EFT-based methods for classical gravity

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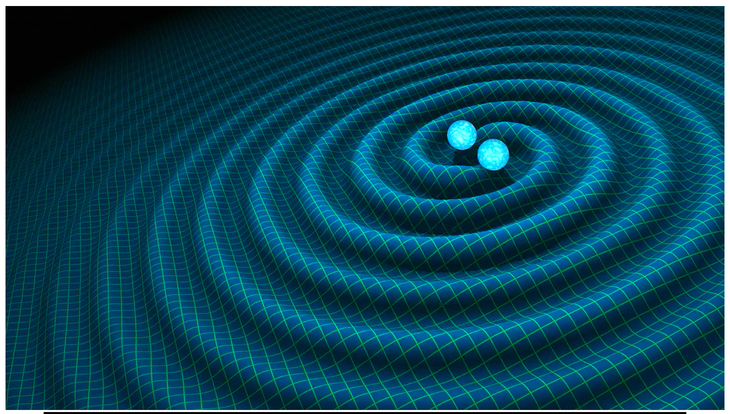
GdR Ondes Gravitationnelles: réunion du groupe de travail «Formes d'Onde»
24/09/2024

Based on works with: Vanhove, Bernard, Dones, Vernizzi, Riva, Levi, Vieira

[2407.09448],[2405.14421],[2310.19679],[2204.06556], [2102.08339],[2010.08882],[1912.06276]

Motivation



Binary Coalescence

Gravitational Wave

LIGO

[1602.03837]

GW150914

Nobel Prize 2017
Thorne, Barish, Weiss

Gravitational wave era

Motivation

Gravitational wave era

1. Multi-messenger Astronomy from the largest particle collider
2. Observational window on “strong” gravity
3. Search for “exotic” objects + new physics

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Weak signal (much noise)

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Weak signal (much noise) — ***requires*** → **Accurate Prediction**

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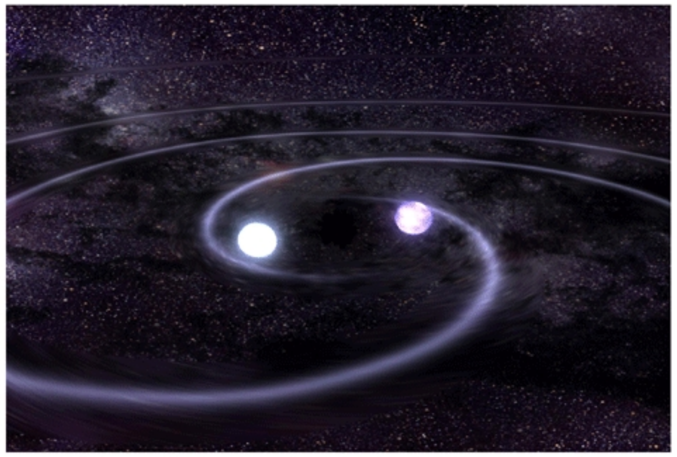
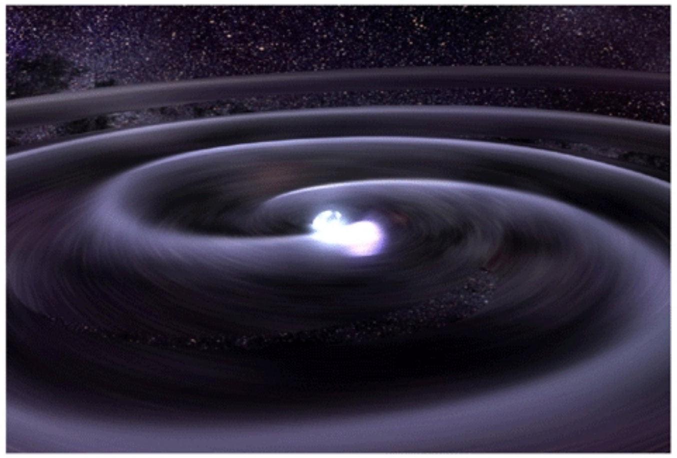
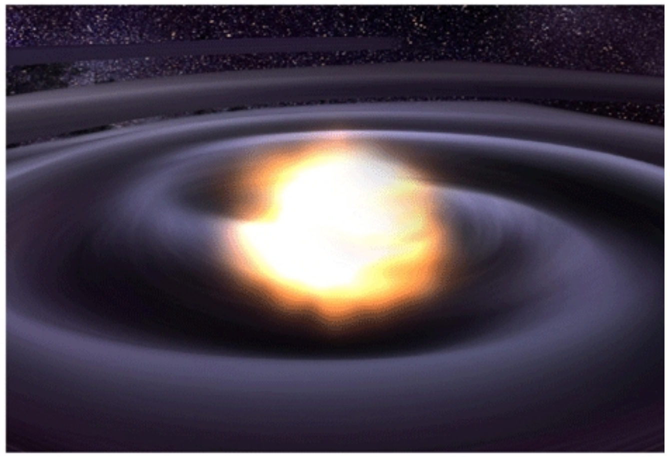
2-body problem in Gravity

Motivation

2-body problem in Gravity



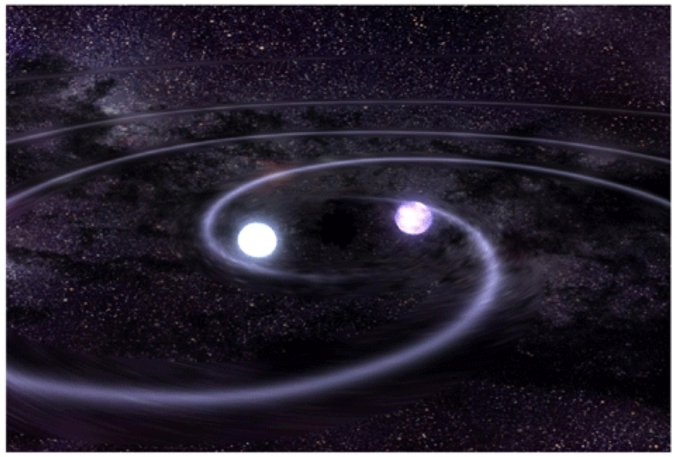


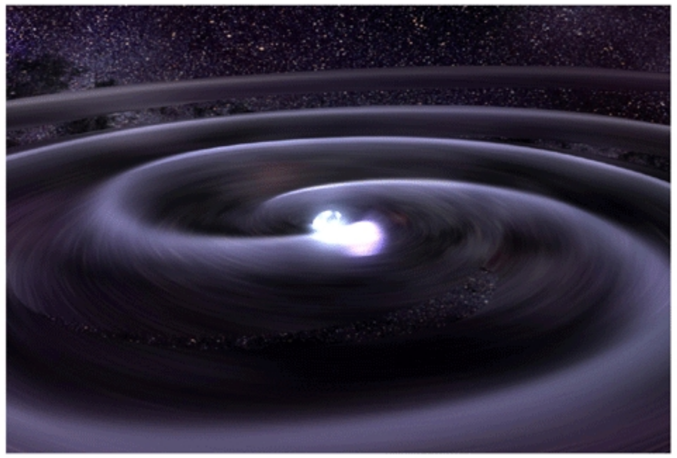


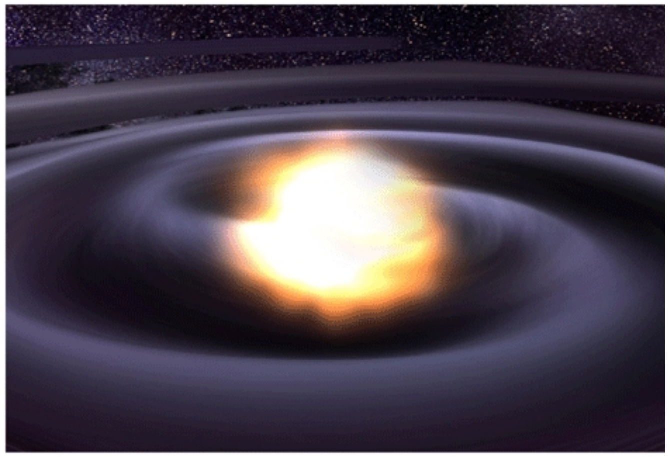
Inspiral **Merger** **Ringdown**

Motivation

2-body problem in Gravity





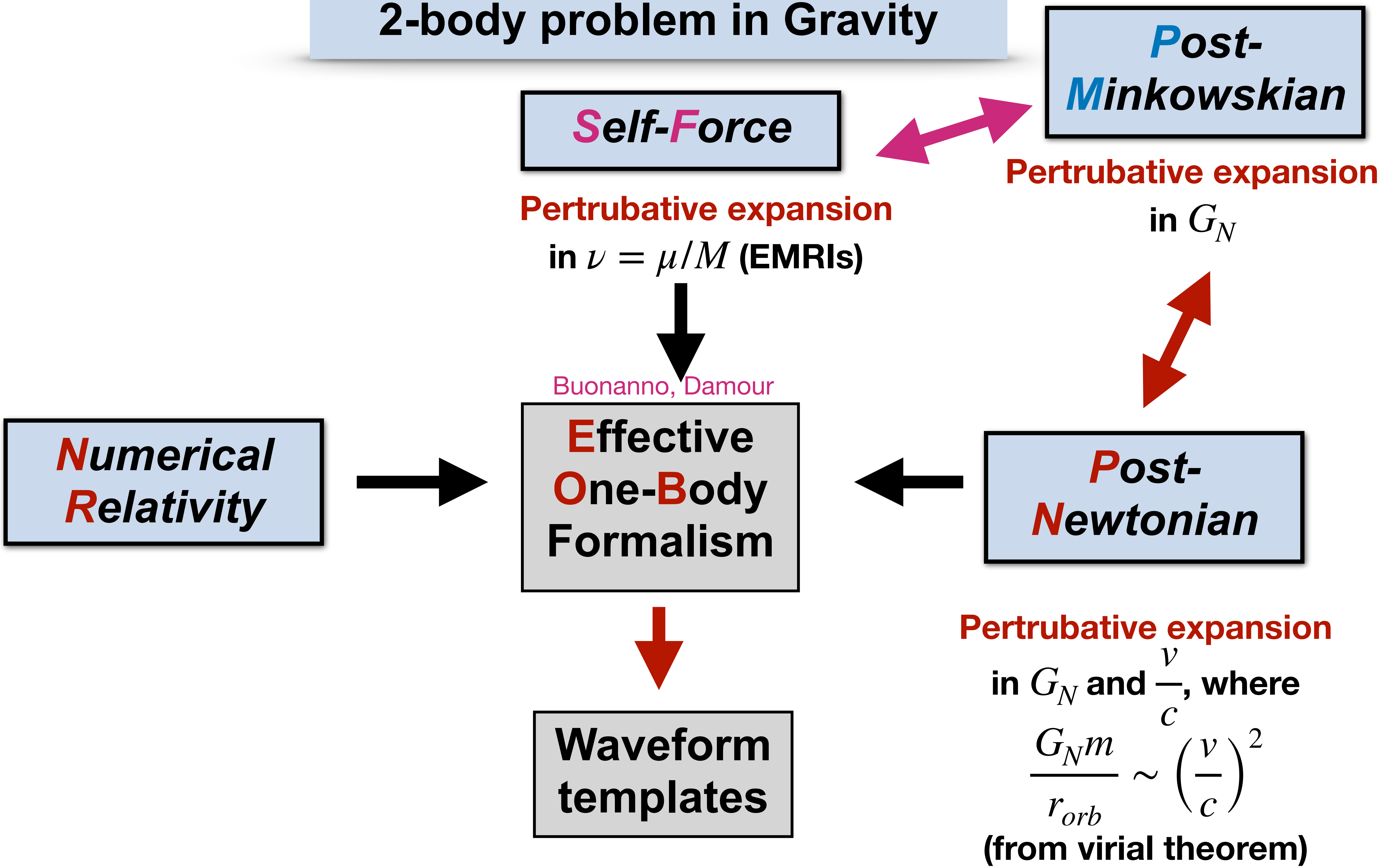


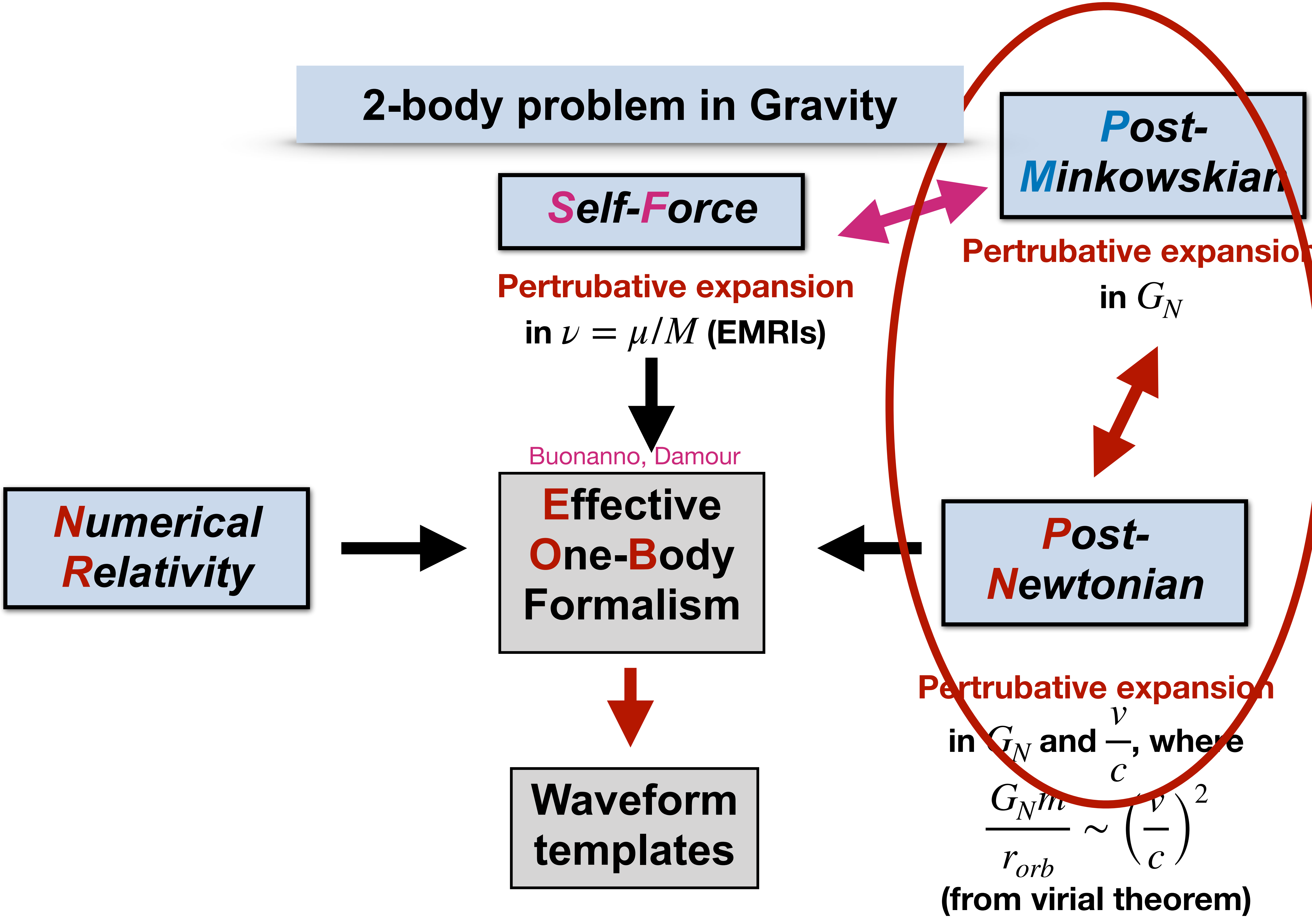
Inspiral
Analytic treatment

Merger
Numerical Relativity

Ringdown
BHPT

2-body problem in Gravity

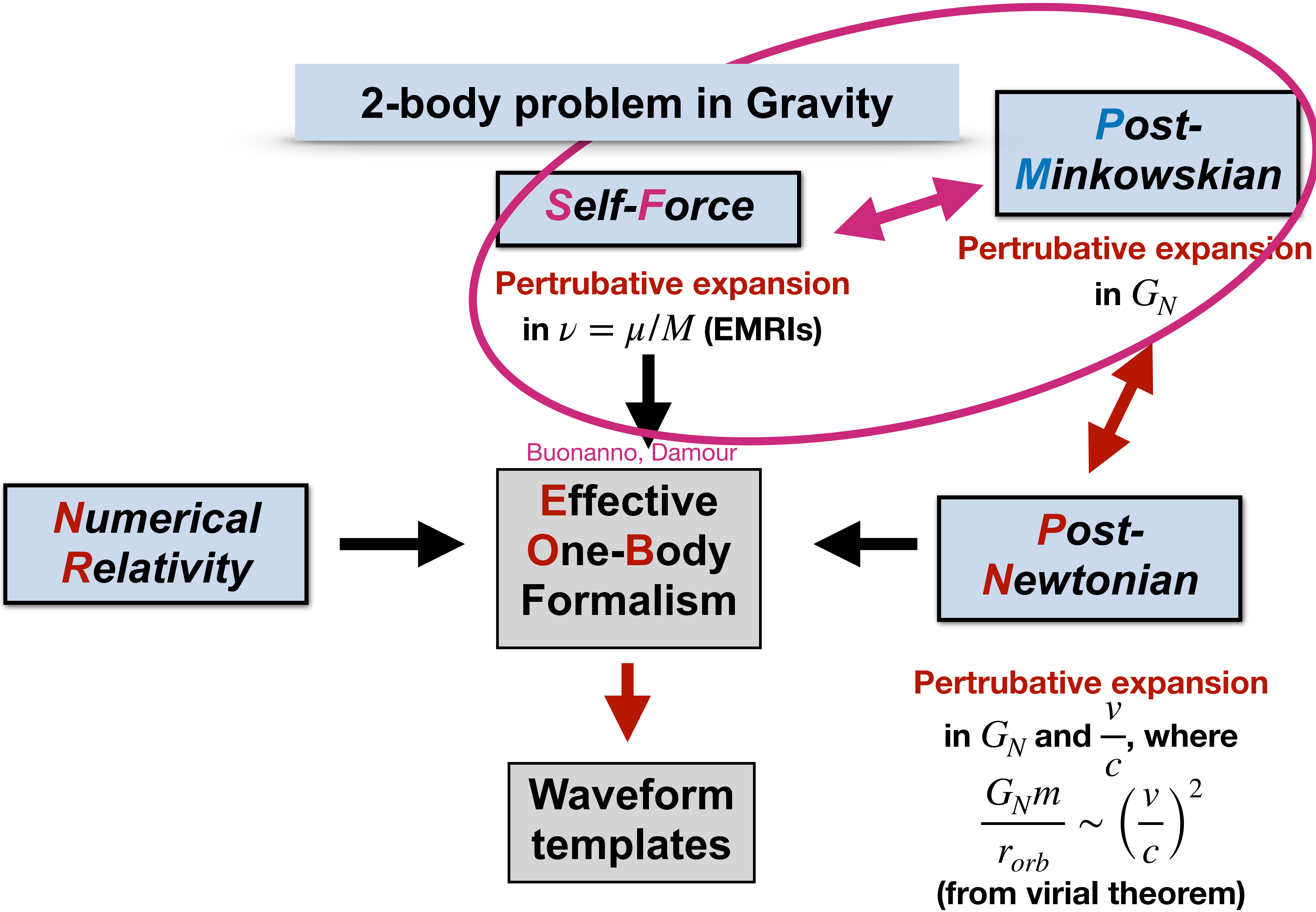



2-body problem in Gravity
Self-Force
Pertrubative expansion
in $\nu = \mu/M$ (EMRIs)
Post-Minkowskian
Pertrubative expansion
in G_N
Buonanno, Damour
Effective One-Body Formalism
Numerical Relativity
Post-Newtonian
Pertrubative expansion
in G_N and $\frac{v}{c}$, where
$\frac{G_N m}{r_{orb}} \sim \left(\frac{v}{c}\right)^2$
(from virial theorem)
Waveform templates

2-body problem in Gravity

Post-Minkowskian vs **Post-Newtonian**

	0PN	1PN	2PN	3PN	4PN	...	
1PM	$[1$	$+v^2$	$+v^4$	$+v^6$	$+v^8$	$+\ldots]$	$\times G$
2PM		$[1$	$+v^2$	$+v^4$	$+v^6$	$+\ldots]$	$\times G^2$
3PM			$[1$	$+v^2$	$+v^4$	$+\ldots]$	$\times G^3$
4PM				$[1$	$+v^2$	$+\ldots]$	$\times G^4$



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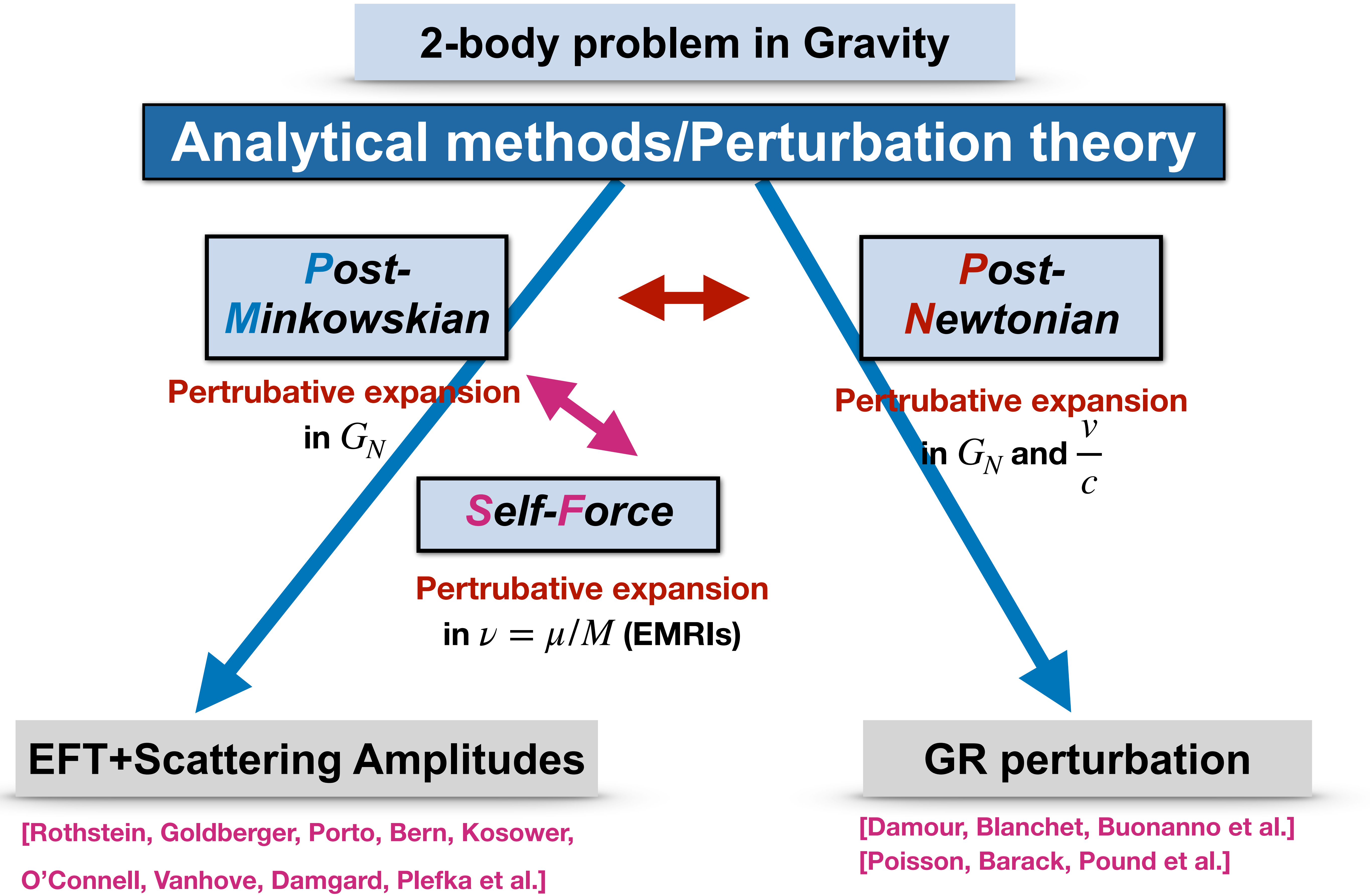
2-body problem in Gravity

Post-Minkowskian vs **Self-Force**

	1PM	2PM	3PM	4PM	5PM	...	
0SF	$\big[(G\,m_1)$	$+(G\,m_1)^2$	$+(G\,m_1)^3$	$+(G\,m_1)^4$	$+(G\,m_1)^5$	$+\ldots\big]$	$\times\, m_2$
1SF			$\big[(G\,m_1)^2$	$+(G\,m_1)^3$	$+(G\,m_1)^4$	$+\ldots\big]$	$\times\, G\, m_2^2$
2SF					$\big[(G\,m_1)^3$	$+\ldots\big]$	$\times\, G^2\, m_2^3$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Analytical methods/Perturbation theory
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Self-Force
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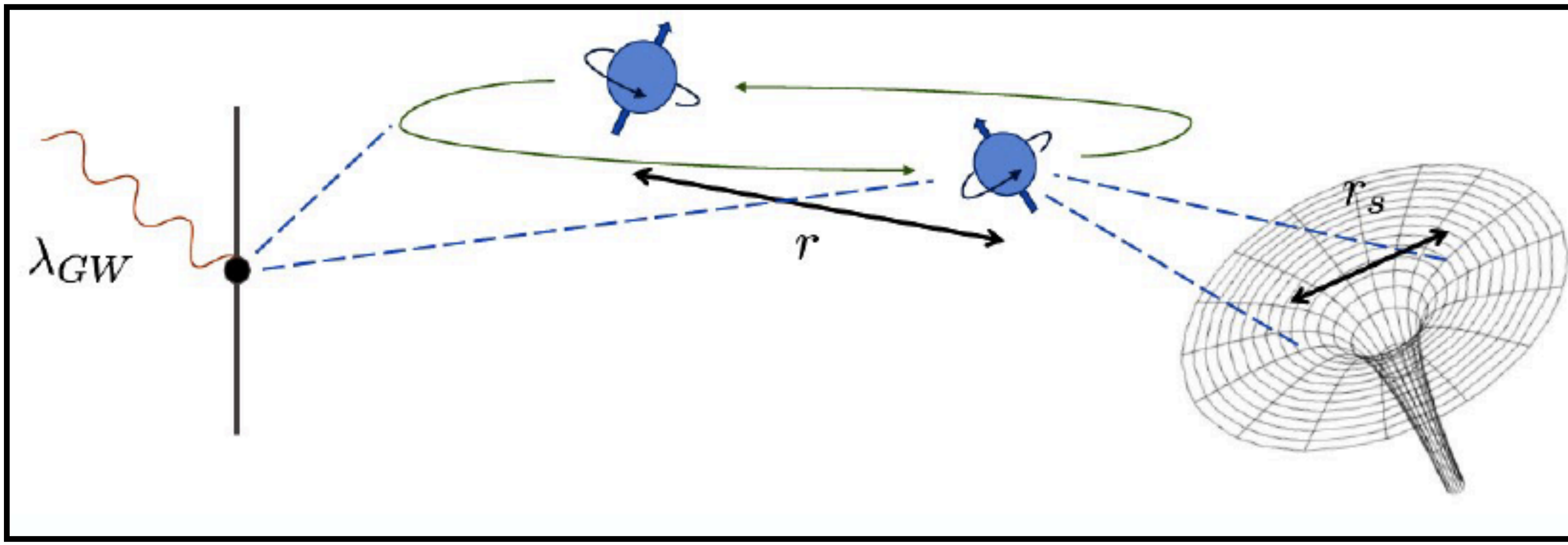
in $\nu = \mu/M$ (EMRIs)
EFT+Scattering Amplitudes
[Rothstein, Goldberger, Porto, Bern, Kosower,
O'Connell, Vanhove, Damgard, Plefka et al.]
GR perturbation
[Damour, Blanchet, Buonanno et al.]
[Poisson, Barack, Pound et al.]

2-body problem in Gravity

EFT+Scattering Amplitudes (quick review)

***Tower of EFTs* (focus on worldline approaches)** [1601.04914] Porto [1807.01699] Levi



Hierarchy of scales: $r_s \ll r \ll \lambda_{rad}$

2-body problem in Gravity

EFT+Scattering Amplitudes (quick review)

Rothstein
Goldberger
Porto,
Steinhoff et al.

1) Point-particle approximation:

$$\mathcal{S}_{p.p.} = -\int d\tau \left[m\sqrt{u^2} + \frac{1}{2} S_{\mu\nu}\Omega^{\mu\nu}\right] + \int d\tau\, Q_E^{ij}(\tau) E_{ij}(x) + \ldots + (E \to B)$$

point particle with spin **finite size**

2) GR as EFT:

$$\mathcal{S}_{eff} = \frac{1}{16\pi G_N} \int d^d x \sqrt{g} R + \mathcal{O}(R^2, R_{\mu\nu}R^{\mu\nu,\ldots})\;,\; g_{\mu\nu} = \eta_{\mu\nu} + \sum_{n=1}^{\infty} h_{\mu\nu}^{(n)}$$

DeWitt
t'Hooft, Veltman
Donoghue et al.

generic and adaptable to modifications of GR

3) Feynman rules + loop-Diagrams: → **Multi-loop Feynman Integrals**

4) Mappings between kinematics regions:

Boundary to Bound (B2B) map

[1910.03008]
[1911.09130]
Kalin, Porto

[2109.05994]
Saketh, Vines
Steinhoff, Buonanno

Hamiltonian reconstruction /EFT matching

[1808.02489] Cheung, Rothstein, Solon
[1906.01579]
Cristofoli, Bjerrum-Bohr, Damgaard, Vanhove

2-body problem in Gravity

EFT+Scattering Amplitudes

(2014-2015) ***Post-Minkowskian***

Pertrubative expansion

in G_N

Post-Newtonian (2004)

Pertrubative expansion

in G_N and $\frac{v}{c}$

Self-Force (2023-2024)

Pertrubative expansion

in $\nu = \mu/M$ (EMRIs)

WHY QFT methods?

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- Recycle knowledge from particle physics + Efficiency
- Clean setup to treat divergencies (Dim. Reg., distinction of IR-UV)
- Tools for integrand construction (Generalized unitarity+Double Copy)
- Solving Feynman integrals instead of D.E.s (IBPs+Reverse Unitarity+D.E.)

2-body problem in Gravity

EFT+Scattering Amplitudes

Post-Minkowskian

Gravitational Bremsstrahlung: Concrete example of "advantage" of Amplitude approach

THE GENERATION OF GRAVITATIONAL WAVES.
IV. BREMSSTRAHLUNG*†‡

SÁNDOR J. KOVÁCS, JR.
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AND
KIP S. THORNE
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W. K. Kellogg Radiation Laboratory, California Institute of Technology
Received 1977 October 21; accepted 1978 February 28

ABSTRACT

This paper attempts a definitive treatment of "classical gravitational bremsstrahlung"—i.e., of the gravitational waves produced when two stars of arbitrary relative mass fly past each other with arbitrary relative velocity v, but with large enough impact parameter that

$$\text{(angle of gravitational deflection of stars' orbits)} \ll (1 - v^2/c^2)^{1/2} .$$

$$P^{\mu}_{rad} = \frac{G^3 m_1^2 m_2^2}{b^3} \frac{u_1^{\mu} + u_2^{\mu}}{\gamma + 1} \epsilon(\gamma) + \mathcal{O}(G^4)$$

no general closed form for $\epsilon(\gamma)$!!

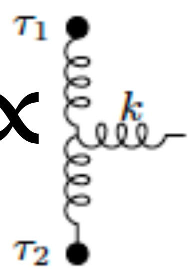
2-body problem in Gravity

EFT+Scattering Amplitudes

Post-Minkowskian

Gravitational Bremsstrahlung: Concrete example of "advantage" of Amplitude approach

$$P^{\mu}_{\rm rad} = \int d\Omega\, du\, r^2\, n^{\mu}\, \dot{h}_{ij}\dot{h}_{ij} = \sum_{\lambda} \int_k \hat{\delta}_+(k^2) k^{\mu} \left| \mathcal{A}_{\lambda} \overset{k}{\sim\!\sim} \right|^2$$

$\mathcal{A}^{(2)}_{\lambda}(k) \propto$ 

plugging in doesn't give closed solution. (similar problem as Kovacs-Thorne)

2-body problem in Gravity

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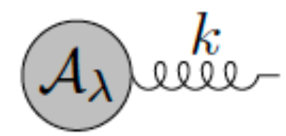
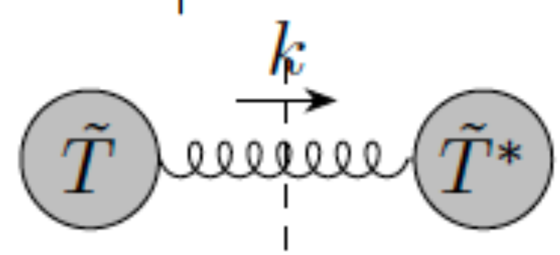
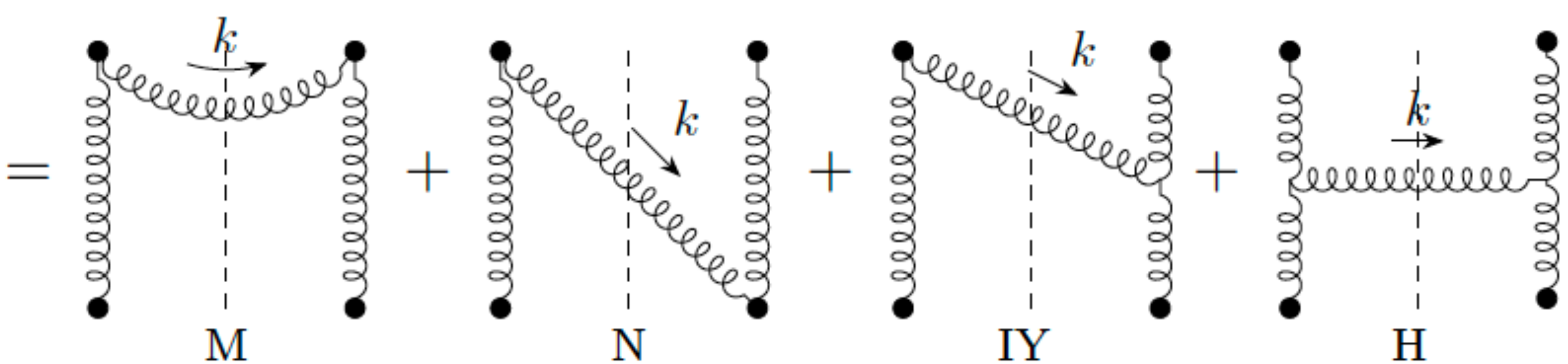
$$P^{\mu}_{\rm rad} = \int d\Omega\, du\, r^2\, n^{\mu}\, \dot{h}_{ij}\dot{h}_{ij} = \sum_{\lambda}\int_k \hat{\delta}_+(k^2)k^{\mu} \left| \mathcal{A}_{\lambda} \right|^2$$



[2110.10140] Riva, Vernizzi

BUT if we think like: $P^{\mu}_{\rm rad} = \sum_{\lambda}\int_k k^{\mu}$





on-shell 2-loop amplitude → **explicit solution**

2-body problem in Gravity

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Post-Minkowskian

Gravitational Bremsstrahlung: Concrete example of "advantage" of Amplitude approach

THE GENERATION OF GRAVITATIONAL WAVES.
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[2101.07255] Hermann et al.

[**2102.08339**] **S.M.** et al.

[2101.12688] Jakobsen et al.

[2104.03256] DVHRV

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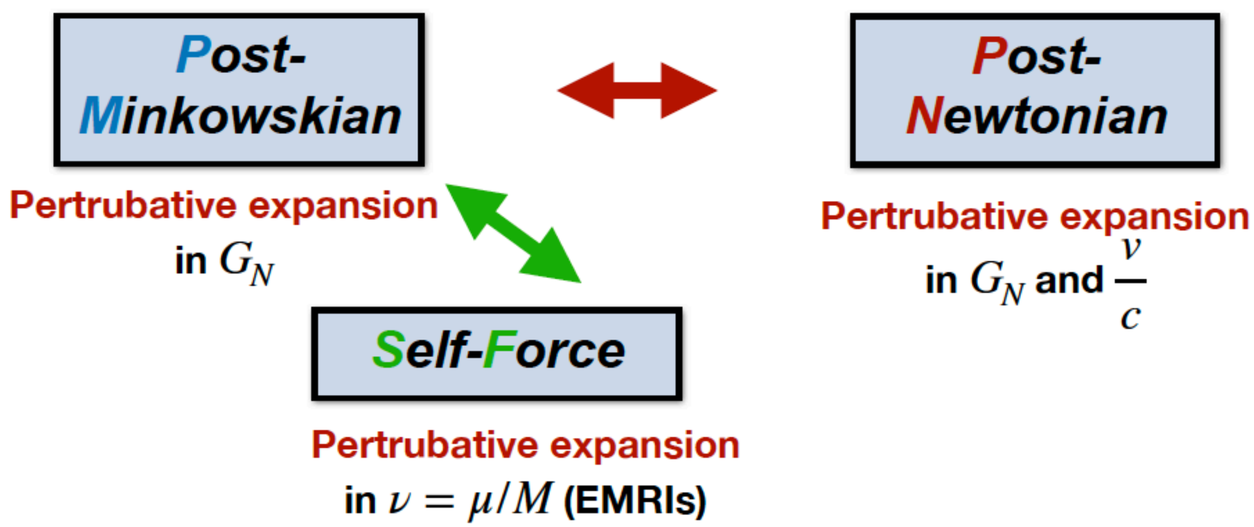
no general closed form for $\epsilon(\gamma)$!!

UNLESS recast as 2-loop on-shell+Reverse Unitarity

2-body problem in Gravity

EFT+Scattering Amplitudes



WHY QFT methods?

- Recycle knowledge from particle physics + Efficiency
- Clean setup to treat divergencies (Dim. Reg., distinction of IR-UV)
- Tools for integrand construction (Generalized unitarity+Double Copy)
- Solving Feynman integrals instead of D.E.s (IBPs+Reverse Unitarity+D.E.)
- **Recasting the computation might overcome problems of traditional methods!!!**

Schwarzschild from Amplitudes to all orders in G

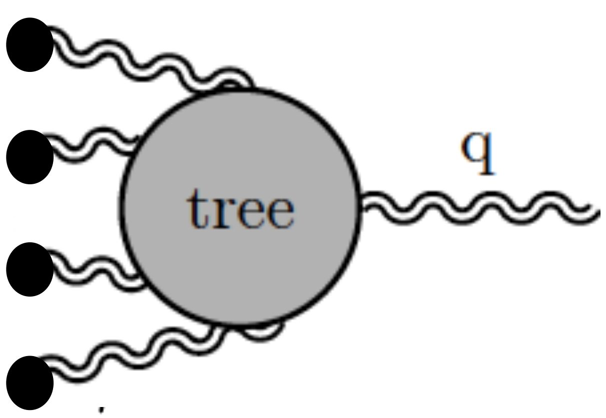
[2407.09448],[2405.14421] S.M., P. Vanhove

GREFT ⟷ General Relativity

Equivalence up to:

1. **Choice of DOFs** → $g_{\mu\nu} = \eta_{\mu\nu} + \sum_{n=1}^{\infty} h_{\mu\nu}^{(n)}$
2. **Choice of Gauge** → **harmonic gauge**

$\mathcal{M}_3(p_1, q) =$



$$i\mathcal{M}_3^{(l)}(p_1, q) = -\frac{i\sqrt{32\pi G_N}}{2}\langle T^{(l)\,\mu\nu}(q^2)\rangle \epsilon_{\mu\nu}$$

Duff (1974)

$$h_{\mu\nu}^{(l+1)}(\vec{x}) = -16\pi G_N \int \frac{d^d\vec{q}}{(2\pi)^d} e^{i\vec{q}\cdot\vec{x}} \frac{1}{\vec{q}^2}\left(\langle T_{\mu\nu}^{(l)}\rangle^{\text{class.}}(q^2) - \frac{1}{d-1}\eta_{\mu\nu}\langle T^{(l)}\rangle^{\text{class.}}(q^2)\right)$$

Expectation: n-loop diagrams generate G_N^{n+1} terms of the metric

Schwarzschild from Amplitudes

[2407.09448],[2405.14421] **S.M.**, P. Vanhove

GREFT ⟷ **General Relativity**

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Too complicated!!

Main problems from previous attempts:

[2010.08882] **S.M**, Vanhove

1. **Infinite tower of *non-minimal couplings* (due to intermediate UV-divs)**
2. **No algorithm for higher loops (3-loops was already complicated)**

Schwarzschild from Amplitudes

[2407.09448],[2405.14421] **S.M.**, P. Vanhove

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Take a step back

Schwarzschild from Amplitudes

[2407.09448],[2405.14421] S.M., P. Vanhove

Cubic formulation of GREFT

[1705.00626] Cheung, Remmen

Choice of DOFs: 1)Gothic metric: $\mathfrak{g}^{ab} = \sqrt{-g} g^{ab} = \eta^{ab} - \sqrt{32\pi G_N} h^{ab}$

(GR people do it, we should pay more attention)

2) Extra Auxiliary field : $A^a_{bc} = \Gamma^a_{bc} - \frac{1}{2}\delta^a_{(b}\Gamma^d_{c)d}.$

(unorthodox but necessary to constrain to 3pt vertices)

Choice of Gauge: harmonic gauge*(non unique)

+ couple to worldline $\mathcal{L}_{p.p.} = -\frac{m}{2}\int d\tau \ \left(e^{-1} g^{\mu\nu} v_\mu v_\nu + e\right) = -\frac{m}{2}\int d\tau \ \left(\frac{\mathfrak{g}^{\mu\nu} v_\mu v_\nu}{(\sqrt{-\mathfrak{g}})^{\frac{2}{D-2}}} + 1\right)$

Schwarzschild from Amplitudes

[2407.09448],[2405.14421] **S.M.**, P. Vanhove

Cubic formulation of GREFT

Complete ansatz:

Single multi-loop Master Integral

$$\sqrt{32\pi G_N} h_{\mu\nu}^{(n)}(\mathbf{x}) = \int d^{D-1}x \; e^{i\mathbf{k}\mathbf{x}} \; J_{\mu\nu}^{(n)}(\mathbf{k}),$$

$$\sqrt{32\pi G_N} A_{bc}^{a\;(n)}(\mathbf{x}) = \int d^{D-1}x \; e^{i\mathbf{k}\mathbf{x}} \; Y_{bc}^{a\;(n)}(\mathbf{k}),$$

$$J_{\mu\nu}^{(n)}(\mathbf{k}) = \rho(|\mathbf{k}|, D, n) \left(\underline{\chi_1^{(n)}} \delta_\mu^0 \delta_\nu^0 + \underline{\chi_2^{(n)}} \eta_{\mu\nu} + \underline{\chi_3^{(n)}} \frac{k_\mu k_\nu}{\mathbf{k}^2} \right)$$

$$Y_{bc}^{a\;(n)}(\mathbf{k}) = -i\rho(|\mathbf{k}|, D, n) \left(k_{(b} \left(\underline{\chi_7^{(n)}} \delta_{c)}^0 \delta_0^a + \underline{\chi_8^{(n)}} \delta_{c)}^a \right) + k^a \left(\underline{\chi_4^{(n)}} \delta_b^0 \delta_c^0 + \underline{\chi_5^{(n)}} \eta_{bc} + \underline{\chi_6^{(n)}} \frac{k_b k_c}{\mathbf{k}^2} \right) \right).$$

$$\rho(|\mathbf{k}|, D, n) = \frac{\Gamma\left(\frac{2-(D-3)(n-1)}{2}\right)}{\Gamma\left(\frac{n(D-3)}{2}\right)} \frac{\left(\Gamma\left(\frac{D-3}{2}\right) G_N m\right)^n}{(|\mathbf{k}|/(2\sqrt{\pi}))^{2-(D-3)(n-1)}};$$

form factors

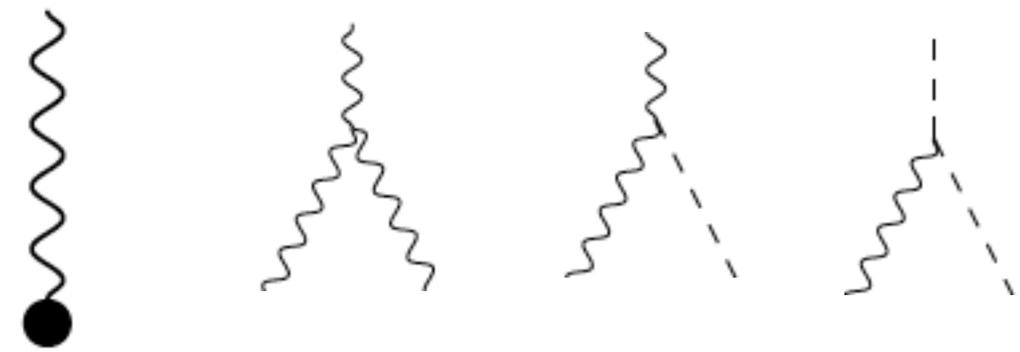
$$\boldsymbol{\chi}^{(n)}(D) = (\chi_1^{(n)}, \ldots, \chi_8^{(n)})$$

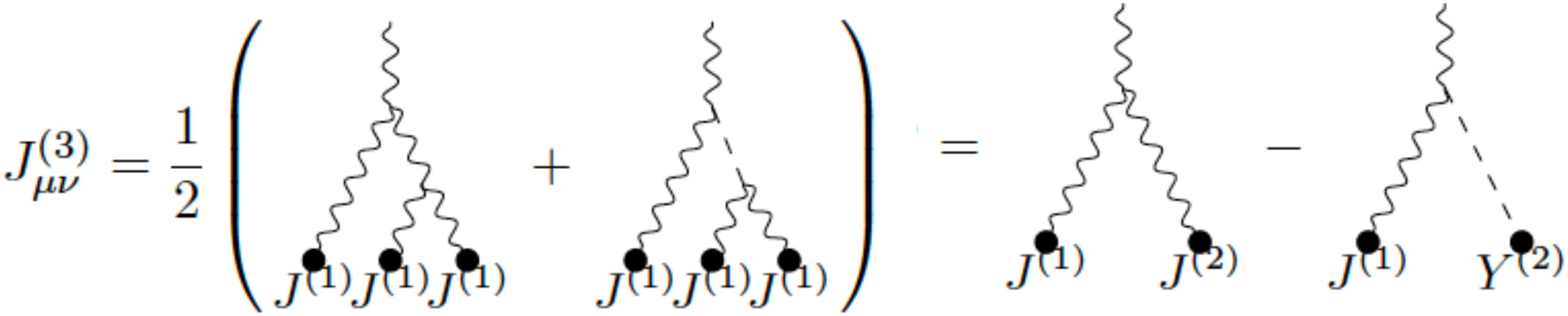
Schwarzschild from Amplitudes

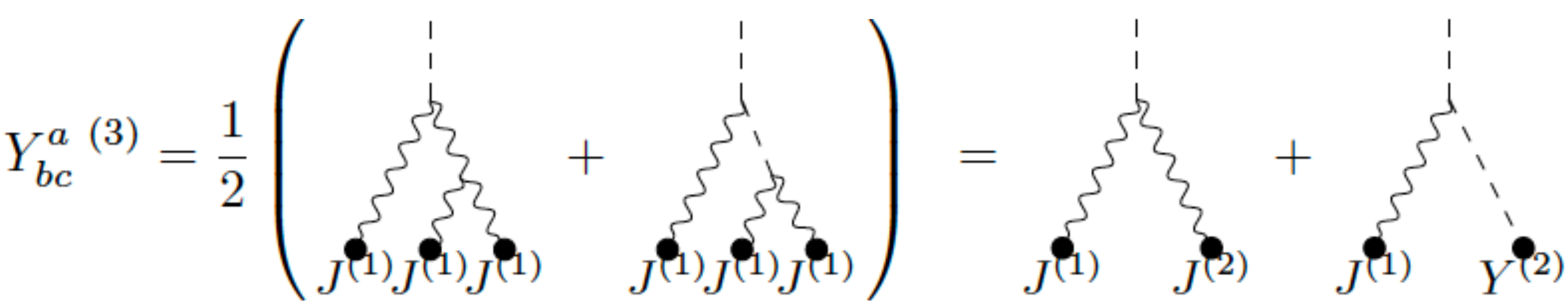
[2407.09448],[2405.14421] **S.M.**, P. Vanhove

Cubic formulation of GREFT

Feynman rules:







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Cubic formulation of GREFT

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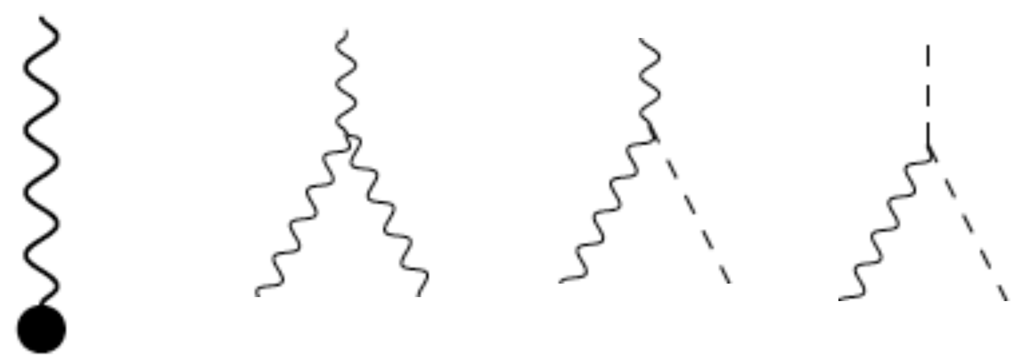
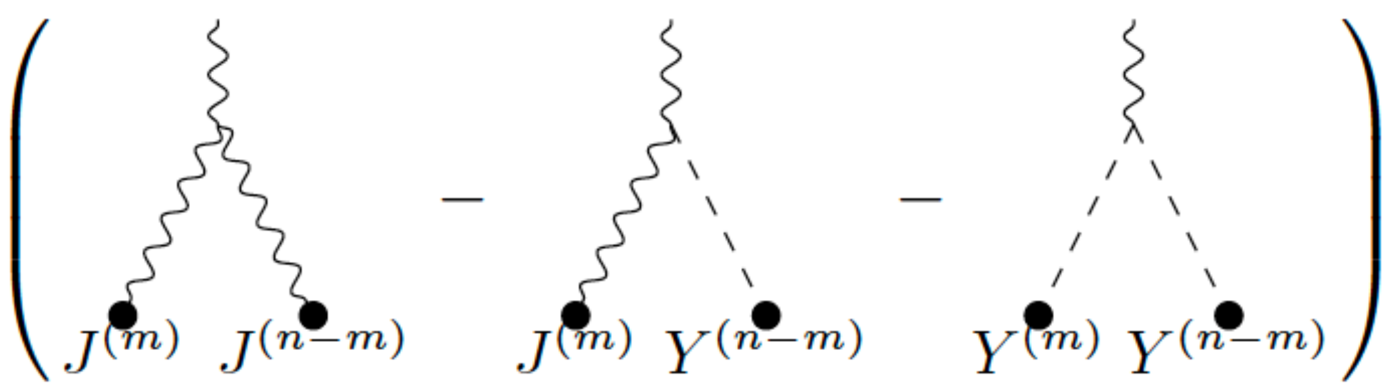
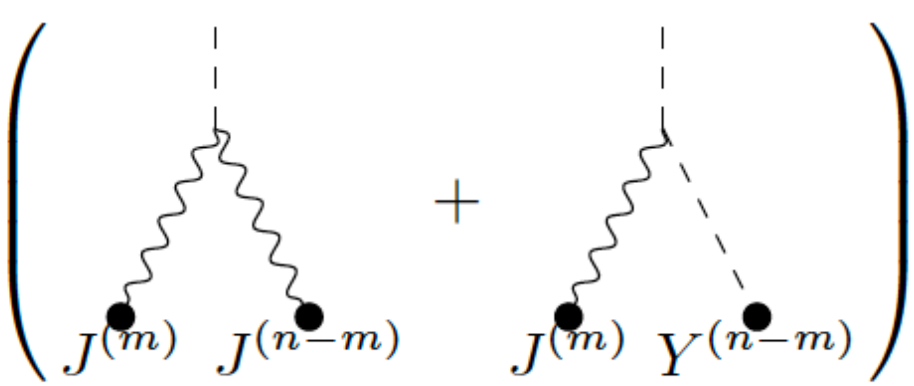


$$J^{(n)} = \sum_{m=1}^{n-1}$$



$$Y^{(n)} = \sum_{m=1}^{n-1}$$



Iterative structure to all orders!!!
due to 3pt interactions

Schwarzschild from Amplitudes

[2407.09448],[2405.14421] S.M., P. Vanhove

Metric to all orders in G

Recursion relations

$$\chi_k^{(n)}(D) = \sum_{i,j=1}^{8} \sum_{m=1}^{n-1} \chi_i^{(m)}(D)\chi_j^{(n-m)}(D)M_k^{ij}(D)$$

Solvable at D=4 →

$$\chi^{(n)}(4) = \Big(8, 0, 0, 4 + n(-1)^n + \frac{1+3(-1)^n}{2n(n+2)}, 2 + \frac{1+3(-1)^n}{2n(n+2)}, \frac{1+3(-1)^n}{2n(n+2)}(n-3), \frac{1}{n} - 4 - \frac{1+3(-1)^n}{2n(n+2)}(n+1), \frac{1+3(-1)^n}{2n(n+2)}\Big)$$

no UV-divs!!!

→ **Resums to GR solution** (**GREFT computation "picks" the simplest harmonic gauge**)

Schwarzschild from Amplitudes

[2407.09448],[2405.14421] **S.M.**, P. Vanhove

Geodesic motion (0SF)

SF expansion: $\mathcal{S} = \mathcal{S}_{EH} + \mathcal{S}_l + \mathcal{S}_H$; $\quad \mathcal{S}_H = -\frac{M}{2}\int d\tau_H \left(\frac{\mathfrak{g}^{\mu\nu} v_{H\mu} v_{H\nu}}{(\sqrt{-\mathfrak{g}})^{\frac{2}{D-2}}} + 1 \right)$

n-point graviton vertices contrary to PM-expansion

$$e^{i\mathcal{S}_{\text{eff}}[x_l, x_H]} = \int \mathcal{D}h\, \mathcal{D}A\; e^{i\mathcal{S}_{EH}[h,A] + i\mathcal{S}_{GF}[h] + i\mathcal{S}_l[x_l,h] + i\mathcal{S}_H[x_H,h]}$$

integrate-out via diagrams →

$$\mathcal{S}_{\text{eff}} = -\frac{M}{2}\int d\tau_H\, \eta^{\mu\nu} v_{H\mu} v_{H\nu} + M \sum_{n=0}^{\infty} \int d\tau_l \left(\frac{m}{M}\right)^{n+1} \mathcal{L}_n[x_l(\tau_l), x_H(\tau_H)]$$

+SF expand trajectories

$$x_l^\mu(\tau_l) \equiv x^\mu(\tau) = \sum_{n=0}^{\infty} \left(\frac{m}{M}\right)^n \delta x^{(n)\,\mu}(\tau), \quad x_H^\mu(\tau_H) = u_H^\mu \tau_H + \sum_{n=1}^{\infty} \left(\frac{m}{M}\right)^n \delta x_H^{(n)\,\mu}(\tau_H)$$

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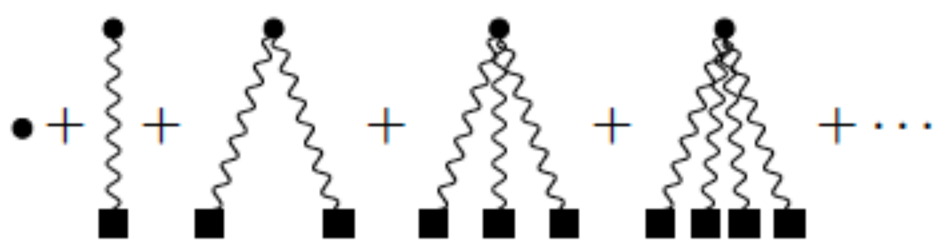
Geodesic motion (0SF)

SF expansion: $\mathcal{S}_{\rm eff} = -\frac{M}{2}\int d\tau_H\; \eta^{\mu\nu} v_{H\mu} v_{H\nu} + M\sum_{n=0}^{\infty} \int d\tau_l\; \left(\frac{m}{M}\right)^{n+1} \mathcal{L}_n[x_l(\tau_l), x_H(\tau_H)]$

$\mathcal{L}_0[x^\mu(\tau), u^\mu_H \tau_H] =$  $+ \cdots$

1) Infinite tower of n-graviton worldlines vertices

where $J_{\mu\nu}(\mathbf{k}) =$  $= \sum_{n=1}^{\infty} J^{(n)}_{\mu\nu}(\mathbf{k})$

2) Effective 1pt contains an infinite series -*dressed graviton emission*-, already known

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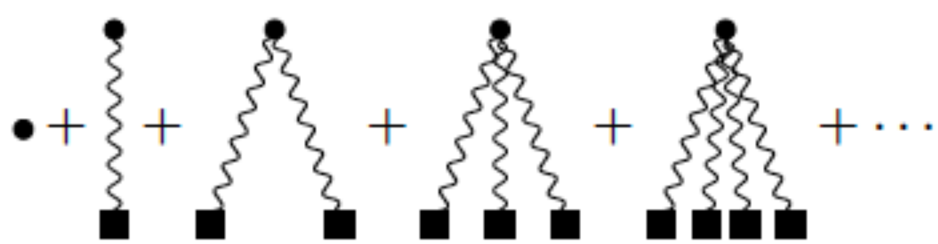
Geodesic motion (0SF)

We managed to perform explicitly the resummation of the above infinite diagrams where each one contains an infinite sum-*<u>double resummation</u>*!

$$\mathcal{L}_0[x^\mu(\tau), u_H^\mu \tau_H] =$$



$$= -\frac{1}{2} v_\mu(\tau) v_\nu(\tau) g^{\mu\nu}(|\mathbf{x}|(\tau))$$

trivially gives geodesic eq.

Conclusion

Classical GR through the lens of QFT approach

1. Efficiently exploits previous knowledge from particle physics
2. Possibly overcomes shortcomings of other methods
3. Easily applicable to GR extensions

4. Done well in PN, PM. Needs to be extended for SF

5. Non-perturbative, EFT-based approach can be used for questions regarding BHs (Love numbers, quantum effects etc.)

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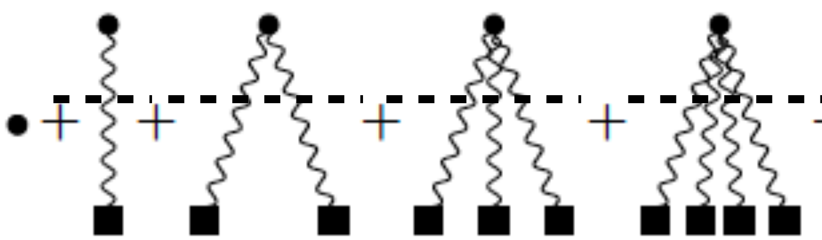
Geodesic motion (0SF)

$$\mathcal{L}_0[x^\mu(\tau), u^\mu_H \tau_H] = \bullet + \ldots$$



$$+ \cdots = \frac{1}{2} v_\mu(\tau) v_\nu(\tau) \left(-\eta^{\mu\nu} + \sum_{L=1}^{\infty} \mathcal{I}^{(L)}_{\alpha_1\beta_1,\ldots,\alpha_L\beta_L} \mathcal{T}^{\mu\nu\alpha_1\beta_1,\ldots,\alpha_L\beta_L}_{(L)} \right)$$

1) <u>Infinite</u> tower of n-graviton worldlines vertices

$$t^{\alpha_1\beta_1,\ldots,\alpha_n\beta_n}_{l\,(n)} = (32\pi G_N)^{n/2} \frac{im}{2} v_{l\mu} v_{l\nu} \mathcal{T}^{\mu\nu\alpha_1\beta_1,\ldots,\alpha_n\beta_n}_{(n)}$$

$$\mathcal{T}^{\mu\nu\alpha_1\beta_1,\ldots,\alpha_n\beta_n}_{(n)} = \eta^{\mu\alpha_n}\eta^{\nu\beta_n} \mathcal{P}^{\alpha_1\beta_1,\ldots,\alpha_{n-1}\beta_{n-1}}_{(n-1)} - \eta^{\mu\nu} \mathcal{P}^{\alpha_1\beta_1,\ldots,\alpha_n\beta_n}_{(n)}$$

defined by:

$$\frac{1}{(\sqrt{-\mathfrak{g}})^{\frac{2}{D-2}}} = 1 + \sum_{n=1}^{\infty} (32\pi G_N)^{\frac{n}{2}} \mathcal{P}^{\alpha_1\beta_1,\ldots,\alpha_n\beta_n}_{(n)} h_{\alpha_1\beta_1} \times \cdots \times h_{\alpha_n\beta_n}$$

2) Effective 1pt contains an <u>infinite</u> series

-*<u>dressed graviton emission</u>*-

$$\mathcal{I}^{(L)}_{\alpha_1\beta_1,\ldots,\alpha_L\beta_L} = \int_{\mathbb{R}^3} \frac{d^3\mathbf{k}}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{x}(\tau)} \int_{\mathbb{R}^{3L}} \prod_{i=1}^{L} \frac{d^3\mathbf{q}_i}{(2\pi)^3} \delta^{(3)}\left(\sum_{i=1}^{L} \mathbf{q}_i - \mathbf{k}\right)$$
$$\times \sum_{n_1=1}^{\infty} \cdots \sum_{n_L=1}^{\infty} \rho(|\mathbf{q}_i|, n_i) \left(\chi_1^{(n_i)} \delta^0_{\alpha_i} \delta^0_{\beta_i} + \chi_2^{(n_i)} \left(\eta_{\alpha_i\beta_i} - \frac{q_{i,\alpha_i} q_{i,\beta_i}}{\mathbf{q}_i^2} \right) \right)$$

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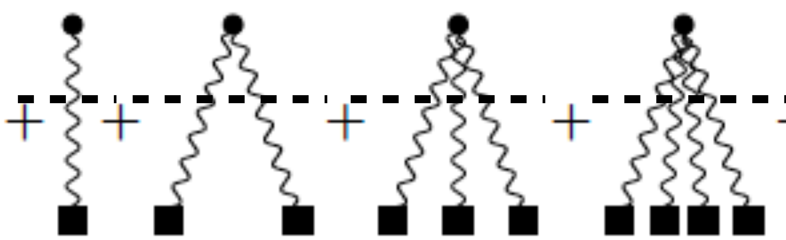
Geodesic motion (0SF)

$\mathcal{L}_0[x^\mu(\tau), u^\mu_H \tau_H] = \bullet +$  $+ \cdots = \frac{1}{2} v_\mu(\tau) v_\nu(\tau) \left(-\eta^{\mu\nu} + \sum_{L=1}^{\infty} \mathcal{I}^{(L)}_{\alpha_1\beta_1,\ldots,\alpha_L\beta_L} \mathcal{T}^{\mu\nu\alpha_1\beta_1,\ldots,\alpha_L\beta_L}_{(L)} \right)$

1) **Infinite tower of n-graviton worldlines vertices**

$$t^{\alpha_1\beta_1,\ldots,\alpha_n\beta_n}_{l\,(n)} = (32\pi G_N)^{n/2} \frac{im}{2} v_{l\mu} v_{l\nu} \mathcal{T}^{\mu\nu\alpha_1\beta_1,\ldots,\alpha_n\beta_n}_{(n)}$$

$$\mathcal{T}^{\mu\nu\alpha_1\beta_1,\ldots,\alpha_n\beta_n}_{(n)} = \eta^{\mu\alpha_n}\eta^{\nu\beta_n}\mathcal{P}^{\alpha_1\beta_1,\ldots,\alpha_{n-1}\beta_{n-1}}_{(n-1)} - \eta^{\mu\nu}\mathcal{P}^{\alpha_1\beta_1,\ldots,\alpha_n\beta_n}_{(n)}$$

defined by:

$$\frac{1}{(\sqrt{-\mathfrak{g}})^{\frac{2}{D-2}}} = 1 + \sum_{n=1}^{\infty} (32\pi G_N)^{\frac{n}{2}} \mathcal{P}^{\alpha_1\beta_1,\ldots,\alpha_n\beta_n}_{(n)} h_{\alpha_1\beta_1} \times \cdots \times h_{\alpha_n\beta_n}$$

2) **Effective 1pt contains an infinite series**
-dressed graviton emission-

Fourier before loop integration decouples the multi-loops!!

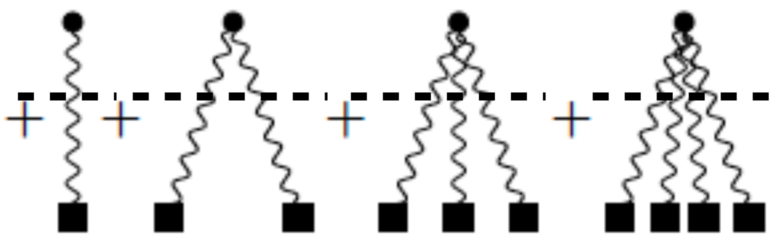
$$\mathcal{I}^{(L)}_{\alpha_1\beta_1,\ldots,\alpha_L\beta_L} = \rho^{2L} \prod_{i=1}^{L} \left(\delta^0_{\alpha_i}\delta^0_{\beta_i} \left(\frac{4(1+\rho)}{\rho(1-\rho)} - 1 \right) + n_{\alpha_i} n_{\beta_i} \right)$$

Schwarzschild from Amplitudes

[2407.09448],[2405.14421] **S.M.**, P. Vanhove

Geodesic motion (0SF)

$$\mathcal{L}_0[x^\mu(\tau), u^\mu_H \tau_H] = \bullet +$$



$$+ \cdots = \frac{1}{2} v_\mu(\tau) v_\nu(\tau) \left(-\eta^{\mu\nu} + \sum_{L=1}^{\infty} \mathcal{I}^{(L)}_{\alpha_1\beta_1,\ldots,\alpha_L\beta_L} \mathcal{T}^{\mu\nu\alpha_1\beta_1,\ldots,\alpha_L\beta_L}_{(L)} \right)$$

1) **Infinite tower of n-graviton worldlines vertices**

$$t^{\alpha_1\beta_1,\ldots,\alpha_n\beta_n}_{l\,(n)} = (32\pi G_N)^{n/2} \frac{im}{2} v_{l\mu} v_{l\nu} \mathcal{T}^{\mu\nu\alpha_1\beta_1,\ldots,\alpha_n\beta_n}_{(n)}$$

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defined by:

$$\frac{1}{(\sqrt{-\mathfrak{g}})^{\frac{2}{D-2}}} = 1 + \sum_{n=1}^{\infty} (32\pi G_N)^{\frac{n}{2}} \mathcal{P}^{\alpha_1\beta_1,\ldots,\alpha_n\beta_n}_{(n)} h_{\alpha_1\beta_1} \times \cdots \times h_{\alpha_n\beta_n}$$

2) **Effective 1pt contains an infinite series**
-*dressed graviton emission*-

Recursion:

$$\mathcal{I}^{(L)}_{\alpha_1\beta_1,\ldots,\alpha_L\beta_L} = \mathcal{I}^{(L-1)}_{\alpha_1\beta_1,\ldots,\alpha_{L-1}\beta_{L-1}} \rho^2 \left(\delta^0_{\alpha_L} \delta^0_{\beta_L} \left(\frac{4(1+\rho)}{\rho(1-\rho)} - 1 \right) + n_{\alpha_L} n_{\beta_L} \right)$$

Schwarzschild from Amplitudes

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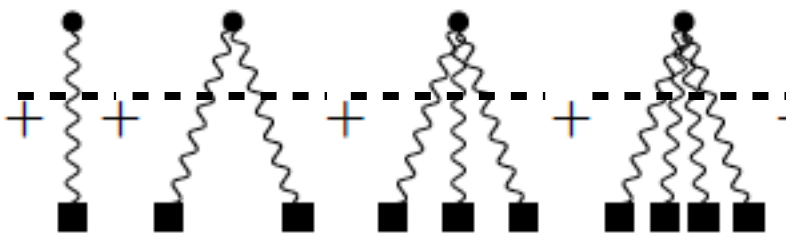
Geodesic motion (0SF)

$$\mathcal{L}_0[x^\mu(\tau), u^\mu_H \tau_H] = \bullet + $$



$$+ \cdots = \frac{1}{2} v_\mu(\tau) v_\nu(\tau) \left(-\eta^{\mu\nu} + \sum_{L=1}^{\infty} \mathcal{I}^{(L)}_{\alpha_1\beta_1,\ldots,\alpha_L\beta_L} \mathcal{T}^{\mu\nu\alpha_1\beta_1,\ldots,\alpha_L\beta_L}_{(L)} \right)$$

__Recursion:__

$$\mathcal{I}^{(L)}_{\alpha_1\beta_1,\ldots,\alpha_L\beta_L} = \mathcal{I}^{(L-1)}_{\alpha_1\beta_1,\ldots,\alpha_{L-1}\beta_{L-1}} \rho^2 \left(\delta^0_{\alpha_L} \delta^0_{\beta_L} \left(\frac{4(1+\rho)}{\rho(1-\rho)} - 1 \right) + n_{\alpha_L} n_{\beta_L} \right)$$

+

$$\mathcal{T}^{\mu\nu\alpha_1\beta_1,\ldots,\alpha_n\beta_n}_{(n)} = \eta^{\mu\alpha_n} \eta^{\nu\beta_n} \mathcal{P}^{\alpha_1\beta_1,\ldots,\alpha_{n-1}\beta_{n-1}}_{(n-1)} - \eta^{\mu\nu} \mathcal{P}^{\alpha_1\beta_1,\ldots,\alpha_n\beta_n}_{(n)}$$

$$\longrightarrow \mathcal{I}^{(L)}_{\alpha_1\beta_1,\ldots,\alpha_L\beta_L} \mathcal{T}^{\mu\nu\alpha_1\beta_1,\ldots,\alpha_L\beta_L}_{(L)} = -\eta^{\mu\nu} \mathcal{I}^{(L)}_{\alpha_1\beta_1,\ldots,\alpha_L\beta_L} \mathcal{P}^{\alpha_1\beta_1,\ldots,\alpha_L\beta_L}_{(L)}$$
$$+ \mathcal{I}^{(L-1)}_{\alpha_1\beta_1,\ldots,\alpha_{L-1}\beta_{L-1}} \mathcal{P}^{\alpha_1\beta_1,\ldots,\alpha_{L-1}\beta_{L-1}}_{(L-1)} \rho^2 \left(\delta^\mu_0 \delta^\nu_0 \left(\frac{4(1+\rho)}{\rho(1-\rho)} - 1 \right) + n^\mu n^\nu \right)$$

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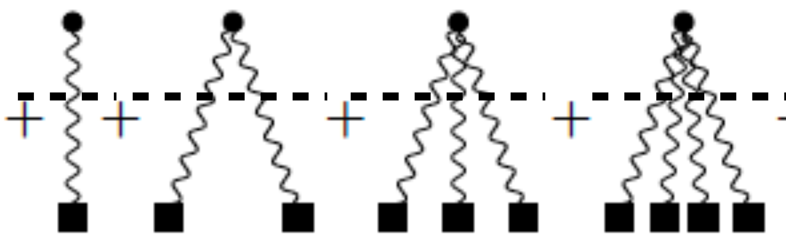
Geodesic motion (0SF)

$$\mathcal{L}_0[x^\mu(\tau), u_H^\mu \tau_H] = \bullet +$$



$$+ \cdots = \frac{1}{2} v_\mu(\tau) v_\nu(\tau) \left(-\eta^{\mu\nu} + \sum_{L=1}^{\infty} \mathcal{I}^{(L)}_{\alpha_1\beta_1,\ldots,\alpha_L\beta_L} \mathcal{T}^{\mu\nu\alpha_1\beta_1,\ldots,\alpha_L\beta_L}_{(L)} \right)$$

$$\mathcal{I}^{(L)}_{\alpha_1\beta_1,\ldots,\alpha_L\beta_L} \mathcal{T}^{\mu\nu\alpha_1\beta_1,\ldots,\alpha_L\beta_L}_{(L)} = -\eta^{\mu\nu} \mathcal{I}^{(L)}_{\alpha_1\beta_1,\ldots,\alpha_L\beta_L} \mathcal{P}^{\alpha_1\beta_1,\ldots,\alpha_L\beta_L}_{(L)}$$
$$+ \mathcal{I}^{(L-1)}_{\alpha_1\beta_1,\ldots,\alpha_{L-1}\beta_{L-1}} \mathcal{P}^{\alpha_1\beta_1,\ldots,\alpha_{L-1}\beta_{L-1}}_{(L-1)} \rho^2 \left(\delta_0^\mu \delta_0^\nu \left(\frac{4(1+\rho)}{\rho(1-\rho)} - 1 \right) + n^\mu n^\nu \right)$$

and from definition: $$\sum_{L=1}^{\infty} \mathcal{P}^{\alpha_1\beta_1,\ldots,\alpha_L\beta_L}_{(L)} \mathcal{I}^{(L)}_{\alpha_1\beta_1,\ldots,\alpha_L\beta_L} = \left(-\det \left[\eta^{\mu\nu} - \rho^2 \left(\delta_0^\mu \delta_0^\nu \left(\frac{4(1+\rho)}{\rho(1-\rho)} - 1 \right) + n^\mu n^\nu \right) \right] \right)^{-\frac{1}{2}} - 1$$

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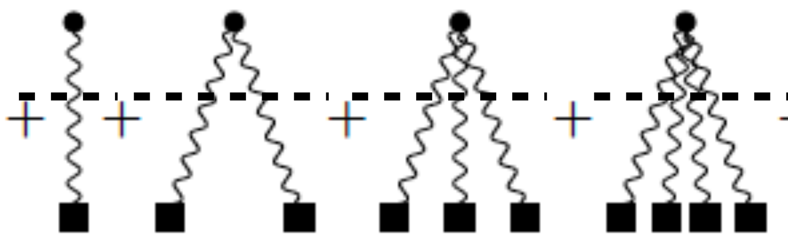
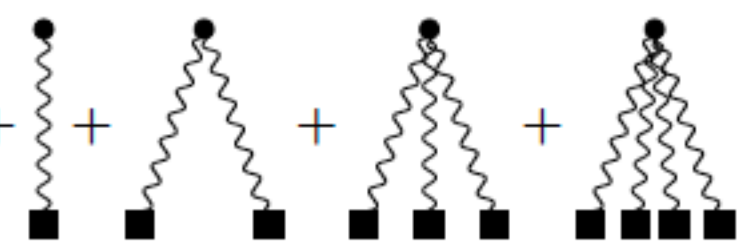
Geodesic motion (0SF)



$$\mathcal{L}_0[x^\mu(\tau), u_H^\mu \tau_H] = \bullet + \cdots = \frac{1}{2} v_\mu(\tau) v_\nu(\tau) \left(-\eta^{\mu\nu} + \sum_{L=1}^{\infty} \mathcal{I}^{(L)}_{\alpha_1\beta_1,\ldots,\alpha_L\beta_L} \mathcal{T}^{\mu\nu\alpha_1\beta_1,\ldots,\alpha_L\beta_L}_{(L)}\right)$$

$$\sum_{L=1}^{\infty} \mathcal{P}^{\alpha_1\beta_1,\ldots,\alpha_L\beta_L}_{(L)} \mathcal{I}^{(L)}_{\alpha_1\beta_1,\ldots,\alpha_L\beta_L} = \frac{1}{(1+\rho)^2} - 1.$$

$$\sum_{L=1}^{\infty} \mathcal{I}^{(L)}_{\alpha_1\beta_1,\ldots,\alpha_L\beta_L} \mathcal{T}^{\mu\nu\alpha_1\beta_1,\ldots,\alpha_L\beta_L}_{(L)} = \frac{\rho}{(1+\rho)^2} \left(\delta^\mu_0 \delta^\nu_0 \left(\frac{4(1+\rho)}{(1-\rho)} - \rho\right) + (\rho+2)\,\eta^{\mu\nu} + \rho\, n^\mu n^\nu\right)$$



$$\mathcal{L}_0[x^\mu(\tau), u_H^\mu \tau_H] = \bullet + \cdots = -\frac{1}{2} v_\mu(\tau) v_\nu(\tau) g^{\mu\nu}(|\mathbf{x}|(\tau))$$