GdR Ondes Gravitationnelles:
réunion du groupe de travail "Formes d'Onde"

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Nonlinear tidal effects and nonlinear deformability of compact objects

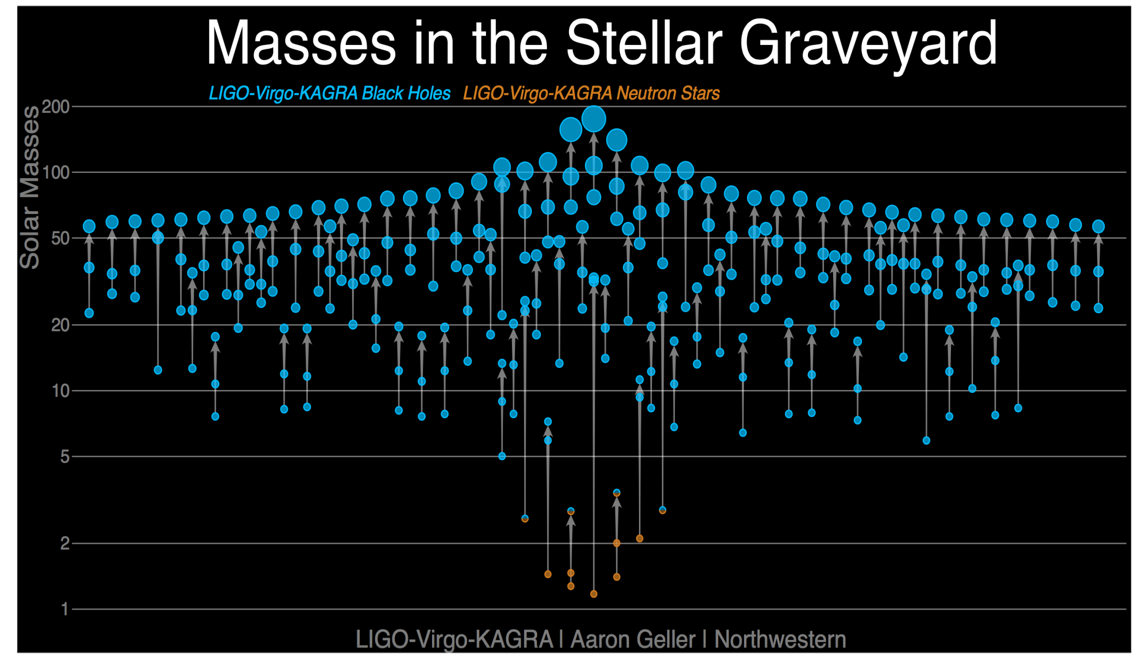
mostly based on arXiv: 2312.05065 and 2409.xxxxx
with S. Iteanu, M. M. Riva, N. Savić and F. Vernizzi

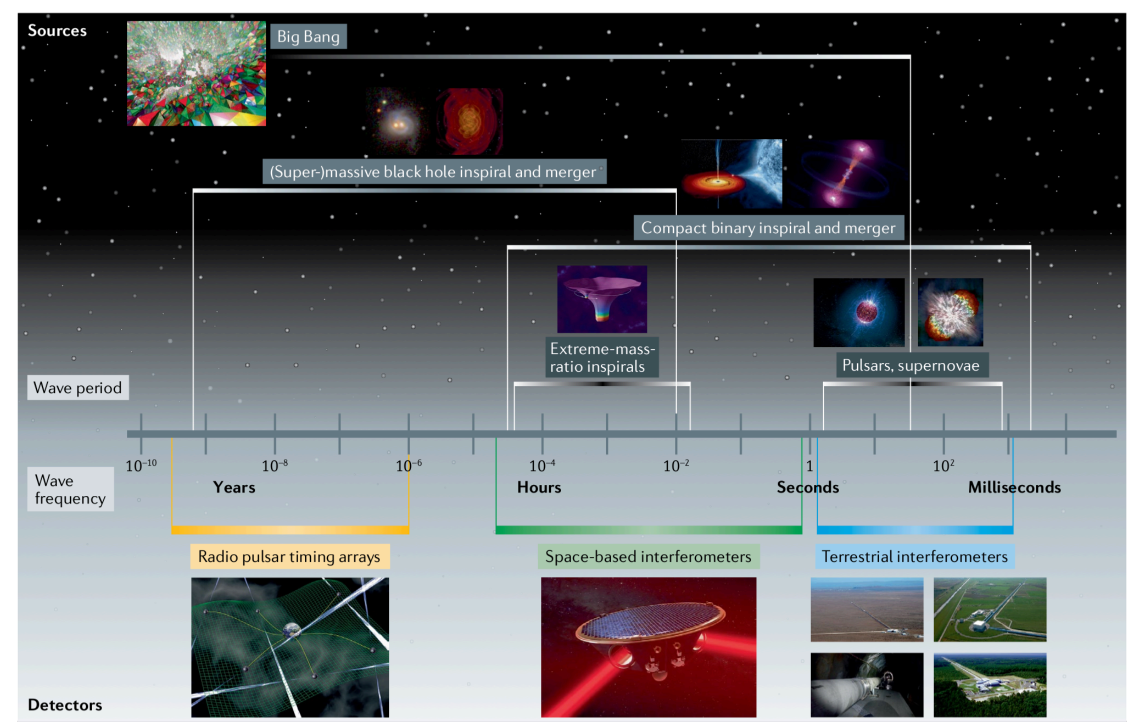
Toward the era of precision physics with gravitational waves

- Upcoming era of precision physics with gravitational waves holds the promise of important breakthroughs and new discoveries.

- To maximize the science return from the increasing number of gravitational-wave observations and the discovery potential of future detectors, ever more accurate waveform templates are necessary. [Maggiore et al. '19], [Sathyaprakash et al. '19], [Kalogera et al. '21], [Berti et al. '21], […]

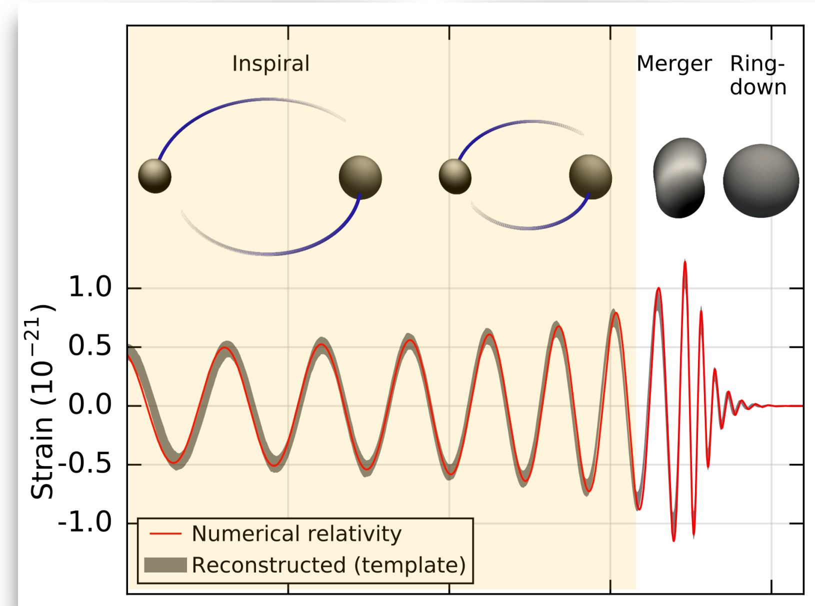
- This requires in particular to have a precise understanding of the conservative and dissipative dynamics of two-body systems, which includes tidal effects. [Buonanno et al. '22], [Flanagan and Hinderer '07], [Henry, Faye and Blanchet '20], [Kälin, Liu, and Porto '20], […]





[Nature Reviews Physics, 3, 344–366 (2021)]

Tidal deformability of compact objects



- Tidal effects change the dynamics during the inspiral.

- The tidal deformability is described in terms of a set of coefficients, which capture the *conservative* and *dissipative* induced response.
  [Fang and Lovelace '05], [Damour and Nagar '09], [Binnington and Poisson '05]

- Conservative coefficients (often called *Love numbers*) and dissipative numbers (or, *tidal heating*) carry relevant information about the object's structure and interior dynamics:

  _ equation of state of neutron stars; [Flanagan and Hinderer '07], [Vines, Flanagan and Hinderer '11], [Bini, Damour and Faye '12], [Baiotti and Rezzolla '17], […]

  _ physics at the horizon of black holes and fundamental aspects of gravity in strong-field regime; [Hui, Joyce, Penco, LS and Solomon '21, '22], [Charalambous, Dubovsky and Ivanov '21], […]

  _ new physics and existence of new types of compact objects.
  [Franzin et al. '17], [Cardoso et al. '17], [Cardoso and Pani '19], […]

Tidal deformability of compact objects

- The measurement of tidal deformation is challenging with current detectors.

- Example of constraints on tidal heating from LKV O1-O3 data: [Chia, Zhou and Ivanov '24].
  Constraints on black hole dissipative coeffs. still 2 orders of magnitudes larger than theoretical value.

- We can search for exotic compact objects (e.g., boson stars, DM stars…) with large λ.
  Exotic compact objects can easily have Love numbers that are orders of magnitude larger than standard objects.
  Example of matched-filtering search for binaries with compact objects with $10^2 \lesssim \lambda \lesssim 10^6$: [Chia et al. '23].

- Precise measurements of tidal coefficients possible in the future:
  _ an EMRI detection by LISA could set constraints on the (dimensionless) Love numbers of highly-spinning central objects at $\sim 10^{-2}$–10^{-3} level; [Piovano, Maselli, Pani '22]
  _ the Einstein Telescope will be able to observe the onset of tidal effects close to the merger and pin down very precisely the EoS of neutron stars. [Maggiore et al '19], [Iacovelli et al '23], […]

Outline

I will discuss theoretical aspects of tidal Love numbers of compact objects.
I will focus on a particular set of subleading tidal effects.

I. Nonlinear corrections to the tidal deformability of compact objects
II. Vanishing of nonlinear Love numbers of black holes

Subdominant tidal effects in binary inspirals

- In the Post-Newtonian (PN) regime of the inspiral, leading-order tidal effects are associated with tidal heating, and start from 2.5PN order for spinning objects and 4PN order for non-rotating ones. [Poisson and Sasaki '94], [Tagoshi et al. '97], [Blanchet '13], […]

- Conservative effects start from 5PN. [Damour '83], [Porto '16], […]

- Various reasons to study subdominant effects:
  _ put leading-order results on firmer grounds;
  _ neglecting them might introduce systematics in parameter estimation;
  _ could be useful to recalibrate effective models, improve waveform, break degeneracy among observables;
  _ could reveal interesting information on the fundamental aspects of gravity;
  _ especially relevant when leading-order effects vanish, making the next-to-leading-order corrections the actual dominant contributions (e.g., black holes).

- Two types of subleading corrections to Love Numbers (LNs):
  → $\omega \neq 0$ (*dynamical* LNs)
  → nonlinearities

Tidal Love numbers of black holes

- It is well known that the conservative tidal response of Black Holes (BHs) in zero:
  _ Schwarzschild BHs: [Fang and Lovelace ’05], [Binnington and Poisson ‘09], [Damour and Nagar ’09]
  _ Kerr BHs: [Le Tiec, Casals ’20], [Le Tiec, Casals, Franzin ’20], [Chia ’20], [Charalambous, Dubovsky, Ivanov ‘21]
  _ Reissner–Nordström BHs: [Cardoso et al. ’17], [Rai and LS ’24]

- This is special of 4D general relativity: Love numbers are nonzero for different objects, modified gravity theories, BHs in higher spacetime dimensions.

- The vanishing of the BH Love numbers has been understood in terms of symmetries in general relativity: [Hui, Joyce, Penco, LS and Solomon ’21, ’22], [Charalambous, Dubovsky and Ivanov ’21, ’22], [Rai and LS ’24]

- All these results have been obtained in the context of *linear perturbation theory*. However, general relativity is intrinsically *nonlinear*.

Nonlinear tidal effects

- Nonlinearities in the Einstein field equations affect the tidal response of a compact object.

- Similarly to electromagnetism, where the nonlinear polarization of an optical medium can be studied in nonlinear polarization theory, we can ask a similar question here.

  - *What is the nonlinear static response of a black hole?*
  - *Are the symmetries of* [Hui, Joyce, Penco, LS and Solomon '21, '22] *an 'accident' of linear response theory, or do they admit an extension to nonlinear order?*

- Previous results in this context focused on a particular subset of static and axisymmetric perturbations: [Gürlebeck '15], [Poisson '20]

- To date a full answer to the previous questions is still unknown.

Nonlinear tidal effects

- A clean way of defining tidal deformation of compact objects – which systematically accounts for nonlinearities and is not affected by ambiguities due to gauge freedom in the theory – is in terms of the point-particle Effective Field Theory (EFT):
[Goldberger and Rothstein '04, '05, '06], [Porto '16], [Blanchet '13], [Levi '18], [...]

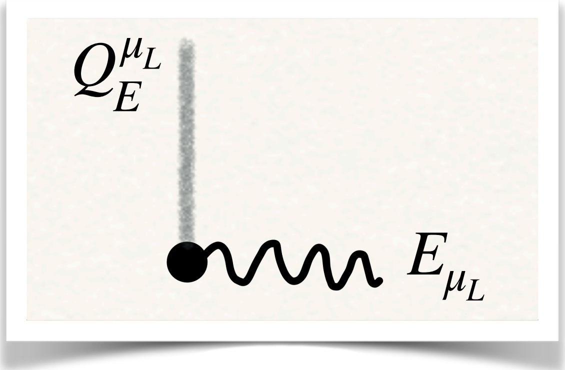
$$S = S_{\rm EH} + S_{\rm pp} + S_{\rm int}$$

$$S_{\rm EH} = \frac{M_{\rm Pl}^2}{2}\int {\rm d}^4x\sqrt{-g}\,R\,, \qquad S_{\rm pp} = -M\int {\rm d}\tau\,, \qquad S_{\rm int} = \sum_{\ell=2}^{\infty}\int {\rm d}\tau\left(Q_E^{\mu_L}E_{\mu_L} + Q_B^{\mu_L}B_{\mu_L}\right),$$

where $\mu_L \equiv \mu_1\cdots\mu_\ell$ and, schematically,

$$E_{\mu_1\cdots\mu_\ell} \sim \nabla_{\mu_1}\cdots\nabla_{\mu_{\ell-2}}E_{\mu_{\ell-1}\mu_\ell}\,, \qquad B_{\mu_1\cdots\mu_\ell} \sim \nabla_{\mu_1}\cdots\nabla_{\mu_{\ell-2}}B_{\mu_{\ell-1}\mu_\ell}\,,$$

$$E_{\mu\nu} \equiv C_{\mu\rho\nu\sigma}u^\rho u^\sigma, \qquad B_{\mu\nu} \equiv \frac{1}{2}\varepsilon_{\gamma\mu}{}^{\alpha\beta}C_{\nu\delta\alpha\beta}u^\delta u^\gamma\,.$$



Nonlinear tidal Love numbers

- $Q^{\mu_L}_{E,B}$ carry information about the object's finite-size properties and microscopic physics: absorption across the horizon, dissipation, tidal response…

- To define nonlinear response, we shall proceed as in EM. Focusing e.g. on E-sector:

$$\langle Q_E^{i_L}(\tau)\rangle = \sum_{n=1}^{\infty}\int \mathrm{d}\tau_1 \cdots \int \mathrm{d}\tau_n \, {}^{(n)}\mathscr{R}^{i_L|i_{L_1}\cdots i_{L_n}}(\tau-\tau_1,\ldots,\tau-\tau_n)E_{i_{L_1}}(\tau_1)\cdots E_{i_{L_n}}(\tau_n) \ ,$$

where ${}^{(n)}\mathscr{R}$ is the n$^{\mathrm{th}}$-order response function.

- To leading-order in the derivative expansion (quadrupole), considering non-rotating objects and conservative sector,

$${}^{(n)}\mathscr{R}^{ij|i_1j_1\cdots i_nj_n} \supset \delta^i_{j_n}\delta^j_{i_1}\delta^{i_2}_{j_1}\delta^{j_2}_{i_3}\cdots\delta^{j_{n-1}}_{i_n}\delta(\tau-\tau_1)\cdots\delta(\tau-\tau_n) + \text{perms.}$$

and S_{int} boils down to *local* contact operators:

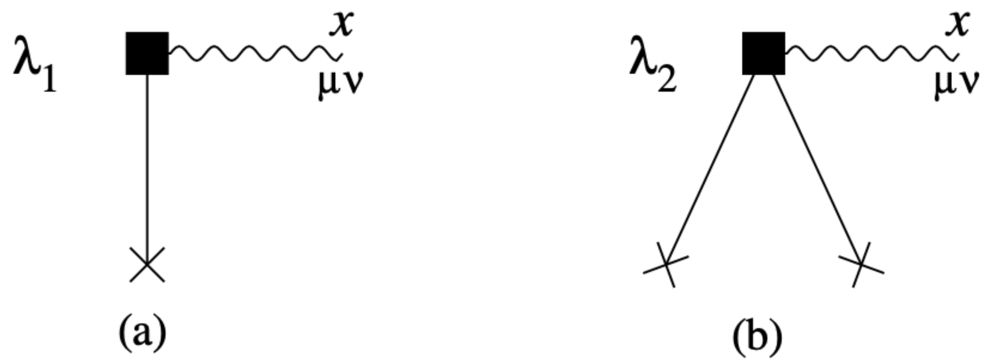
$$S_{\mathrm{int}} = \int \mathrm{d}\tau \sum_{n=1}^{\infty}\left[\lambda_n^{(E)} E_{\mu_1}{}^{\mu_2}\cdots E_{\mu_{n+1}}{}^{\mu_1} + \text{higher multipoles}\right] \sim \sum_{n,l}\int \mathrm{d}\tau\, \lambda^{(E)}_{l,n}\partial^l E^{n+1} \ .$$

Nonlinear tidal Love numbers

$$S_{\text{int}} = \int \mathrm{d}\tau \sum_{n=1}^{\infty} \left[\lambda_n^{(E)} E_{\mu_1}{}^{\mu_2} \cdots E_{\mu_{n+1}}{}^{\mu_1} + \text{higher multipoles}\right] \sim \sum_{n,l} \int \mathrm{d}\tau\, \lambda_{l,n}^{(E)} \partial^l E^{n+1}$$

- $\lambda_{l,n}^{(E)}$ are the (nonlinear) Love numbers.
- As in any EFT, $\lambda_{l,n}^{(E)}$ can be either constrained experimentally or determined via matching to some explicit UV model.
- In [2312.05065] and [2409.xxxxx] we computed the quadratic Love numbers of BHs by matching with GR:

$$S_{\text{int}} = \int \mathrm{d}\tau \left[\lambda_1^{(E)} E_{ij} E^{ij} + \lambda_1^{(B)} B_{ij} B^{ij} + \lambda_2^{(E)} E^i{}_j E^j{}_k E^k{}_i + \lambda_2^{(EB)} E^i{}_j B^j{}_k B^k{}_i + \text{higher multipoles}\right]$$



Feynman diagrams for the (a) linear and (b) nonlinear tidal deformation.
These vertices scale as $\sim 1/r^{\ell+1}$.

Nonlinear equations for static perturbations

- There are two expansion parameters:
  _ $\kappa \equiv 1/M_{\rm Pl}$ which controls nonlinearities of gravity;
  _ $\mathscr{E}$: the amplitude of the external tidal field.

- At quadratic order, the (static) equations are schematically: $\mathscr{D}\delta g \sim \kappa\, \delta g^2$.
  Solving perturbatively as $\delta g_{\mu\nu} = \delta g^{(1)}_{\mu\nu} + \kappa \delta g^{(2)}_{\mu\nu}$ and imposing regularity at the horizon, one can find the nonlinear solution for $\delta g_{\mu\nu}$.

- $\delta g_{\mu\nu} = \delta g^{\rm even}_{\mu\nu} + \delta g^{\rm odd}_{\mu\nu}$. As an example, let's focus on $\delta g^{\rm even}_{\mu\nu}$.
  In the Regge–Wheeler gauge, the most general parametrization of $\delta g^{\rm even}_{\mu\nu}$ is:

$$\delta g^{\rm even}_{\mu\nu}(r,\theta,\phi) = \sum_{\ell m} {\rm diag}\left[\left(1-\frac{r_s}{r}\right) H_0(r), H_2(r), K(r),\ \sin^2\theta\, K(r)\right] Y^m_\ell(\theta,\phi)\ .$$

- After some algebra, the Einstein equations boil down to

$$H_0'' + \frac{2r-r_s}{r(r-r_s)}H_0' - \frac{\ell(\ell+1)r(r-r_s)+r_s^2}{r^2(r-r_s)^2}H_0 = S_{H_0}\,, \qquad \text{where } S_{H_0} \sim O(\delta g^2).$$

Nonlinear equations for static perturbations

- Let's assume that the external tidal field is a quadrupole: asymptotically, $H_0^{(\ell=2,m)} \sim \mathscr{E} r^2$.

- The solution to the linearized equation is $H_0^{(\ell=2,m)} = \mathscr{E}\left(r^2 - r_s r\right)$, which is notoriously a polynomial (i.e., no induced linear static response).

- $S_{H_0} \sim O(\delta g^2)$ contains a product of 3 spherical harmonics:

$$\mathscr{G}^{\ell,\ell_1,\ell_2}_{m,m_1,m_2} \equiv \int Y_\ell^{m*}(\theta,\phi) Y_{\ell_1}^{m_1}(\theta,\phi) Y_{\ell_2}^{m_2}(\theta,\phi) \sin\theta \mathrm{d}\phi \mathrm{d}\theta \,,$$

  which enforces the standard angular momentum selection rule $\ell = \ell_1 \otimes \ell_2$, i.e. $|\ell_1 - \ell_2| \leq \ell \leq \ell_1 + \ell_2$ and $m = m_1 + m_2$.

- If $\ell_1 = \ell_2 = 2 \ \Rightarrow \ \ell = 0, 2, 4$. At the next order:

$$H_0^{(\ell=2,m')} = \mathscr{E}\left(r^2 - r_s r\right)\left[\delta_m^{m'} - \frac{\mathscr{E}}{4r_s^2}\mathscr{G}^{2,2,2}_{m',m,m} r(2r + 3r_s)\right] .$$

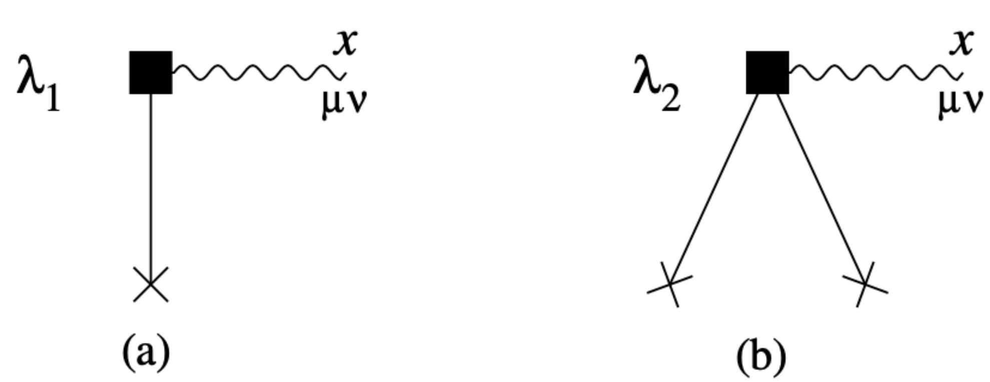
- Note that the quadratic terms in $\mathscr{E}$ are small corrections as long as $\mathscr{E} r^2 \ll r_s^2$.

Matching with point-particle EFT

- The full quadratic static solution

$$H_0^{(\ell=2,m')} = \mathcal{E}\left(r^2 - r_s r\right)\left[\delta_m^{m'} - \frac{\mathcal{E}}{4r_s^2}\mathcal{G}^{2,2,2}_{m',m,m} r(2r+3r_s)\right]$$

should then be compared (after a suitable gauge transformation) with



$$\sim \frac{1}{r^{\ell+1}}\,.$$

- The result of the matching (in the region $r_s \ll r \ll r_s/\sqrt{\mathcal{E}}$) is:

$$\lambda_1 = \lambda_2 = 0\,.$$

[Riva, LS, Savic, Vernizzi '23]

Vanishing of nonlinear Love numbers

- In [2409.xxxxx] we extend the result to the odd sector and to all multipoles.

$$H_0'' + \frac{2r - r_s}{r(r-r_s)} H_0' - \frac{\ell(\ell+1) r(r-r_s) + r_s^2}{r^2(r-r_s)^2} H_0 = S_{H_0} \, ,$$

$$h_0'' - \frac{\ell(\ell+1) r - 2r_s}{r^2(r-r_s)} h_0 = S_{h_0} \, ,$$

where $S_{H_0}, S_{h_0} \sim O(\delta g^2)$.

- We show that, for all ℓ's, the structure of the eqs in GR is so special that, up to second order, the solutions H_0 and h_0 are simple polynomials in r.

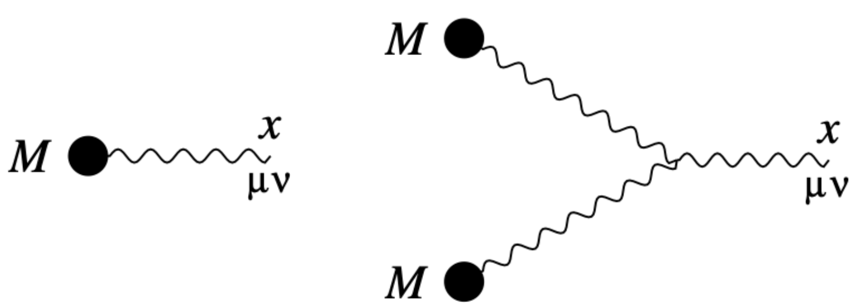
- In the EFT, all linear and quadratic Love number couplings vanish:

$$S_{\rm int} \sim \int {\rm d}\tau \left[\lambda_{1,n}^{(E)} (\partial^n E)^2 + \lambda_{1,n}^{(B)} (\partial^n B)^2 + \lambda_{2,nml}^{(E)} \partial^n E \, \partial^m E \, \partial^l E + \lambda_{2,nml}^{(EB)} \partial^n E \, \partial^m B \, \partial^l B + \dots \right]$$

$$\boxed{\lambda_{1,n}^{(E)} = \lambda_{1,n}^{(B)} = \lambda_{2,nml}^{(E)} = \lambda_{2,nml}^{(EB)} = 0 \, .}$$

Vanishing of nonlinear Love numbers

- The point-particle EFT can be matched to the full solution without turning on Love number couplings.
  Nonlinear corrections to the static solution in GR can be reconstructed from the EFT, to all orders in r_s, via just graviton bulk nonlinearities.



Feynman diagrams that reconstruct the Schwarzschild metric up to $O(r_s^2)$.

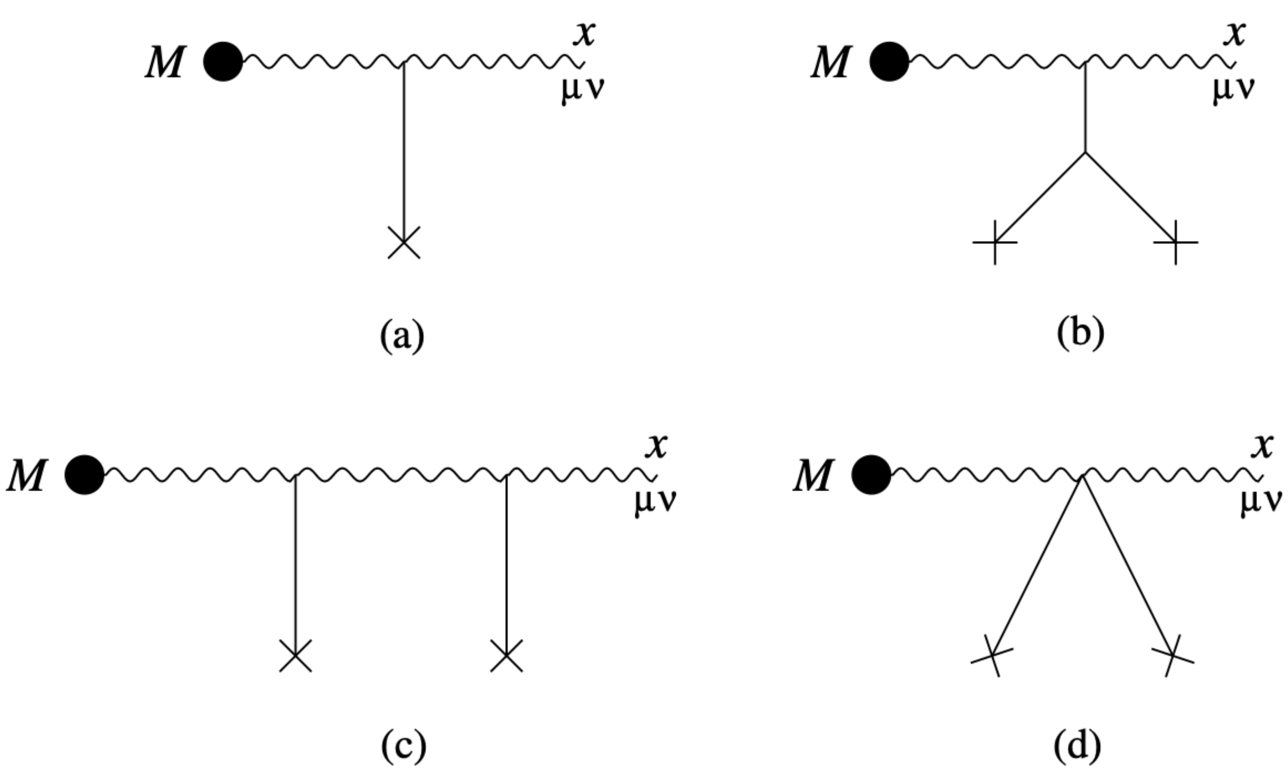


Diagram (a) yields the order-r_s correction to the linear tidal field solution.

Diagrams (b), (c) and (d) represent order-r_s corrections to the tidal source at second order in the external field amplitude.

Vanishing of nonlinear Love numbers

- Following 't Hooft's naturalness principle, the vanishing of the Love numbers is a naturalness puzzle from an EFT perspective. [Rothstein '14], [Porto '16]

- $\lambda_1^{(E)} = \lambda_1^{(B)} = 0$ can be explained in terms of ladder symmetries: [Hui, Joyce, Penco, LS and Solomon '21], [Rai and LS '24]

- How about $\lambda_2^{(E)} = \lambda_2^{(EB)} = 0$?

- The simple form of the source and the vanishing of quadratic Love numbers suggest:
  1. possible resummation/reorganization of perturbation theory;
  2. nonlinear extension of ladder symmetries.

Vanishing of nonlinear Love numbers

- Example of resummation: static and axisymmetric (even) perturbations.

- Weyl metric in spherical coordinates:

$$\mathrm{d}s^2 = -\,\mathrm{e}^{2U}\mathrm{d}t^2 + \mathrm{e}^{2k-2U}\left(\frac{\mathrm{d}r^2}{1-\frac{r_s}{r}} + r^2\mathrm{d}\theta^2\right) + r(r-r_s)\sin^2\theta\,\mathrm{e}^{-2U}\mathrm{d}\varphi^2\,.$$

- Hidden symmetries emerge from dimensional reduction of GR to two spacetime dimensions.

  In [Combaluzier-Szteinsznaider, Hui, LS and Solomon, *in preparation*] we show that there are nonlinear ladder symmetries which imply:

$$S_{\rm int} \sim \sum_{n\geq 2}\sum_{l\geq 0}\int \mathrm{d}\tau\,\lambda^{(E)}_{l,n}\partial^l E^n$$

$$\boxed{\lambda^{(E)}_{l,n} = 0\,, \qquad \text{for all } l, n.}$$

Parity-even nonlinear Love number couplings vanish to *all* orders in perturbation theory.

Conclusions

Conclusions and open directions

- Black holes of GR display astonishing simplicity properties also beyond linear perturbation theory.
- In the static regime of the ppEFT, Schwarzschild BHs look effectively like simple point particles at all orders. How about Kerr?
- To see nonlinear (dissipative) effects one needs either nonzero spin, $a \neq 0$, or $\omega \neq 0$.
- Impact of subdominant effects on waveforms?
  Nonlinear (dissipative) response starts from 5.5PN order for Kerr BHs.
- Resummation of perturbations and nonlinear symmetries for odd sector?
- BHs with charge? Higher spacetime dimensions?