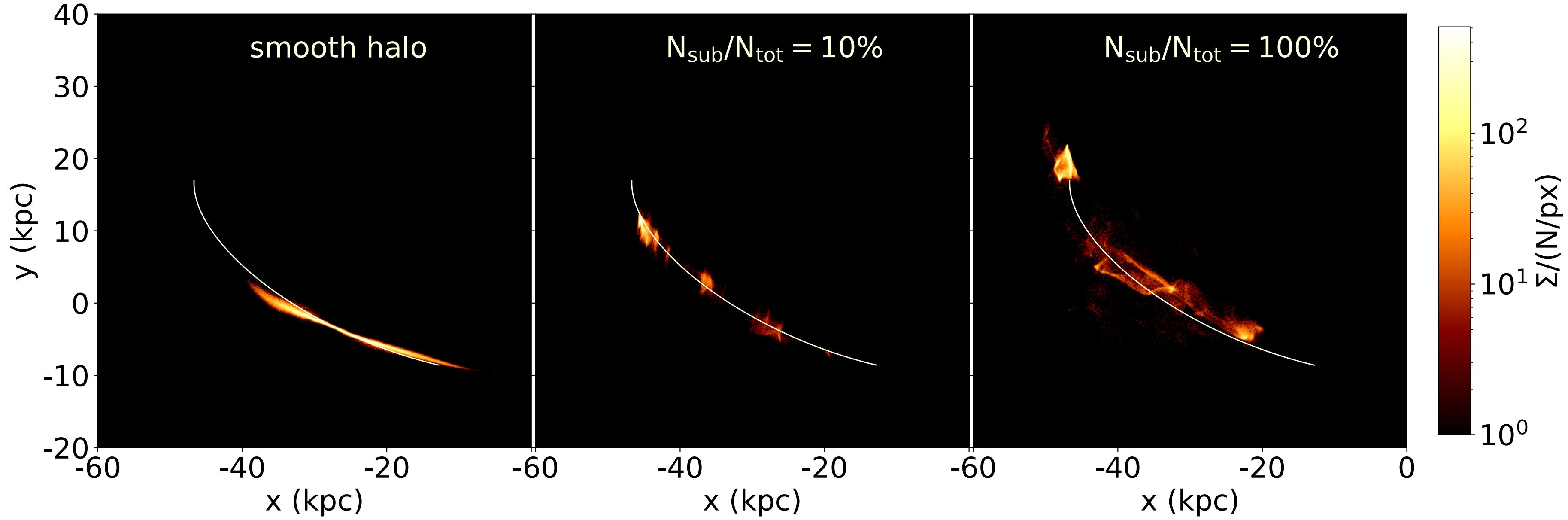
cf. also Delos & Schmidt 2022

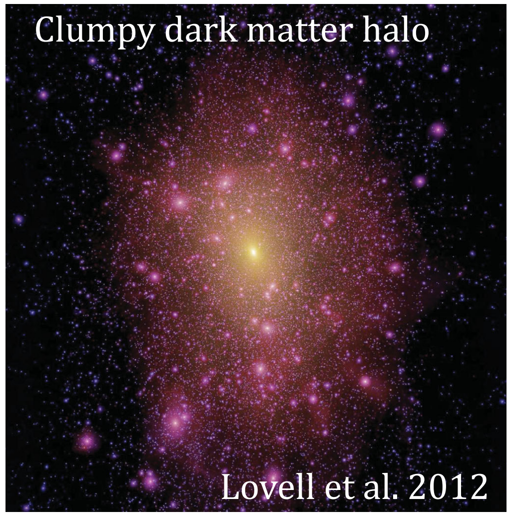


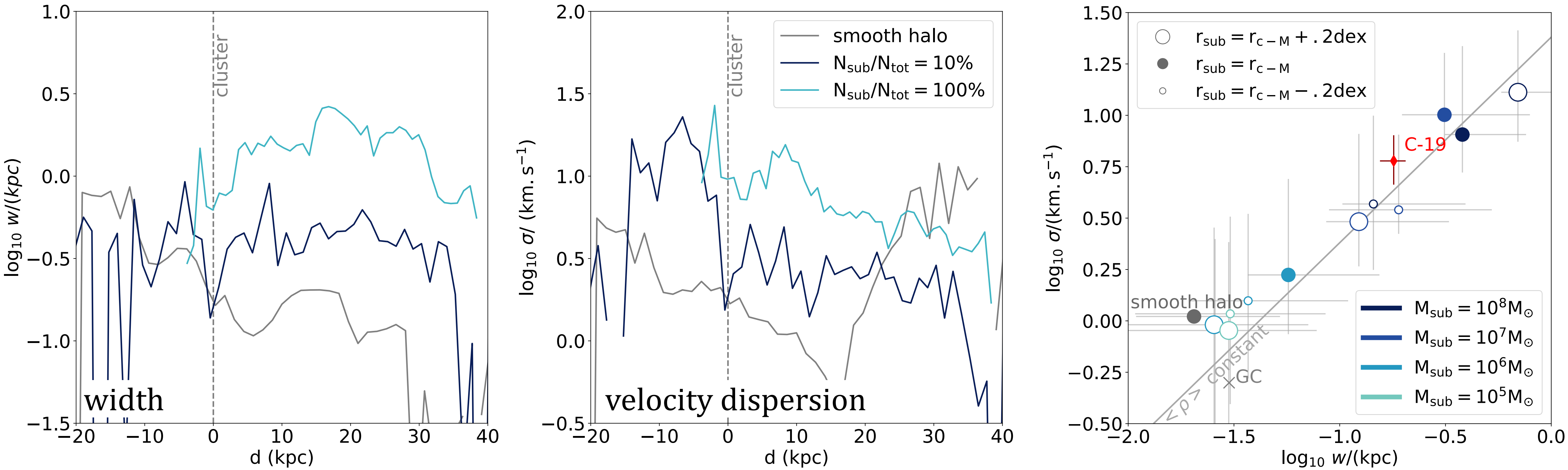
Stellar streams around the Milky Way (Gaia DR2)

But cold dark matter substructures also affect stellar streams

Evolution of a tidally stripped globular cluster in a clumpy halo







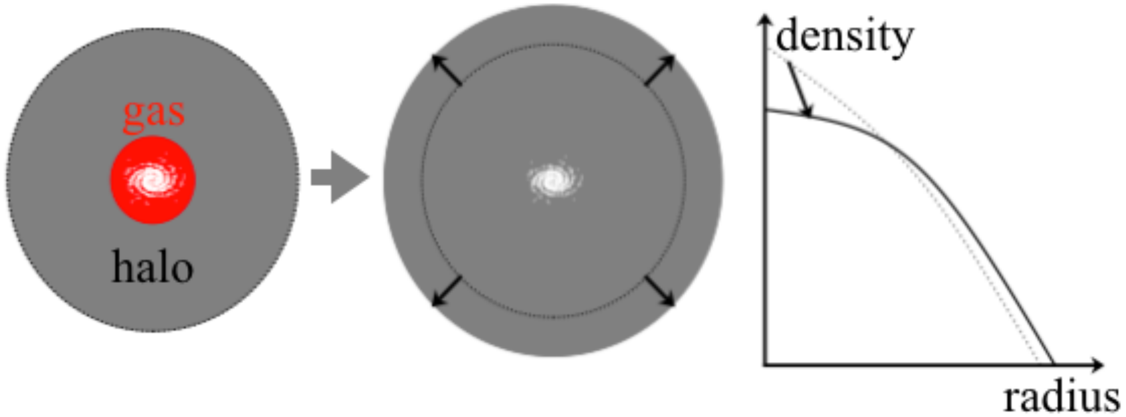
M2 internship of Margot Pernet
co-supervised with Raphael Errani

The high velocity dispersion and width of C-19 are consistent with a globular cluster progenitor evolving in a clumpy dark matter halo

Conclusion

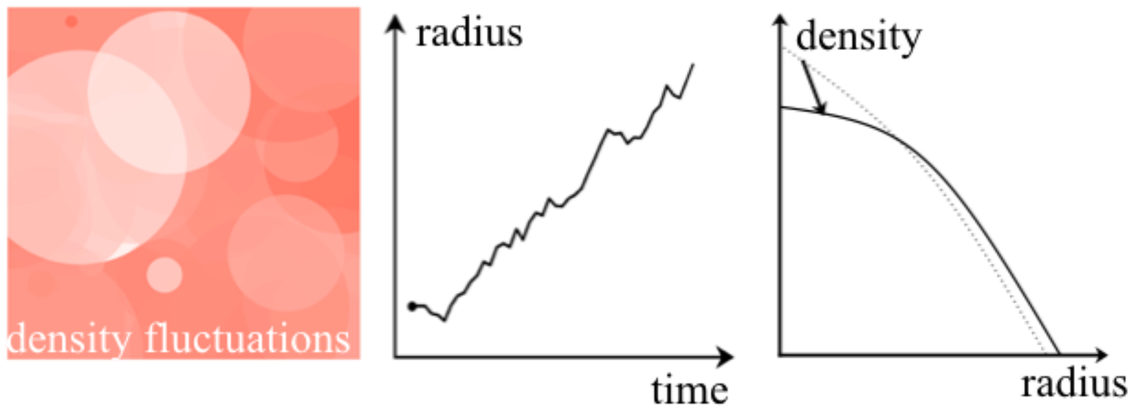
Modeling core formation from feedback processes

✦ From bulk outflows



Freundlich+2020a,b, Dekel+2021, Li+2022

✦ From stochastic density fluctuations



El-Zant, Freundlich & Combes 2016, Hashim+2022

✦ A new mass-dependent dark matter halo profile

✦ Constraining feedback processes through the diversity of halo shapes?

Constraining fuzzy dark matter (FDM) with dynamical heating

✦ A model to describe the effect of FDM density fluctuations on stellar structures (relaxation time, diffusion coefficients)

✦ Possible constraints on the FDM particle mass from the velocity dispersion of the MW and the existence of the central stellar cluster of Eridanus II

El-Zant, Freundlich, Combes & Hallé (2020)

✦ Simulations of the effect of FDM fluctuations on galactic disks

Hallé et al. in prep.

✦ Constraining FDM with stellar streams?