

Shell Model Photo-Strength Functions and Pygmy Dipole Resonance

E1 response of sd-shell nuclei using the configuration interaction shell model

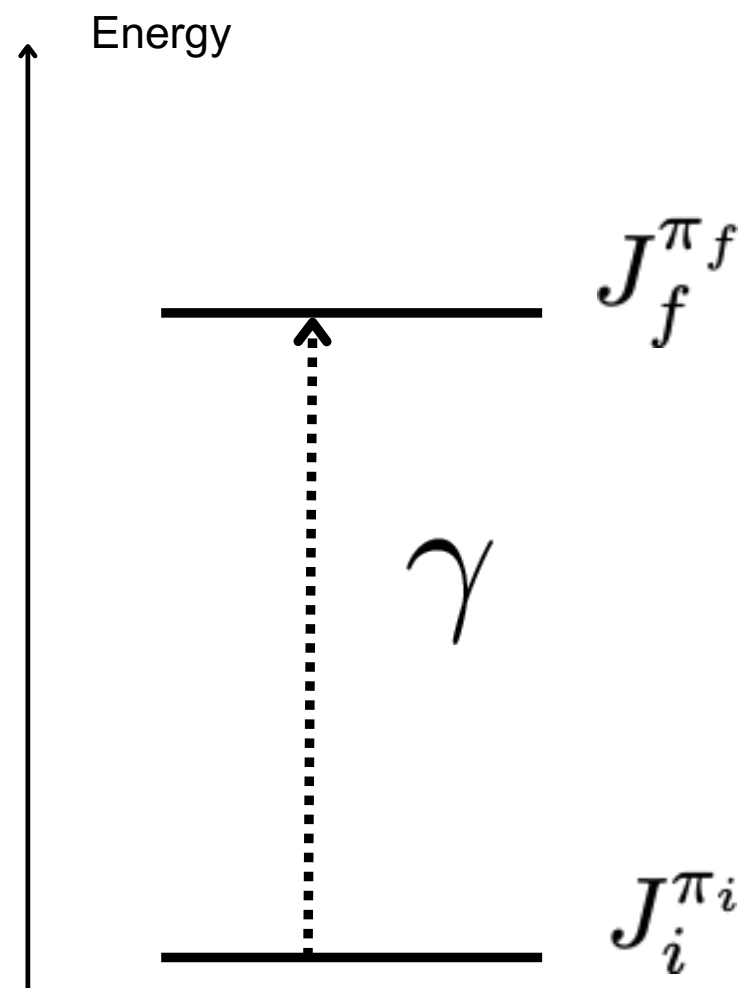
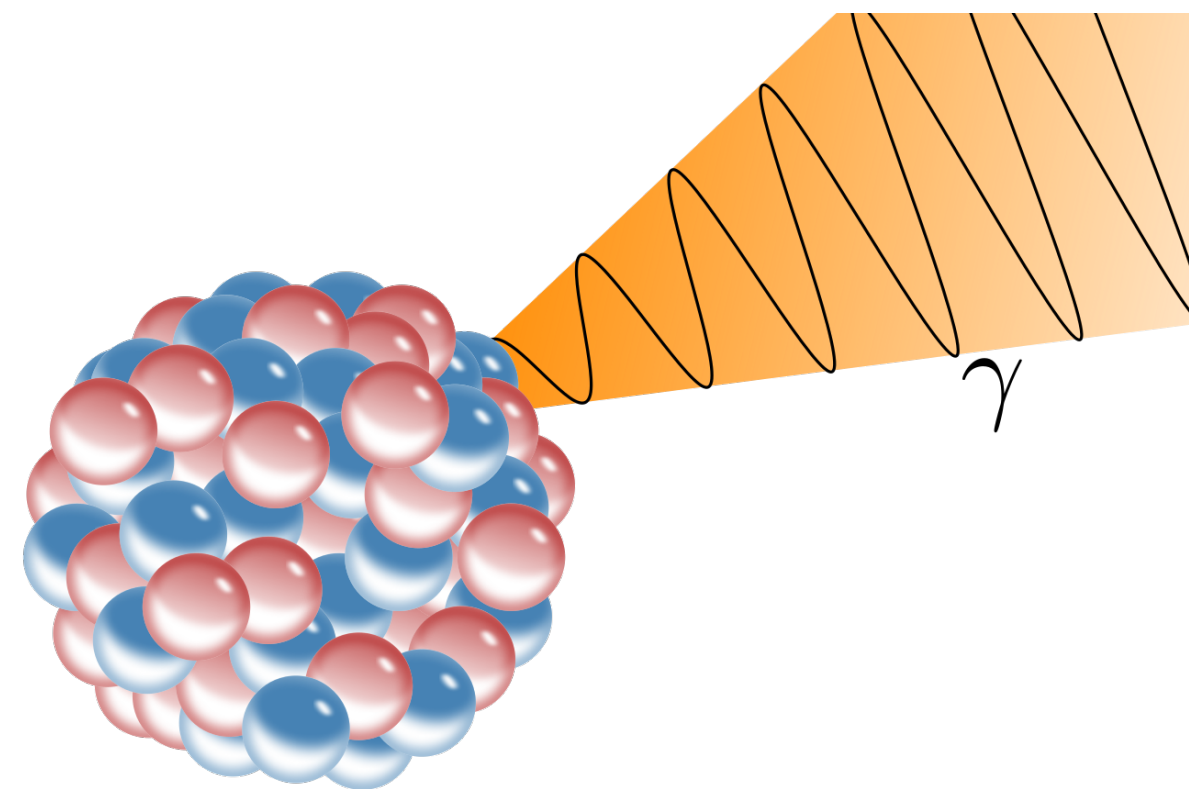
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Under the supervision of Kamila Sieja

Photo-strength function (PSF) definition

Nuclear electromagnetic transitions

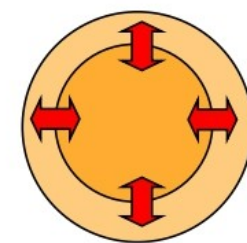


Transition probability rate

$$\Gamma_{fi} \propto B_{fi}(X\lambda)$$

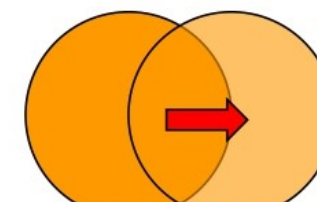
$$B_{fi}(X\lambda) = \frac{1}{2J_i + 1} |\langle \psi_f | \mathcal{O}_\lambda | \psi_i \rangle|^2$$

Monopole



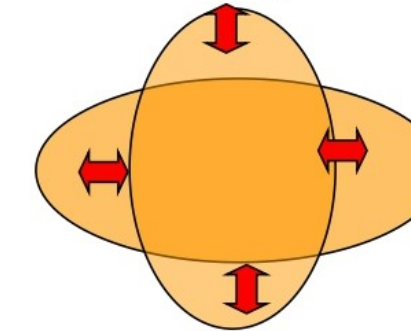
IS GMR

Dipole



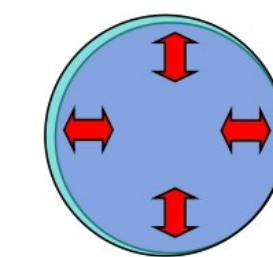
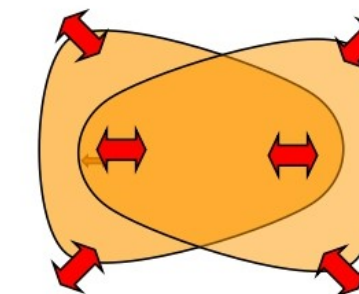
spurious state

Quadrupole

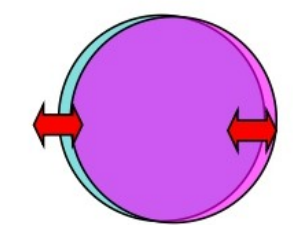


GQR

Octupole



IV GMR



IV GDR

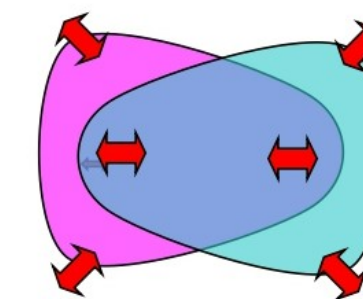
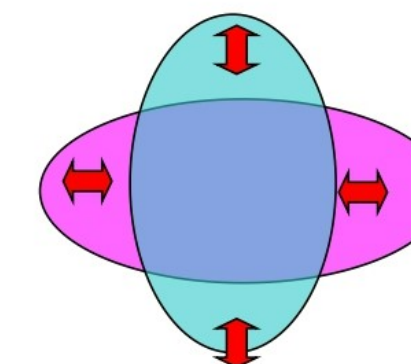
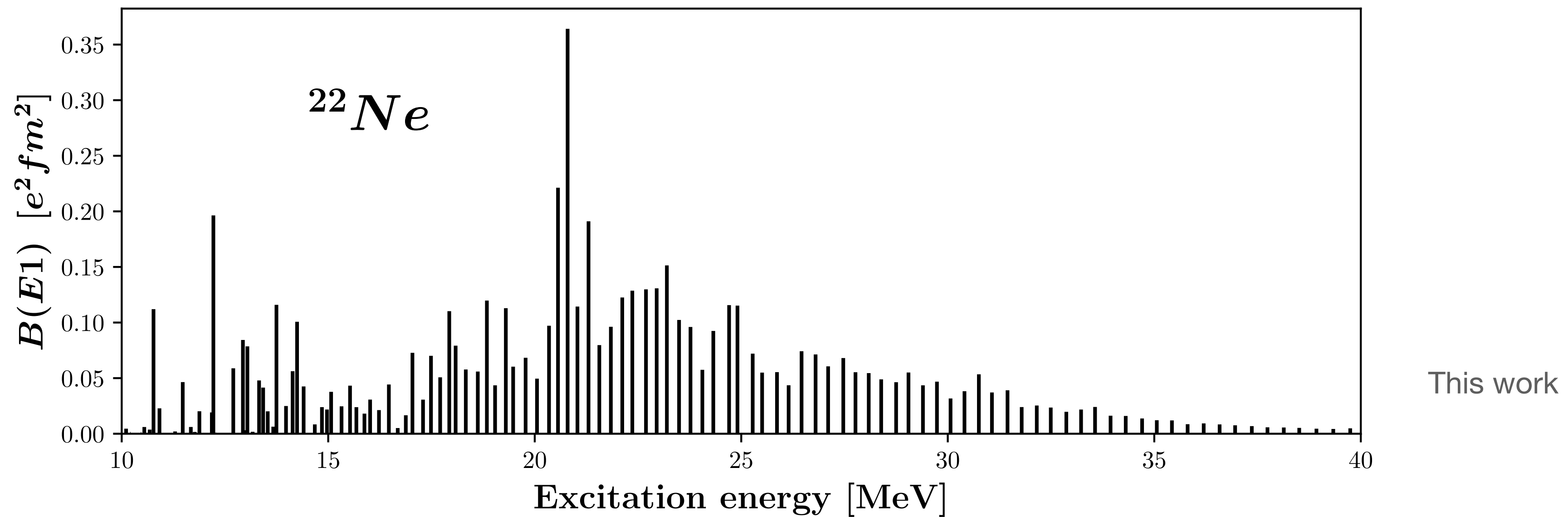


Photo-strength function (PSF) definition

Photo-strength function (PSF)

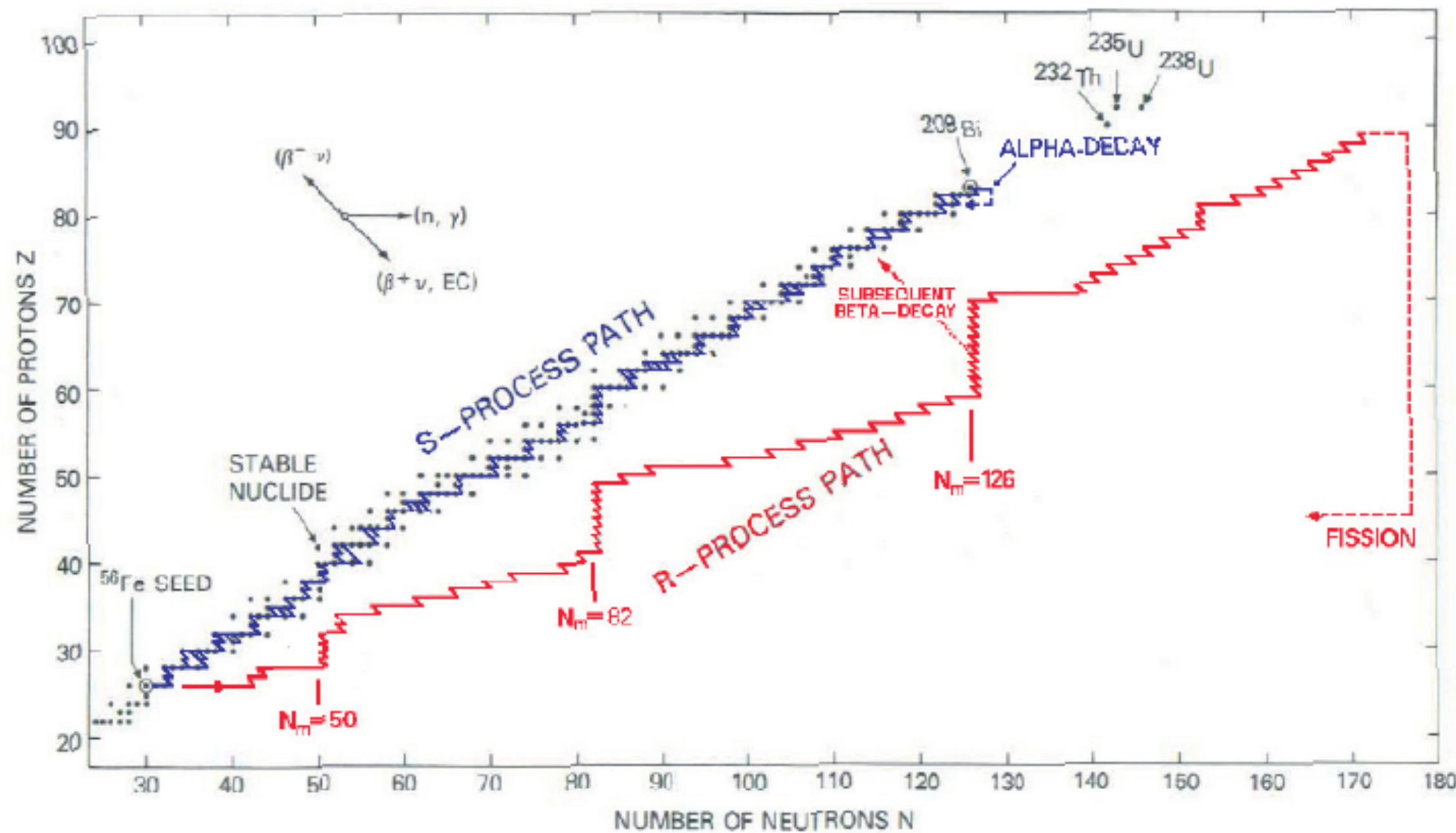
$$S(\varepsilon) = \sum_f B_{fi} \delta(\varepsilon - \varepsilon_f)$$



Global response to an EM perturbation of a given type and multipolarity e.g. E1

Why is it interesting ?

R-process nucleosynthesis

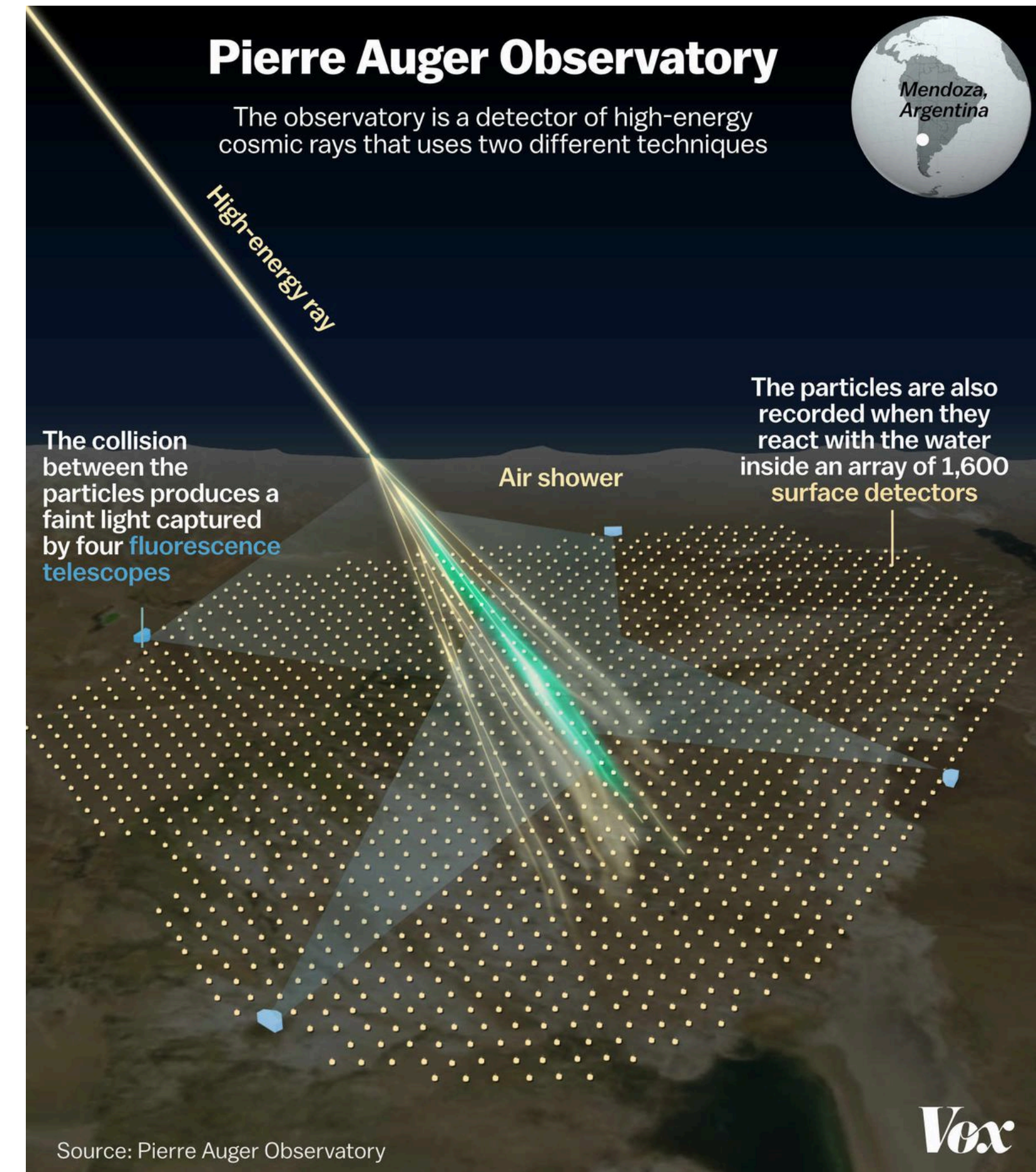


M.L Pumo. The s-process nucleosynthesis in massive stars: current status and uncertainties due to convective overshooting. (2012) ISBN 978-953-51-0473-5

Hauser-Feshbach model for (n,γ) cross sections

$$\sigma_{(n,\gamma)} \sim T_{\gamma}(\varepsilon, X\lambda) \propto S(\varepsilon, X\lambda)$$

Ultra High Energetic Cosmic Rays (UHECR)



Source: Pierre Auger Observatory

Vox

Why is it interesting ?

Neutron matter equation of state (EOS)

$$\mathcal{E}(\rho, \alpha) = \mathcal{E}_{\text{SNM}}(\rho) + \alpha^2 \mathcal{S}(\rho) + \mathcal{O}(\alpha^4)$$

$$\rho = (\rho_n + \rho_p) \quad \alpha = (\rho_n - \rho_p) / \rho$$

where the **density-dependent symmetry energy** is:

$$\mathcal{S}(\rho) = J + L \frac{(\rho - \rho_0)}{3\rho_0} + \dots$$

symmetry energy
at saturation
density

slope parameter,
related to **pressure**
of pure neutron
matter at saturation
density

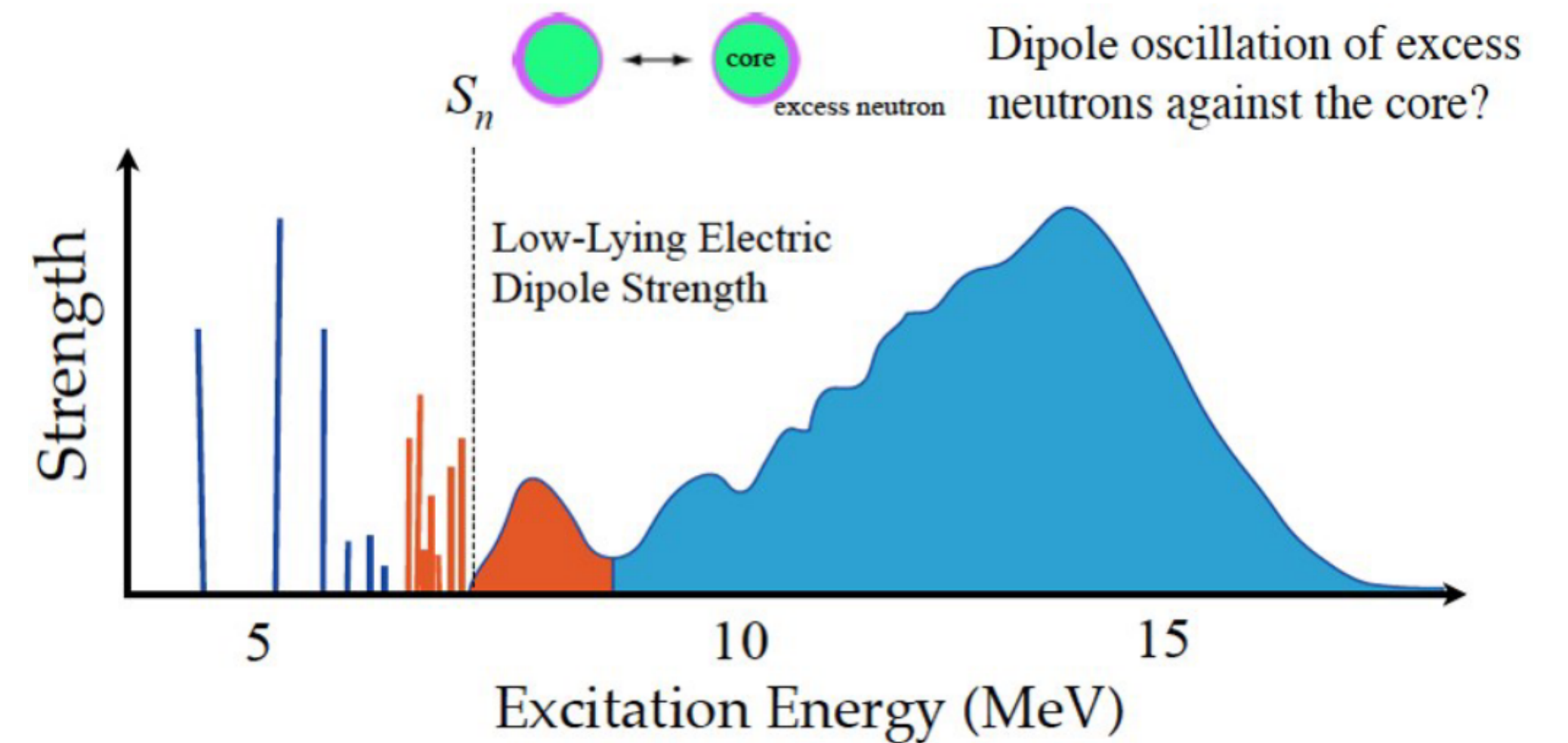
FRANCESCA BONAITI, JGU MAINZ RISING RESEARCHERS SEMINAR SERIES (2023)

- Neutron skin thickness
- Electric dipole polarizability

$$\alpha = \frac{8\pi}{9} \int d\varepsilon \frac{S(\varepsilon)}{\varepsilon}$$

Nuclear structure

Pigmy Dipole Resonance (PDR)



Tamii (2016). Adelaide International Conference

Model and Method

- QRPA approaches
- Generator coordinates method
- Time-dependent Hartree-Fock

Configuration Interaction Shell Model

$$S(\varepsilon) = \sum_f B_{fi} \delta(\varepsilon - \varepsilon_f)$$

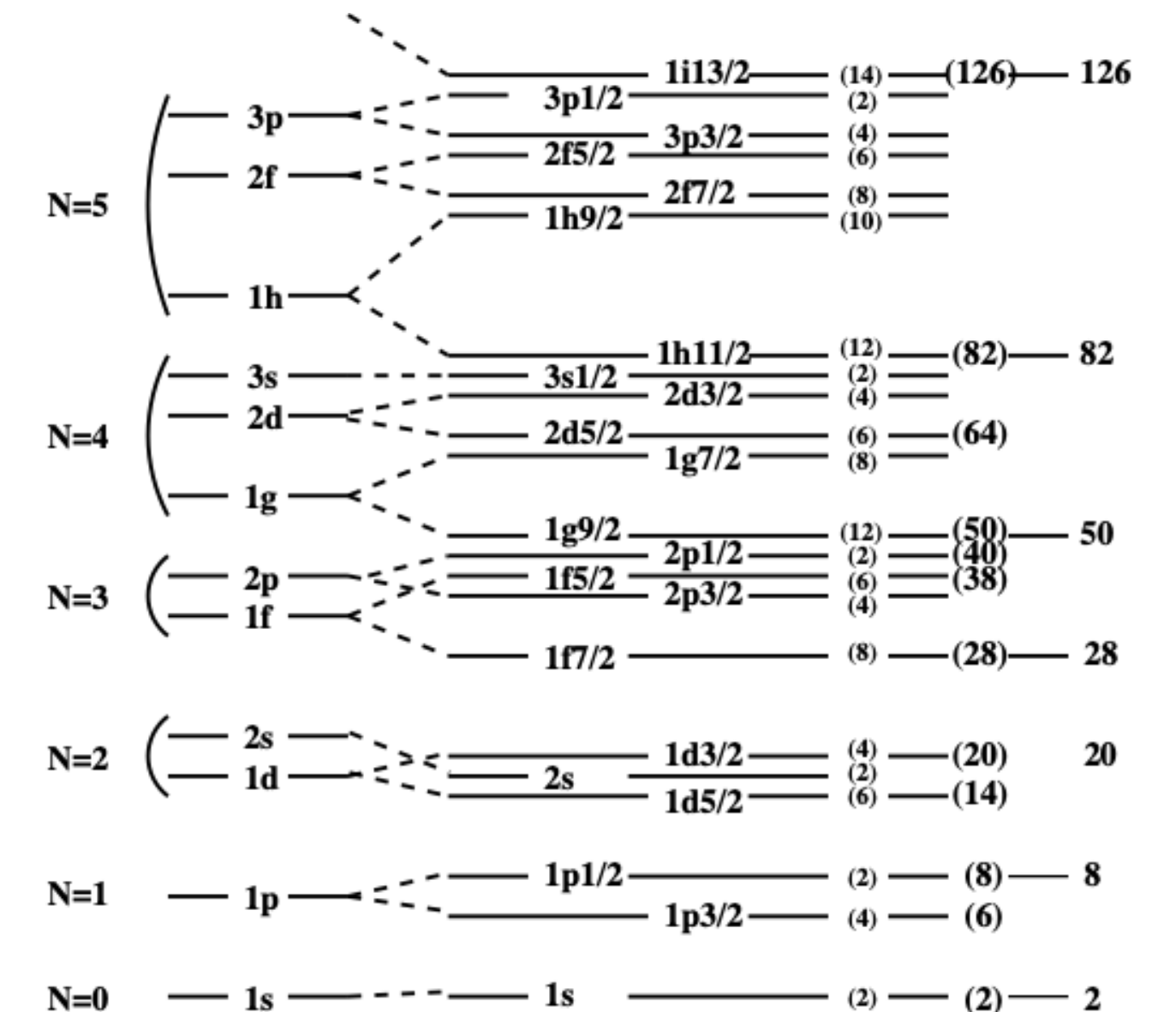
A-Body Quantum Problem

$$B_{fi}(X\lambda) = \frac{1}{2J_i + 1} |\langle \psi_f | \mathcal{O}_\lambda | \psi_i \rangle|^2$$

$$i\hbar \frac{\partial}{\partial t} |\psi\rangle = \mathcal{H} |\psi\rangle$$

Hilbert space basis : product of one body harmonic oscillator states

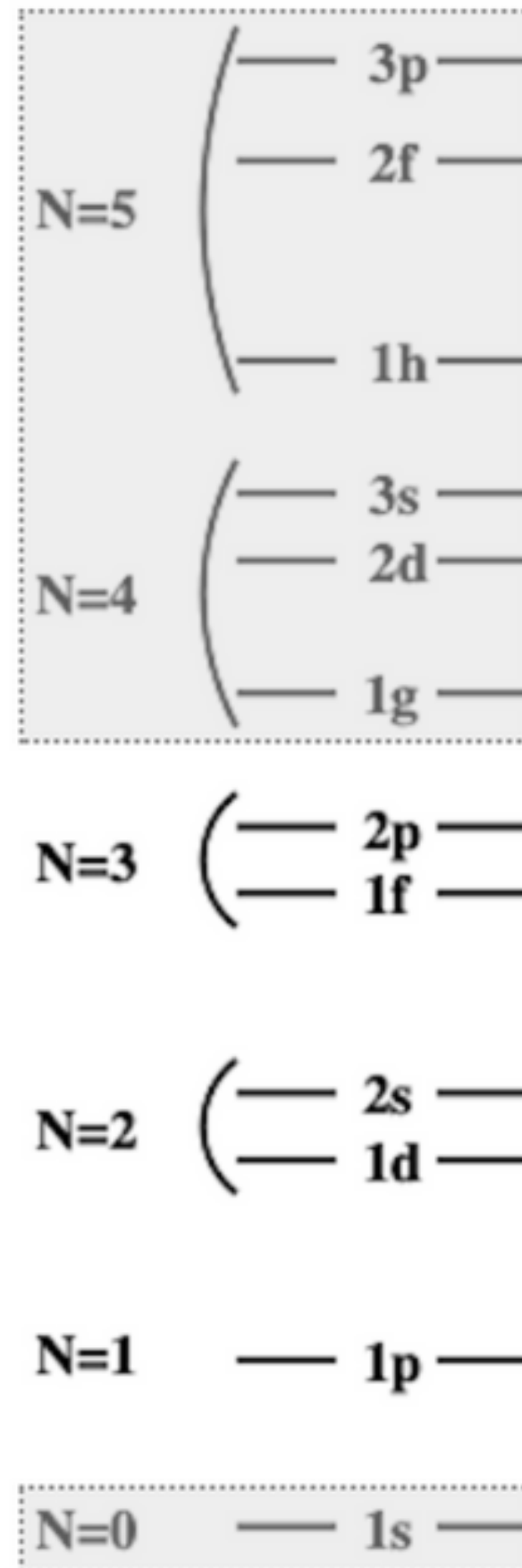
$$|n, l, j, m_j, \tau_3\rangle$$



Model and Method

Configuration Interaction Shell Model

Restriction to a sub space : the valence space



sd shell nuclei : psdpf model space :

$$\mathcal{H}|\psi\rangle = E|\psi\rangle \quad \text{Mapped to} \quad \mathcal{H}_{eff}|\tilde{\psi}\rangle = E|\tilde{\psi}\rangle$$

- Lee-Suzuki
- Similarity renormalization group
- + phenomenological fit



Available online at www.sciencedirect.com

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Nuclear Physics A 864 (2011) 113–127



www.elsevier.com/locate/nucphysa

A PSDPF interaction to describe the $1 \hbar\omega$ intruder states in sd shell nuclei

M. Bouhelal ^{a,*}, F. Haas ^b, E. Caurier ^b, F. Nowacki ^b, A. Bouldjedri ^c

Model and Method

Configuration Interaction Shell Model

Iterative diagonalization procedure : Lanczos Algorithm

$$H |\psi\rangle = E |\psi\rangle \quad H = \sum_i \epsilon_i c_i^\dagger c_i + \sum_{ijkl} V_{ijkl} c_i^\dagger c_j^\dagger c_l c_k + \beta H_{COM} \quad H = U D U^\dagger$$

Bare electric dipole operator

$$\hat{O}_{1\mu} = -e \frac{Z}{A} \sum_{i=1}^N r_i Y_{1\mu}(\hat{r}_i) + e \frac{N}{A} \sum_{i=1}^Z r_i Y_{1\mu}(\hat{r}_i)$$

Initial pivot as $\frac{\hat{O}_{10} |GS\rangle}{\sqrt{S_0}}$

$$S(\varepsilon) = \sum_f U_{1f}^2 S_0 \delta(\varepsilon - \varepsilon_f) = \sum_f B_{fi} \delta(\varepsilon - \varepsilon_f)$$

Reduced transition probability

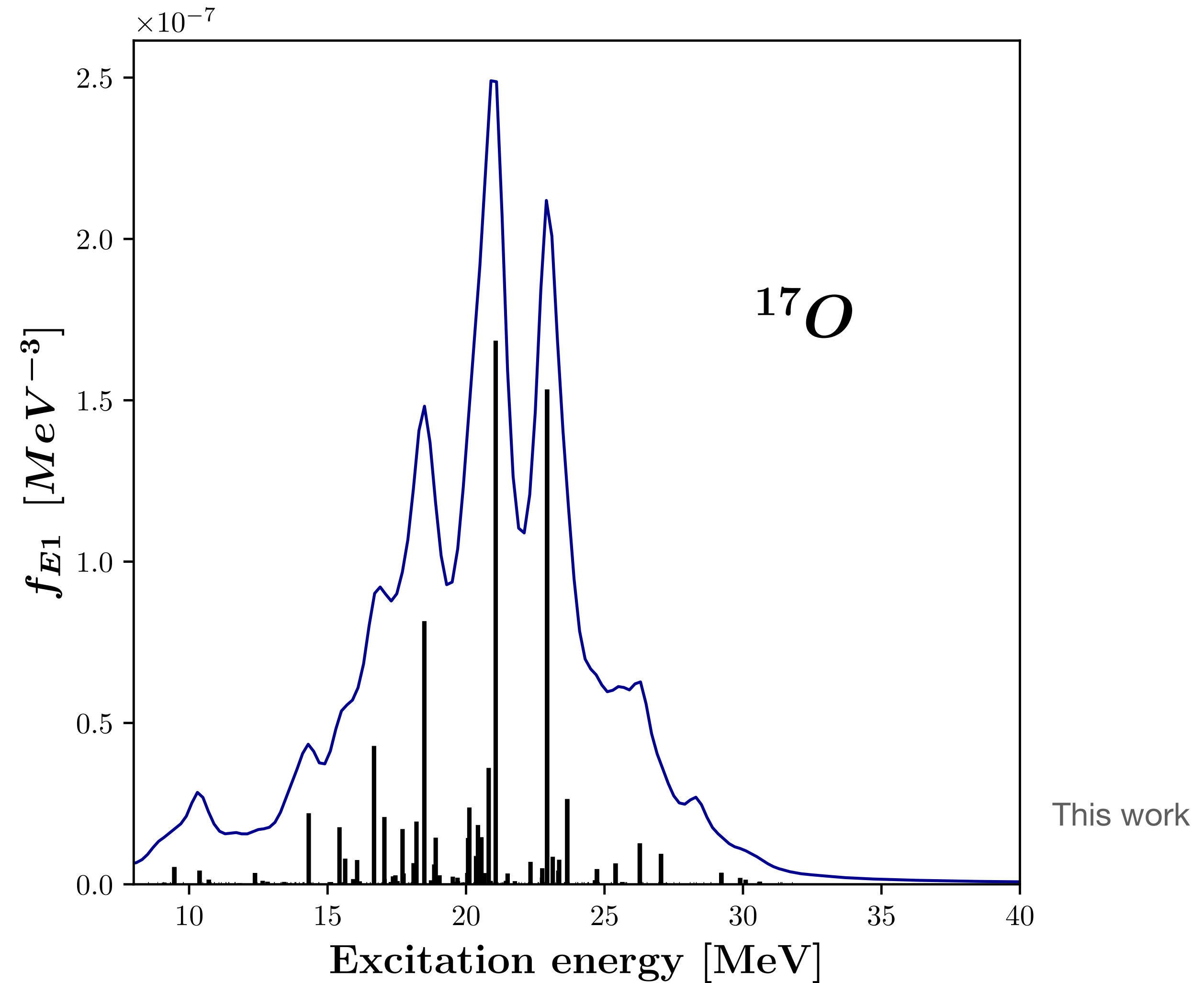
$$B_{fi}(X\lambda) = \frac{1}{2J_i + 1} |\langle \psi_f || \mathcal{O}_\lambda || \psi_i \rangle|^2$$

Model and Method

To compare to experiments and to account for the quasi continuum : Lorentzian folding

$$f(\varepsilon) = \sum_f \frac{1}{2\pi} \frac{\Gamma}{(\varepsilon - \varepsilon_f)^2 + \frac{\Gamma^2}{4}} B_{fi}$$

$$\Gamma = 1 \text{ MeV}$$



Model and Method

E1 Quenching Factor

Systematic strength overestimation

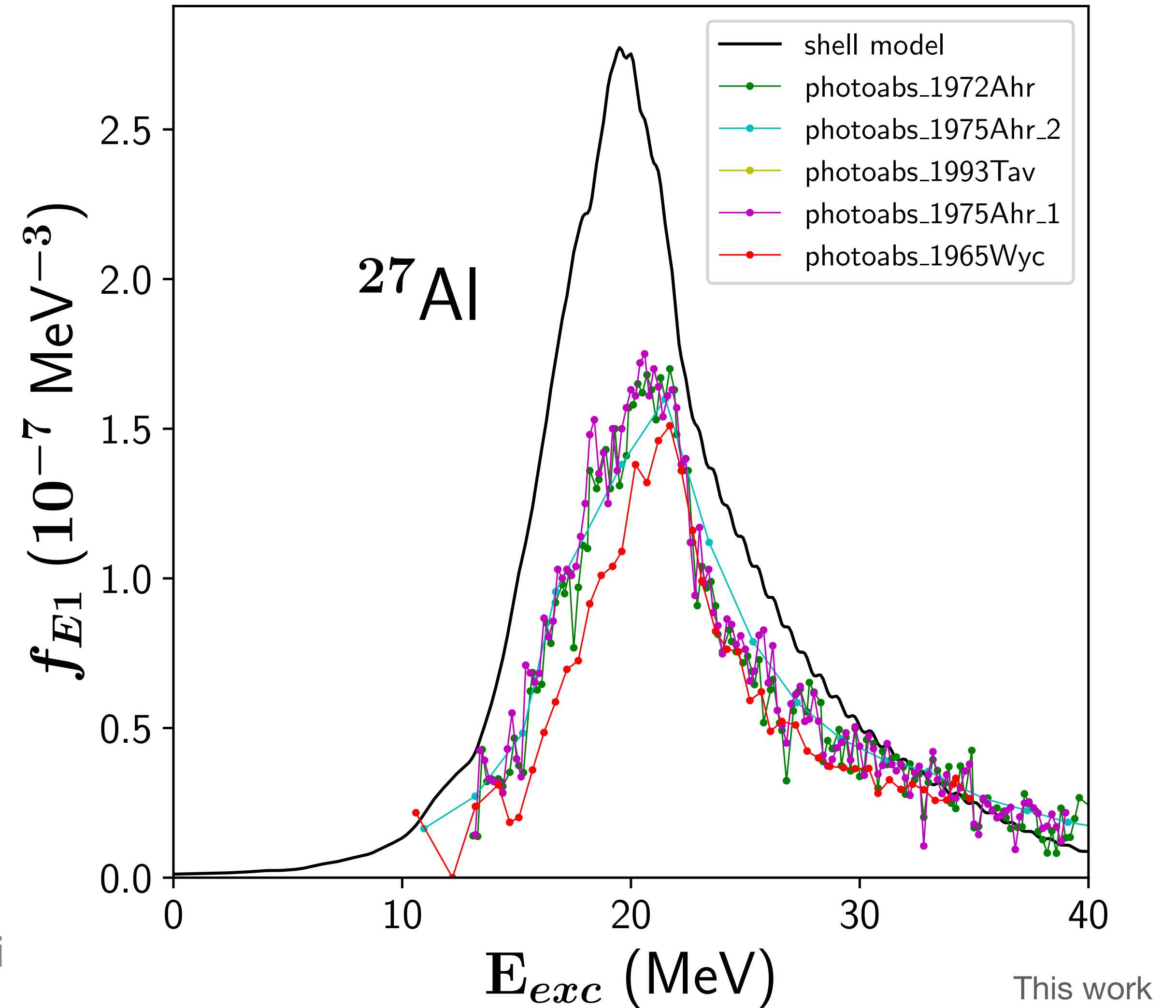
Non-renormalization of E1 operator : bare \hat{O}_1

Known issue e.g. Gamow-Teller, M1 cases

Caurier et al.
The Shell Model as Unified View of Nuclear Structure (2005)

To account for the core and external spaces correlations :

- Many body perturbation theory on E1 operator (e.g. Lee-Suzuki approach)
- Phenomenological Quenching : scaling to experimental data
- Scaling to TRK-values (E1)



One experimental data
set discarded !

Model and Method

E1 Quenching Factor

Energy Weighted Sum Rule (EWSR)

$$S_1 = \sum_{\nu} \varepsilon_{\nu} |\langle \nu | \hat{O} | 0 \rangle|^2$$

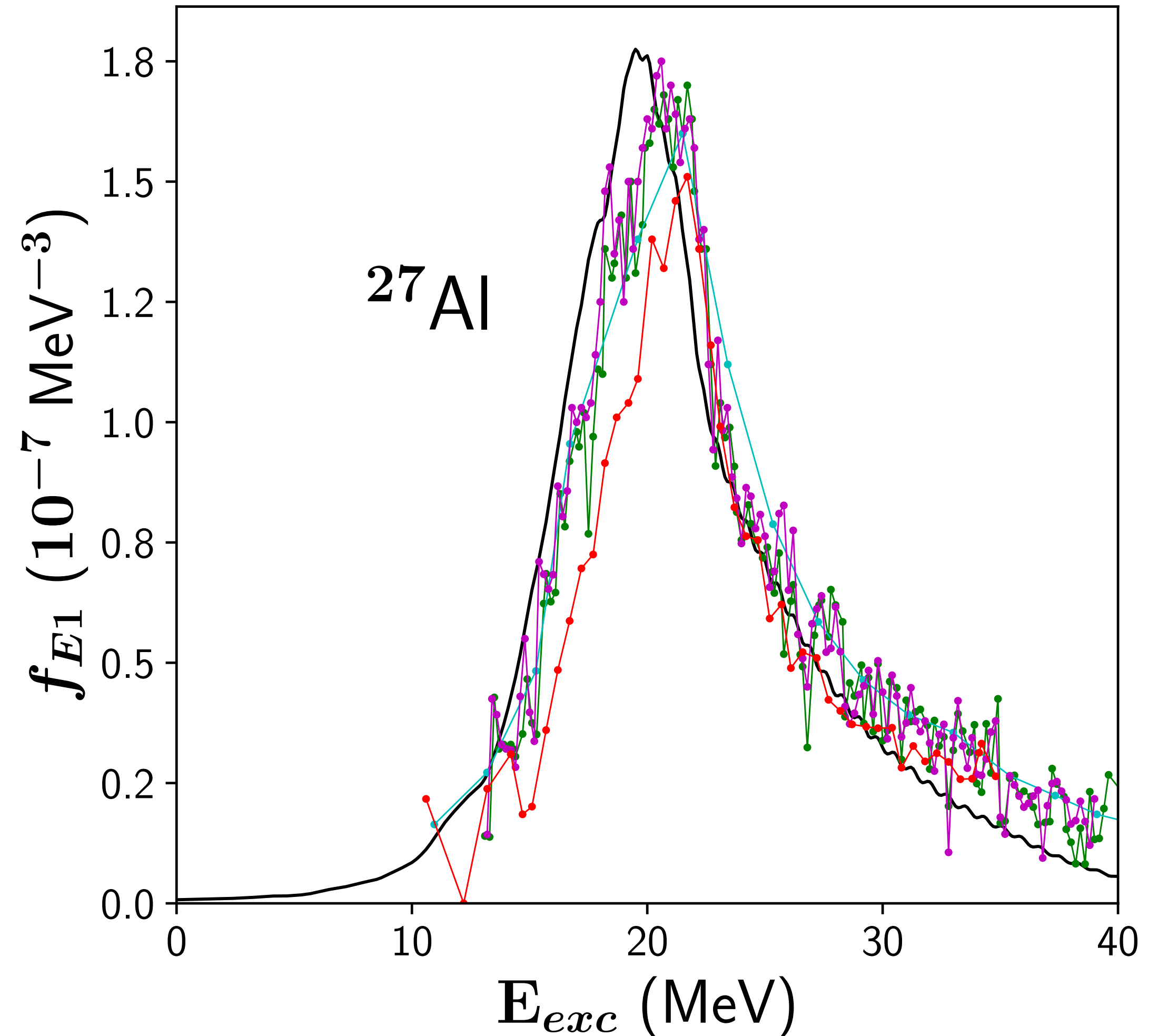
$$Q^2 = 0.64$$

TRK Sum Rule (lower bound)

$$S_1^{TRK} = \frac{9\hbar^2 e^2}{8\pi m} \frac{NZ}{A}$$

Quenching Factor

$$Q_{TRK}^2 = \frac{S_1^{TRK}}{S_1^{SM}} \quad Q_{EXP}^2 = \frac{S_1^{EXP}}{S_1^{SM}}$$



Results : PSF for all long lived sd nuclei

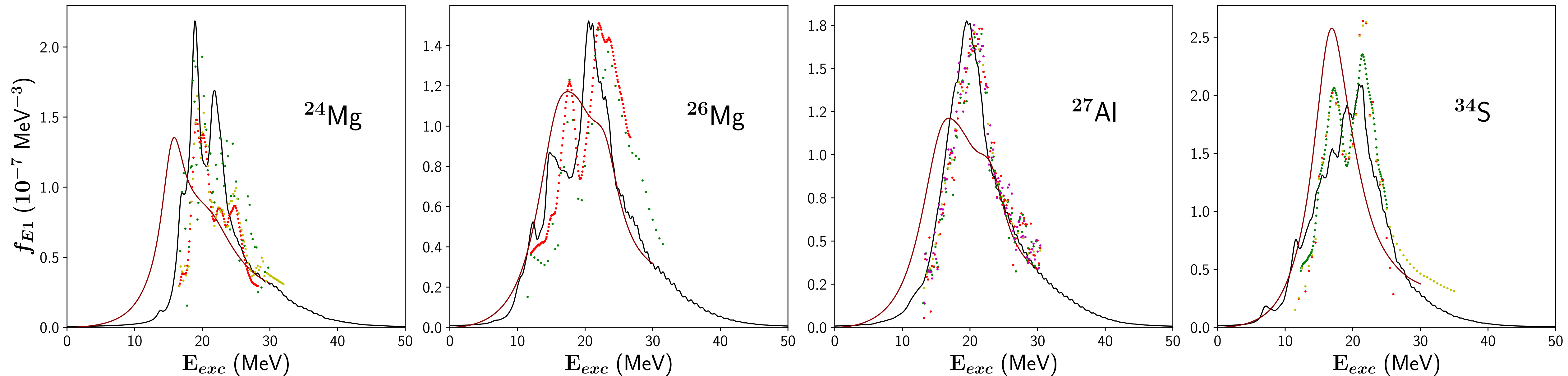
●●● Exp data

— Quenched CISM Prediction

— QRPA Prediction

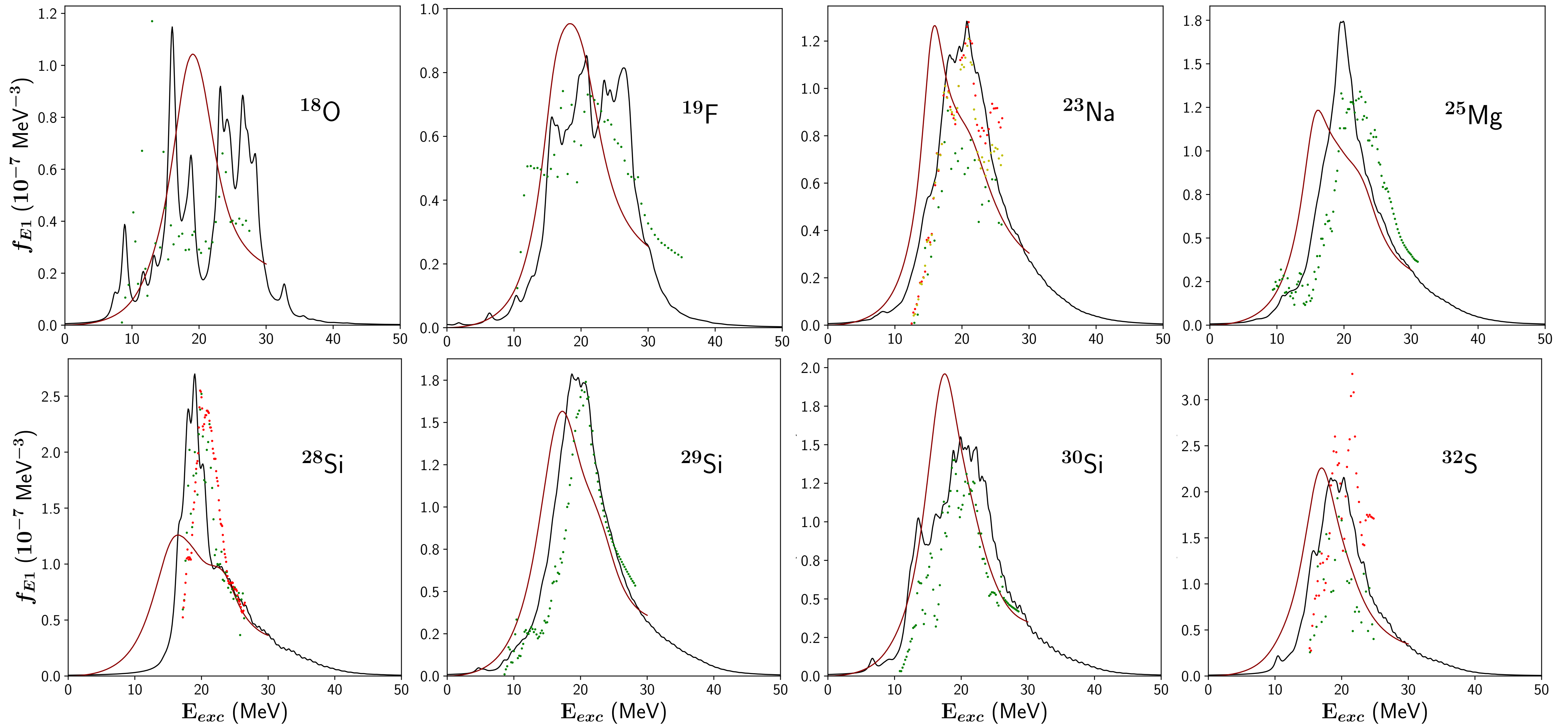
Gogny-HFB+QRPA

S. Goriely, S. Hilaire, S. Péru, K. Sieja, [Phys. Rev. C 98, 014327 \(2018\)](#).



- V.V.Varlamov, M.E. Stepanov, V.V. Chesnokov; *Izv. Ros. Akad. Nauk, Ser.Fiz.* 67, 656(2003); (*Bull.Rus.Acad.Sci.Phys.* 67, 724 (2003))
- J.Ahrens, H.Borchert, H.B.Eppler, H.Gimm, H.Gundrum; *Conf.Nucl.Structure Studies, Sendai, Japan*, 213, (1972)
- B.S.Ishkhanov, I.M.Kapitonov, E.I.Lileeva, et al.,; *Moscow State Univ. Inst. of Nucl. Phys. Reports*, No.2002, 27/711(2002)

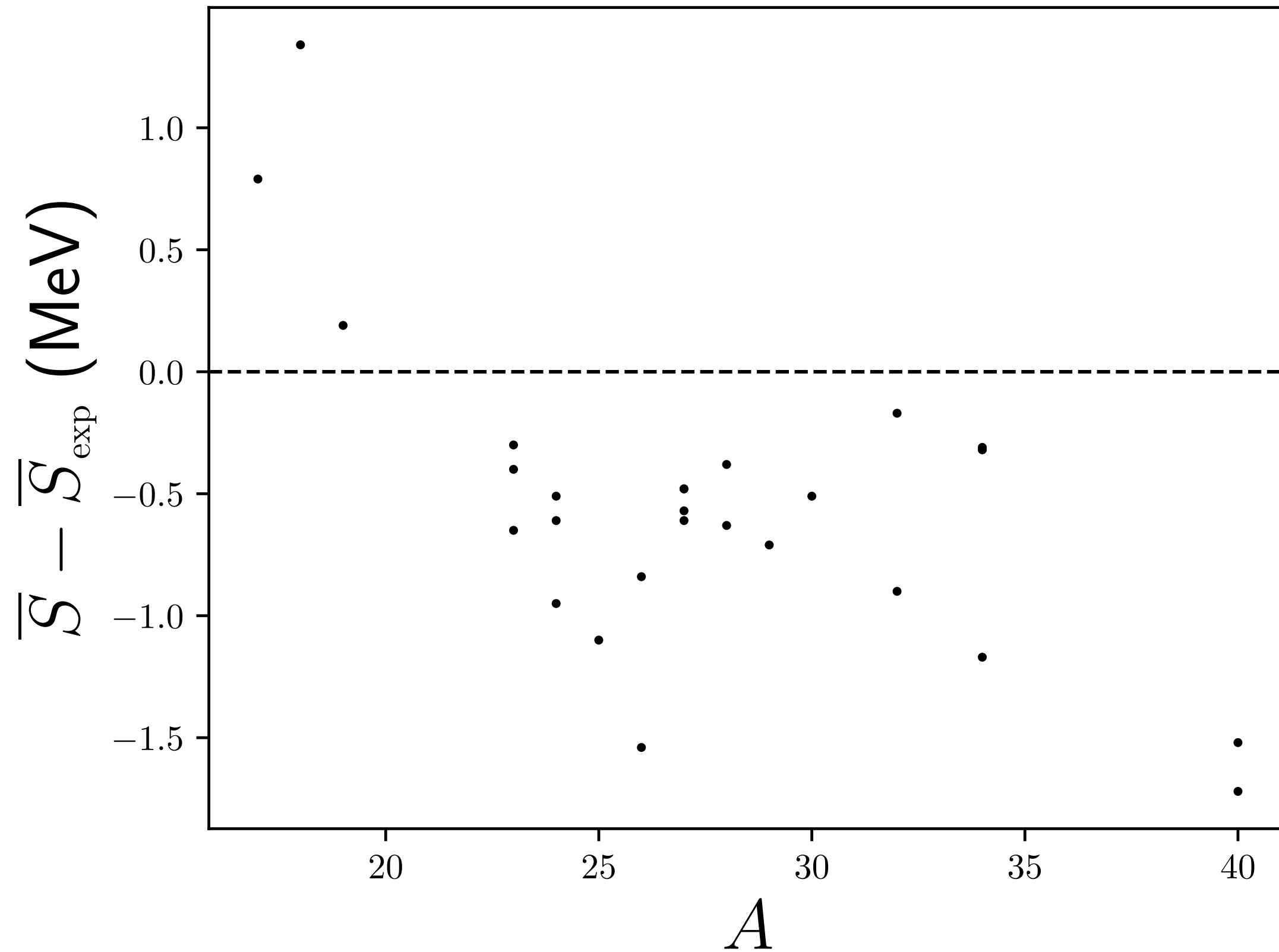
Results : PSF for all long lived sd nuclei



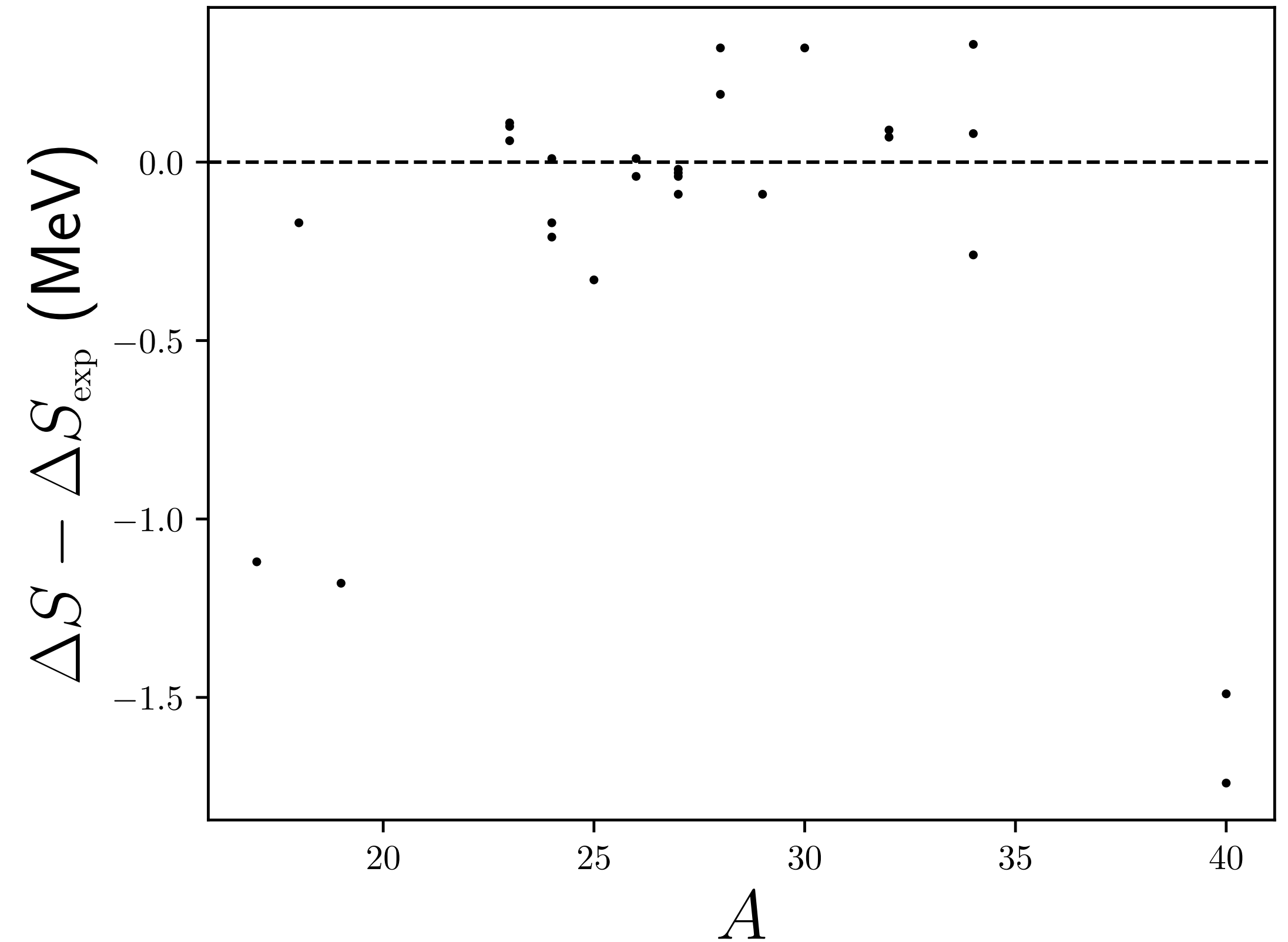
Results : PSF for all long lived sd nuclei

Centroids $\bar{S} = \frac{S_1}{S_0}$

Width $\Delta S = \sqrt{\frac{S_2}{S_0} - \bar{S}^2}$

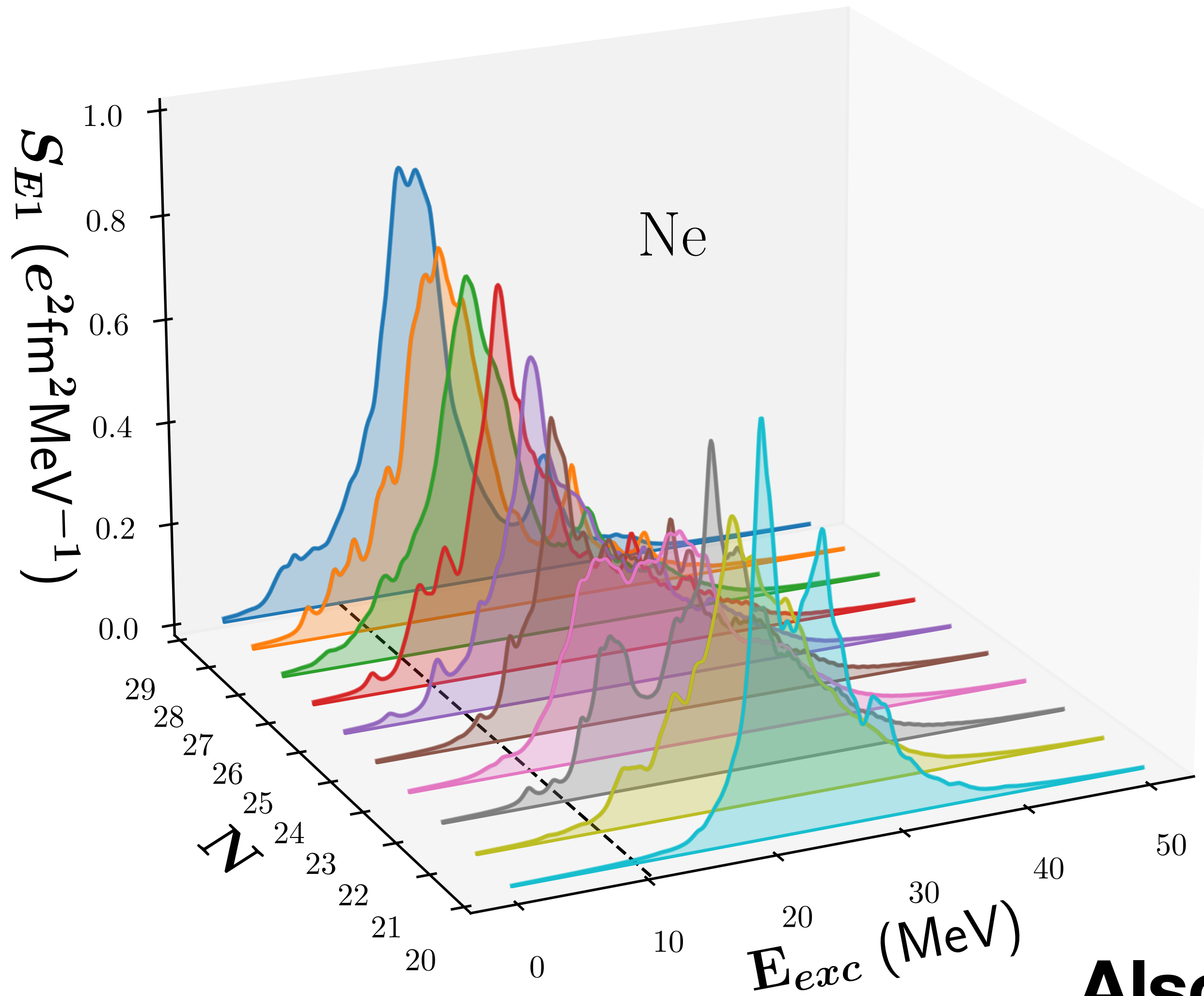


RMS 0.84 MeV



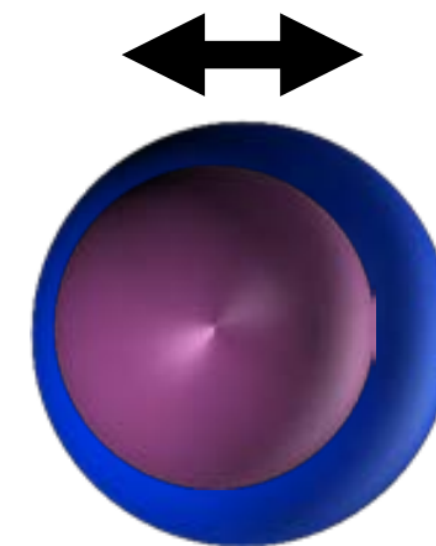
RMS 0.56 MeV

Results II : Pygmy Dipole resonance in ^{26}Ne



Can the PDR be said to be collective ?

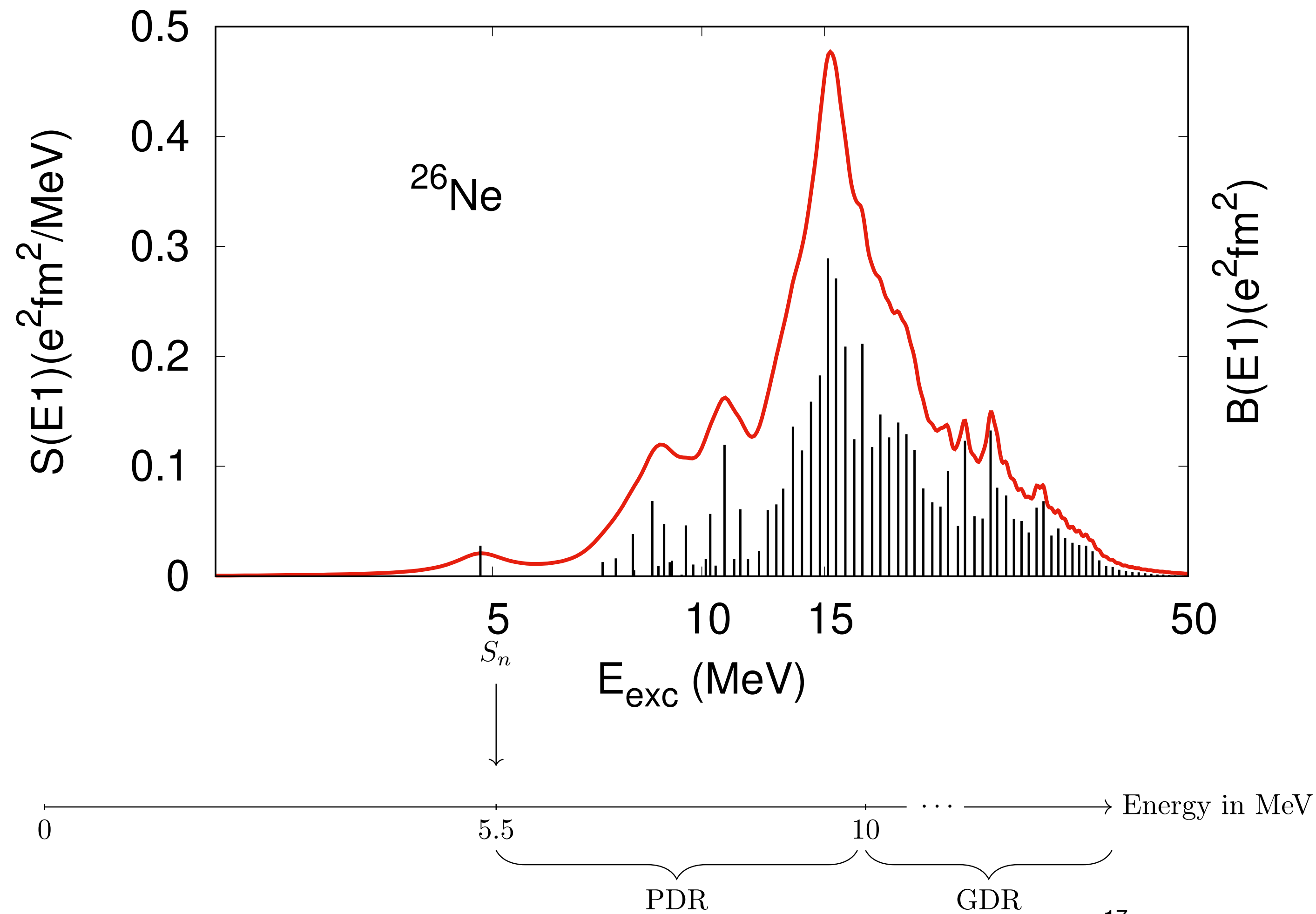
Can we give a classical interpretation to this resonance ?



Also called kramplouz resonance

Results II : Pygmy Dipole resonance in ^{26}Ne

Focus on ^{26}Ne



Consistent with :

- Suzuki et al, Phys. Atom. Nuclei 2004 (SM, WBP10)
- Kimura, Phys. Rev. C 2017 (AMD)
- Cao et al, Phys. Rev C 2005 (RQRPA)
- Yoshida, Phys. Rev. C 2008 (QRPA)

Gibelin et al

Phys. Rev. Lett. 2008

Results II : Pygmy Dipole resonance in ^{26}Ne

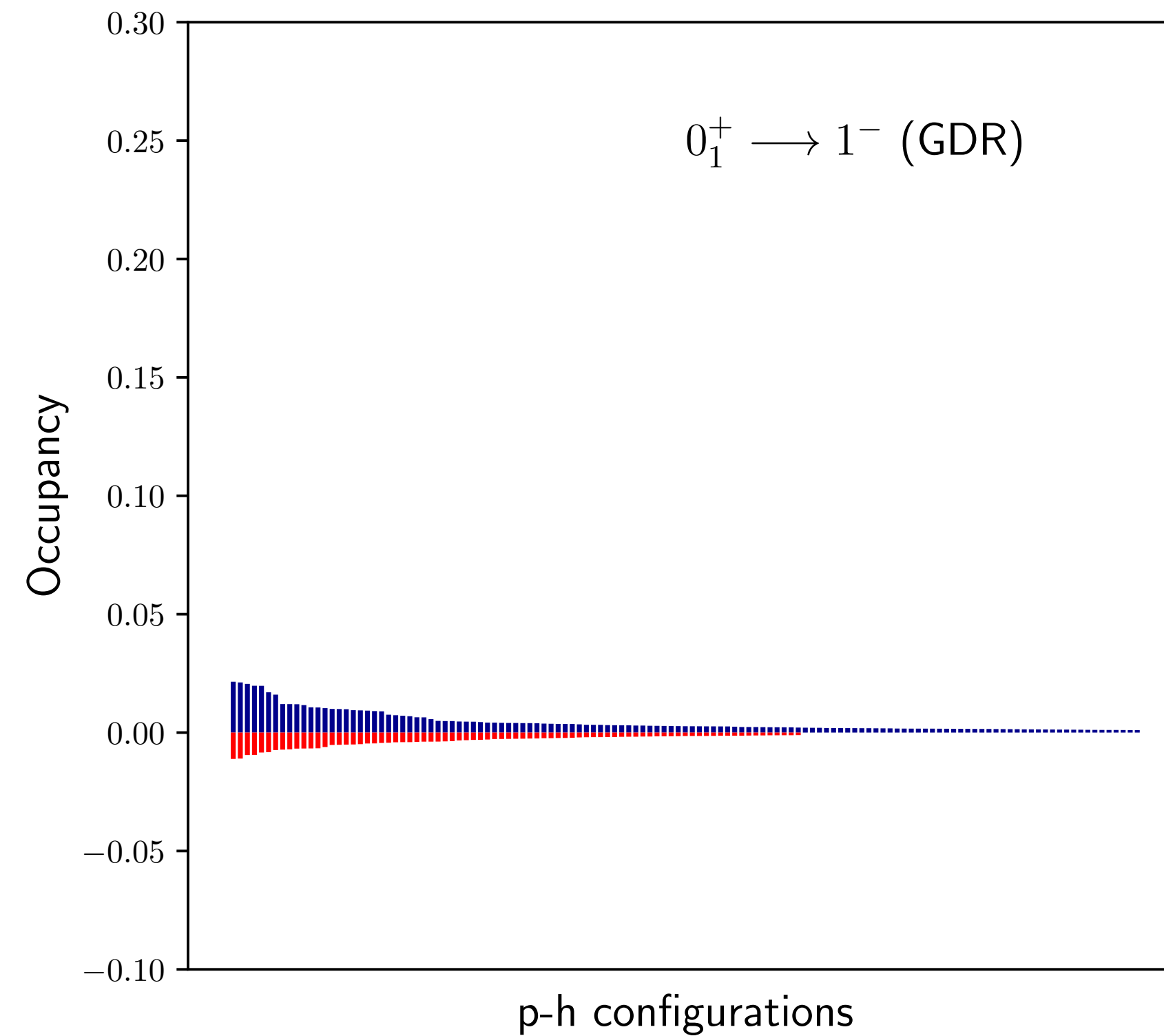
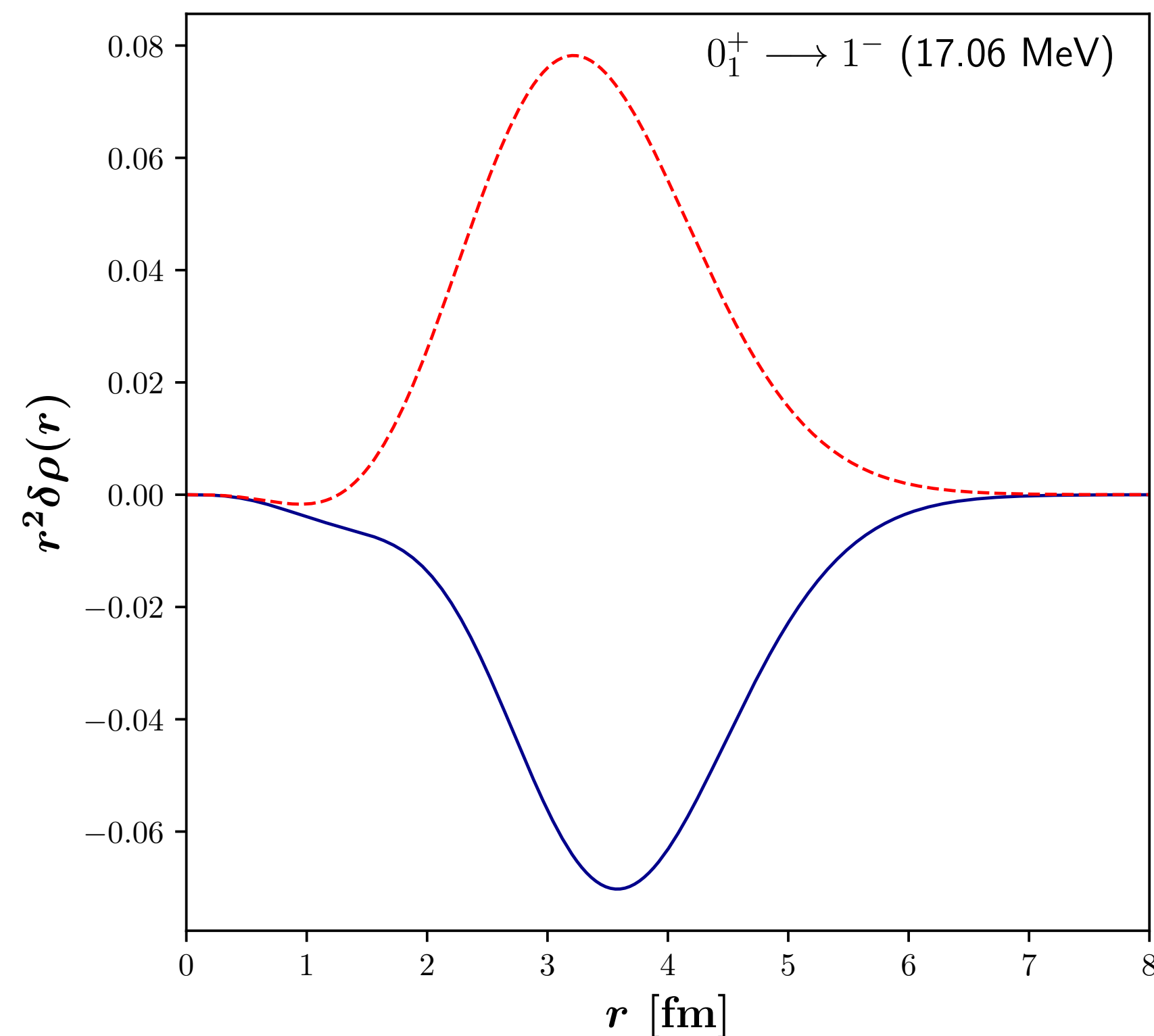
Collectivity indicators :

- Great regularity over large area of the nuclear chart
- Display rather classical properties, smooth dependence on (N,Z)
- Similar structures (transition densities, ν and π ratio)
- Imply 'many' p-h contributions (occupancy spread)
- Coherence => collective strength (% of TRK)

Method : We investigate GDR states to have a reference point in terms of collectivity and then compare to PDR states

Results II : Pygmy Dipole resonance in ^{26}Ne

GDR



Transition densities = Fourier modes of the shape dynamics

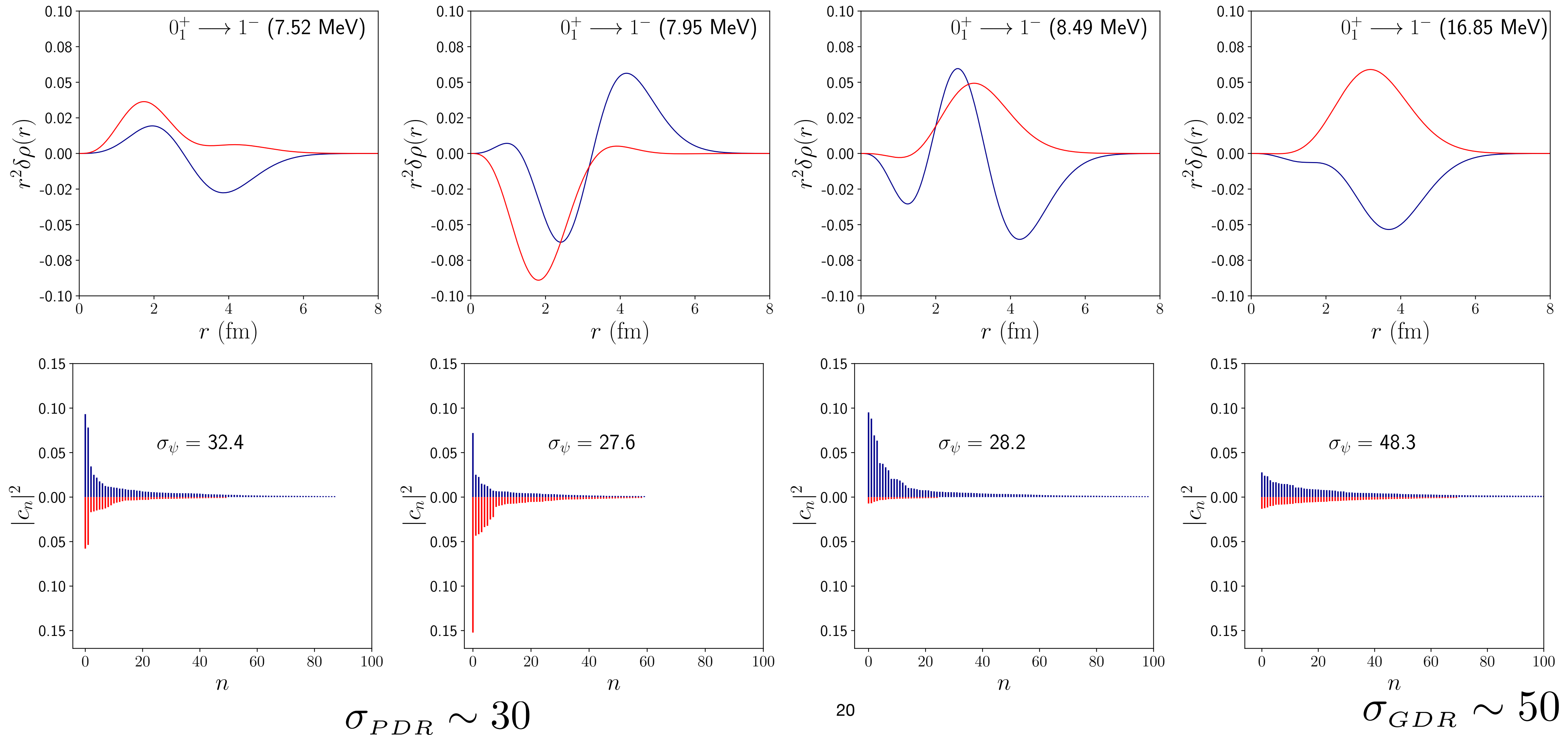
$$\delta \rho_\psi(\mathbf{r}, t) = \rho_\psi(\mathbf{r}, t) - \rho_0(\mathbf{r}) = \sum_{\nu \neq 0} b_\nu \langle 0 | \hat{\rho}(\mathbf{r}) | \nu \rangle e^{-\frac{i}{\hbar} \varepsilon_\nu t}$$

$$\delta \rho_{0\nu}(\mathbf{r}) = \langle 0 | \hat{\rho}(\mathbf{r}) | \nu \rangle$$

$$\sigma_{GDR} \sim 50$$

Results II : Pygmy Dipole resonance in ^{26}Ne

PDR Region : 6-10 MeV



Conclusion

E1 PSF predicted for 36 sd-shell long-lived isotopes : will be added to PSF database

Will be tested in TALYS reaction code (S. Goriely's team, ULB)

Pigmy Dipole Resonance



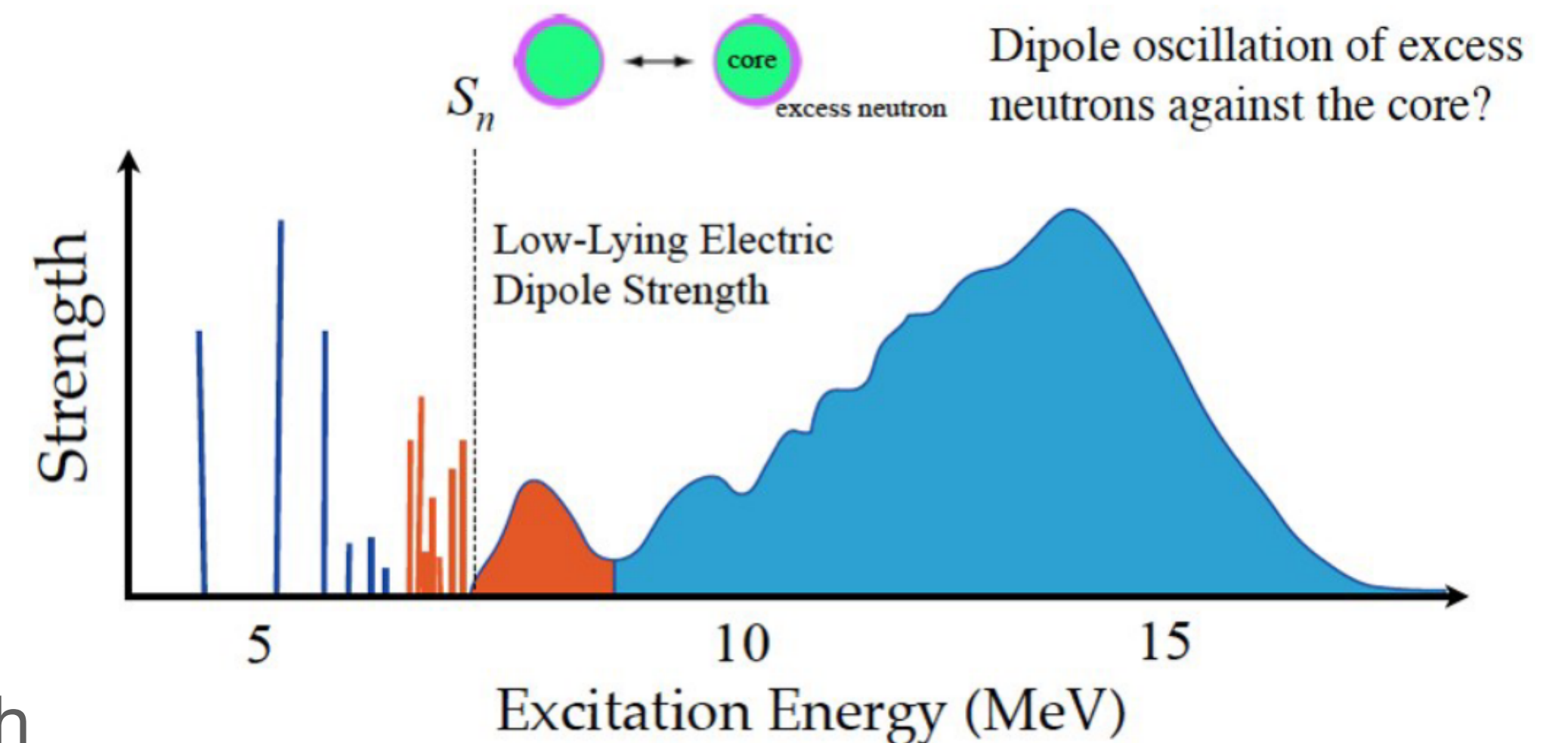
Pygmy Dipole Resonance

PDR differs from single-particle-like excitations

Not tail GDR

Some collectivity

Classical interpretation of a neutron skin oscillation seems to h



Tamii (2016). Adelaide International Conference

Opening : E1 Strength Function from PGCM

Why PGCM ?

Shell Model Dimensionality Curse :
factorial increase

Caurier et al.
The Shell Model as Unified View of Nuclear
Structure (2005)

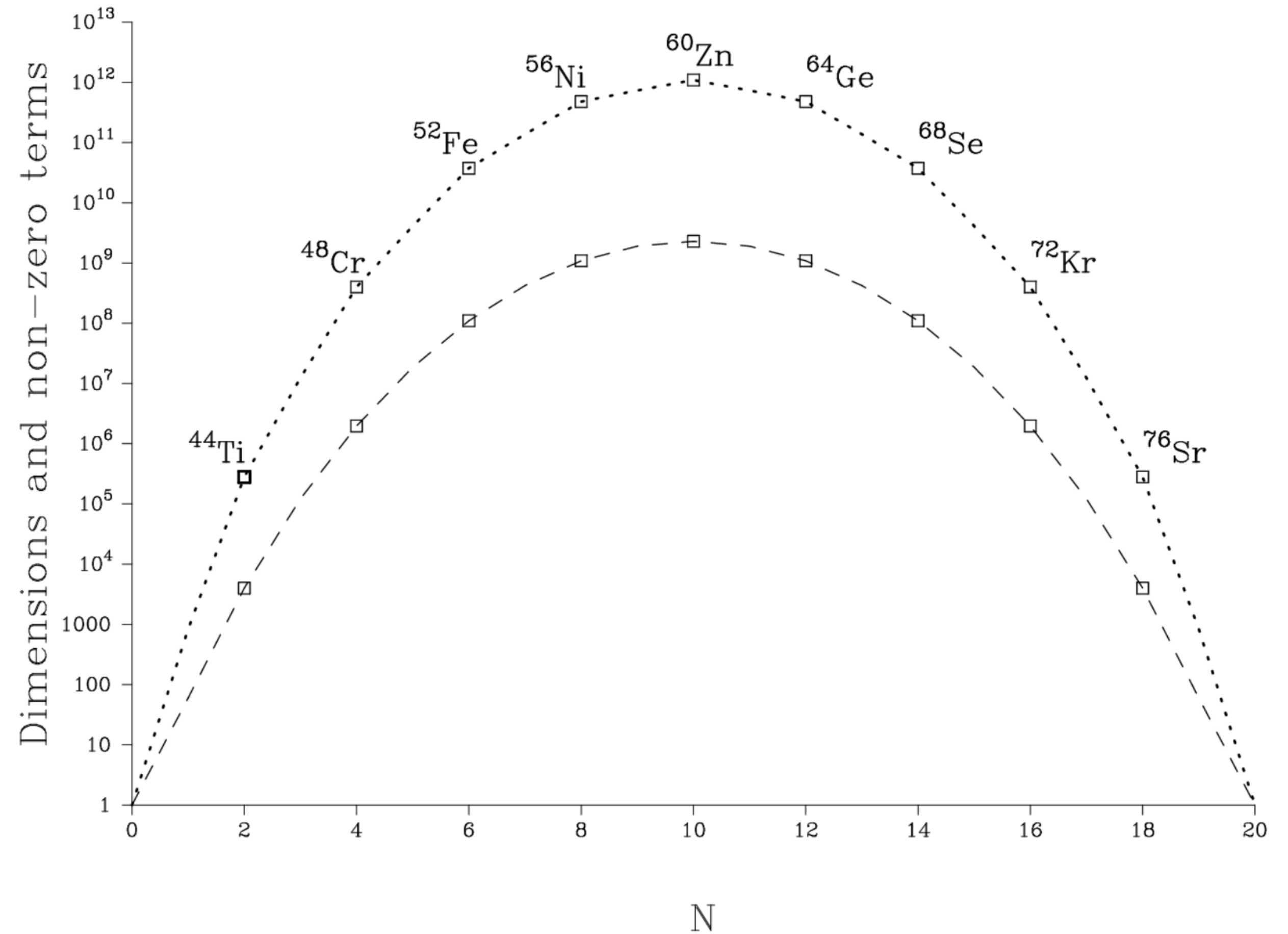


FIG. 8 m -scheme dimensions and total number of non-zero matrix elements in the pf -shell for nuclei with $M = T_z = 0$

E1 Strength Function from PGCM

Projected Generator Coordinate Method (PGCM)

Reference states : Set of HFB states
with constrains $a = \{a_1, \dots, a_n\}$

$$\{|\Phi_a\rangle, a\}$$

PGCM Variational Ansatz

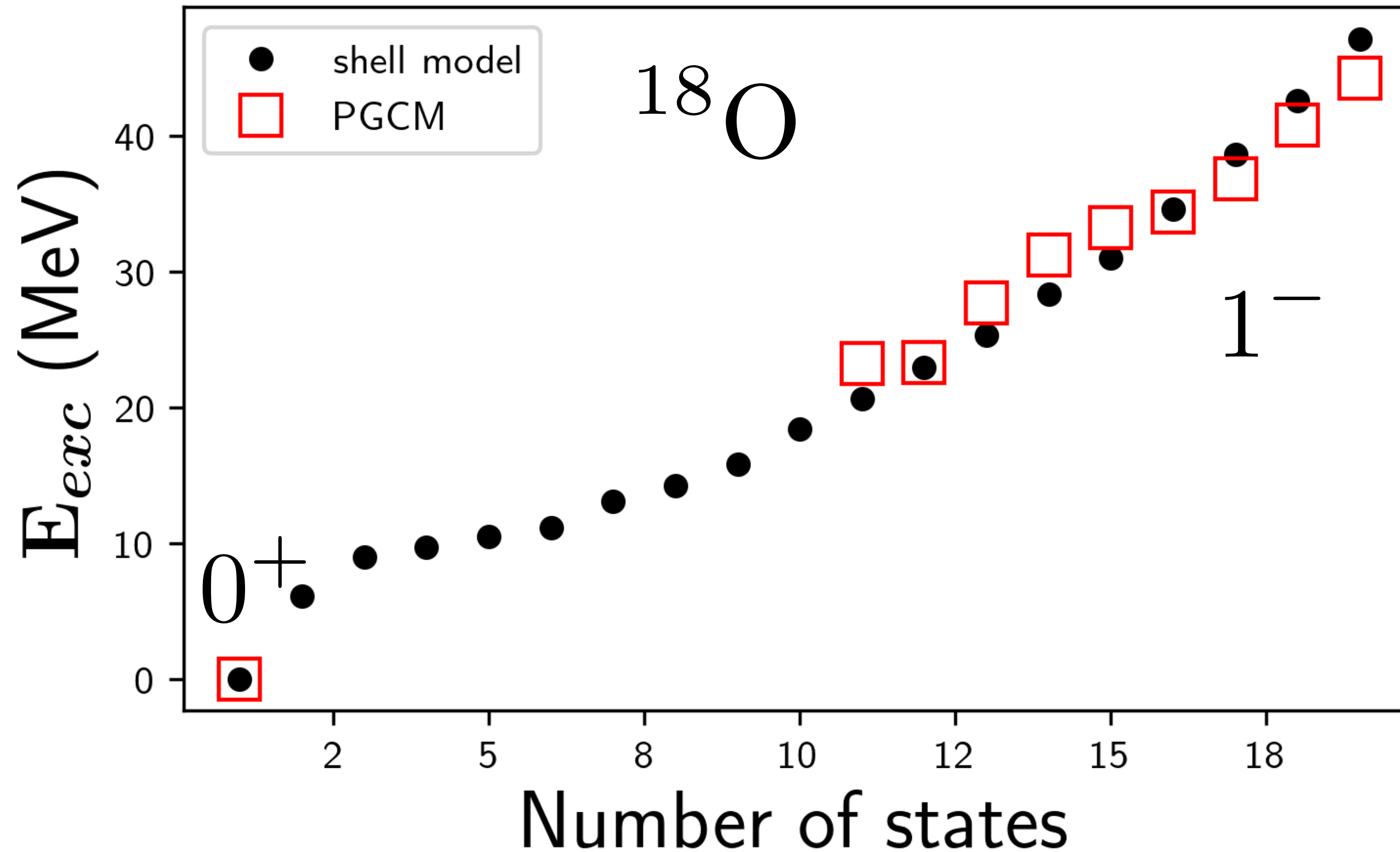
$$|\Psi_{\Sigma\varepsilon}^{\lambda i}\rangle = \sum_a^\Sigma \sum_j^{d_\lambda} f_{\Sigma a \varepsilon}^{\lambda j} P_{ij}^\lambda |\Phi_a\rangle$$

Generalized eigenvalue problem

$$H^{\lambda\Sigma} \mathbf{f} = E N^{\lambda\Sigma} \mathbf{f}$$

E1 Strength Function from PGCM

Choice of the collective coordinates



9 reference HFB states

Collective variables :

Q_{10}

Q_{11}