Structure Formation Constraints on Ultralight Dark Matter

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Collapsing Cosmological Condensates

1. Ultralight axion dark matter

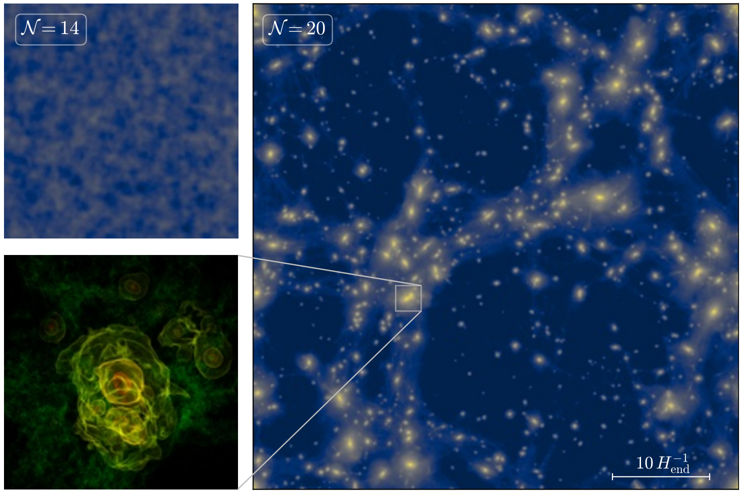
   late-time cosmology (matter & vacuum domination)

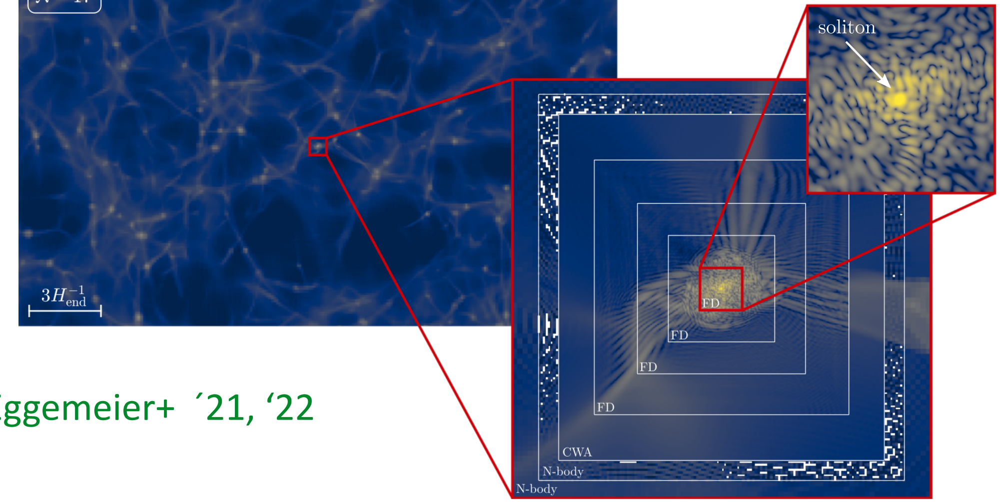
2. QCD axion dark matter

   radiation & early matter domination

3. Inflaton

   post-inflationary matter domination





Eggemeier+ ´21, '22

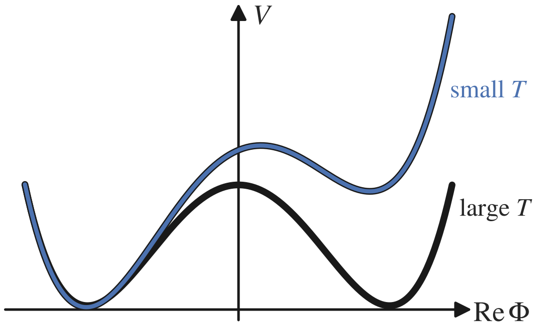
Axion-like (ALP) Dark Matter

ALPs are light, weakly coupled (pseudo)scalar particles motivated by the strong CP problem and string theory. They are excellent dark matter candidates.

In the context of structure formation, we usually assume that ALPs are

- classical (but waves)
- nonrelativistic
- only gravitationally coupled

→ m is the only free parameter



Two classes of axion DM:

1. ultralight / fuzzy DM: e.g. from string theory
2. QCD axion DM: from strong CP problem / Peccei-Quinn mechanism

unique small-scale phenomenology from wavelike effects (1.) or large isocurvature perturbations (2.)

- In the Newtonian limit, ULAs obey the Schrödinger-Poisson (SP) equations:

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2a^2 m}\nabla^2 \psi + mV\psi$$

$$\nabla^2 V = 4\pi G a^2 \delta\rho = \frac{4\pi G}{a}\rho_0(|\psi|^2 - 1)$$

- Dynamics of gravitationally interacting random waves is equivalent to collisionless matter on large scales.
- Madelung / fluid formulation:

$$\dot{\rho} + \nabla(\rho \mathbf{v}) = 0 \qquad \dot{\mathbf{v}} + (\mathbf{v}\cdot\nabla)\mathbf{v} = -\nabla(Q+V)$$

$$\mathbf{v} = m^{-1}\nabla S \qquad Q = -\frac{\hbar^2}{2m^2}\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}$$

„quantum pressure“

- „Quantum Reynolds number“ (compare advection and quantum pressure terms):

$$\frac{Q/R}{v^2/R} \simeq \frac{\hbar^2}{m^2 R^3}\frac{R}{v^2} = \left(\frac{\lambda_{\rm dB}}{R}\right)^2 = \epsilon^2$$

$$\lambda_{\rm dB} \simeq 0.2\left(\frac{10^{-22}\,{\rm eV}}{m}\right)\left(\frac{100\,{\rm km/s}}{v}\right)\,{\rm kpc}$$

- Gravitational relaxation / condensation time:

$$\tau \sim 10^{-2} \times \left(\frac{\lambda_{\rm dB}}{R}\right)^{-3} t_{\rm cr}$$

From waves to particles

- Kinetic formalism (Levkov+ '18):
  - ensemble of waves with random phases and only gravitational interactions
  - **Wigner function:**

$$f_W(\mathbf{x}, \mathbf{p}) = \int \frac{d^3\xi}{(\pi\hbar)^3} \, e^{-2i\mathbf{p}\xi/\hbar} \langle \psi(\mathbf{x}+\xi)\psi^*(\mathbf{x}-\xi)\rangle$$

  - use SP equation, expand in ϵ:

$$\partial_t f_W + \nabla_p \mathcal{H} \, \nabla_x f_W - \nabla_x \mathcal{H} \, \nabla_p f_W + O(\epsilon^2) = \mathrm{St}\, f_W$$

$$\mathcal{H} = \frac{p^2}{2a^2 m} + m\langle V_N \rangle \quad , \quad \nabla^2 \langle V_N \rangle = \frac{4\pi G m}{a}\left(\int d^3p \, f_W - n\right)$$

  - **scattering integral:** $\mathrm{St}\, f_W \simeq f_W/\tau \sim O(\epsilon^3)$

use N-body methods

Simulations with bosonic dark matter

Different scales / physics require different numerical methods.

1. **N-body with modified initial conditions**:

   CDM-like dynamics, linear / weakly nonlinear scales: useful for large-scale structure constraints on FDM (Ly alpha forest, reionization, high-z luminosity functions etc.) or QCD axion miniclusters

2. **Madelung (fluid) formulation** (SPH, PM, or finite volume):

   same as above, includes „quantum pressure“ effects, resolution requirements and validity unclear

3. **Schrödinger formulation** (finite difference or pseudo-spectral):

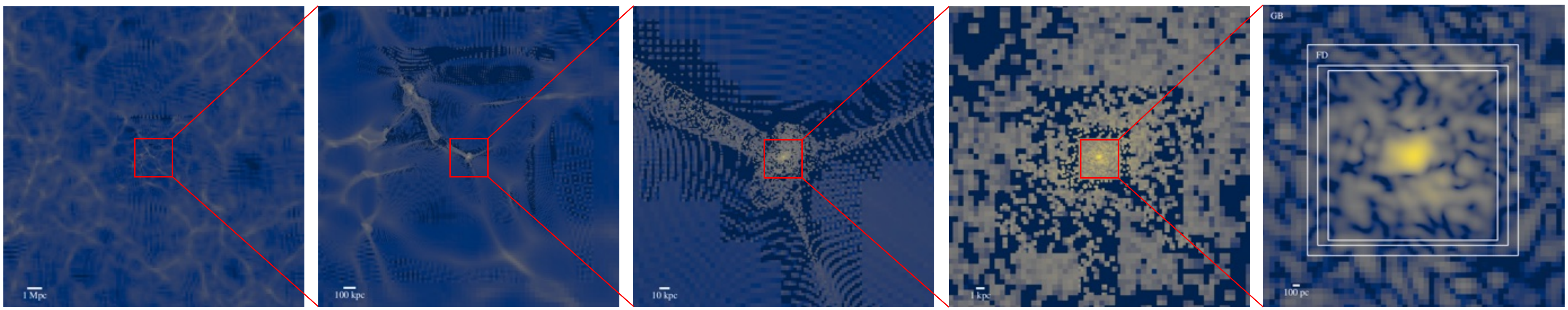
   full wave-like dynamics, requires phase resolution, can only handle relatively small boxes, nonlinear scales: useful for isolated halos or small cosmological boxes

4. **Hybrid zoom-in method** (N-body on coarse grids, Schrödinger on finest grid):

   dynamics CDM-like on large scales, wave-like on small (nonlinear) scales: useful for zoom-in simulations in cosmological boxes

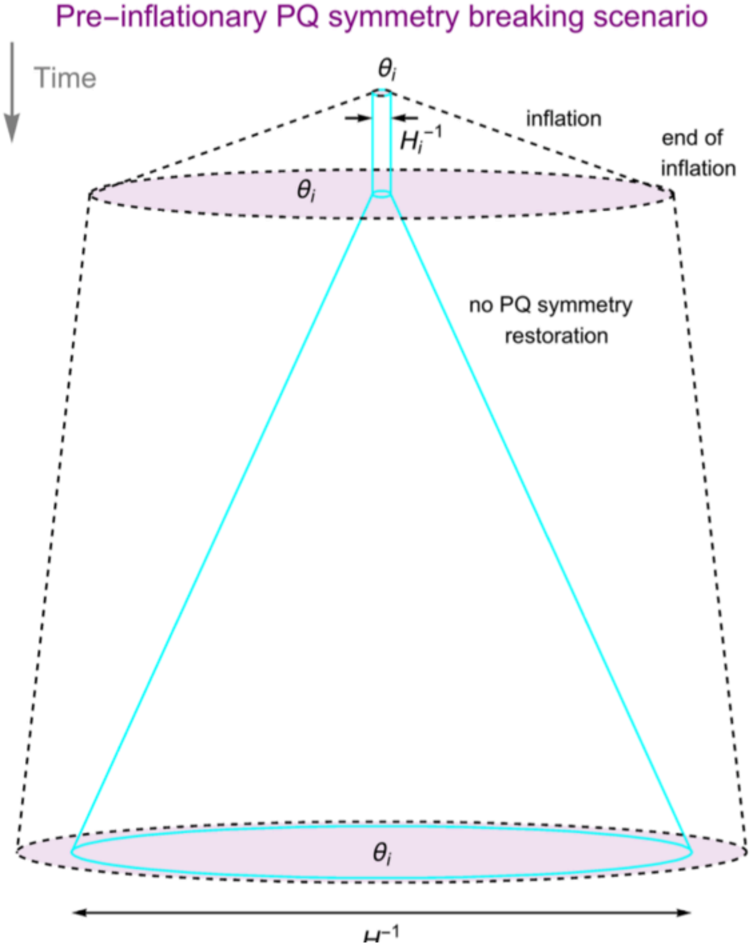
Hybrid N-body / Gaussian Beam Method

(Schwabe, JN ’22, PRL 128, 181301)

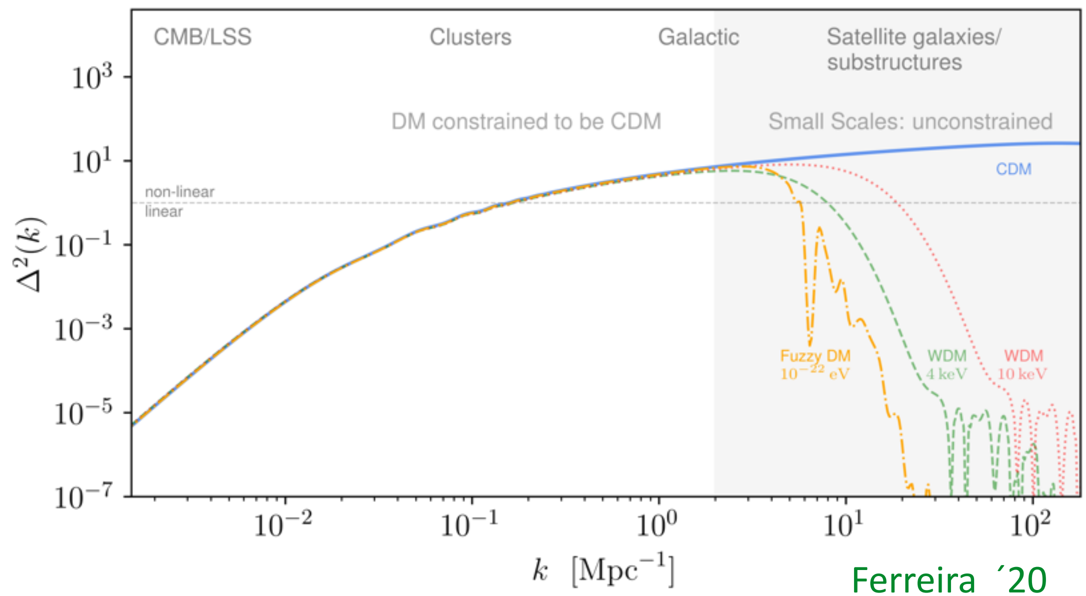


- AGORA proof-of-concept halo (Kim+ ’14)
- $M_{\rm vir} = 1.7 \times 10^{11}\, M_\odot$ in $(60 h^{-1}\,{\rm Mpc})^3$ box
- $m = 2.5 \times 10^{-22}\,{\rm eV}$
- wave function reconstructed from particle phases at 11th AMR level, additional 3 levels using Schrödinger solver
- effective resolution of 20 pc

I. Ultralight axions

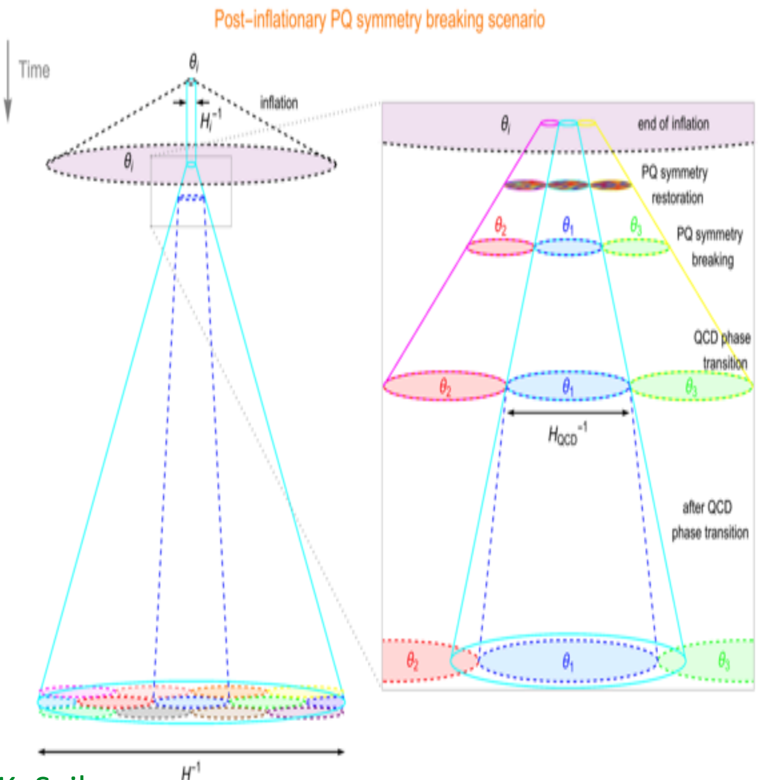


K. Saikawa



Ferreira ´20

II. (Mostly) QCD axions



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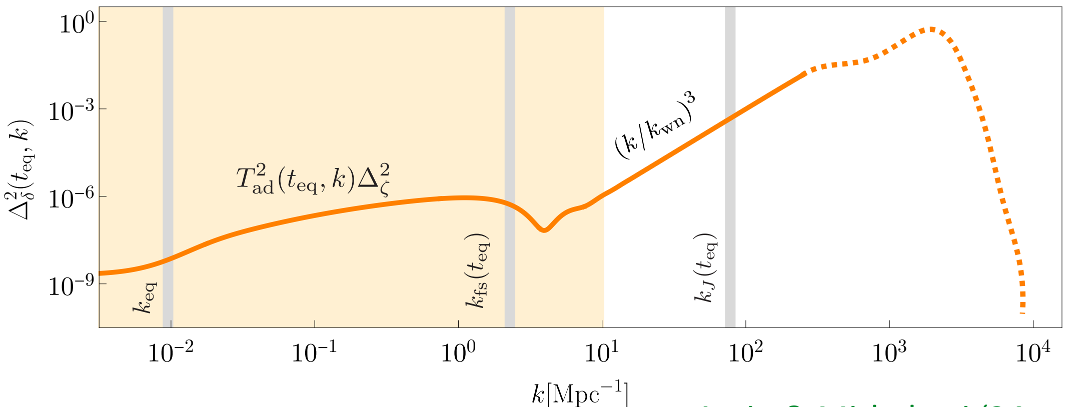


Amin & Mirbabayi '24

- Post-inflationary symmetry breaking ($f < H_i \lesssim 10^{14}\mathrm{GeV}$) is mostly relevant for higher m (e.g. QCD axions) to avoid fine-tuning of DM abundance

- For ULA, observation of nearly scale-invariant density perturbations implies m > 10^{-19} eV (Amin & Mirbabayi '24)

Ultralight axions (ULA), „fuzzy dark matter“ (FDM) (Hu+ ´00)

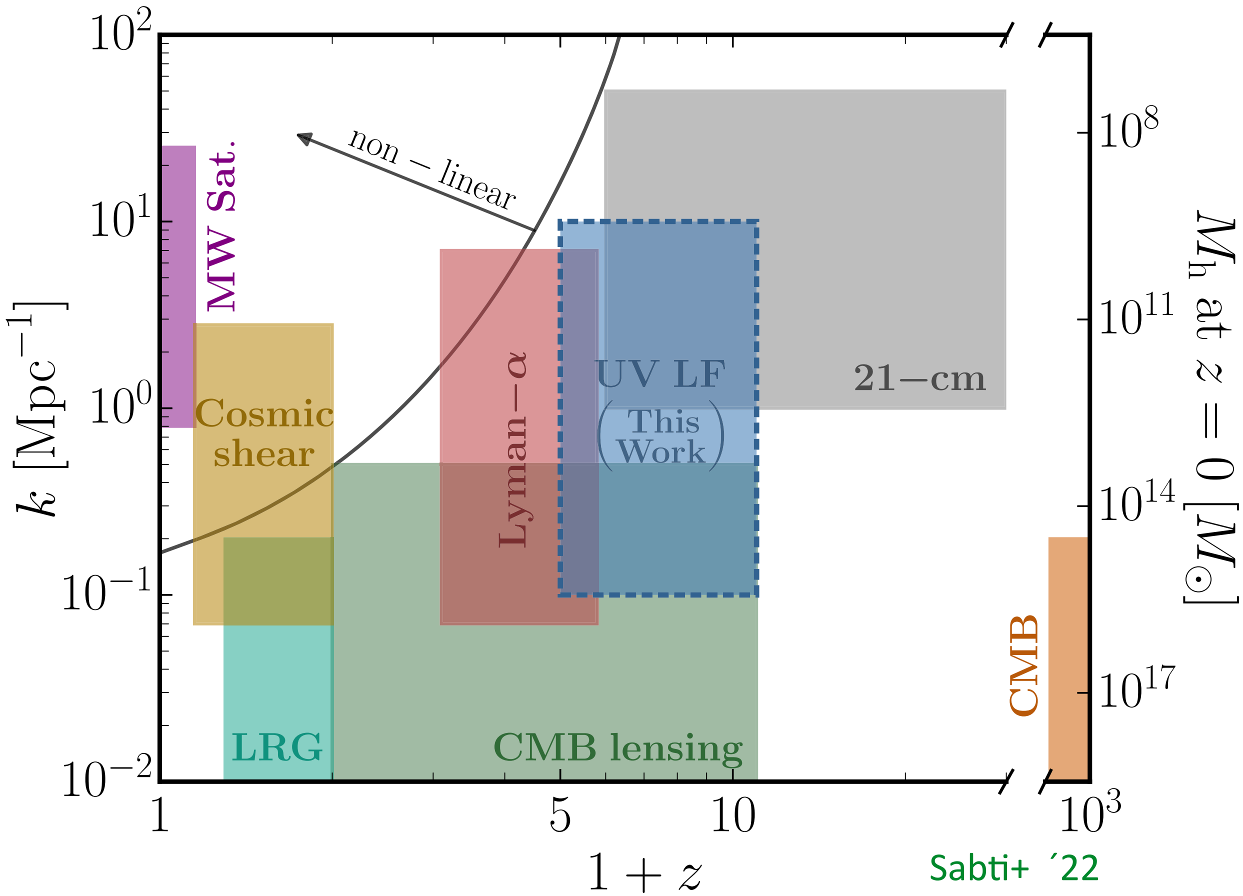
1. Linear effects: suppression of small-scale perturbations

CMB , galaxy clustering, weak lensing (Hlozek+ ’18, Dentler+ ‘22, Rogers+ ’23)

high-z luminosity functions, reionization (Bozek+ ’15, Schive+ ’16, Corasaniti+ ’17, Menci+ ’17, Schneider ’18; Lidz, Hui ’18, Winch+24, Lazare+ ’24, Sipple+ ‘24)

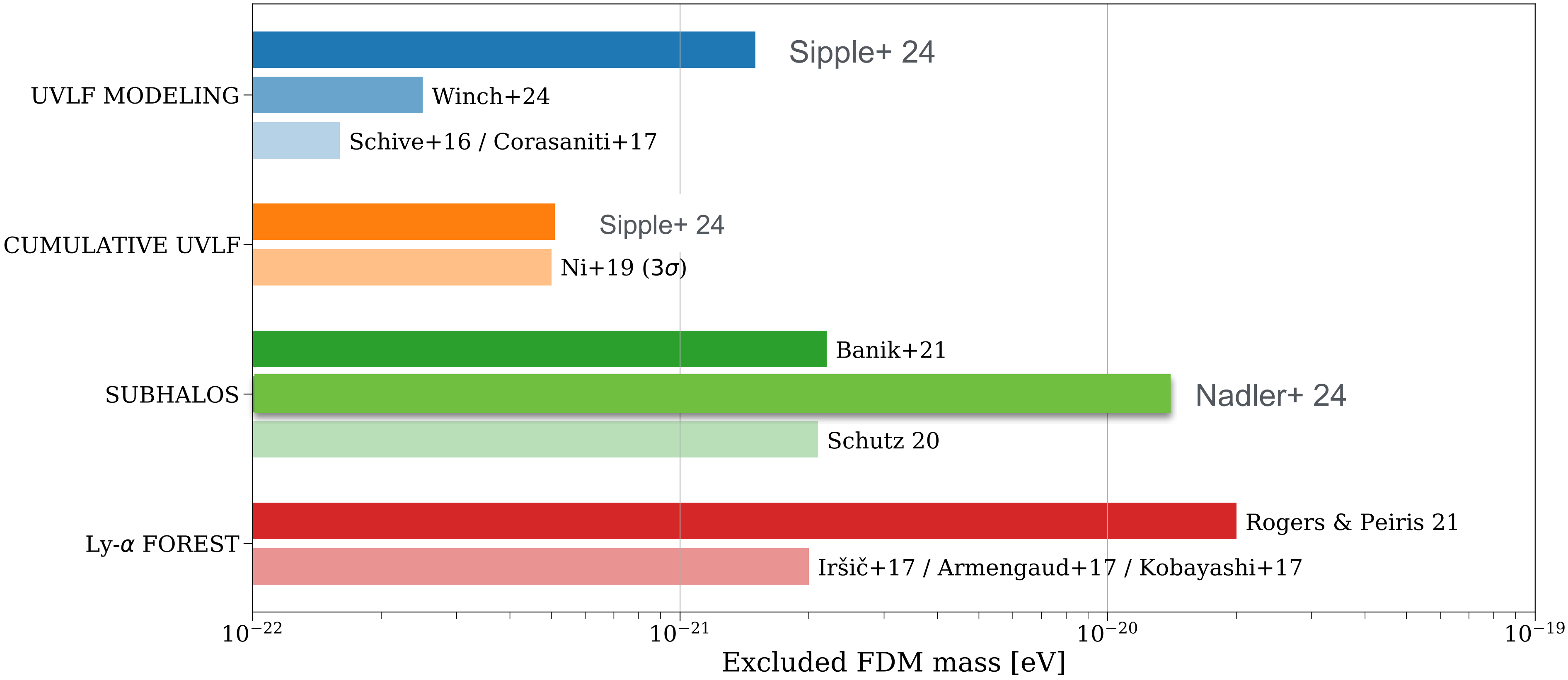
Lyman-α forest (Iršič+ ’17, Armengaud+ ’17, Rogers+ ’20, Rogers & Poulin ‘23)

MW satellites (Nadler+ ‘21, Nadler+ ’24)



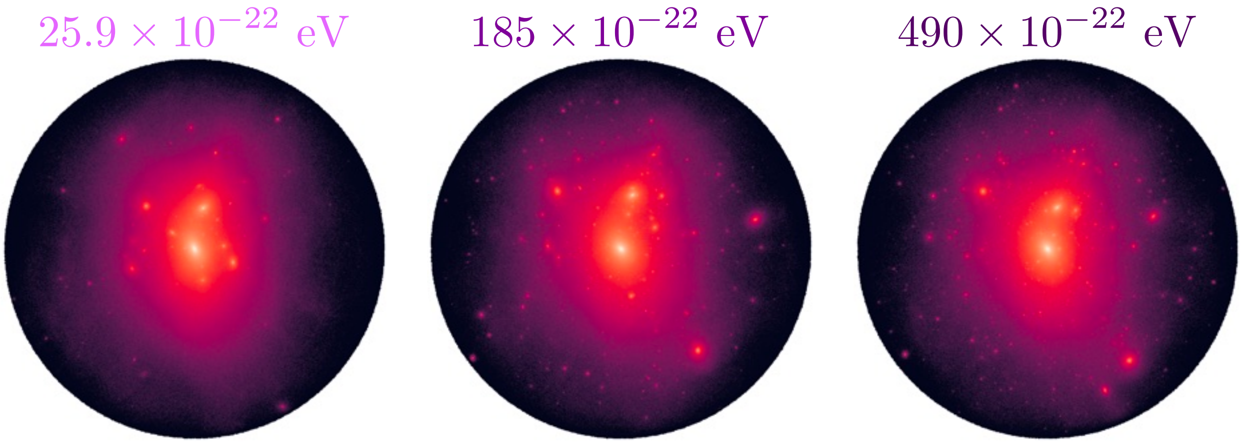
Sabti+ ´22

Pure FDM constraints



from Sipple+ ´24

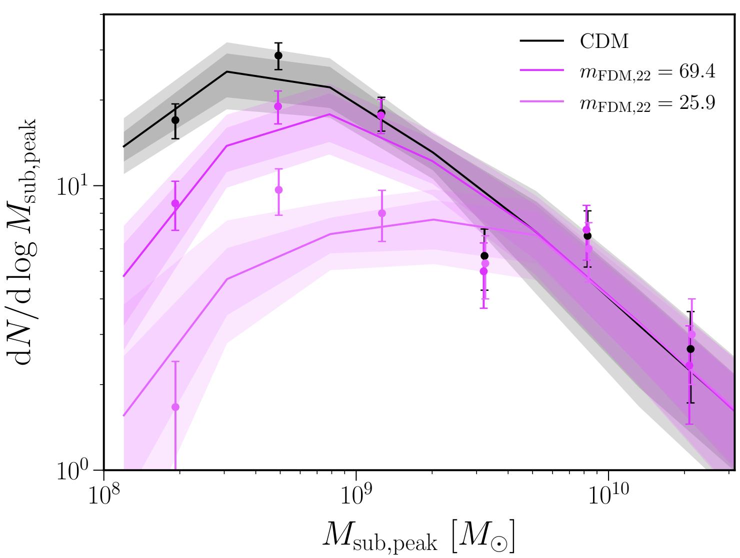
Constraints from MW subhalos: (Nadler+ ‘21, ‘24)
DES, PanSTARRS1



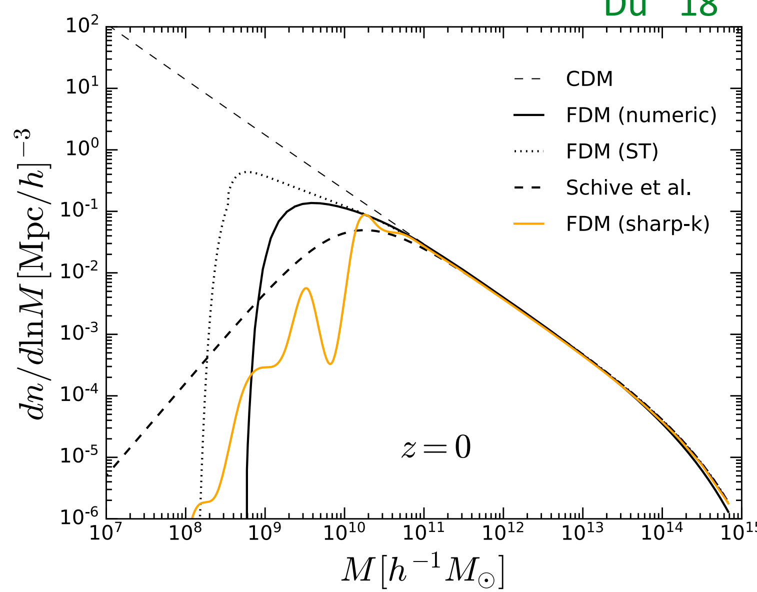
Nadler+ ´24

m > 1.4 x 10^{-20} eV

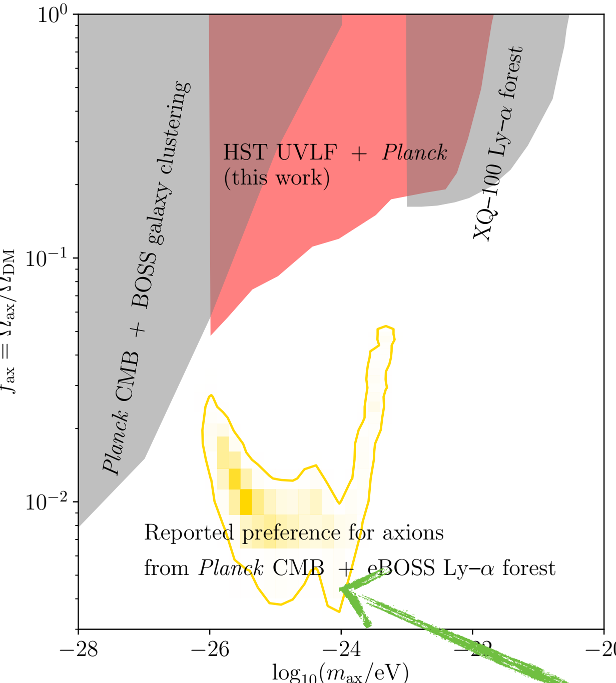
N-body subhalo mass function:



Du ´18



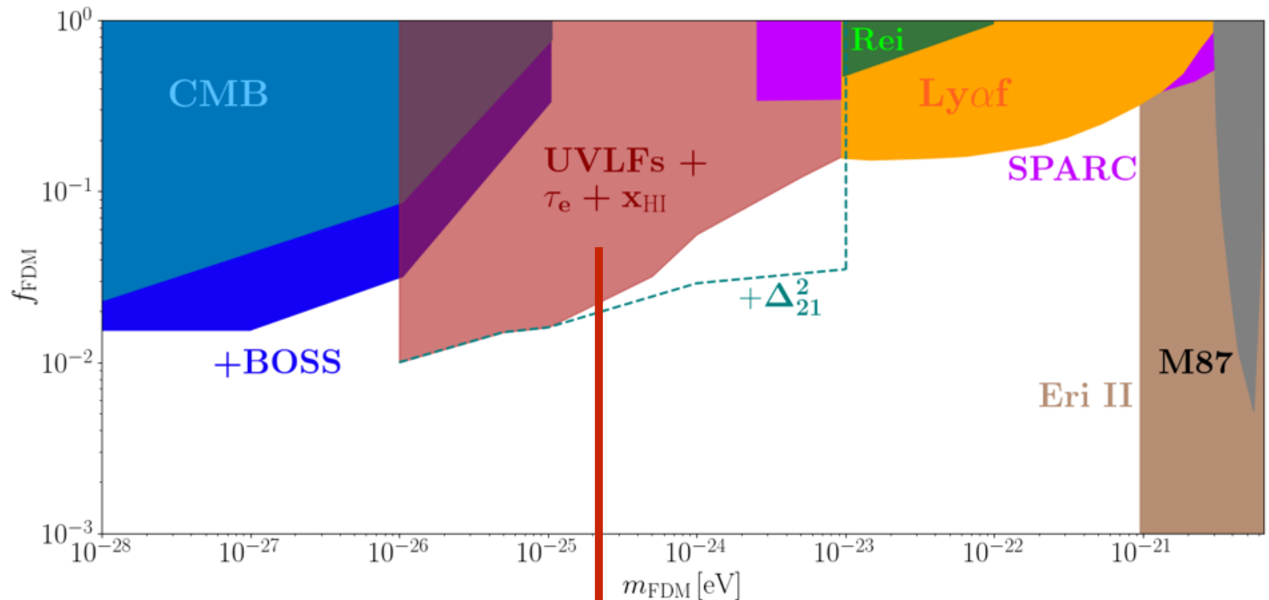
Mixed ULA / CDM constraints (and hints): Planck, BOSS, HST UVLFs, Ly-α forest

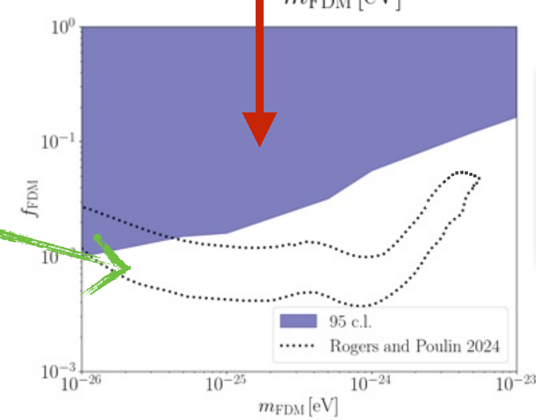


Winch+ ´24

$f_{\rm ULA} < 0.22\ldots0.04$ for $-26 < \log(m) < -23$

Rogers & Poulin ´24





Lazare+ ´24

$f_{\rm ULA} < 0.16\ldots0.01$ for $-26 < \log(m) < -23$

Ultralight axions (ULA), „fuzzy dark matter“ (FDM)

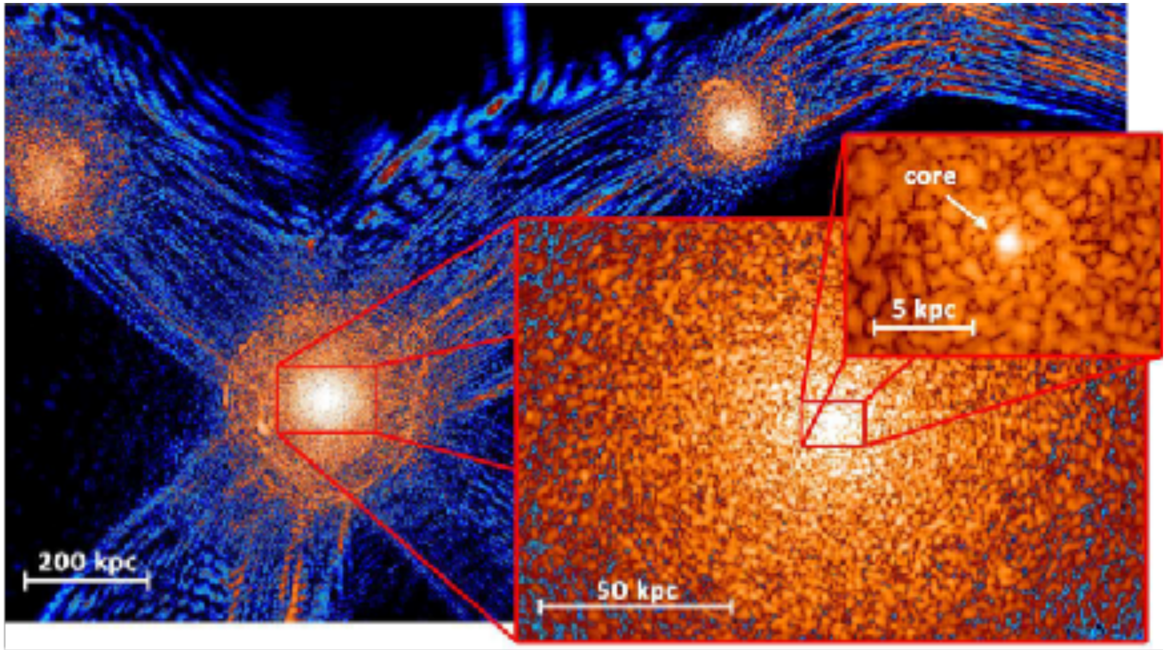
2. Nonlinear effects: large density fluctuations from wave interference

Formation of (solitonic) halo cores

cusp-core etc. (Marsh,Silk ’13, Schive+ ’14, Marsh,Pop ’15, Calabrese,Spergel ’16, …, Zimmermann+ ‘24)

Incoherent interference patterns and granularity

„quasi-particle relaxation“ → dynamical friction / heating / diffusion (Hui+ ’17, Bar-Or ´18, Marsh & JN ´18, Dalal & Kravtsov ’22, …)

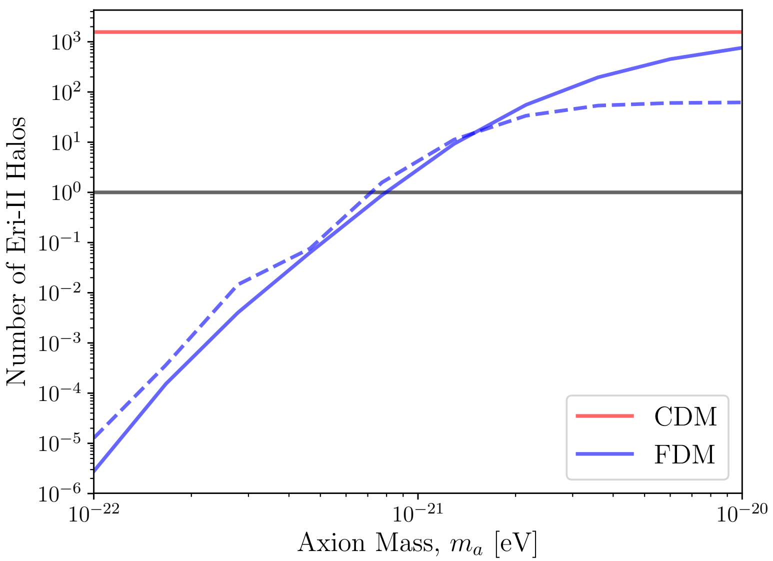


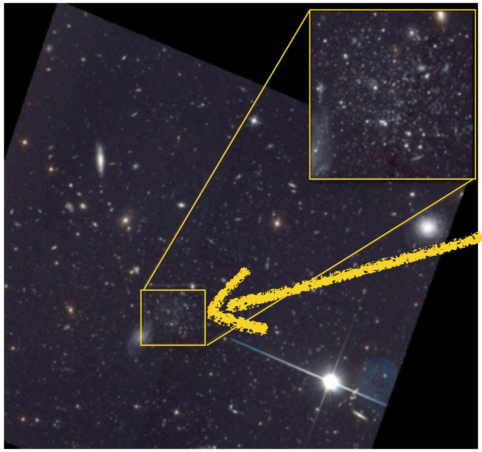
Schive+ ’14

Gravitational heating constraints: Star cluster in UFD Eridanus II

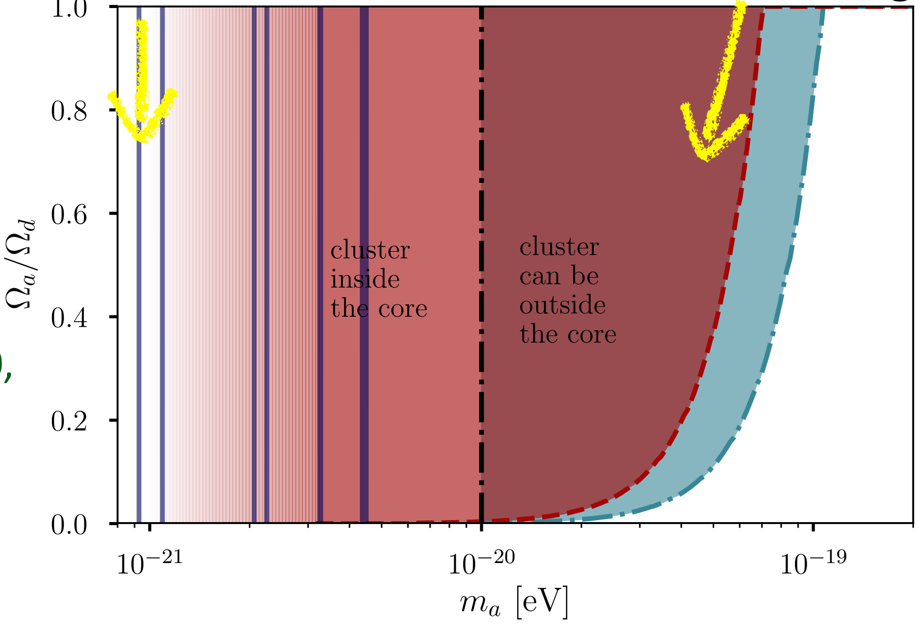
(Marsh & JN ´19, PRL 123, 051103)

expected number of subhalos:





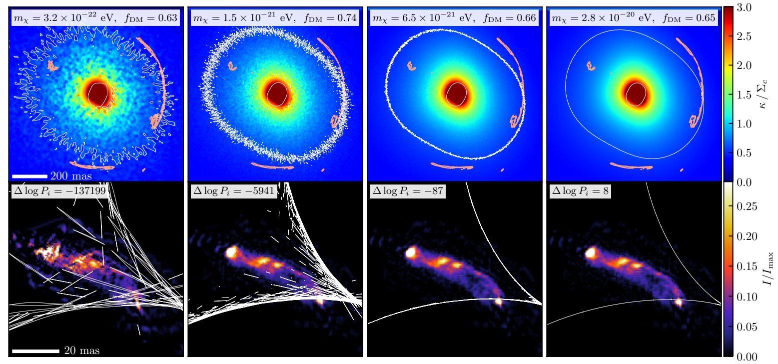
Simon+ ´20



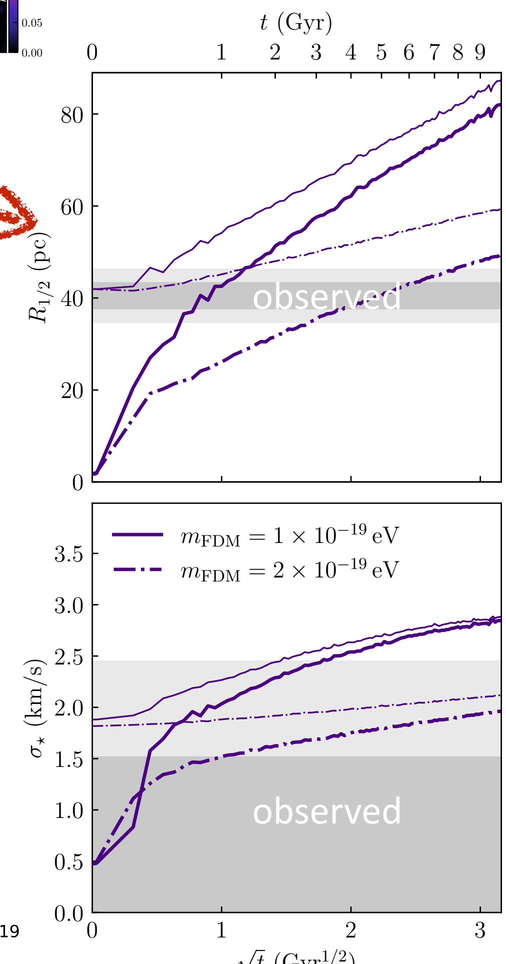
Open questions (loopholes?) wrt. soliton random walk, soliton fluctuations, and tidal stripping (e.g. Schive+ '20, Chiang+ '20, Dalal & Kravtsov '22)

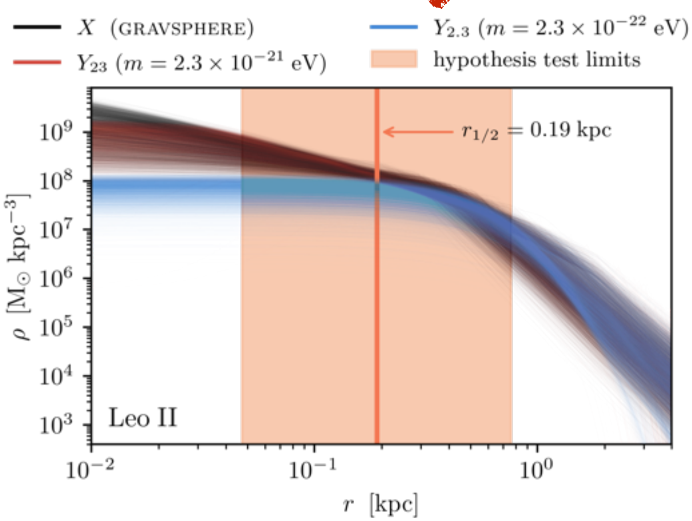
→ need simulations with realistic subhalo+soliton evolution

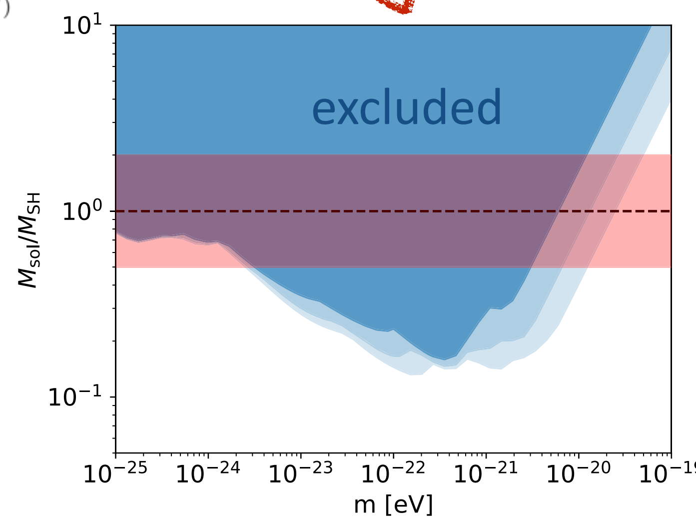
Nonlinear constraints (halo profiles, substructure, heating)



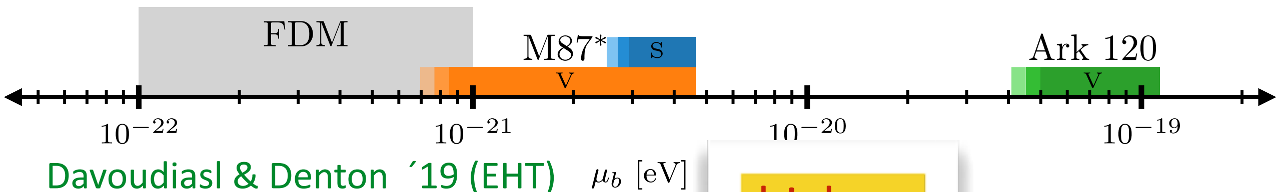
- VLBI jet lensing (Powell+ '23):
  m > 4.4 x 10^{-21} eV
- UFD stellar velocity dispersion heated by wave granules (Dalal & Kravtsov '22):
  m > 3 x 10^{-19} eV
- SPARC rotation curves with central solitons (Bar+ '22)
- Leo II satellite stellar kinematics (Zimmermann+ '24) :
  m > 2.2 x 10^{-21} eV



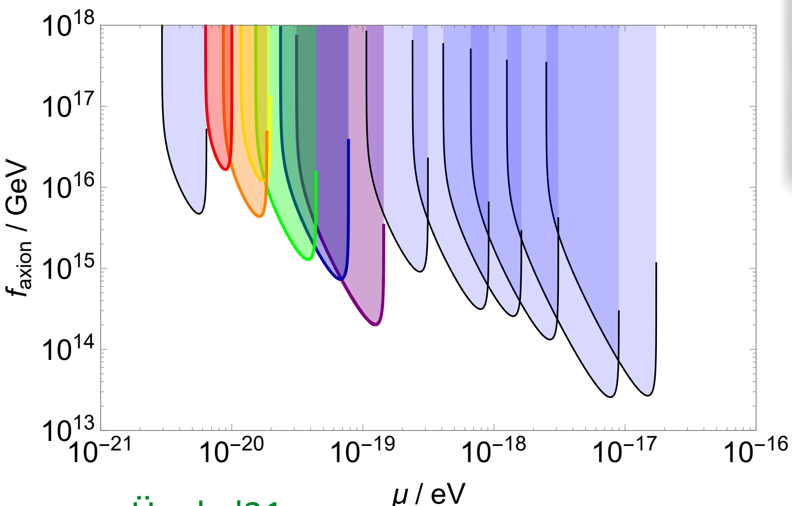




BH superradiance, pulsar timing



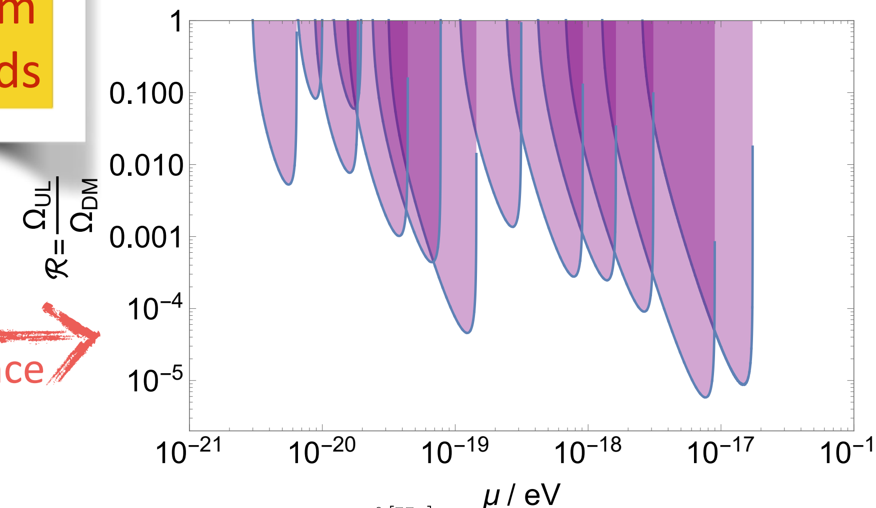
Davoudiasl & Denton ´19 (EHT)

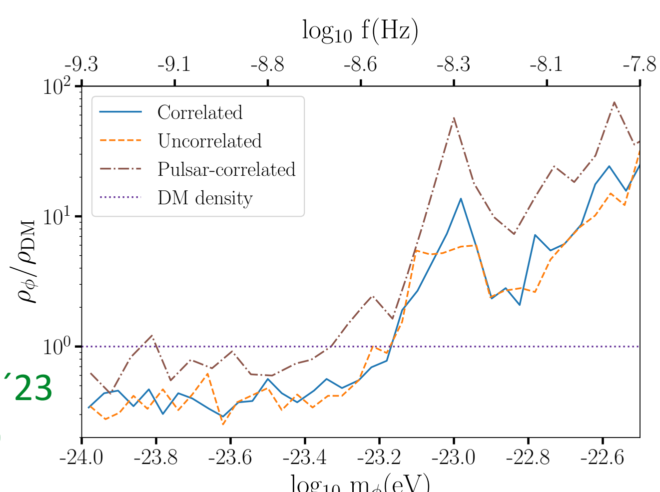


Ünal+ '21

high-m bounds

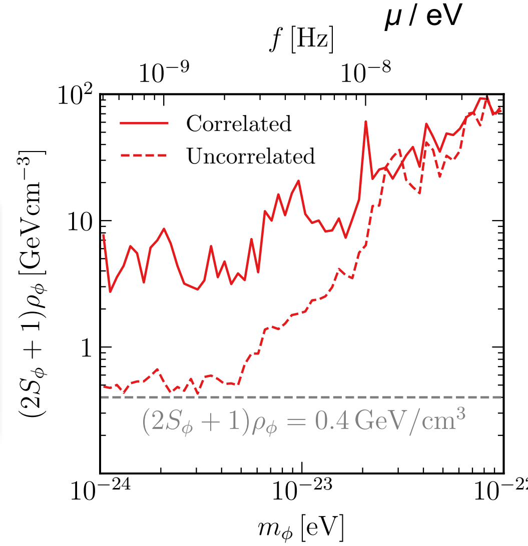
via DM abundance





Smarra+ ´23 (EPTA)

low-m bounds



Afzal+ ´23 (NANOGrav)

Summary:
Constraints on ULA dark matter

- Pure ULA:
  - $m > 10^{-20}$ eV for $f_{ULA} = 1$ from suppression of linear transfer function (Ly-α forest (Rogers & Peiris '21), MW subhalos (Nadler+ '24))
  - possible theoretical uncertainty: use of CDM (small-ε) approximation for nonlinear tracers, in particular the subhalo mass function
  - consistent with constraints from wave-like halo structure (strong lensing (Powell+ '23), stellar kinematics (Dalal & Kravtsov '22, Zimmermann+ '24))
  - effectively closes the high-m window to (non-cosmological but mildly model dependent) superradiance constraints (Ünal+ '21)

- Mixed ULA / CDM (natural scenario in string axiverse):
  - $f_{ULA} < 0.1$ for $m < 10^{-23}$ eV (Planck + HST UVLF + x_{HI} + HERA (Winch+ '24, Lazare+ '24))
  - expect improvement from future JWST UVLF and 21cm (but better galaxy / SF modeling required)
  - consistent with EPTA and NANOGrav pulsar timing constraints (Smarra+ '23, Afzar+ '23)
  - hint for $f_{ULA} \approx 0.02$ at $\log(m) = -26...-24$ (Planck + eBOSS Ly-α forest (Rogers & Poulin '23))