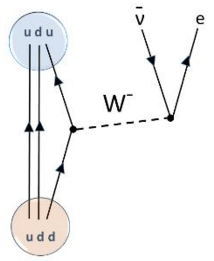
Isospin-symmetry breaking correction to nuclear beta decay

Latsamy Xayavong [1], Nadezda A. Smirnova [2]

1 - Department of Physics, Yonsei University, Seoul, South Korea
2 - Laboratoire de Physique des Deux Infinis de Bordeaux
(LP2IB, CNRS/IN2P3 – University of Bordeaux, France



Workshop on **Vud from pion, nucleon and nuclear beta decay**
GANIL, Caen, France, 5-6 November 2024

Isospin-symmetry breaking correction to nuclear beta decay

- Shell-Model Formalism for isospin-symmetry breaking correction δ_C

- Upgrades and results from the Shell Model + WS wave functions

  L. Xayavong, N. Smirnova, Phys. Rev. C97, 024324 (2018)

- New results from the Shell Model + Hartree-Fock wave functions

  L. Xayavong, N. Smirnova, Phys. Rev. C105, 044308 (2022)

- Higher-order terms in δ_C

  L. Xayavong, N. Smirnova, Phys. Rev. C109, 014317 (2024)

- Conclusions and perspectives

Fundamental interactions studies

Fermions in the Standard Model

$$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix}$$
$$\begin{pmatrix} e \\ \nu_e \end{pmatrix} \begin{pmatrix} \mu \\ \nu_\mu \end{pmatrix} \begin{pmatrix} \tau \\ \nu_\tau \end{pmatrix}$$

Testing grounds:

- at colliders: search for direct production
- at low energies in nuclear beta decay: in precision experiments

Nuclear Matrix elements are needed !

Cabibbo - Kobayashi - Maskawa (CKM) quark-mixing matrix

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

N. Cabibbo (1963); M. Kobayasi, T. Maskawa (1973).

What does the Standard Model prescribe?

- V_{ik} to be extracted from experiment
- Standard Model: V is unitary

$$\sum_k |V_{ik}|^2 = \sum_k |V_{ki}|^2 = 1$$

$\Rightarrow$ constraints on new physics beyond the Standard Model

$|V_{ud}|$ from pion, nucleon and nuclear beta decay

- Superallowed nuclear 0+ -> 0+ beta decay (nuclear structure effects)

- Mirror transitions (F/GT ratio, nuclear structure effects)

- Neutron decay (Lifetime)

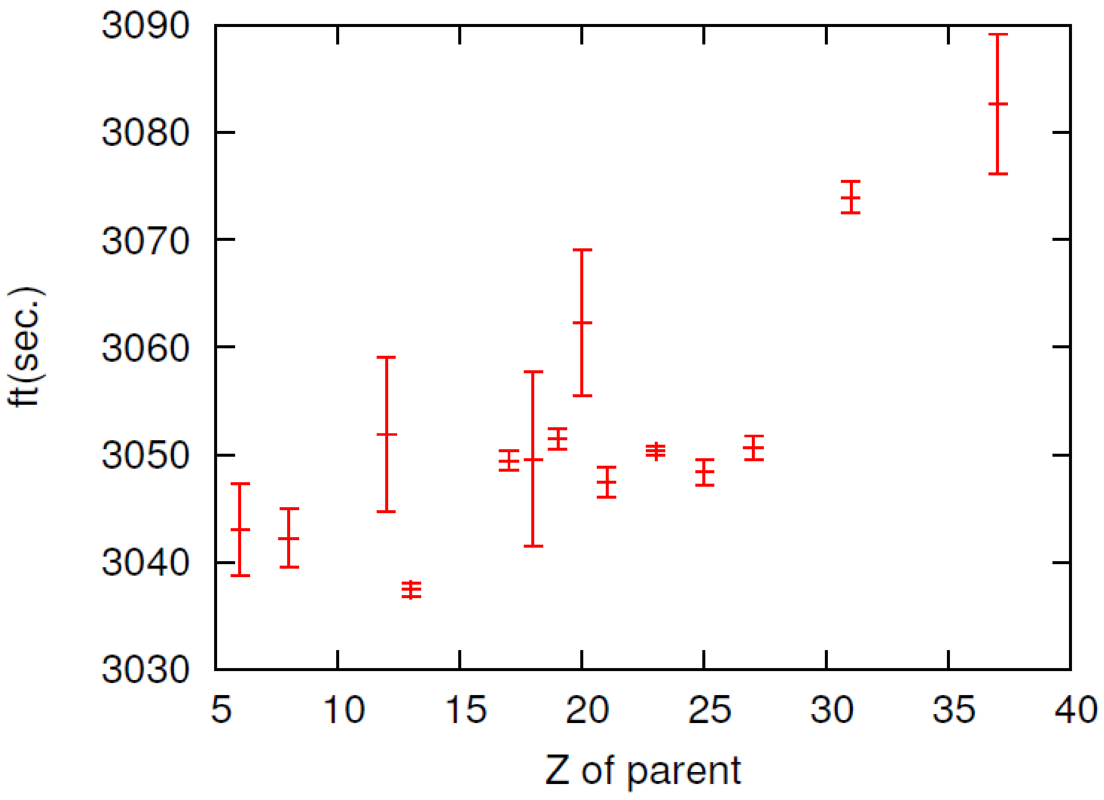
- Pion decay (Branching ratio)

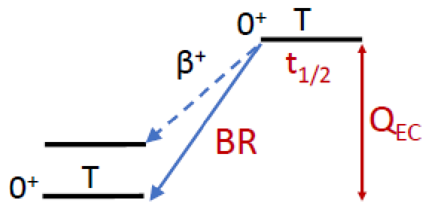
Superallowed $0^+ \rightarrow 0^+$ beta decay

15 best known $T = 1$ emitters ($ft^{0^+ \rightarrow 0^+}$-value known with a precision $\lesssim 0.4\%$):

^{10}C, ^{14}O, ^{22}Mg, ^{26m}Al, ^{26}Si, ^{34}Cl, ^{34}Ar, ^{38m}K, ^{38}Ca, ^{42}Sc, ^{46}V, ^{50}Mn, ^{54}Co, ^{62}Ga, ^{74}Rb

J.C. Hardy, I.S. Towner, PRC102, 045501 (2020)





Statistical rate function:

$$f = f(Z, Q_{EC})$$

Partial halflife:

$$t = \frac{t_{1/2}}{BR}$$

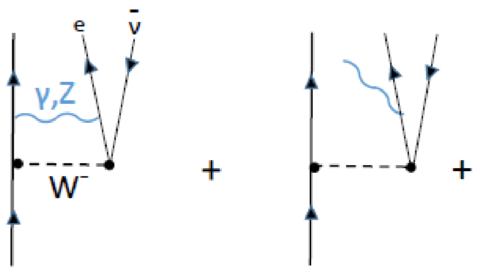
Talks by B. Blank and B.Rebeiro

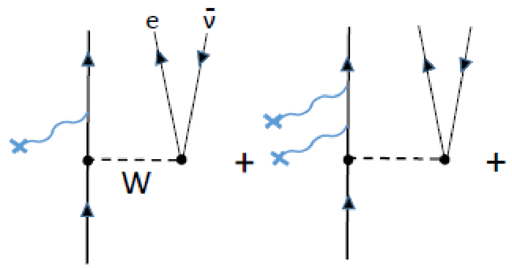
Superallowed $0^+ \to 0^+$ beta decay

Absolute *Ft* value

$$Ft^{0^+\to 0^+} \equiv ft^{0^+\to 0^+}(1+\delta'_R)(1+\delta_{NS}-\delta_C) = \frac{K}{|M_F^0|^2 G_V^2 (1+\Delta_R)}$$

J.C. Hardy, I.S. Towner, PRC102, 045501 (2020)





Radiative corrections

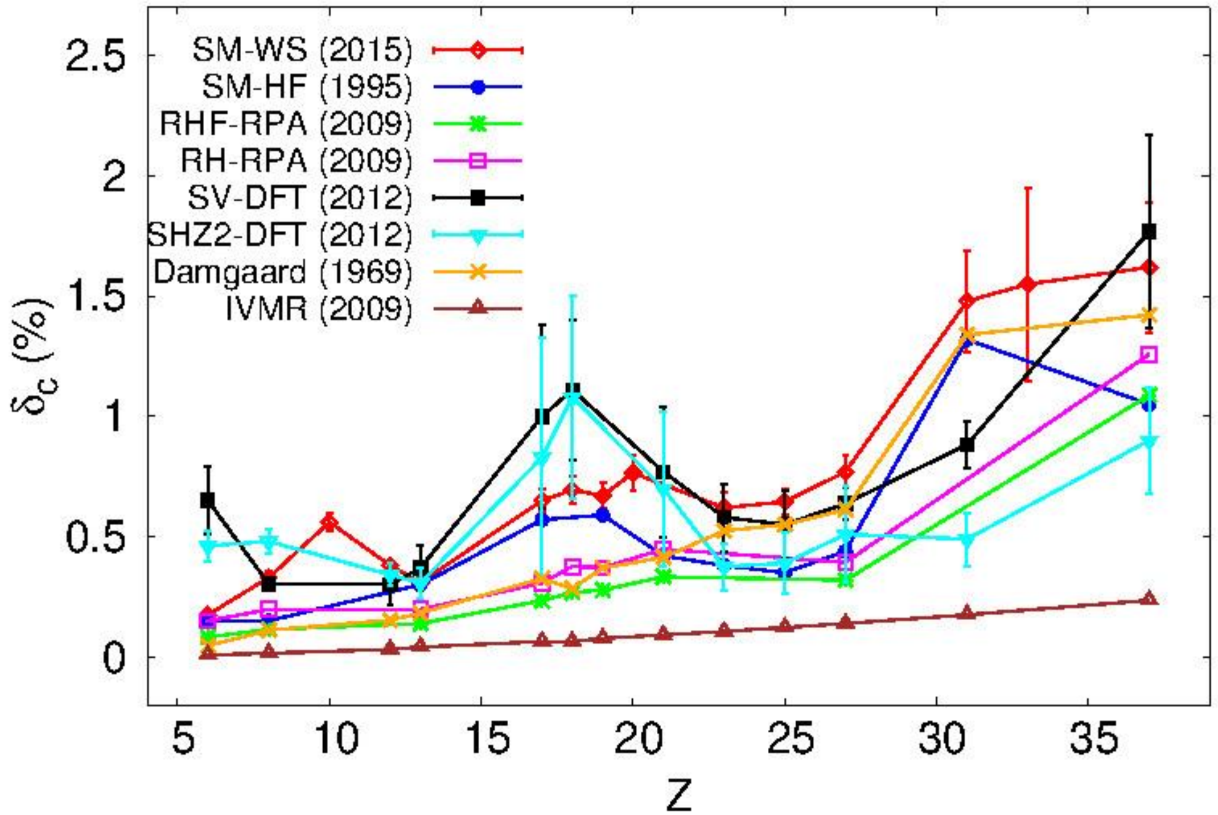
$$\Delta_R^V = (2.454 \pm 0.019)\%$$
$$\delta'_R \sim (1.50 \pm \sim 0.12)\%$$
$$|\delta_{NS}| \lesssim 0.4\%$$

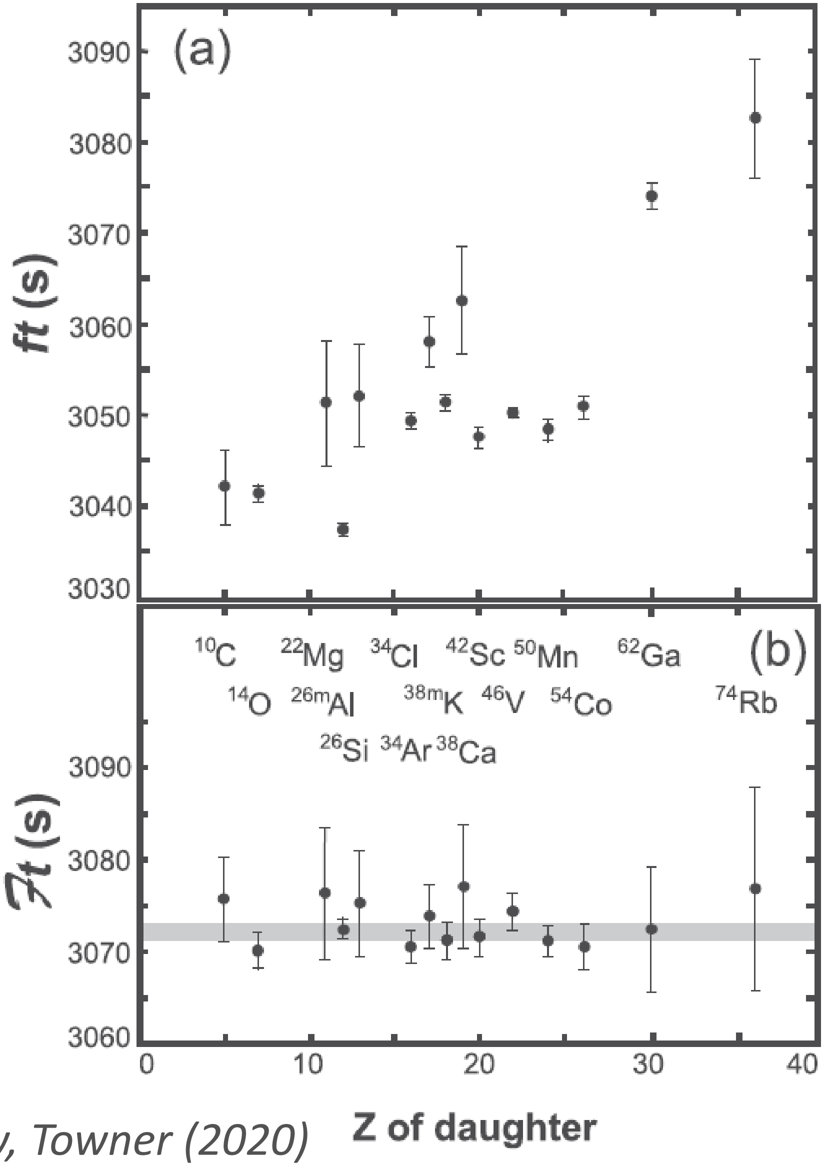
Nuclear-structure correction

$$|M_F|^2 = |M_F^0|^2(1-\delta_C)$$
$$|M_F^0|^2 = T(T+1) - T_{zi}T_{zf}$$
$$\delta_C \approx 0.1 - 2.0\%$$

Talks by L. Hayen and M. Drissi

Isospin-symmetry breaking correction





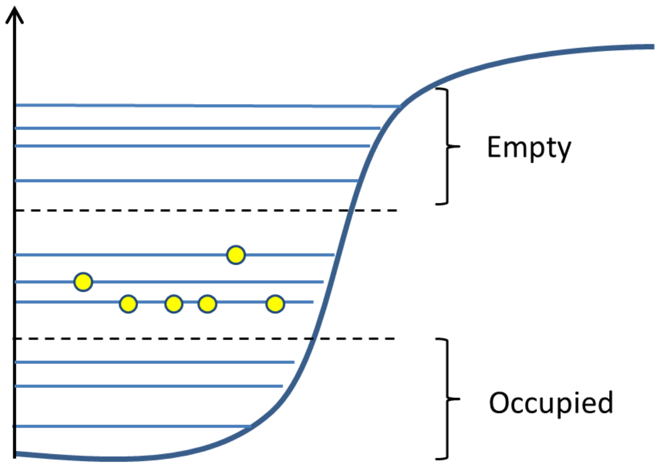
SM-WS : Towner, Hardy (2015)
SM-HF : Ormand, Brown (1989, 1995)
RHF-RP : Liang et al (2009)
SV/SHZ2-DFT : Satula et al (2012)
IVMR : Auerbach (2009)

CVC test
Towner, Hardy
PRC82 (2010)

Hardy, Towner (2020)

Shell model (full configuration-interaction approach)

Resolution of the nuclear many-body problem by Hamiltonian matrix diagonalization



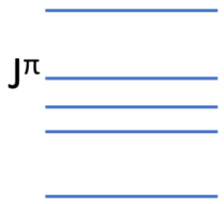
$$H = T + V = \underbrace{(T + U)}_{\text{exactly solvable}} + \underbrace{(V - U)}_{\text{residual interaction}} = H_0 + V_{res}$$

$$H|\Psi_p\rangle = E_p|\Psi_p\rangle$$

$$|\Psi_p\rangle = \sum c_{kp}\,|\Phi_k\rangle$$

$$\langle\Phi_k|\Phi_l\rangle = \delta_{kl}$$

$$\sum_{k=1}^{d}\langle\Phi_l|H|\Phi_k\rangle\, c_{kp} = E_p c_{lp} \;\Rightarrow\; \begin{pmatrix} H_{11} & H_{12} & \cdots & H_{1d} \\ H_{21} & H_{22} & \cdots & H_{2d} \\ \vdots & & \ddots & \\ H_{d1} & H_{d2} & \cdots & H_{dd} \end{pmatrix} \;\Rightarrow$$



Avantages of the theoretical approach:

- Conservation of symmetries of the full Hamiltonian (rotational, translation invariance, parity, particle number, etc)
- Precise information on low-energy states and transitions
- Excellent description with appropriate interactions and in a suitable model space

Challenges :

- Basis dimensions !

Fermi matrix element within the shell model

Fermi β-decay matrix element

$$H|\Psi\rangle = E|\Psi\rangle \quad \Rightarrow \quad \begin{array}{l} |\Psi_i\rangle = \sum c_{ki}|\Phi_k\rangle \\ |\Psi_f\rangle = \sum c_{kf}|\Phi_k\rangle \end{array}$$

$$\boxed{M_F = \langle\Psi_f|T_+|\Psi_i\rangle = \sum_\alpha \langle\Psi_f|a^\dagger_{\alpha_n} a_{\alpha_p}|\Psi_i\rangle\langle\alpha_n|t_+|\alpha_p\rangle}$$

$$\langle\Psi_f|a^\dagger_{\alpha_n} a_{\alpha_p}|\Psi_i\rangle \equiv \rho_\alpha \quad (\neq \rho^T_\alpha)$$

Miller, Schwenk PRC (2009,2010): Radial excitations !

$$\langle\alpha_n|t_+|\alpha_p\rangle = \int_0^\infty R_{\alpha_n}(r)R_{\alpha_p}(r)r^2dr \equiv \Omega_\alpha \quad (\neq 1)$$

$$\alpha = (n_\alpha, l_\alpha, j_\alpha, m_\alpha)$$

- Large-scale calculations
- Global parameterization of INC forces
- Revisit WS procedure
- Implicate HF wave functions

I.S. Towner, J.C. Hardy; (1973 – 2020)
W.E. Ormand, B.A. Brown (1985 – 1995)

Fermi matrix element within the shell model

Isospin-symmetry limit

$$M_F^0 = \sum_\alpha \rho_\alpha^T \Omega_\alpha^T = \pm\sqrt{T(T+1) - T_{zi}T_{zf}}\,, \qquad \Omega_\alpha^T = 1$$

Realistic model (Coulomb and charge-symmetry breaking effective nuclear forces)

$$M_F = \sum_\alpha \rho_\alpha \Omega_\alpha$$

In first order perturbation theory:

$$|M_F|^2 \approx |M_F^0|^2 \Big[1 - \underbrace{\frac{2}{M_F^0} \sum_\alpha \left(\rho_\alpha^T - \rho_\alpha \right)}_{\delta_{C1}} - \underbrace{\frac{2}{M_F^0} \sum_\alpha \rho_\alpha^T (1 - \Omega_\alpha)}_{\delta_{C2}} \Big],$$

$$\delta_C \approx \delta_{C1} + \delta_{C2}$$

- δ_{C1} is the *isospin-mixing* part
- δ_{C2} is the *radial-overlap* part

Shell-model calculations

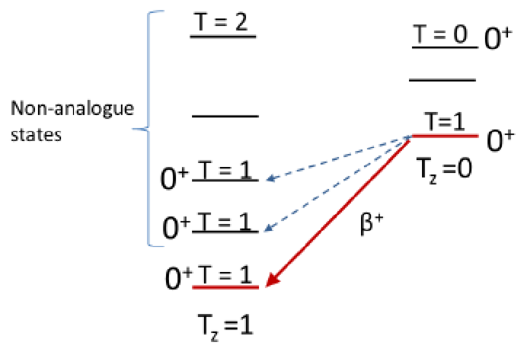
^{10}C, ^{14}O, ^{18}Ne, ^{22}Mg, ^{26m}Al, ^{26}Si, ^{34}Cl, ^{34}Ar, ^{38m}K, ^{38}Ca, ^{42}Sc, ^{42}Ti, ^{46}V, ^{50}Mn, ^{54}Co, ^{62}Ga, ^{66}As

Model spaces and effective interactions (+ charge-dependence)

- p-shell: CKPOT *(Cohen-Kurath, 1965)*
- $(p_{1/2}sd_{5/2})$-shell: ZBM's *(Zuker et al, 1969)*, REWIL *(Reehal, Wildenthal, 1973)*
- sd-shell: USD *(Wildenthal, 1984)*, USDA/USDB *(Brown, Richter, 2006)*
- $(sd_{3/2}f_{7/2}p_{3/2})$-shell: ZBM2 *(Nowacki et al, 2014)*
- pf-shell: KB3G *(Poves et al, 2004)*, GXPF1A *(Honma et al, 2004)*
- $pf_{5/2}g_{9/2}$: JUN45 *(Honma et al, 2009)*, MRG *(Nowacki et al, 1996)*

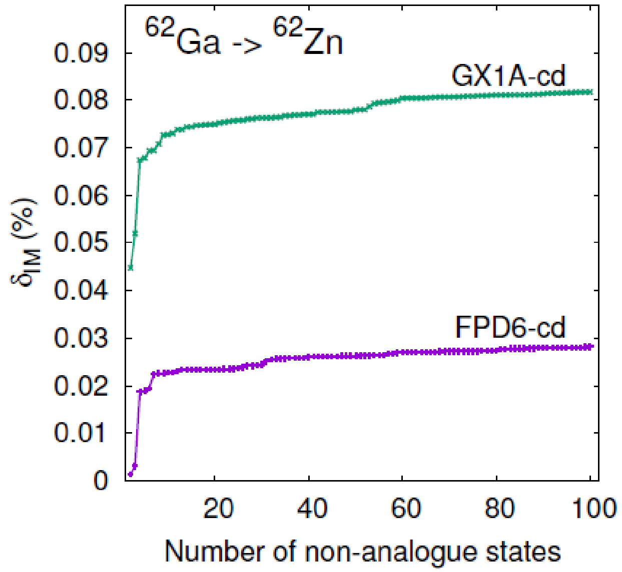
NuShellX@MSU shell-model code *(B.A. Brown, W.D.M. Rae, Nucl. Data Sheets 120, (2014)*.

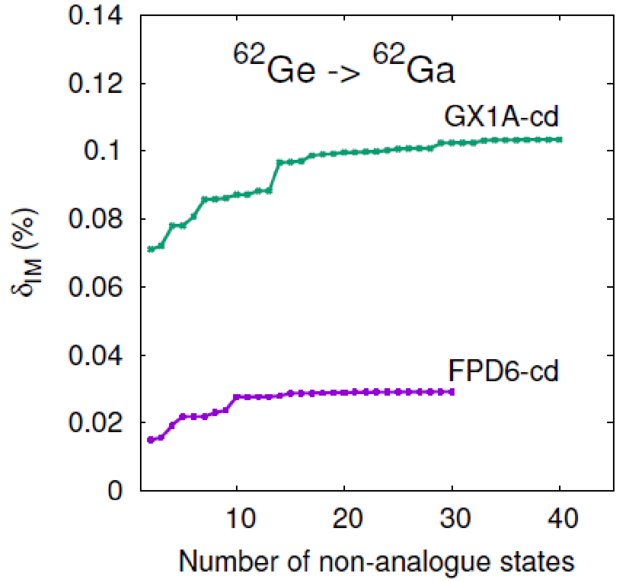
Shell-model evaluation of δ_{C1}

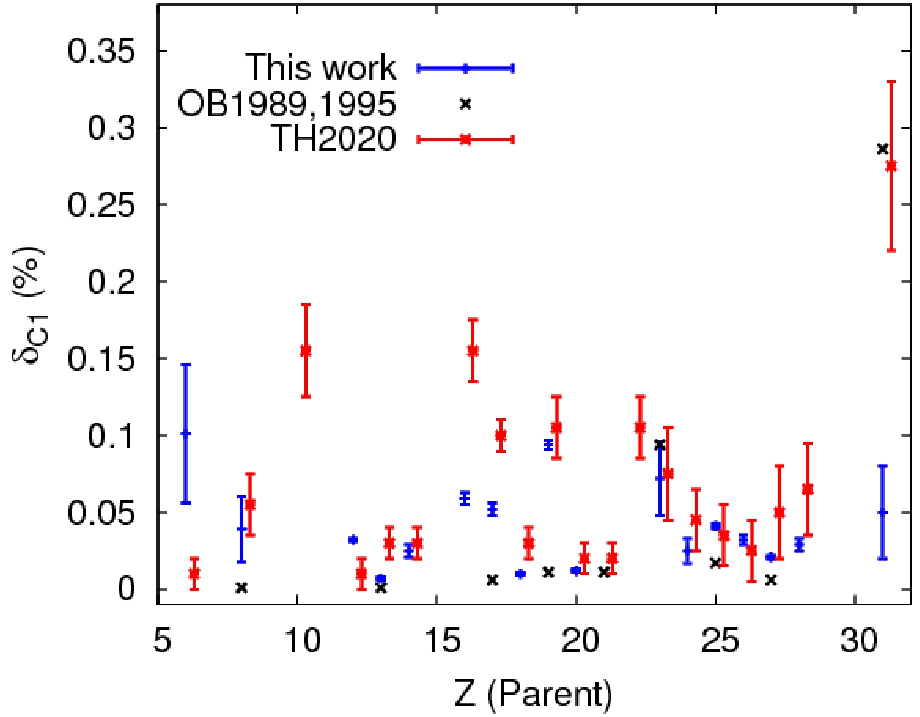


We sum over the Fermi strengths to non-analogue states!
⇒ experimental information on non-analogue strength is highly desirable!

Preliminary !







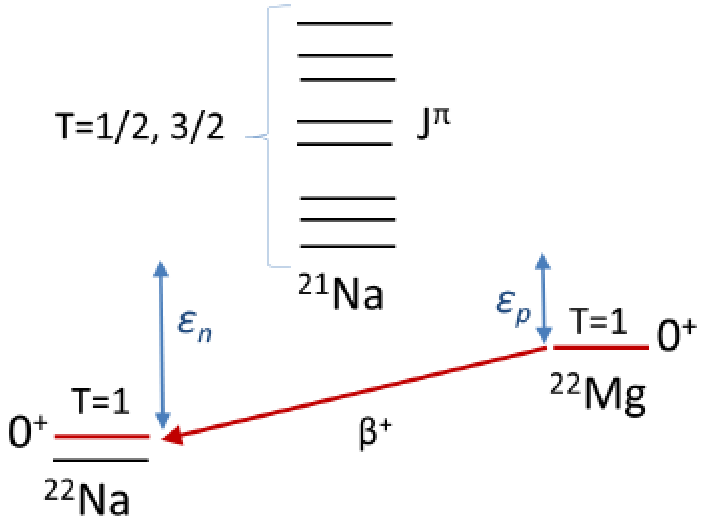
Shell-model evaluation of δ_{C2} beyond the closure approximation

$$\delta_{C2} = \frac{2}{M_F^0} \sum_{\alpha} \langle \Psi_f | a^\dagger_{\alpha_n} a_{\alpha_p} | \Psi_i \rangle^T (1 - \Omega_\alpha)$$

$$\Downarrow$$

$$\delta_{C2} = \frac{2}{M_F^0} \sum_{\alpha,\pi} \langle \Psi_f | a^\dagger_{\alpha_n} | \pi \rangle^T \langle \pi | a_{\alpha_p} | \Psi_i \rangle^T (1 - \Omega_\alpha^\pi)$$

Experimental information on
Relevant spectroscopic
Factors is highly desirable !



- *Spectroscopic amplitudes* (from the shell-model):

$$\langle \Psi_f | a^\dagger_{\alpha_n} | \pi \rangle = \frac{\langle \Psi_f || a^\dagger_{\alpha_n} || \pi \rangle}{\sqrt{2J_f + 1}}$$

- *Radial-overlap integrals* (from a realistic single-particle potential)

$$\Omega_\alpha^\pi = \int_0^\infty R^\pi_{\alpha_n}(r) R^\pi_{\alpha_p}(r) r^2 dr$$

Shell-model + Woods-Saxon evaluation of δ_{C2}

L. Xayavong, PhD thesis, U. Bordeaux (2016)

L. Xayavong, N. Smirnova, Phys. Rev. C97, 024324 (2018)

Parameterization

$$V_{WS}(r) = -\frac{V_0}{1+\exp\left(\frac{r-R}{a}\right)} - V_{ls}(r)\vec{l}\cdot\vec{\sigma} + V_C(r)$$

- A. Bohr, B.R. Mottelson modified (BM_m) from *Nuclear Structure, Vol. I.*
- N. Schwierz, I. Wiedenhöver, A. Volya (SWV) from *nucl-th:0709.3525*

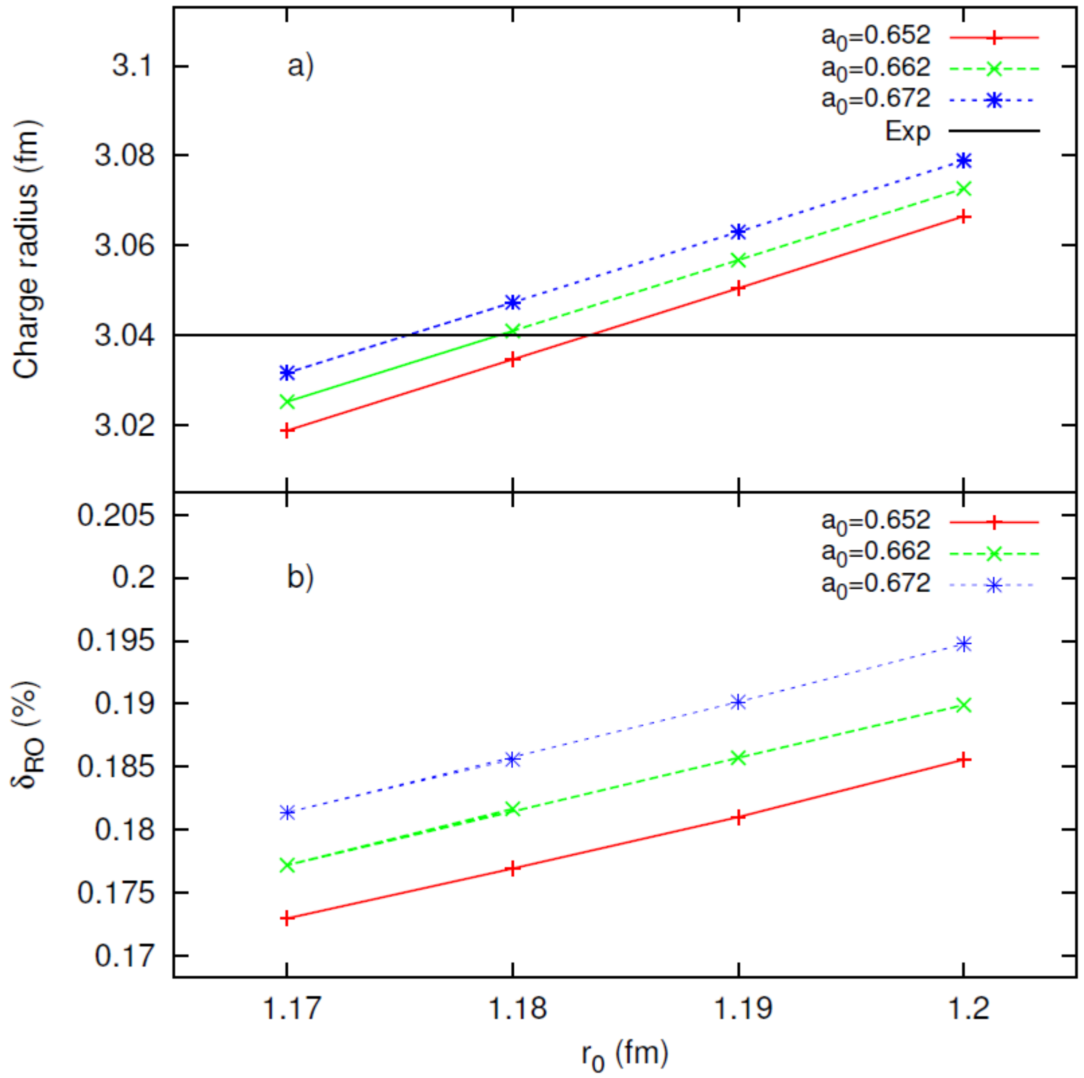
Under stringent experimental constraints:

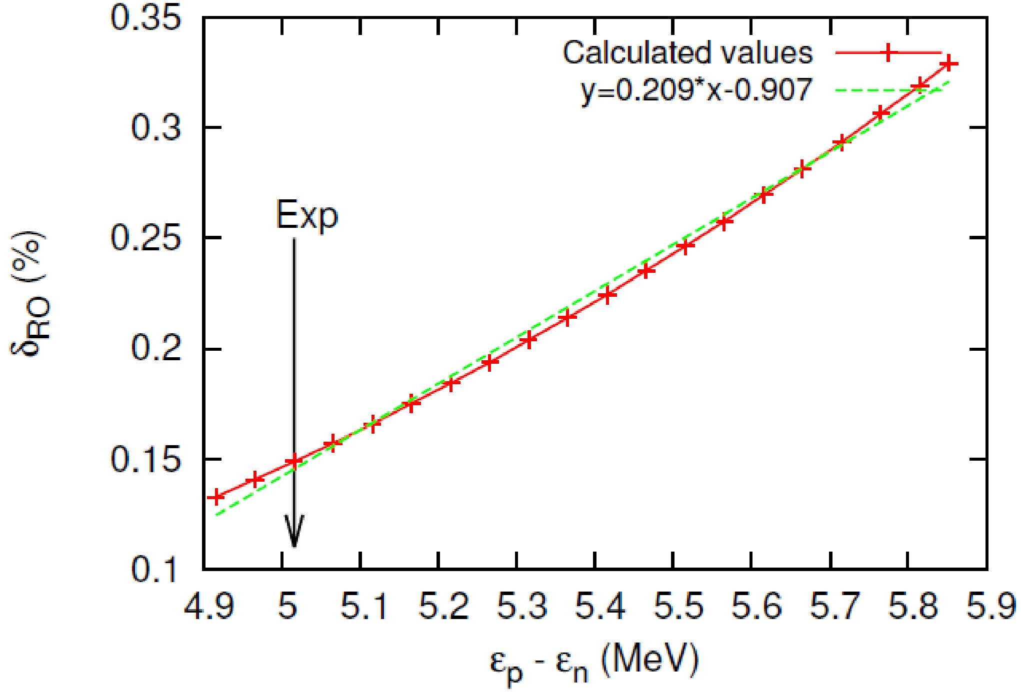
- (V_0,r_0) are adjusted simultaneously to reproduce experimental **nucleon separation energies** and **charge radii**

$$\psi(r) \rightarrow \exp\left(-\frac{\sqrt{2m|\epsilon|}r}{\hbar}\right)$$

- A new approach to nuclear charge radii beyond the closure approximation: $\langle r^2\rangle_{ch} = \frac{1}{Z}\sum_{\pi\alpha}\langle\alpha|r^2|\alpha\rangle^{\pi}|\langle\Psi_i||a^\dagger_\alpha||\pi\rangle|^2$
- Thorough **propagation of uncertainties**.

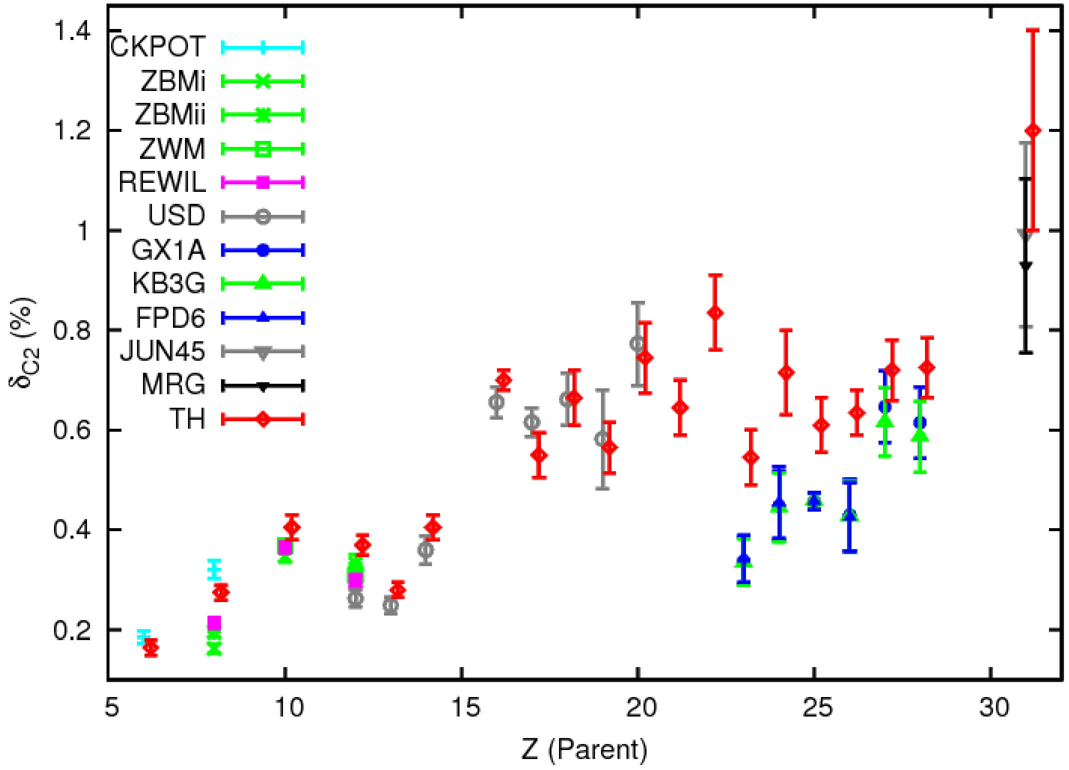
WS Adjustment



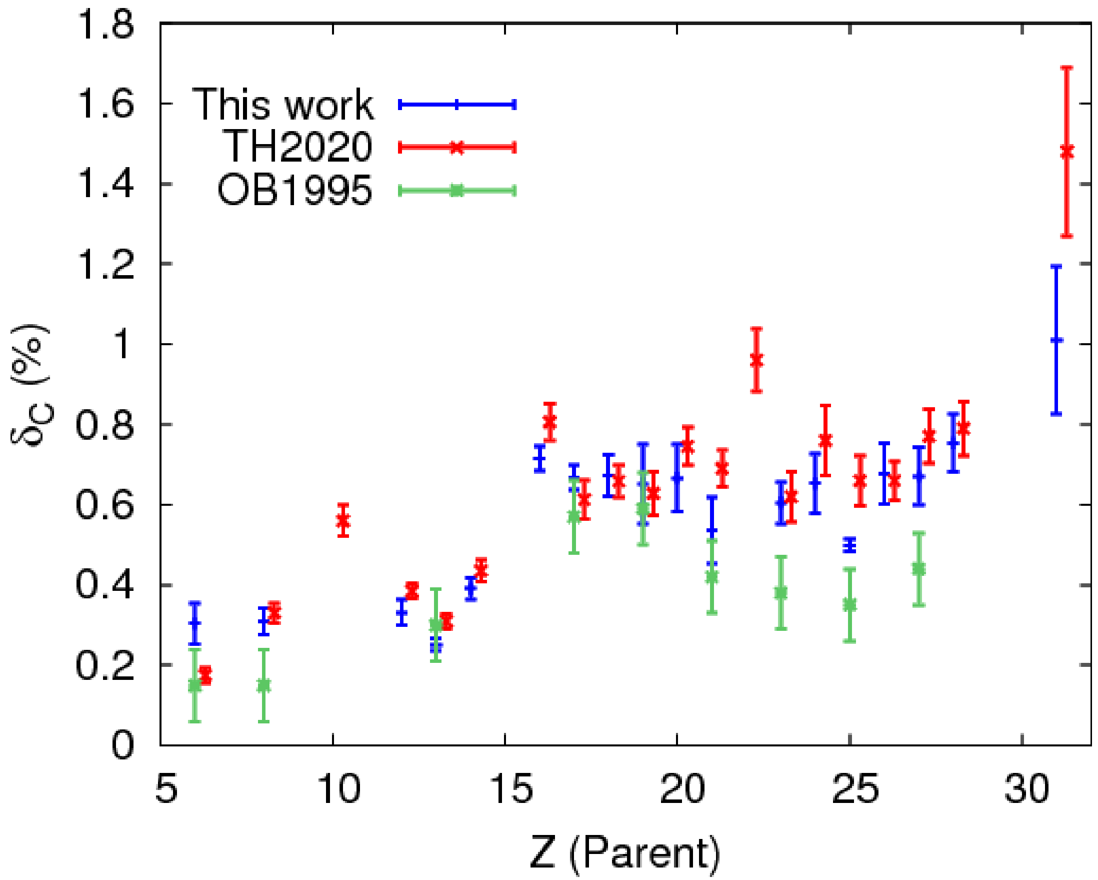


L. Xayavong, PhD thesis, U. Bordeaux (2016)

Shell-model + Woods-Saxon evaluation of δ_{C2} and δ_C



Preliminary !



Uncertainties are mainly due to the charge radii uncertainties !

N. Smirnova, L. Xayavong, Proc. NTSE2018.

Shell Model + Hartree-Fock radial wave functions for δ_{C2}

Skyrme force calculations

SkM* parameterization *J. Bartel et al, NPA386 (1982).*

Optimization

- Energy-dependent *local equivalent potential*
  C.B.Dover, N. Van Giai, NPA190 (1972):

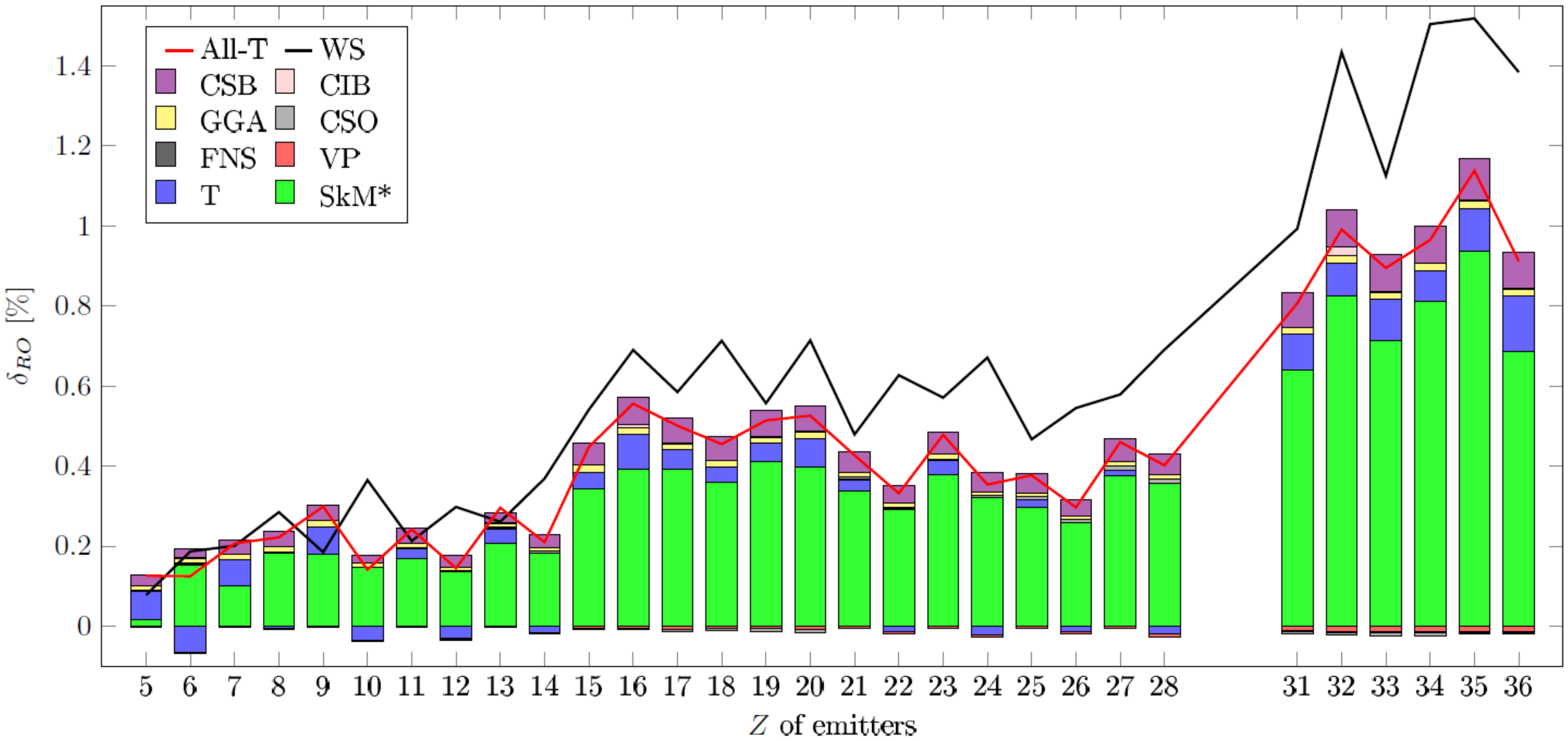
$$V^{LE}(r,\epsilon_\alpha) = V^0(r,\epsilon_\alpha) + V^{so}(r)\langle\vec{l}\cdot\vec{s}\rangle + V_C(r) \quad \text{for} \quad R_\alpha(r) = N\sqrt{\frac{m^*(r)}{m}}\,R_\alpha^{LE}(r).$$

- Adjustment of the central term $V^0(r,\epsilon_\alpha)$ by a scaling factor to match the experimental proton and neutron separation energies
- Effects from CIB and CSB terms, spurious isospin mixing, Slater approximation for Coulomb term, two-body center-of-mass correction, finite nucleon size, etc. are investigated.

W.E. Ormand, B.A. Brown, NPA440 (1985); PRL62 (1989); PRC52 (1995)
L. Xayavong, N. Smirnova, PRC105 (2022).

Shell Model + Hartree-Fock radial wave functions for δ_{C2}

L. XAYAVONG AND N. A. SMIRNOVA PHYSICAL REVIEW C **105**, 044308 (2022)



Post HF effects to be considered!

Consistency tests

Test of the CVC hypothesis for δ_C (as was proposed by J.C. Hardy, I. S. Towner, PRC82, 065501 (2010)):

$$\mathcal{F}t = (1+\delta_R)(1+\delta_{NS}-\delta_C)ft = \frac{K}{M_0^2 G_F{}^2 |V_{ud}|^2 (1+\Delta_R)}$$

^{10}C, ^{14}O, ^{22}Mg, ^{26m}Al, ^{26}Si, ^{34}Cl, ^{34}Ar, ^{38m}K, ^{38}Ca, ^{42}Sc, ^{46}V, ^{50}Mn, ^{54}Co, ^{62}Ga, (^{74}Rb)

- Δ_R , δ_R , δ_{NS} Radiative corrections are kept the same

Towner, Hardy (2020)	$\mathcal{F}t = 3072.24 \pm 0.57\ sec$	$\chi2/\nu = 0.47$
This work (WS)	$\mathcal{F}t = 3073.19 \pm 0.71\ sec$	$\chi2/\nu = 1.65$
This work (HF)	$\mathcal{F}t = 3074.81 \pm 0.93\ sec$	$\chi2/\nu = 2.96$

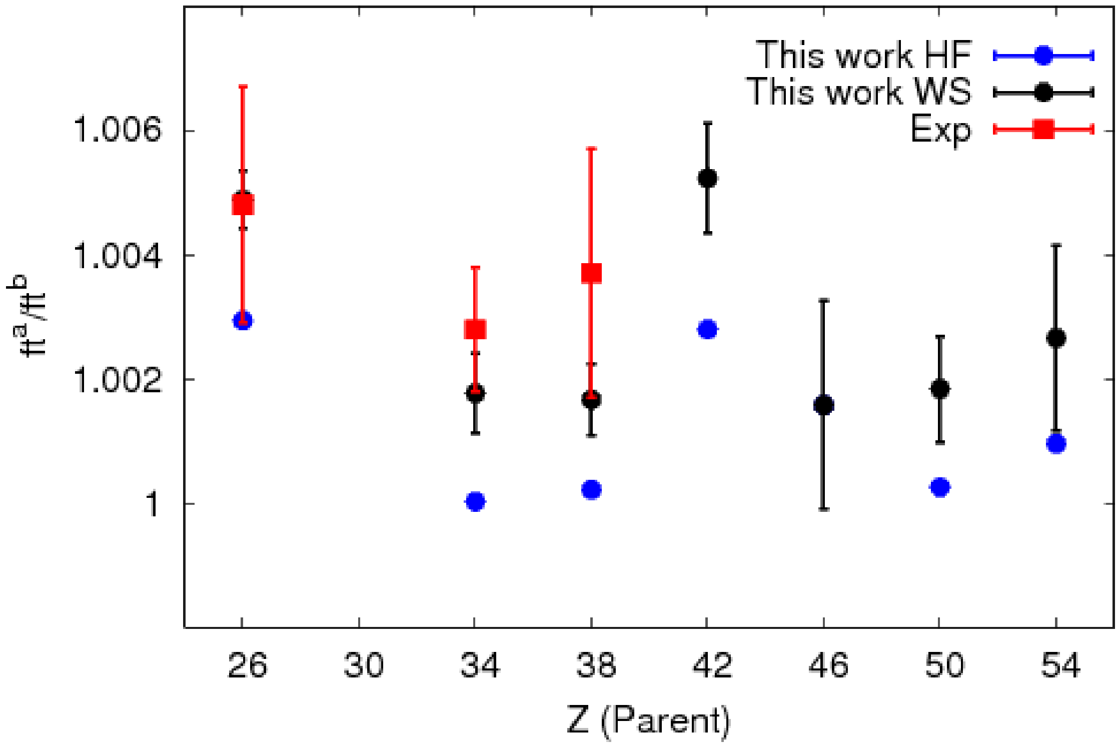
Preliminary !

Consistency tests

Test of the mirror ratio for triplets

$$\frac{ft^a}{ft^b} = 1 + \left(\delta_R^a - \delta_R^b\right) + \left(\delta_{NS}^a - \delta_{NS}^b\right) + \left(\delta_C^a - \delta_C^b\right)$$

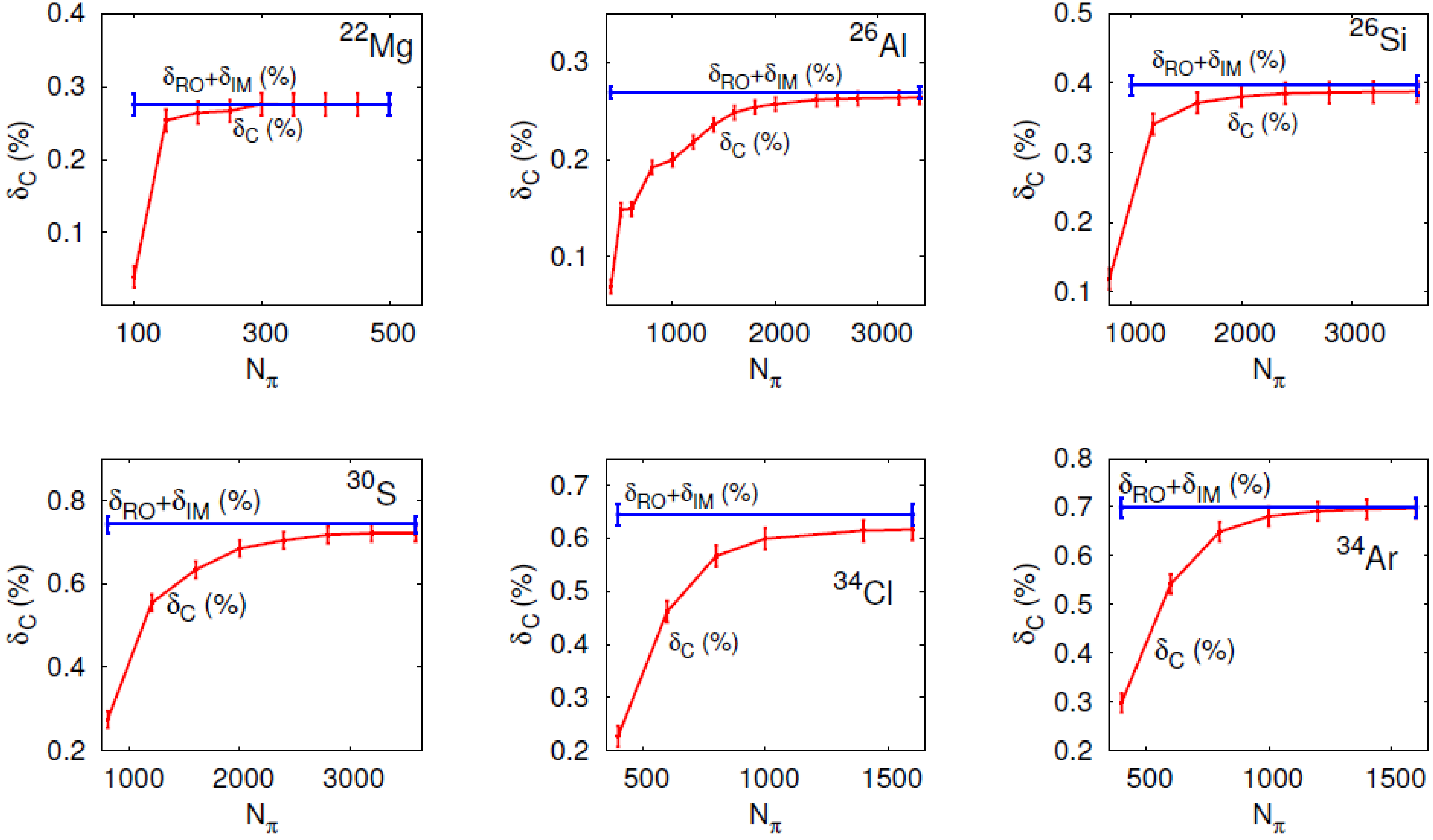
^{26m}Al, ^{26}Si, ^{34}Cl, ^{34}Ar, ^{38m}K, ^{38}Ca, ^{42}Sc, ^{42}Ti, ^{46}V, ^{46}Cr, ^{50}Mn, ^{50}Fe, ^{54}Co, ^{54}Ni



Preliminary !

Test of the separation ansatz of the isospin-symmetry breaking correction

δ_C is compared to $\delta_{C1} + \delta_{C2}$



N. Smirnova, L. Xayavong, Proc. NTSE2018.

Higher-order terms of the isospin-symmetry breaking correction

Exact evaluation of δ_C:

$$M_F = M_F^0\left[1 - \frac{1}{M_F^0}\sum_\alpha\left(\rho_\alpha^T - \rho_\alpha\right) - \frac{1}{M_F^0}\sum_\alpha \rho_\alpha^T\left(1-\Omega_\alpha\right) + \frac{1}{M_F^0}\sum_\alpha\left(\rho_\alpha^T - \rho_\alpha\right)\left(1-\Omega_\alpha\right)\right],$$

$$|M_F|^2 = |M_F^0|^2(1-\delta_C), \quad \delta_C = \sum_{i=1}^{6}\delta_{Ci}$$

Explicit expressions for the six terms:

These terms have similar structure when expressed beyond the closure approximation!

$$\delta_{C1} = \frac{2}{M_F^0}\sum_\alpha\left(\rho_\alpha^T - \rho_\alpha\right) \quad \text{LO}$$

$$\delta_{C2} = \frac{2}{M_F^0}\sum_\alpha \rho_\alpha^T\left(1-\Omega_\alpha\right) \quad \text{LO}$$

$$\delta_{C3} = \frac{2}{M_F^0}\sum_\alpha\left(\rho_\alpha^T - \rho_\alpha\right)\left(1-\Omega_\alpha\right) \quad \text{NLO}$$

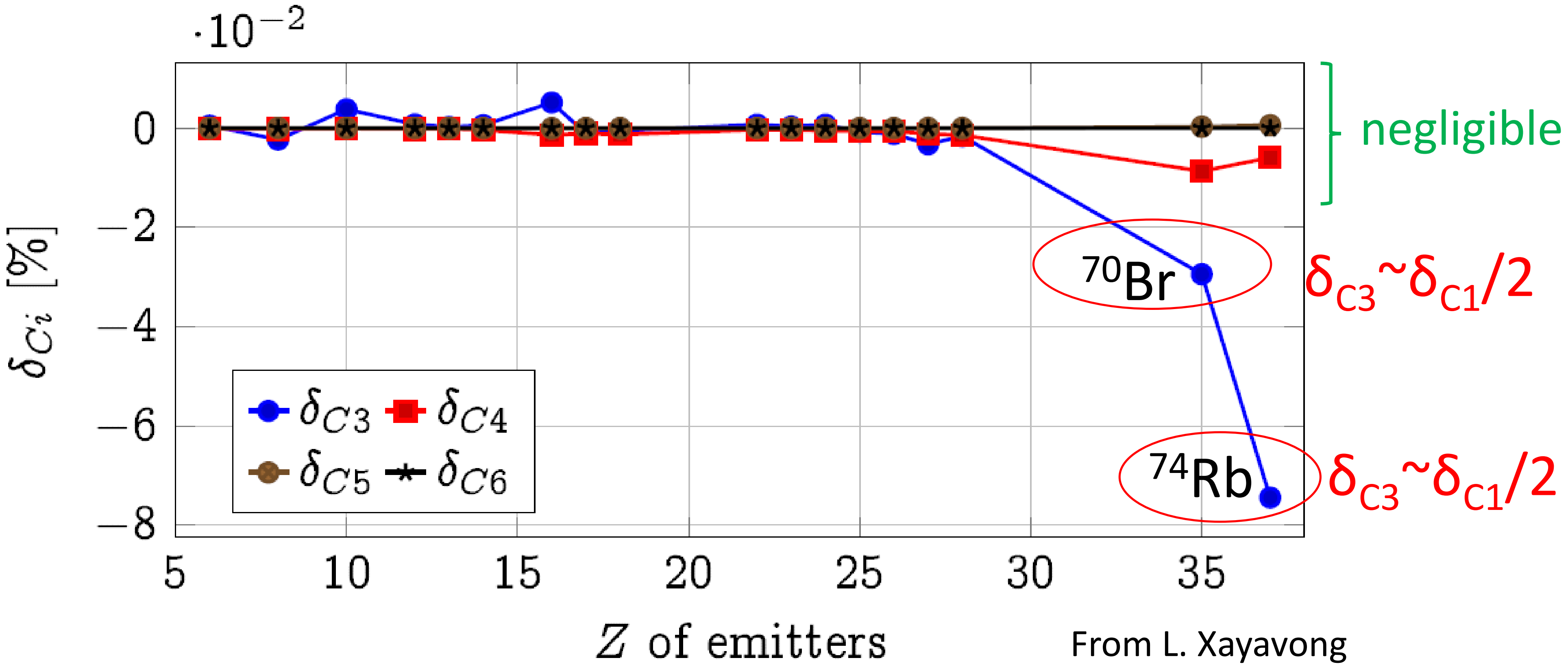
$$\delta_{C4} = -\left(\delta_{C1}+\delta_{C2}\right)^2/4 \quad \text{NLO}$$

$$\delta_{C5} = -\left(\delta_{C1}+\delta_{C2}\right)\delta_{C3}/2 \quad \text{N}^2\text{LO}$$

$$\delta_{C6} = -\delta_{C3}^2/4 \quad \text{N}^3\text{LO}$$

Higher-order terms of the isospin-symmetry breaking correction

- Higher-order contribution (superallowed $0^+ \rightarrow 0^+$ Fermi transition)



From L. Xayavong

L. Xayavong, N. Smirnova, Phys. Rev. C109, 014317 (2024)

Nuclear-structure correction to Fermi β decay

Results :

- Large-scale calculations with global WS parametrization lead to the results similar to those of Towner and Hardy (2015, 2020)

- New calculations with the Skyrme HF wave functions + numerous effects checked. Conclusion: importance of CSB/CIB terms, of elimination of spurious isospin mixing and post-HF effects

- Higher-order terms in the correction are identified and assessed

Perspectives :

- The issue of the exact Fermi operator to be investigated via ab-initio methods (L. Xayavong et al, in progress)

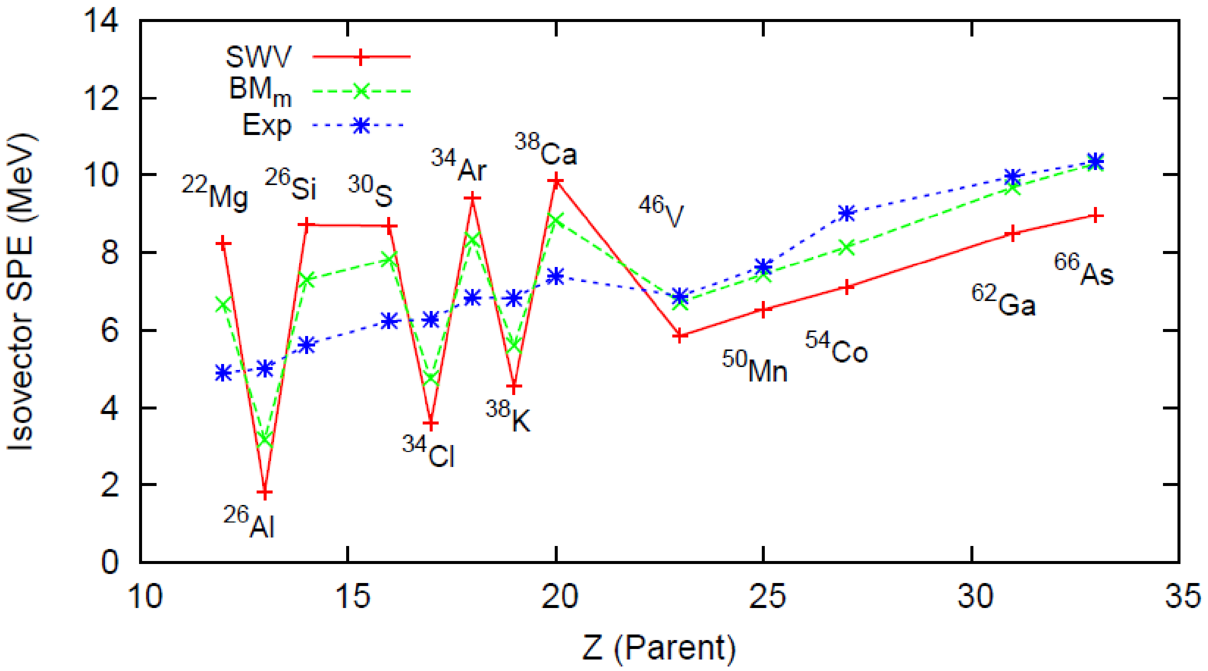
- High-dimensions and heavier nuclei (near A=80)

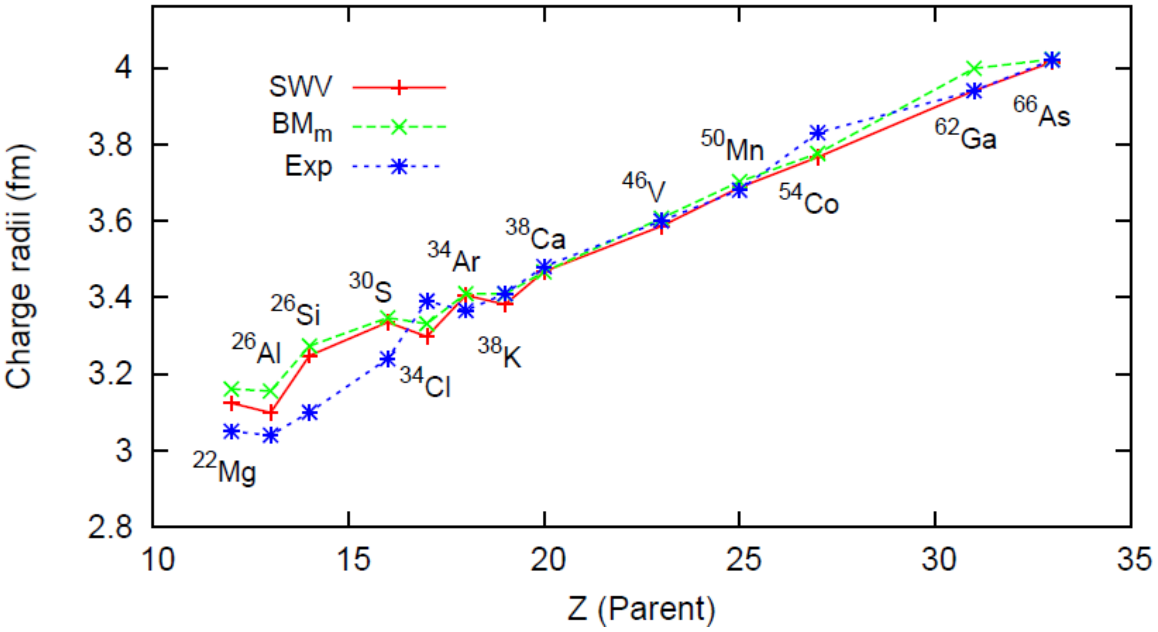
- Improved charge-dependent interactions

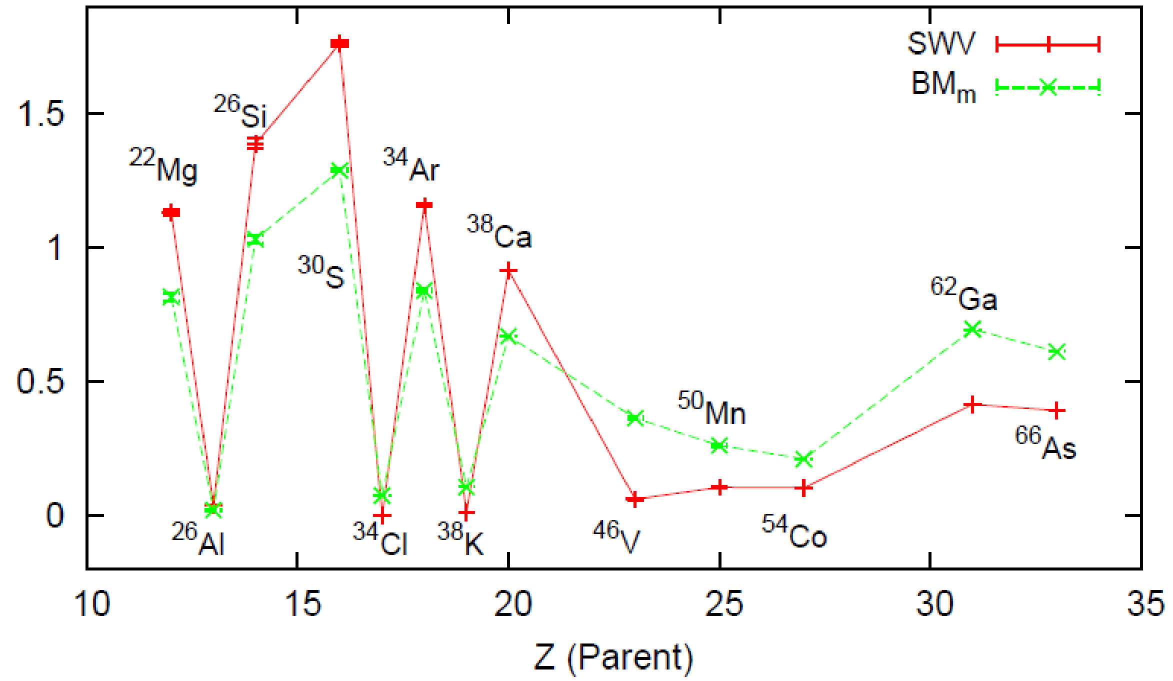
- Mirror decays (T=1/2)

BACK-UP SLIDES

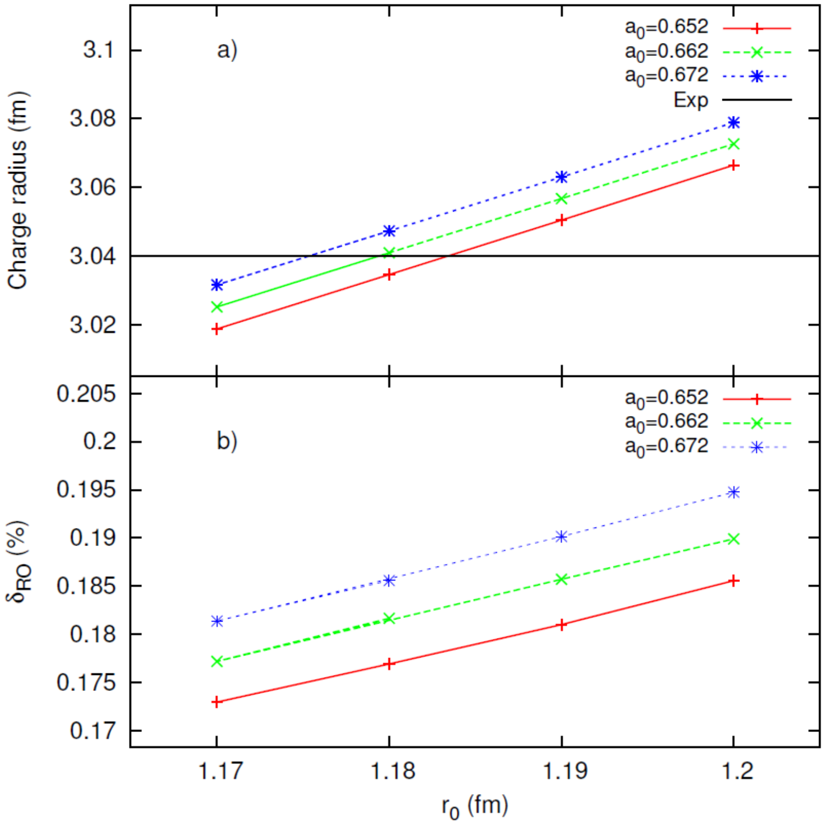
WS Non-adjusted !



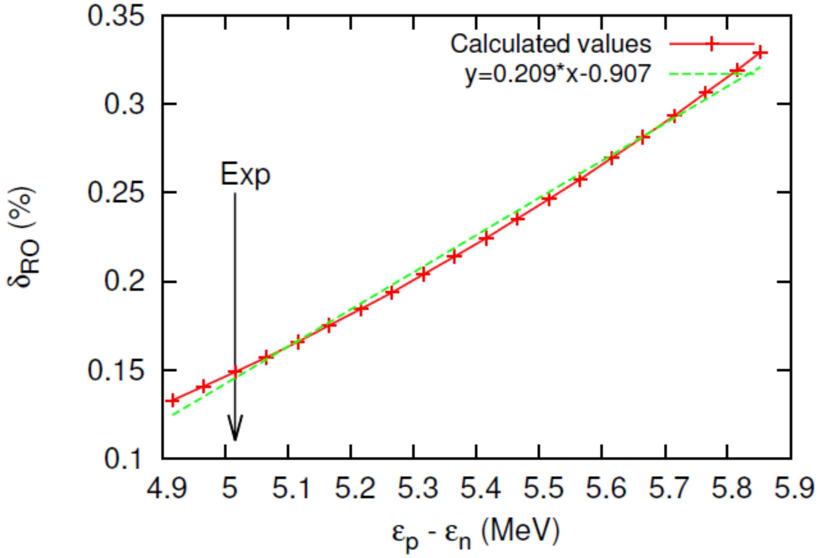




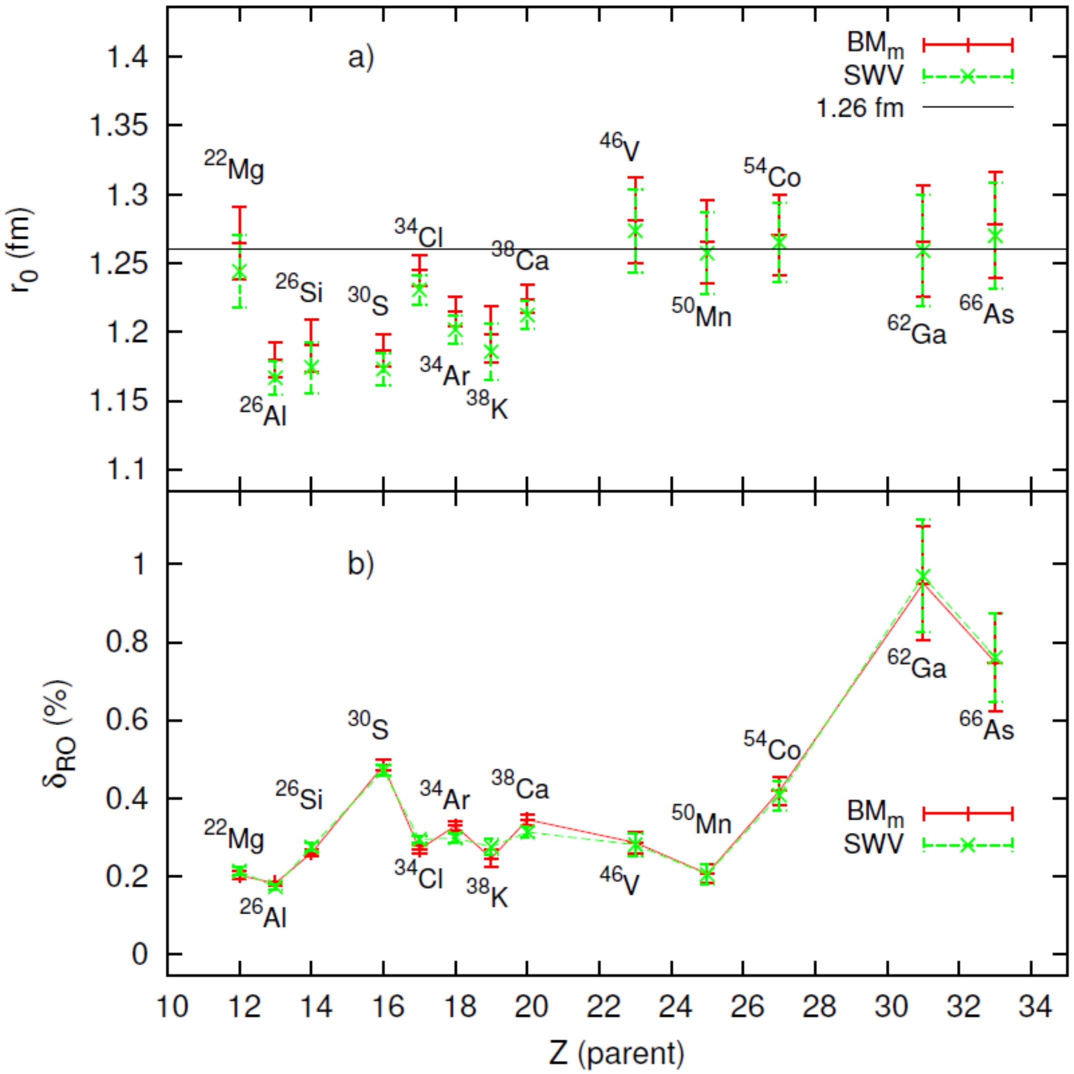
WS Adjusted !



a)
a0=0.652
a0=0.662
a0=0.672
Exp
Charge radius (fm)
3.1
3.08
3.06
3.04
3.02
b)
δRO (%)
0.205
0.2
0.195
0.19
0.185
0.18
0.175
0.17
1.17
1.18
1.19
1.2
r0 (fm)

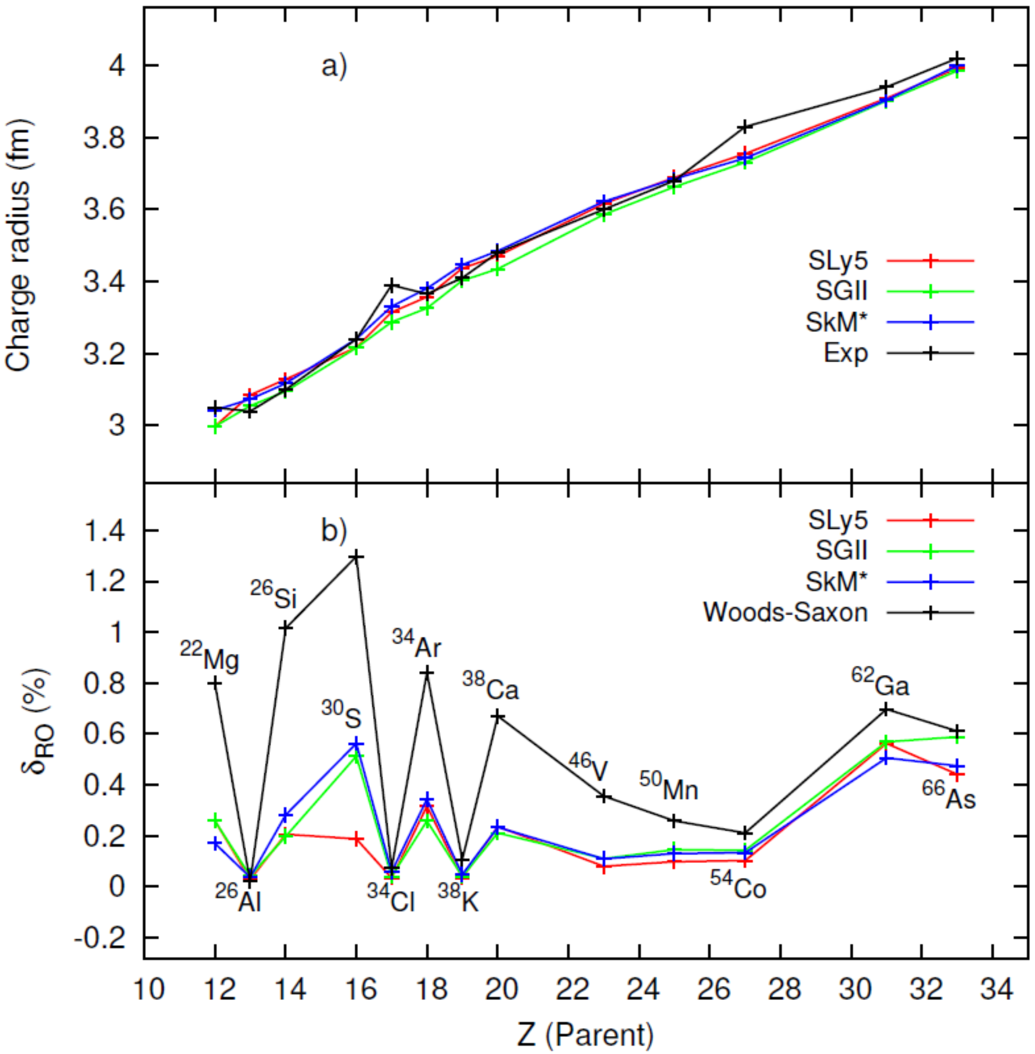


Calculated values
y=0.209*x-0.907
Exp
δRO (%)
0.35
0.3
0.25
0.2
0.15
0.1
4.9
5
5.1
5.2
5.3
5.4
5.5
5.6
5.7
5.8
5.9
εp - εn (MeV)

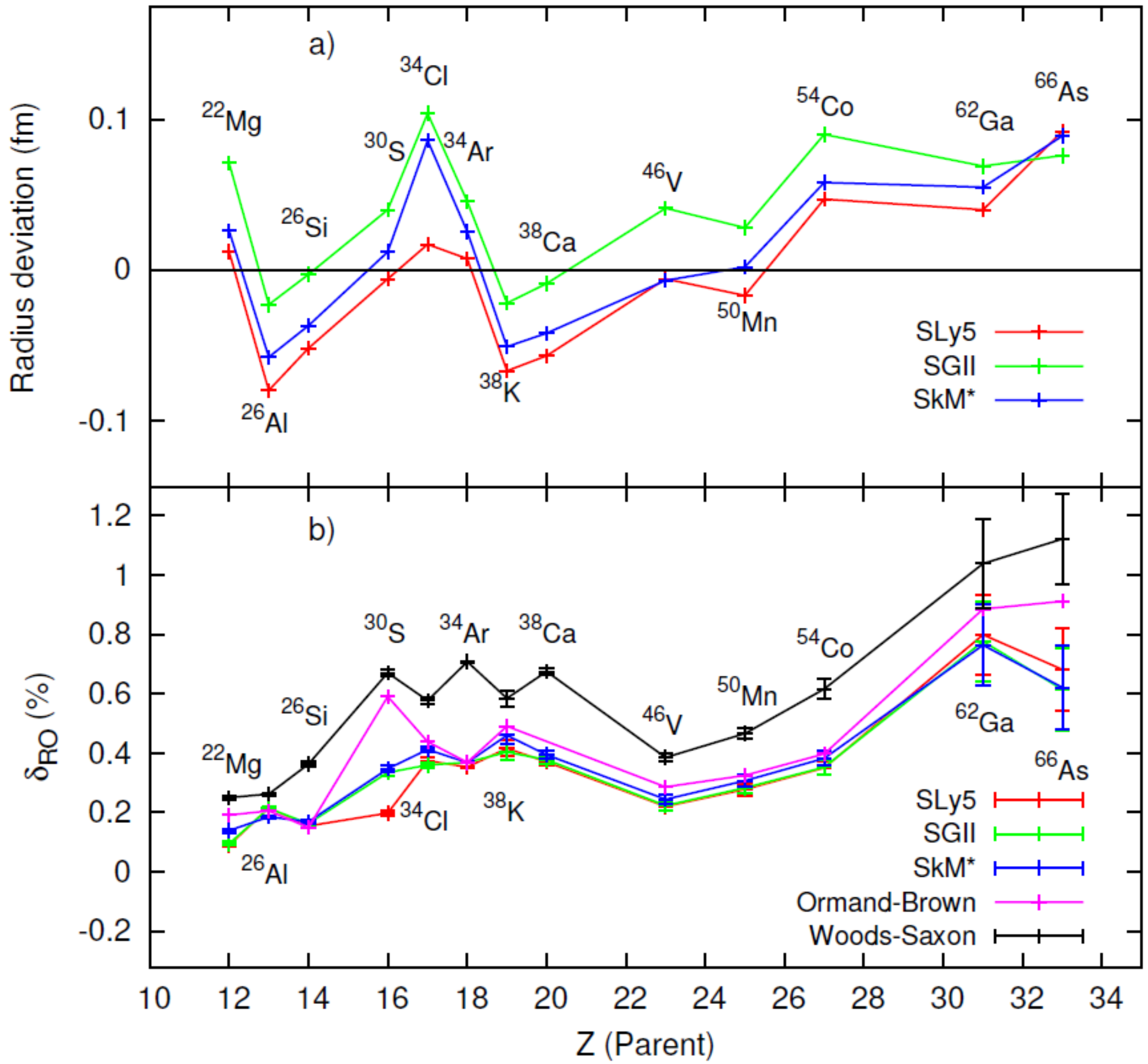


a)
BMm
SWV
1.26 fm
r0 (fm)
1.4
1.35
1.3
1.25
1.2
1.15
1.1
22Mg
26Al
26Si
30S
34Cl
34Ar
38K
38Ca
46V
50Mn
54Co
62Ga
66As
b)
δRO (%)
1
0.8
0.6
0.4
0.2
0
10
12
14
16
18
20
22
24
26
28
30
32
34
Z (parent)

WS and HF Non-adjusted !



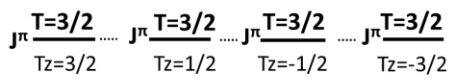
WS and HF Adjusted !



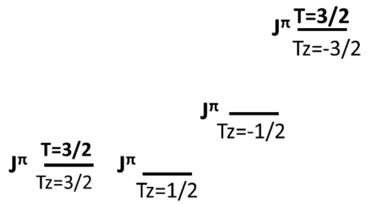
Nuclei along N=Z line: isospin-symmetry breaking

Isospin non-conserving Hamiltonian
(Coulomb + effective charge-dependent components)

$$H_{INC} = H_0 + V_{res} + V_{CD}$$

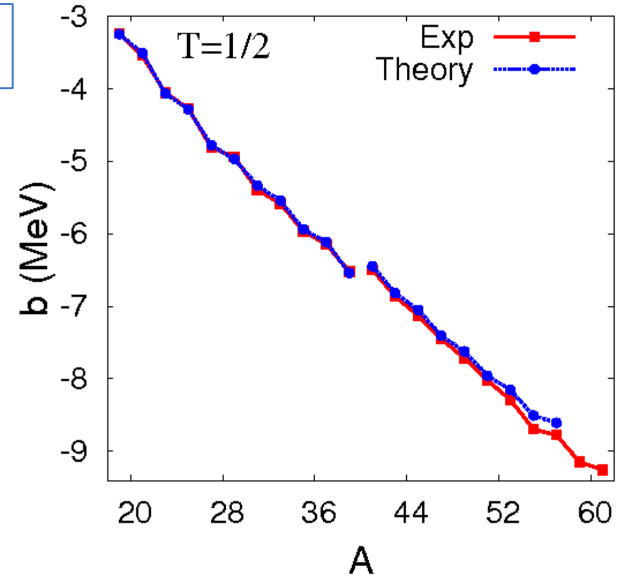


Isospin-symmetry limit

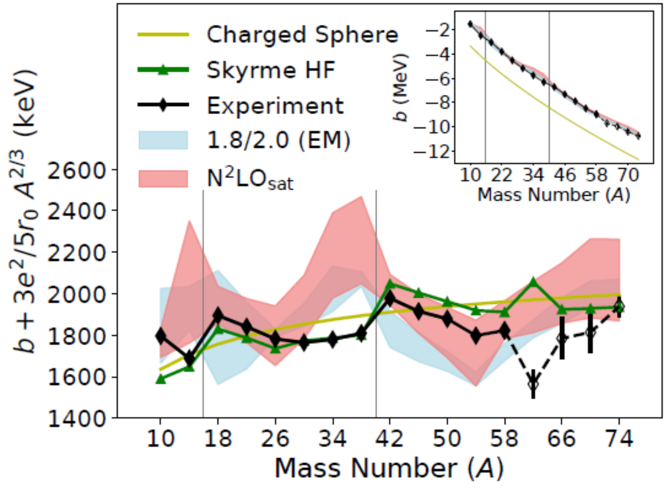


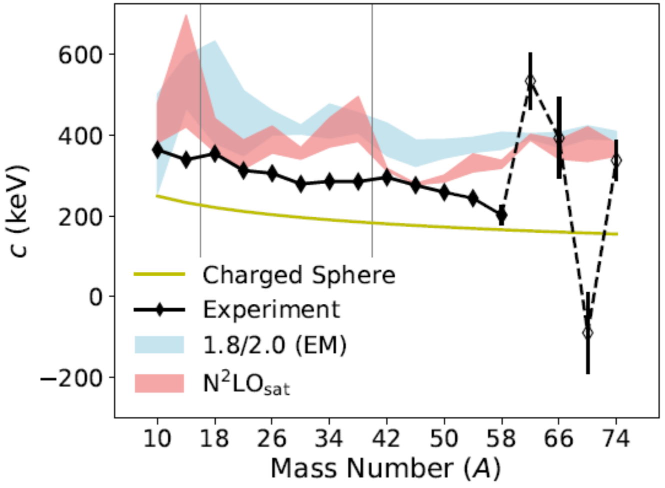
Realistic situation

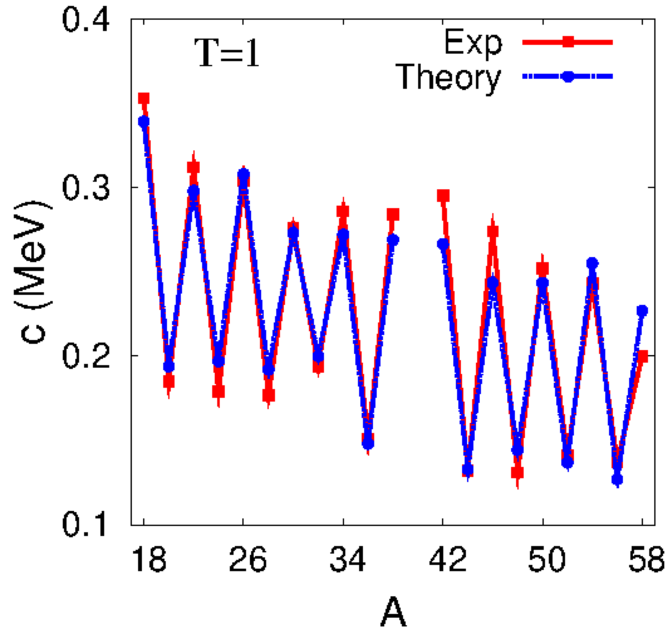
$$M(\eta, T, T_z) = a(\eta, T) + b(\eta, T)T_z + c(\eta, T)T_z^2$$



From IMSRG, Martin et al (2021)







IMME b and c coefficients of lowest and excited multiplets

Fine structure (staggering) of b and c coefficients

$$M_{T_z} = a + bT_z + cT_z^2$$

Importance :

- Prediction of masses and excited levels in proton-rich nuclei, e.g. if *Tz*>0 :

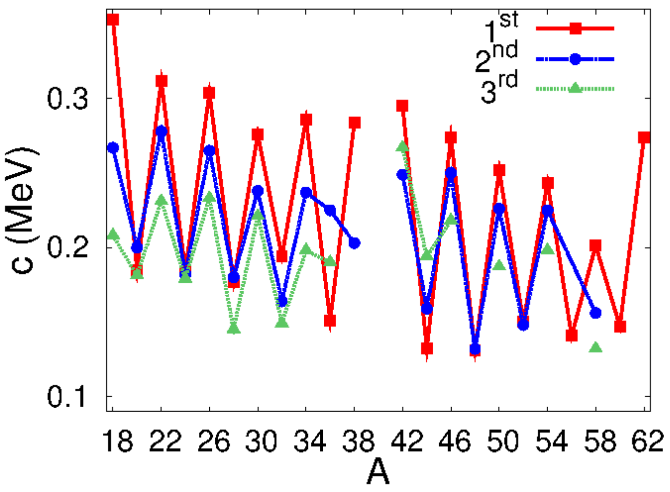
$$M_{-T_z} = M_{T_z}^{exp} + 2b^{th}T_z$$

- Particular case of triplets :

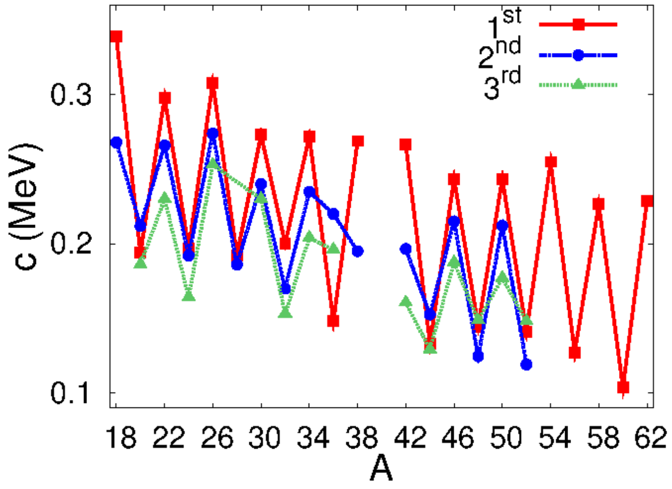
$$M_{-1} = 2M_0^{exp} - M_1^{exp} + 2c^{th}$$

Important for nuclear astrophysics applications!

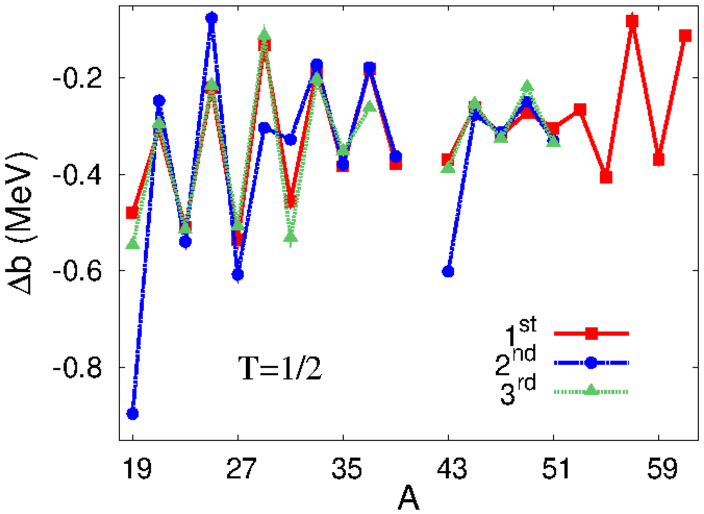
Experiment



Theory



Experiment



Theory