Correlating $0\nu\beta\beta$ decays and flavor observables in leptoquark models

Luighi P. S. Leal (University of São Paulo, University Paris-Saclay)

In colaboration with Svjetlana Fajfer, Olcyr Sumensari and Renata Z. Funchal

[arXiV: 2406.20050]

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Overview

- **Introduction**
  - Neutrino oscillations
  - Neutrinoless double-beta decay
- **Our Framework**
  - Scalar Leptoquark models
  - Neutrino masses
  - Neutrinoless double-beta decay
- **Phenomenology**
- **Results**
- **Conclusions**

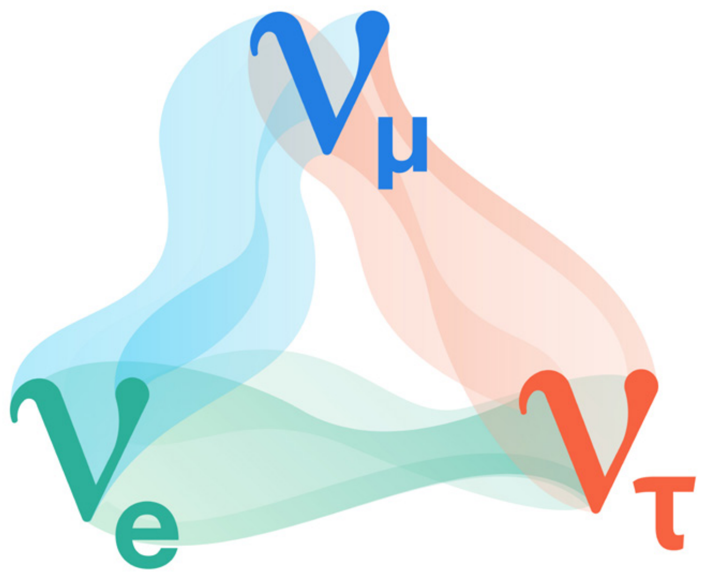
Neutrino Oscillations

❖ **Mass and flavor states:**

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

PMNS matrix

Neutrino Oscillations



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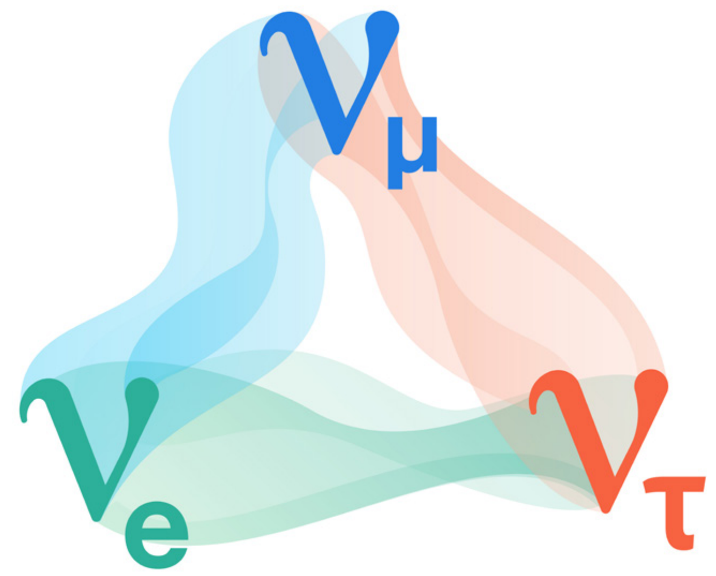
$$\nu_\alpha = \sum_i U_{\alpha i} \nu_i$$

$$\nu_\alpha(L) = \sum_i U_{\alpha i} \nu_i \, e^{-i\left(p + \frac{m_i^2}{2p}\right)L}$$

Neutrino Oscillations



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$$\Delta m^2_{ij} = m_i^2 - m_j^2$$

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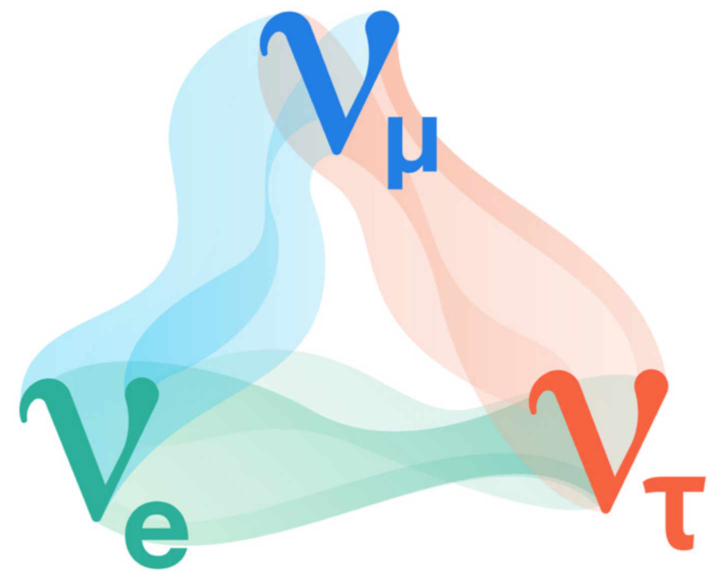
- Neutrinos are massive particles!!

$$\Delta m^2_{ij} = m_i^2 - m_j^2$$

Open problems in neutrino physics



- Neutrinos are the least known particles in SM:
  - Normal or inverted **ordering**?

$$\Delta m_{ij}^2 = m_i^2 - m_j^2$$

Experimental results: $\Delta m_{21}^2 \simeq 7 \times 10^{-5}\,\mathrm{eV}^2$

$$m_2 > m_1$$

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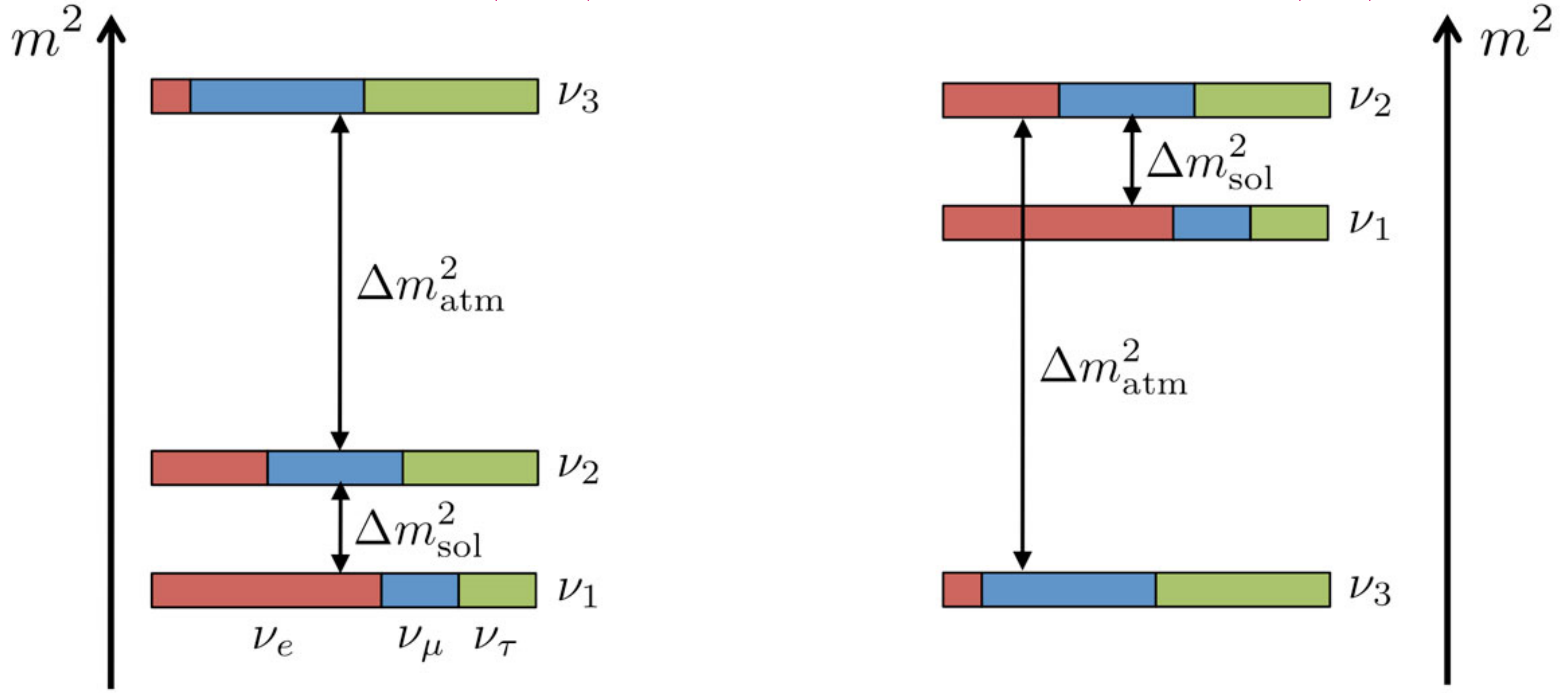
$$m_3 > m_2 > m_1$$

or

$$m_2 > m_1 > m_3$$

Normal Ordering (NO) **Inverted Ordering (IO)**



[NuFIT 5.3 (2024), www.nu-fit.org.]

Open problems in neutrino physics

- Neutrinos are the least known particles in SM:
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  - What is the **absolute scale** of neutrino masses?

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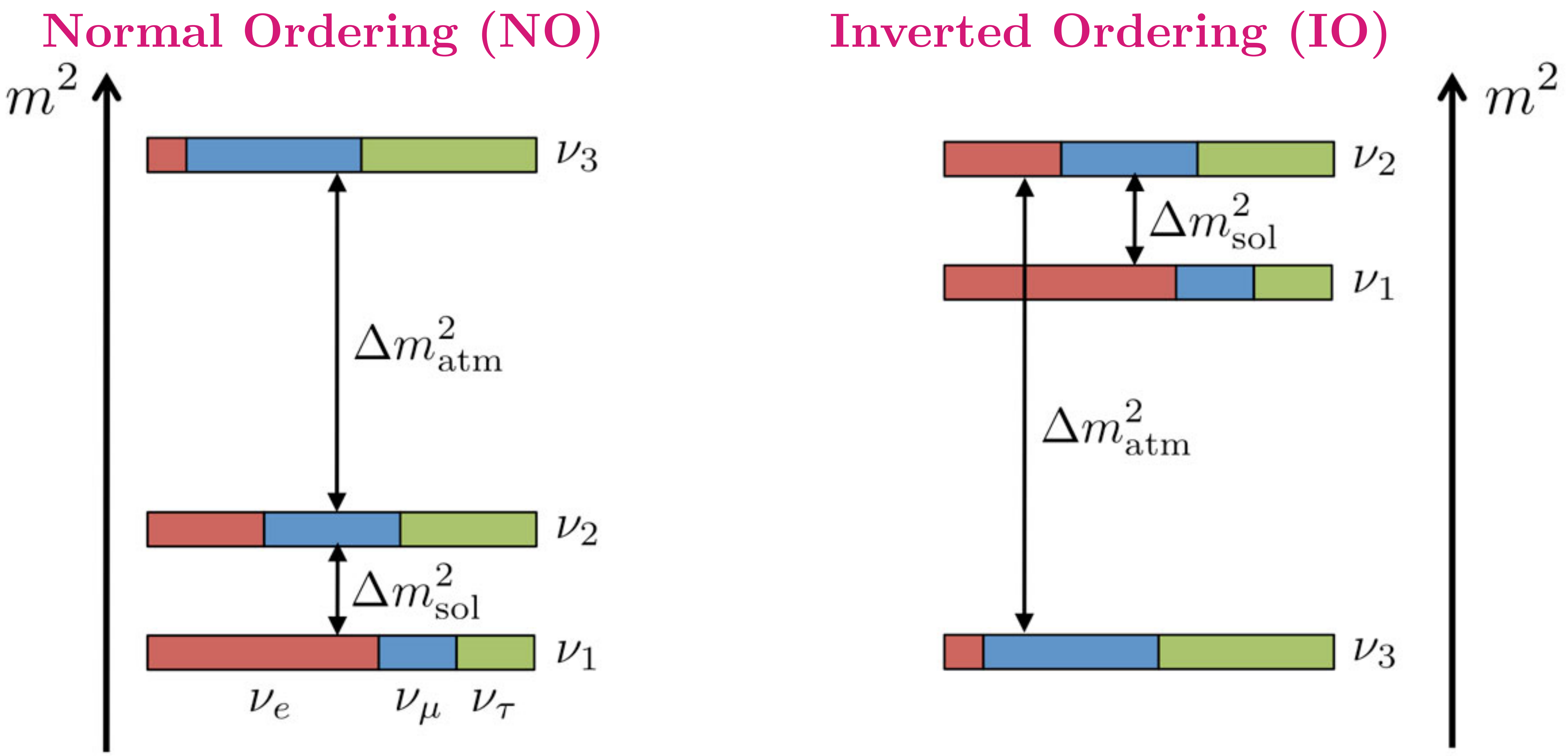

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  - **Majorana or Dirac** particles? (Is **lepton number** conserved?)

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  - What is the **absolute scale** of neutrino masses?
  - **Majorana or Dirac** particles? (Is **lepton number** conserved?)
  - Is **CP violated** in leptonic sector?

- 3 mixing angles
- 1 CP phase
- 2 Majorana phases

$$\nu_\alpha = \sum_i U_{\alpha i} \nu_i$$

$$U_{\text{PMNS}} \equiv U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$s_{ij} = \sin\theta_{ij} \qquad c_{ij} = \cos\theta_{ij}$$

Open problems in neutrino physics

❖ Neutrinos are the least known particles in SM:

- Normal or inverted **ordering**?
- What is the **absolute scale** of neutrino masses?
- **Majorana or Dirac** particles? (Is **lepton number** conserved?)
- Is **CP violated** in leptonic sector?
- How **neutrino masses** are generated?

Huge experimental/theoretical effort to answer these questions and to measure precisely the PMNS matrix

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Neutrino masses

- **Neutrinos are massive and oscillate** → Evidence of Physics Beyond Standard Model (BSM)

- **Standard Model effective field theory (SMEFT)**

$$\mathscr{L}_{\text{SMEFT}} = \sum_{d\geq 5} \sum_{a} \frac{\mathscr{C}_a^{(d)}}{\Lambda^{d-4}} \mathscr{O}_a^{(d)}$$

Operators invariant under $SU(3)_C \times SU(2)_L \times U(1)_Y$

EFT cutoff

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EFT cutoff

- **Neutrino masses described by the Weinberg operator (d = 5)**

$$\mathscr{L}^{d=5} = \frac{C_{ij}^{(5)}}{\Lambda} (\overline{L_i^C} \tilde{H}^*)(\tilde{H}^\dagger L_j) + h.c. \rightarrow (m_\nu)_{ij} = \frac{C_{ij}^{(5)} v^2}{\Lambda}$$

Lepton Number (L) is broken

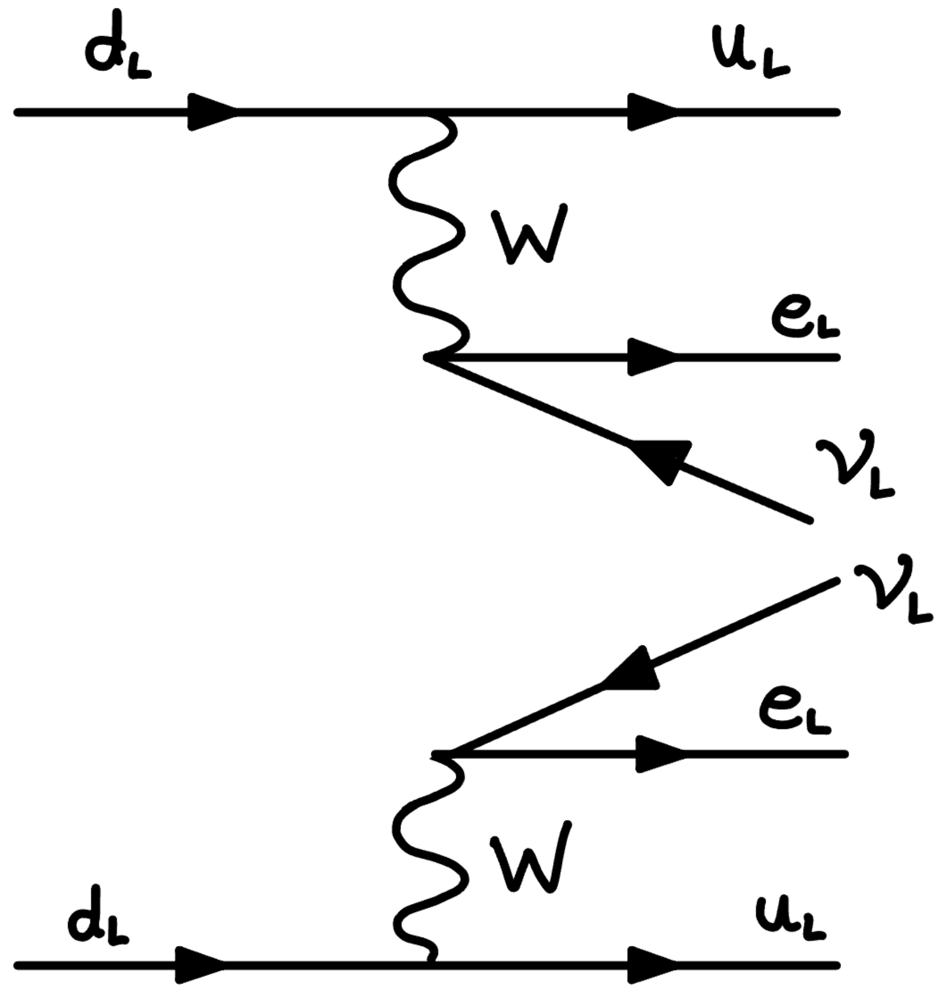
Majorana Neutrino Masses

[S. Weinberg, Phys. Rev. Lett. 43]

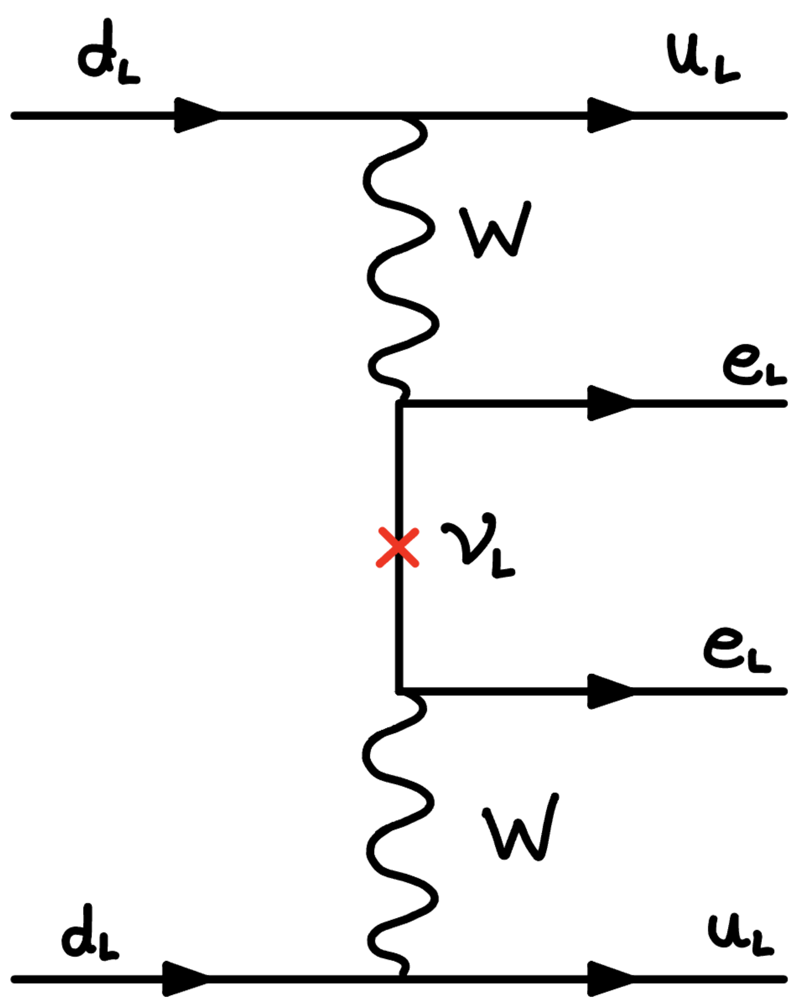
Lepton Number Violation (LNV)

❖ Neutrinoless double-beta decay ($0\nu\beta\beta$)

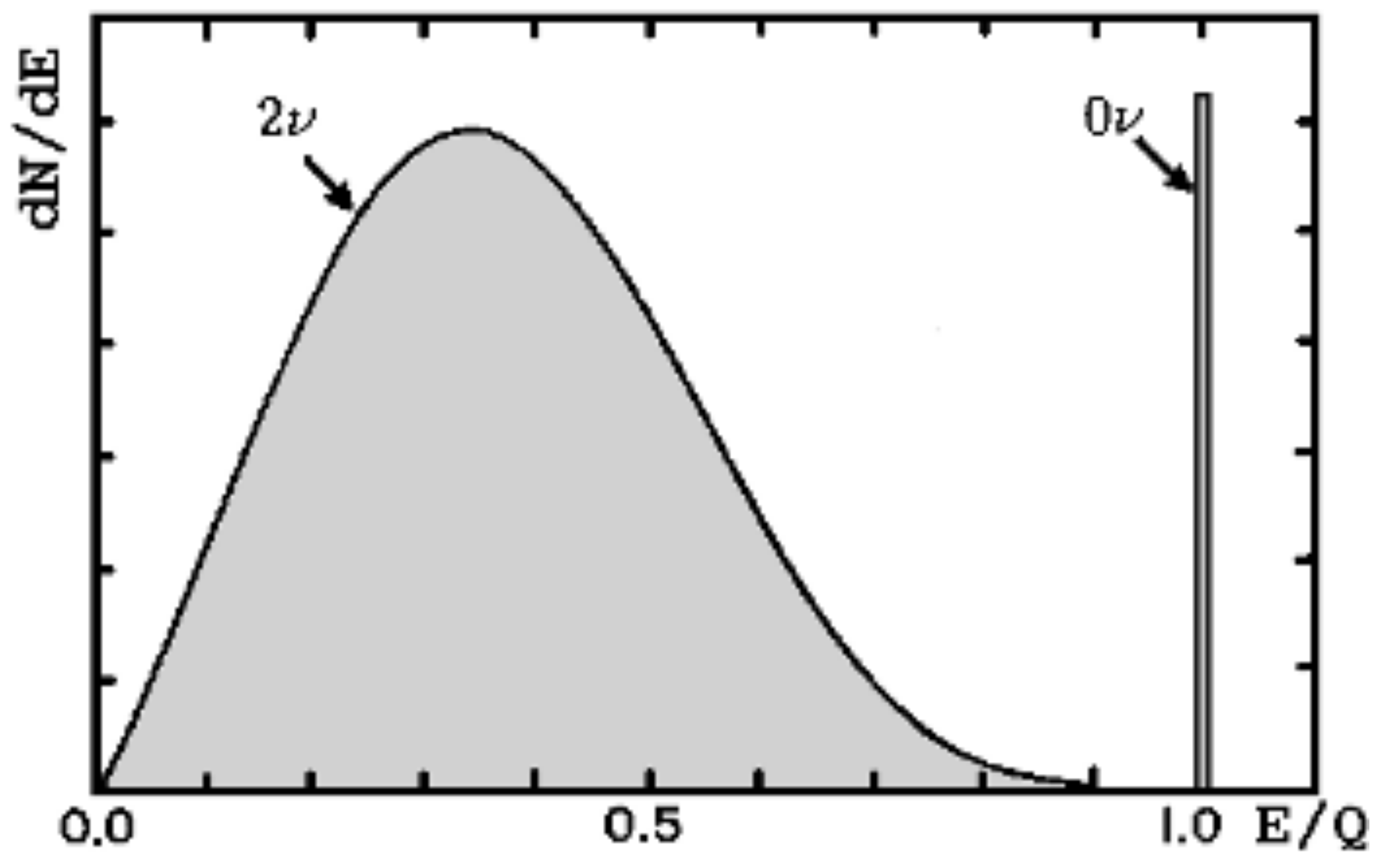
Induced by $\mathbf{d = 5}$ **operator**



VS



Experimental searches



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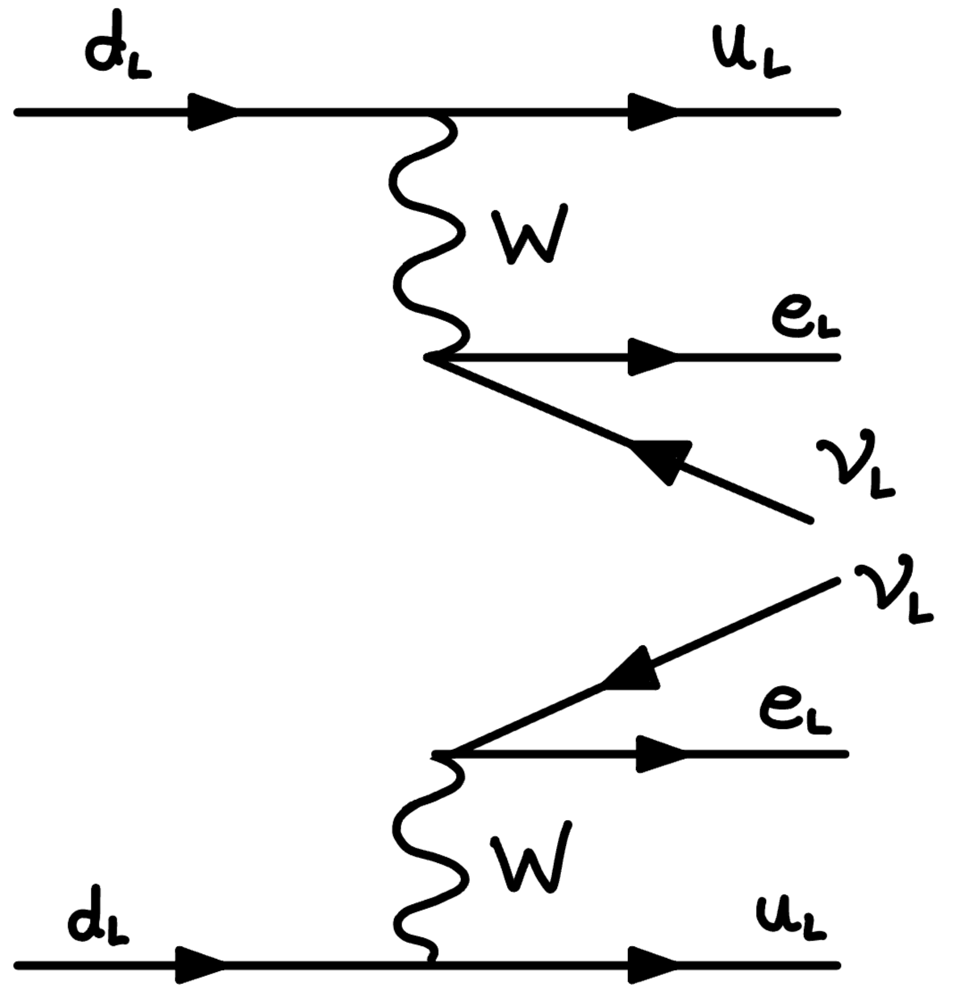
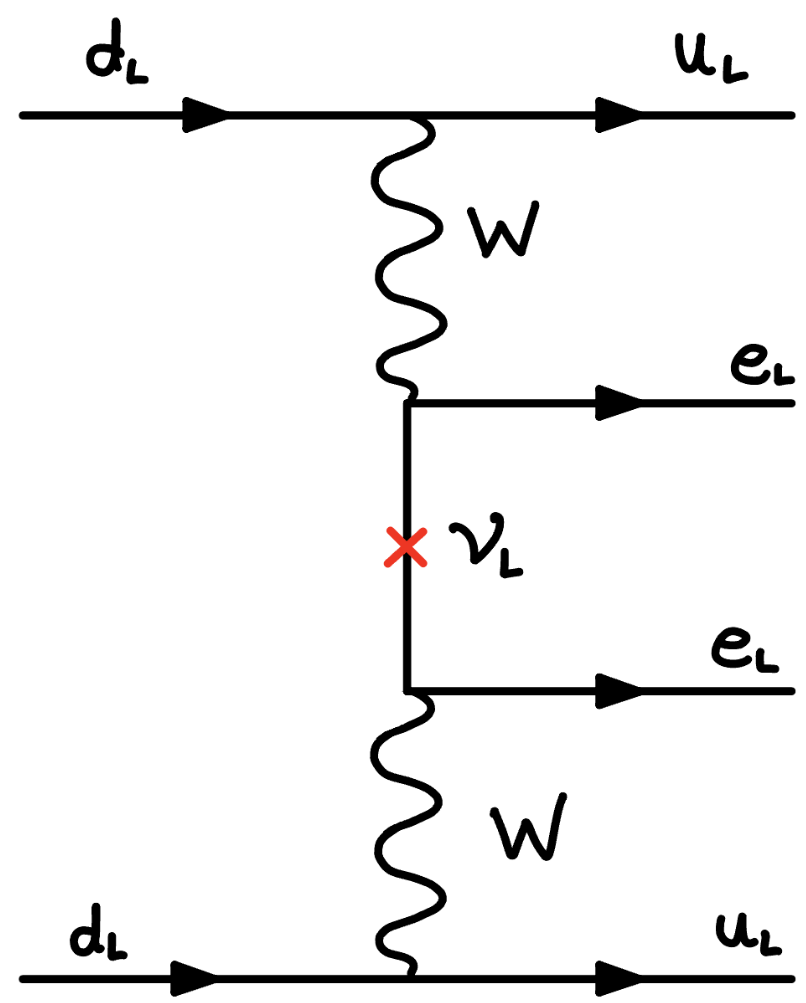


vs



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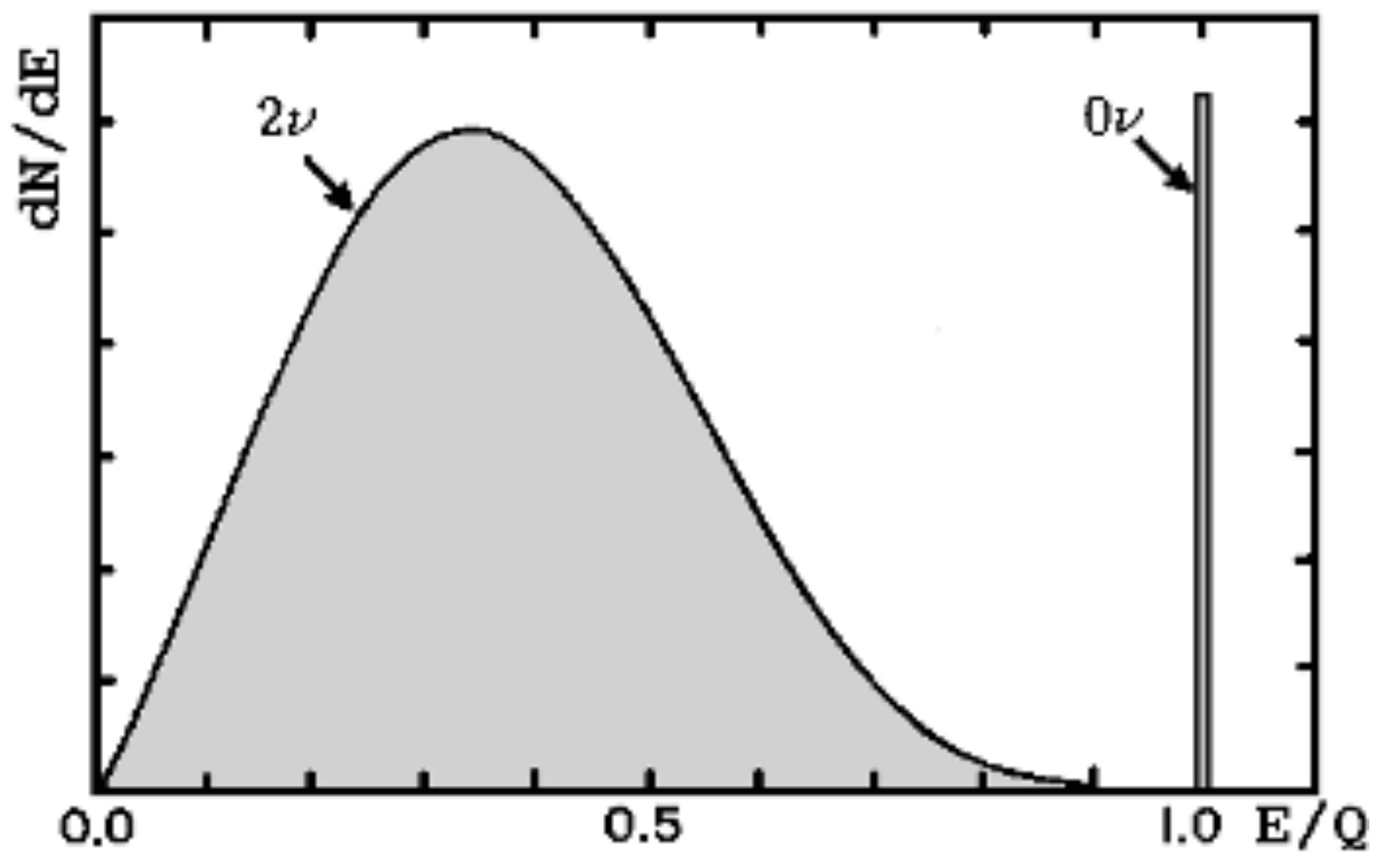


Half-life

$$\left(T^{0\nu}_{1/2}\right)^{-1} = \frac{g_A^4 G_{01}}{m_e^2} \, |m_{\beta\beta}|^2 \, |\mathscr{M}_\nu^{(3)}|^2$$

with $m_{\beta\beta} = \sum_i m_{\nu_i} U_{ei}^2$

Phase space factor

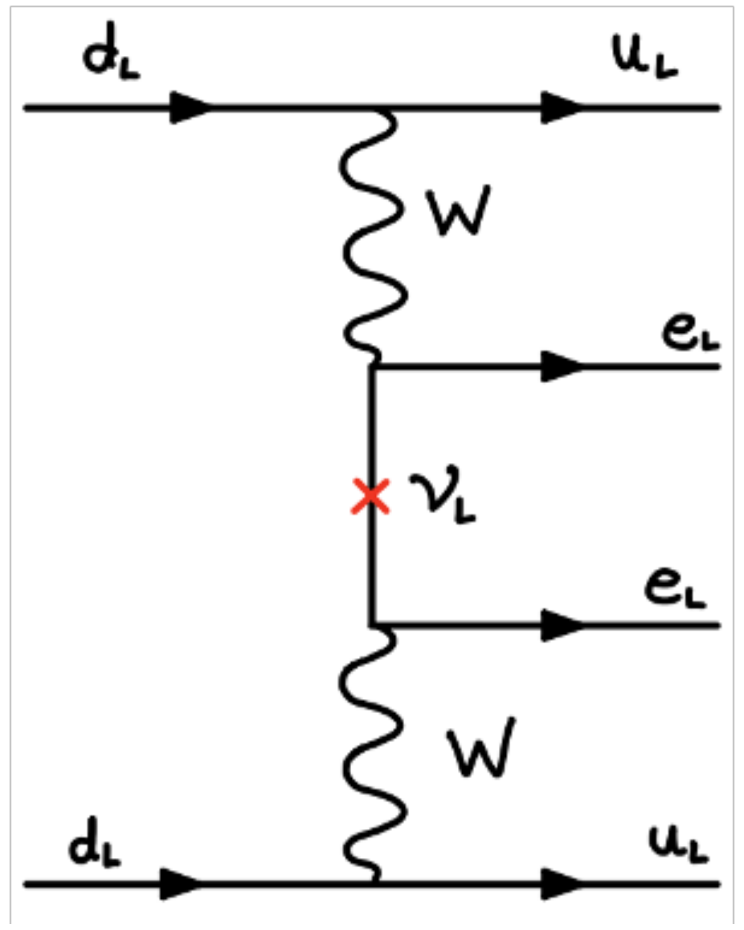
Nuclear Matrix Element

PMNS

Lepton Number Violation (LNV)

❖ Concrete BSM models for neutrino masses can induce additional contributions to $0\nu\beta\beta$:

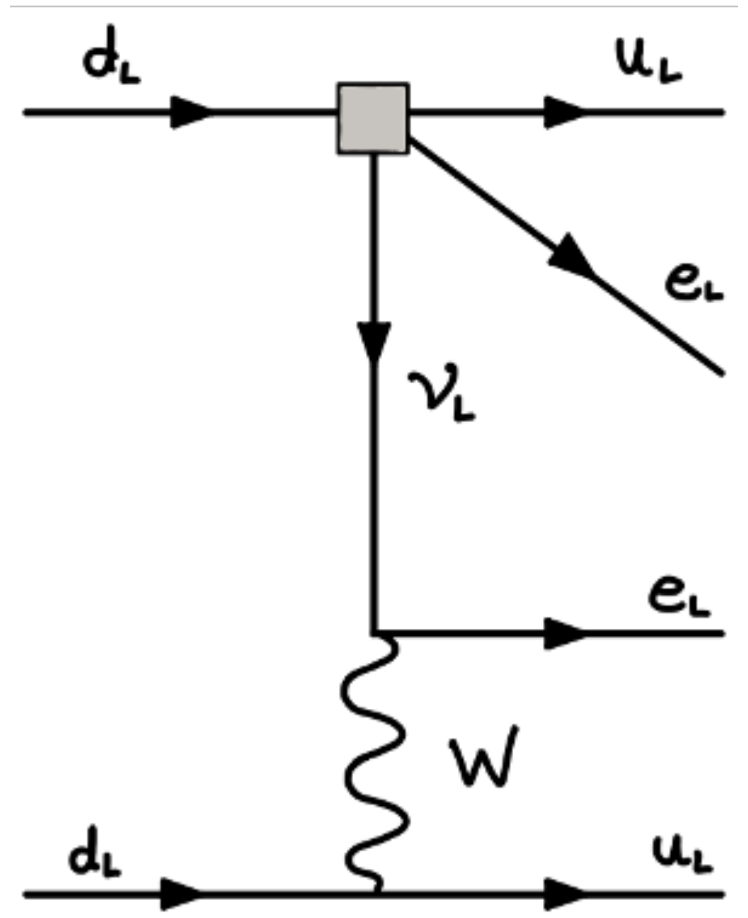
(d = 5)



($\propto m_\nu$)

+

(d > 5)



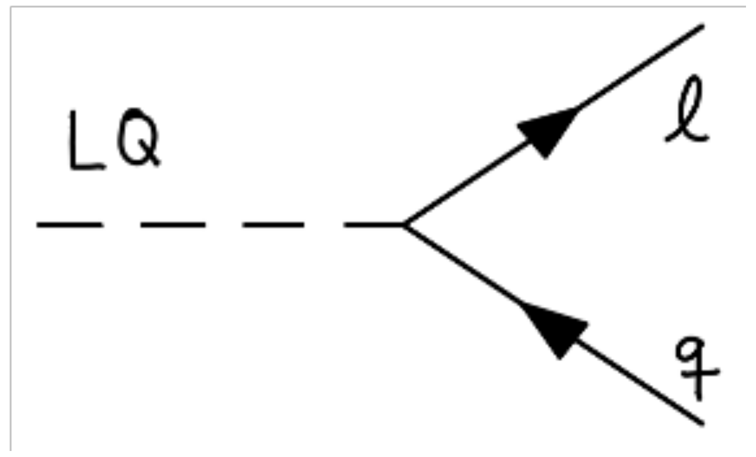
Are d > 5 operators suppressed with respect to d = 5?

❖ We are going to explore these effects in a minimal Leptoquark (LQ) model

[V. Cirigliano et al, JHEP 12]

$SU(3)_C \times SU(2)_L \times U(1)_Y$

Leptoquarks and $0\nu\beta\beta$



- Leptoquarks: colored particles that couple to quarks and leptons
- Models: $S_3 \sim (\overline{3},3,1/3)$ and $\tilde{R}_2 \sim (3,2,1/6)$ or $S_1 \sim (\overline{3},1,1/3)$ and $\tilde{R}_2 \sim (3,2,1/6)$

[C. K Chua et al. Phys. Lett. B, 479]
[U. Mahanta Phys. Rev D, 62]

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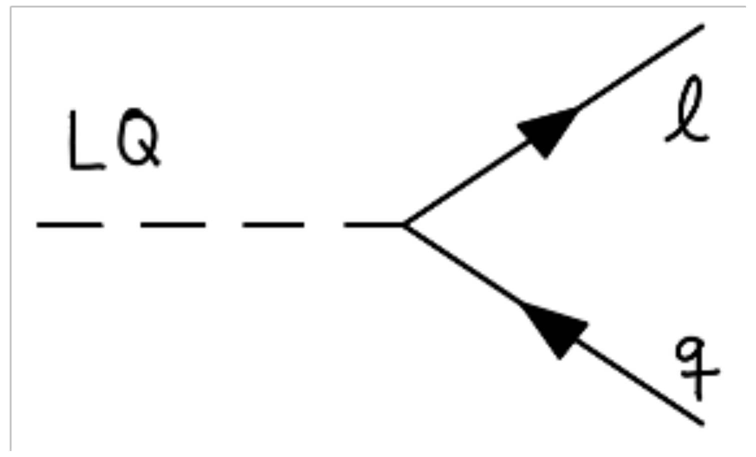

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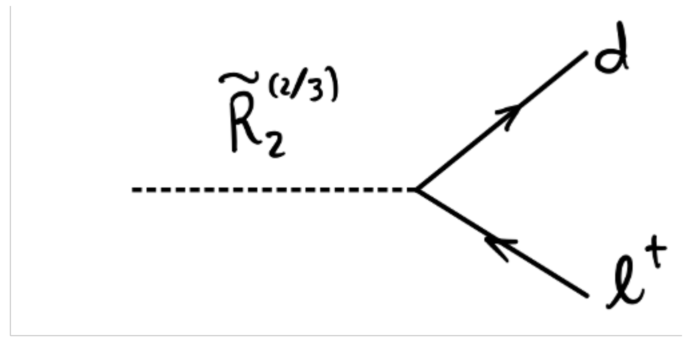
$$\mathscr{L}_{\tilde{R}_2\&S_3} \supset (D_\mu \tilde{R}_2)^\dagger (D_\mu \tilde{R}_2) + (D_\mu S_3^a)^\dagger (D_\mu S_3^a) - m^2_{\tilde{R}_2} \tilde{R}_2^\dagger \tilde{R}_2 - m^2_{S_3} S_3^{a\,\dagger} S_3^a$$

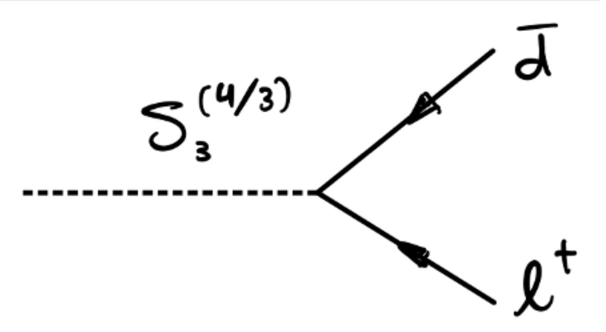
$$y_{3L}^{ij} \overline{Q_i^C} i\tau_2 (\tau^a S_3^a) L_j - y_{2L}^{ij} \bar{d}_{Ri} \tilde{R}_2 i\tau_2 L_j - \lambda_3 \tilde{R}_2^\dagger (\tau^a S_3^a)^\dagger H + \mathrm{h.c.}$$

F = 2

F = 0

Mixing induces Lepton Number Violation





$$\tilde{R}_2 = \begin{pmatrix} \tilde{R}_2^{(2/3)} \\ \tilde{R}_2^{(-1/3)} \end{pmatrix} \qquad S_3 = \begin{pmatrix} S_3^{(4/3)} \\ S_3^{(1/3)} \\ S_3^{(-2/3)} \end{pmatrix} \qquad Q = \begin{pmatrix} u \\ d \end{pmatrix}_L \qquad L = \begin{pmatrix} \nu \\ \ell \end{pmatrix}_L$$

[C. K Chua et al. Phys. Lett. B, 479]
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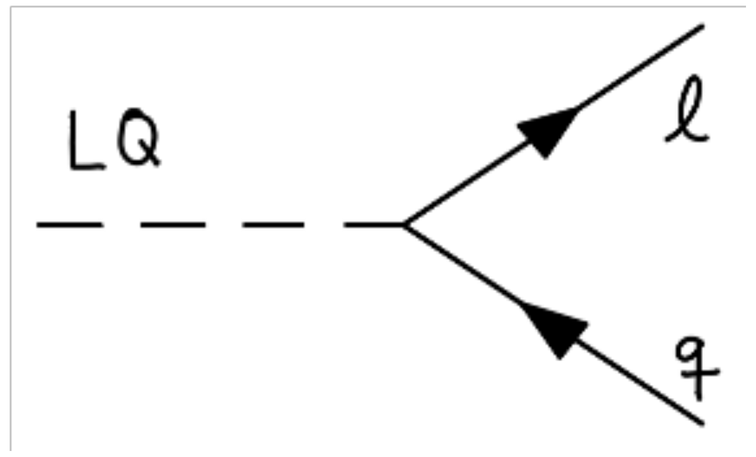
$SU(3)_C \times SU(2)_L \times U(1)_Y$

Leptoquarks and $0\nu\beta\beta$



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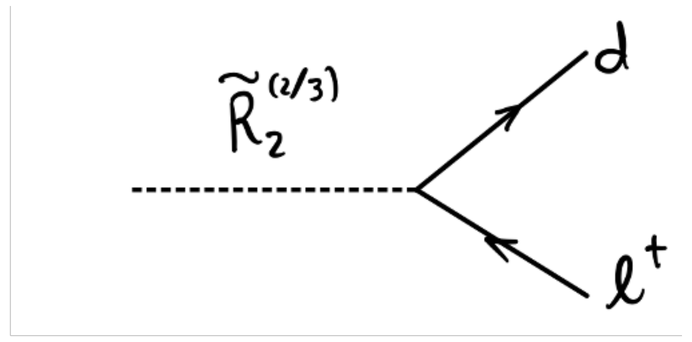

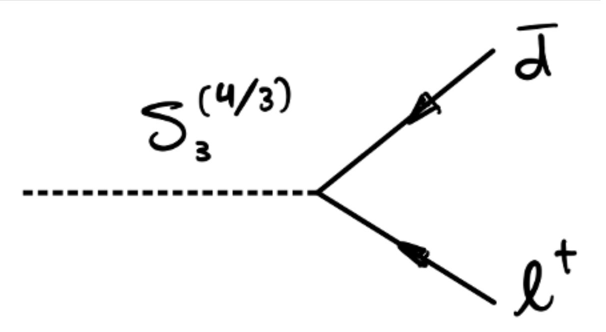

Mixing induces Lepton Number Violation

- Interaction to mass bassis:

$$f_L \to U_{f_L} f_L, \qquad f_R \to U_{f_R} f_R, \quad \text{where} \quad f \in \{\nu, \ell, d, u\}$$

$$V_{\text{CKM}} \equiv V = U_{u_L}^\dagger U_{d_L}$$
$$U_{\text{PMNS}} \equiv U = U_{\ell_L}^\dagger U_{\nu_L}$$

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[C. K Chua et al. Phys. Lett. B, 479]
[U. Mahanta Phys. Rev D, 62]

Neutrino masses

❖ Q = -1/3 components of LQs can mix to generate neutrino masses

$$\mathscr{L}_{\tilde{R}_2\&S_3} \supset (D_\mu \tilde{R}_2)^\dagger (D_\mu \tilde{R}_2) + (D_\mu S_3^a)^\dagger (D_\mu S_3^a) - m_{\tilde{R}_2}^2 \tilde{R}_2^\dagger \tilde{R}_2 - m_{S_3}^2 S_3^{a\,\dagger} S_3^a$$

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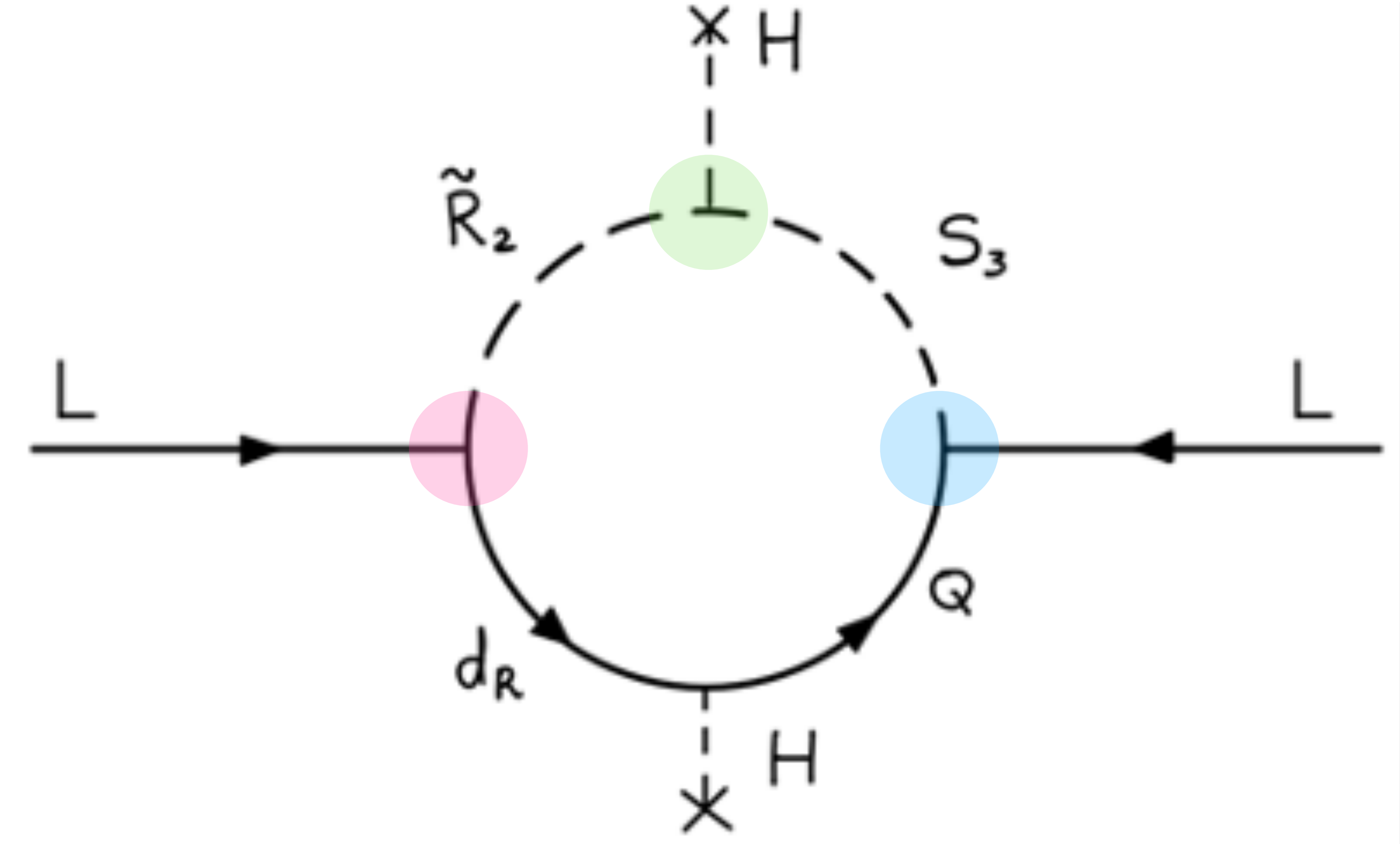
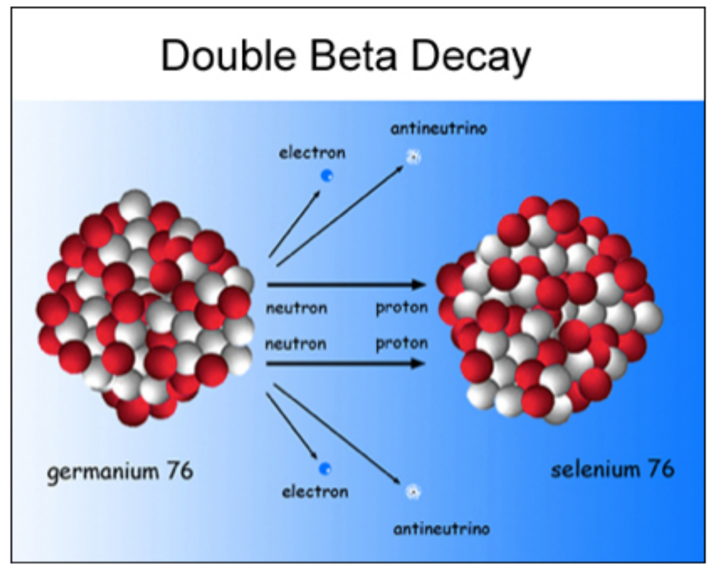


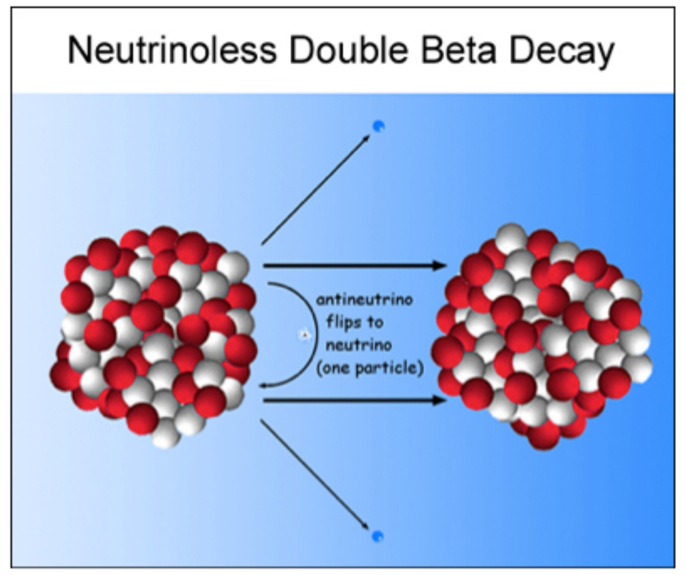
$$m_\nu \simeq -\frac{3\lambda_3}{16\sqrt{2}\pi^2} \frac{v^2}{M^2} \left(y_{2L}^T \cdot \hat{y}_d \cdot y_{3L} + y_{3L}^T \cdot \hat{y}_d \cdot y_{2L} \right)$$

$$m_{\tilde{R}_2} \sim m_{S_{1,3}} \sim M \gg v$$

[D. Aristizabal Sierra, M. Hirsch and S. G. Kovalenko, Phys. Rev. D 77]
[I. Dorsner, S. Fajfer and N. Kosnik, Eur. Phys. J. C 77]

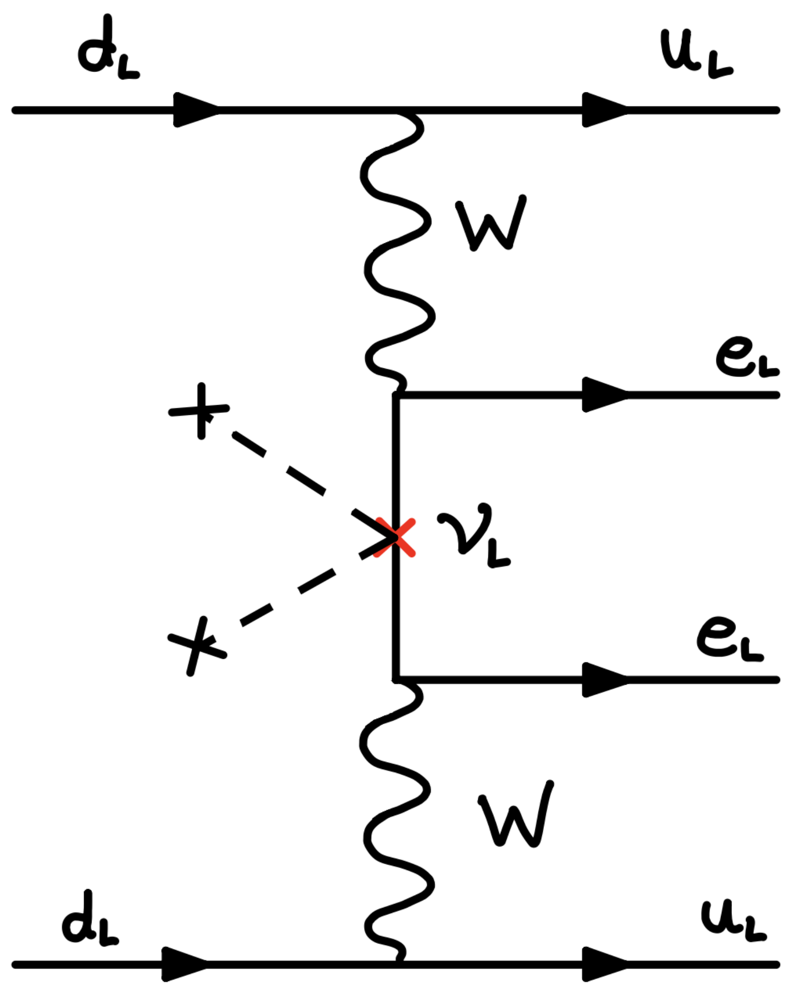
Neutrinoless double-beta decay





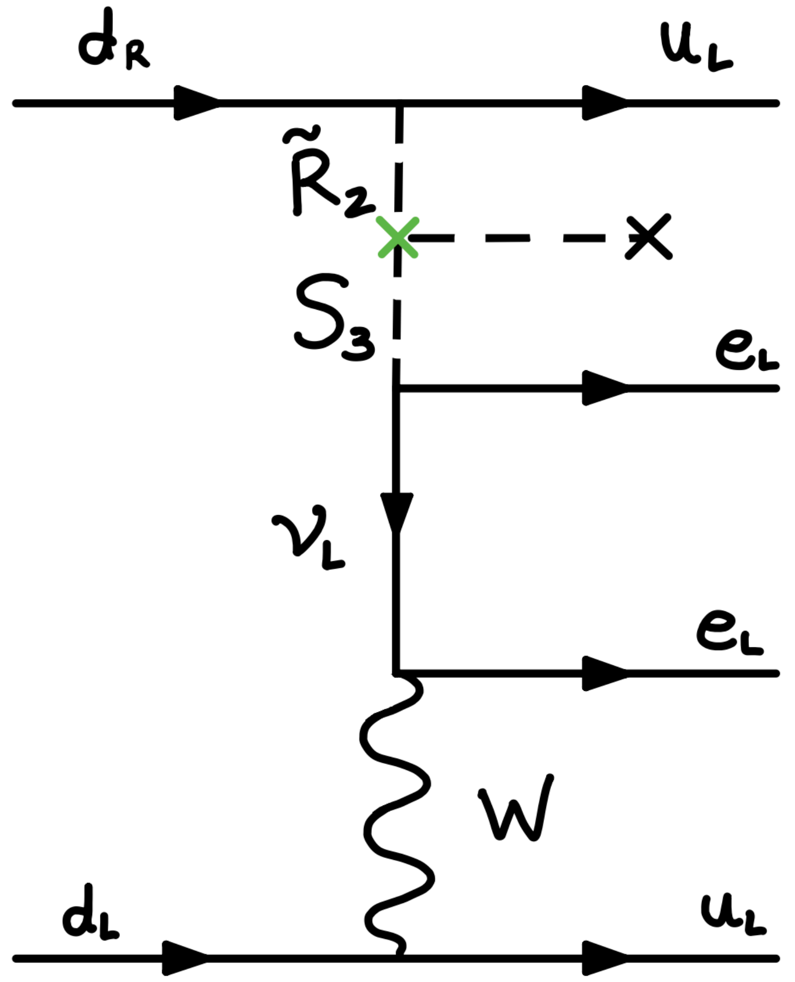
❖ Contributions from d = 5 and d = 7 effective operators

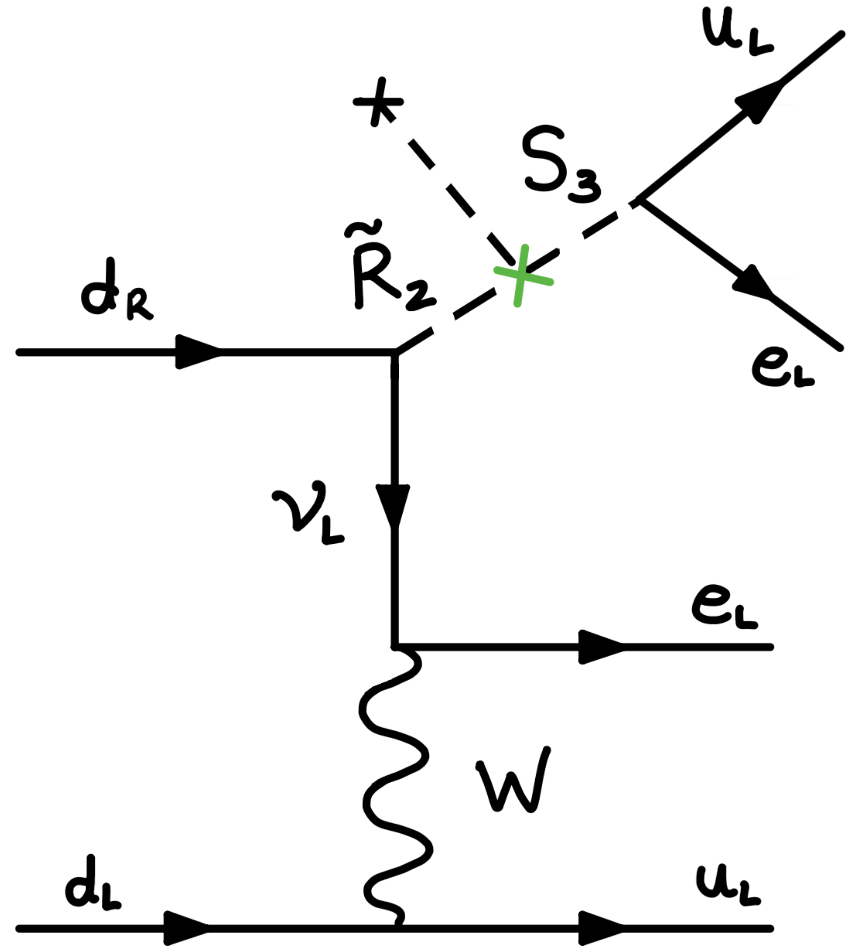
d = 5 contribution
(loop - level)



$(\propto m_\nu)$

d = 7 contributions
(Tree - level)

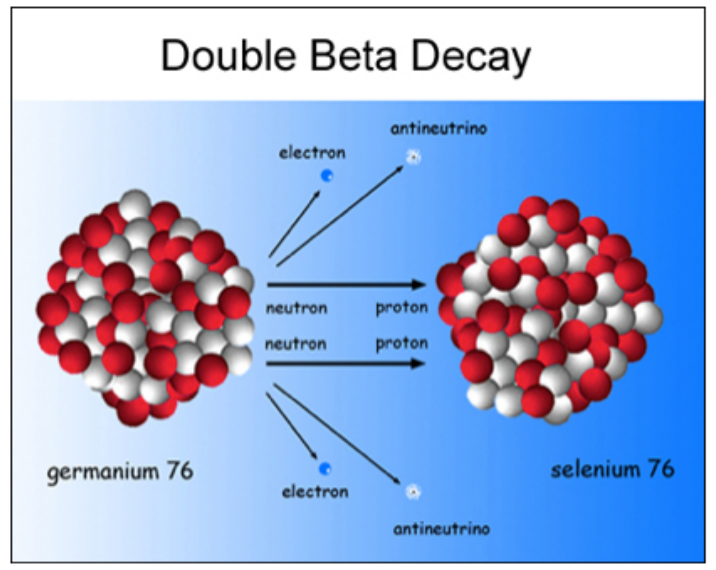


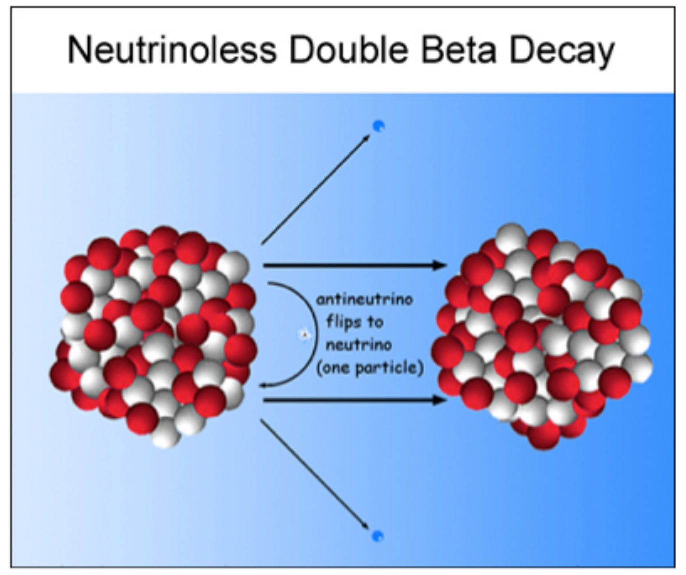


Chirality-enhanced ($\propto E/m_\nu$) contributions
to $0\nu\beta\beta$, with $E \sim 100$ MeV

$\underline{S_3 - \tilde{R}_2}$

Leptoquark mixing
through λ_3 breaks
lepton number

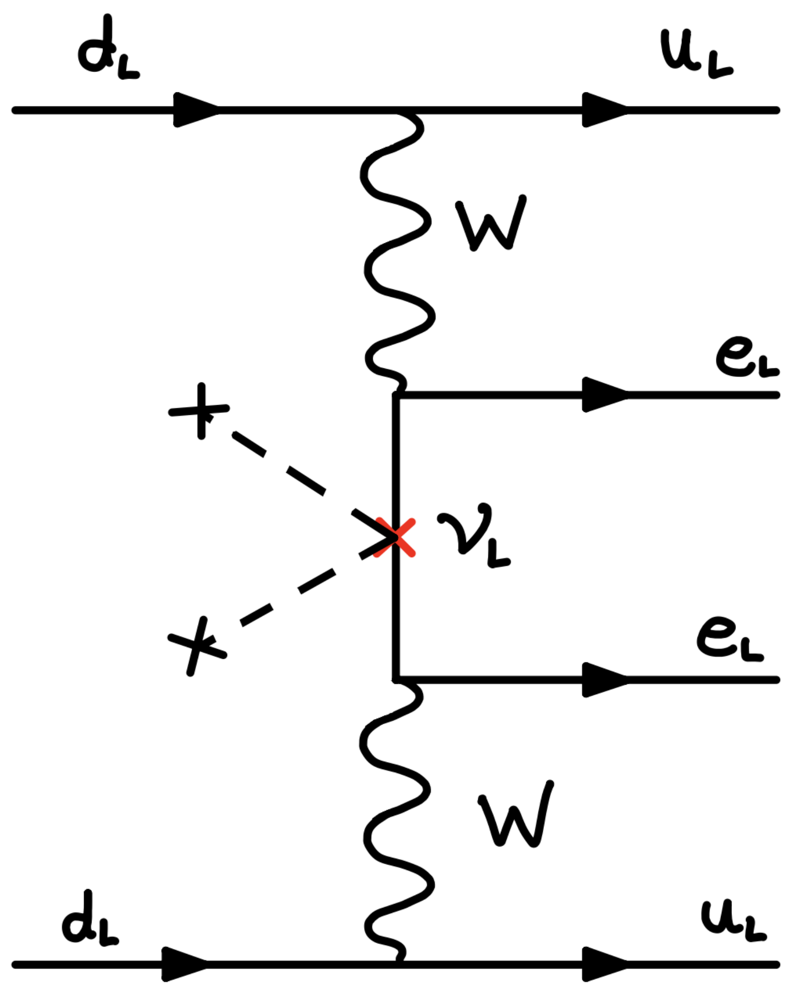




Neutrinoless double-beta decay

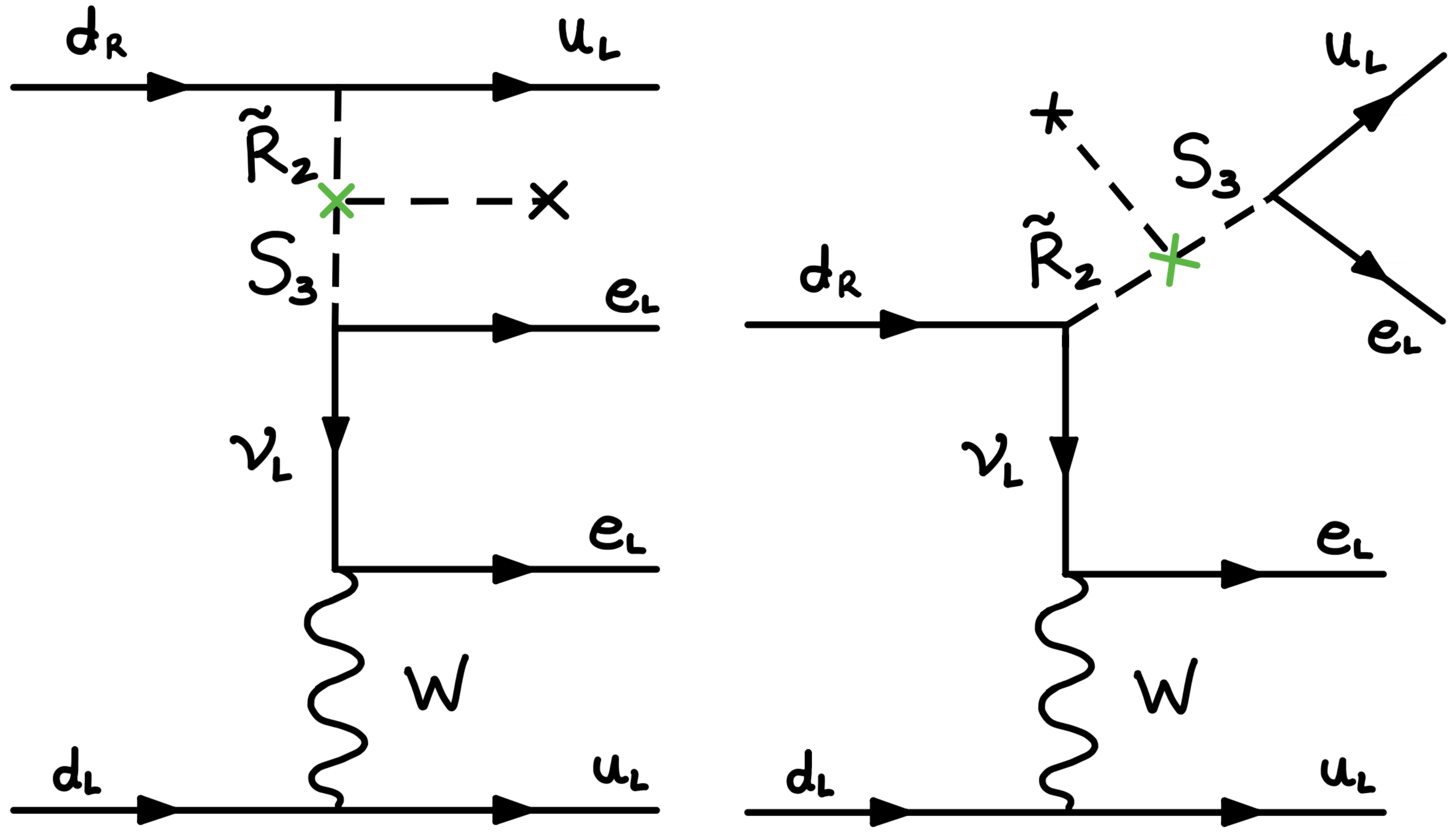
❖ Contributions from d = 5 and d = 7 effective operators

d = 5 contribution
(loop - level)



$(\propto m_\nu)$

d = 7 contributions
(Tree - level)



Chirality-enhanced ($\propto E/m_\nu$) contributions to $0\nu\beta\beta$, with $E \sim 100$ MeV

$\underline{S_3 - \tilde{R}_2}$

Leptoquark mixing through λ_3 breaks lepton number

Upper limits:

$\underline{S_3 - \tilde{R}_2}$ model:

$$\frac{|\lambda_3 y'^{11}_{3L} y'^{11}_{2L}|}{M^4} \lesssim (340\ \text{TeV})^{-3}$$

$y'_L = y_L U_{\ell L}$ where $U_{\ell L}$ is the left-handed charged lepton rotation

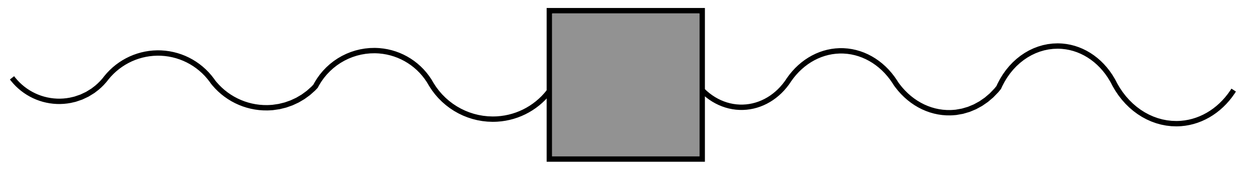
❖ Effective Field theory approach with running effects

[V. Cirigliano et al, JHEP 12]
[KamLAND-Zen Collaboration]

Phenomenology

❖ Electroweak Precision Tests

- T parameter: corrections to the gauge-boson two point functions



Scalar Leptoquarks

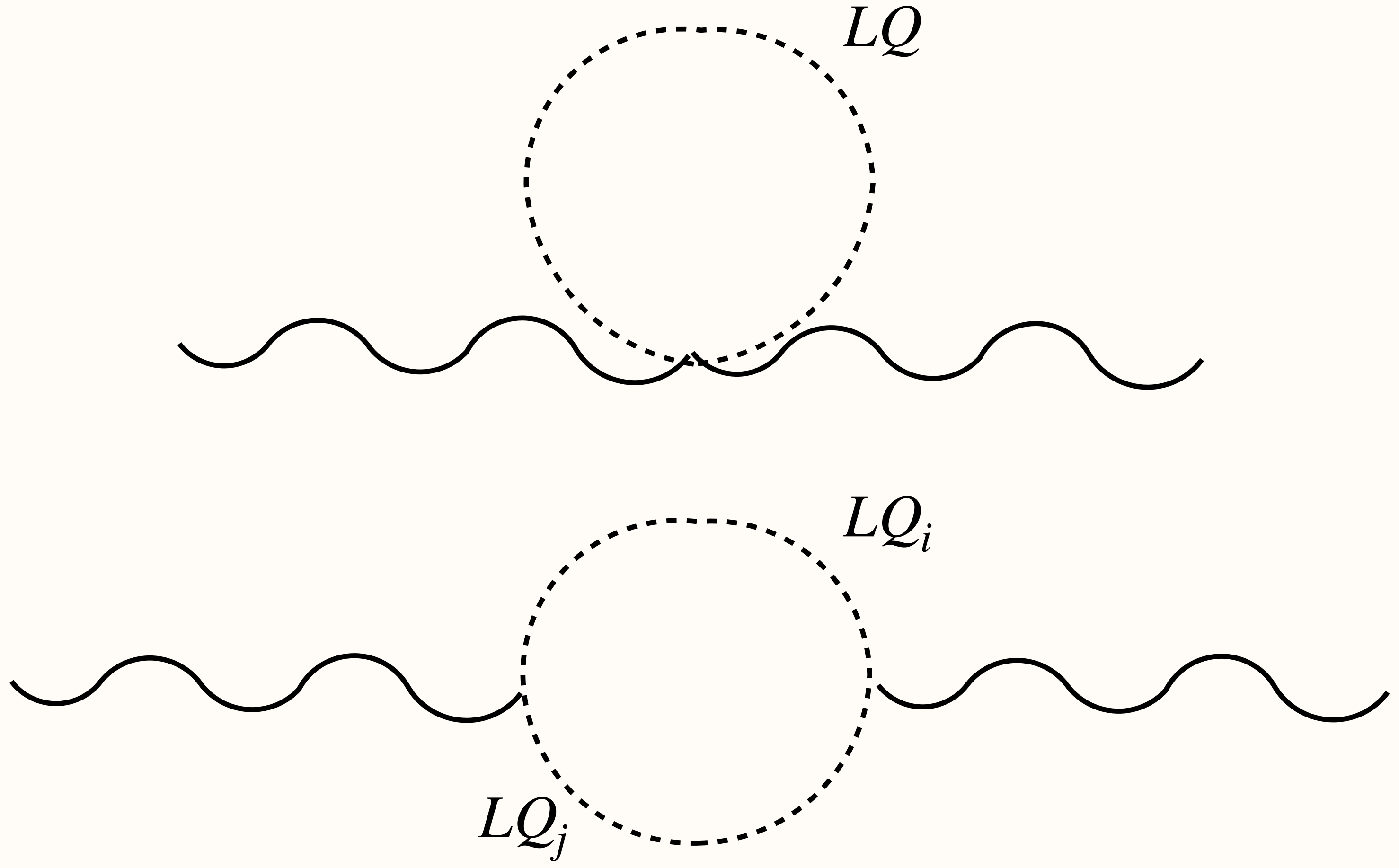
- Mass spliting between leptoquarks induces additional contributions to T

$(D_\mu \tilde{R}_2)^\dagger (D_\mu \tilde{R}_2)$
$(D_\mu S_3)^\dagger (D_\mu S_3)$

→ give rise to $W - LQ - LQ$ and $W - W - LQ - LQ$ interactions

→ Affected when we rotate LQs to mass basis

New Contributions

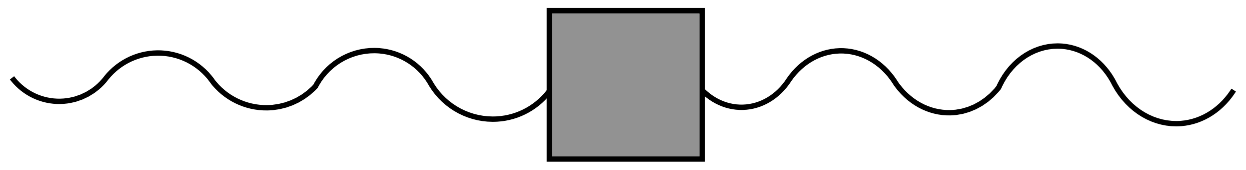


[M. Baak et al., Eur. Phys. J. C 72]

Phenomenology

❖ Electroweak Precision Tests

- T parameter: corrections to the gauge-boson two point functions



Scalar Leptoquarks

- Mass spliting between leptoquarks induces additional contributions to T

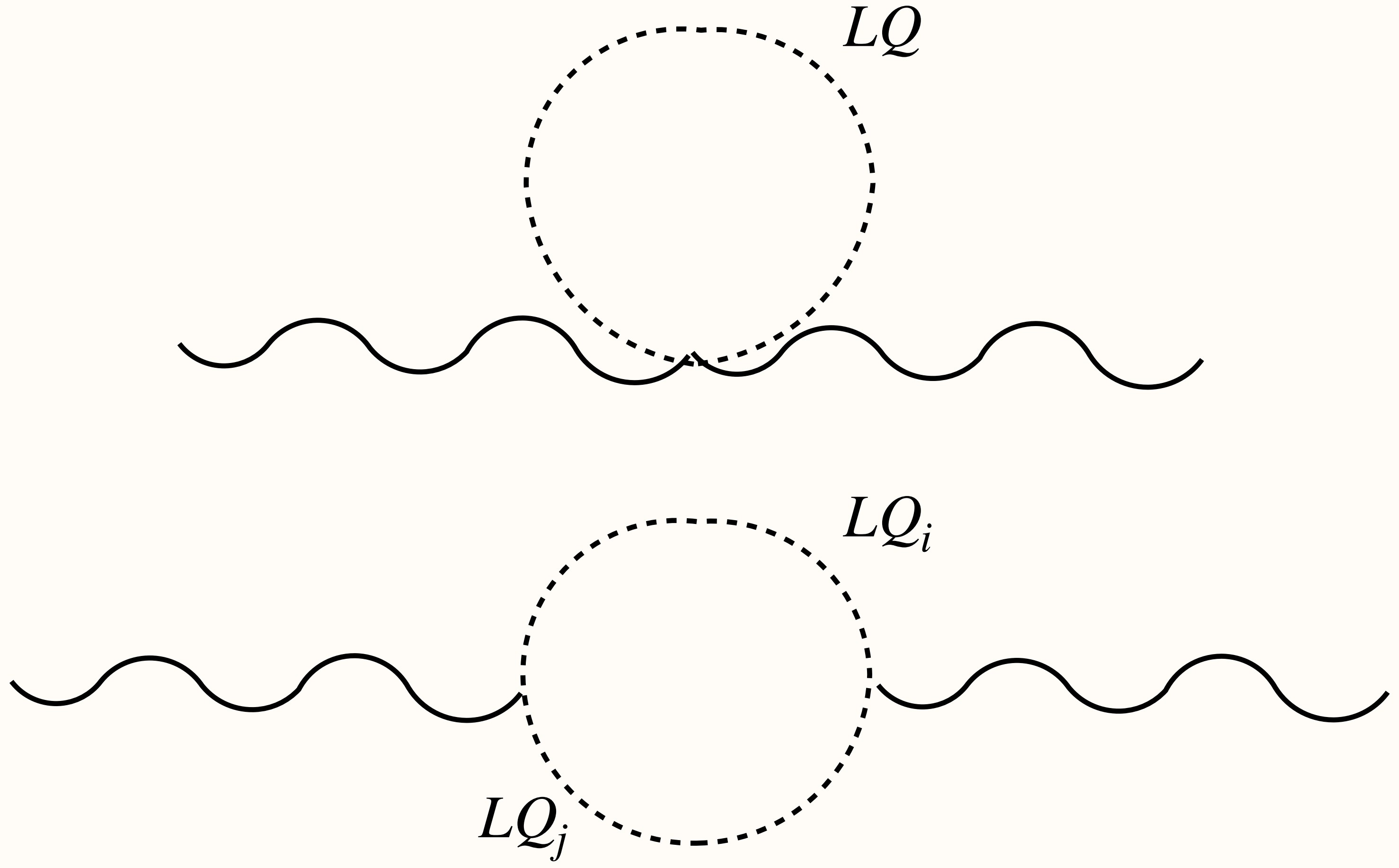
Bounds:

$$T = 0.05 \pm 0.12 \implies \frac{|\lambda_3|}{M} \lesssim 0.7$$

(Electroweak fit)

$\implies$ We choose $\lambda_3 = M/2$ in our analysis

New Contributions



[M. Baak et al., Eur. Phys. J. C 72]

Phenomenology

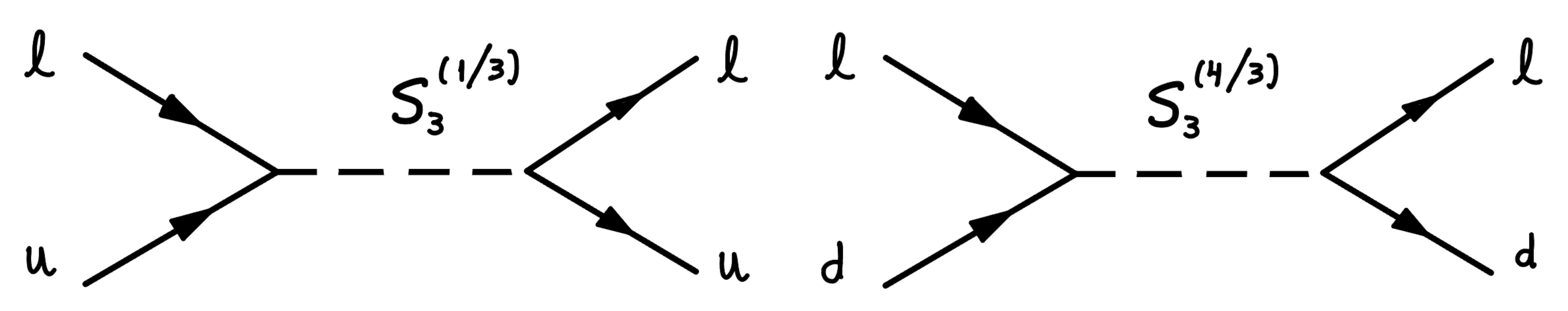
❖ **Flavor Physics**

- $\mu N \to eN$

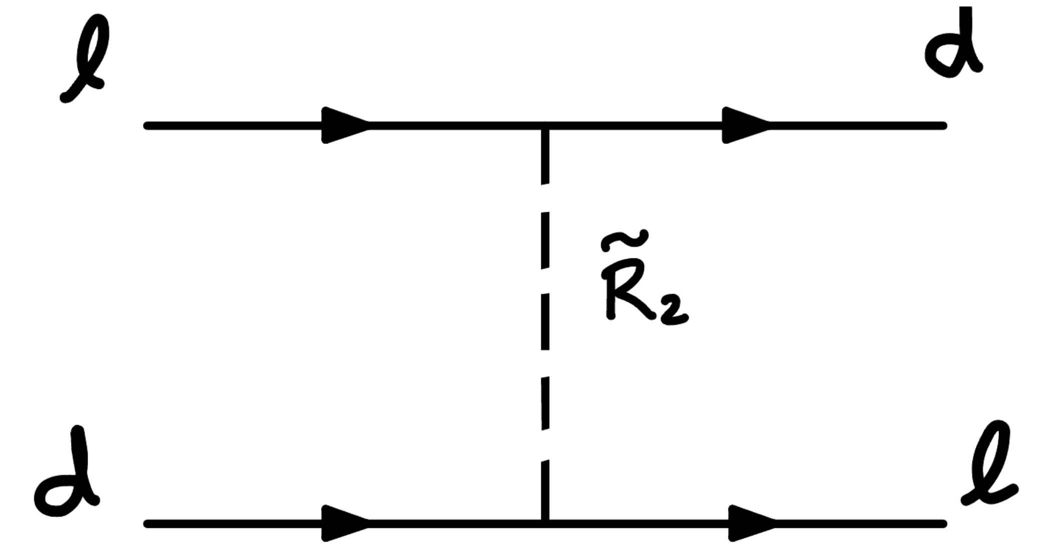
Yukawa Lagrangian:

- $\mathcal{L}^{\tilde{R}_2}_{\text{Yuk.}} = -\left(y'_{2L}\right)_{ij} \bar{d}_{Ri}\ell_{Lj}\tilde{R}_2^{(2/3)} + \left(y'_{2L}\right)_{ij} \bar{d}_{Ri}\nu_{Lj}\tilde{R}_2^{(-1/3)} + \text{h.c.},$
- $\mathcal{L}^{S_3}_{\text{Yuk.}} = -\left(y'_{3L}\right)_{ij} \bar{d}^C_{Li}\nu_{Lj}S_3^{(1/3)} - \sqrt{2}\left(y'_{3L}\right)_{ij} \bar{d}^C_{Li}\ell_{Lj}S_3^{(4/3)}$
  $+\sqrt{2}\left(V^* y'_{3L}\right)_{ij} \bar{u}^C_{Li}\nu_{Lj}S_3^{(-2/3)} - \left(V^* y'_{3L}\right)_{ij} \bar{u}^C_{Li}\ell_{Lj}S_3^{(1/3)} + \text{h.c.},$

S_3



$\tilde{R}_2$



Bounds:

SINDRUM II : $B^{(Au)}_{\mu e} < 7 \times 10^{-13}$ (90% C.L)

$$\frac{|y'^{12}_{1L}\, y'^{11*}_{1L}|}{M^2} < (470\ \text{TeV})^{-2}, \qquad \frac{|y'^{12}_{3L}\, y'^{11*}_{3L}|}{M^2} < (830\ \text{TeV})^{-2},$$

$$\frac{|y'^{12}_{2L}\, y'^{11*}_{2L}|}{M^2} < (500\ \text{TeV})^{-2}$$

Future Experiments:

Mu2E (Fermilab) and COMET (J-PARC):

$B^{(Al)}_{\mu e} < \mathcal{O}(10^{-17})$

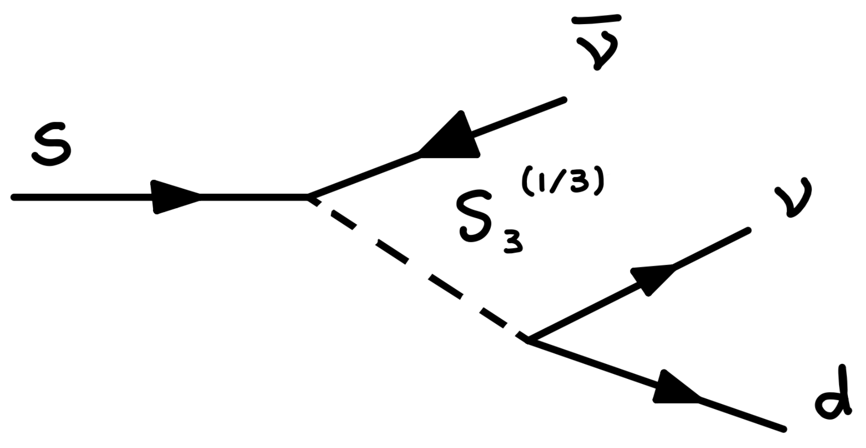
[R. L. Workman et al. [Particle Data Group]]

Phenomenology

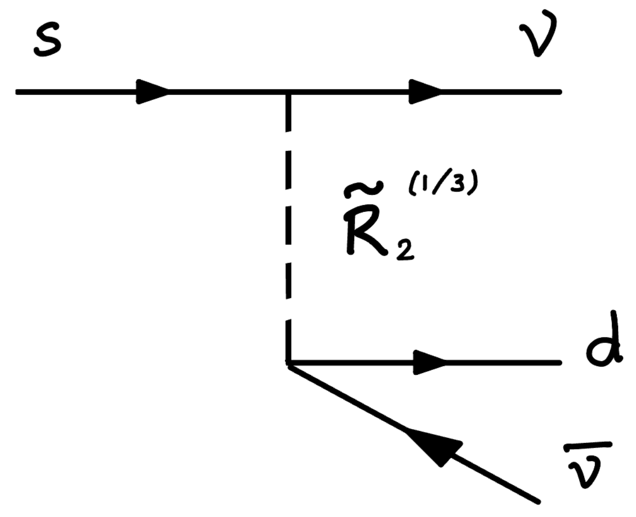
❖ **Flavor Physics**

- $K^+ \to \pi^+ \nu \bar{\nu}$

S_3



$\tilde{R}_2$



Yukawa Lagrangian:

- $\mathcal{L}^{\tilde{R}_2}_{\text{Yuk.}} = -\left(y'_{2L}\right)_{ij} \bar{d}_{Ri} \ell_{Lj} \tilde{R}_2^{(2/3)} + \left(y'_{2L}\right)_{ij} \bar{d}_{Ri} \nu_{Lj} \tilde{R}_2^{(-1/3)} + \text{h.c.},$
- $\mathcal{L}^{S_3}_{\text{Yuk.}} = -\left(y'_{3L}\right)_{ij} \bar{d}^C_{Li} \nu_{Lj} S_3^{(1/3)} - \sqrt{2}\left(y'_{3L}\right)_{ij} \bar{d}^C_{Li} \ell_{Lj} S_3^{(4/3)}$
  $+\sqrt{2}\left(V^* y'_{3L}\right)_{ij} \bar{u}^C_{Li} \nu_{Lj} S_3^{(-2/3)} - \left(V^* y'_{3L}\right)_{ij} \bar{u}^C_{Li} \ell_{Lj} S_3^{(1/3)} + \text{h.c.},$

Bounds:

- NA62: $B(K^+ \to \pi^+ \nu \bar{\nu})^{\text{exp}} = \left(1.14^{+0.40}_{-0.33}\right) \times 10^{-10}$
- SM: $B(K^+ \to \pi^+ \nu \bar{\nu})^{\text{SM}} = (7.7 \pm 0.6) \times 10^{-11}$

$$\frac{|y'^{2i}_L\, y'^{1i*}_L|}{M^2} < (57\ \text{TeV})^{-2}, \quad i \in \{1,2,3\}$$

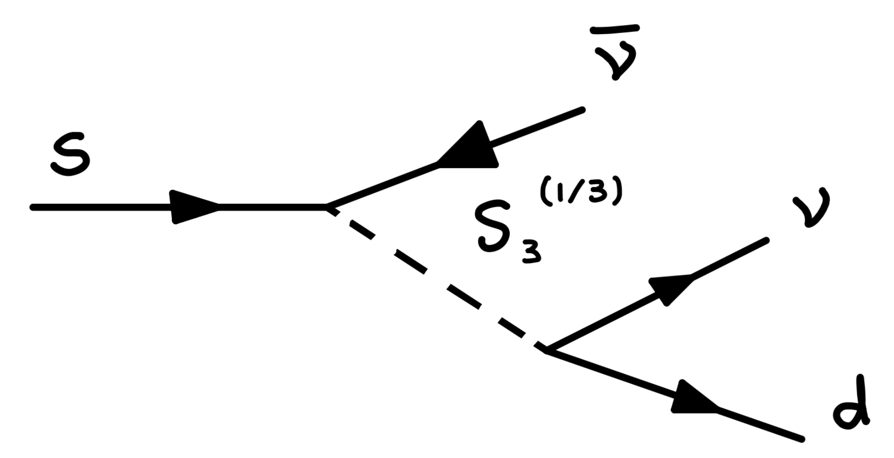
[C. Bobeth and A. J. Buras, JHEP 02]
[J. Brod, M. Gorbahn and E. Stamou, PoS BEAUTY2020, 056]

Phenomenology

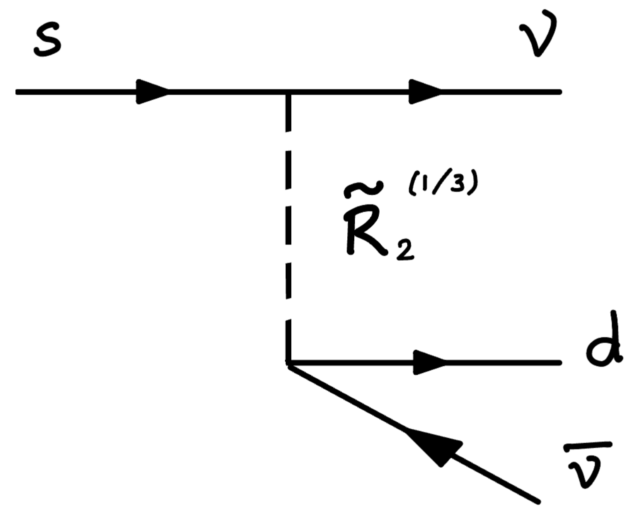
❖ Flavor Physics

- $K^+ \to \pi^+ \nu \bar{\nu}$

S_3



$\tilde{R}_2$



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  $+\sqrt{2}\left(V^* y'_{3L}\right)_{ij} \bar{u}^C_{Li} \nu_{Lj} S_3^{(-2/3)} - \left(V^* y'_{3L}\right)_{ij} \bar{u}^C_{Li} \ell_{Lj} S_3^{(1/3)} + \text{h.c.},$

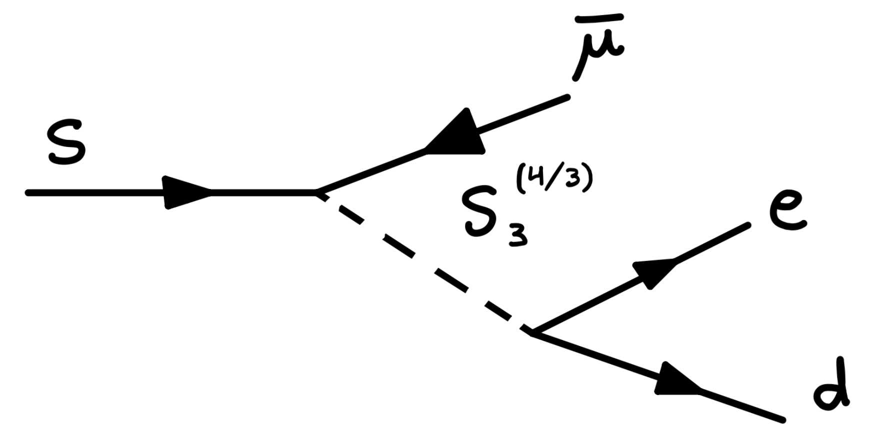
Bounds:

- NA62: $B(K^+ \to \pi^+ \nu \bar{\nu})^{\text{exp}} = \left(1.14^{+0.40}_{-0.33}\right) \times 10^{-10}$
- SM: $B(K^+ \to \pi^+ \nu \bar{\nu})^{\text{SM}} = (7.7 \pm 0.6) \times 10^{-10}$

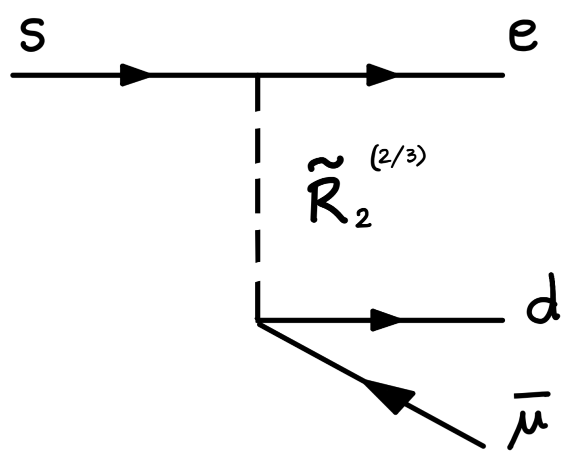
$$\frac{|y'^{2i}_L y'^{1i*}_L|}{M^2} < (57\ \text{TeV})^{-2}, \quad i \in \{1,2,3\}$$

- $K_L \to \mu^\pm e^\mp$ and $K^+ \to \pi^+ \mu^+ e^-$

S_3



$\tilde{R}_2$



Bounds:

$$B(K_L \to \mu^\pm e^\mp) < 4.7 \times 10^{-12}\ (90\%\ \text{C.L})$$
$$B(K^+ \to \pi^+ \mu^+ e^-) < 1.3 \times 10^{-11}\ (90\%\ \text{C.L})$$

$$\frac{|y'^{21}_{2L} y'^{12*}_{2L}|}{M^2} < (208\ \text{TeV})^{-2}, \qquad \frac{|y'^{21}_{3L} y'^{12*}_{3L}|}{M^2} < (290\ \text{TeV})^{-2}$$

[R. L. Workman et al. [Particle Data Group]]

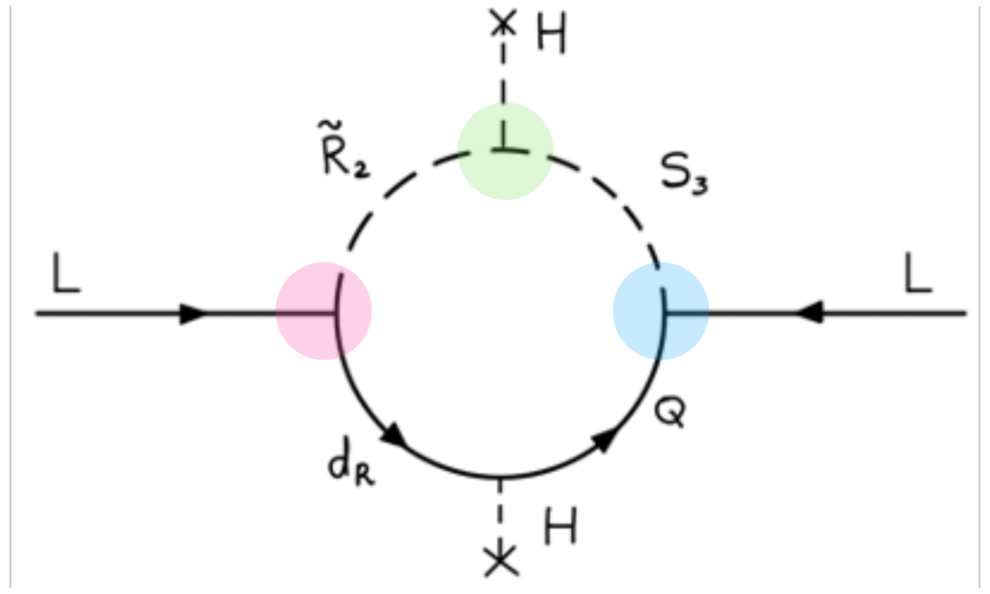
Recap

❖ Scalar Leptoquarks with mixing

$$\mathscr{L}_{\tilde{R}_2\&S_3} \supset (D_\mu \tilde{R}_2)^\dagger (D_\mu \tilde{R}_2) + (D_\mu S_3^a)^\dagger (D_\mu S_3^a) - m_{\tilde{R}_2}^2 \tilde{R}_2^\dagger \tilde{R}_2 - m_{S_3}^2 S_3^{a\,\dagger} S_3^a$$

$$y_{3L}^{ij} \overline{Q_i^C} i\tau_2 (\tau^a S_3^a) L_j - y_{2L}^{ij} \bar{d}_{Ri} \tilde{R}_2 i\tau_2 L_j - \lambda_3 \tilde{R}_2^\dagger (\tau^a S_3^a)^\dagger H + \mathrm{h.c.}$$

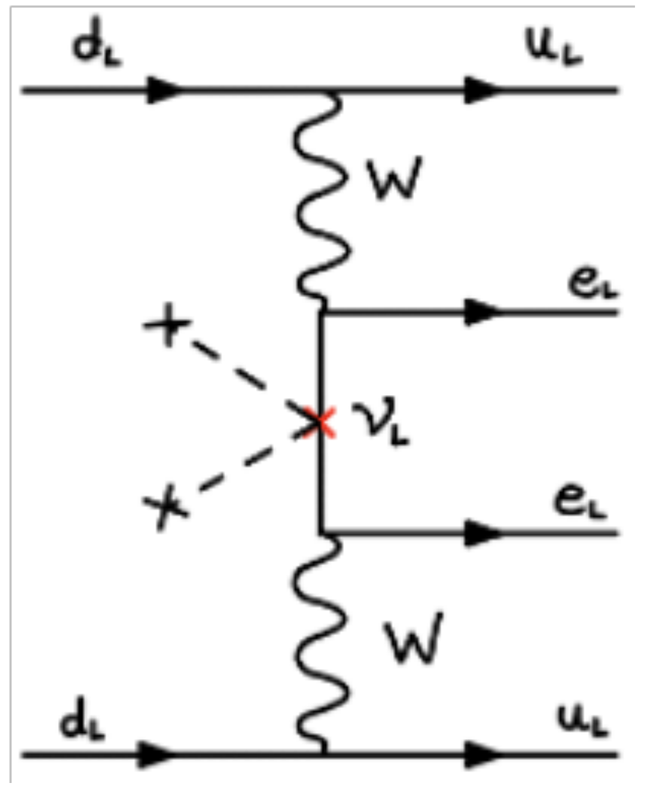
Neutrino Masses



$0\nu\beta\beta$

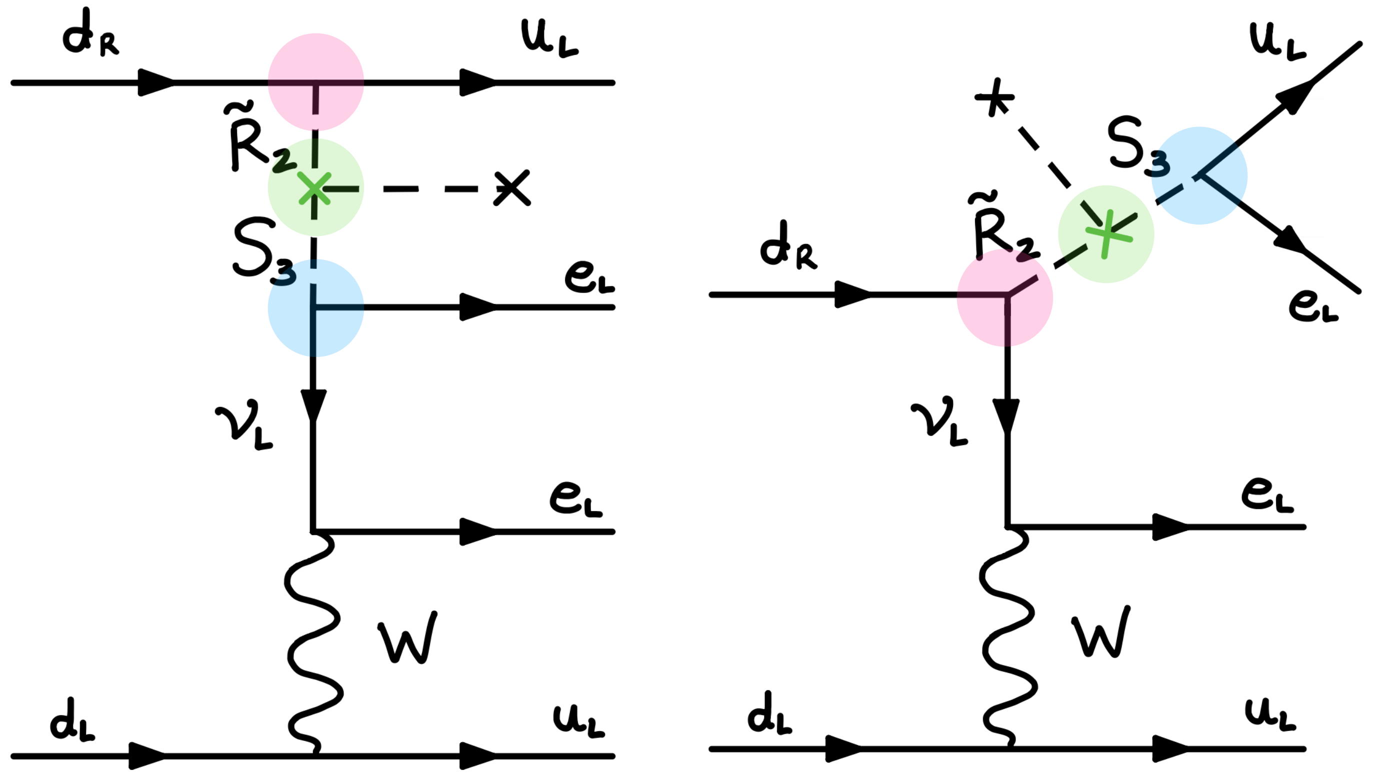
d = 5 contribution

(loop - level)



d = 7 operators

(tree - level)

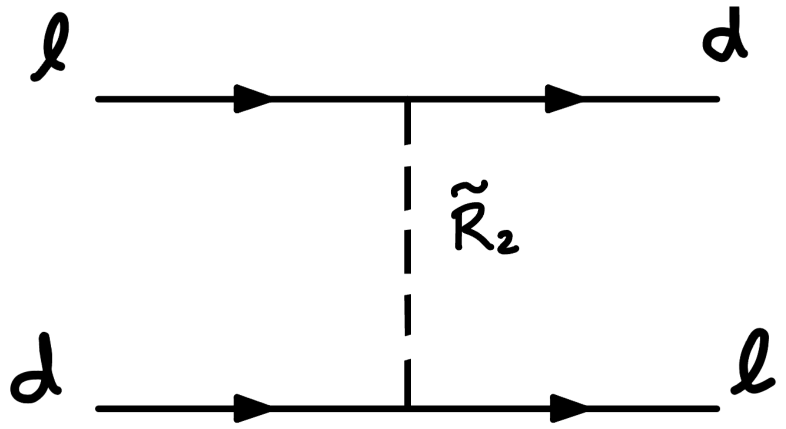


Chirality-enhanced ($\propto E/m_\nu$) contributions to $0\nu\beta\beta$, with $E \sim 100$ MeV

Flavor Physics

d = 6 operators

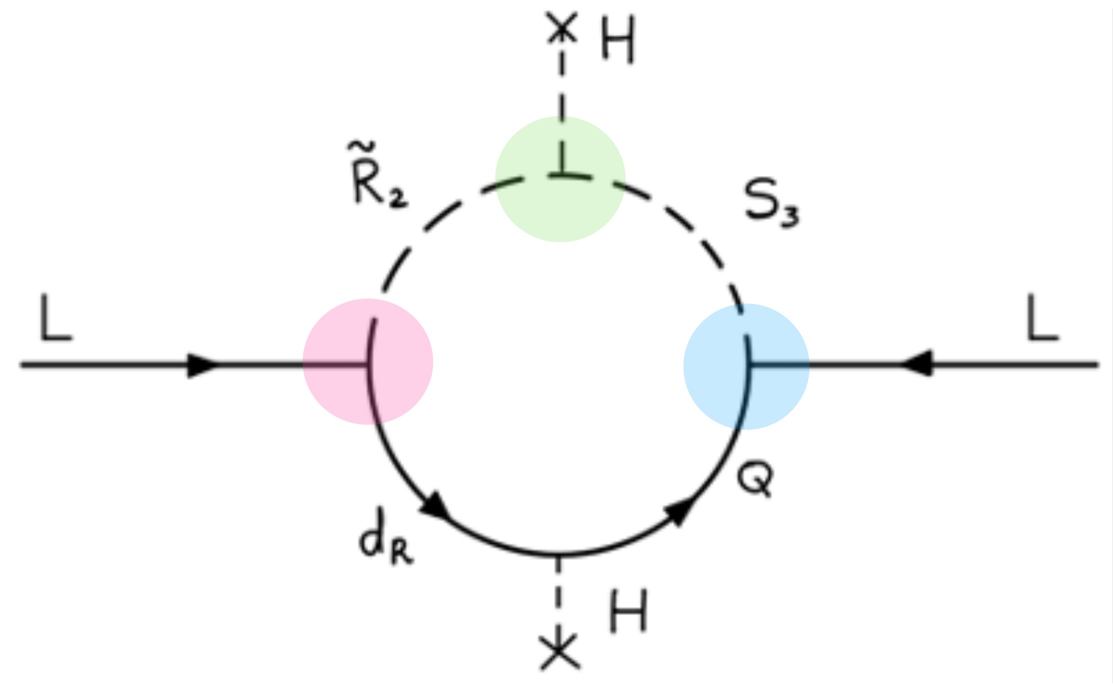
(tree - level)



Numerical Results

❖ **Neutrino masses**

$$m_\nu = -\frac{3\lambda_3}{16\sqrt{2}\pi^2}\frac{v^2}{M^2}\left(y_{2L}^T \cdot \hat{y}_d \cdot y_{3L} + y_{3L}^T \cdot \hat{y}_d \cdot y_{2L}\right)$$



❖ $\lambda_3 = M/2$ and $M = 100$ TeV

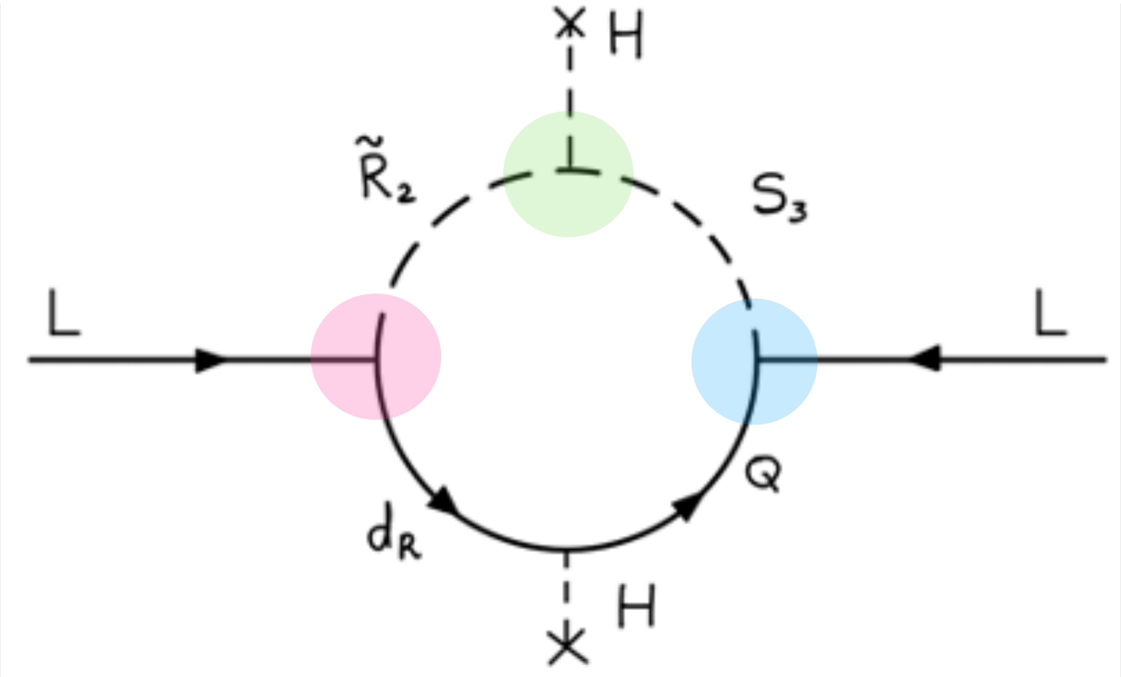
Restricts our phenomenological analysis of flavor processes

Numerical Results

❖ **Neutrino masses**

varied within perturbativity range

$$m_\nu = -\frac{3\lambda_3}{16\sqrt{2}\pi^2}\frac{v^2}{M^2}\left(y_{2L}^T \cdot \hat{y}_d \cdot y_{3L} + y_{3L}^T \cdot \hat{y}_d \cdot y_{2L}\right)$$



❖ $\lambda_3 = M/2$ and $M = 100$ TeV

Restricts our phenomenological analysis of flavor processes

❖ Requirement that we need to reproduce results from oscillation experiments:

$\Delta m_{21}^2 = (7.4 \pm 0.2) \times 10^{-5}$ eV^2, $|\Delta m_{3\ell}^2| = (2.51 \pm 0.03) \times 10^{-3}$ eV^2, with $\ell = 1\,(2)$ for NO (IO)

❖ PMNS matrix within the allowed 3σ range

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

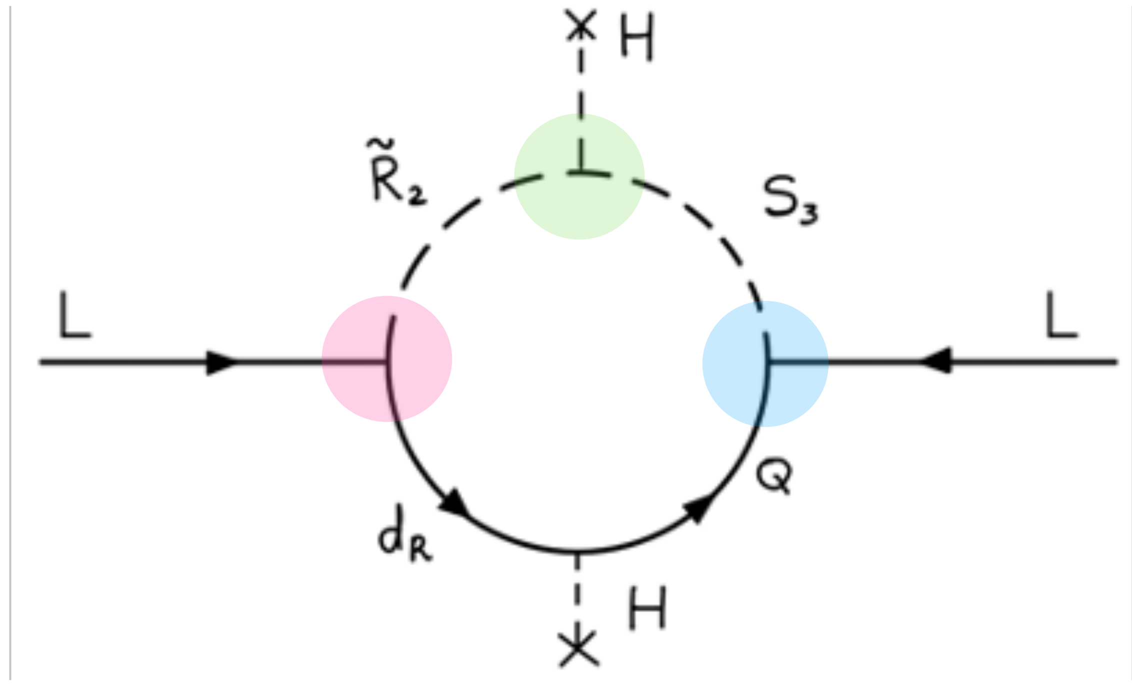
[N. Aghanim et al. [Planck], Astron. Astrophys. 641]
[NuFIT 5.3 (2024), www.nu-fit.org.]

Numerical Results

❖ **Neutrino masses**

varied within perturbativity range

$$m_\nu = -\frac{3\lambda_3}{16\sqrt{2}\pi^2}\frac{v^2}{M^2}\left(y_{2L}^T \cdot \hat{y}_d \cdot y_{3L} + y_{3L}^T \cdot \hat{y}_d \cdot y_{2L}\right)$$



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❖ PMNS matrix within the allowed 3σ range

❖ Random Majorana phases in the range $0 \leq \alpha_{1,2} \leq \pi$

❖ Cosmology: $\sum_i m_{\nu_i} \lesssim 0.12\,\text{eV}$

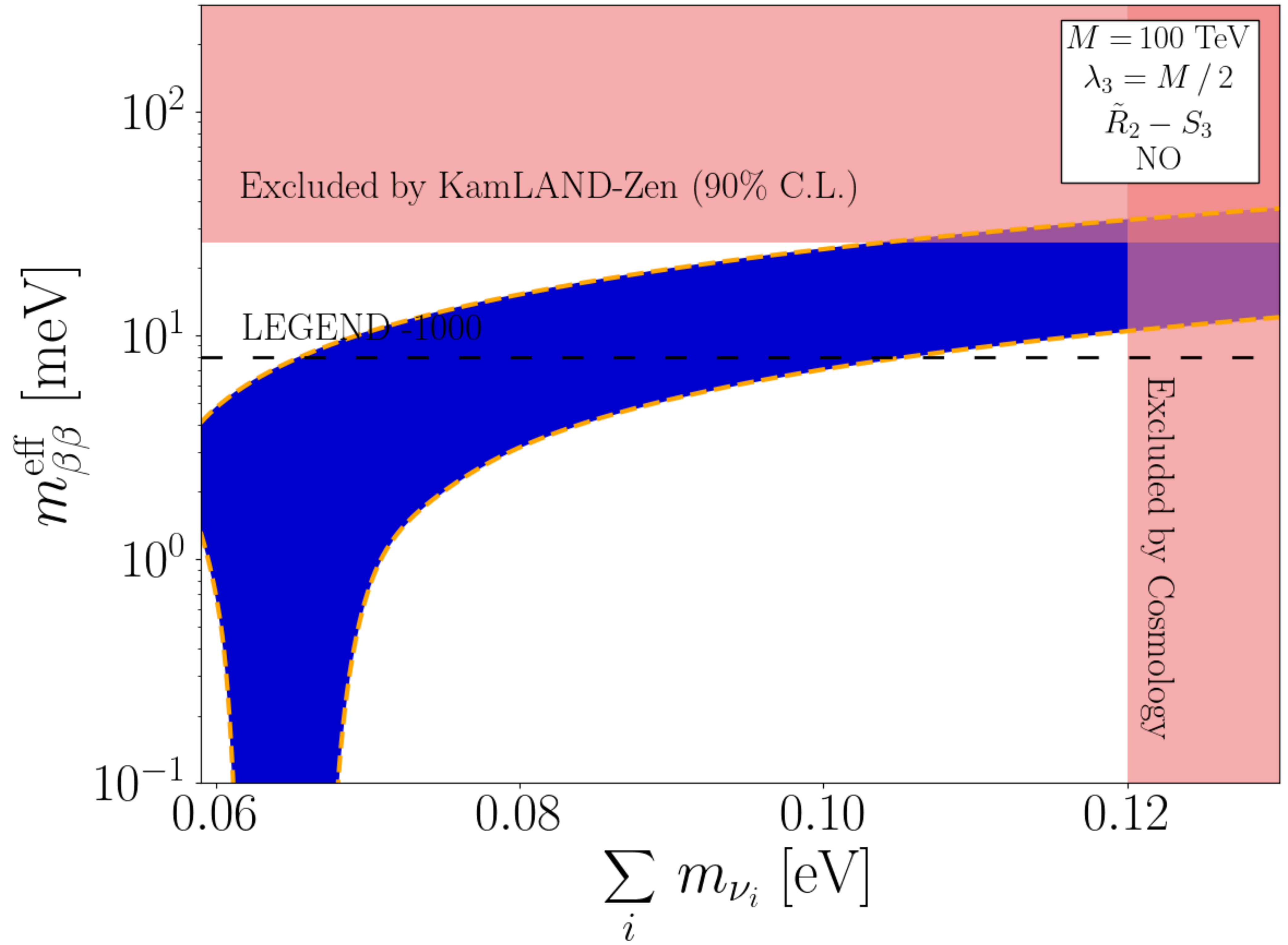
$$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = \begin{pmatrix}U_{e1} & U_{e2} & U_{e3}\\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3}\\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3}\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}$$

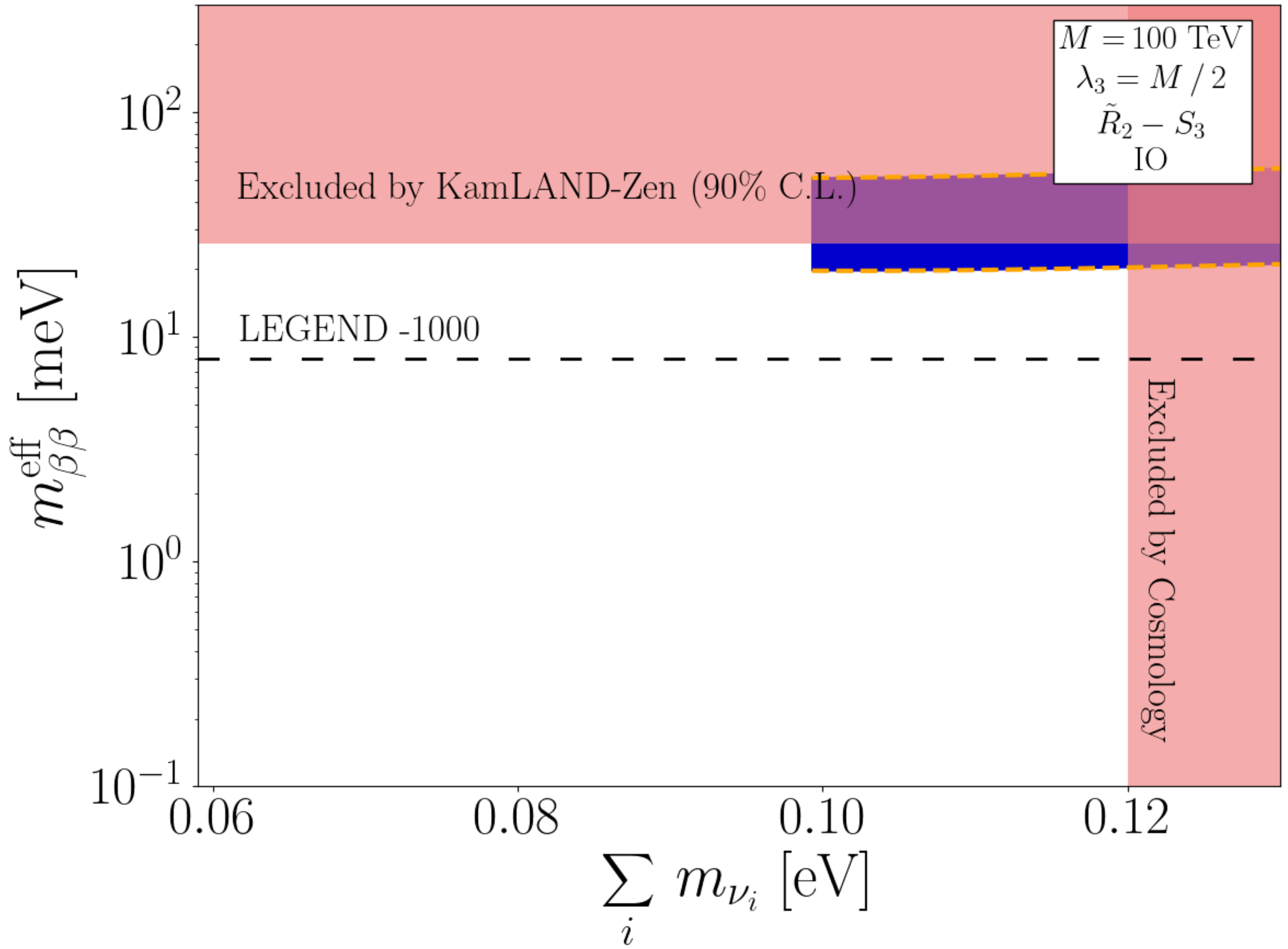
[N. Aghanim et al. [Planck], Astron. Astrophys. 641]
[NuFIT 5.3 (2024), www.nu-fit.org.]

Neutrinoless double-beta decay

$$\left(m_{\beta\beta}^{\text{eff}}\right)^2 \equiv m_{\beta\beta}^2 + \delta m_{\beta\beta}^{2\,(\text{int})} + \delta m_{\beta\beta}^{2\,(\text{LQ})} \quad ; \quad m_{\beta\beta} = \sum_i U_{ei}^2 m_{\nu_i}$$

d = 5





NO: $m_1 < m_2 < m_3$

$\Delta m_{21}^2 = (7.4 \pm 0.2) \times 10^{-5}$ eV^2

$\Delta m_{31}^2 = (2.51 \pm 0.03) \times 10^{-3}$ eV^2

IO: $m_3 < m_1 < m_2$

$\Delta m_{21}^2 = (7.4 \pm 0.2) \times 10^{-5}$ eV^2

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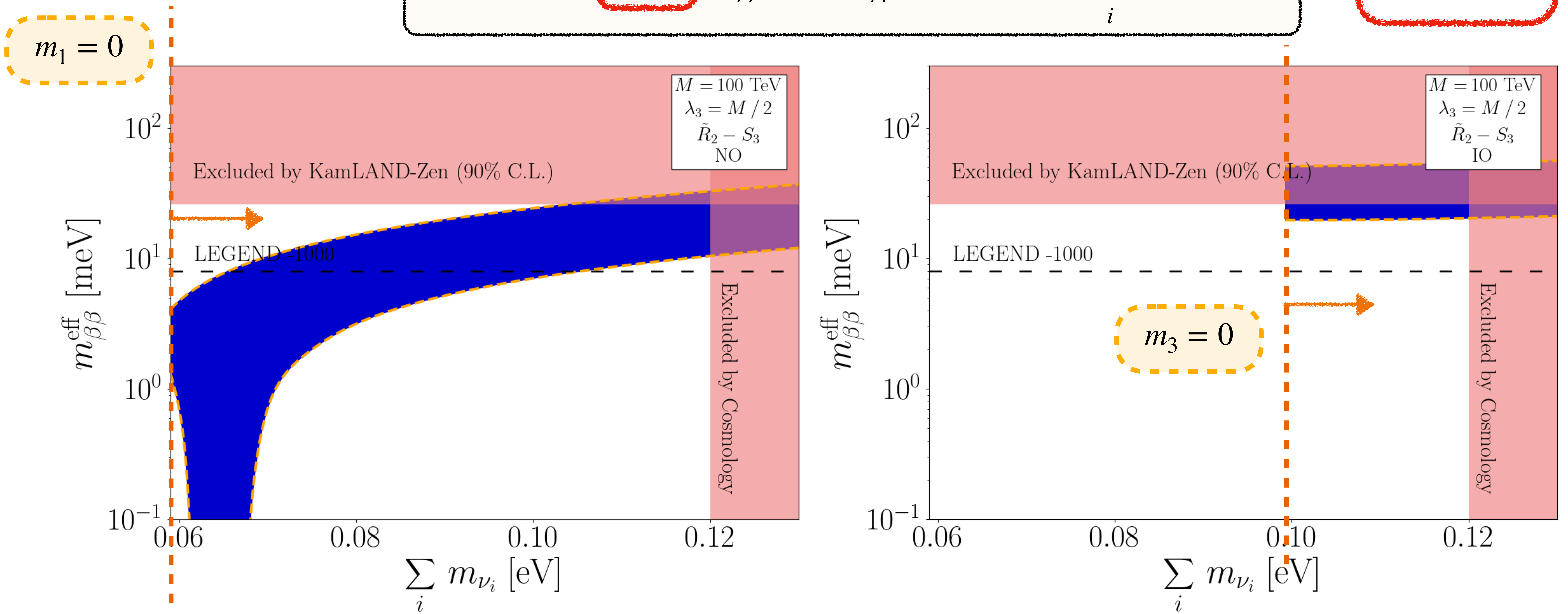
Neutrinoless double-beta decay

$$\left(m_{\beta\beta}^{\rm eff}\right)^2 \equiv m_{\beta\beta}^2 + \delta m_{\beta\beta}^{2\,(\rm int)} + \delta m_{\beta\beta}^{2\,(\rm LQ)} \quad ; \quad m_{\beta\beta} = \sum_i U_{ei}^2 m_{\nu_i}$$

d = 5



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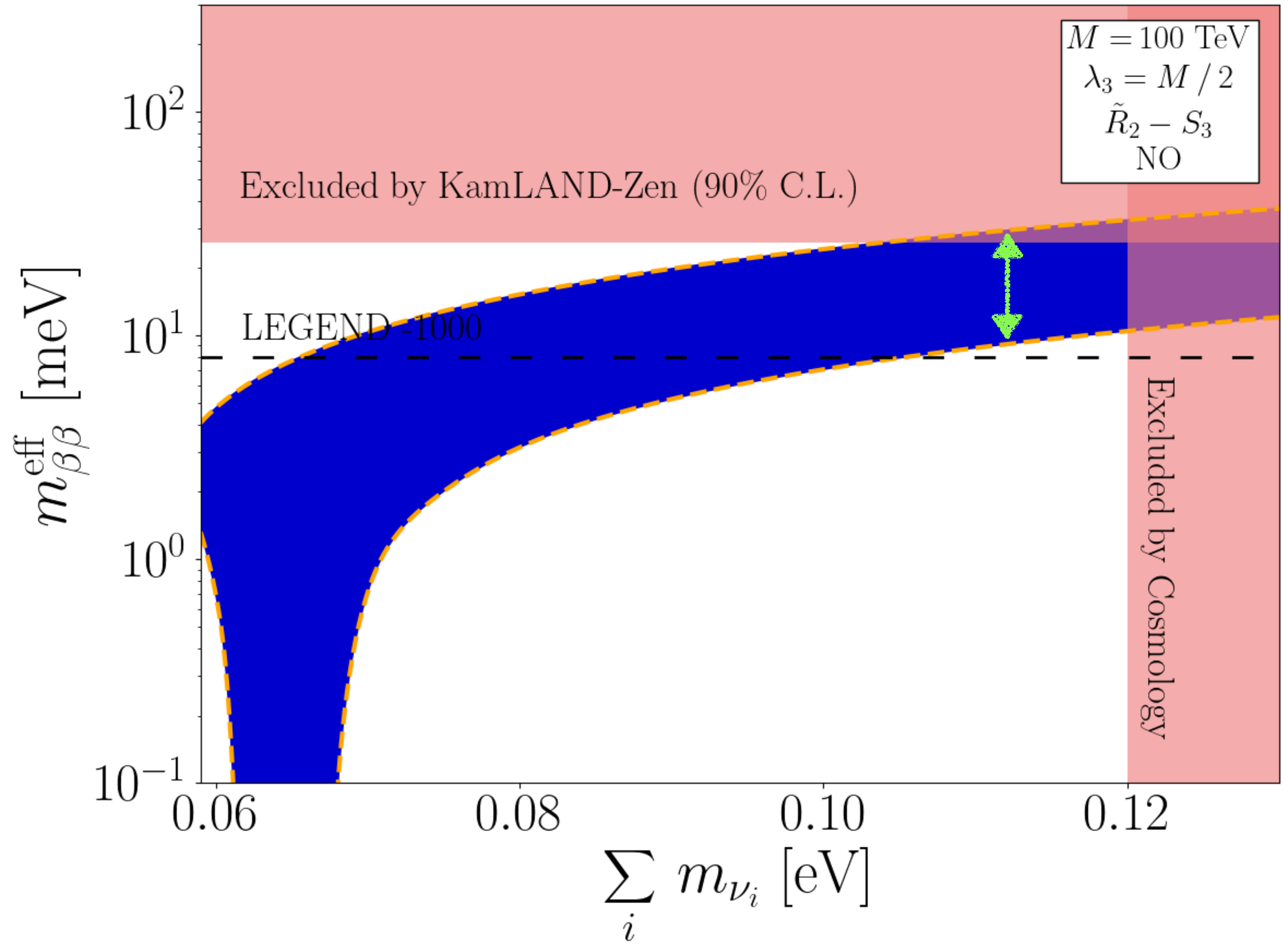
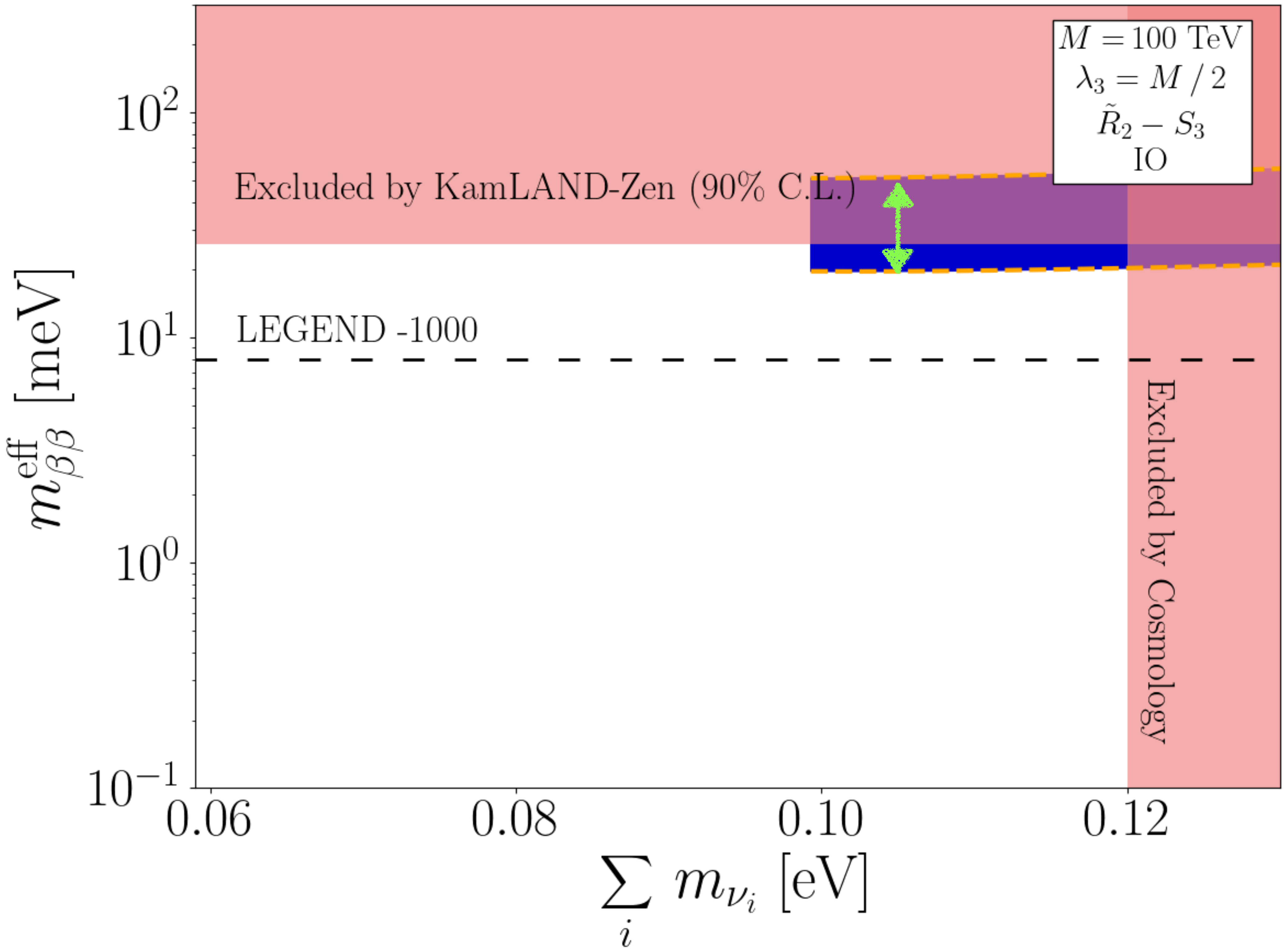
Neutrinoless double-beta decay

- 3 mixing angles
- 1 CP phase
- 2 Majorana phases

$$\left(m_{\beta\beta}^{\text{eff}}\right)^2 \equiv m_{\beta\beta}^2 + \delta m_{\beta\beta}^{2\,(\text{int})} + \delta m_{\beta\beta}^{2\,(\text{LQ})} \quad ; \quad m_{\beta\beta} = \sum_i U_{ei}^2 m_{\nu_i}$$

d = 5





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Neutrinoless double-beta decay

- 3 mixing angles
- 1 CP phase
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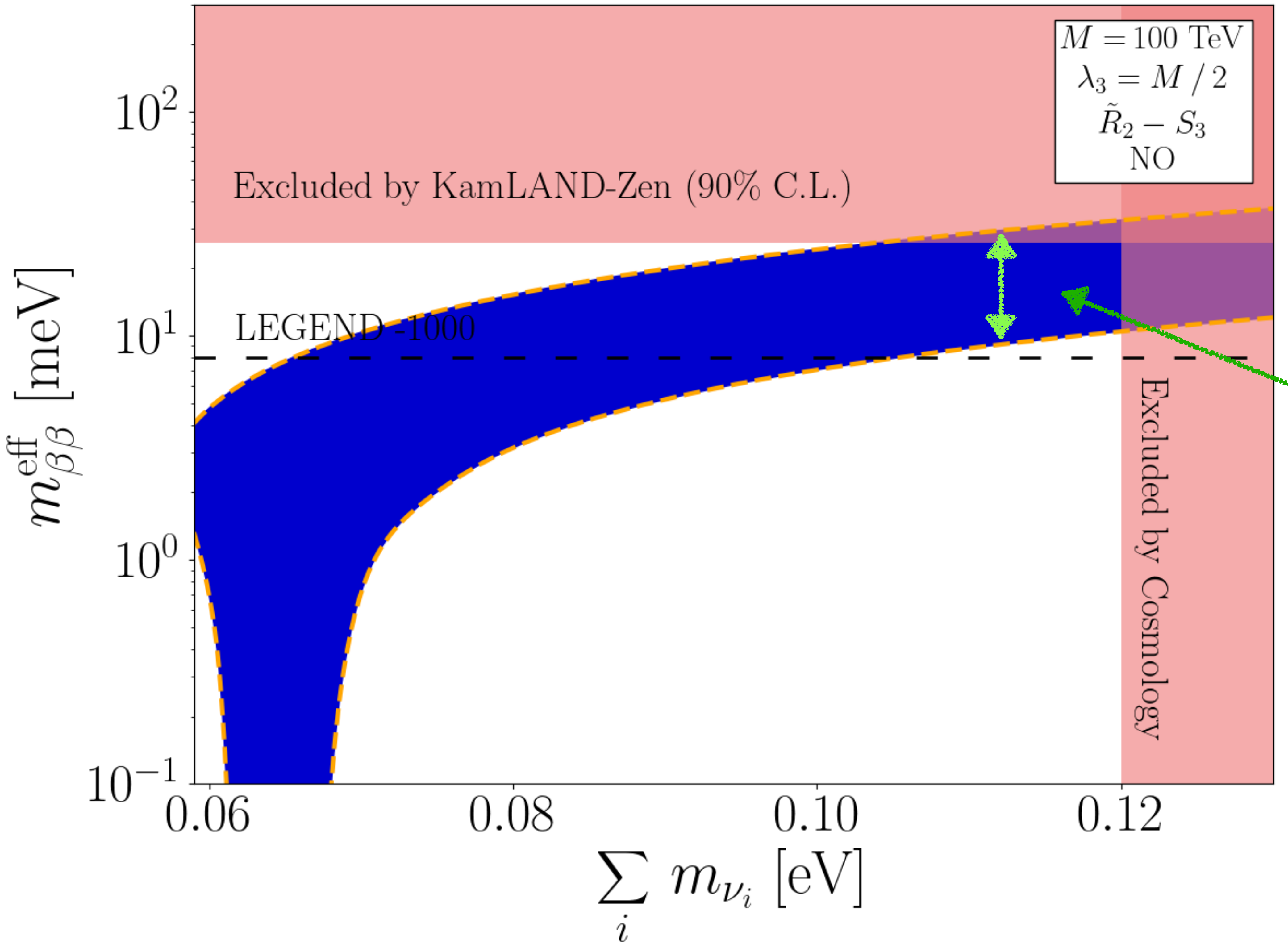
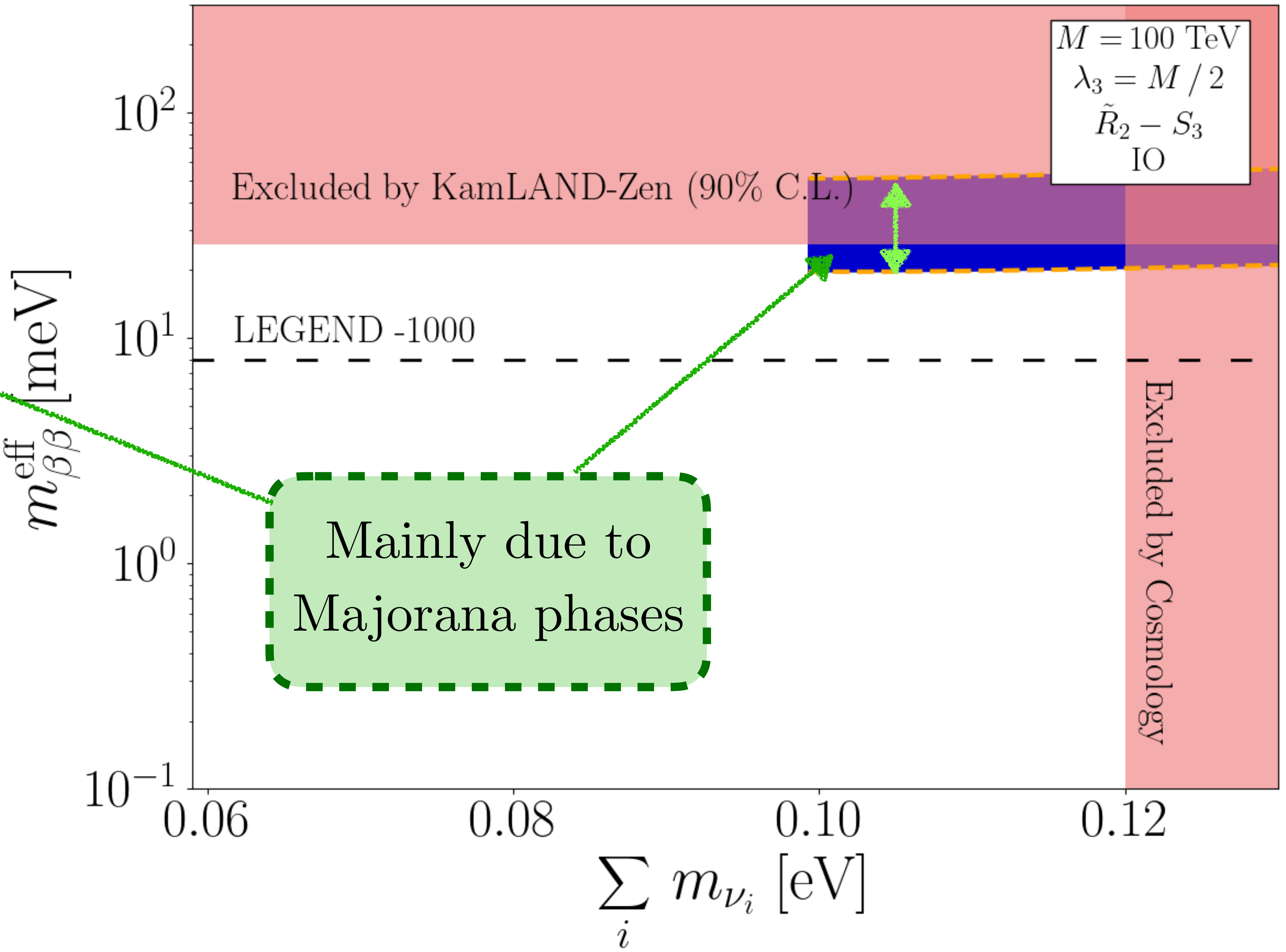
d = 5





Mainly due to Majorana phases

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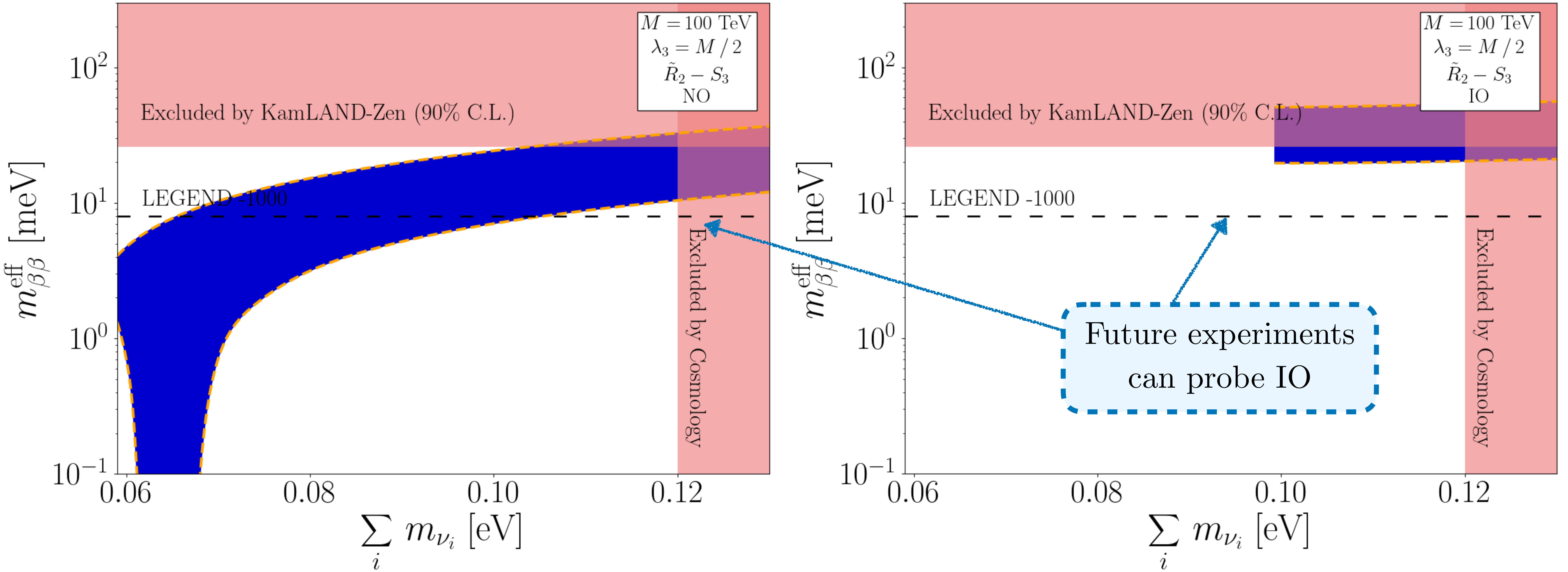
d = 5



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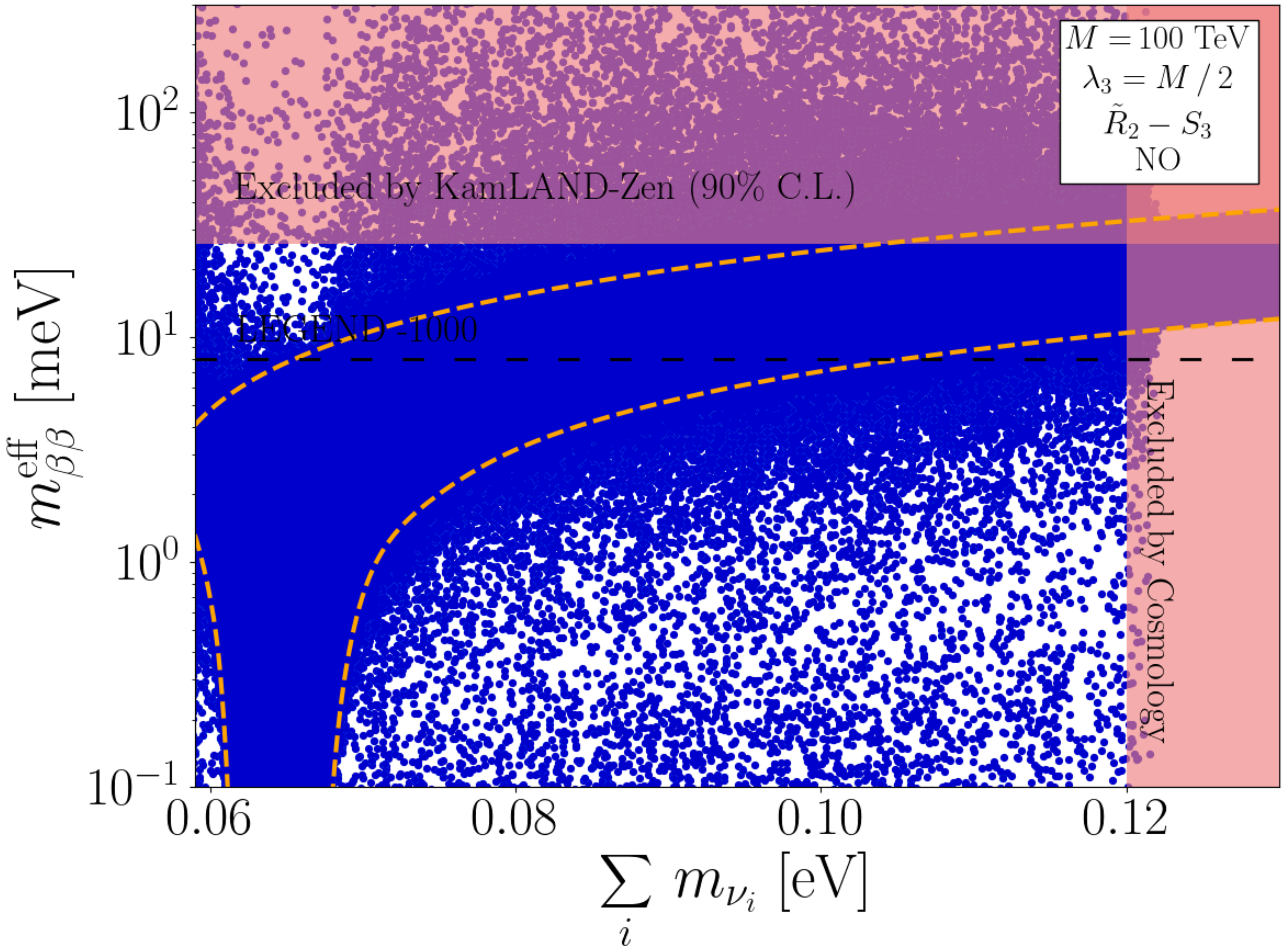
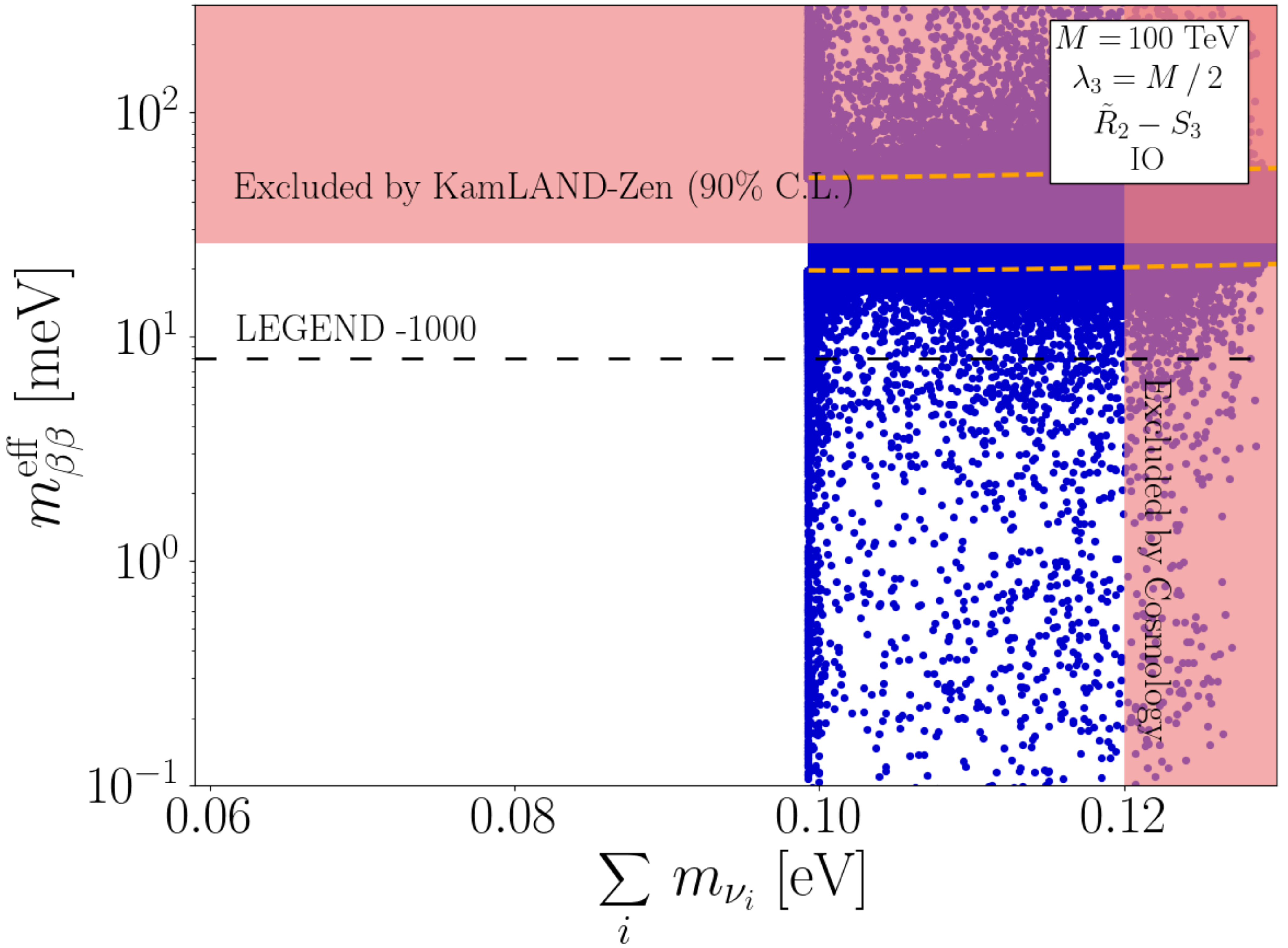
Neutrinoless double-beta decay

M = 100 TeV

Contributions up to d = 7

$$\left(m_{\beta\beta}^{\text{eff}}\right)^2 \equiv m_{\beta\beta}^2 + \delta m_{\beta\beta}^{2\,(\text{int})} + \delta m_{\beta\beta}^{2\,(\text{LQ})} \quad ; \quad m_{\beta\beta} = \sum_i U_{ei}^2 m_{\nu_i}$$





NO: $m_1 < m_2 < m_3$

$\Delta m_{21}^2 = (7.4 \pm 0.2) \times 10^{-5}$ eV^2

$\Delta m_{31}^2 = (2.51 \pm 0.03) \times 10^{-3}$ eV^2

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Neutrinoless double-beta decay

M = 100 TeV

Contributions up to d = 7

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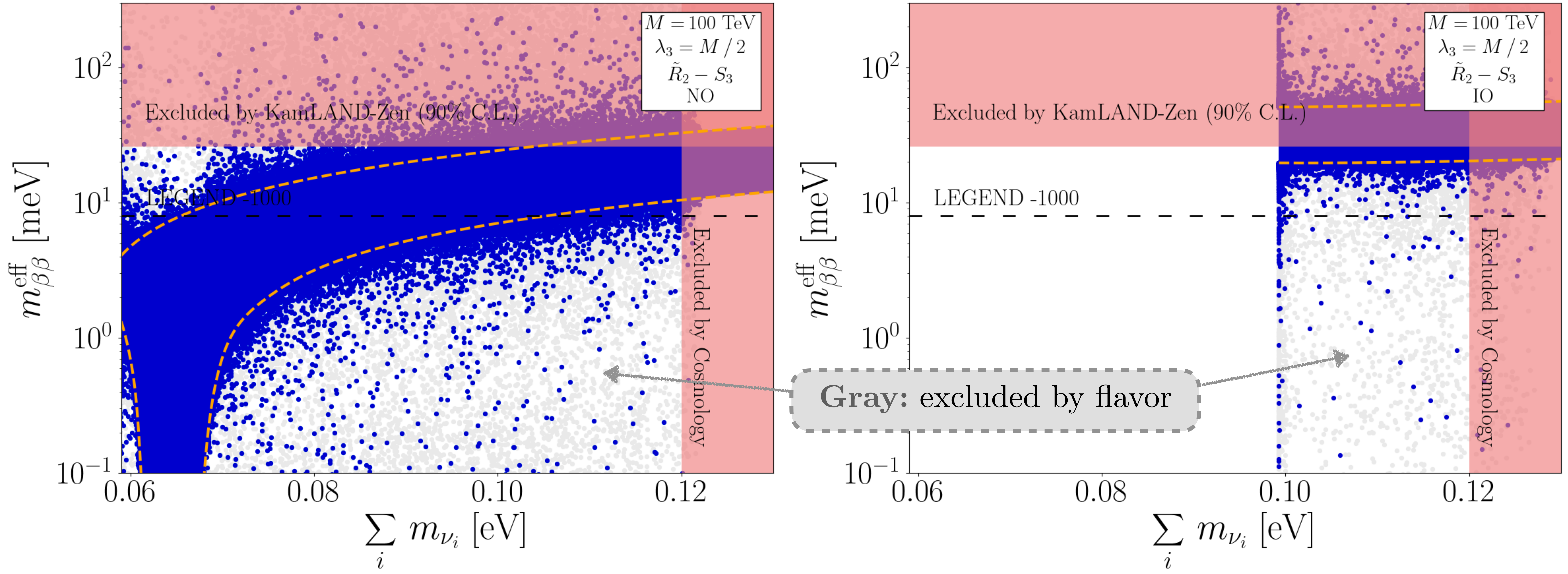


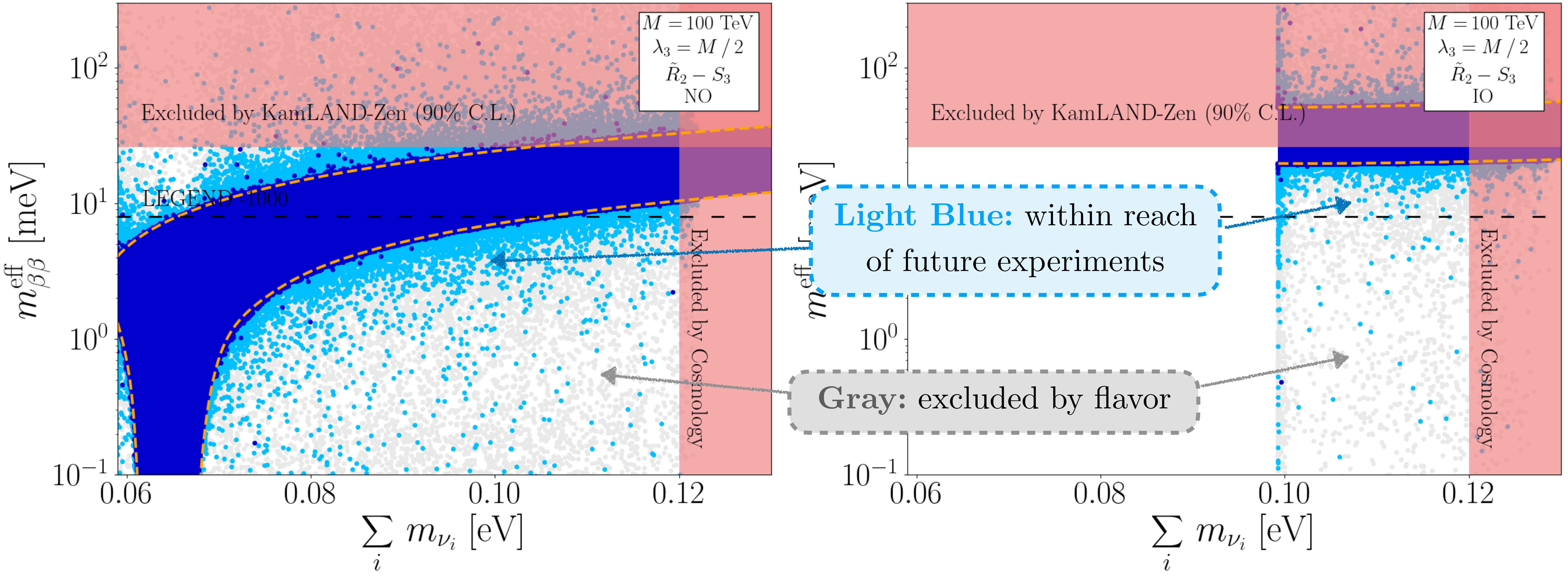
$\Longrightarrow$ **Ambiguity between NO and IO** (more information needed, e.g. oscillation determination of ordering)

Neutrinoless double-beta decay

M = 100 TeV

Contributions up to d = 7

$$\left(m_{\beta\beta}^{\text{eff}}\right)^2 \equiv m_{\beta\beta}^2 + \delta m_{\beta\beta}^{2\,(\text{int})} + \delta m_{\beta\beta}^{2\,(\text{LQ})} \;;\quad m_{\beta\beta} = \sum_i U_{ei}^2 m_{\nu_i}$$



Complementarity between $0\nu\beta\beta$ and LFV probes!

Conclusions

❖ **Neutrino masses** and oscillation are clear **evidence of physics BSM**

❖ **Majorana neutrino masses** can be **generated at one loop** in scalar Leptoquark models:

$\Longrightarrow$ New **chirality-enhanced ($\propto E/m_\nu$) d $=$ 7 contributions** to $0\nu\beta\beta$ at tree-level!

❖ **Ambiguity between NO and IO** (more info is needed, e.g. oscillation)!

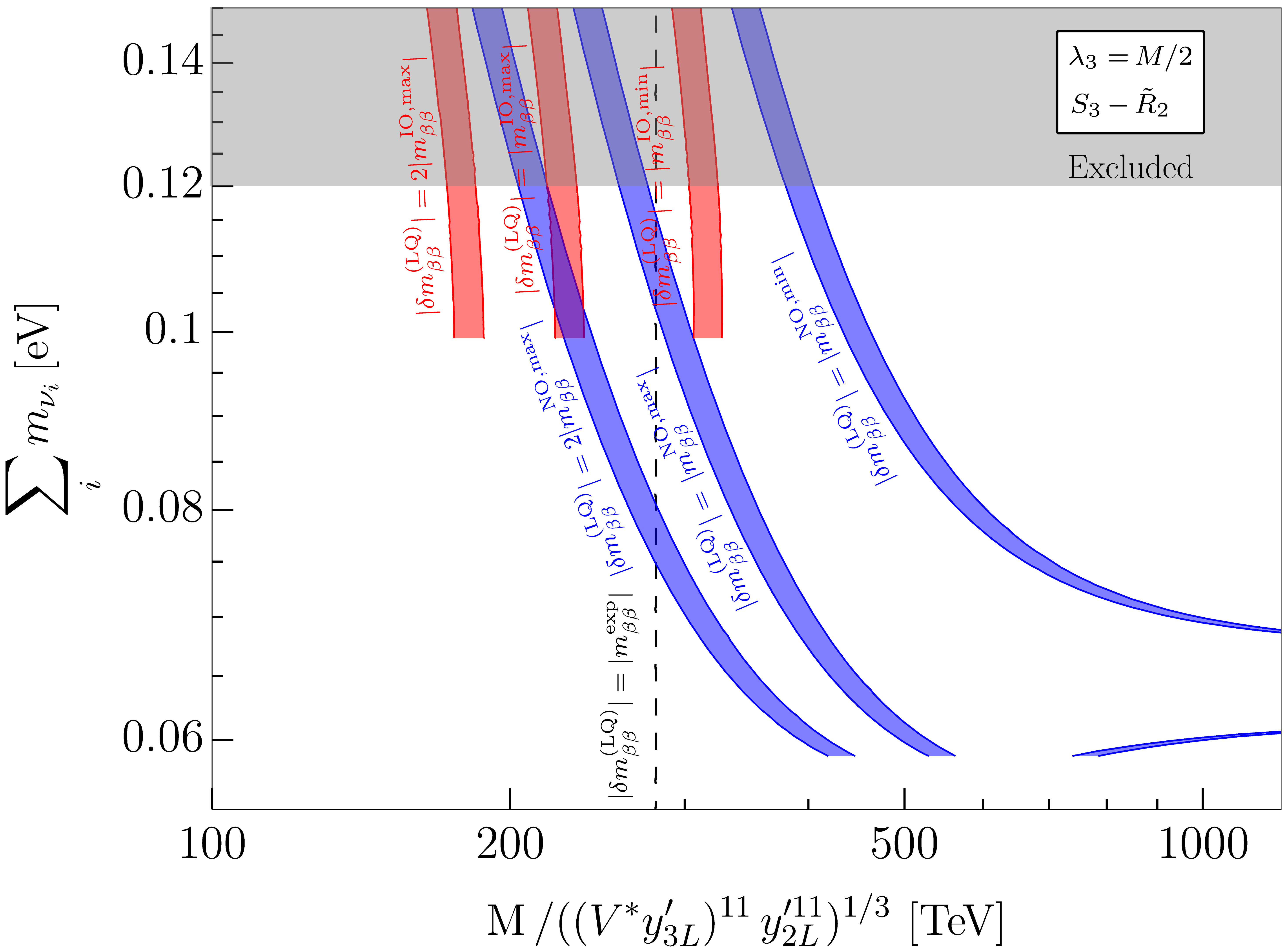
❖ **Flavor constraints** can probe a substantial part of the parameter space

$\Longrightarrow$ Large exp. improvements in $\mu \to e$ conversion in nuclei in the near future

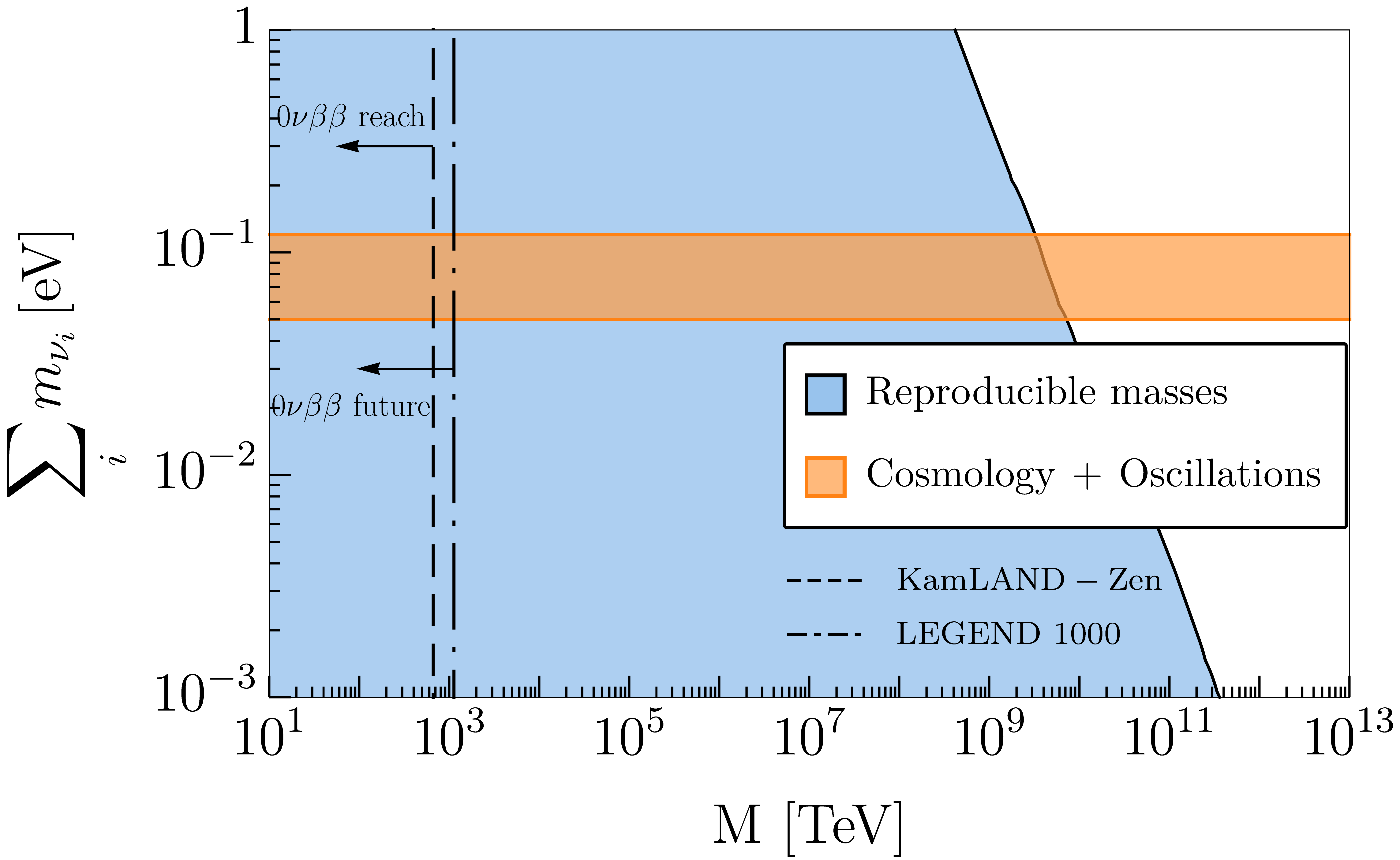
$\Longrightarrow$ **Complementarity to future searches of $0\nu\beta\beta$**

Thank you!

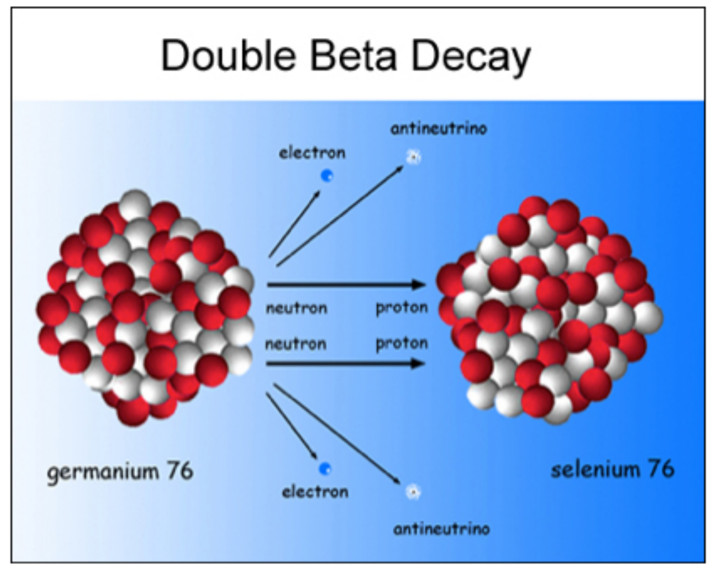
Neutrinoless double-beta decay and m_ν

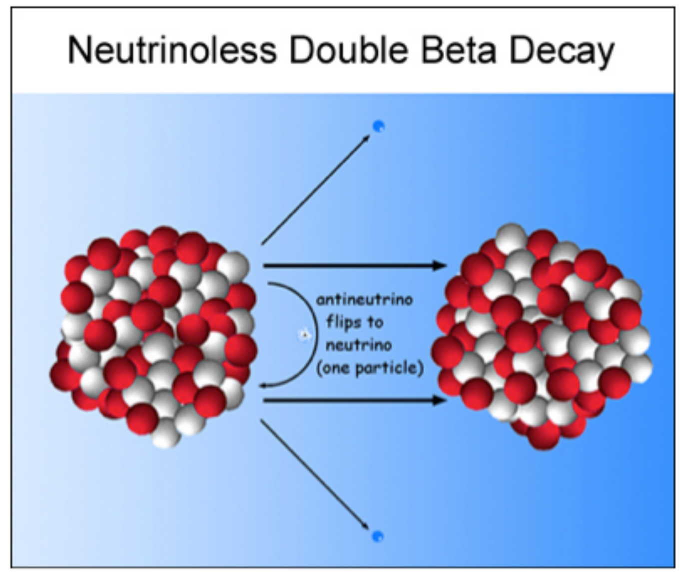


Reproducible m_ν



Neutrinoless double-beta decay





❖ Effective Field theory approach

E

Scalar Leptoquark Model

$E \sim \Lambda$ — Matching

Running — SMEFT

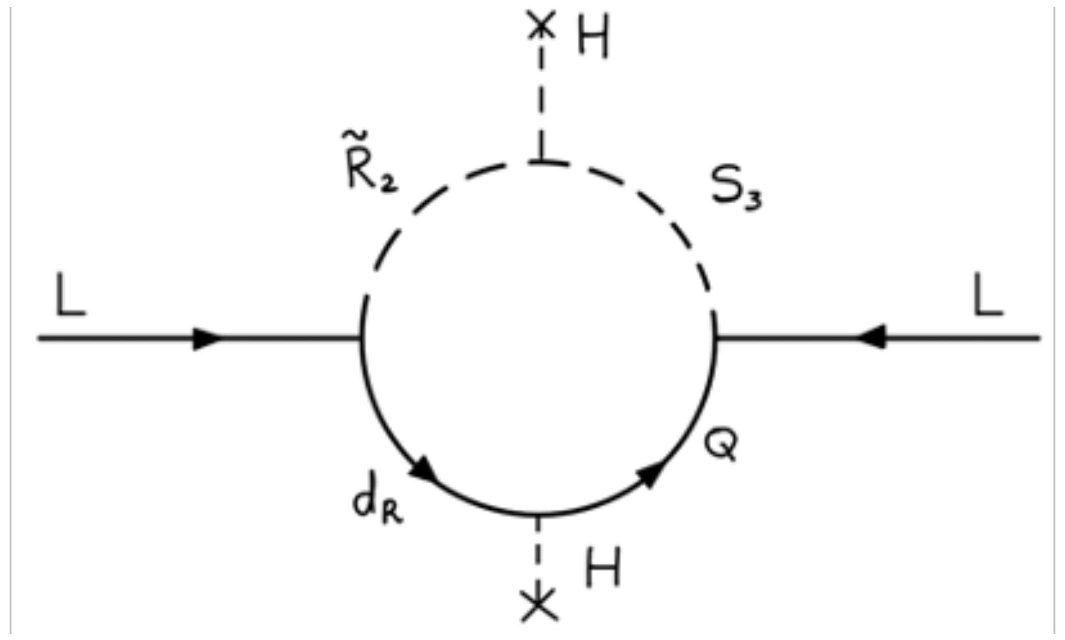
$E \sim m_Z$ — Matching

Running — Low Energy EFT (LEFT)

$E \sim 2$ GeV — Master formula to $0\nu\beta\beta$ from LEFT coefficients

d = 5 operator at loop-level

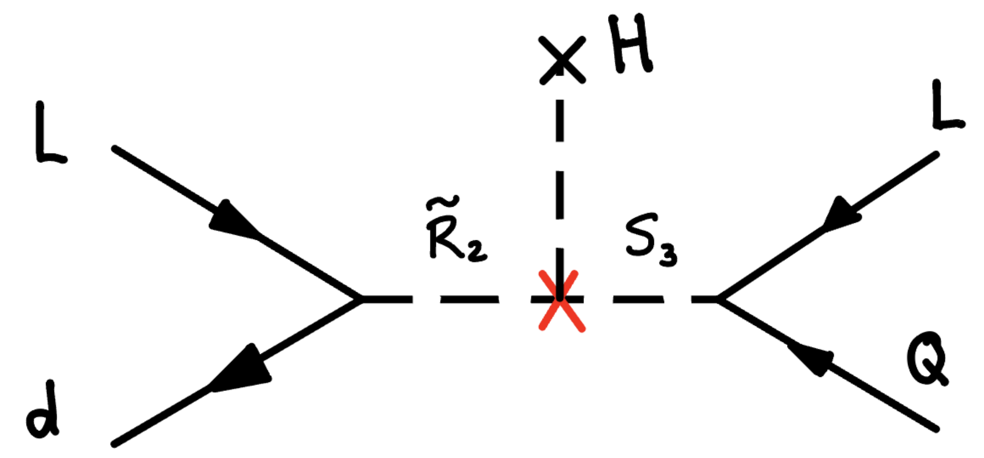
$$\mathcal{O}^{d=5} = \frac{C_{ij}^{(5)}}{\Lambda}(\overline{L_i^C}\tilde{H}^*)(\tilde{H}^\dagger L_j)$$



d = 7 operators at tree-level

$$\left[\mathcal{O}^{(1)}_{LLQ\bar{d}H}\right]_{ijkl} = \epsilon_{ab}\epsilon_{mn}\left(\overline{d_{Rk}}L_i^a\right)\left(\overline{L_j^{m\,C}}Q_l^b\right)H^n,$$

$$\left[\mathcal{O}^{(2)}_{LLQ\bar{d}H}\right]_{ijkl} = \epsilon_{am}\epsilon_{bn}\left(\overline{d_{Rk}}L_i^a\right)\left(\overline{L_j^{m\,C}}Q_l^b\right)H^n$$



Upper limits:

$S_3 - \tilde{R}_2$ model:

$$\frac{|\lambda_3 y'^{11}_{3L} y'^{11}_{2L}|}{M^4} \lesssim (340\ \text{TeV})^{-3}$$

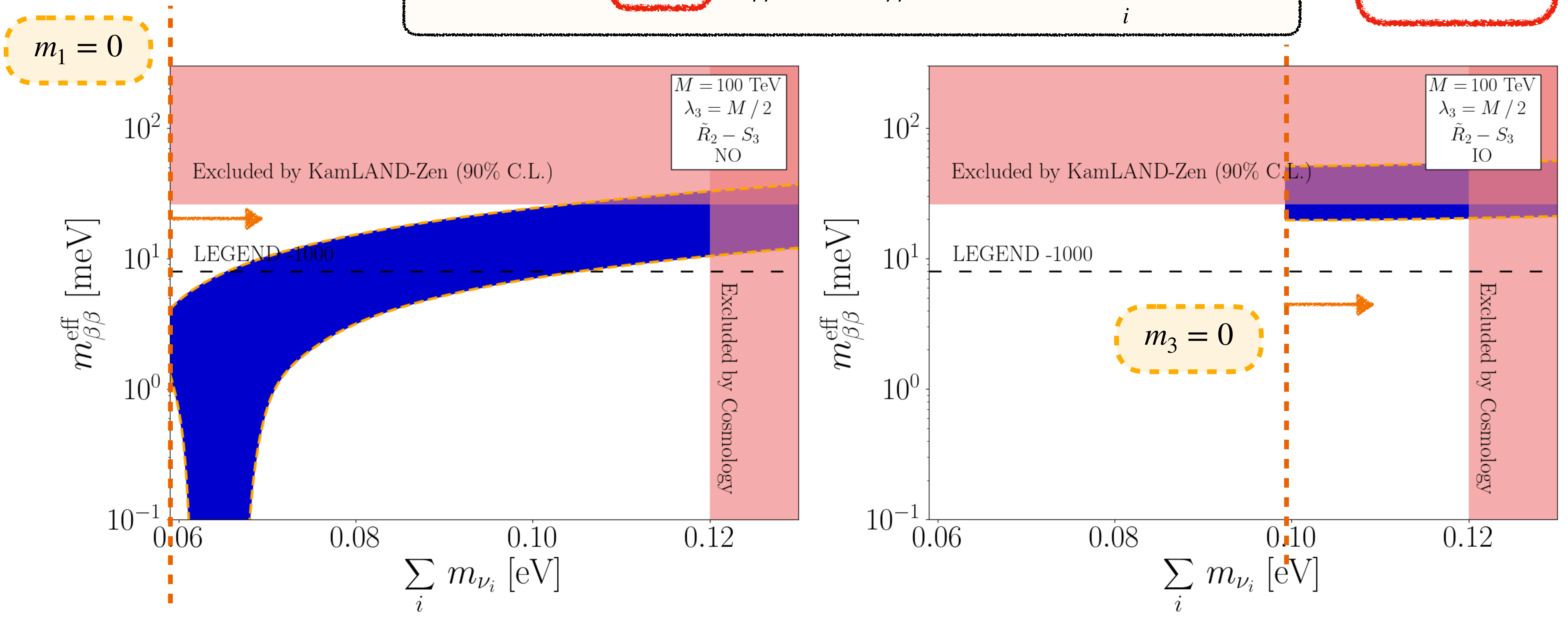
$y'_L = y_L U_{\ell L}$ where $U_{\ell L}$ is the left-handed charged lepton

[V. Cirigliano et al, JHEP 12]

Neutrinoless double-beta decay

$$\left(m_{\beta\beta}^{\rm eff}\right)^2 = \equiv m_{\beta\beta}^2 + \delta m_{\beta\beta}^{2\,(\rm int)} + \delta m_{\beta\beta}^{2\,(\rm LQ)} \quad ; \quad m_{\beta\beta} = \sum_i U_{ei}^2 m_{\nu_i}$$

d = 5



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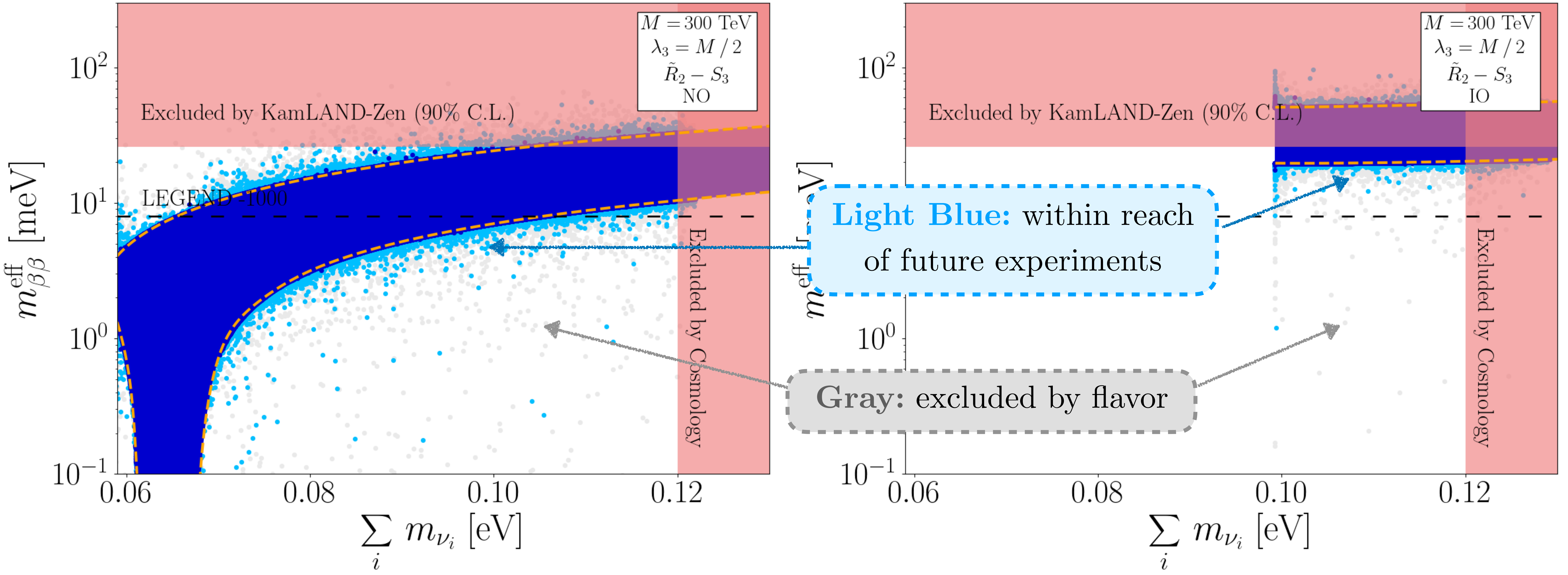
M = 300 TeV



Correlation $0\nu\beta\beta$ vs. $\mu N \to eN$

- Huge improvement in $\mu N \to eN$ experimental sentitivity
  - Present: $B(\mu\mathrm{Au} \to e\mathrm{Au}) < 7 \times 10^{-13}$ (90% C.L.)
  - Future: $B(\mu\mathrm{Al} \to e\mathrm{Al}) \sim \mathcal{O}(10^{-17})$

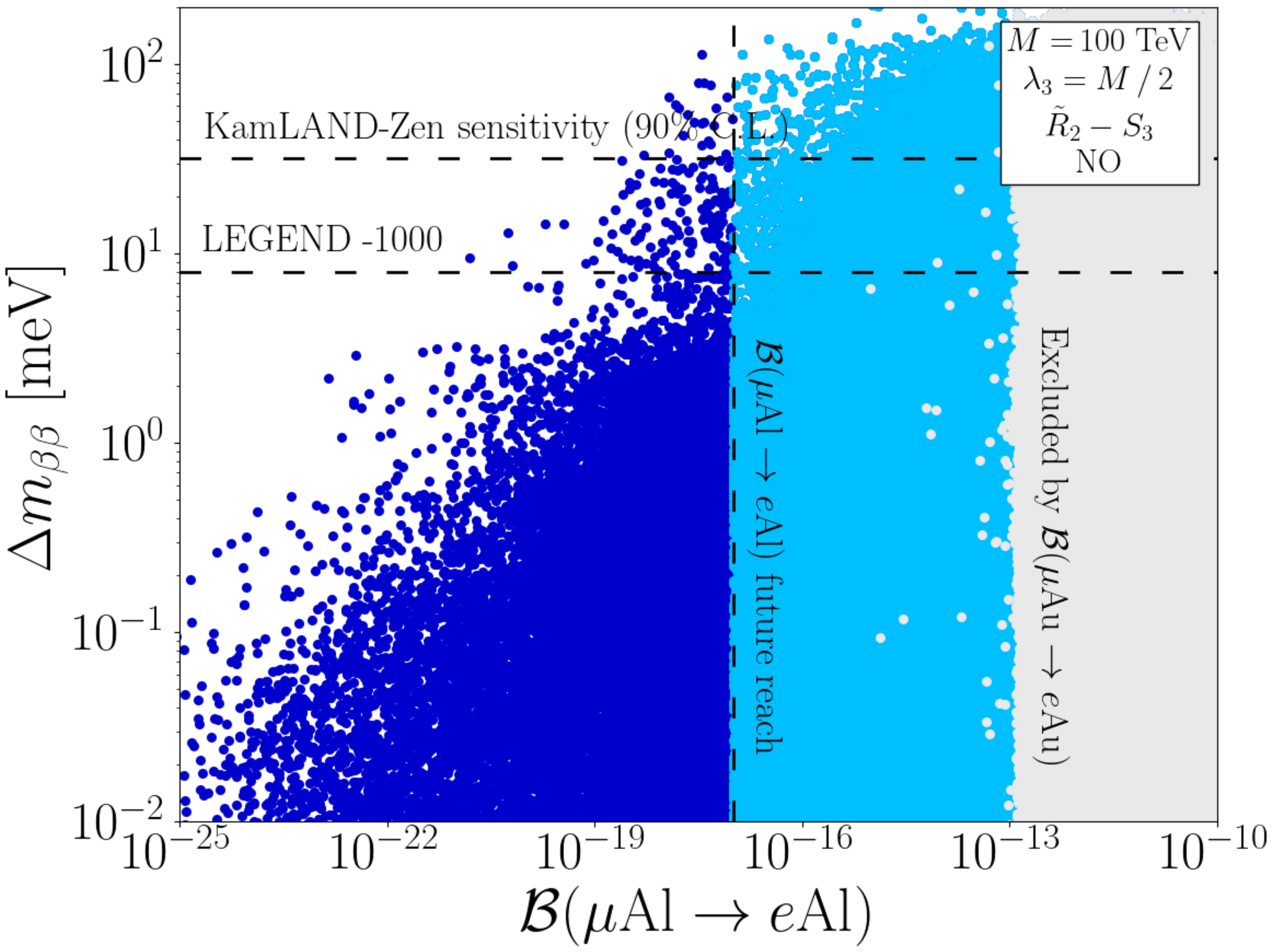
$\Delta m_{\beta\beta} \equiv |m_{\beta\beta}^{\mathrm{eff}\,2} - m_{\beta\beta}^2|^{1/2}$

- Main contributions to $\mu N \to eN$ comes from spin-independent rates

$$\mathcal{B}_{\mu e}^{(N)}\Big|_{\mathrm{SI}} = \frac{2\,G_F^2\,m_\mu^5}{\Gamma_{\mathrm{capt}}^N\,\pi^2}(Z\alpha_{\mathrm{em}})^3\left|Z\,F_p^N(m_\mu)\,c_{VL}^{(p)} + (A-Z)\,F_n^N(m_\mu)\,c_{VL}^{(n)}\right|^2 + (L \leftrightarrow R)$$

- Scales with atomic number Z

Complementarity between $0\nu\beta\beta$ and LFV probes!

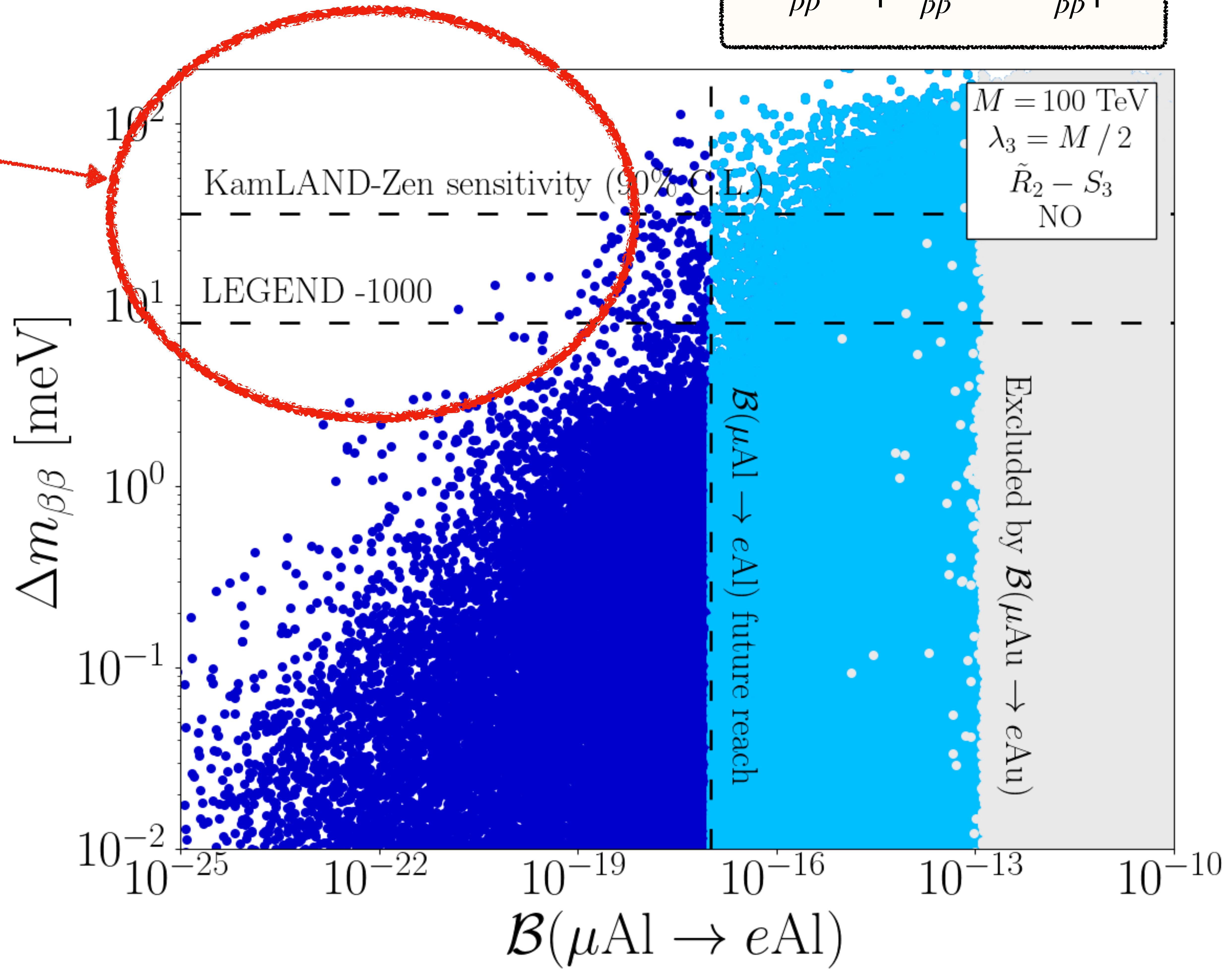


Correlation $0\nu\beta\beta$ vs. $\mu N \to eN$

❖ Huge improvement in $\mu N \to eN$ experimental sentitivity

Can we have suppressed $\mu N \to eN$ while $m_{\beta\beta}$ is large?

$\Delta m_{\beta\beta} \equiv |m_{\beta\beta}^{\mathrm{eff}\,2} - m_{\beta\beta}^2|^{1/2}$



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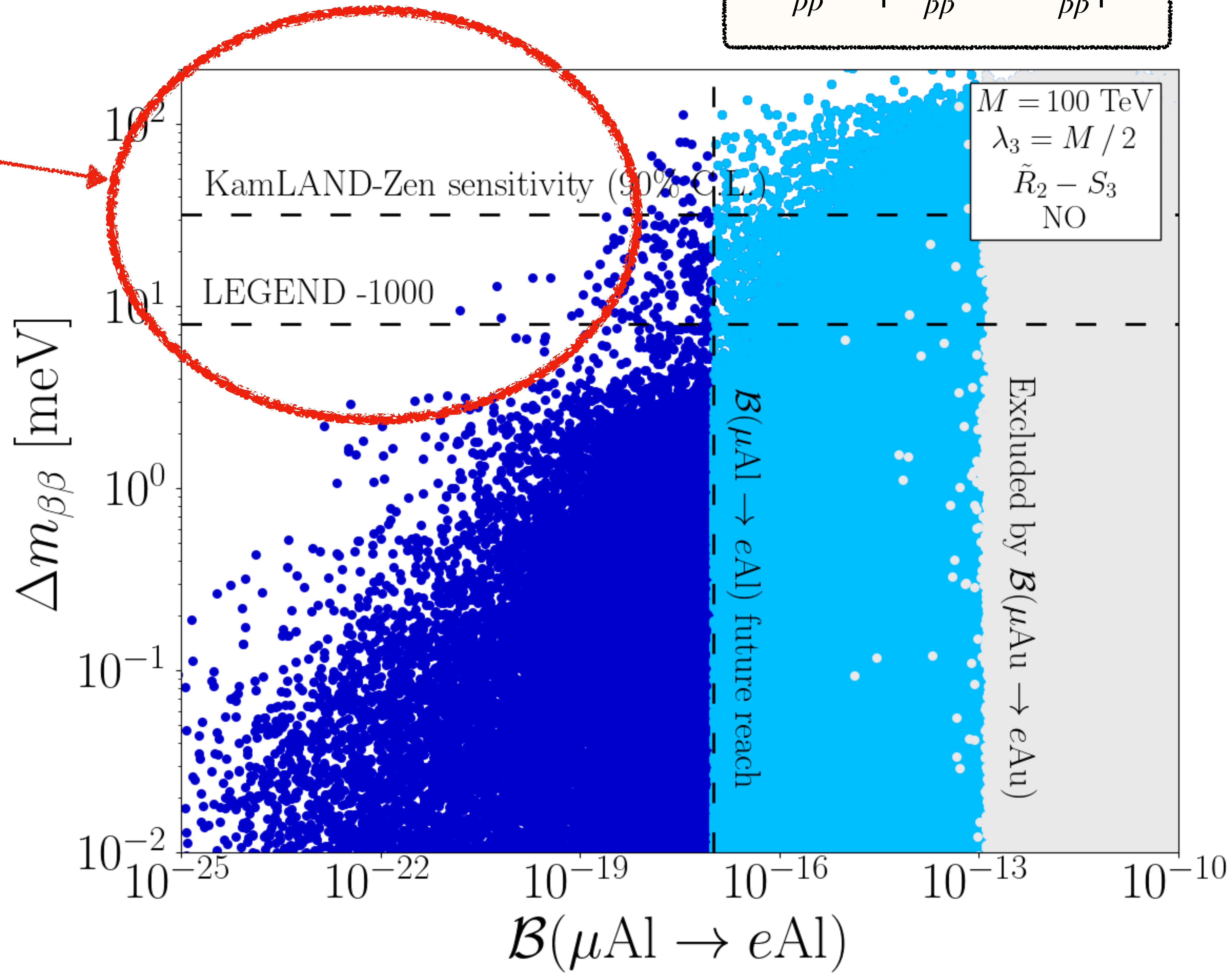
❖ Huge improvement in $\mu N \to eN$ experimental sentitivity

Can we have suppressed $\mu N \to eN$ while $m_{\beta\beta}$ is large?

1) Cancellation between S_3 and $\tilde{R}_2$ contributions

2) Coupling to muons vanish in physical basis

$\Delta m_{\beta\beta} \equiv |m_{\beta\beta}^{\text{eff}\,2} - m_{\beta\beta}^2|^{1/2}$



Correlation $0\nu\beta\beta$ vs. $\mu N \to eN$

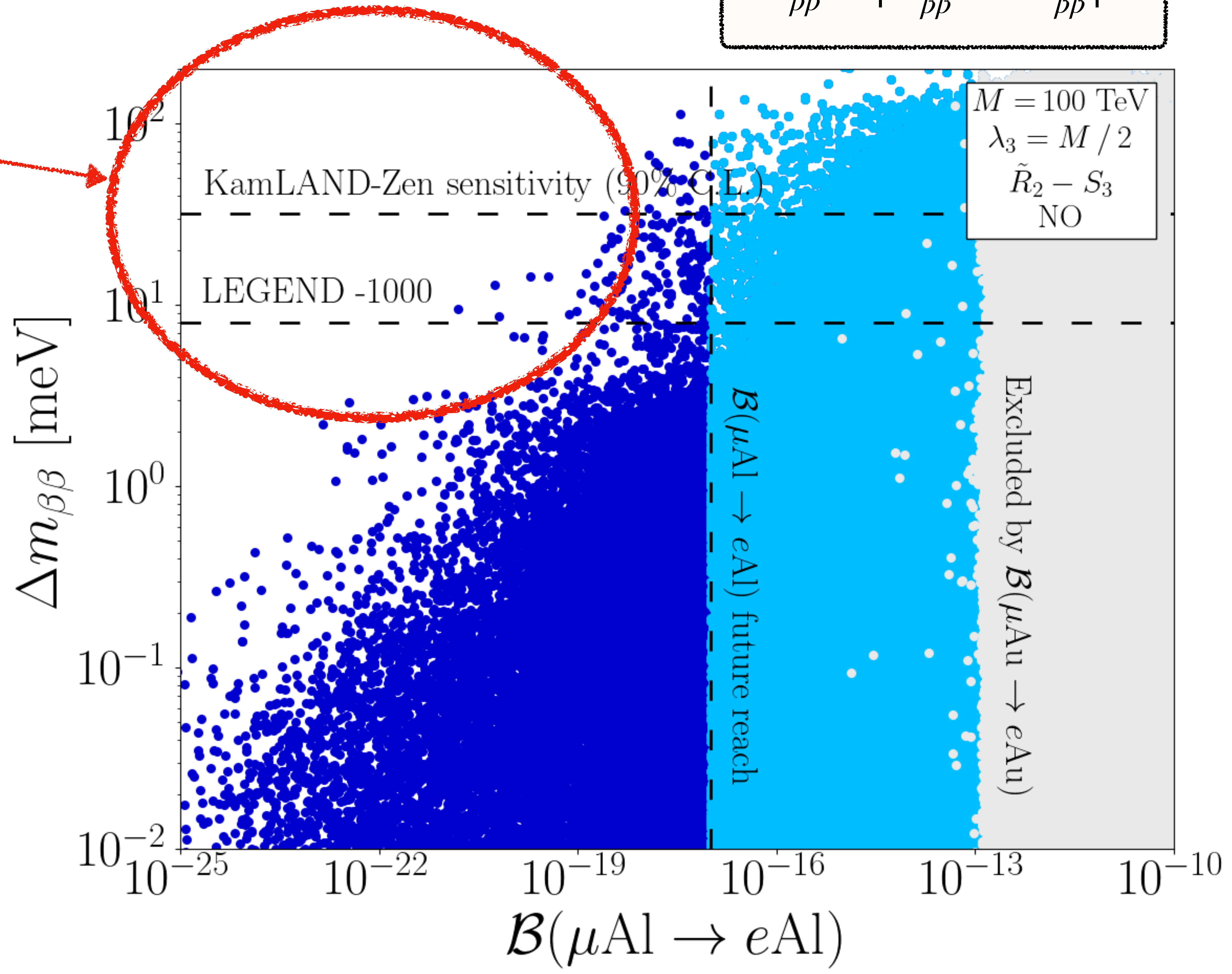
❖ Huge improvement in $\mu N \to eN$ experimental sentitivity

Can we have suppressed $\mu N \to eN$ while $m_{\beta\beta}$ is large?

Condition 1: Cancellation between S_3 and $\tilde{R}_2$ contributions

‣ Additional spin-dependent contributions.
$\Longrightarrow$ enhanced when we have cancelations in SI part

$\Delta m_{\beta\beta} \equiv |m_{\beta\beta}^{\text{eff}\,2} - m_{\beta\beta}^2|^{1/2}$



Correlation $0\nu\beta\beta$ vs. $\mu N \to eN$

❖ Huge improvement in $\mu N \to eN$ experimental sentitivity

<u>Condition 2:</u> Coupling to muons vanish in physical basis

- <u>$S_3 - \tilde{R}_2$:</u> $(y'_{2L})_{12} = (V^* y'_{3L})_{12} = (y'_{3L})_{12} = 0$
- Consequences to neutrino mass matrix structure

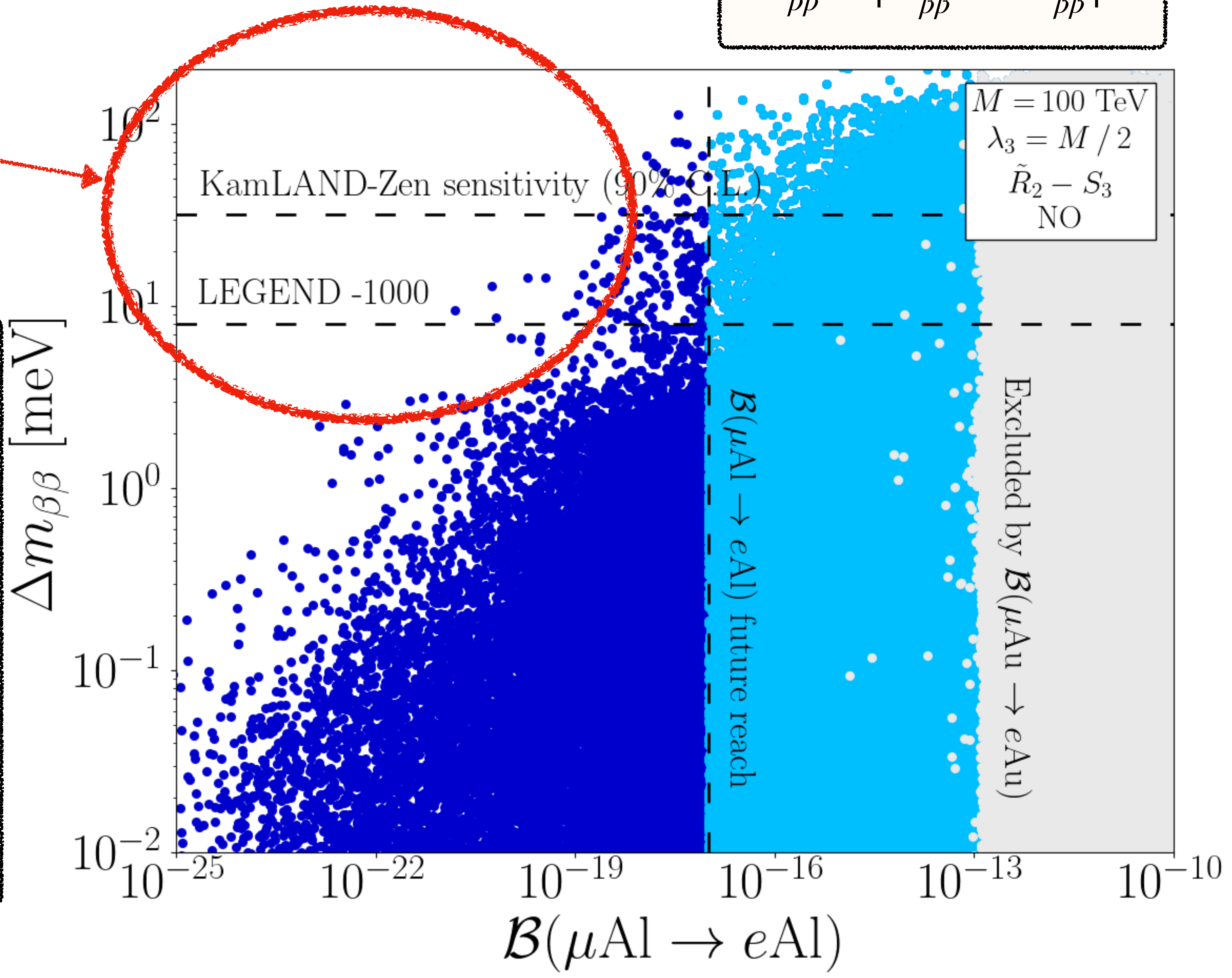
$$m_\nu = -\frac{3\lambda_3}{16\sqrt{2}\pi^2}\frac{v^2}{M^2}\left(y_{2L}^T \cdot \hat{y}_d \cdot y_{3L} + y_{3L}^T \cdot \hat{y}_d \cdot y_{2L}\right)$$

$U = U_{PMNS}$

$y'_L = y_L U_{\ell L}$

$$U \hat{m}_\nu U^T = -\frac{3\lambda_3}{16\sqrt{2}\pi^2}\frac{v^2}{M^2}\left(y'^T_{2L} \cdot \hat{y}_d \cdot y'_{3L} + y'^T_{3L} \cdot \hat{y}_d \cdot y'_{2L}\right)$$

$\Delta m_{\beta\beta} \equiv |m_{\beta\beta}^{\text{eff}\,2} - m_{\beta\beta}^2|^{1/2}$



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- **We did not find solutions compatible with neutrino oscillations**