Radiative Inclusive B Decays

(Based on PRD 110 (2024) 5, 053003)

Dayanand Mishra
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- Requires the understanding of radiative decay rate (for eg $B \to X_c \ell \bar{\nu} \gamma$).

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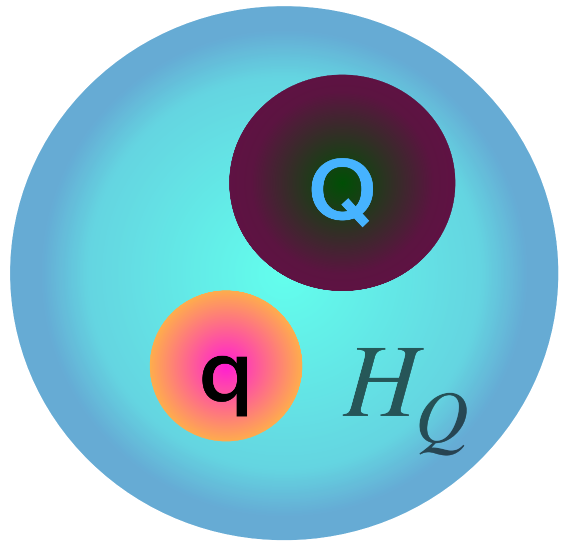

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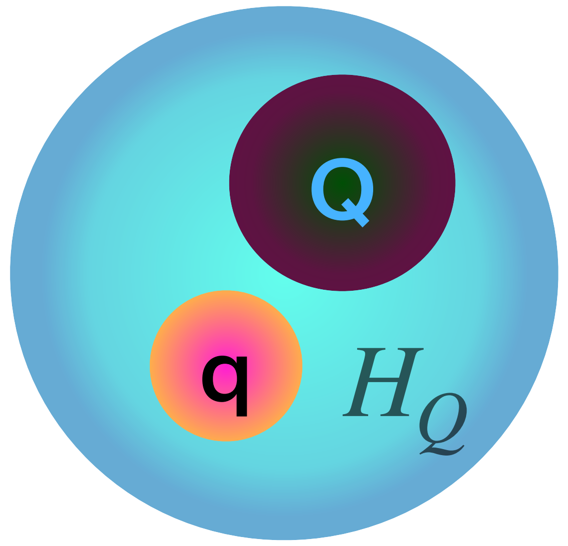

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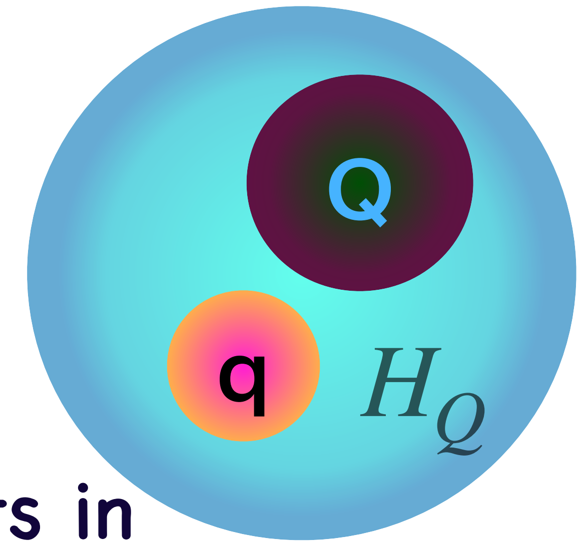

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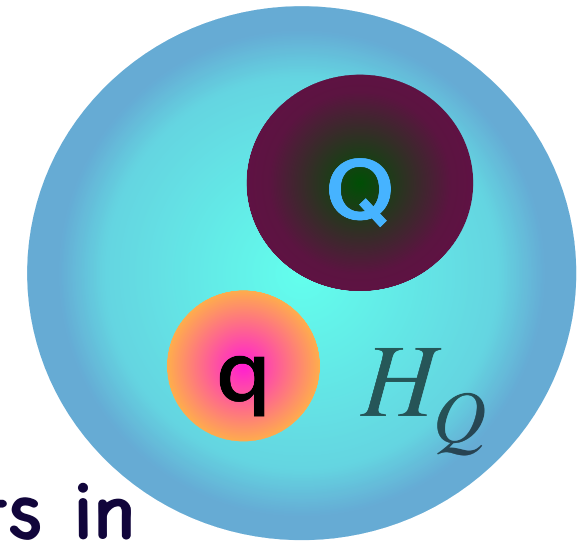

- Heavy Quark size $\sim 1/m_Q$
  1. At short Dist.- heavy quark surrounded by a static coulomb like colour field A_0
  2. At large Dist.- self interaction strengthens, at $R \geq \Lambda_{QCD}^{-1}$ (completely Non-perturbative)

- Weak Hamiltonian density for B meson decays semi-leptonically to final states containing u quarks

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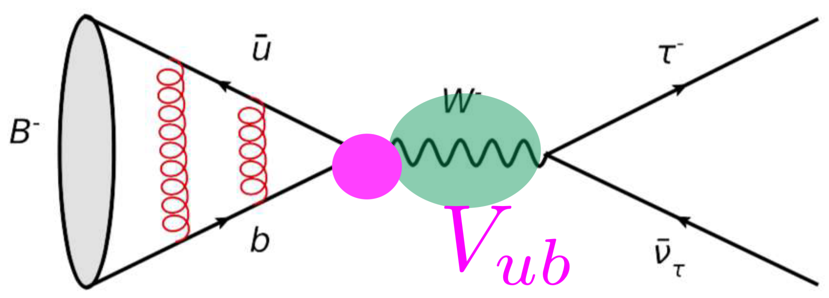
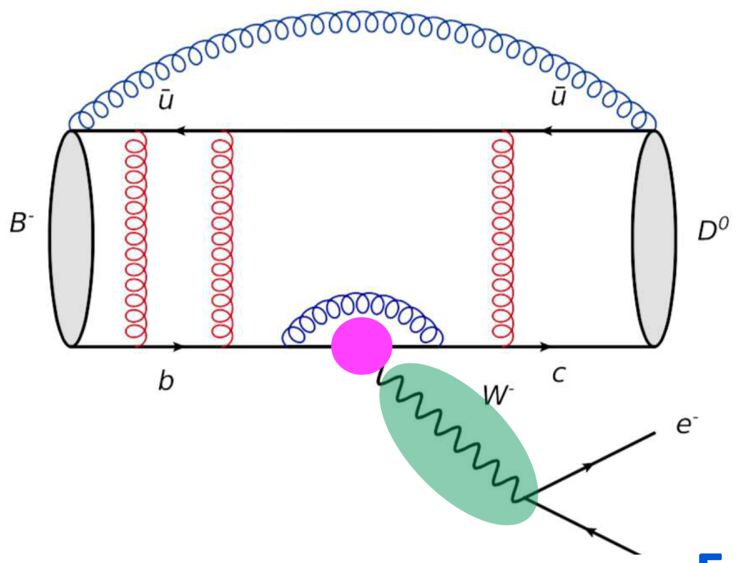


Decay constant

$$\langle 0|\bar{b}\gamma^\mu\gamma_5 u|B_q(p)\rangle = i f_{B_q} p^\mu$$

Semileptonic decays



Form factor

$$\langle D^0(p_D)|\bar{c}\gamma_\mu b|B^-(p_B)\rangle = f_+^{B^-\to D^0}(q^2)\left(p_B^\mu + p_D^\mu - \frac{m_B^2 - m_D^2}{q^2}q^\mu\right) + \mathcal{O}(f_0^{B^-\to D^0})$$

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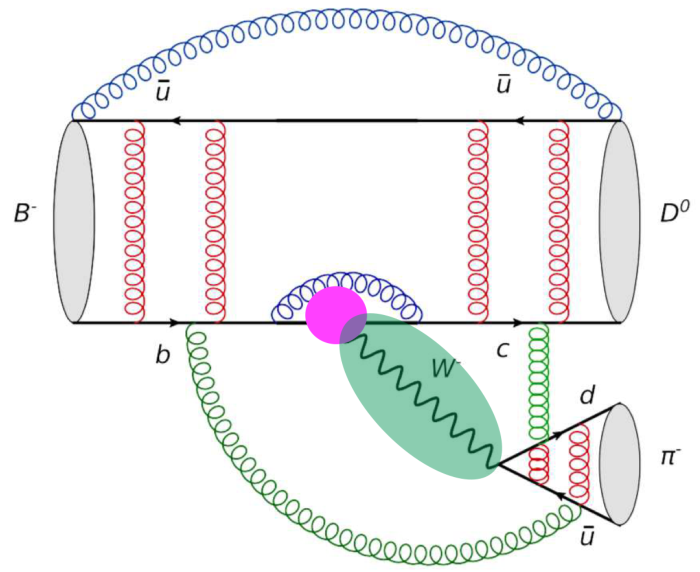

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$$\langle D^0\pi^-|\bar{c}\gamma_\mu(1-\gamma_5)b \cdot \bar{u}\gamma^\mu(1-\gamma_5)d|B^-\rangle \approx \langle D^0|\bar{c}\gamma_\mu(1-\gamma_5)b|B^-\rangle \cdot \langle \pi^-|\bar{u}\gamma^\mu(1-\gamma_5)d|0\rangle$$

(Figs taken from A. Lenz's talk)

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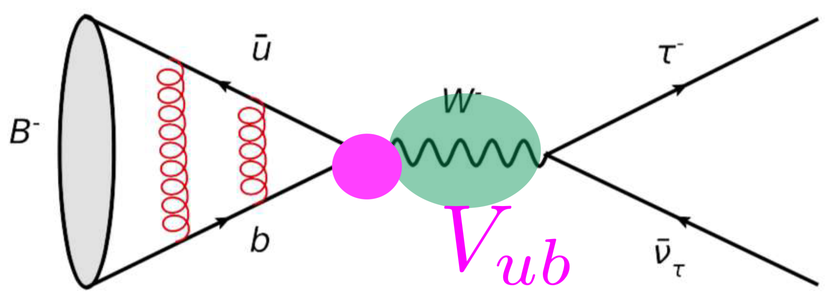

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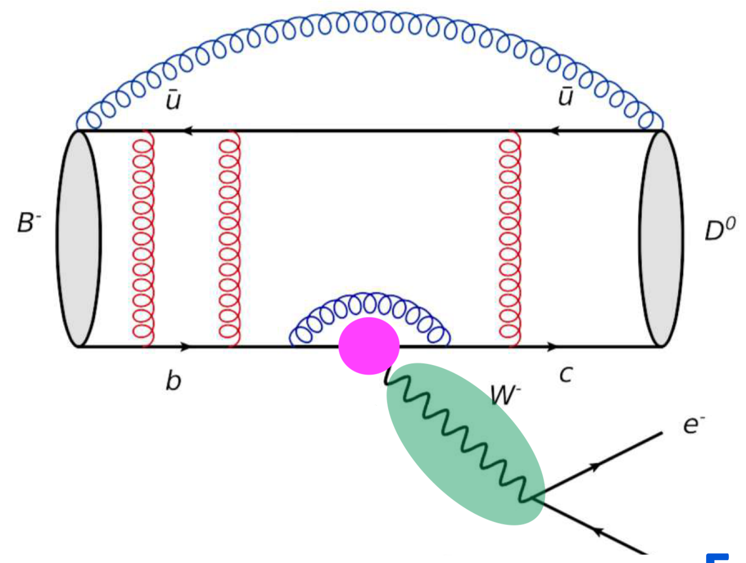

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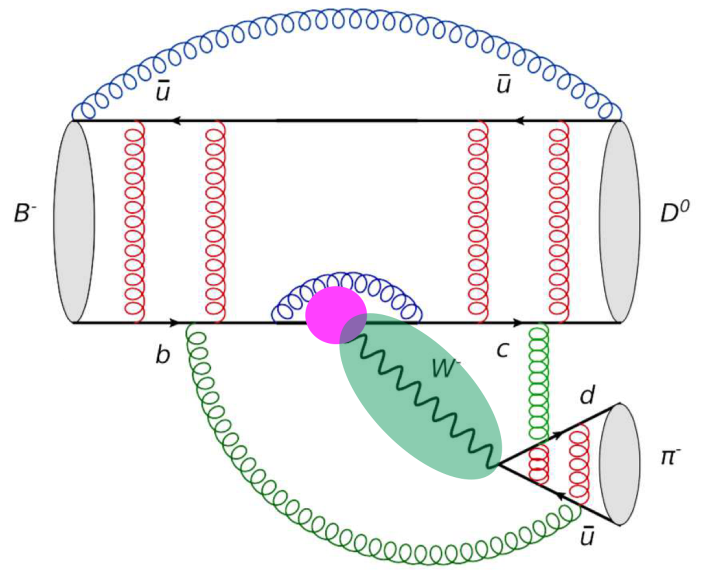

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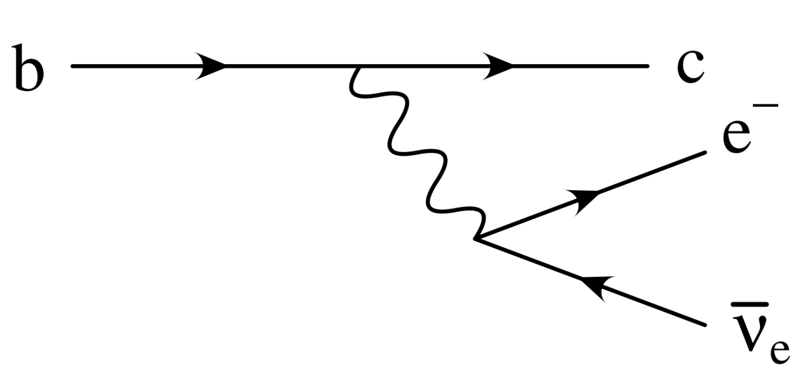

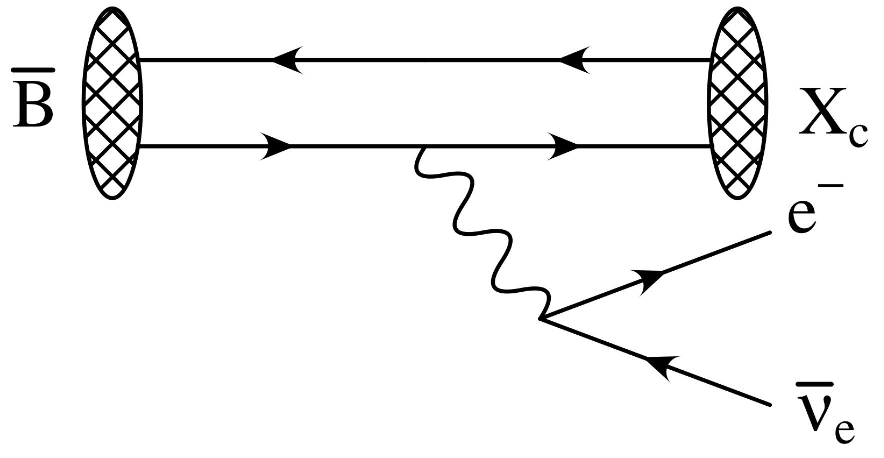

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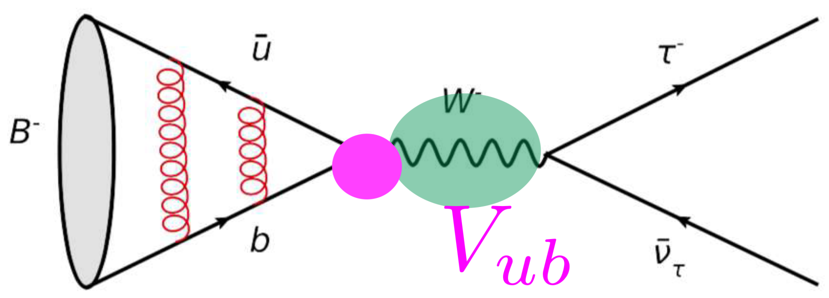

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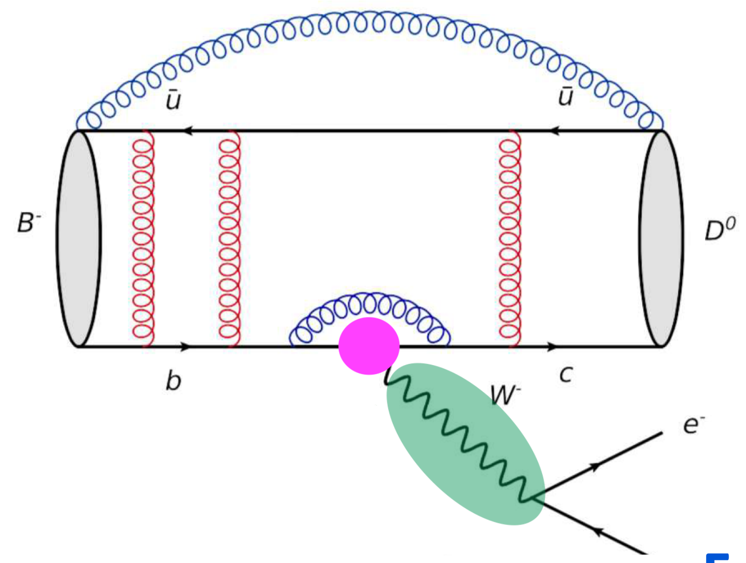

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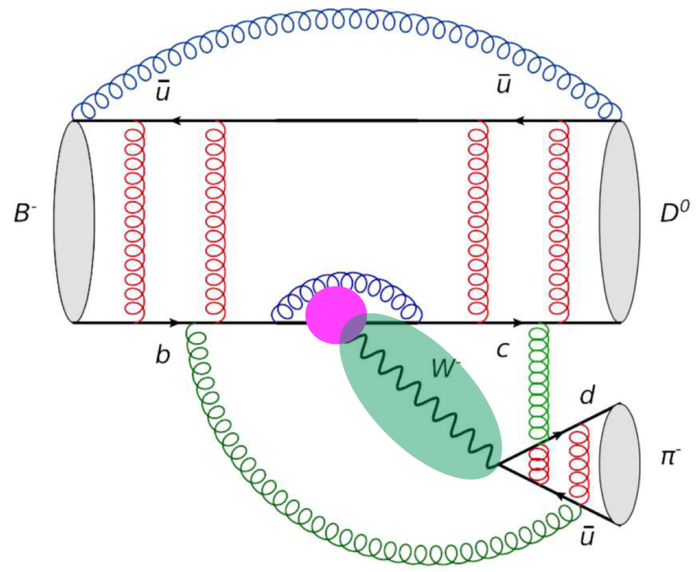

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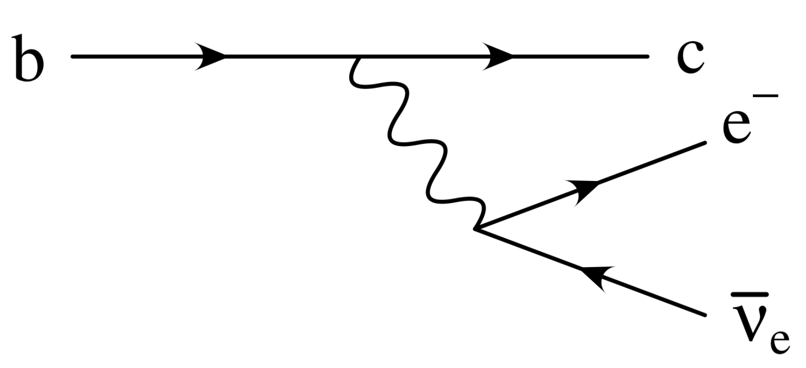

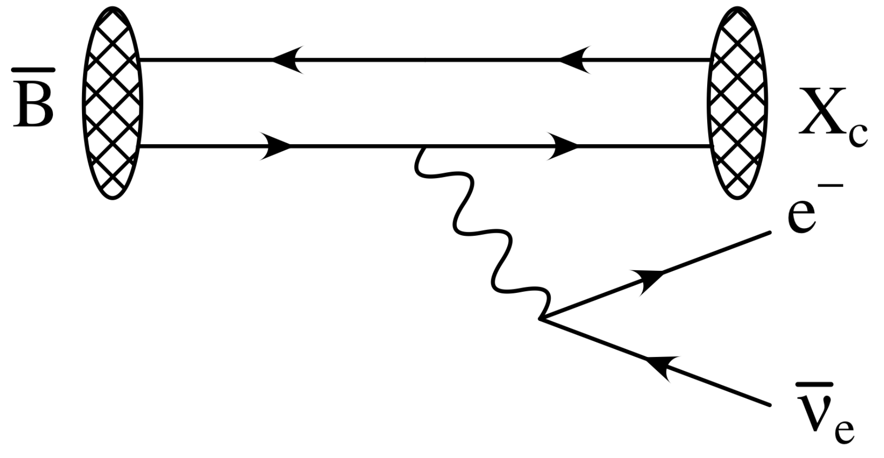

$$\Gamma \propto C_0 + C_1 \frac{\lambda_1}{m_b^2} + C_2 \frac{\lambda_2}{m_b^2} + \ldots$$

$$\lambda_1 = \frac{1}{2}\langle B(v)\,|\,\bar{b}_v (iD)^2 b_v\,|\,B(v)\rangle$$

$$\lambda_2 = -\frac{1}{12}\langle B(v)\,|\,\bar{b}_v g(\sigma\cdot G) b_v\,|\,B(v)\rangle$$

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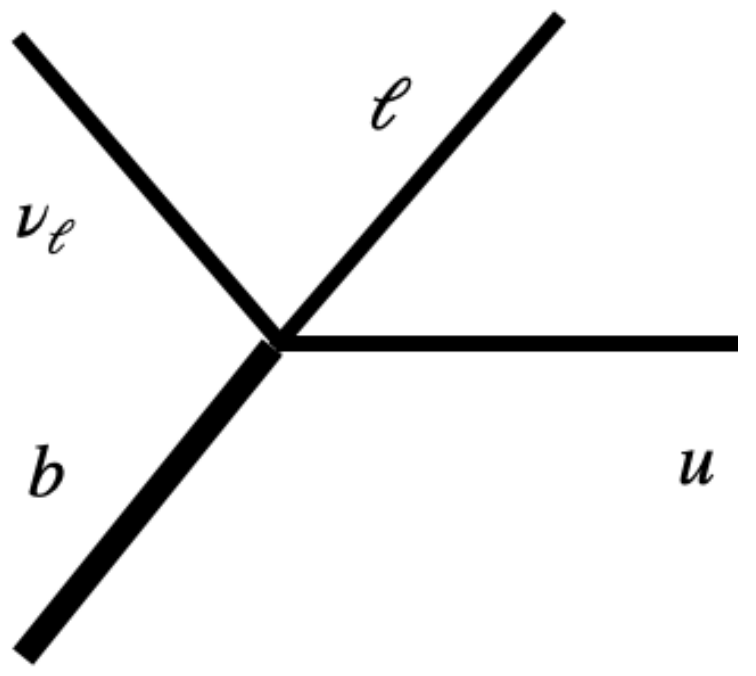

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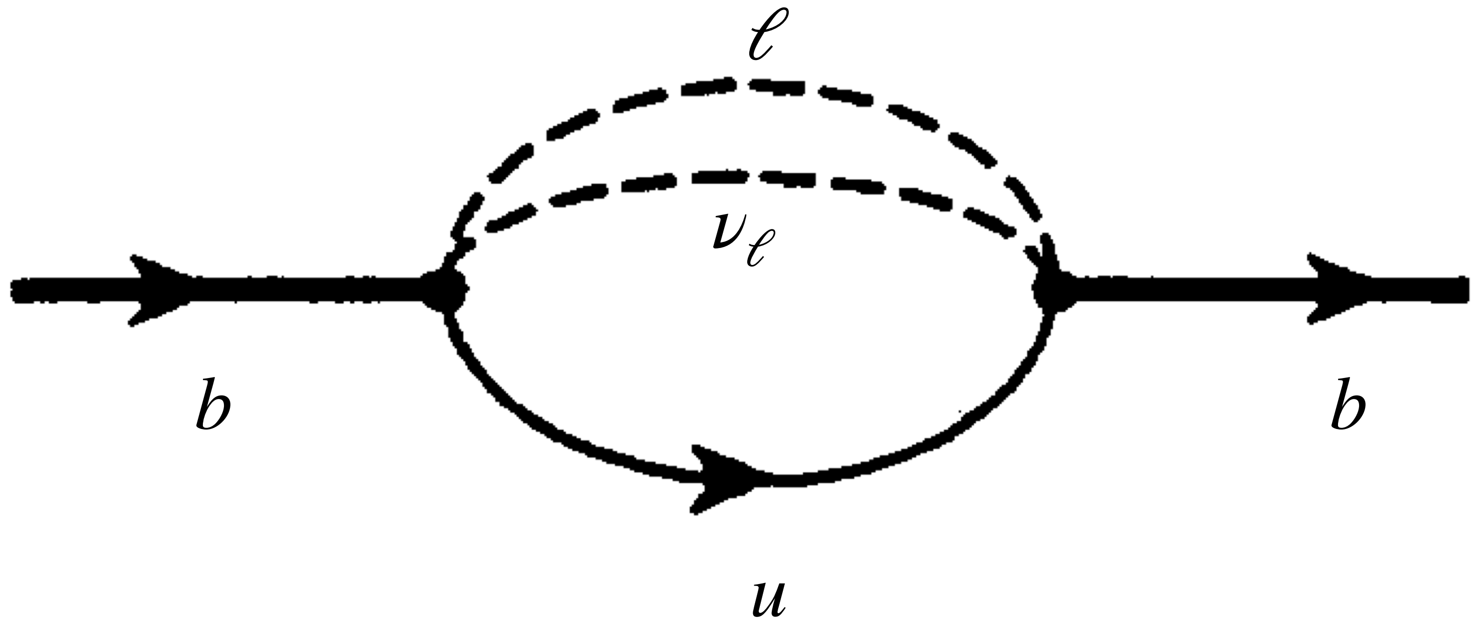

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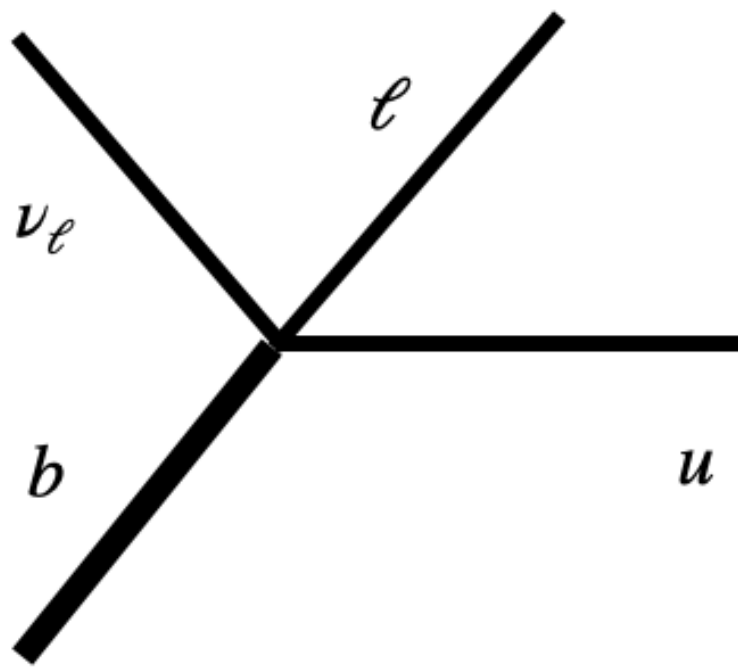

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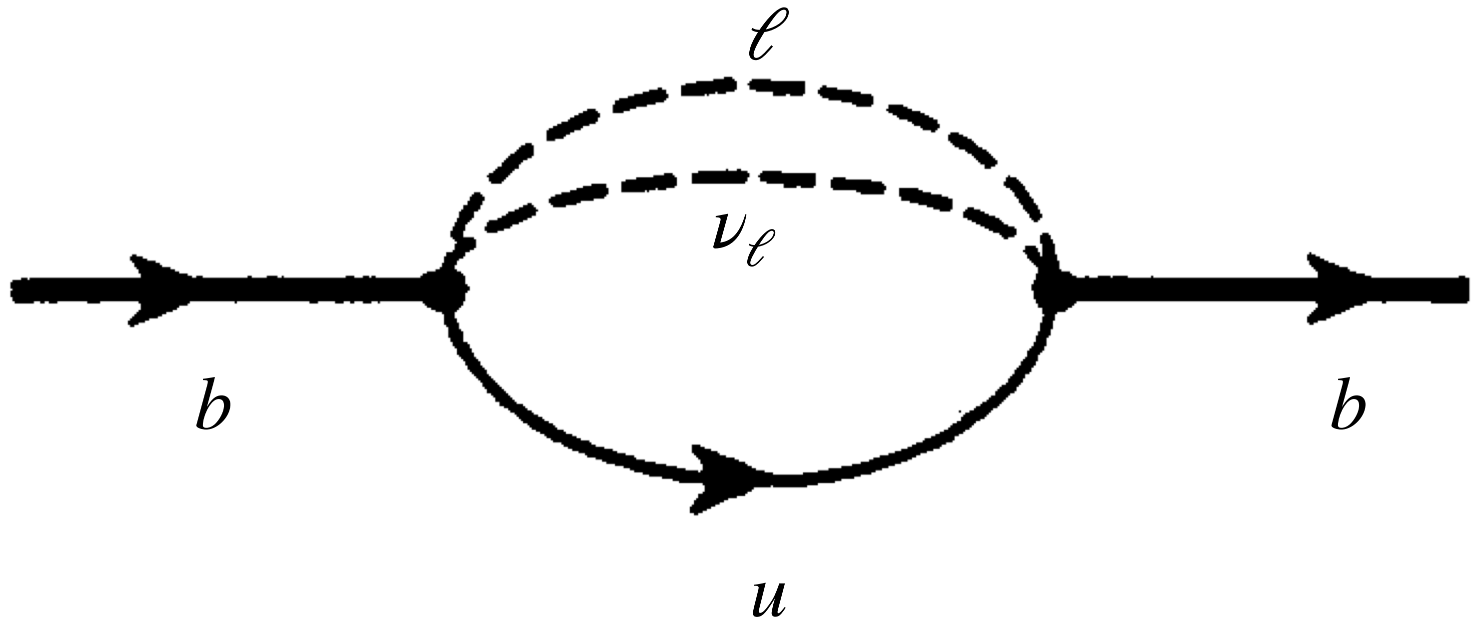

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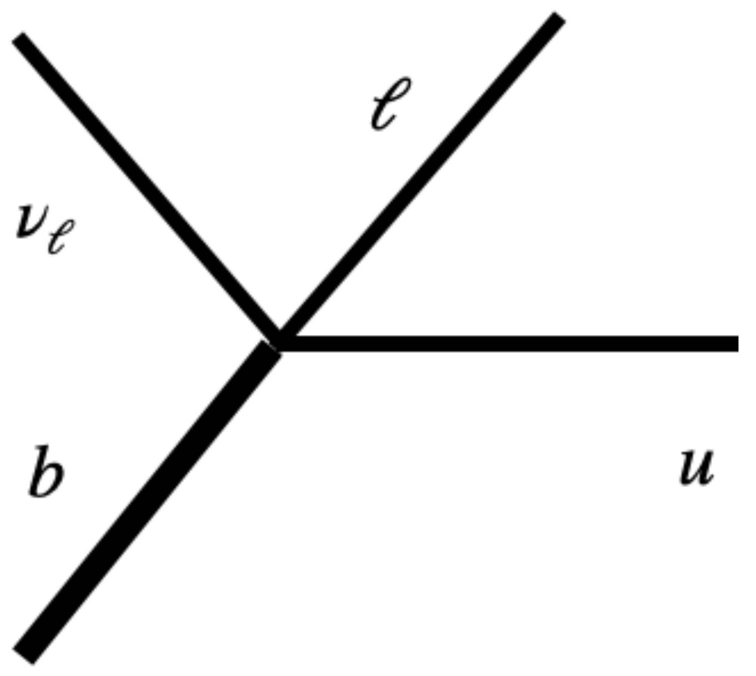

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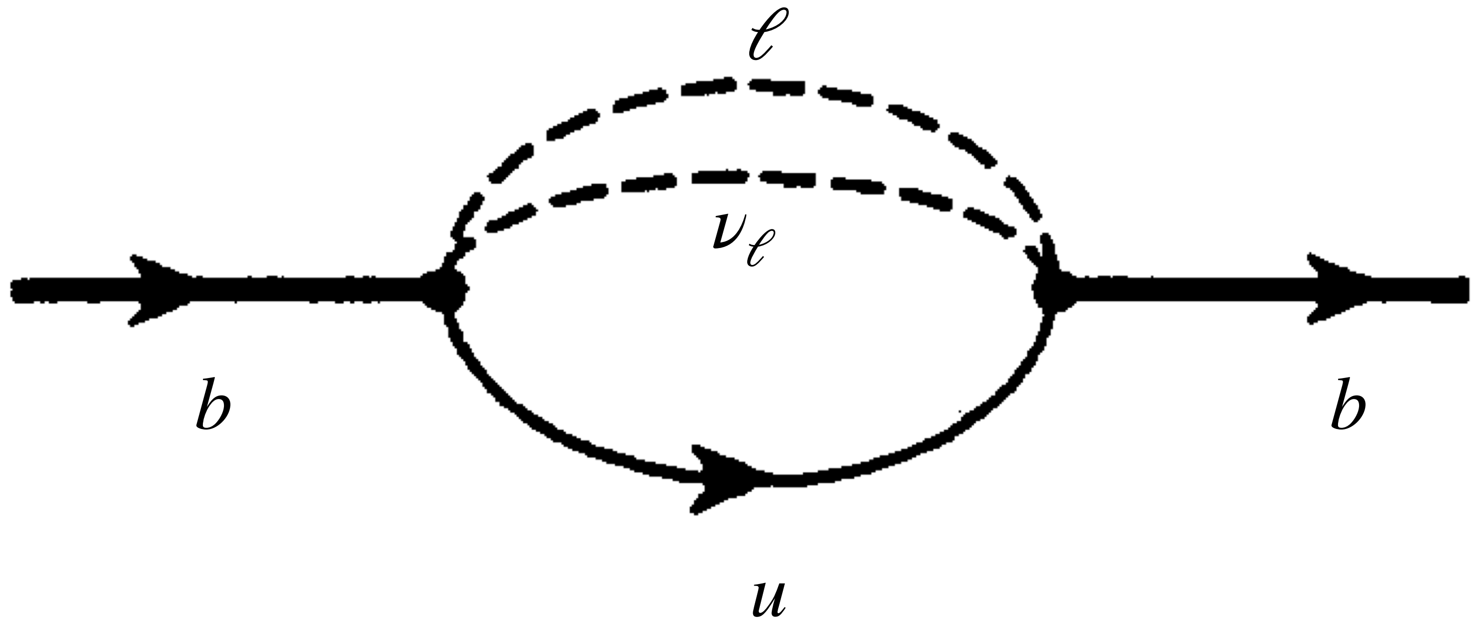

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- Method used to compute this transition operator : Heavy Quark Effective Theory (HQET)

$$\Gamma \propto C_0 + C_1 \frac{\lambda_1}{m_b^2} + C_2 \frac{\lambda_2}{m_b^2} + \ldots$$

Let's look at $B \to X_c \ell \nu_\ell$

- Differential decay width: $$d\Gamma = \frac{1}{8m_B}\sum \delta^4(p_B - p_x - q)\,|\langle X\ell\nu_\ell\,|\,H_{\text{weak}}\,|B\rangle|^2\, dq^2 dE_\ell dE_\nu$$

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Transition operator

$$\left(T(Q \to X \to Q) = \sum C_i \mathcal{O}_i\right)$$

Where, $\underset{\text{(QCD)}}{b(x)} = e^{-im_b v.x} \underset{\text{(HQET)}}{b_v(x)}$

- Operators at linear and quadratic order in Π :

$$\bar{b}_v \gamma^\mu b_v$$

(dim 3 operator)

$$\bar{b}_v \gamma_\mu (iD_\nu) b_v$$

(dim 4 operator: does not contribute)

$$\bar{b}_v (D^2) b_v \quad \text{and} \quad \frac{i}{2} \bar{b}_v (G_{\mu\nu} \sigma^{\mu\nu}) b_v$$

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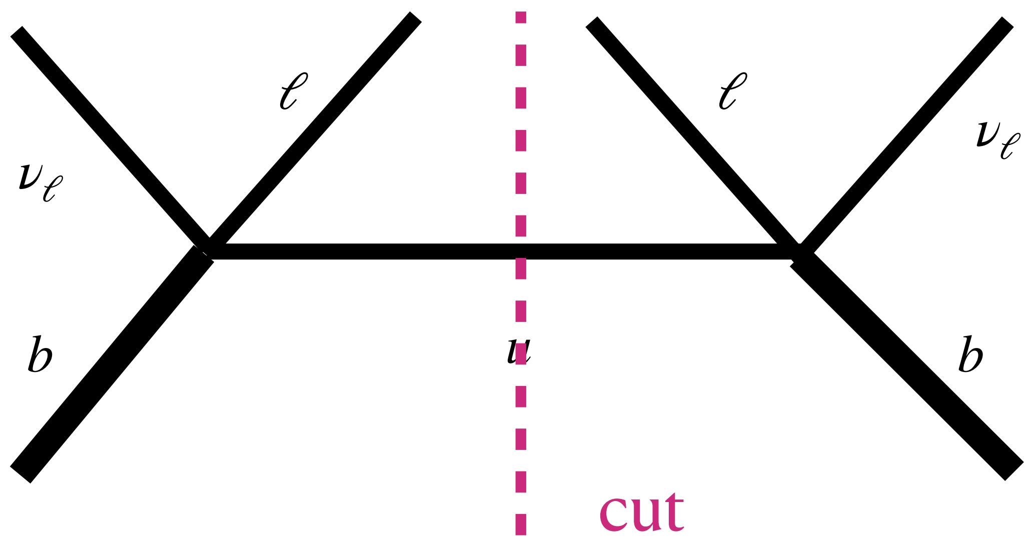

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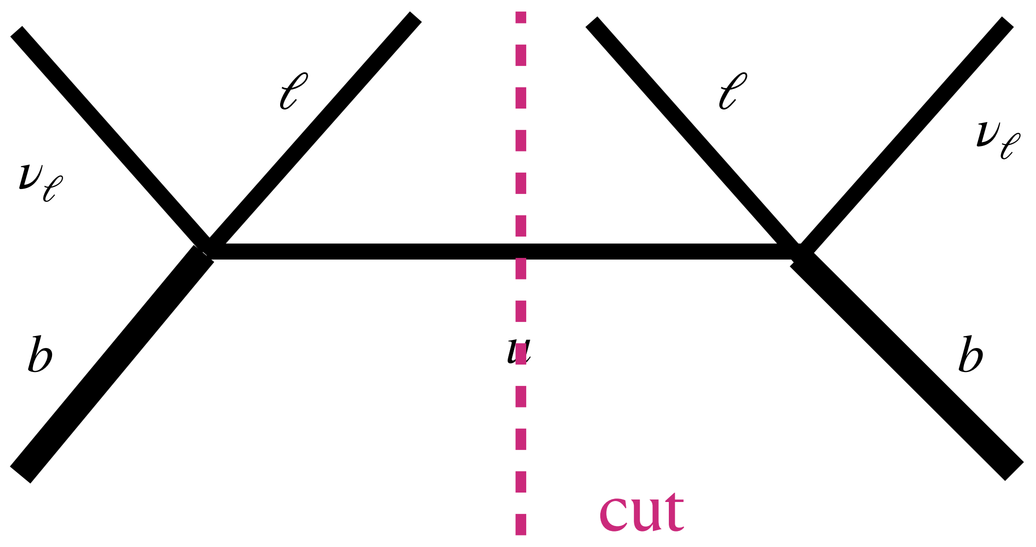

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- Non-perturbation parameters :

$$\lambda_1 \sim -\frac{1}{2} \langle B | \bar{b}_v D^2 b_v | B \rangle \qquad \lambda_2 \sim \frac{1}{2} a(\mu) \langle B | \bar{b}_v g \sigma_{\alpha\beta} G^{\alpha\beta} b_v | B \rangle$$

Where, $a(\mu) = 1 + \mathcal{O}(\alpha_s(m_b))$

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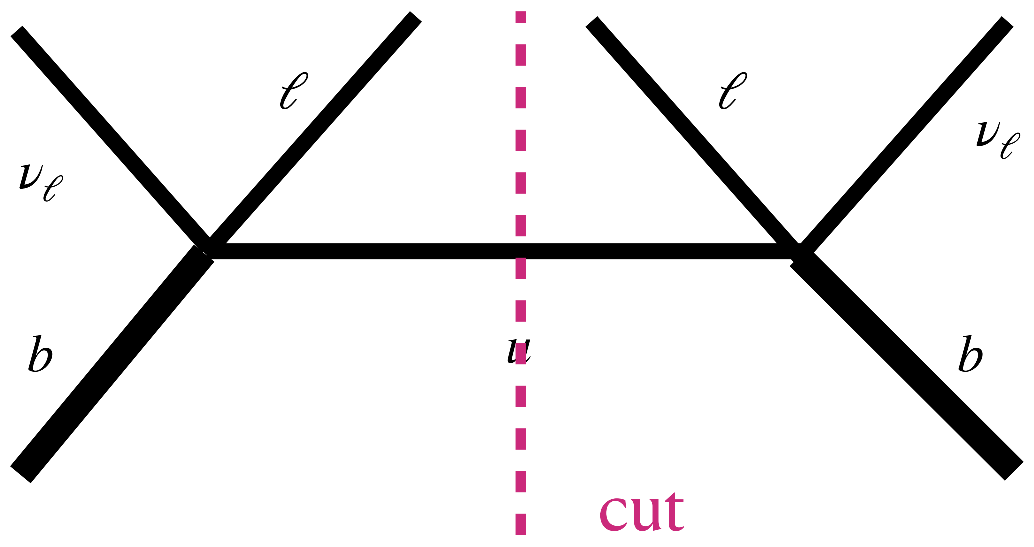

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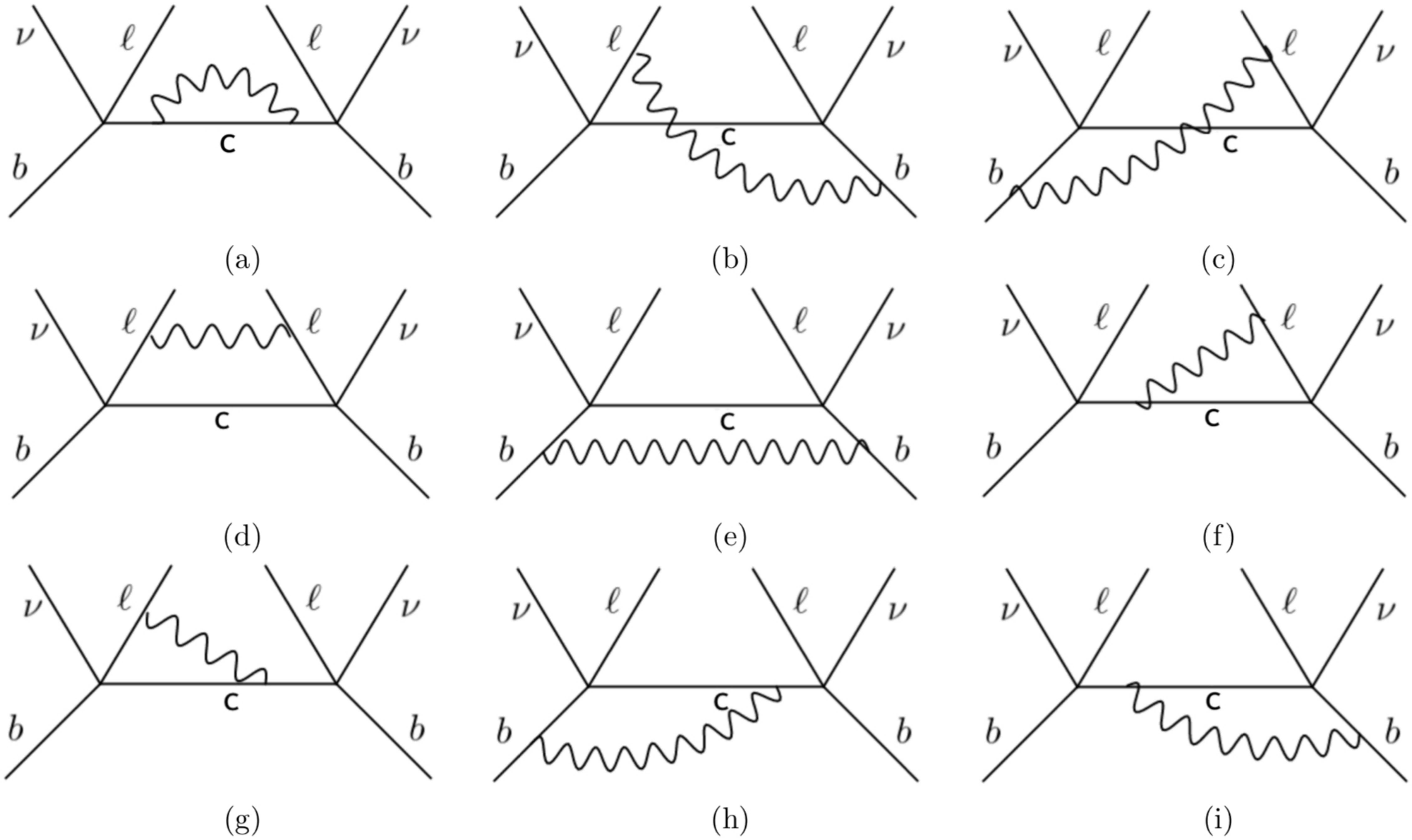
Where, $a(\mu) = 1 + \mathcal{O}(\alpha_s(m_b))$

No first principle method is available to calculate heavy quark parameters λ_1 and λ_2.

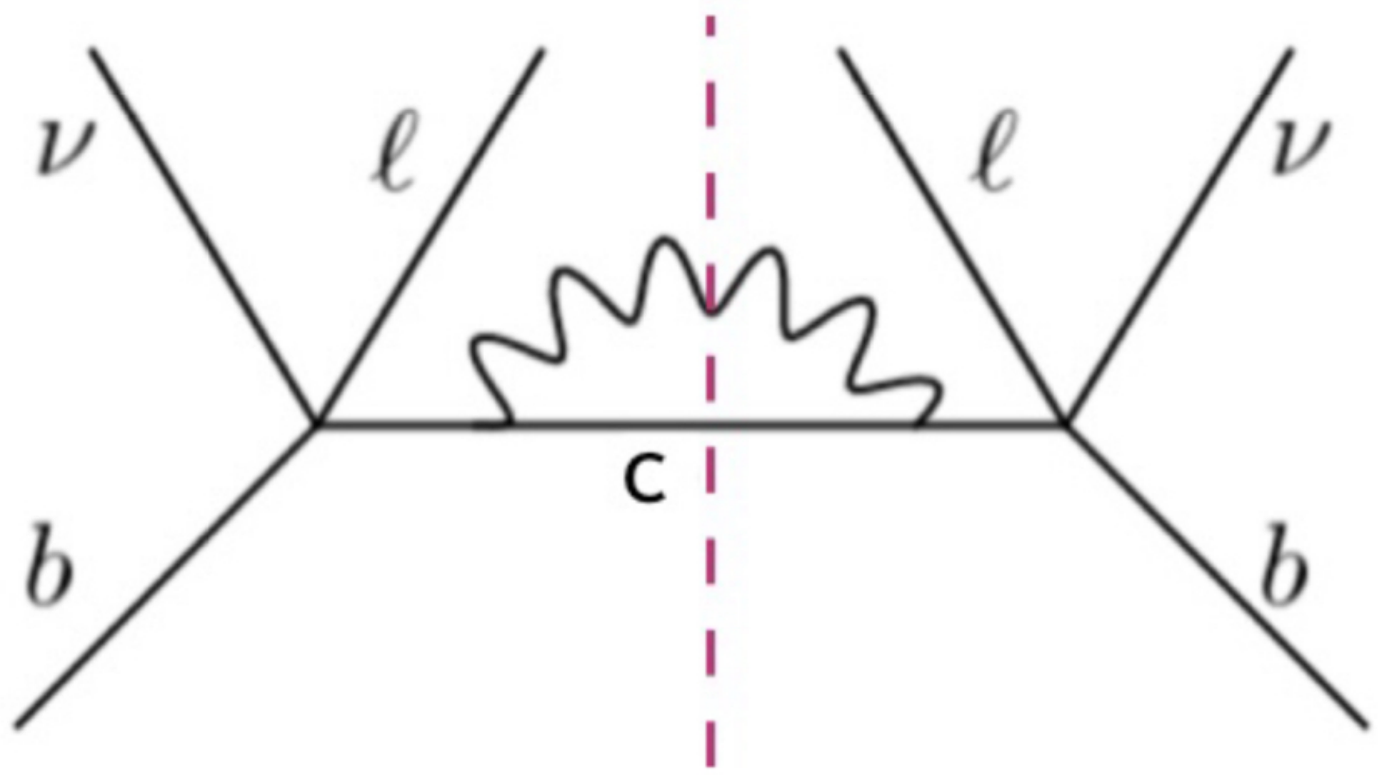
$B \to X_c \ell \nu_\ell \gamma$

- The Relevant Feynman Diagrams are



- **Matrix element**

$$\mathscr{M} = \left(\frac{4G_F}{\sqrt{2}}\right)^2 |V_{cb}|^2 \frac{1}{2m_B} \int \frac{d^3p_l}{(2\pi)^3 2E_l} \int \frac{d^3p_n}{(2\pi)^3 2E_\nu} \int \frac{d^4k}{(2\pi)^4} \langle B | I_m \mathscr{M}^{(m)}_{\mu\nu} \mathscr{L}^{(m)}_{\mu\nu} | B \rangle$$



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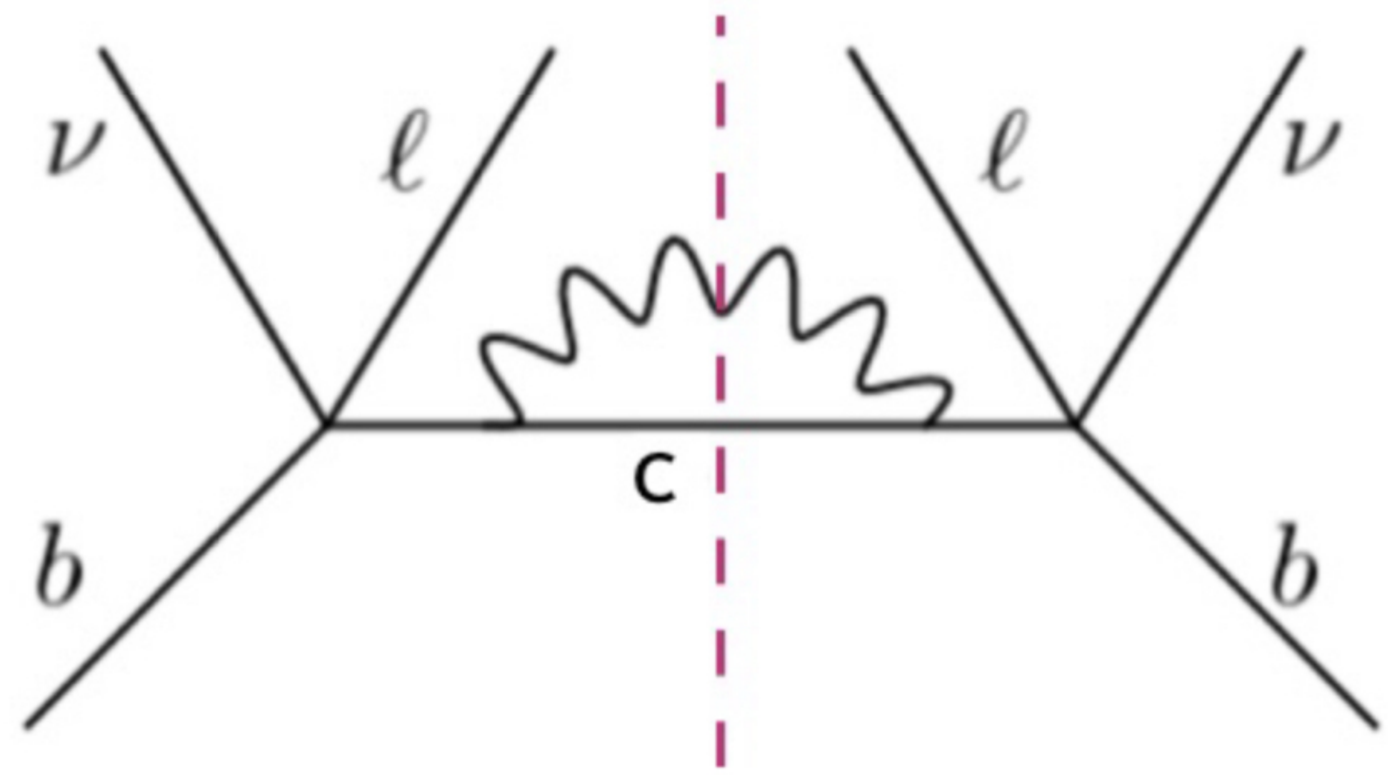

- Numerator part

$$\mathscr{M}^{(1)}_{\mu\nu} = 2(-ig^{\alpha\beta})\bar{b}\gamma^\nu \left(1-\gamma^5\right) i\gamma\,.\left(p_b + \Pi - q\right)(-ieQ_u)\gamma^\alpha i \left(\gamma\,.(p_b + \Pi - k - q) + m_c\right)$$
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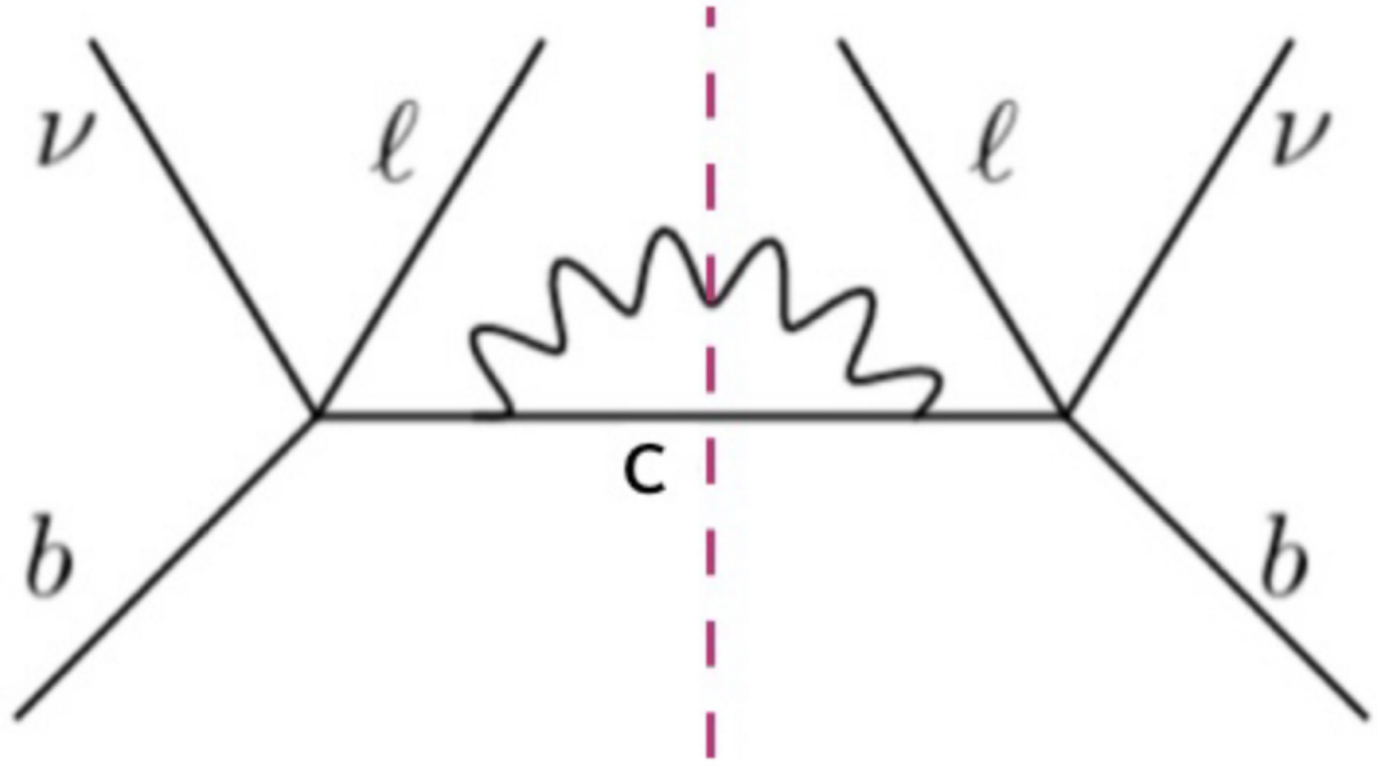

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$$\frac{1}{k^2\left((p_b+\Pi-q)^2-m_c^2\right)\left((p_b+\Pi-q-k)^2-m_c^2\right)\left((p_b+\Pi-q)^2-m_c^2\right)} = \frac{1}{k^2\left((p_b-q-k)^2-m_c^2\right)}\left[\frac{1}{(p_c\,.k)^2} - \frac{2(p_b-q)\,.\Pi}{(p_c\,.k)^3} - \frac{\Pi^2}{(p_c\,.k)^3} + \frac{2\left((p_b-q)\,.\Pi\right)^2}{(p_c\,.k)^4}\right]$$
$$-\frac{1}{k^2\left((p_b-q-k)^2-m_c^2\right)^2}\left[\frac{2p_c\,.\Pi}{(p_c\,.k)^2} - \frac{4(p_c\,.\Pi)(p_b-q)\,.\Pi}{(p_c\,.k)^3} - \frac{\Pi^2}{(p_c\,.k)^2}\right] + \frac{1}{k^2\left((p_b-q-k)^2-m_c^2\right)^3}\frac{2(p_c\,.\Pi)^2}{(p_c\,.k)^2}$$

- **It is the consequence of expansion of denominator in the power of Π up to square order.**

Results

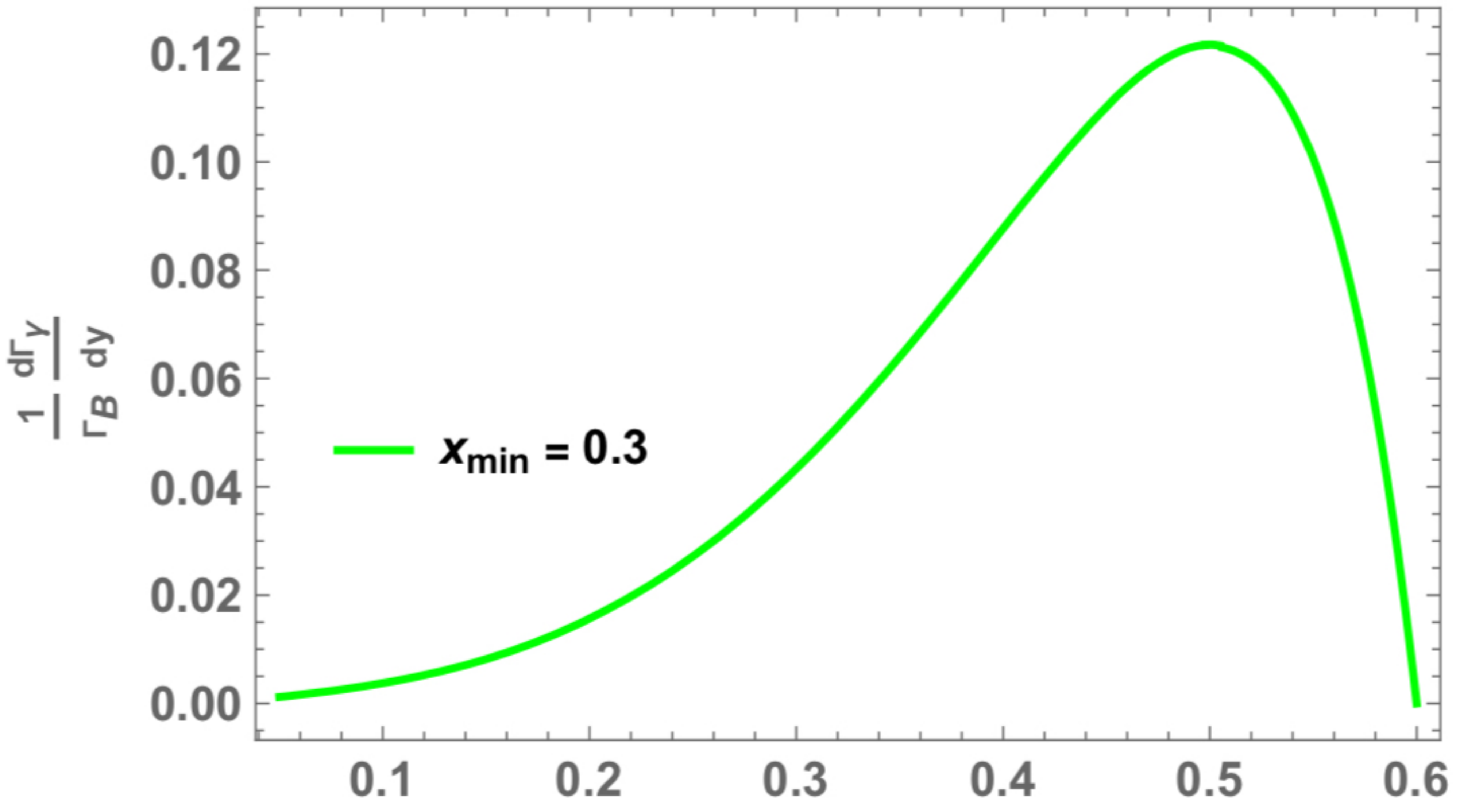

Figure : Plot of differential decay width ($B \to X_c \mu \bar{\nu} \gamma$) with lepton energy in the rest frame of B meson

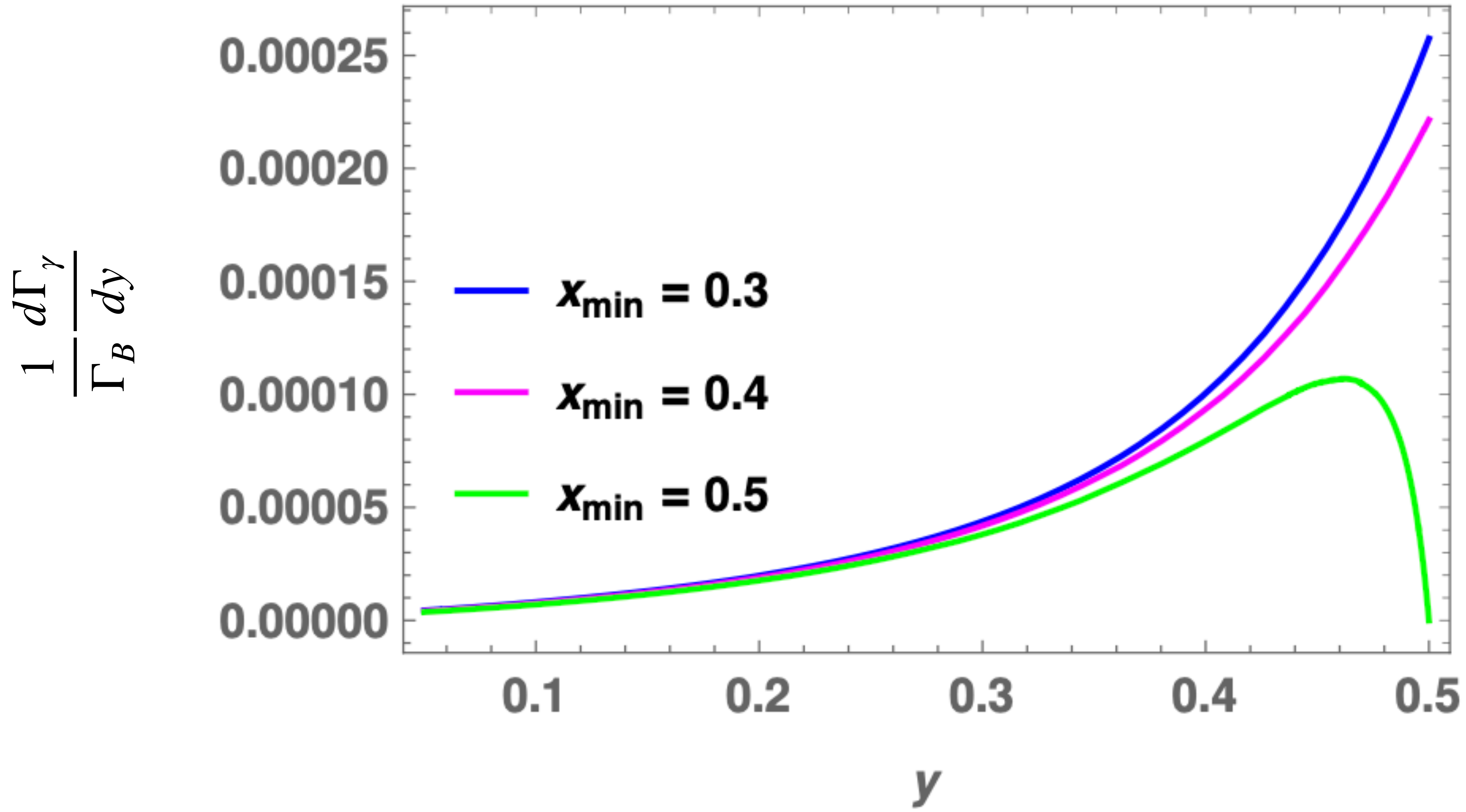

Figure : Plot of differential decay width ($B \to X_u \mu \bar{\nu} \gamma$) with lepton energy in the rest frame of B meson

- x_{min} define hardness of photon
- Decay rate is sensitive to energy of photon.
- As photons become soft, decay rate increases as expected.

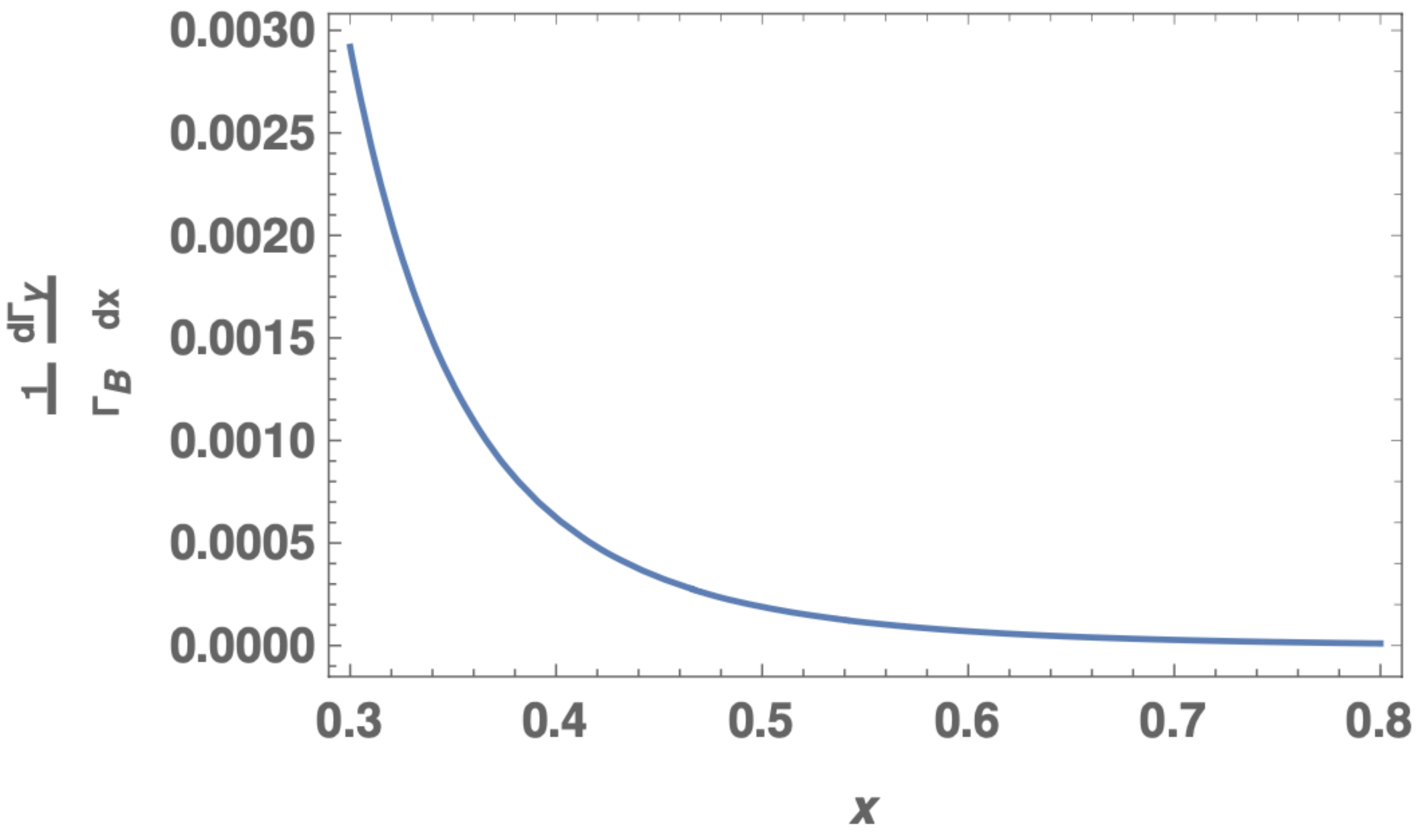

Figure : Plot of differential decay width with photon energy in the rest frame of B meson

- For hard photon : decay rate $\sim \mathcal{O}(\alpha_{em})\Gamma_{B\to X_u \ell \nu_\ell}$
- For sufficient low photon the rate for radiative mode reaches to non-radiative mode

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- Considering higher order in HQE will provide calculation of other non-perturbative parameters.

Thank you

Backup

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  - What are the relevant operators
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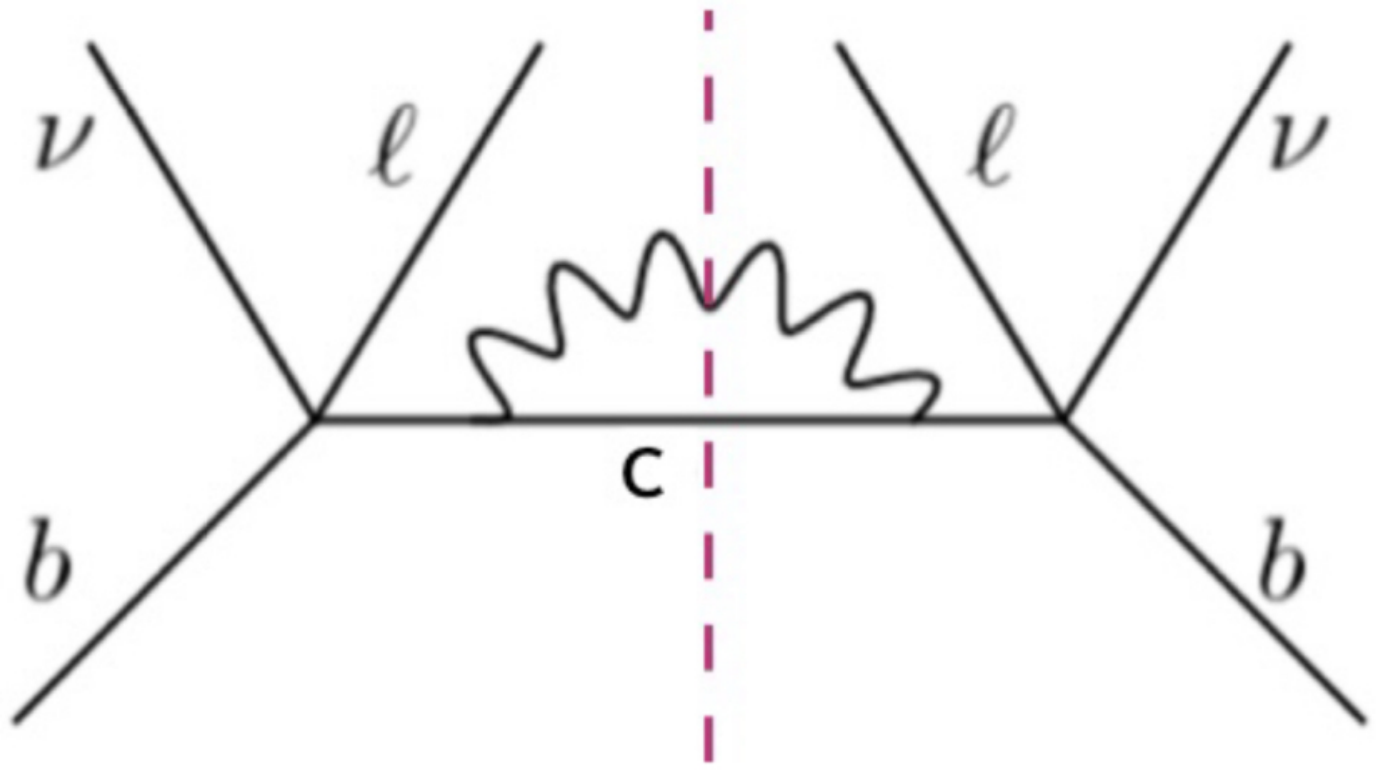
- General form of decay width for $B \to X_c \ell \nu_\ell \gamma$

$$\Gamma(B \to X \ell \nu_\ell \gamma) = \Gamma(b \to q \ell \nu_\ell \gamma) + \mathcal{O}\left(\frac{\Lambda_{QCD}}{m_b}\right) + \ldots$$

- Analysed the gauge invariance for $B \to X_c \ell \nu_\ell \gamma$ ➡ No contact term arises

$\Longrightarrow$ Free from UV divergences problem appear in exclusive decays.

- Cutkosky Rule
  - cut divides diagram into four body decay processes.



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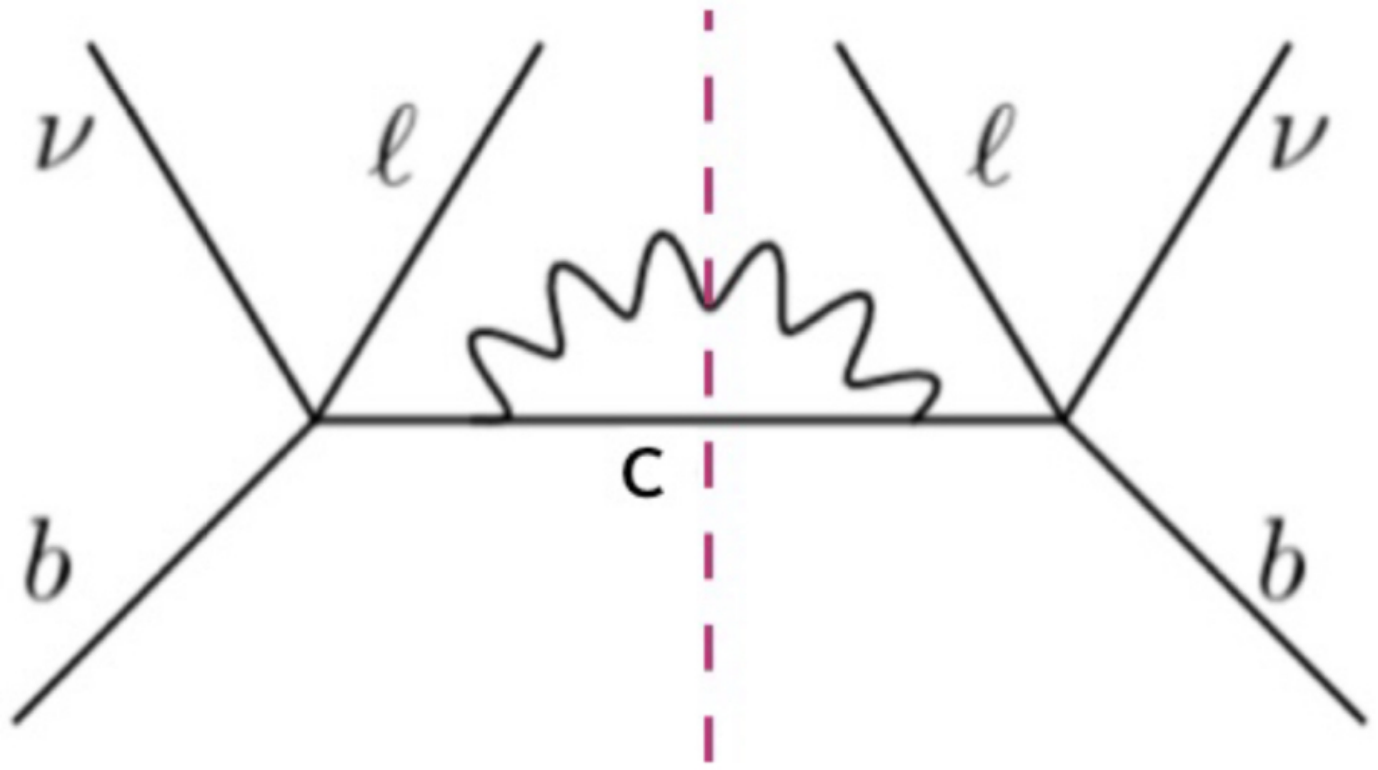

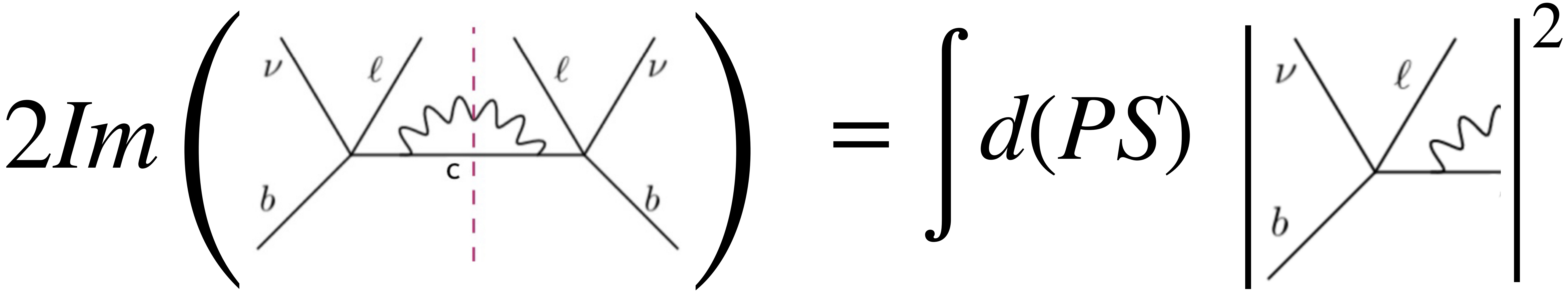

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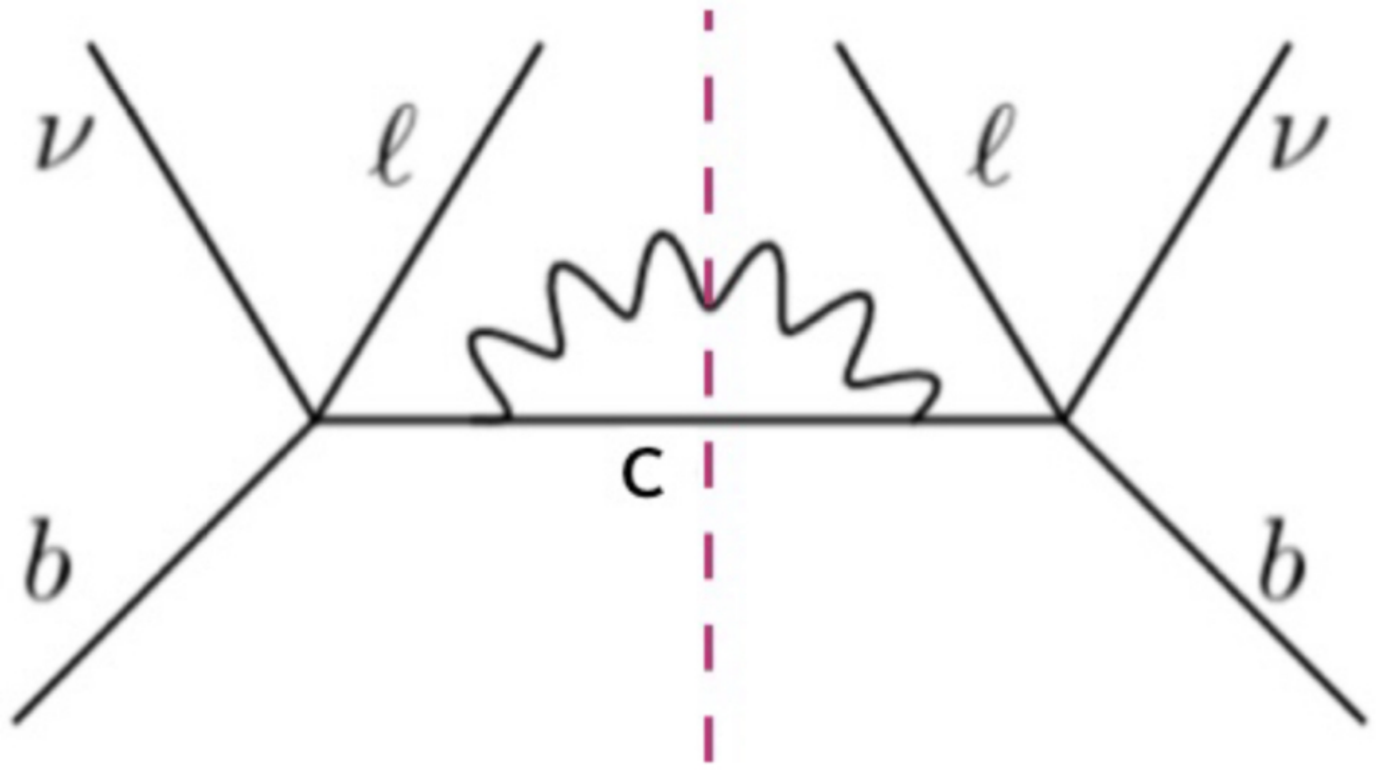
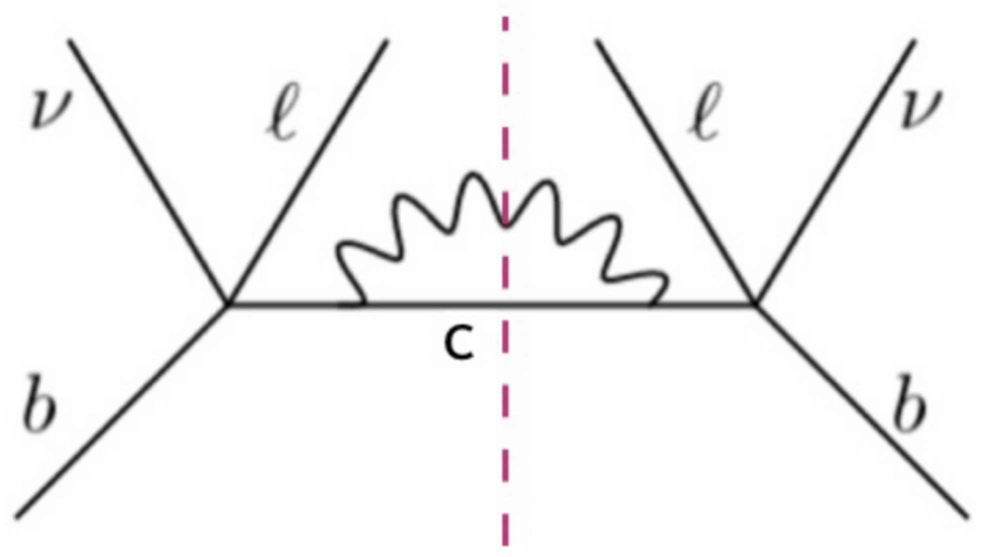
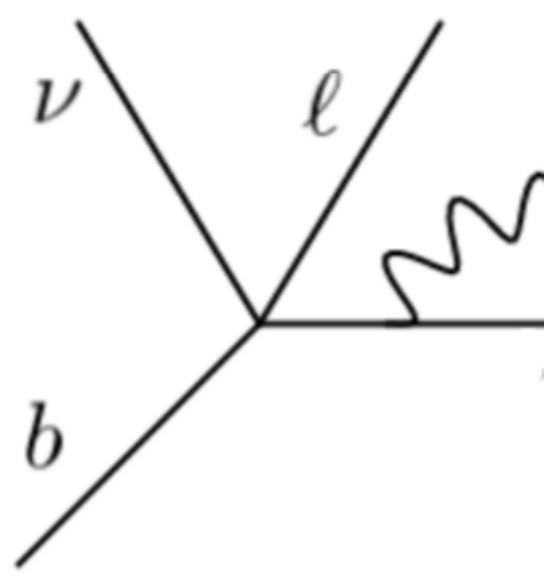


$2Im\Big($  $\Big) = \int d(PS)\, \Big|$  $\Big|^2$

- Expanding delta function in Π :

$$\delta(p_b + \Pi - q - k)^2 = \delta(p_b - q - k)^2 + 2\Pi\,.\,(p_b - q - k)\delta'(p_b - q - k)^2 + \Pi^2\Big(\delta'(p_b - q - k)^2 + 2(p_b - q - k)^2\delta''(p_b - q - k)^2\Big) + \dots$$

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$$\mathbf{q} = \frac{\lambda(q^2, r^2, m_b^2)}{4m_b^2} \qquad q^0 = \frac{m_b^2 + q^2 - r^2}{2m_b}$$ $\implies$ Provides neutrino energy E_ν