

Credit: NASA/Swift Dana Berry



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UNIVERSITÉ
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IN2P3
Les deux infinis

GW and ultra-dense matter

Francesca Gulminelli – Université de
Caen-Normandie

3rd MaNiTou summer school on gravitational waves,
June 30 – July 4, 2025

Lecture plan

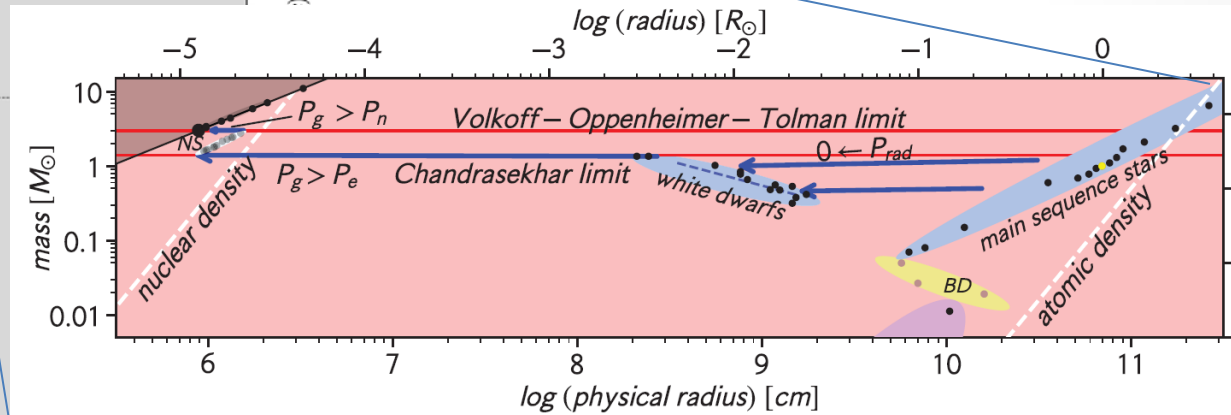
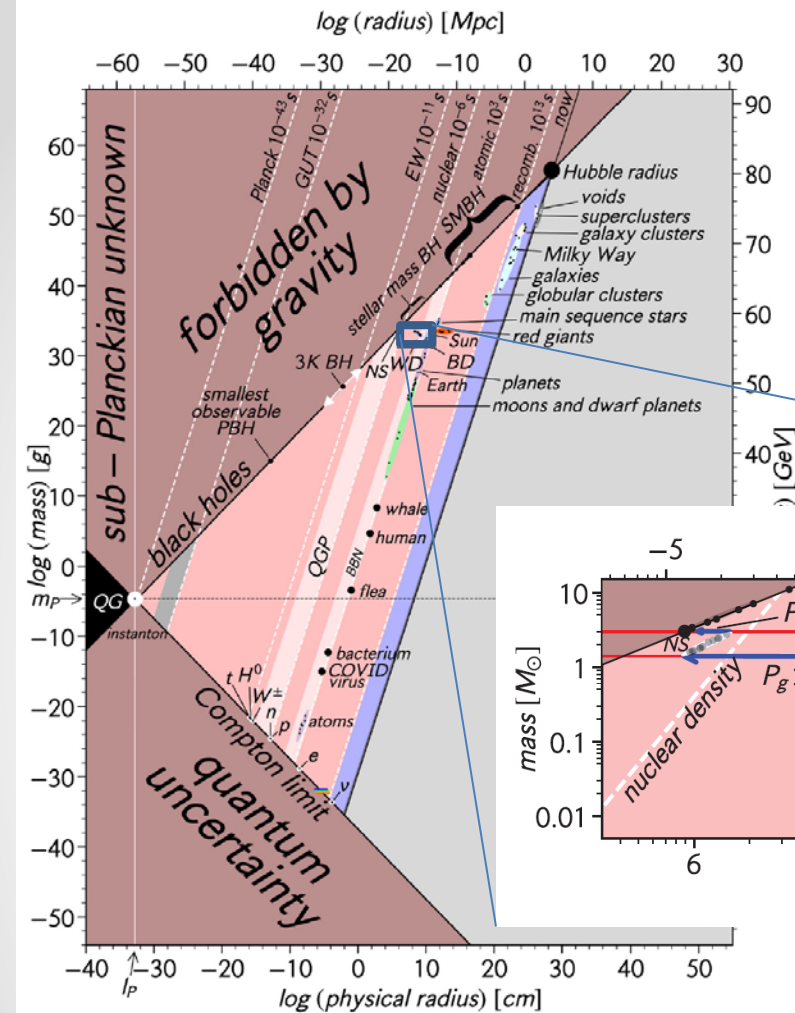
1. Introduction: ultra-dense matter in the universe

- a. The sites
- b. The signals
- c. The big questions

2. The gravitational wave probe for dense matter

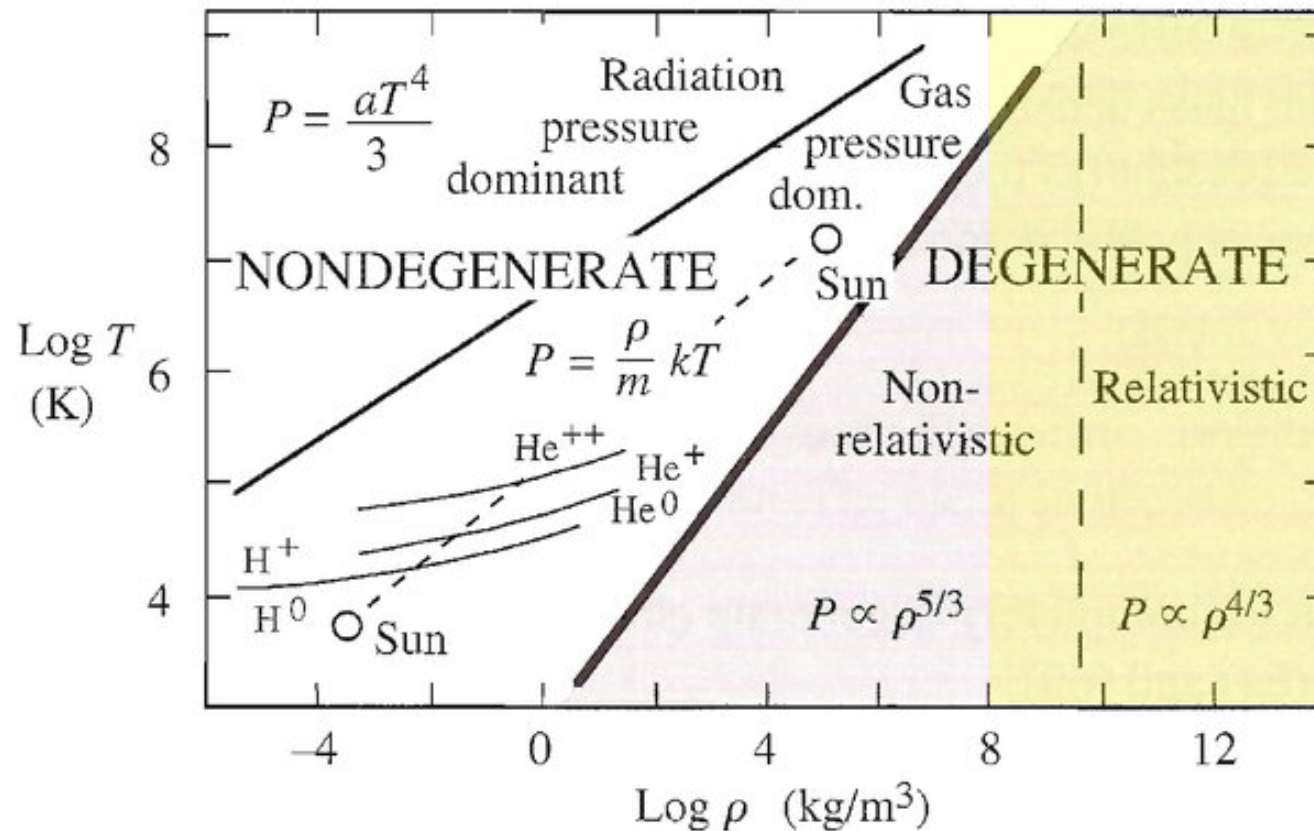
- a. Neutron star modelling and the equation of state (EoS)
- b. Observations
- c. EoS and observables: constraining the parameters

Ultra-dense matter



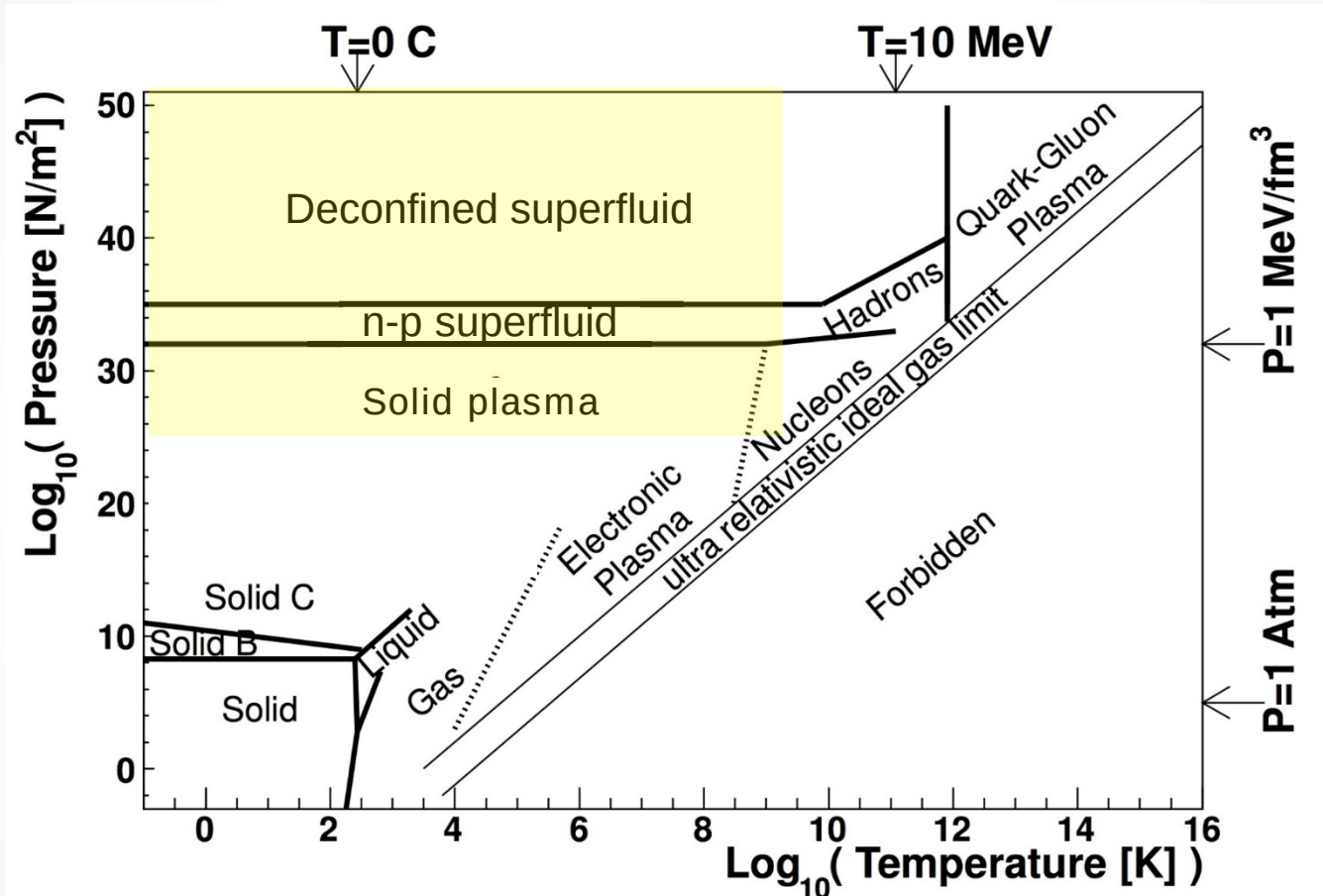
C.H.Lineweaver, V.M.Patel,
Am.J.Phys.91 (2023) 819

Ultra-dense matter



Degenerate and relativistic at all temperatures

Ultra-dense matter

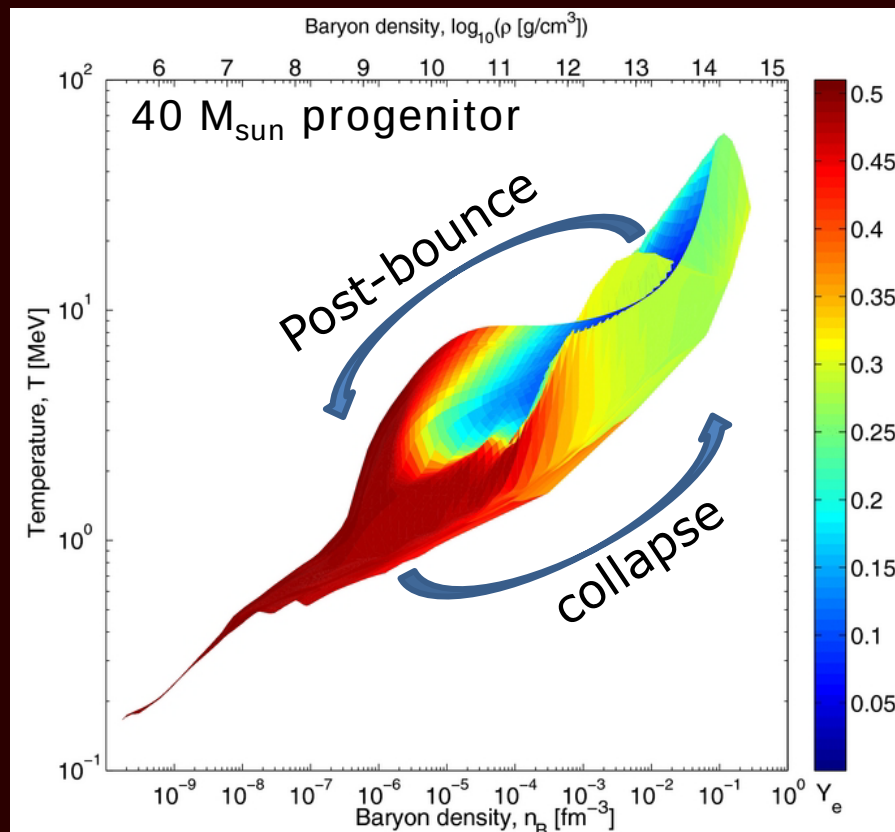
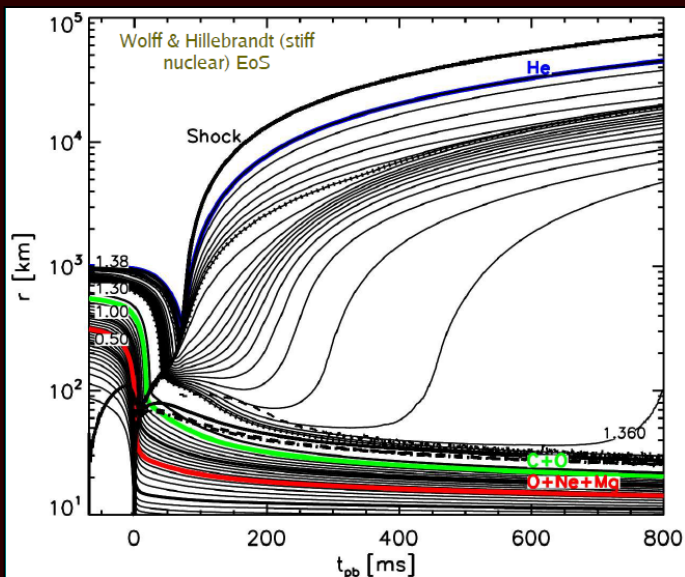


Exotic constituents, exotic phases

Supernova remnant
in Puppis A
MIPS+XMM
IR Credit: NASA)

Dense matter in the Universe: CCSN

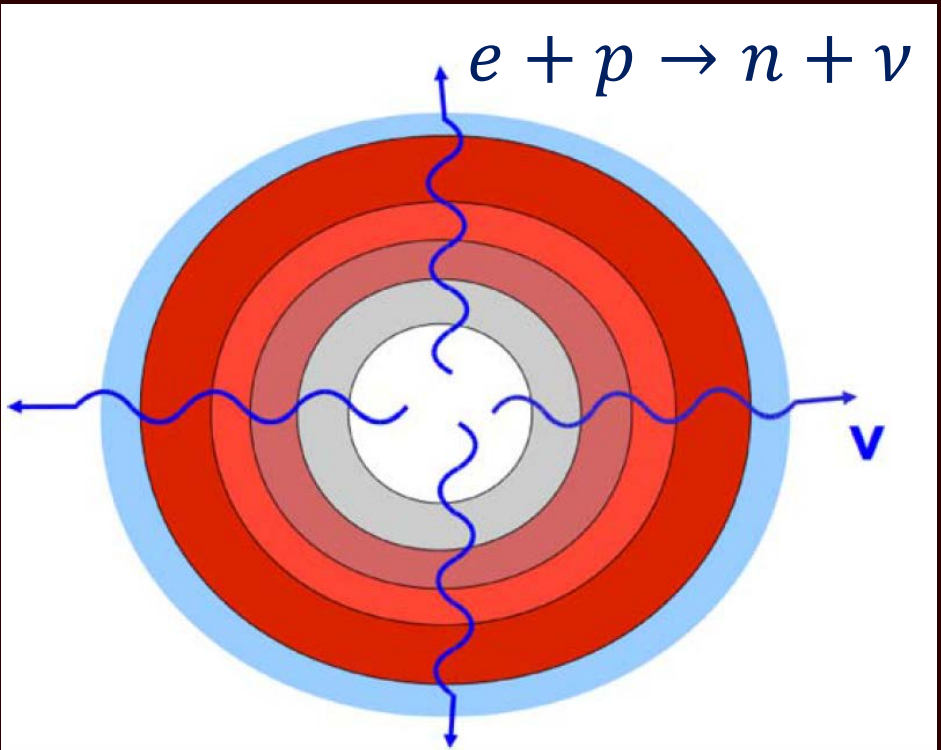
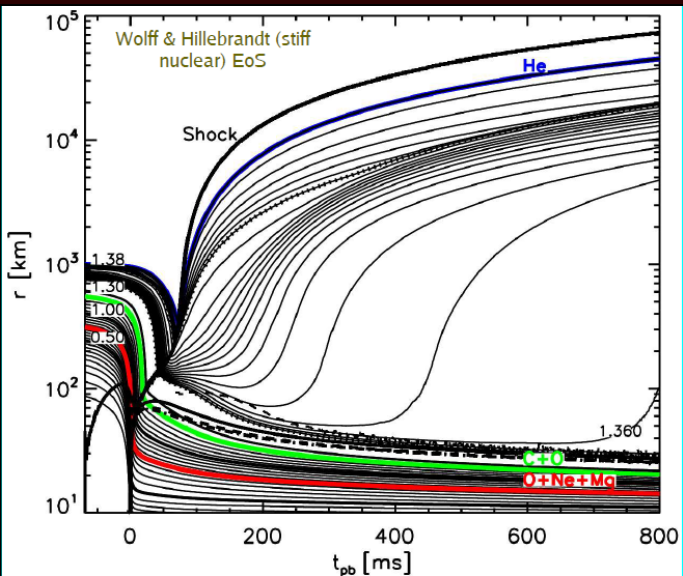
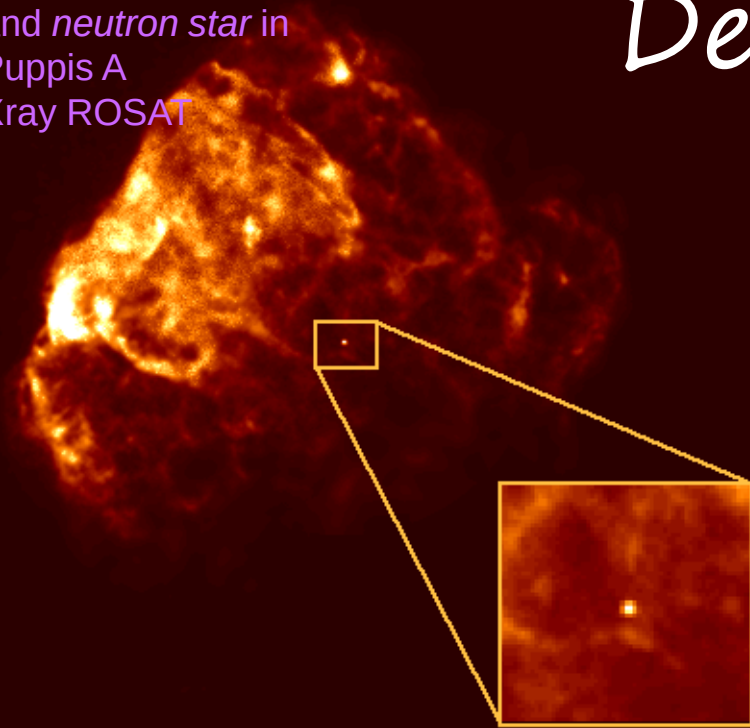
F.S.Kitaura et al, A&A 450 (06) 345



T.Fischer et al, 2011 ApJS 194 39

Supernova remnant
and neutron star in
Puppis A
Xray ROSAT

Dense matter in the Universe: PNS



Supernova remnant
and neutron star in
Puppis A
Xray ROSAT

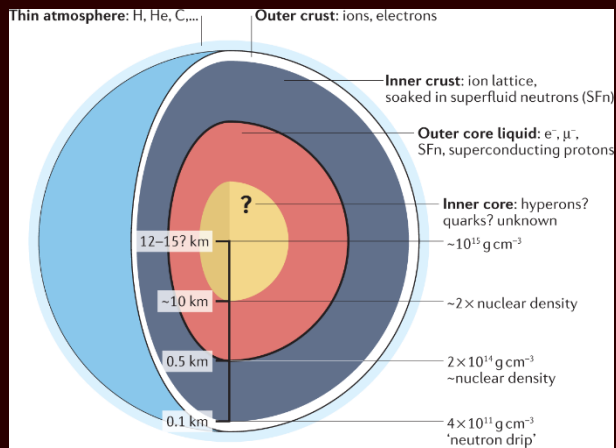
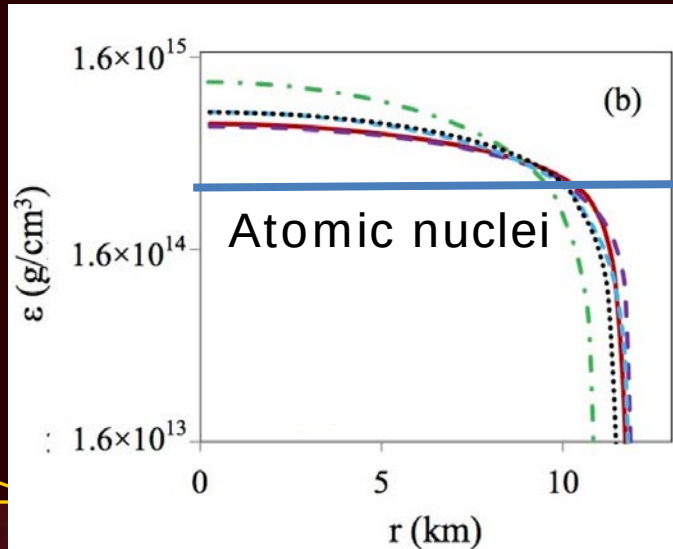
Dense matter in the Universe: NS

$y_e \cong 1/2$
 $T \sim 10^{12} K$

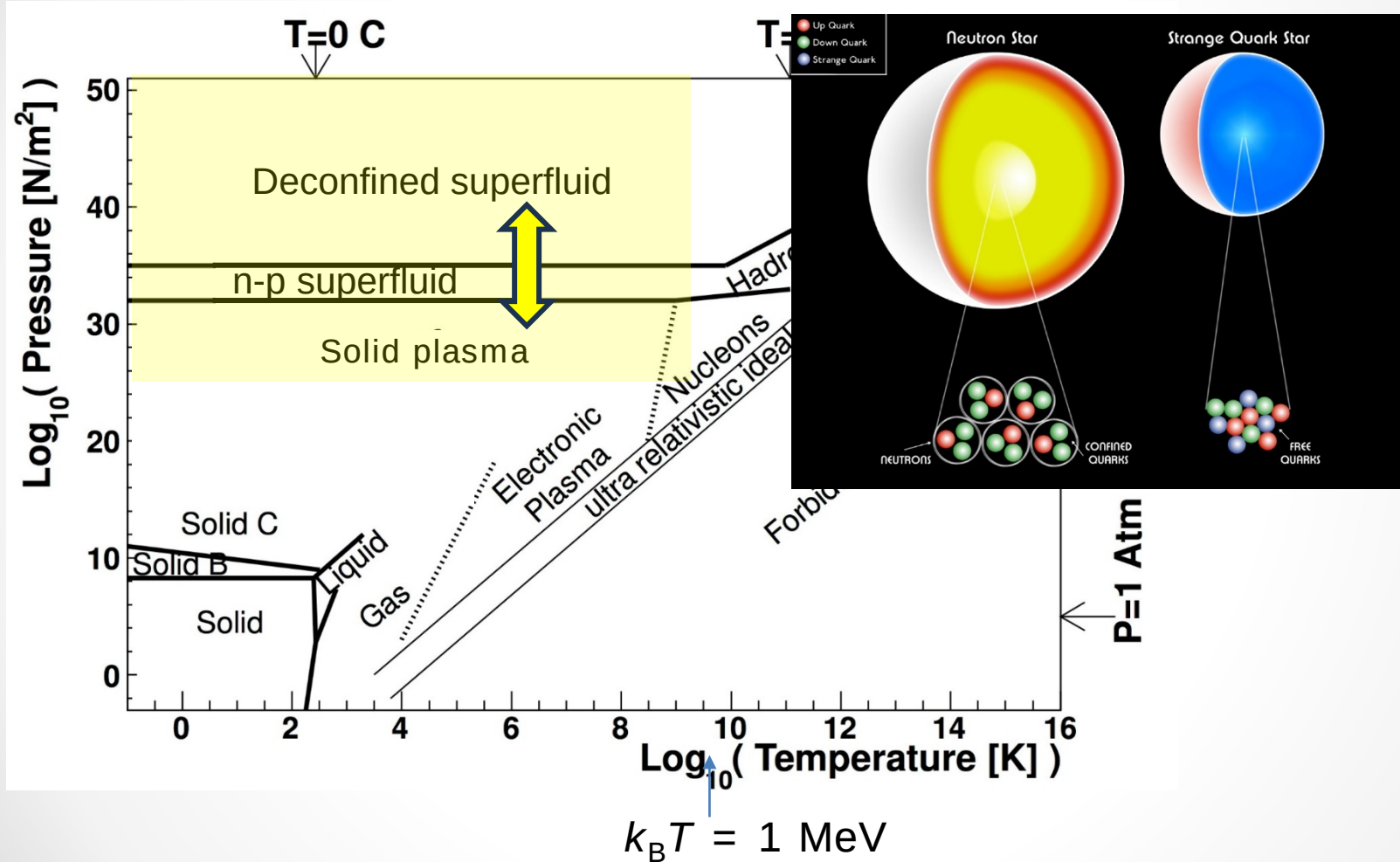
$y_e \cong 1/3$
 $T \sim 10^{11} K$



$y_e \cong 1/5$
 $T \sim 10^6 K$



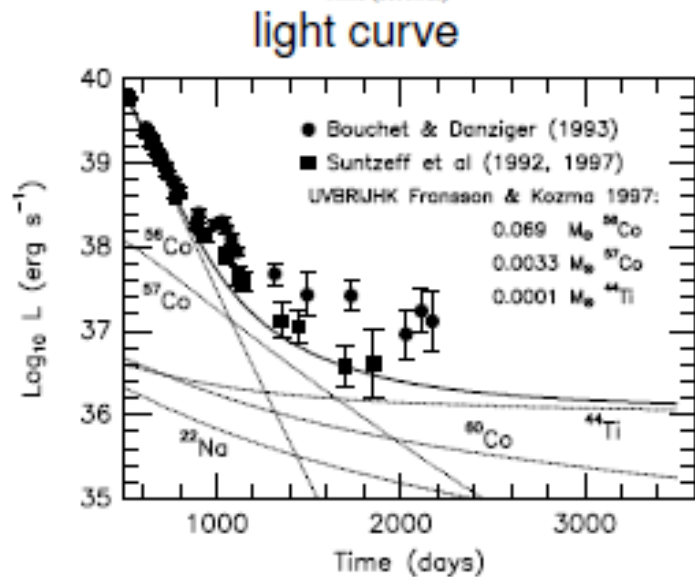
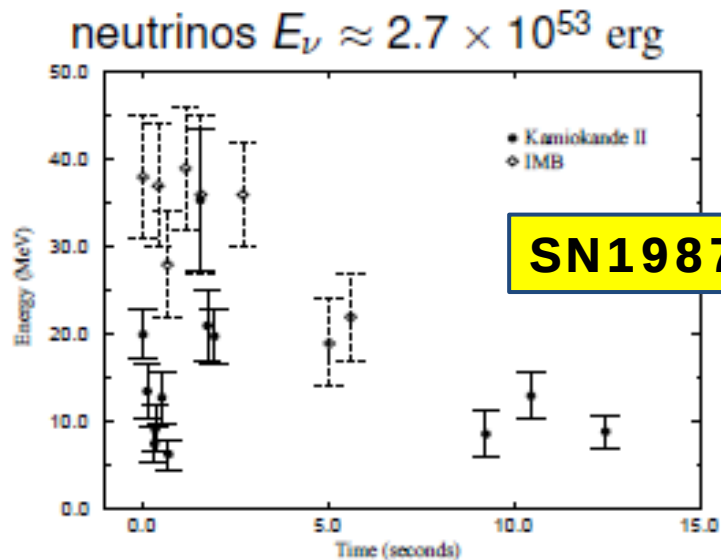
The phases of matter



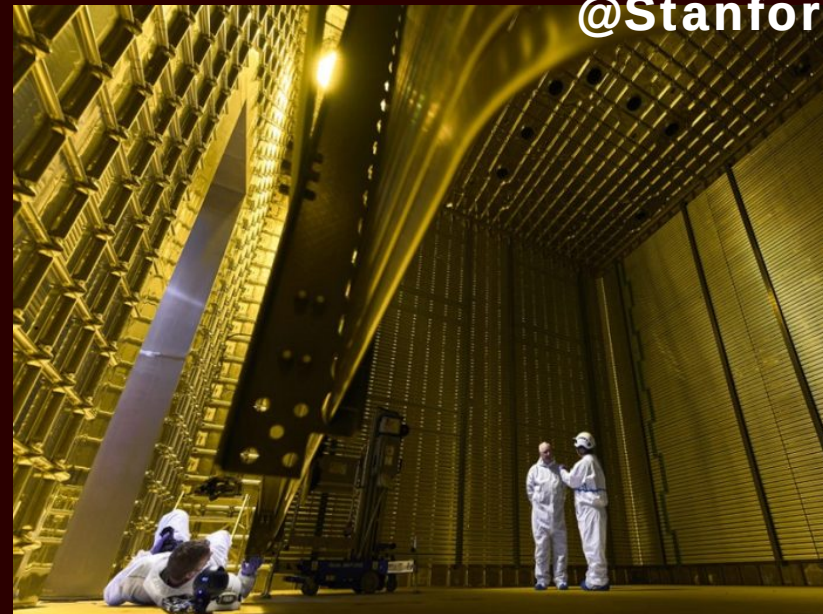
Credits: CXC/M. Weiss

CCSN

Signals

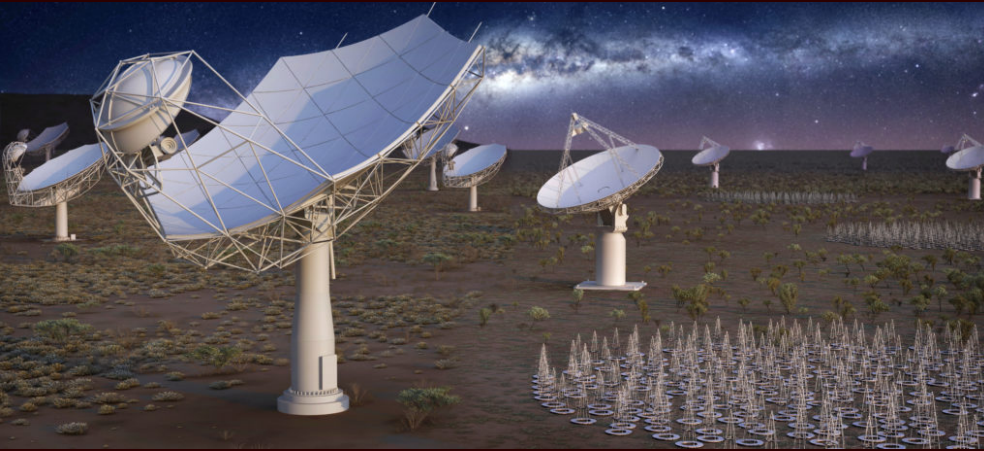


**SuperKamioKande
@Kamioka**



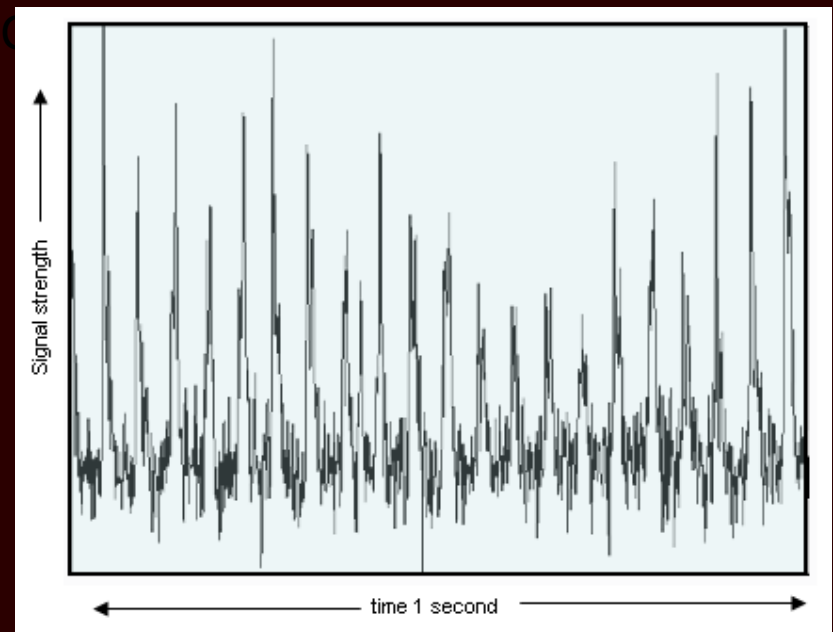
**DUNE
@Stanford**

Signals

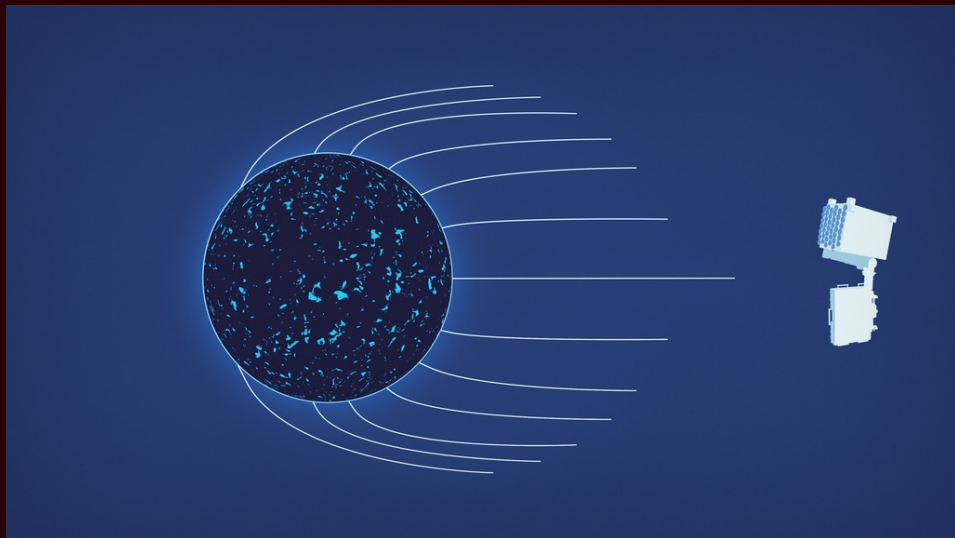


SKA@ South Africa

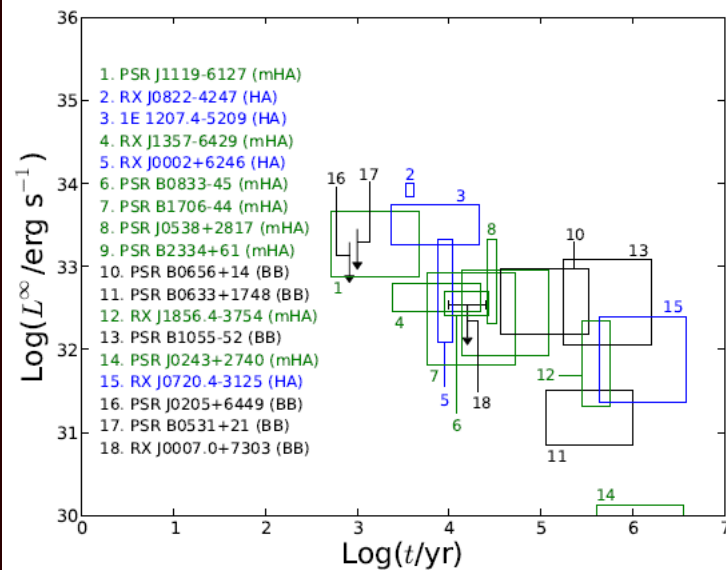
Pulsars



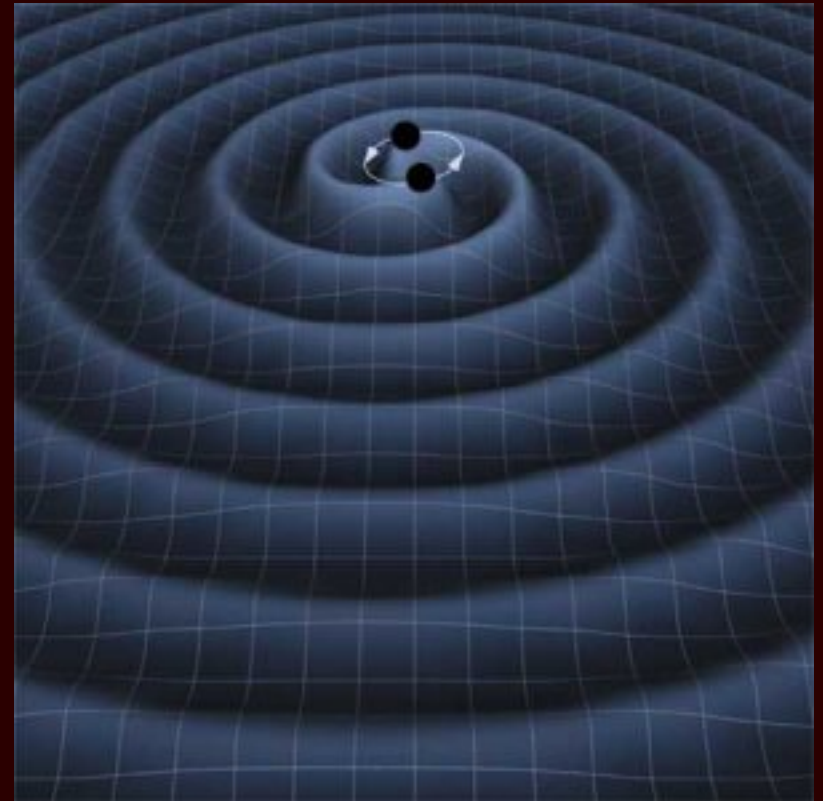
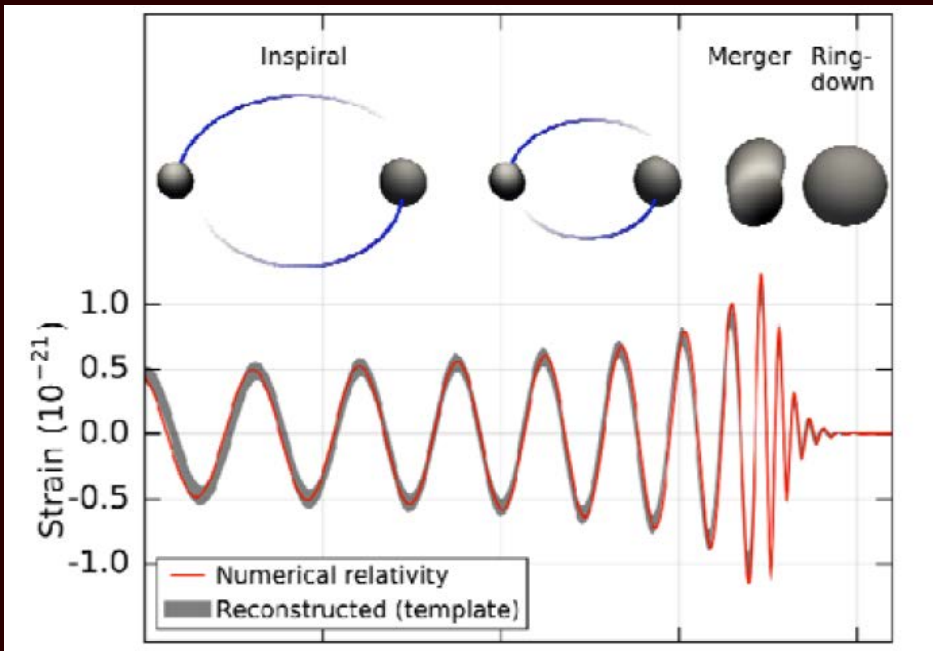
Signals



XR sources

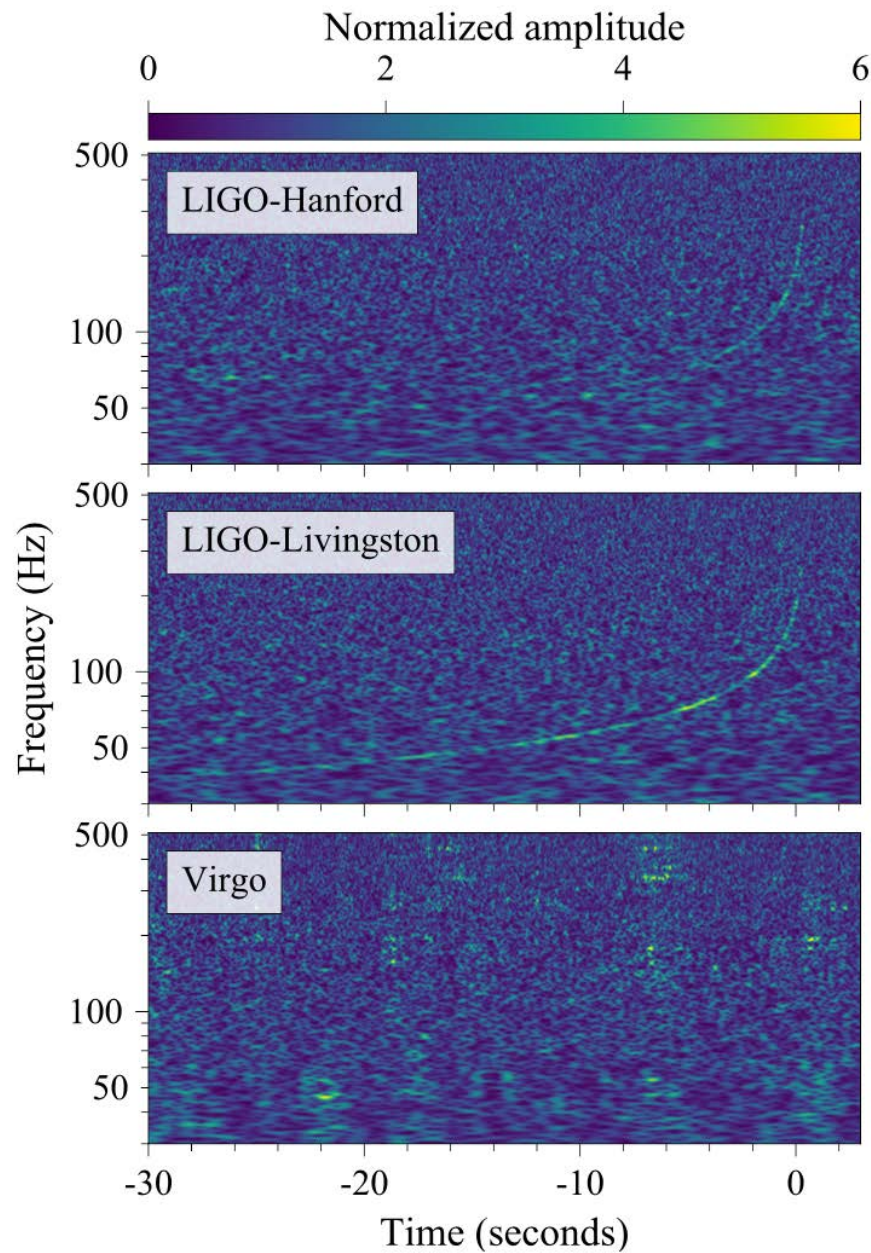


Signals



Mergers

Signals



Mergers



Multi-messenger Observations of a Binary Neutron Star Merger

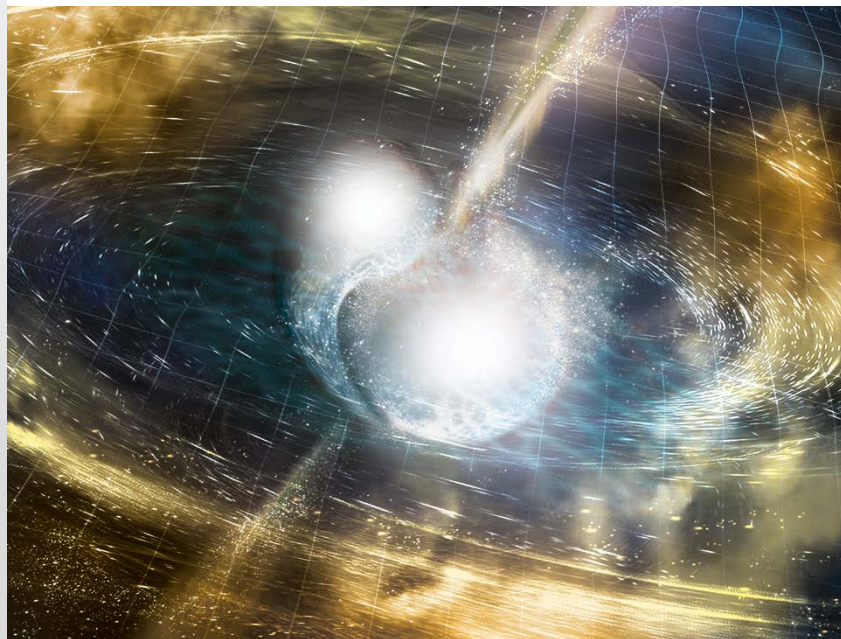
LIGO Scientific Collaboration and Virgo Collaboration, Fermi GBM, INTEGRAL, IceCube Collaboration, AstroSat Cadmium Zinc Telluride Imager Team, IPN Collaboration, The Insight-Hxmt Collaboration, ANTARES Collaboration, The Swift Collaboration, AGILE Team, The 1M2H Team, The Dark Energy Camera GW-EM Collaboration and the DES Collaboration, The DLT40 Collaboration, GRAWITA: GRAVitational Wave Inaf TeAm, The Fermi Large Area Telescope Collaboration, ATCA: Australia Telescope Compact Array, ASKAP: Australian SKA Pathfinder, Las Cumbres Observatory Group, OzGrav, DWF (Deeper, Wider, Faster Program), AST3, and CAASTRO Collaborations, The VINROUGE Collaboration, MASTER Collaboration, J-GEM, GROWTH, JAGWAR, Caltech-NRAO, TTU-NRAO, and NuSTAR Collaborations, Pan-STARRS, The MAXI Team, TZAC Consortium, KU Collaboration, Nordic Optical Telescope, ePESSTO, GROND, Texas Tech University, SALT Group, TOROS: Transient Robotic Observatory of the South Collaboration, The BOOTES Collaboration, MWA: Murchison Widefield Array, The CALET Collaboration, IKI-GW Follow-up Collaboration, H.E.S.S. Collaboration, LOFAR Collaboration, LWA: Long Wavelength Array, HAWC Collaboration, The Pierre Auger Collaboration, ALMA Collaboration, Euro VLBI Team, Pi of the Sky Collaboration, The Chandra Team at McGill University, DFN: Desert Fireball Network, ATLAS, High Time Resolution Universe Survey, RIMAS and RATIR, and SKA South Africa/MeerKAT (See the end matter for the full list of authors.)

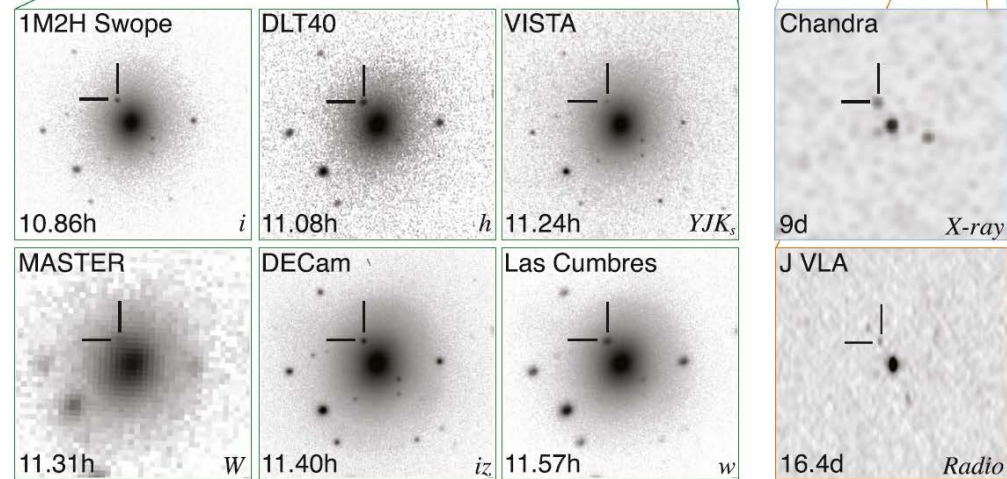
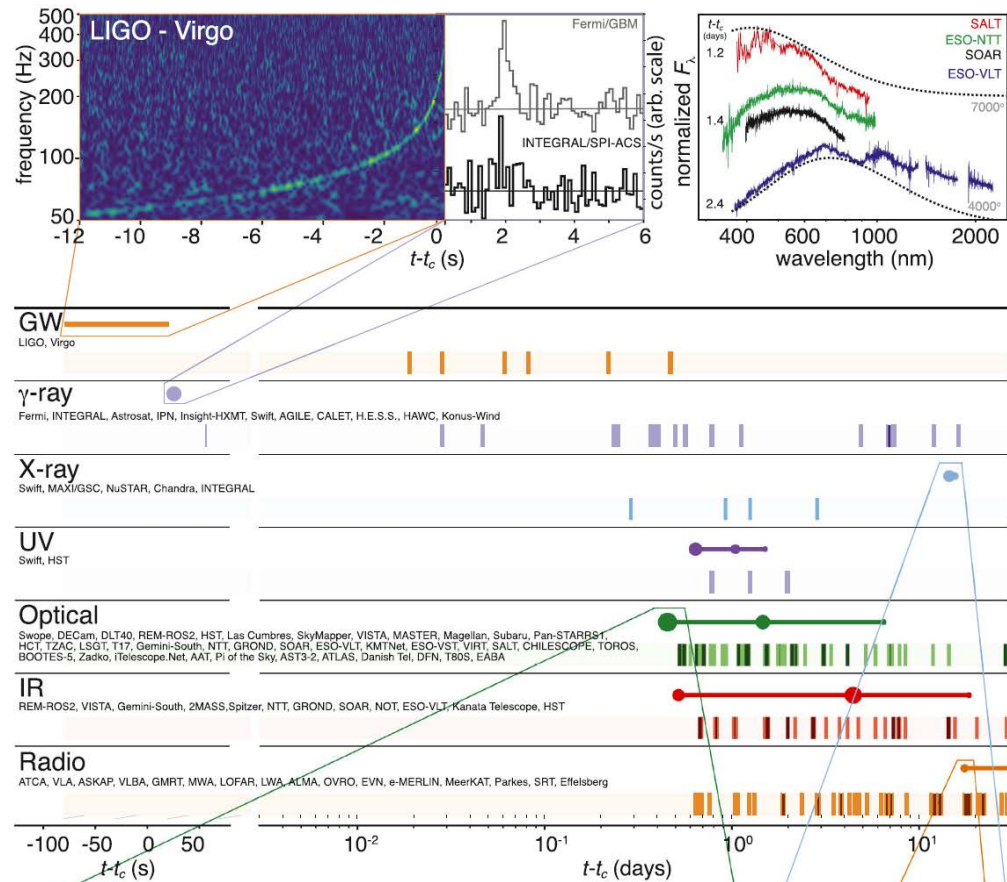
Received 2017 October 3; revised 2017 October 6; accepted 2017 October 6; published 2017 October 16

Foundation
Operated by Caltech and MIT

GW170817 Press Release
LIGO and Virgo make first
detection of gravitational
waves produced by colliding
neutron stars

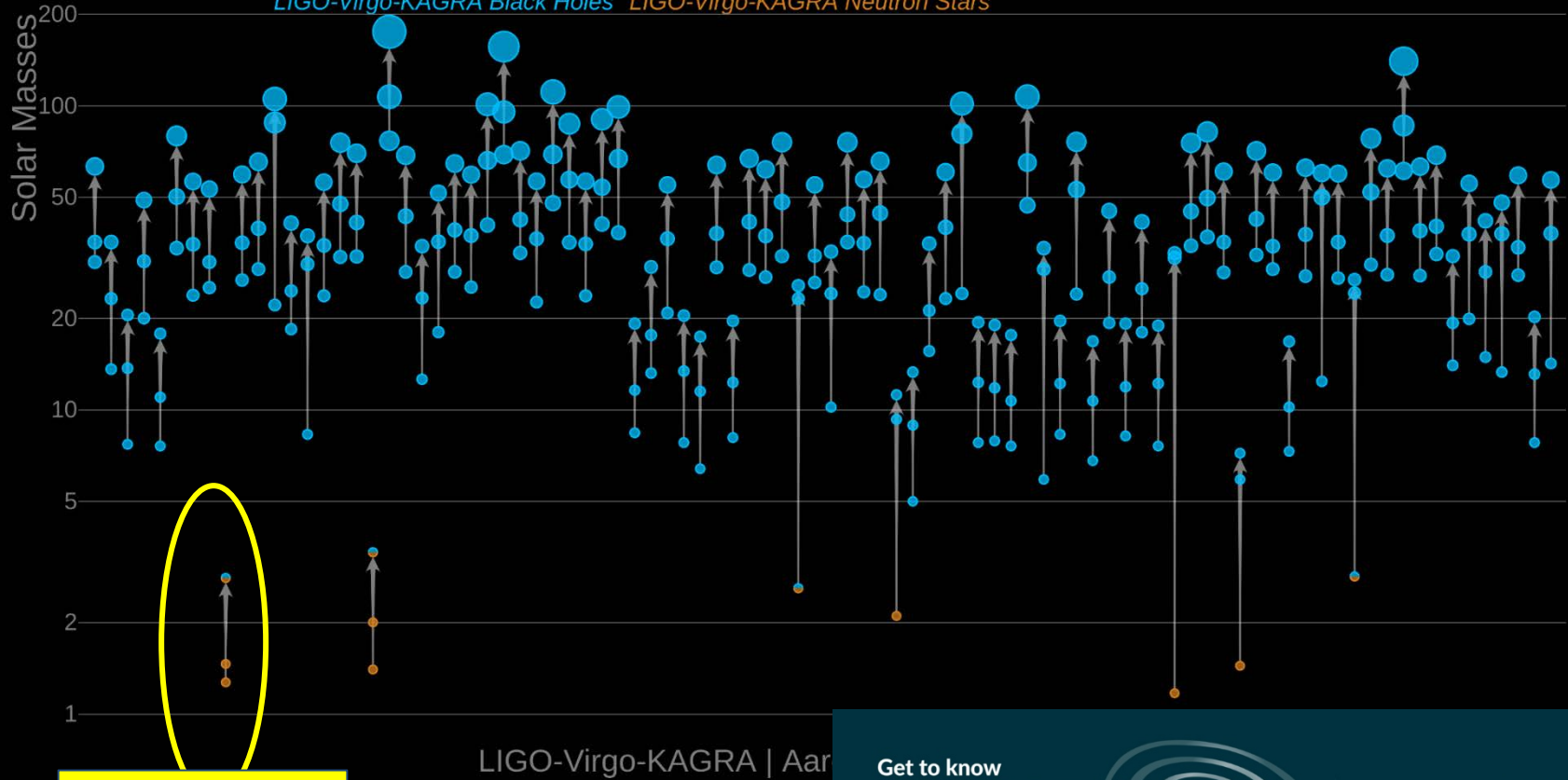
Discovery marks first cosmic event
observed in both gravitational
waves and light.





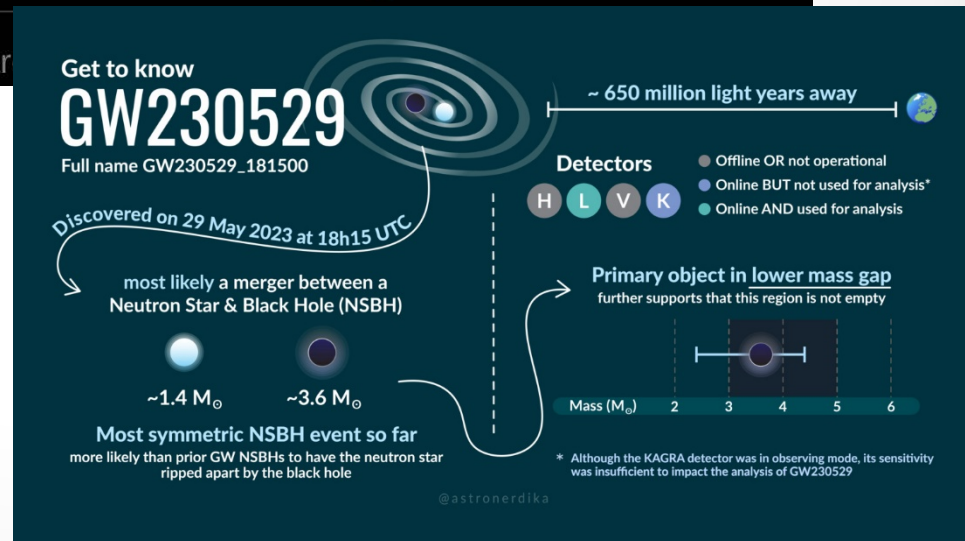
Masses in the Stellar Graveyard

LIGO-Virgo-KAGRA Black Holes LIGO-Virgo-KAGRA Neutron Stars



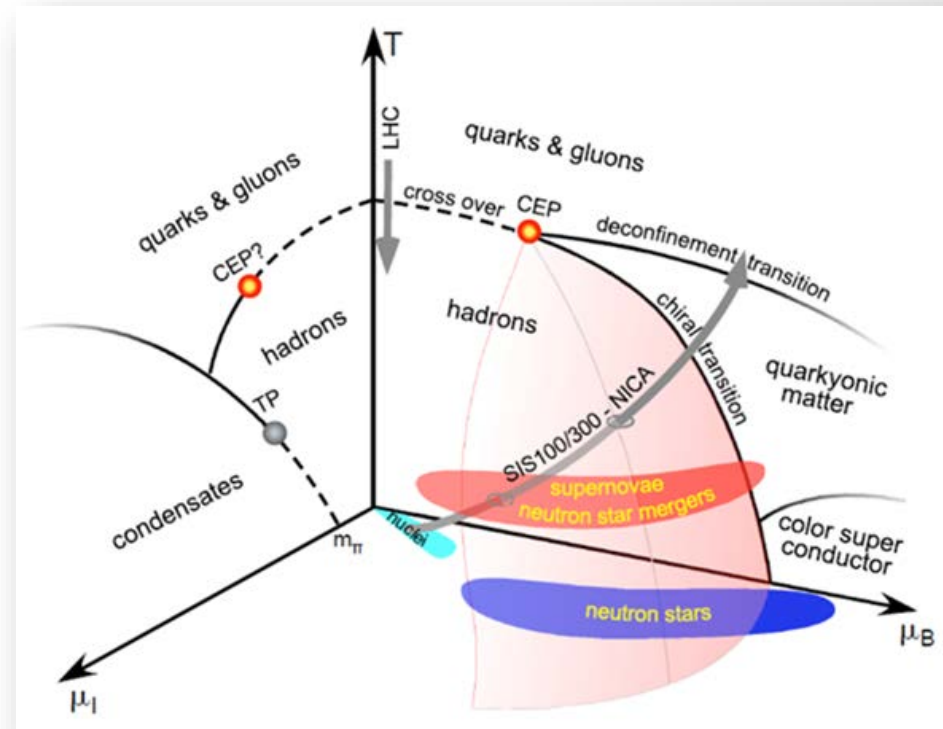
GW170817

10 years of LVK
observations



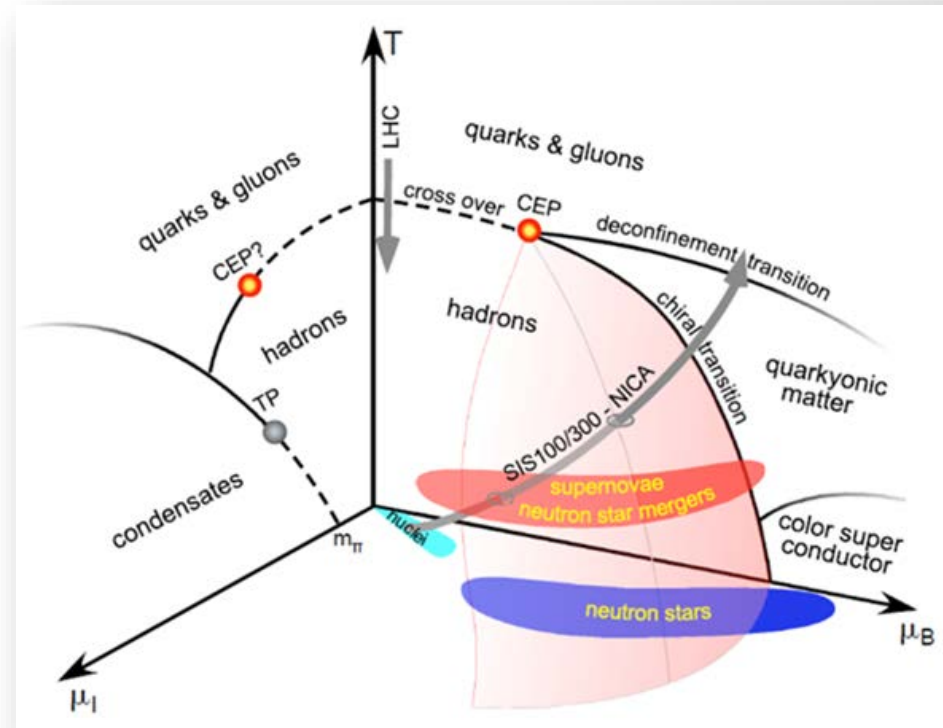
Dense matter: the Questions

1. What is the internal structure of the dense matter in neutron stars, supernova cores and mergers?
2. How does this structure reflect into the observable signals?
3. What can we learn on the underlying nuclear and hadronic physics?



The message I would like to convey:

Gravitational waves, and particularly the inspiral GW signal from NS mergers, opens the possibility of a ***DIRECT*** measurement of the QCD phase diagram



Lecture plan

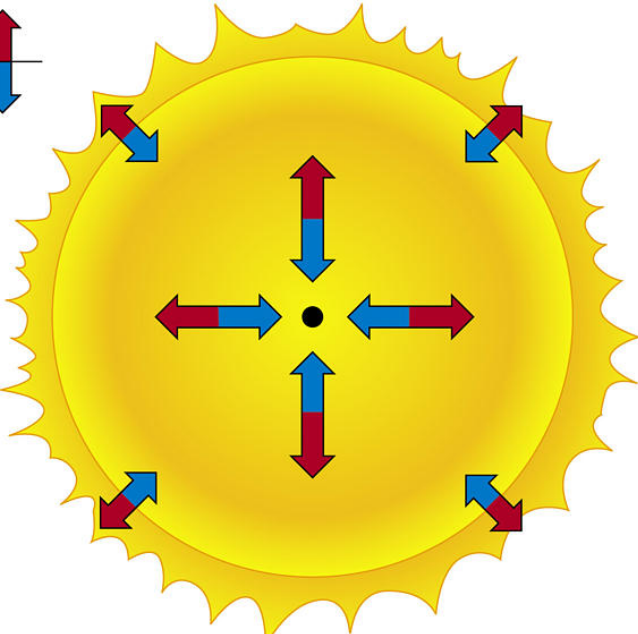
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Modelling (Neutron) Stars: hydrostatics

- Self-gravitation => Tolman Oppenheimer Volkoff (1939):

$$\frac{dP(\rho)}{dr} = -\frac{G}{r^2} \left[\rho(r) + \frac{P(\rho)}{c^2} \right] \left[m(r) + 4\pi r^3 \frac{P(\rho)}{c^2} \right] \left[1 - \frac{2Gm(r)}{rc^2} \right]^{-1}$$

Pressure
out
Gravity
in



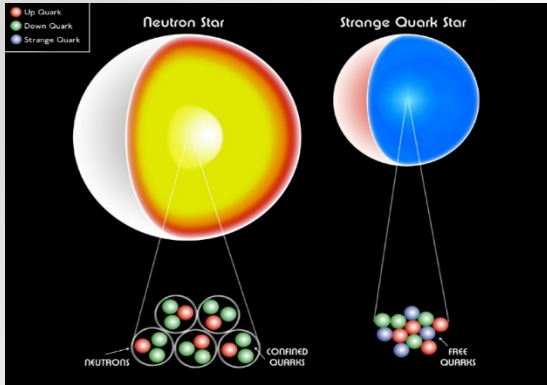
The diagram shows a yellow sphere representing a neutron star with a jagged outer edge. Inside the sphere, there are four pairs of arrows pointing away from the center, representing pressure. Outside the sphere, there are four pairs of arrows pointing towards the center, representing gravity. A legend to the left of the sphere shows a red arrow pointing up labeled 'Pressure out' and a blue arrow pointing down labeled 'Gravity in'.



$$\begin{aligned} \forall \rho_c \\ R=r(P=0) \\ M=m(r=R) \end{aligned}$$

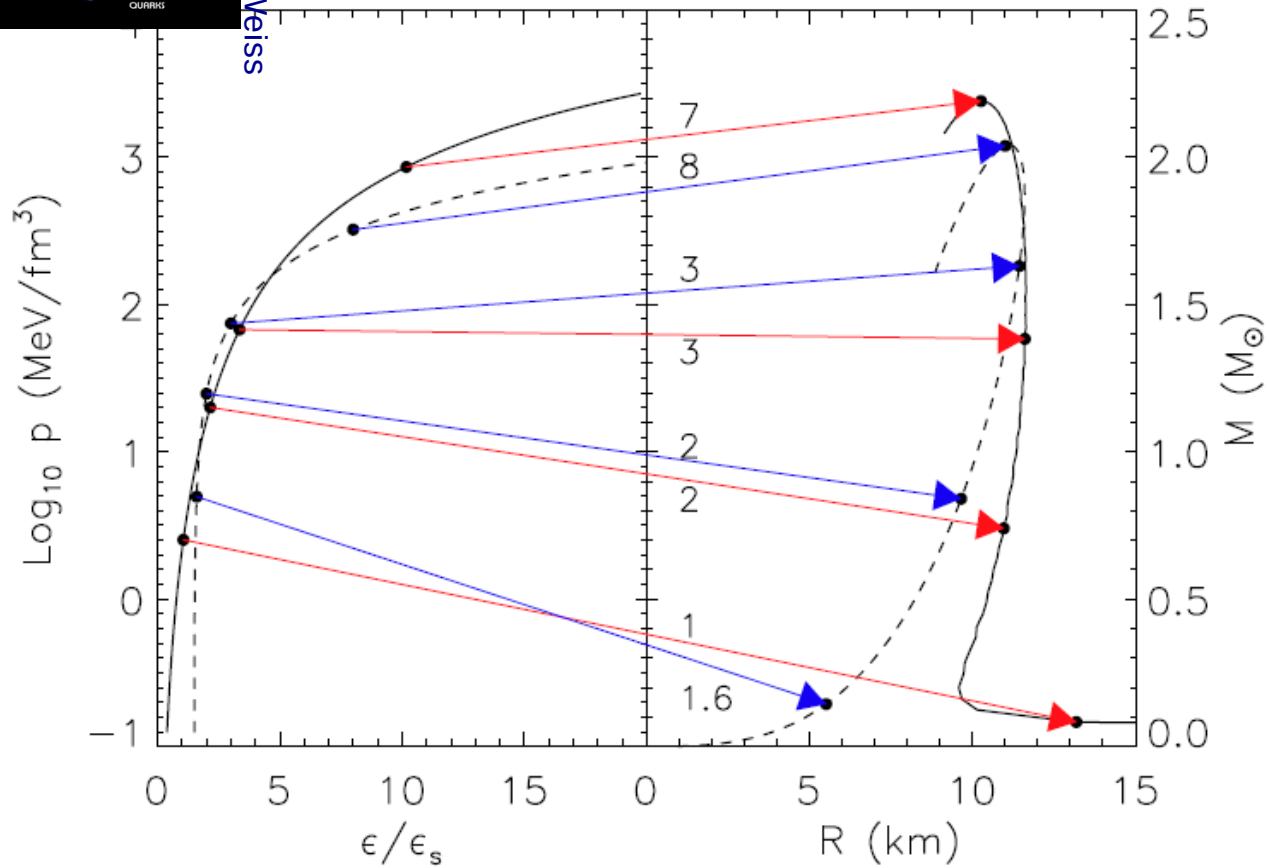
Mass and radius

**Needs ONLY $P(\rho)$
of NS matter**



Credits: CXC/M.Weiss

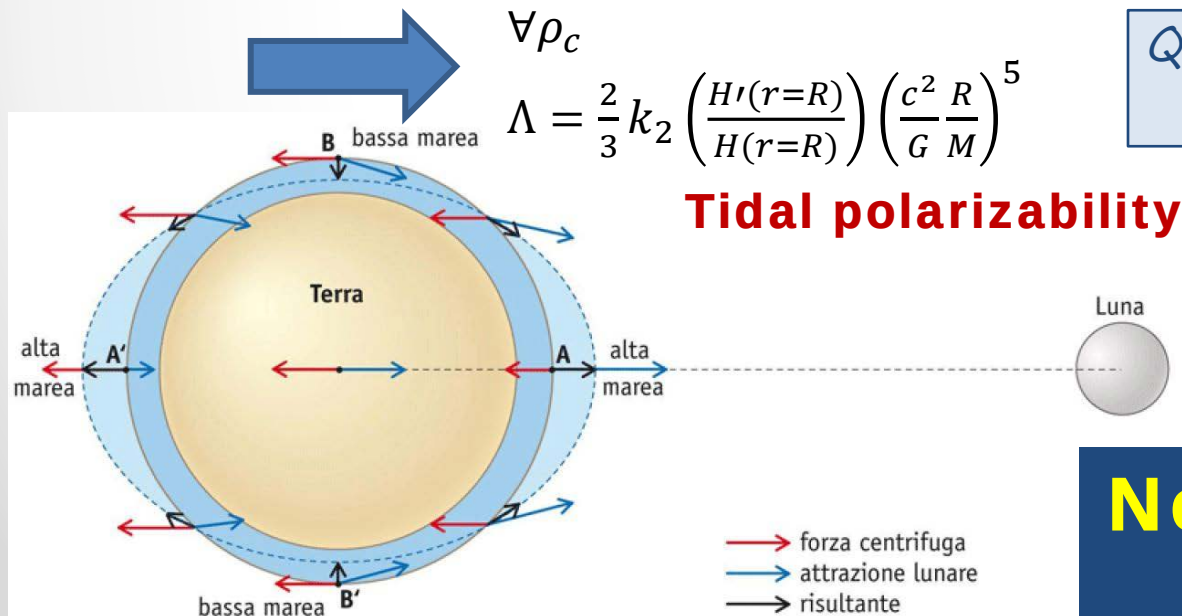
$$P(\rho) \Leftrightarrow M(R)$$



Modelling (Neutron) Stars: hydrostatics

- Influence of a second body => Thorne and Campolattaro (1967):

$$\frac{d^2 H(r)}{dr^2} + \frac{dH(r)}{dr} \left[\frac{2}{r} + e^{\lambda(\mathbf{P}(\rho))} \left(\frac{2m(r)}{r^2} + 4\pi r(\mathbf{P}(\rho) - \rho(r)) \right) \right] + H(r)Q(\mathbf{P}(\rho)) = 0$$



Quadrupole moment

Companion tidal field

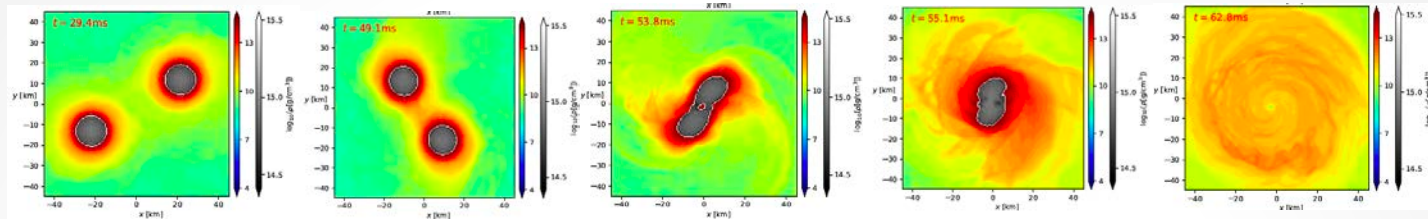
$$\bar{\bar{Q}}^{(i)} = -\Lambda_i \bar{\bar{\mathcal{E}}}^{(j)}$$

LIGO
Livingston

**Needs ONLY $P(\rho)$
of NS matter**

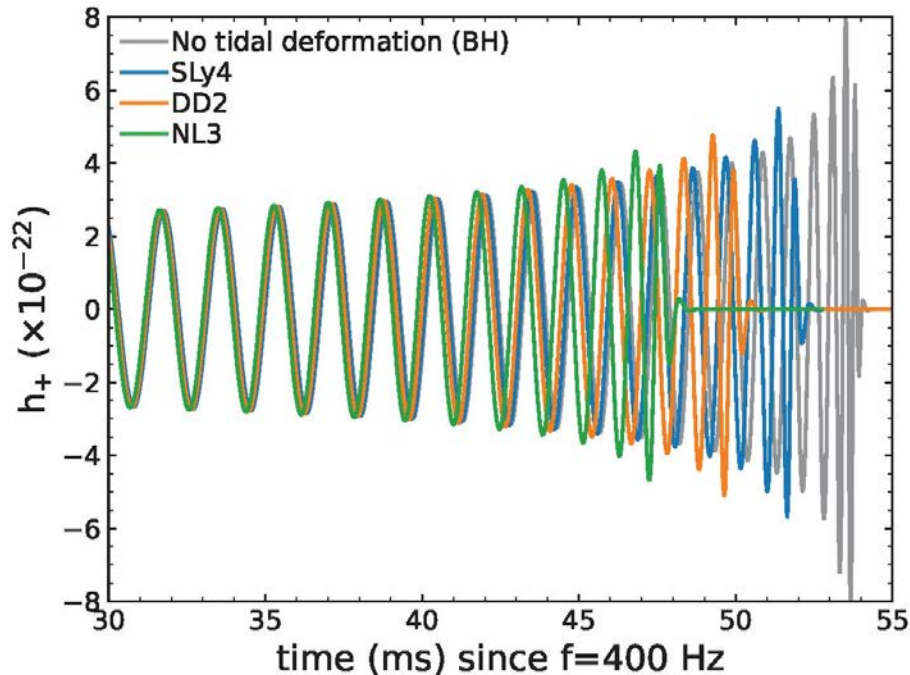


Extraction of the tidal effects



Gieg et al., Particle 2, 23 (2019)

SEOBNRV4T_SURROGATE waveform,
Courtesy J.Read



R.Wade et al., PRD89(2014)103012

$$\delta\psi_{\text{tidal}} = \frac{3}{128\eta x^{5/2}} \left[\left(-\frac{39}{2} \tilde{\Lambda} \right) x^5 + \left(-\frac{3115}{64} \tilde{\Lambda} + \frac{6595}{364} \sqrt{1-4\eta\delta\tilde{\Lambda}} \right) x^6 \right]$$

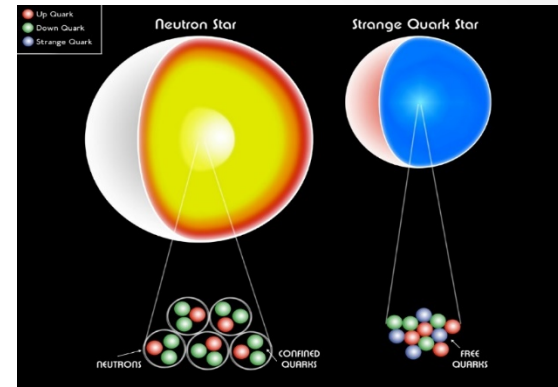
LIGO
Livingston

$$(\Lambda_1, \Lambda_2) \leftrightarrow (\tilde{\Lambda}, \delta\tilde{\Lambda})$$

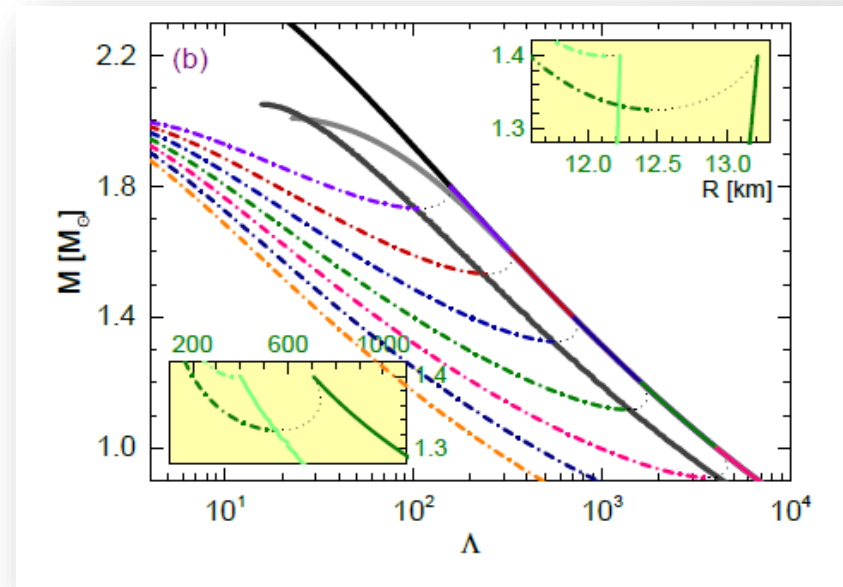
Tidal effects: modification of the gravitational waveform wrt a GR point-particle calculation, due to the star deformation just before merging

...a brief summary $\langle O \rangle \Leftrightarrow P(\rho)$

- GR imposes a 1-to-1 correspondence between the dense matter EoS and static properties of NS ($M(R)$ - $M(\Lambda)$)
- Different compositions $\Rightarrow$ different $M(\Lambda) \Rightarrow$ different gravitational signals!
- Systematics due to the astrophysical modelling in principle under control

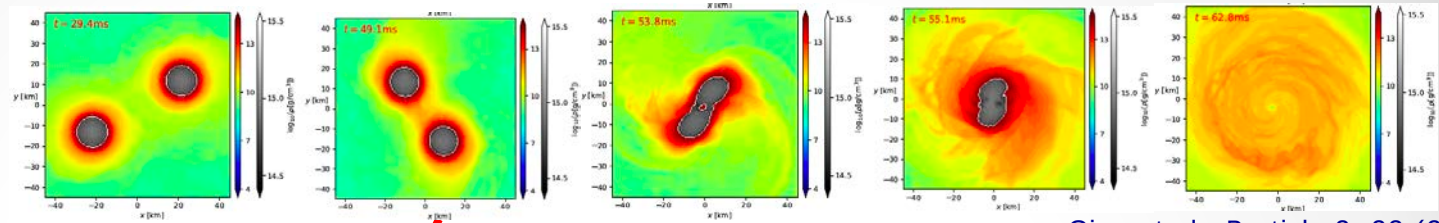


Credits: CXC/M.Weiss

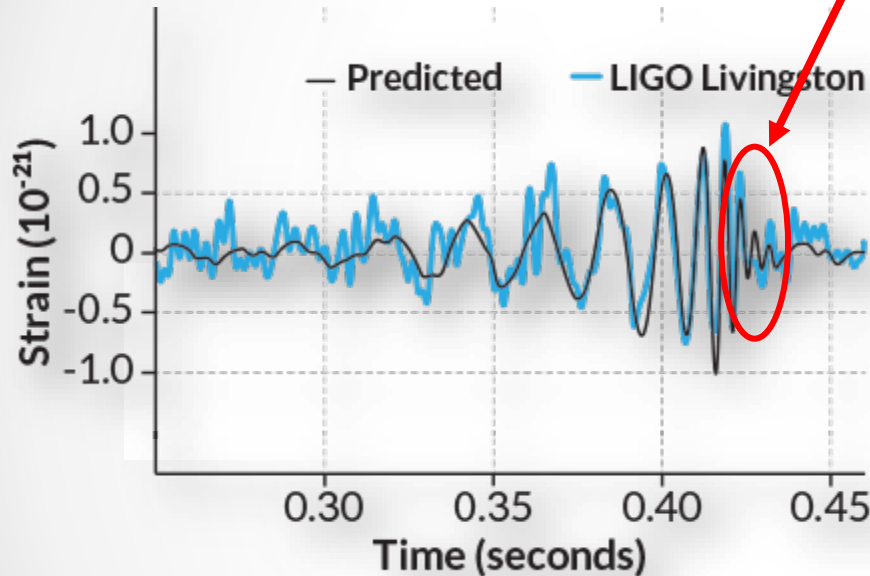


J.J.Li, A.Sedrakian, M.Alford,
PRD101 (2020) 063022

Problem 1 : the Λ info is hard to measure

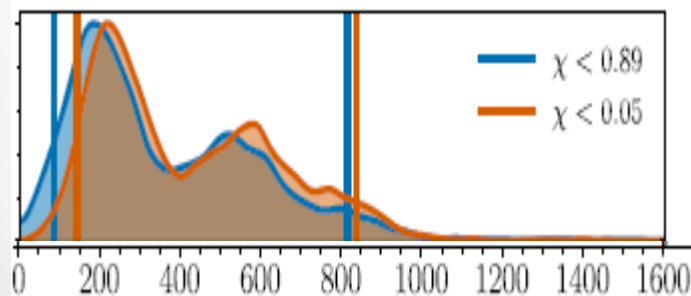


Gieg et al., Particle 2, 23 (2019)



R.Wade et al., PRD89(2014)103012

$$\delta\psi_{\text{tidal}} = \frac{3}{128\eta x^{5/2}} \left[\left(-\frac{39}{2} \tilde{\Lambda} \right) x^5 + \left(-\frac{3115}{64} \tilde{\Lambda} + \frac{6595}{364} \sqrt{1-4\eta\delta\tilde{\Lambda}} \right) x^6 \right]$$



LVC, PRX 9 (2019), 011001

$$(\Lambda_1, \Lambda_2) \leftrightarrow (\tilde{\Lambda}, \delta\tilde{\Lambda})$$

=> Only the combined $\tilde{\Lambda}$ is measurable, and with error bars

Problem II : no ab-initio theory of dense matter

=> Density Functional Approach

$\varepsilon_{tot} = \varepsilon_B + \varepsilon_L$ (baryons and leptons decoupled, leptons free FG)

- **NS core:** $\rho_q(r) = \rho_q$ ($\forall q$ constituent)

- **Effective single particles:** $e_q(k) = \sqrt{m_q^{*2} + k^2} + V_q(\rho_q, \rho_{q'})$

$\Rightarrow m_q^*, V_q$ from a phenomenological Hamiltonian (Skyrme, Gogny, M3Y..) or Lagrangian (RMF)

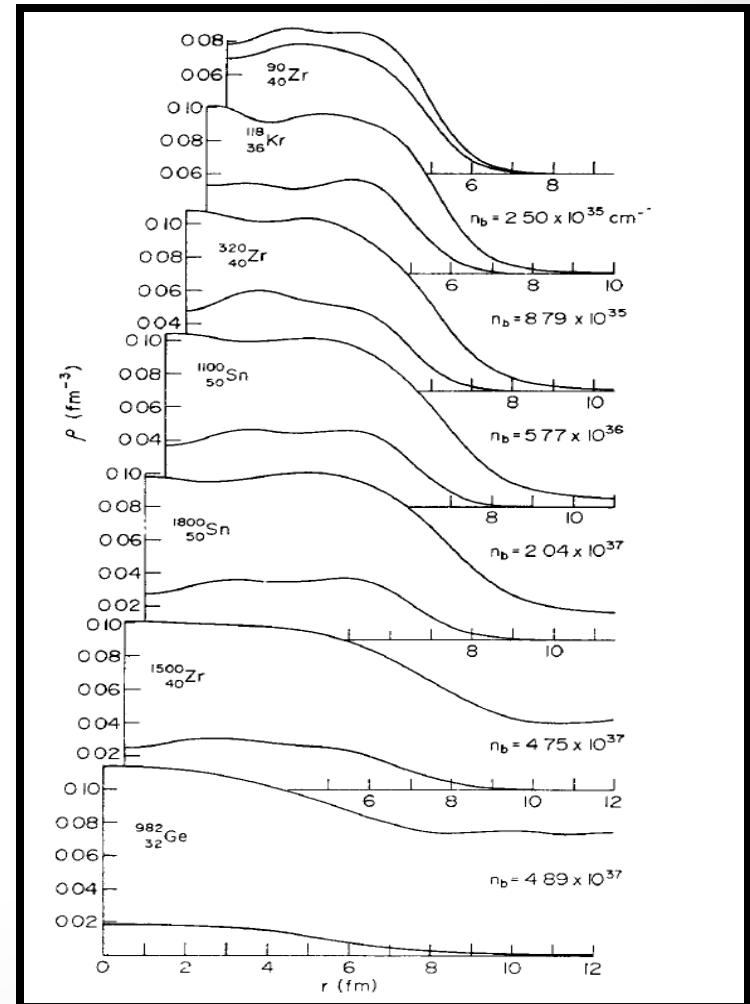
$\Rightarrow$ Parameters fitted on nuclear data and/or ab-initio calculations

$\Rightarrow e(\rho_B, \rho_L, \rho_S) \quad P(\rho) = -\rho_B^2 \frac{\partial e}{\partial \rho_B} \Big|_{\mu_L=0, \mu_S=0},$

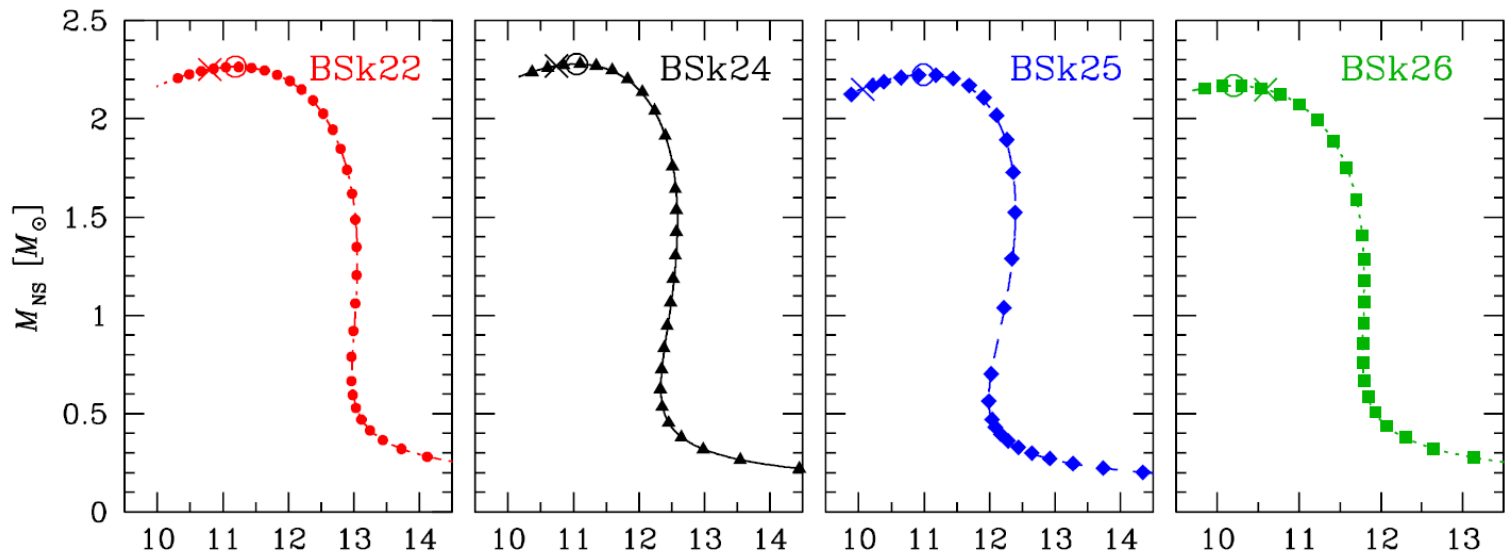
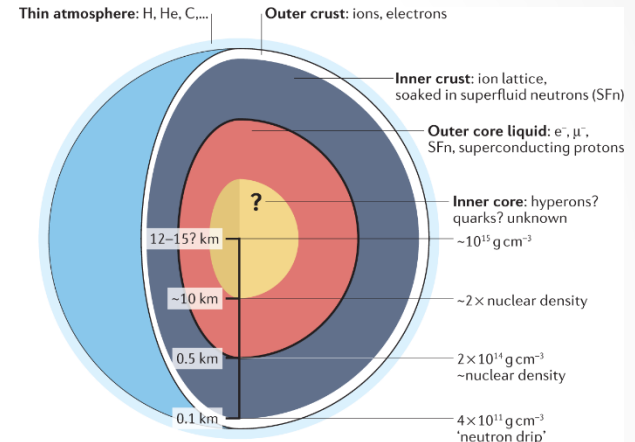
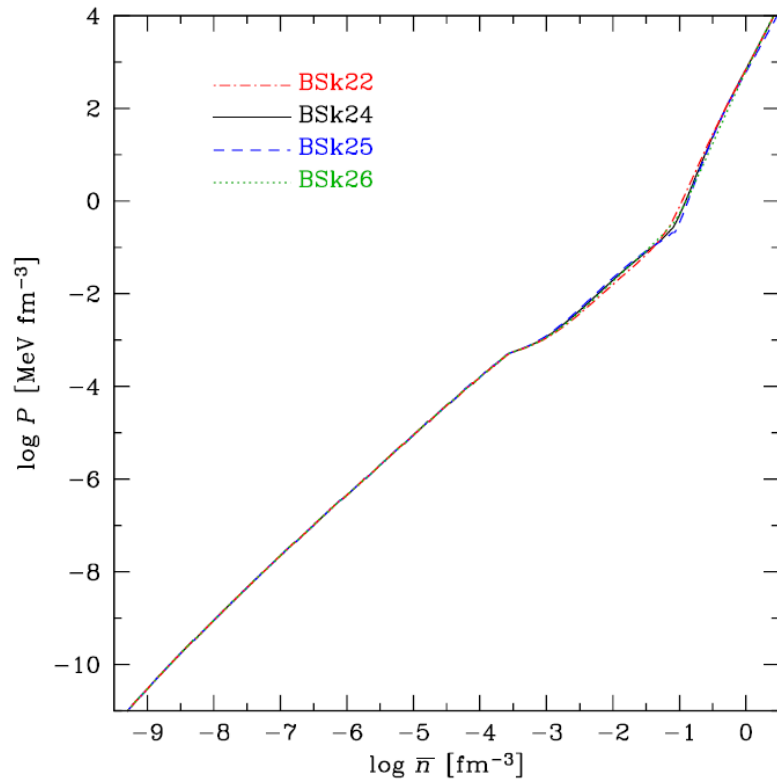
Problem II : no *ab-initio* theory of dense matter

=> Density Functional Approach

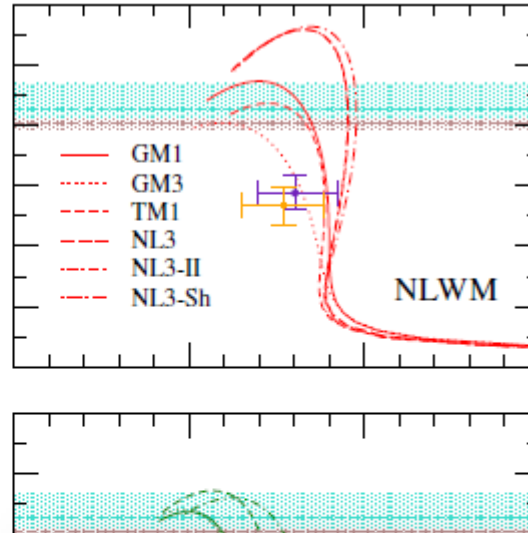
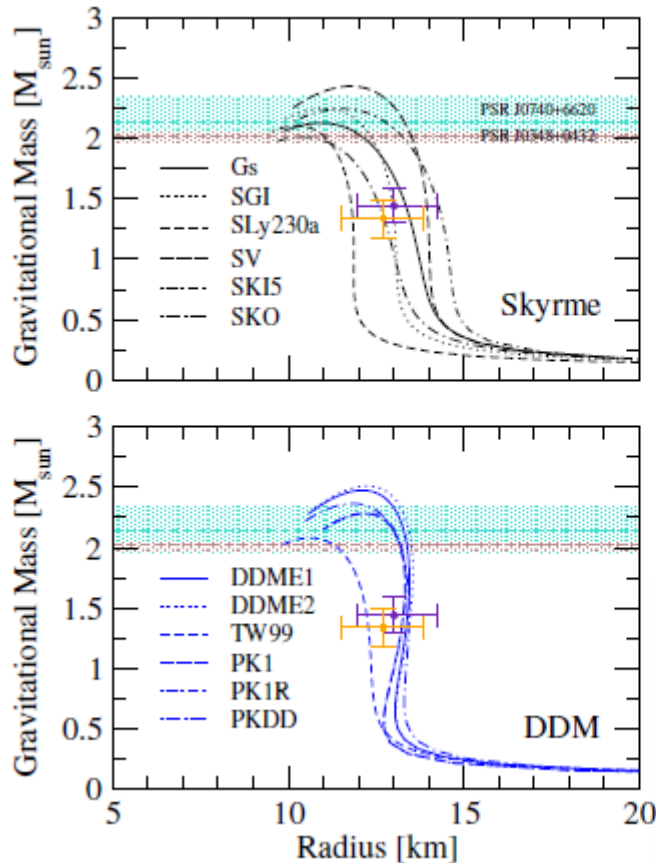
- **NS crust:** $q=n,p$ variational calculation of the single particle wave-functions => $\rho_q(r)$ within the Hartree-Fock-(Bogoliubov) theory



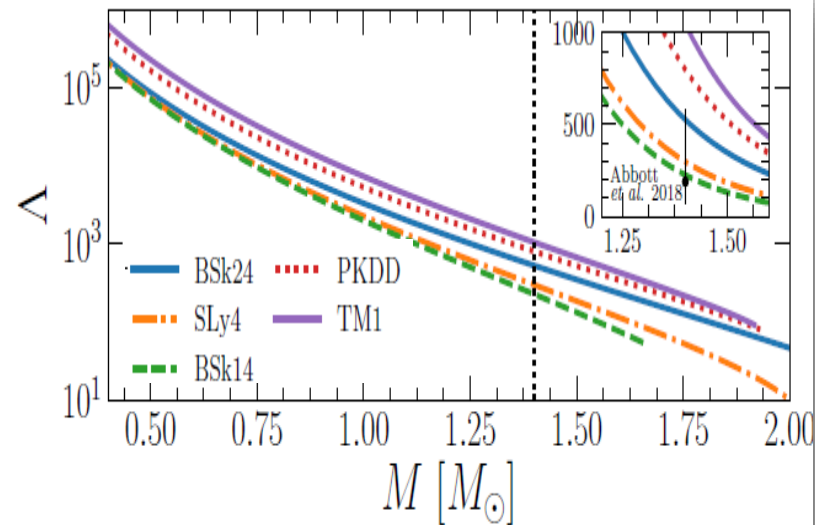
An example: the Brussels-Montreal functionals BSk



Problem 11 : no ab-initio theory of dense matter

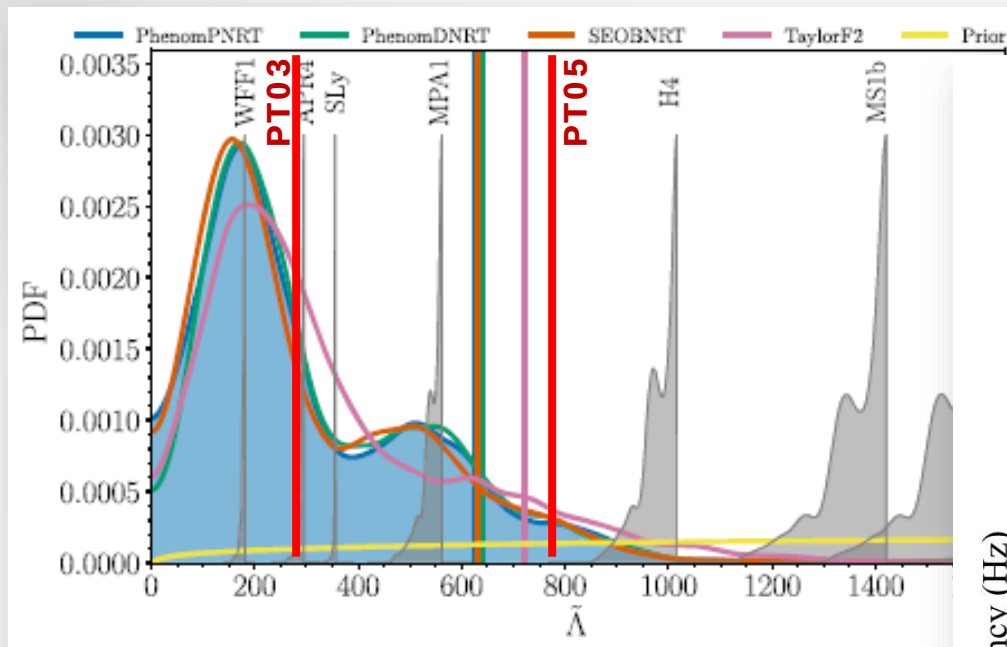


- **Model dependence: choice of functional form (Lagrangian versus Hamiltonian) and extrapolations lead to different predictions**



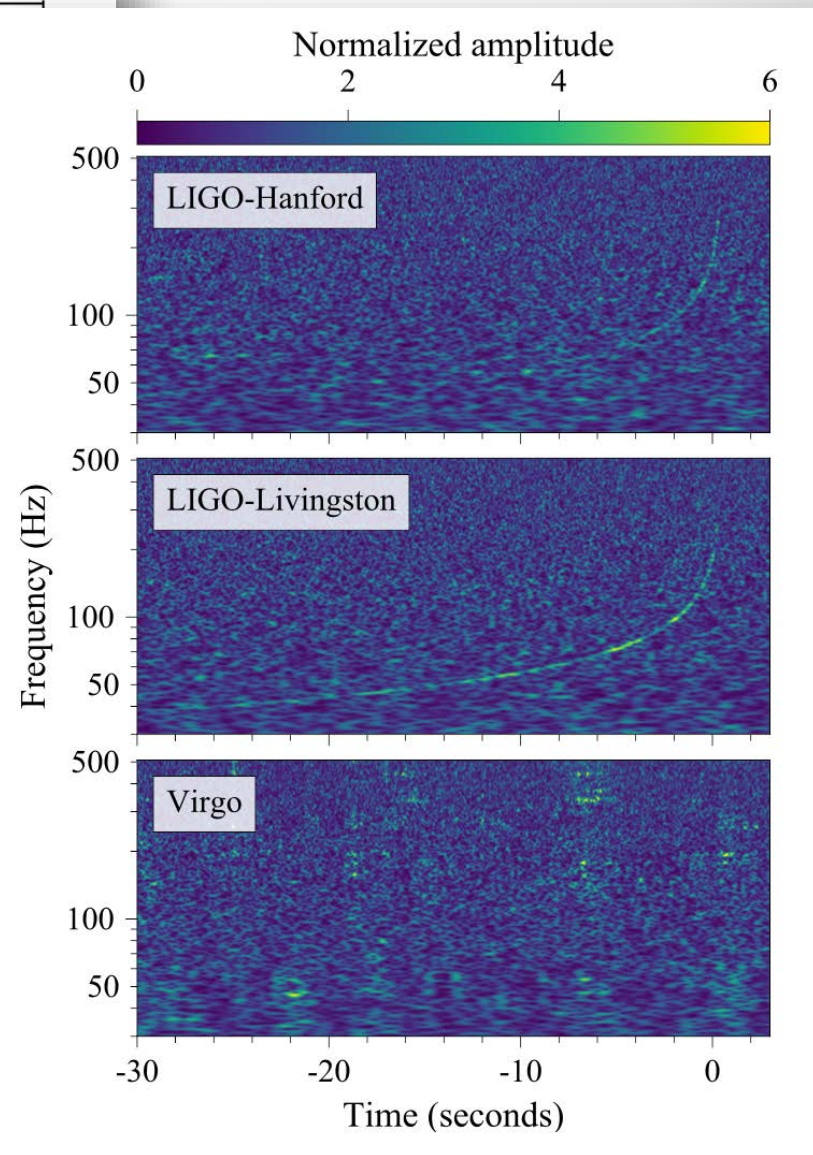
F.Burgio, I.Vidana,
Universe 2020, 6,
119

Present status: GW170817



LVC, APJL2017 & PRX9 2019

- The observation cannot discriminate



Problem 11 : no *ab-initio* theory of dense matter

$\Rightarrow$ Nucleonic « *ab-initio* » approach

$\Rightarrow$ 2- and 3-body interactions from chiral perturbation theory

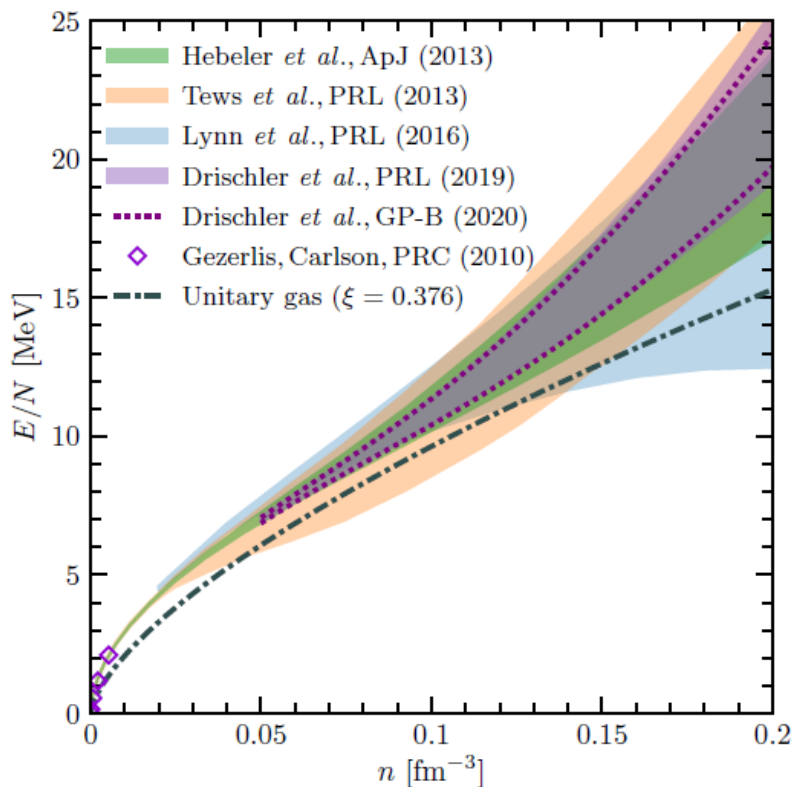
$\Rightarrow$ GS from beyond-MF many body techniques (variational, CC, MBPT, QMC...)

$\Rightarrow$ Contact terms fitted on scattering data and light nuclei

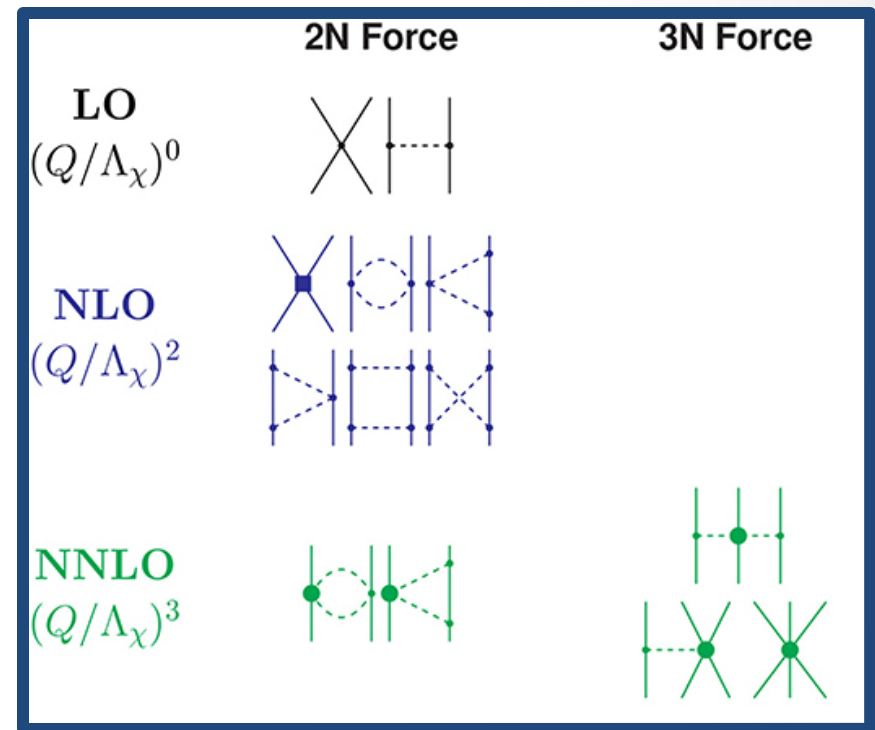
$$\Rightarrow e(\rho_n, \rho_p) \quad P(\rho) = -\rho_B^2 \left. \frac{\partial e}{\partial \rho_B} \right|_{\mu_n - \mu_p = \mu_e},$$

Nucleonic « *ab-initio* » approach

- **Diagrammatic expansion: controlled uncertainties!**
- **Still, power counting & regularization valid only up to $\sim 1,5\rho_0$**
- **Extrapolations needed**

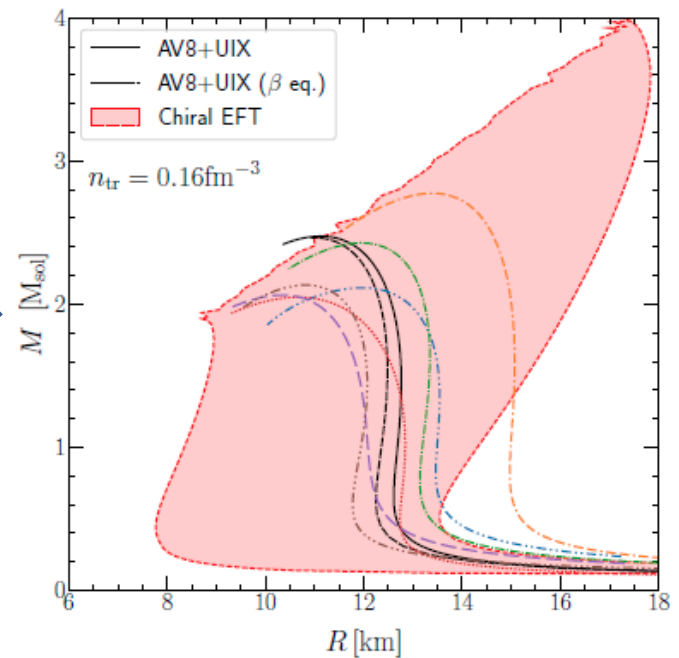
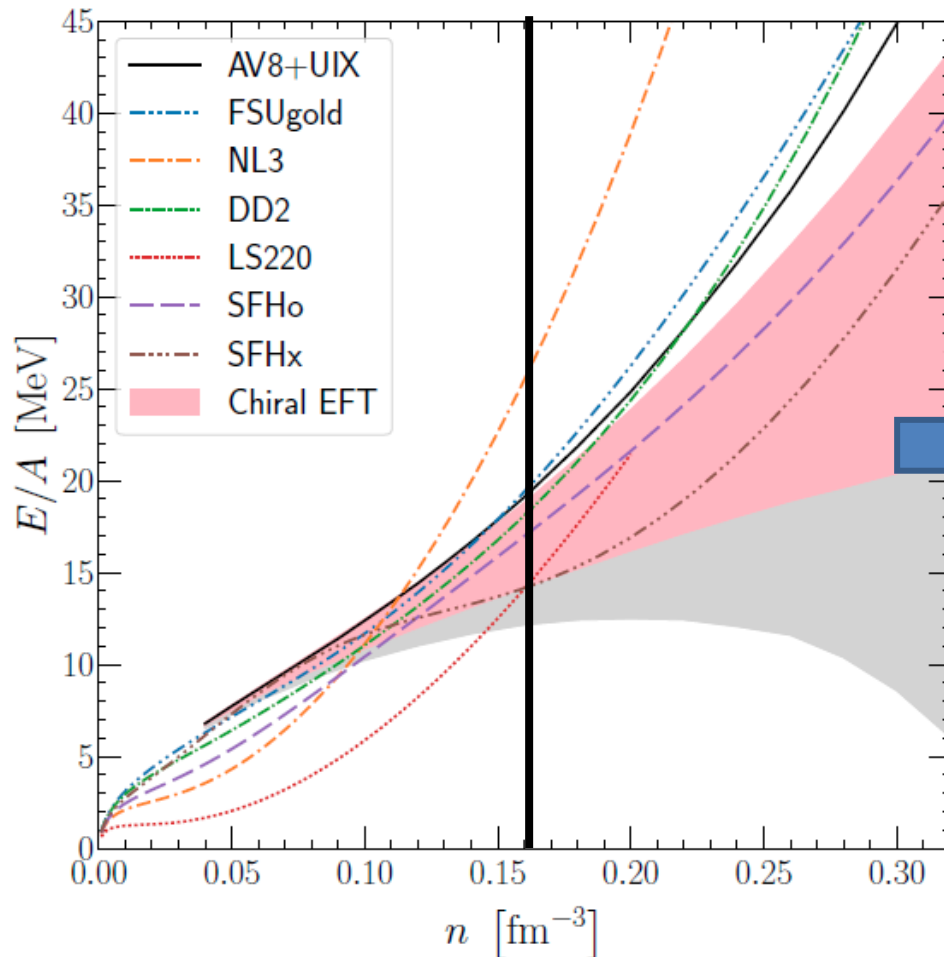


Machleidt R., Int J Mod Phys. (2017)

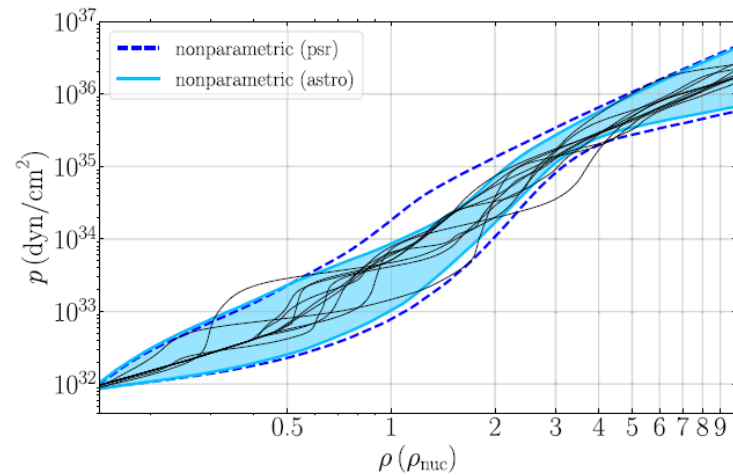
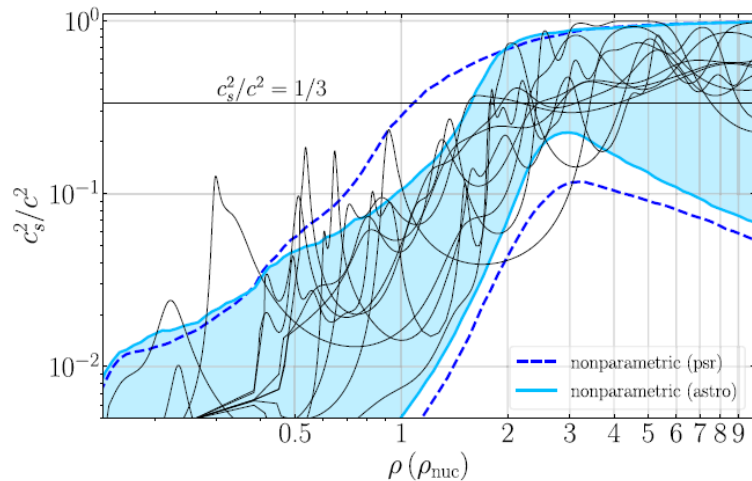


Nucleonic « ab-initio » approach

Tews, Carlson, Gandolfi, Reddy 2018



The inversion strategy



- Agnostic EoS only respecting causality and thermodynamical stability
- Example: non-parametric EoS from Gaussian Process

$$\phi(p) = \log \left(\frac{c^2}{c_s^2} - 1 \right) = N \left(\mu(p_i), K(p_i, p_j) \right)$$

- Bayesian inference

$$P(EoS | \vec{f}) = \frac{P(EoS) \prod_i P(f_i | EoS)}{P(\vec{f})}$$

f₁. max.mass (radio)

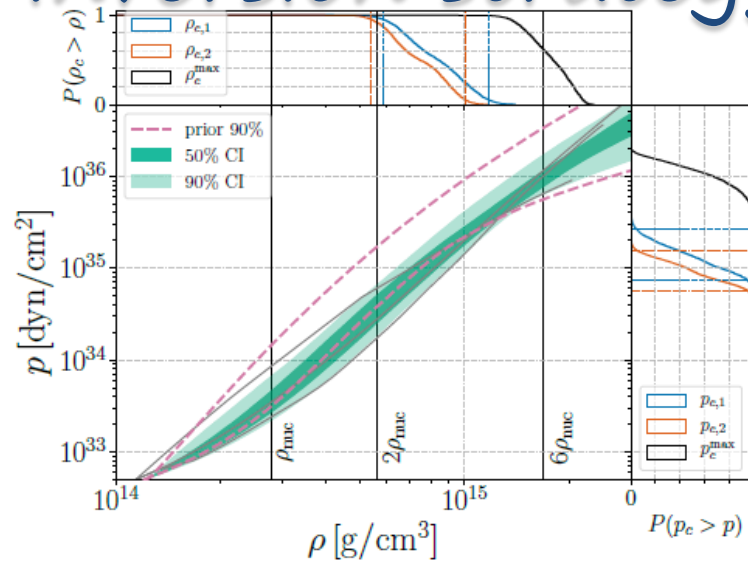
f₂. tidal polarisability (GW)

f₃. radius (X-ray)

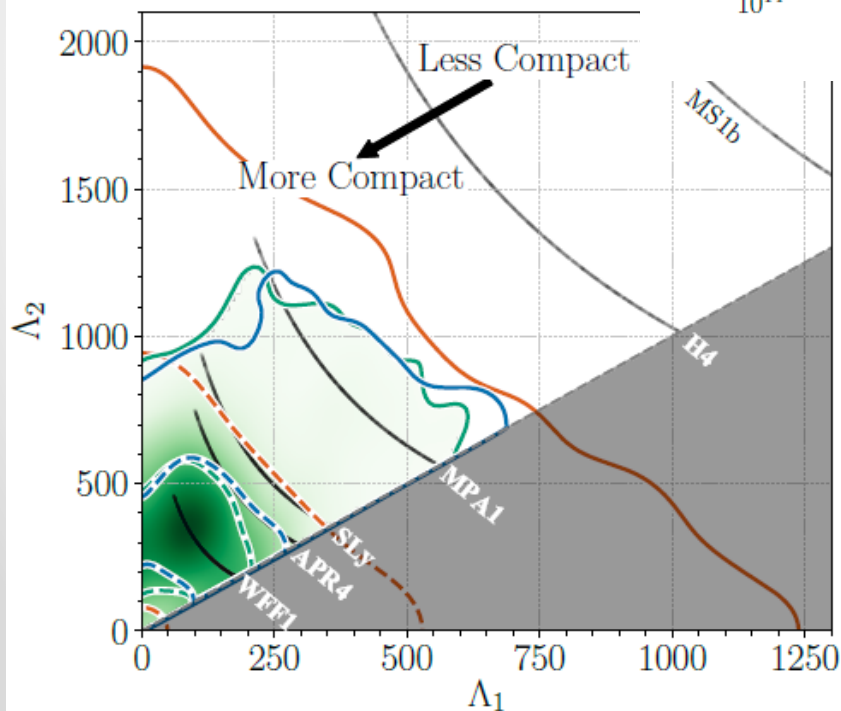
P. Landry and R. Essick, PRD 99, 084049 (2019)

I. Legred et al PRD105, 043016 (2022)

The inversion strategy



A. Abbott et al, PRL 2018

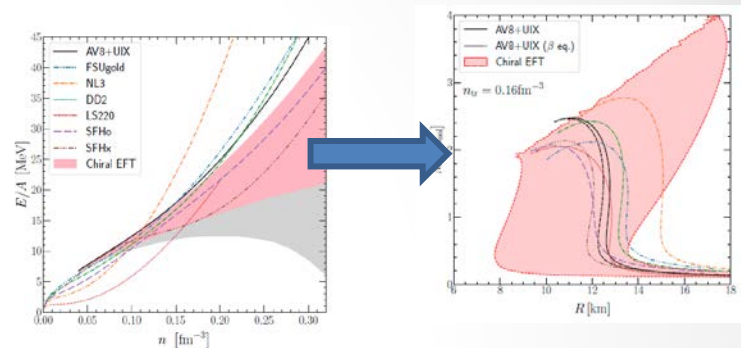


- (Almost) model independent evaluation of the EoS
- Still, we do not learn much about nuclear physics
- We do not exploit our nuclear physics knowledge either

...a brief summary $\langle O \rangle \Leftrightarrow P(\rho)$

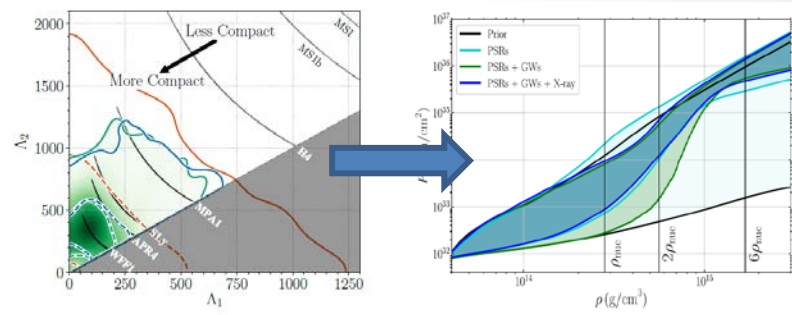
The nuclear physicist viewpoint: $e(\rho_n, \rho_p) \Rightarrow P(\rho) \Rightarrow \langle O \rangle$

- Controlled dof, hypotheses and approximations, exp info included
- Still, the predictive power is limited



The astrophysicist viewpoint: $\langle O \rangle \Rightarrow P(\rho)$

- Non-parametric representation: model independent evaluation of the EoS
- Still, we do not learn much about dense matter



A nuclear-astrophysicist viewpoint:
meeting in the middle...

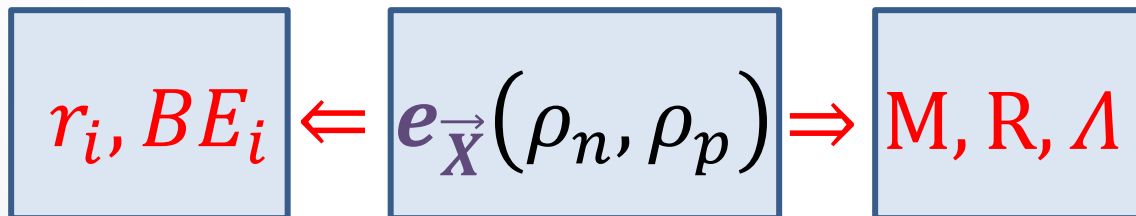
General EoS modelling: the syllabus

- An agnostic (parametric) representation ~~$P(\rho)$~~ $e_{\vec{X}}(\rho_n, \rho_p)$: the variation of the parameter set $\vec{X}$ allows reproducing the different nuclear models and interpolating among them ~ 15 parameters – RMF and EDF versions
- The X_i variation explores the equation of state space compatible with the hypothesis of a matter of neutrons and protons

$$P(\rho) = -\rho^2 \left. \frac{\partial e(\rho_n, \rho_p)}{\partial \rho} \right|_{\mu_L=0} \quad \text{and also}$$

$y_e(\rho)|_{\mu_L=0}, P(\rho, y_e, T)$ for the SN and merger simulations

A. Steiner et al ApJ 2010
A. Bulgac et al 2016
J. Margueron et al PRC 2018
Y. Lim, J.W. Holt, PRL 2018
C. Mondal et al, PRC 2022
P. Char et al, PRD 2023
G. Montefusco et al, A&A 2025
H. Koehn et al, PRX 2025



Laboratory
observables

Nuclear model

Astronomical
observables

Bayesian Inference

$$P(EoS|\vec{f}) = \frac{P(EoS) \prod_i P(f_i|EoS)}{P(\vec{f})} \quad (\text{MCMC or Nested Sampling})$$

- f_1 . nuclear structure data
- f_2 . ab-initio theory (chiral)
- f_3 . HI collisions



Nuclear physics

- f_4 . max.mass (radio)
- f_5 . tidal polarisability (GW)
- f_6 . radius (X-ray)
- f_7 . multi-messenger



Astrophysics

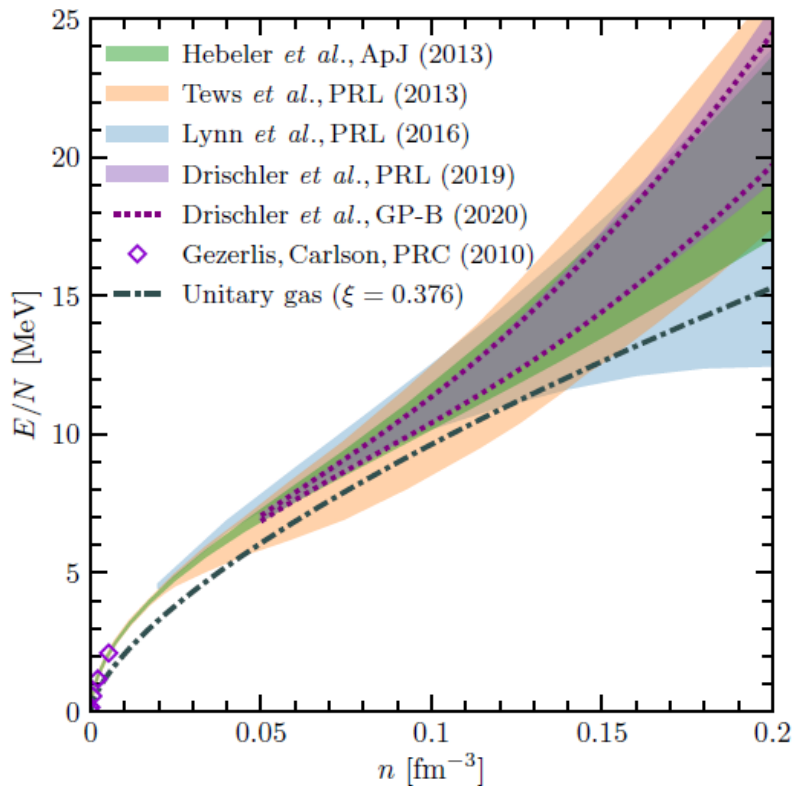
(3) PSR J0348+0432 $M=2.01\pm0.04 M_{\odot}$

(4) GW170817 $\tilde{\Lambda}(M)$ LVK

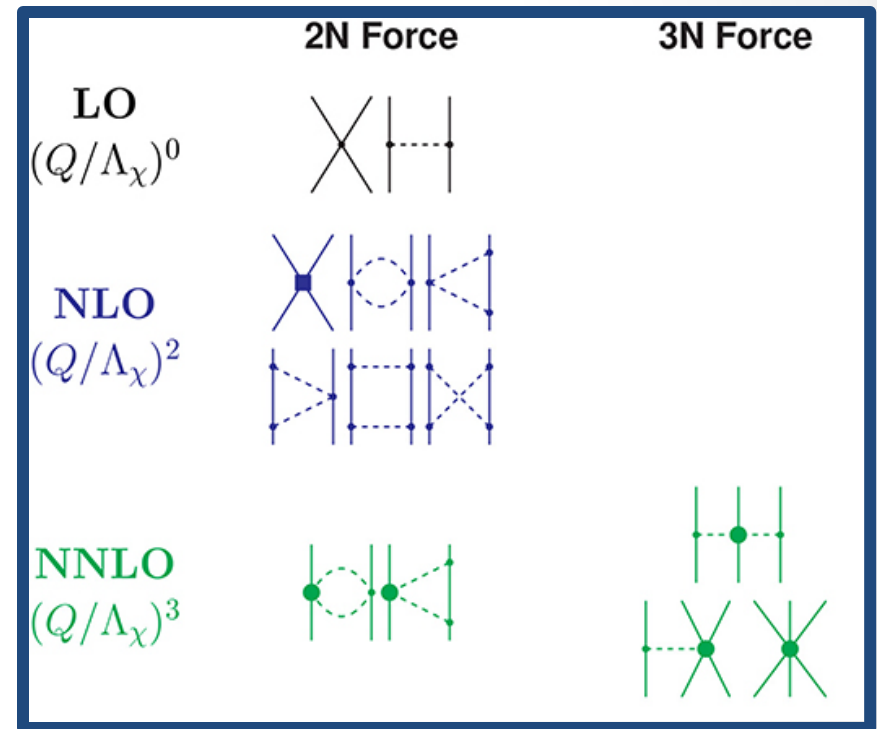
(5) PSRJ0030+0451, PSRJ0740+6620 NICER (+ PSR10030-0451, PSRJ0614-3329)

Ab-initio nuclear theory

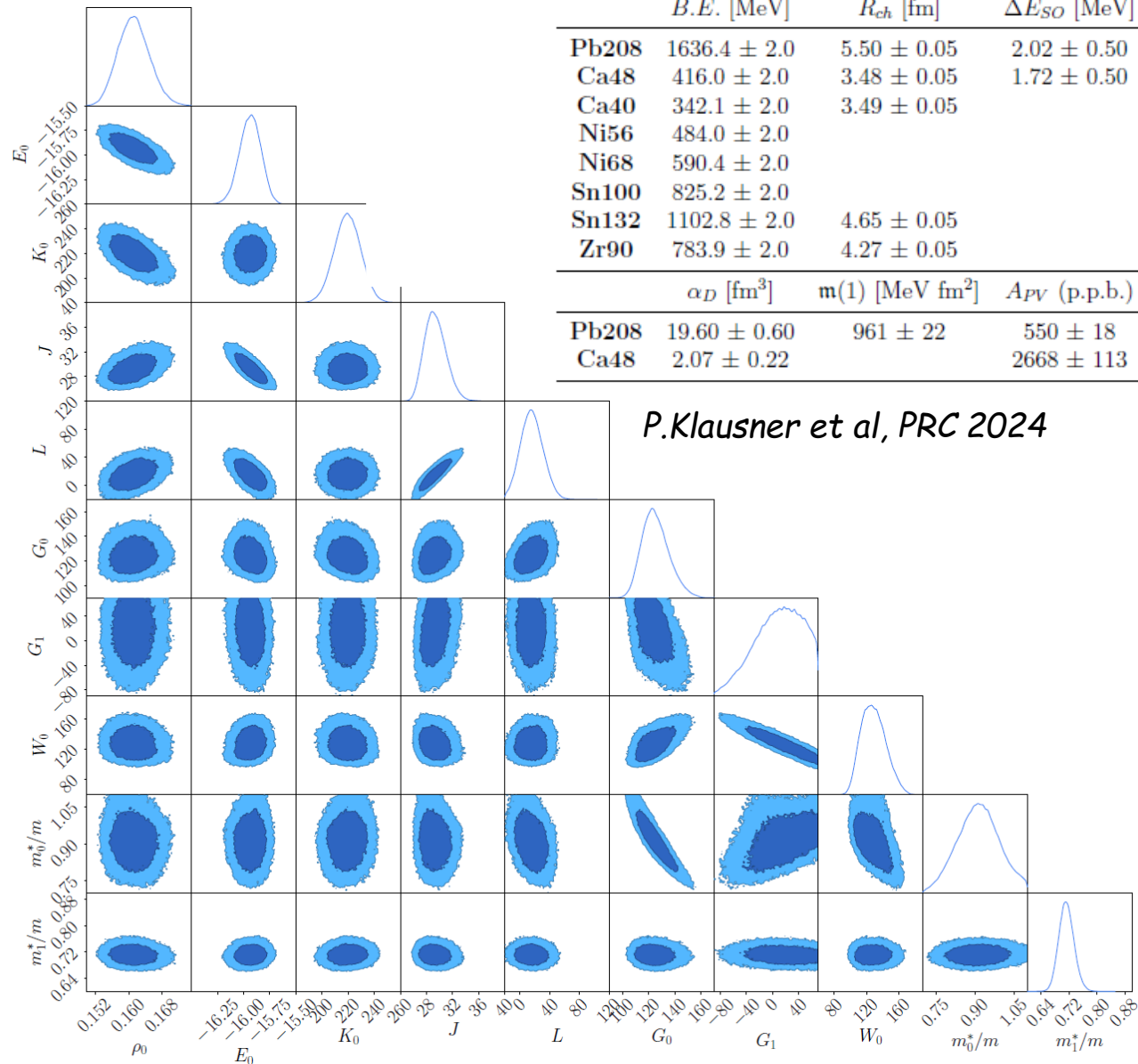
- interaction from χ -EFT, different many body methods (MBPT, AFMC)
- Diagrammatic expansion : controlled truncation errors
- Moment expansion! Only valid at low density



Machleidt R., Int J Mod Phys. (2017)



Laboratory experiments

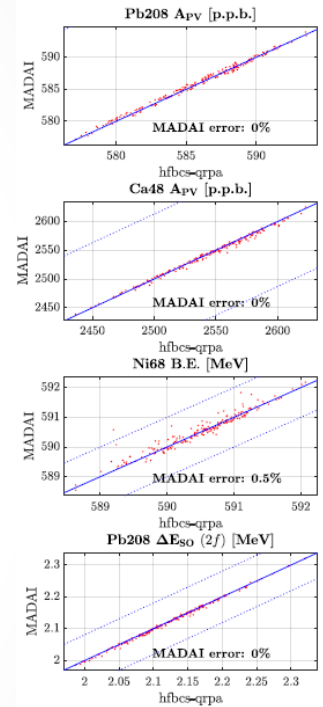


	$B.E.$ [MeV]	R_{ch} [fm]	ΔE_{SO} [MeV]	E_{GMR}^{IS} [MeV]
Pb208	1636.4 ± 2.0	5.50 ± 0.05	2.02 ± 0.50	13.5 ± 0.5
Ca48	416.0 ± 2.0	3.48 ± 0.05	1.72 ± 0.50	
Ca40	342.1 ± 2.0	3.49 ± 0.05		
Ni56	484.0 ± 2.0			
Ni68	590.4 ± 2.0			
Sn100	825.2 ± 2.0			
Sn132	1102.8 ± 2.0	4.65 ± 0.05		
Zr90	783.9 ± 2.0	4.27 ± 0.05		17.7 ± 0.5

	α_D [fm ³]	$m(1)$ [MeV fm ²]	A_{PV} (p.p.b.)	E_{GQR}^{IS} [MeV]
Pb208	19.60 ± 0.60	961 ± 22	550 ± 18	10.9 ± 0.5
Ca48	2.07 ± 0.22		2668 ± 113	

P.Klausner et al, PRC 2024

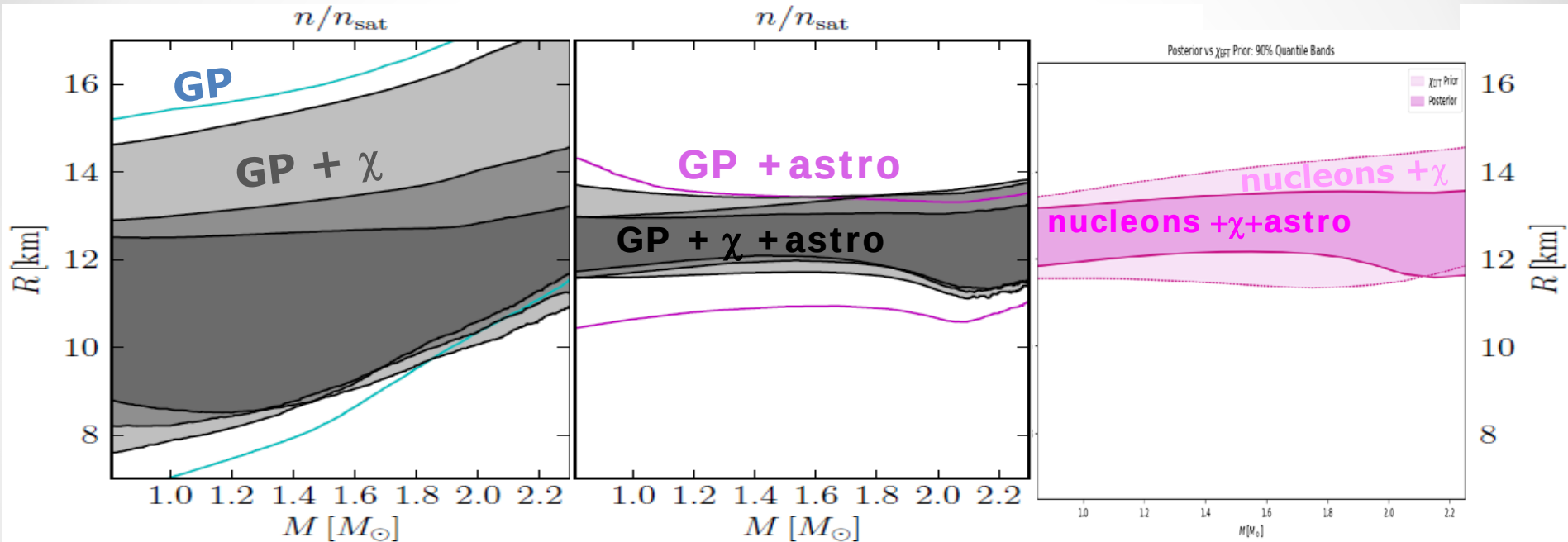
madai.phy.duke.edu



Agnostic versus nucleonic

R.Essick et al, PRC 2020

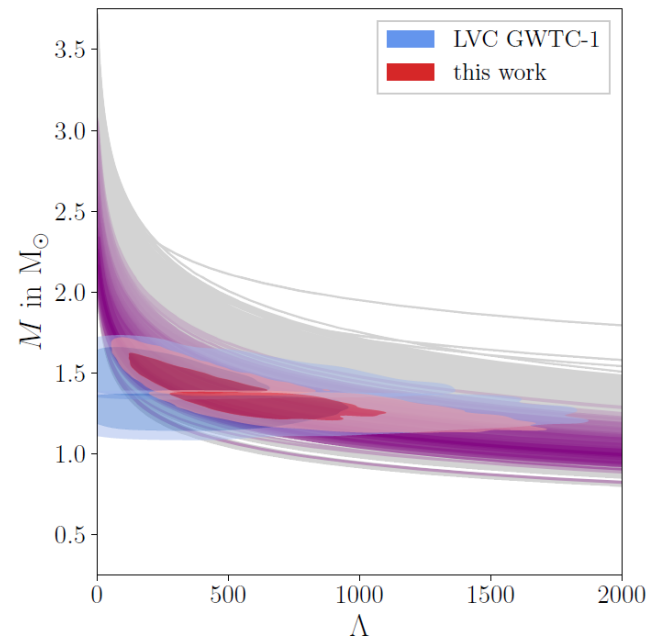
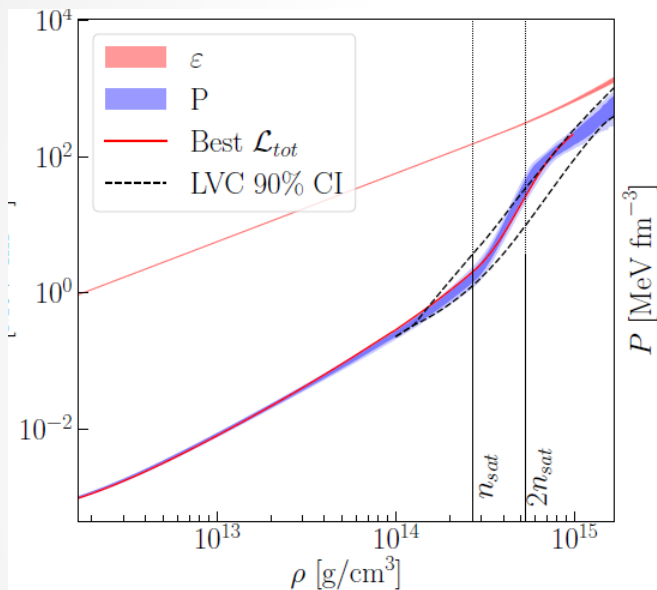
G.Montefusco et al, A&A 2025



- Prior distributions are larger with agnostic models (GP)
- Distributions are very comparable with chiral (χ)+astro $\Rightarrow$ the nucleonic models do cover all the parameter space compatible with the observations ! $\Rightarrow$ the observations are compatible with a pure nucleonic composition

Agnostic versus nucleonic

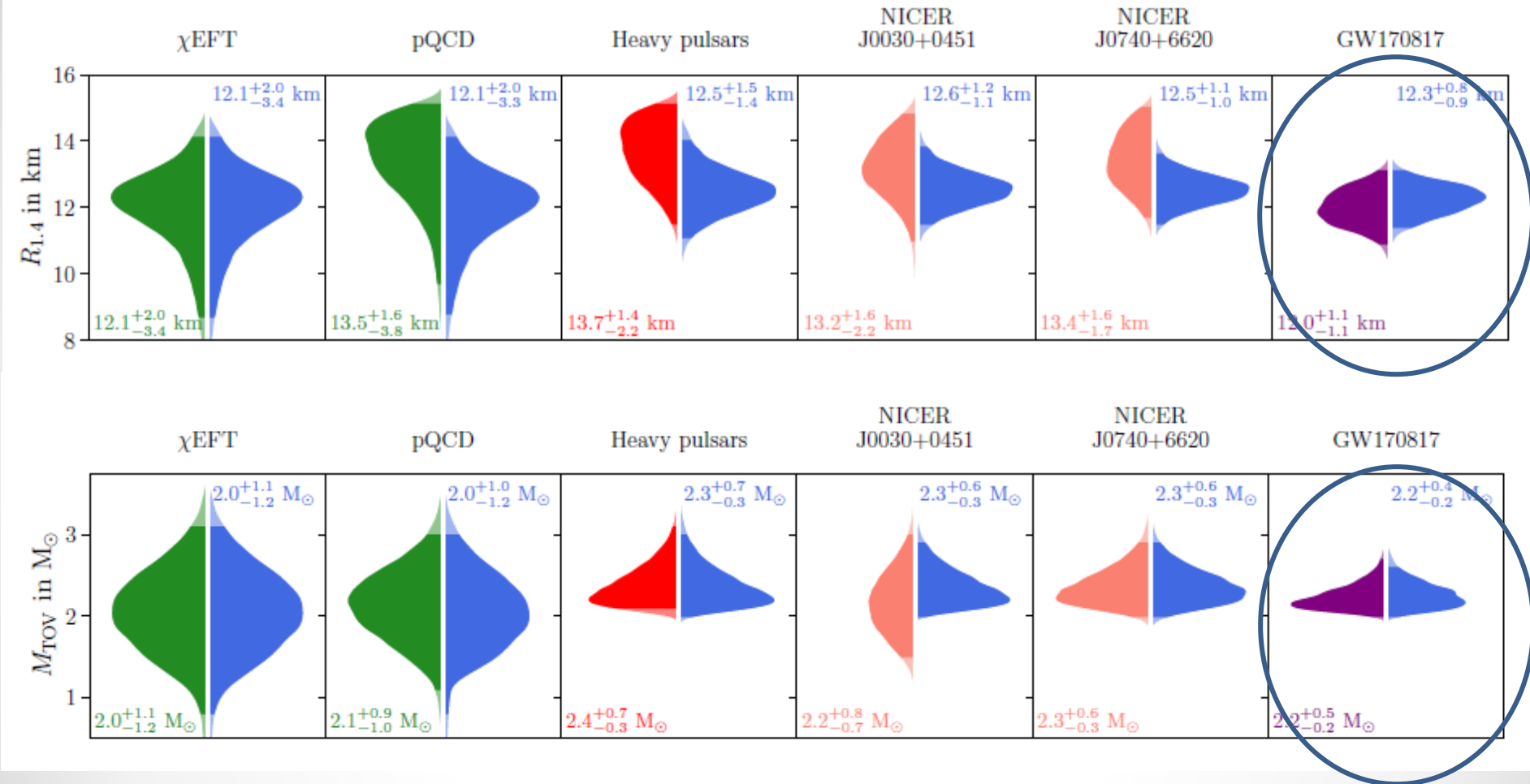
P.Klausner et al, ArXiv 2025



H.Koehn et al, PRX 2025

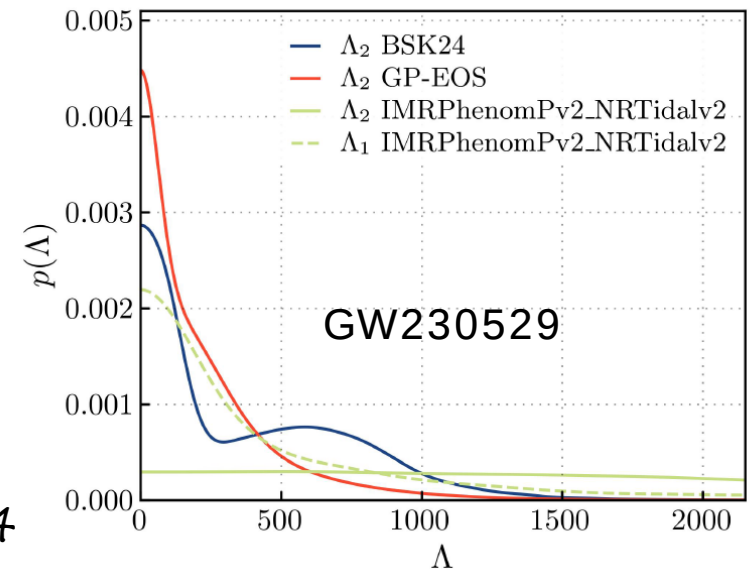
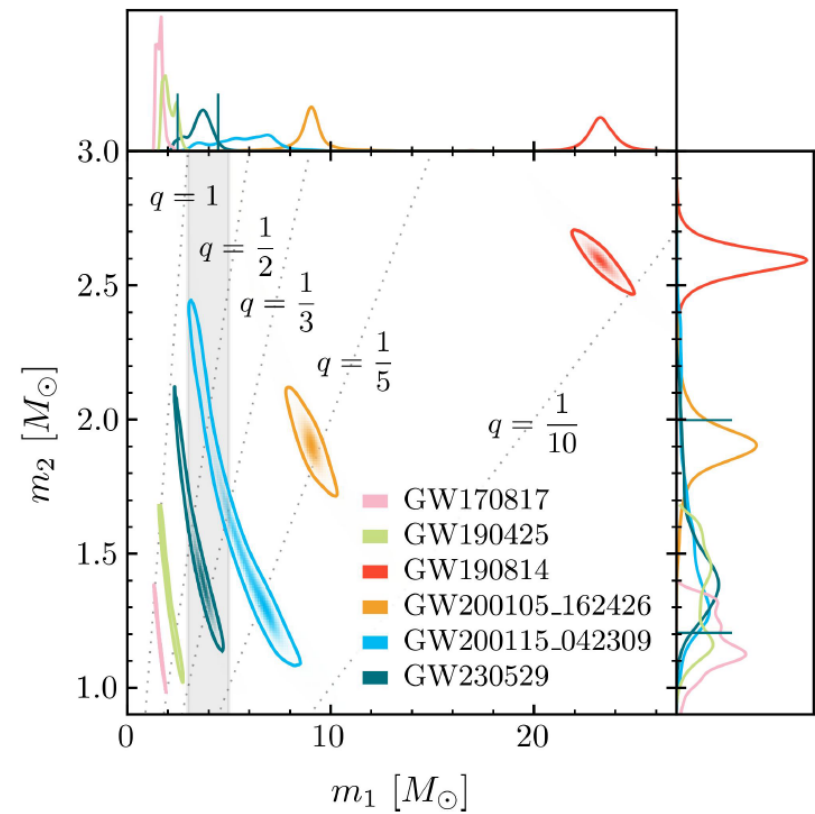
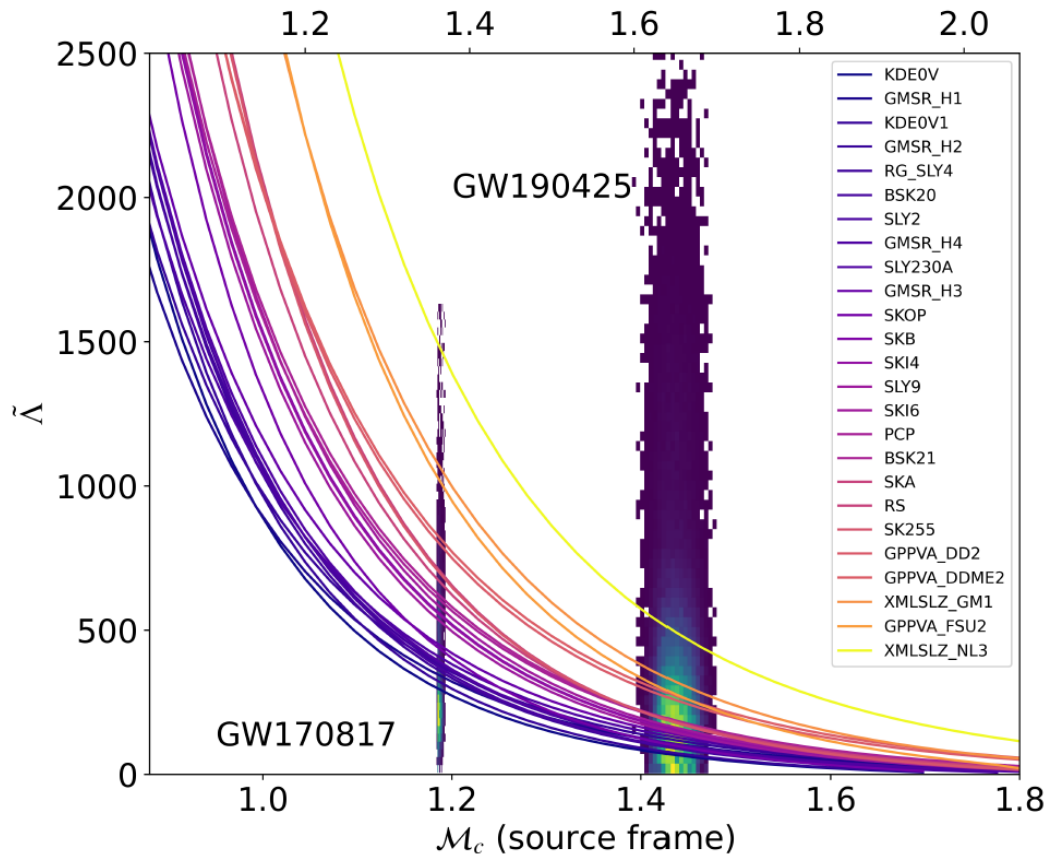
- Prior distributions are larger with agnostic models (GP)
- Distributions are very comparable with chiral (χ)+astro $\Rightarrow$ the nucleonic models do cover all the parameter space compatible with the observations ! $\Rightarrow$ the observations are compatible with a pure nucleonic distribution
- **Distributions get shrinked when nuclear experiments are included $\Rightarrow$ potential challenge of the nucleonic hypothesis!**

The importance of the GW signal



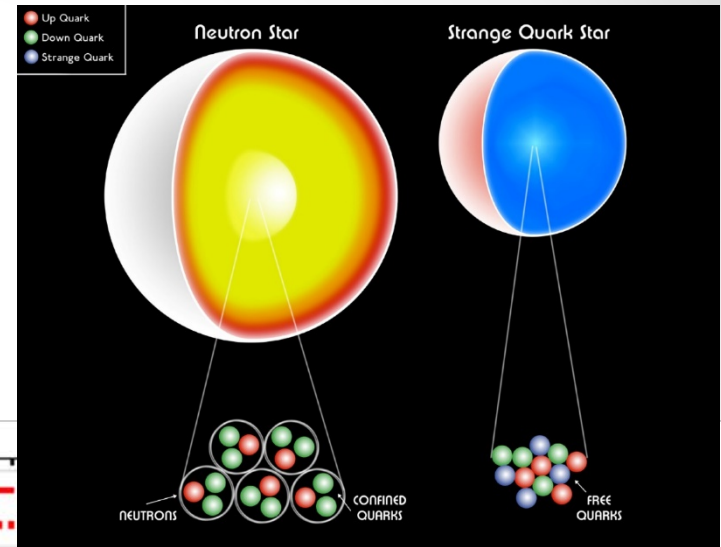
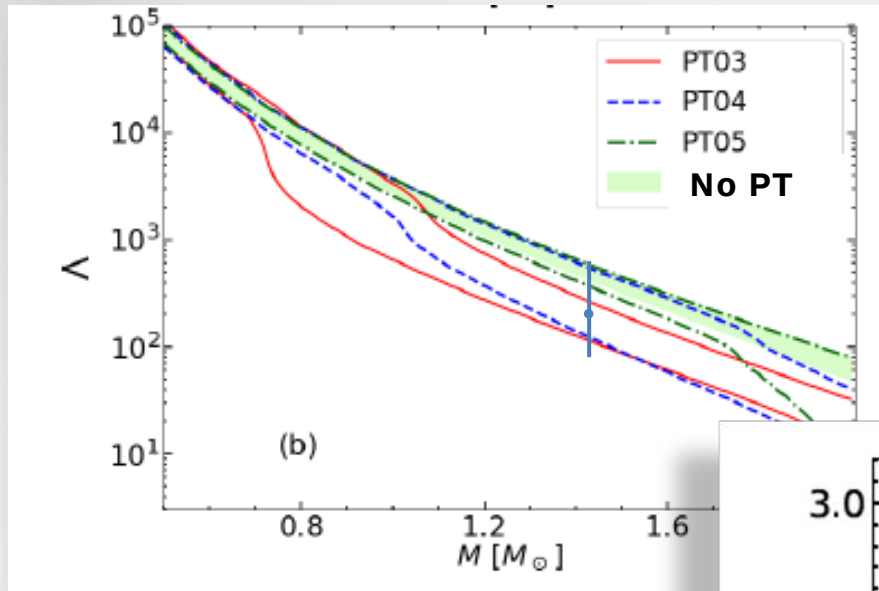
H.Koehn et al, PRX 2025

GW info after GW170817



A. Abac et al, ApJL 2024

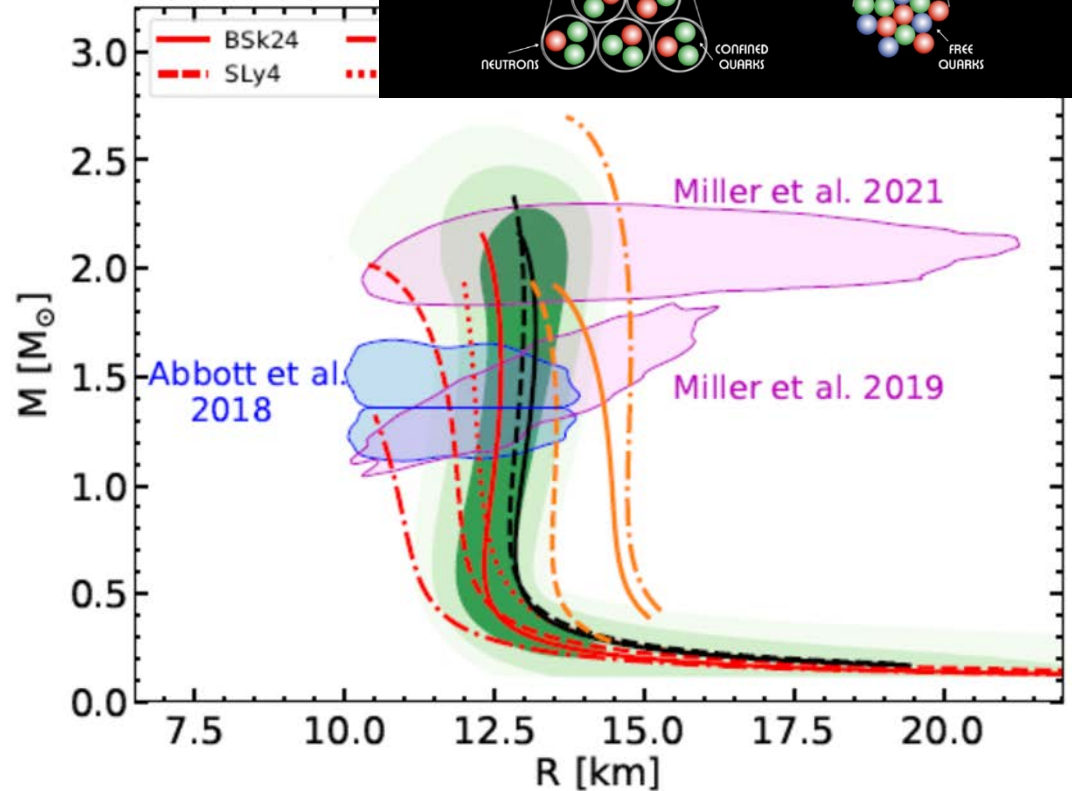
Quarks in the core of neutron stars ?



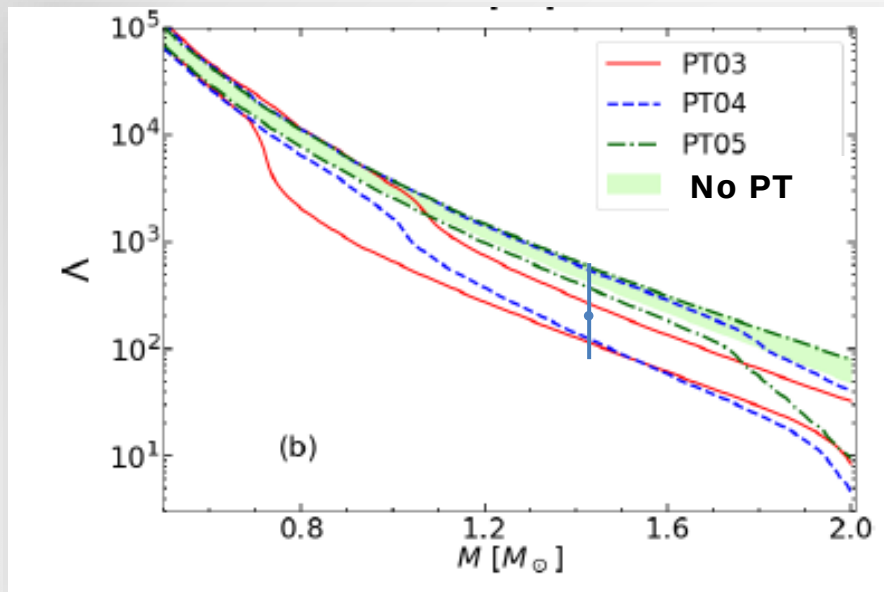
Credits: CXC/M. Weiss

H.Dinh Thi et al, A&A 2021

- The observations are compatible with a pure nucleonic composition



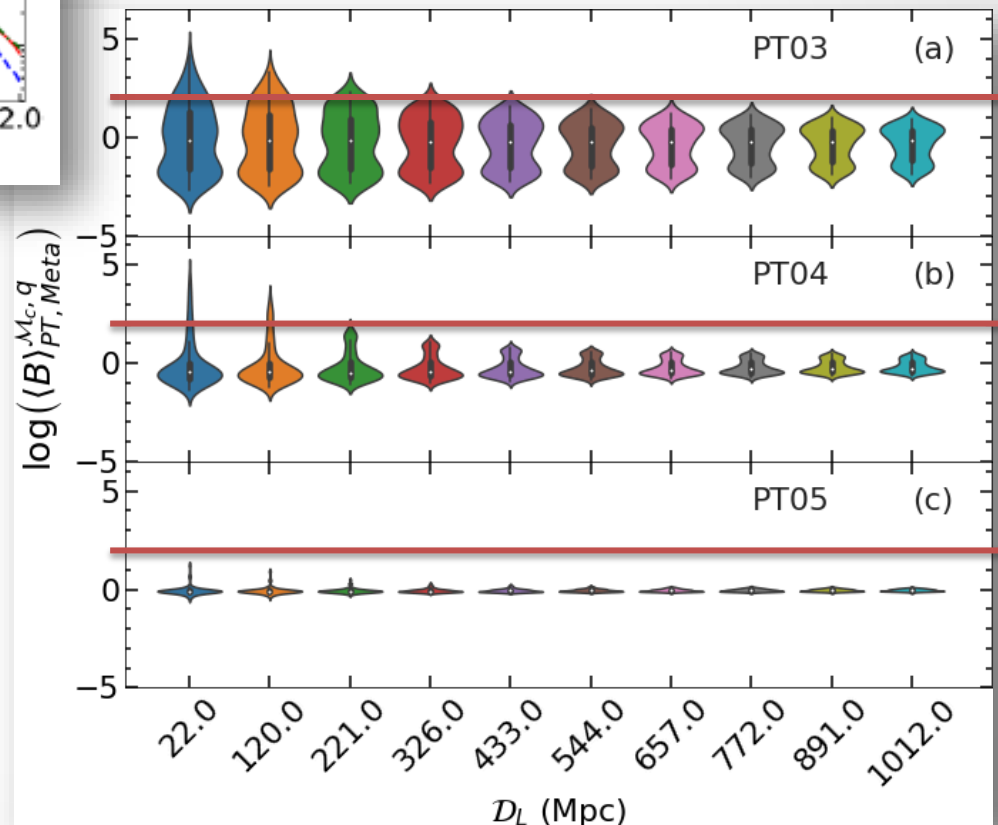
Quarks in the core of neutron stars ?



- Need to reduce the uncertainties!
 - New nuclear observables
 - Multiple detections with better SNR => Einstein Telescope, Cosmic Explorer

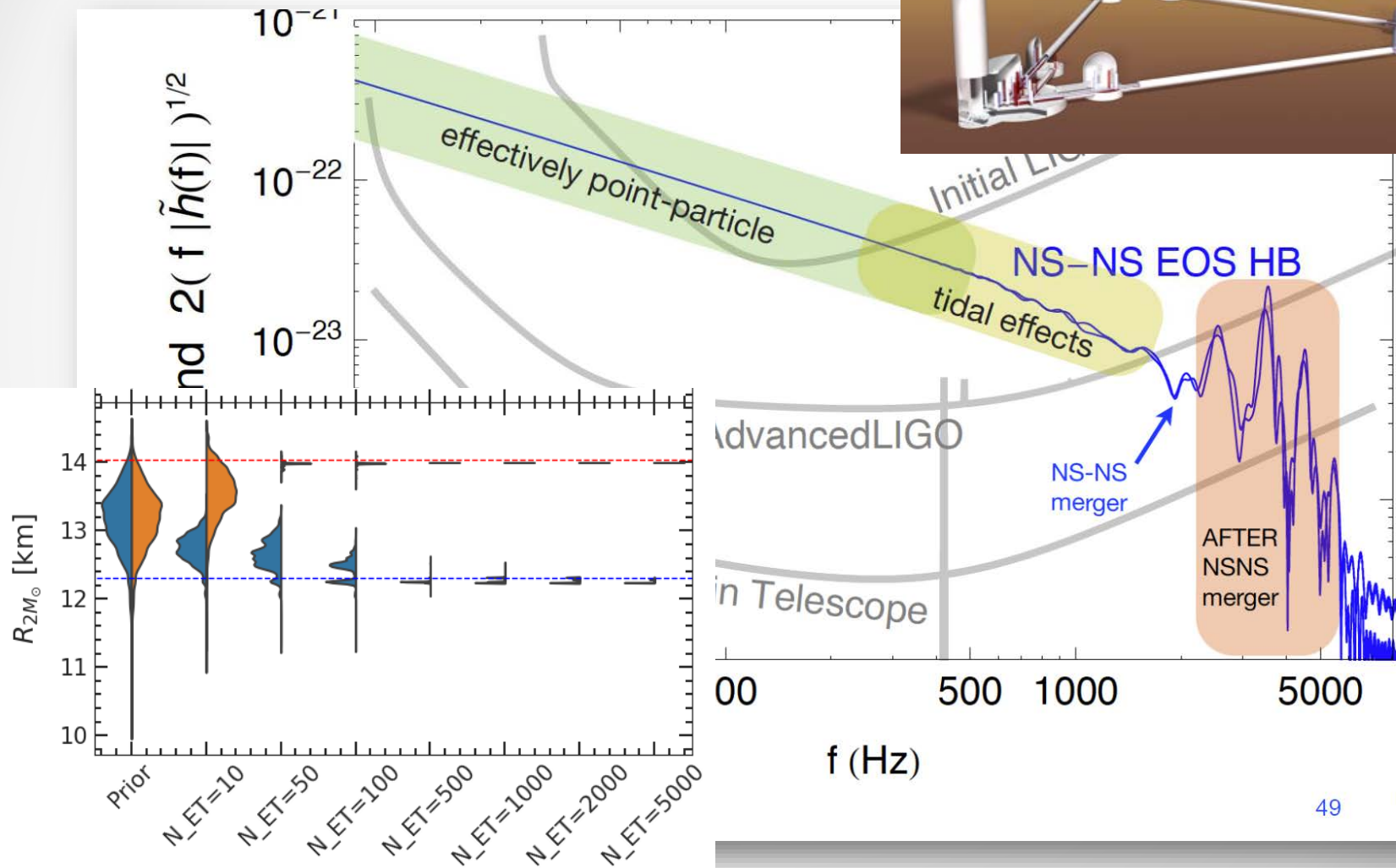
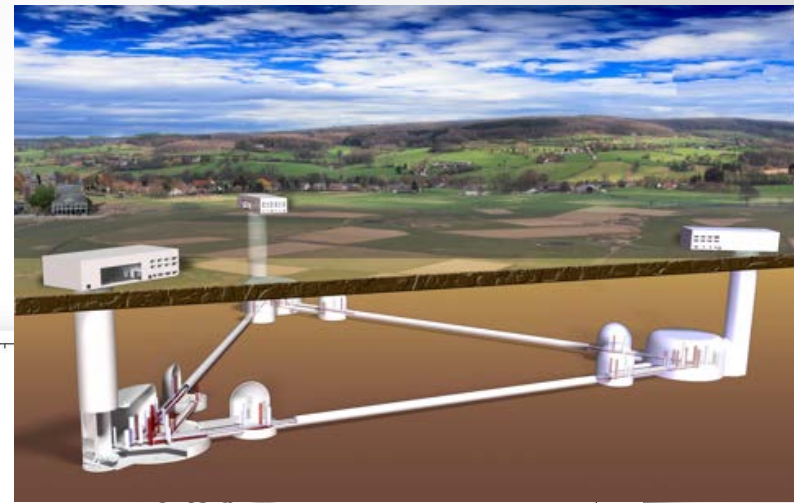
C.Mondal et al, MNRAS 2023

- The observations are compatible with a pure nucleonic composition
- Even with AdV+ sensitivity, only a very close detection would allow identifying deconfined matter, and only if the quark core is large



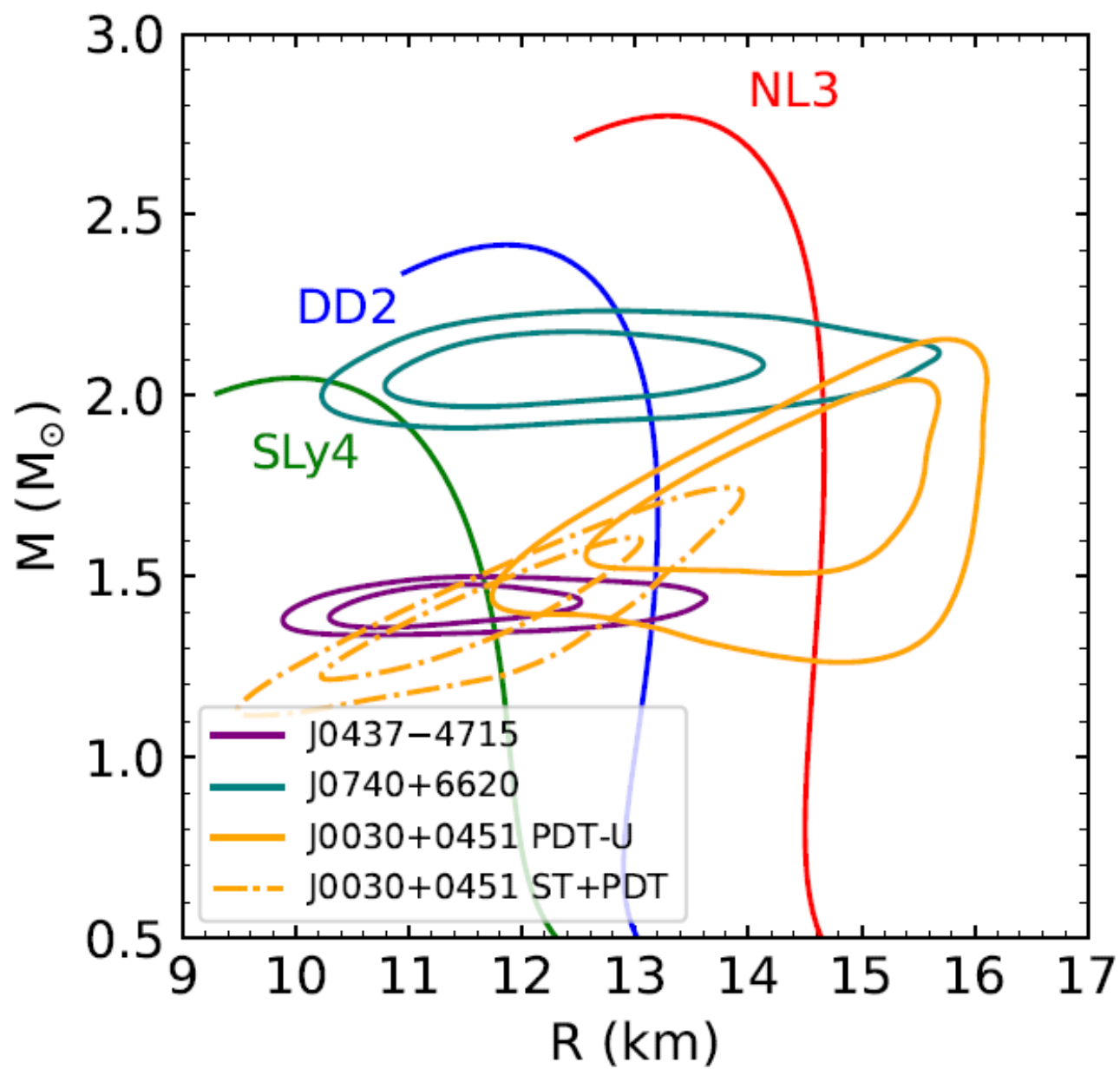
The Einstein Telescope project

F.Iacovelli, C.Mondal et al, PRD 2023

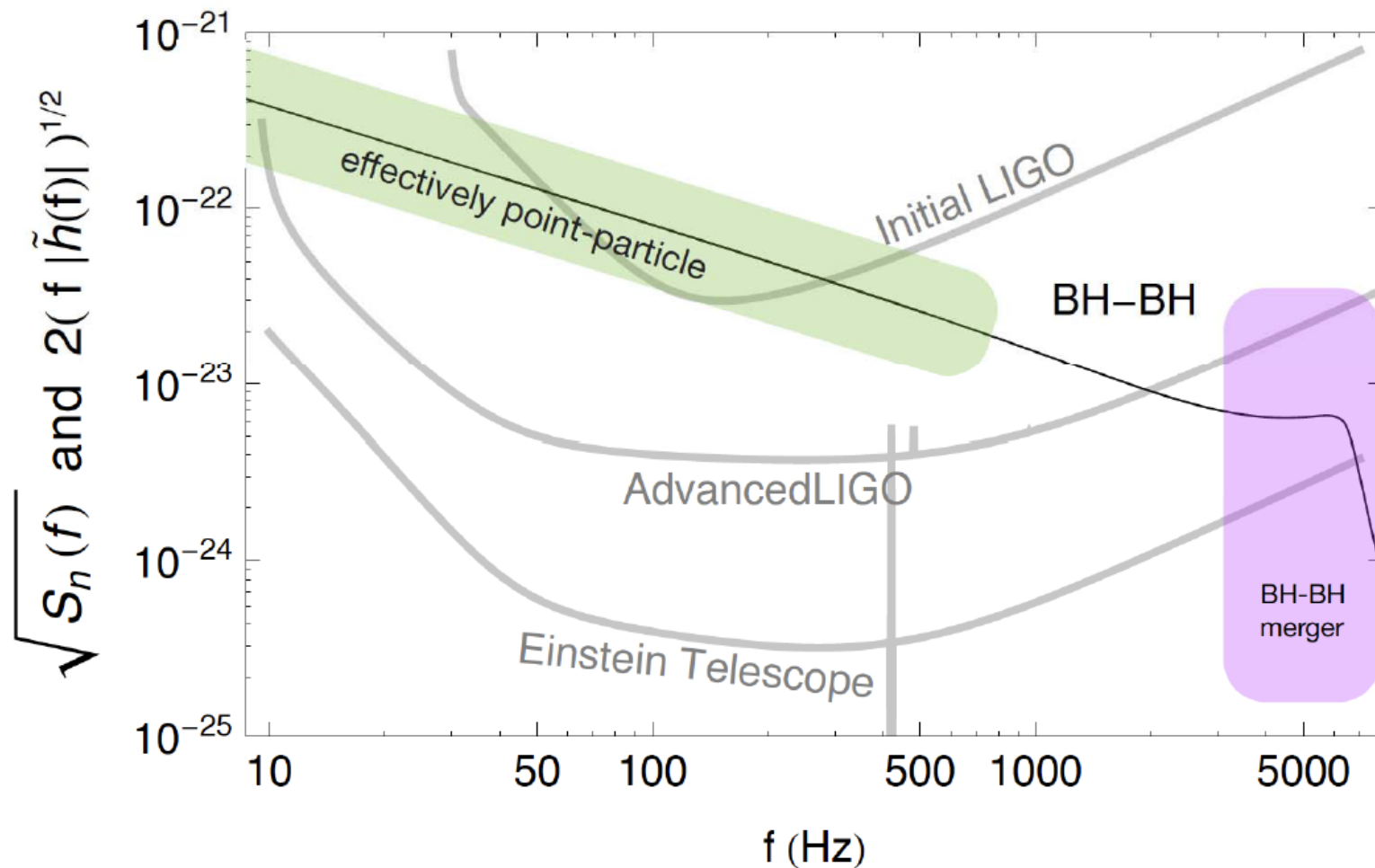
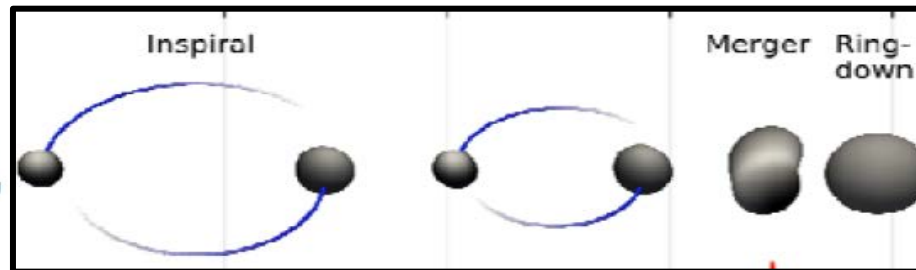


Summary & Conclusions

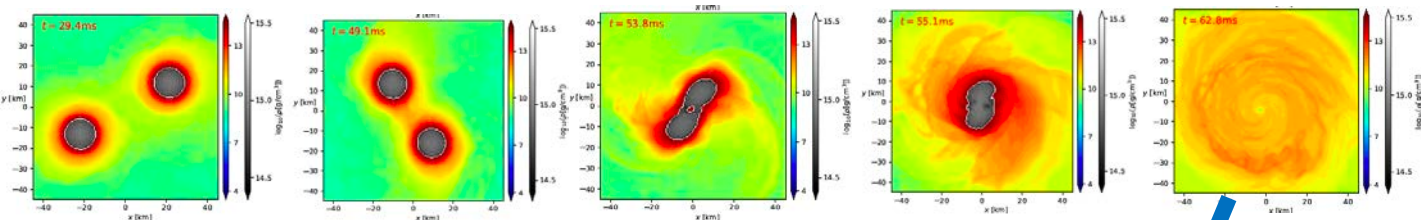
- The Equation of State can be univocally mapped to the static properties of NS $\Leftrightarrow$ to the GW signals from NS mergers
- No ab-initio model of nuclear matter for all densities! But Bayesian techniques allow controlled extrapolations of low density constraints from nuclear theory and experiments
 - => No present indication of exotic degrees of freedom**
- Relatively tight observable prediction within the nucleonic hypothesis
 - => A 1st order phase transition to deconfined matter can potentially be detected with 3G interferometers**



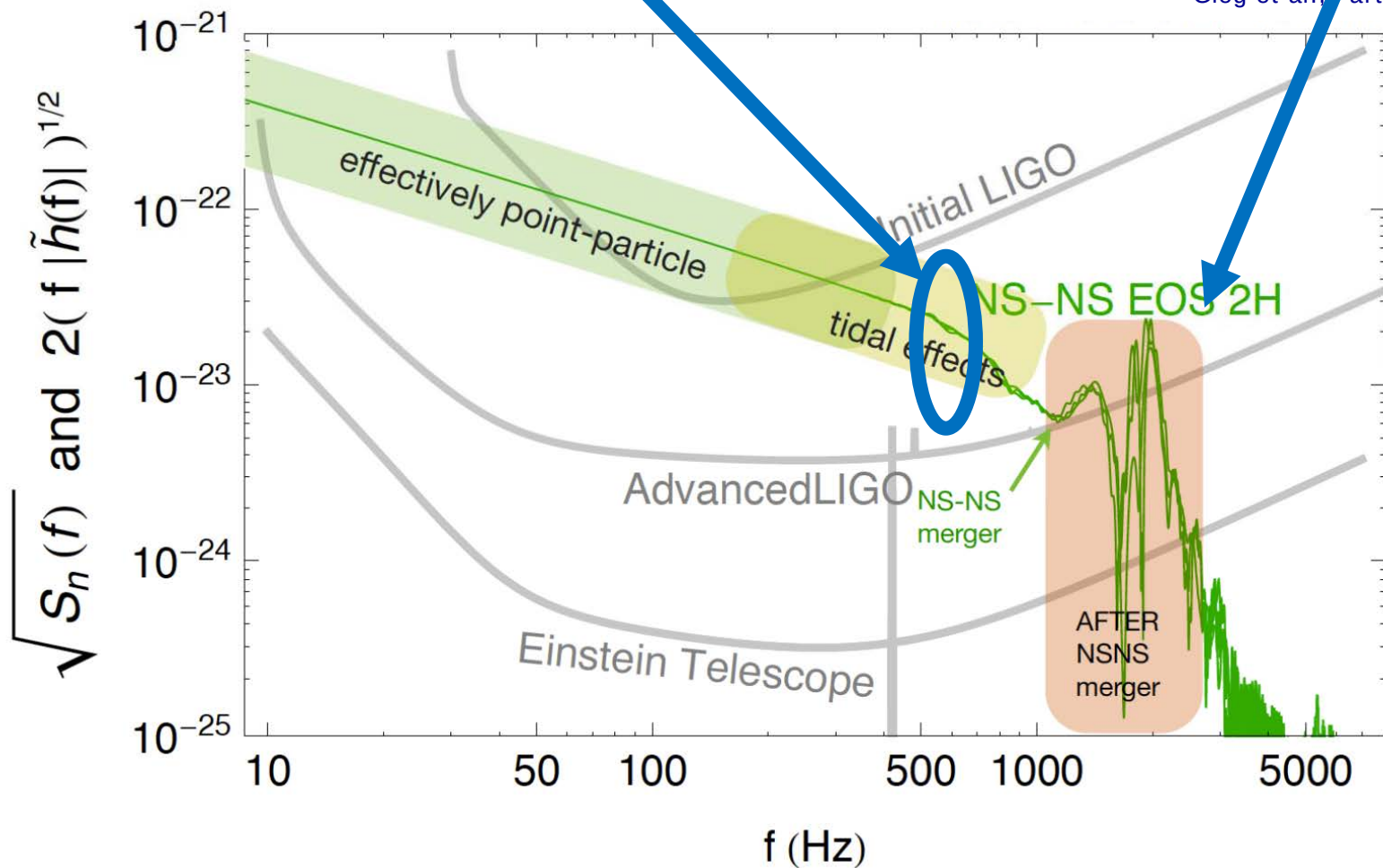
Spectrum of BBH inspiral,



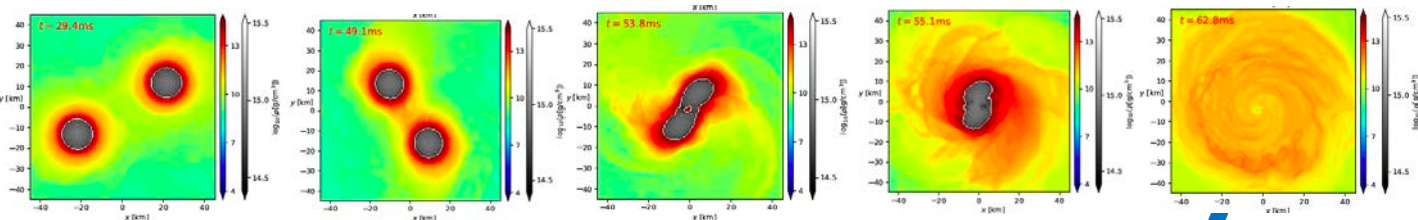
Spectrum



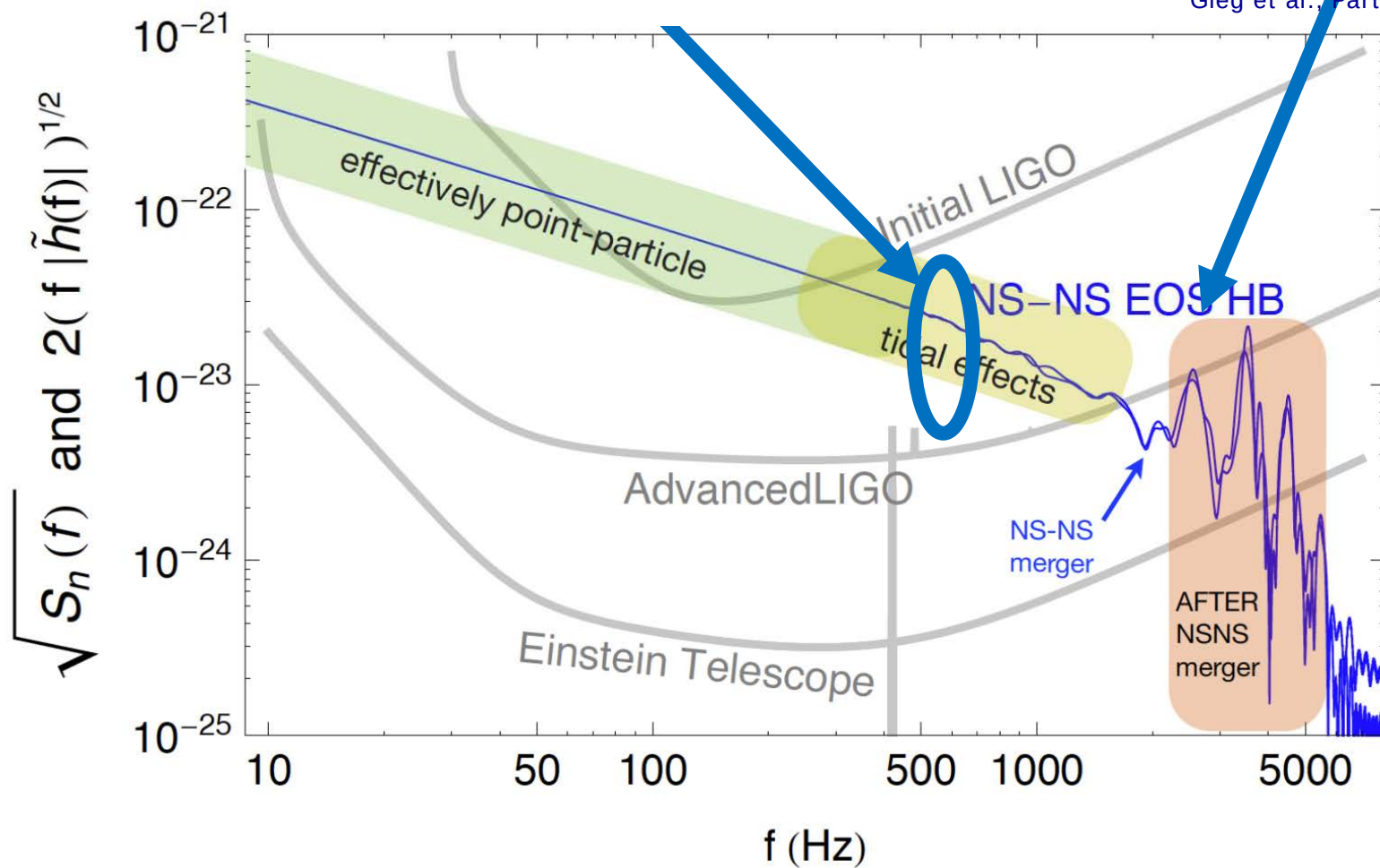
Giee et al., *Article 2, 23* (201...)



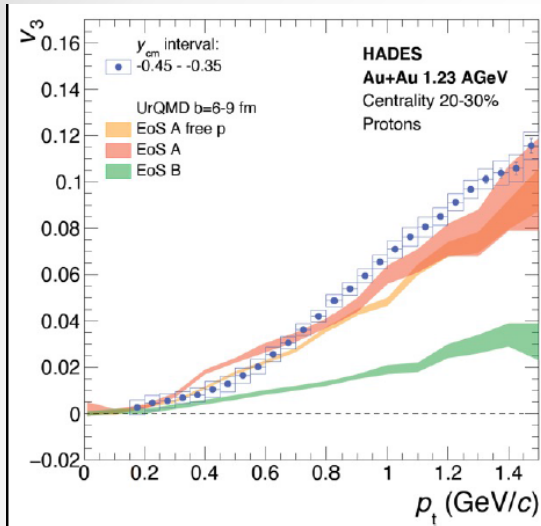
Spectrum



Gieget al., Particle 2, 23 (201

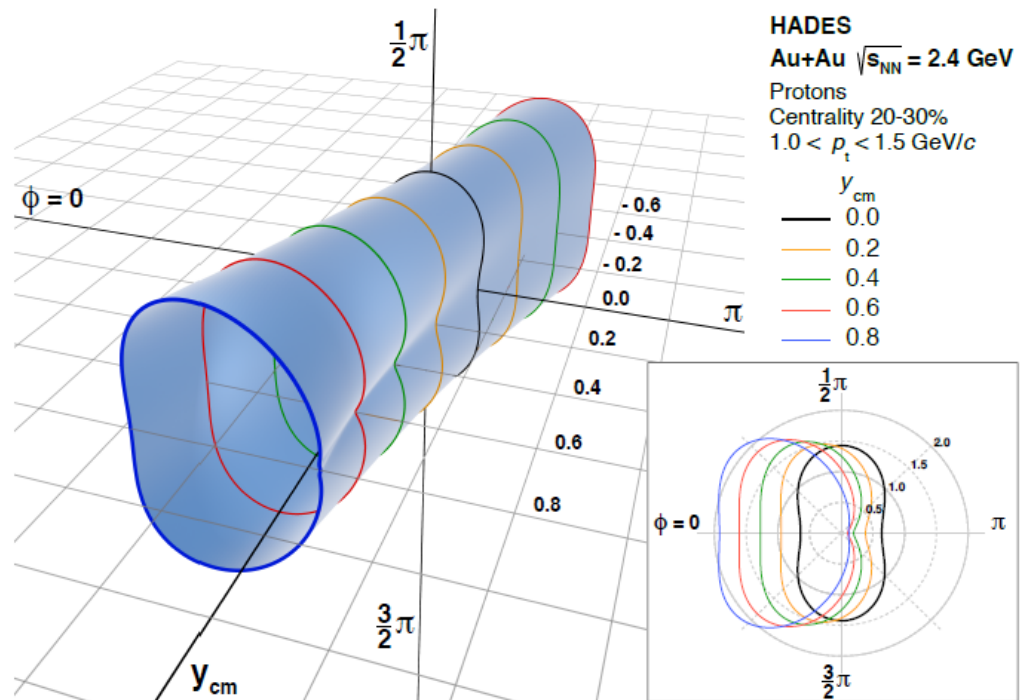
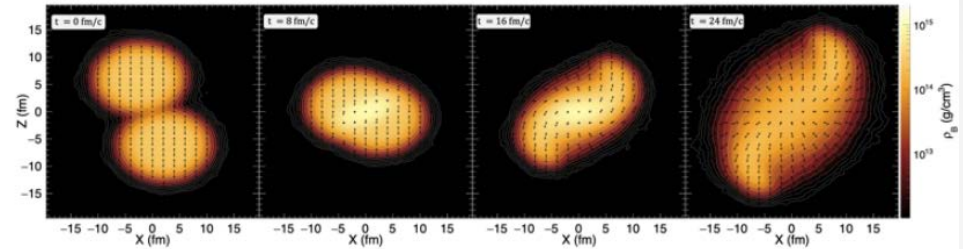


The HI collisions probe

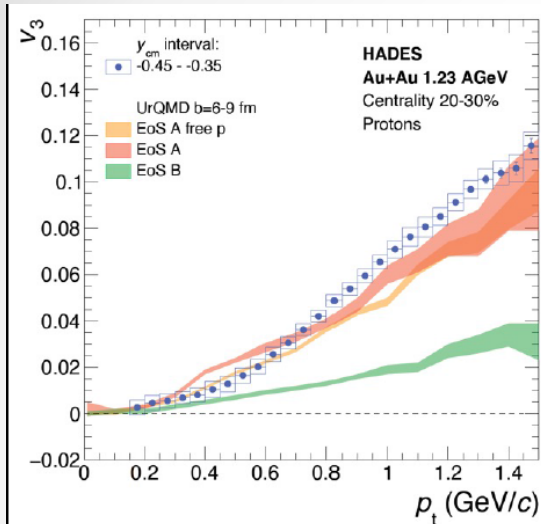


HADES collaboration
PRL 125 (2020)
262301

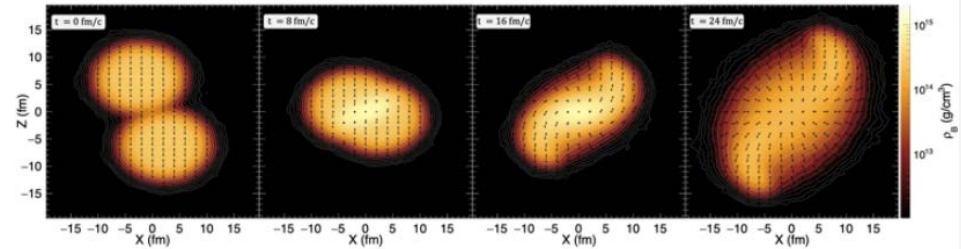
Au+Au $\sqrt{s_{NN}} = 2.4$ GeV (UrQMD)
 $\phi = 16$ fm $\tau \sim 10^{-23}$ s



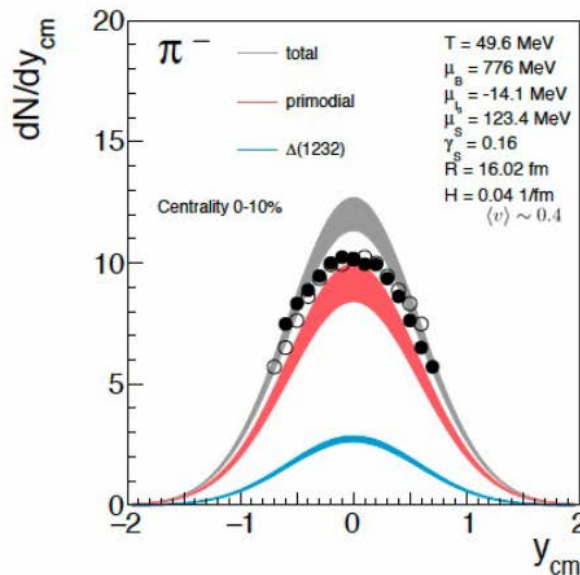
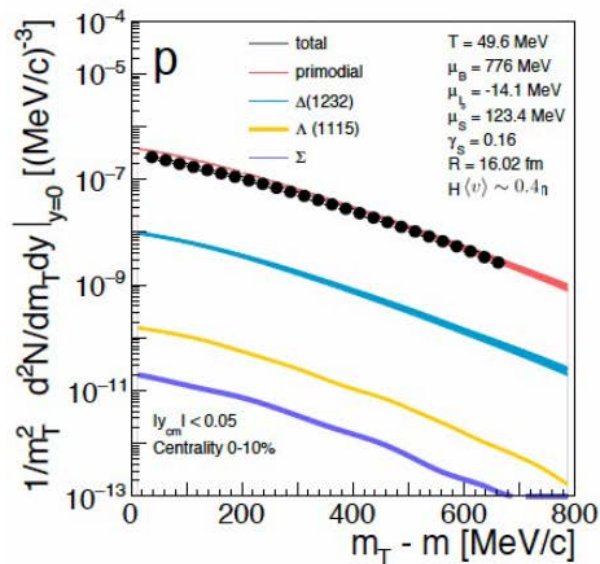
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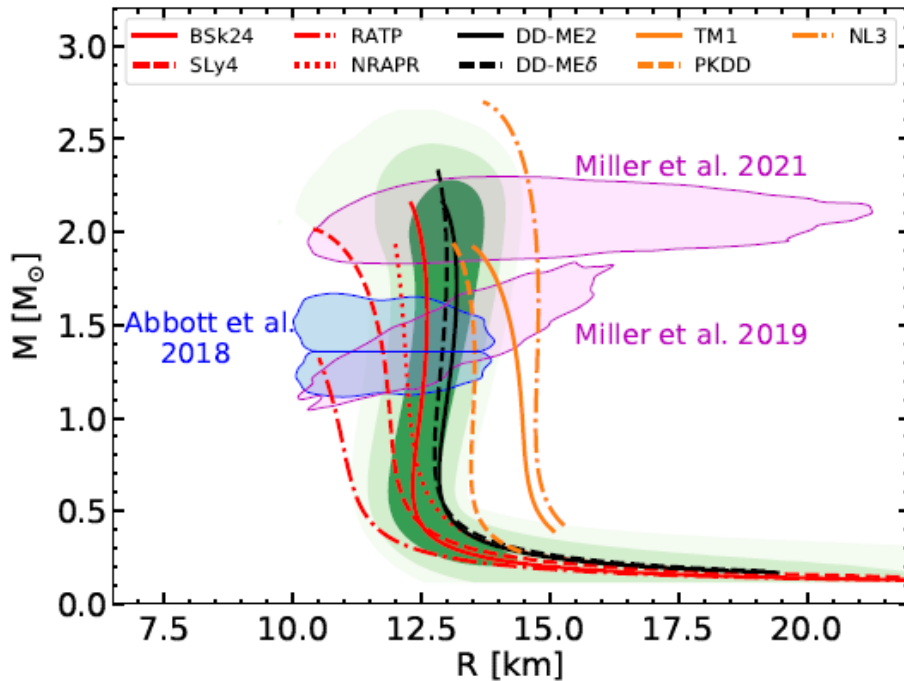
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S.Harabasz PRC102 (2020) 054903

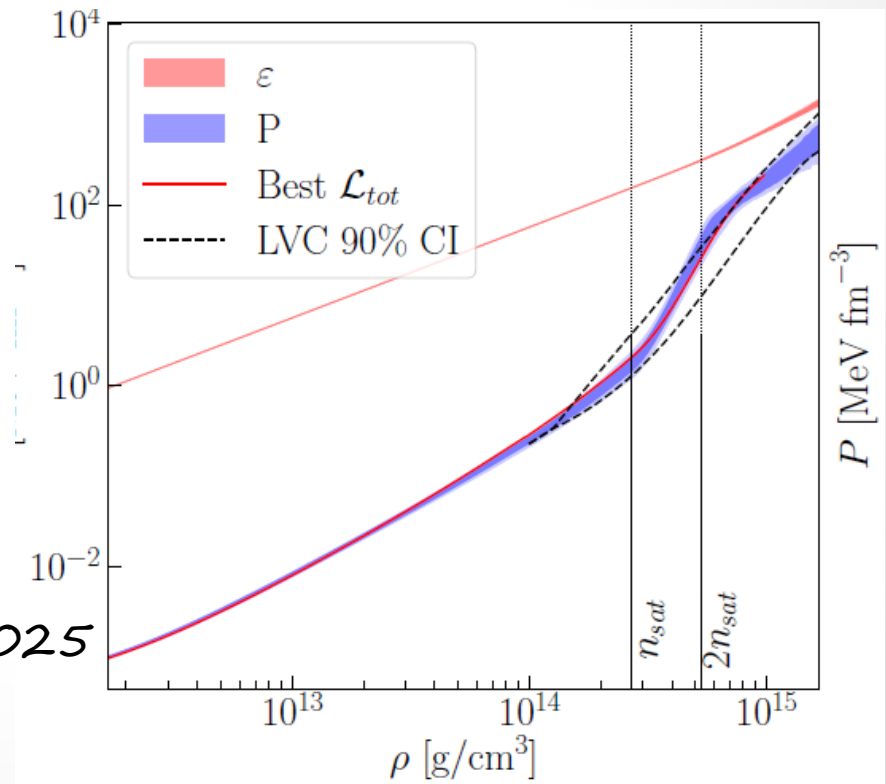


Nuclear physics informed predictions



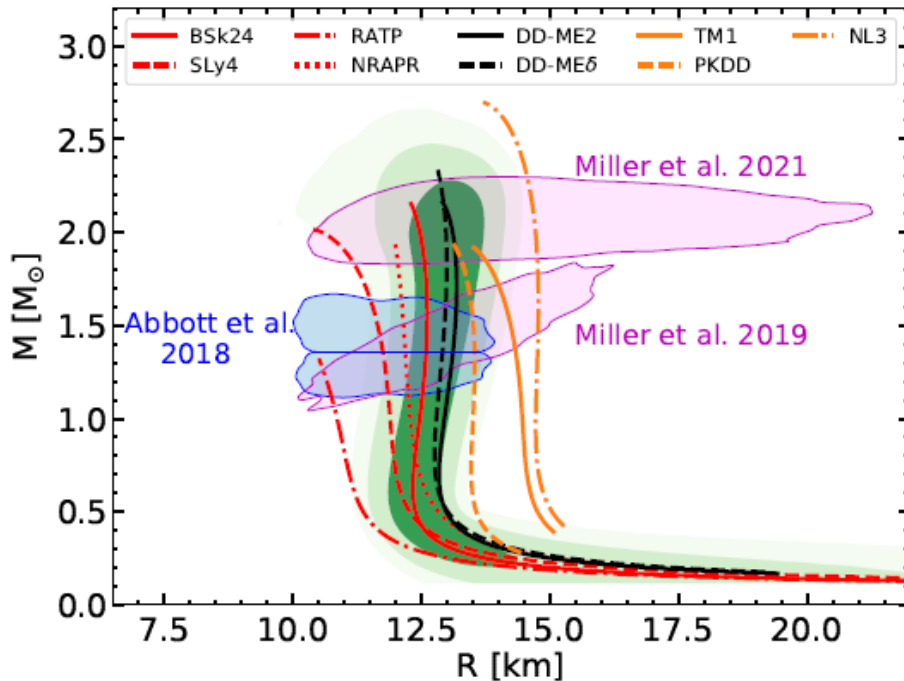
H.Dinh Thi et al, A&A 2021

- Nuclear constraints are very important up to $\sim 2n_{\text{sat}}$
- Many models can be excluded
- A neutrons and protons composition is compatible with the observations



P.Klausner et al, ArXiv 2025

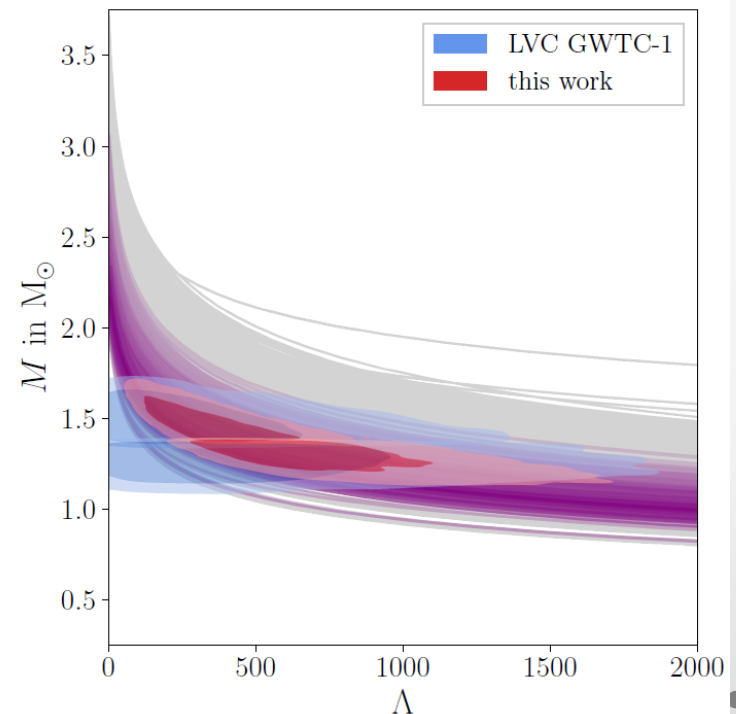
Nuclear physics informed predictions



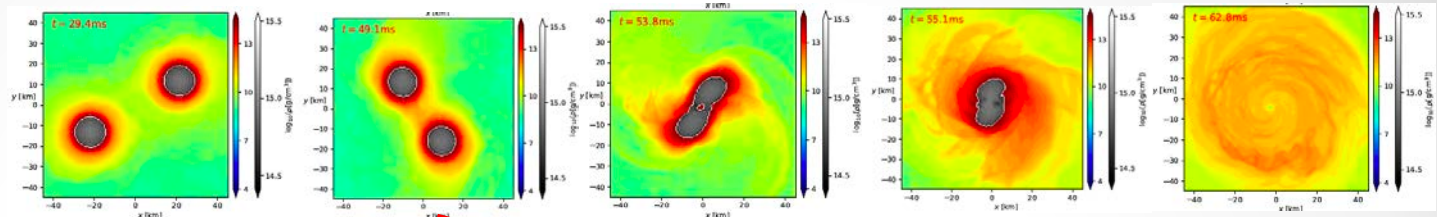
H.Dinh Thi et al, A&A 2021

H.Koehn et al, PRX 2025

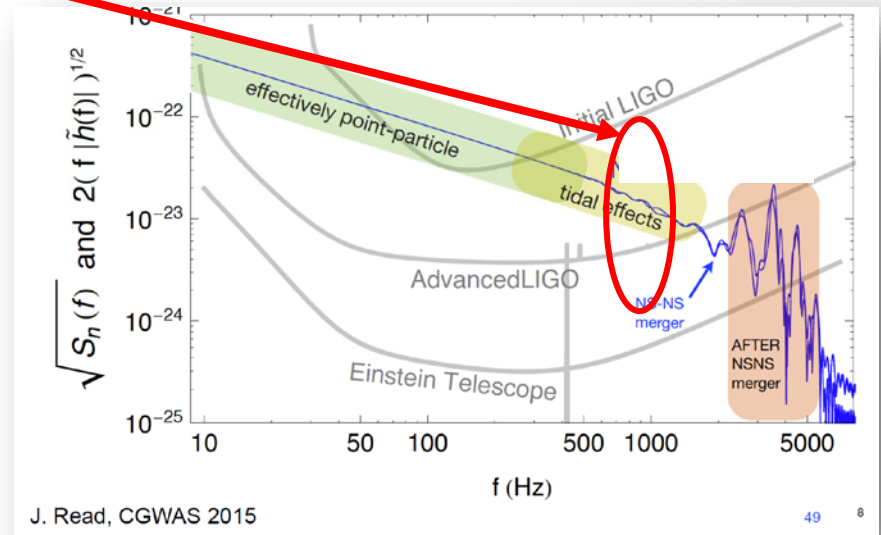
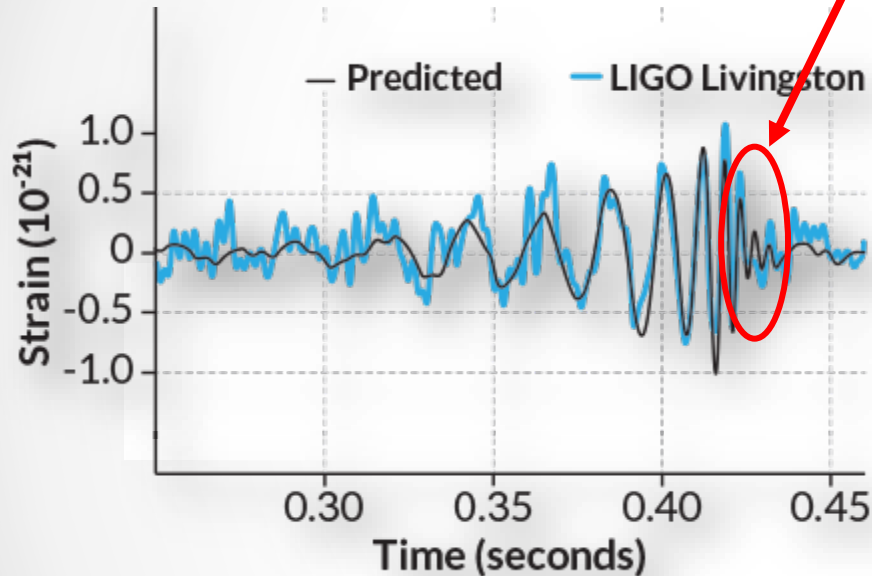
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Extraction of the tidal effects



Gieg et al., Particle 2, 23 (201)



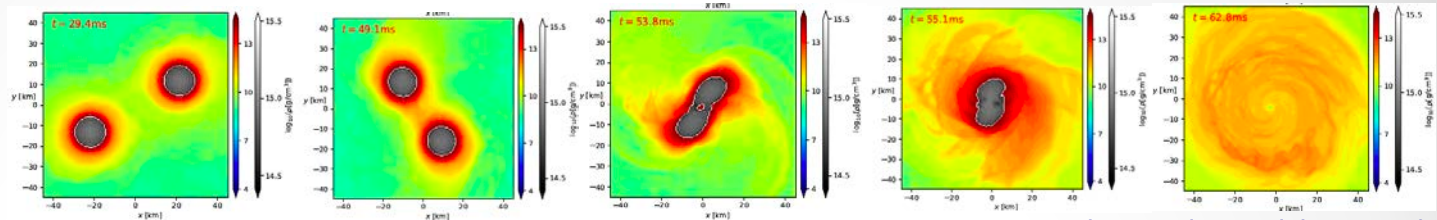
J. Read, CGWAS 2015

49 8

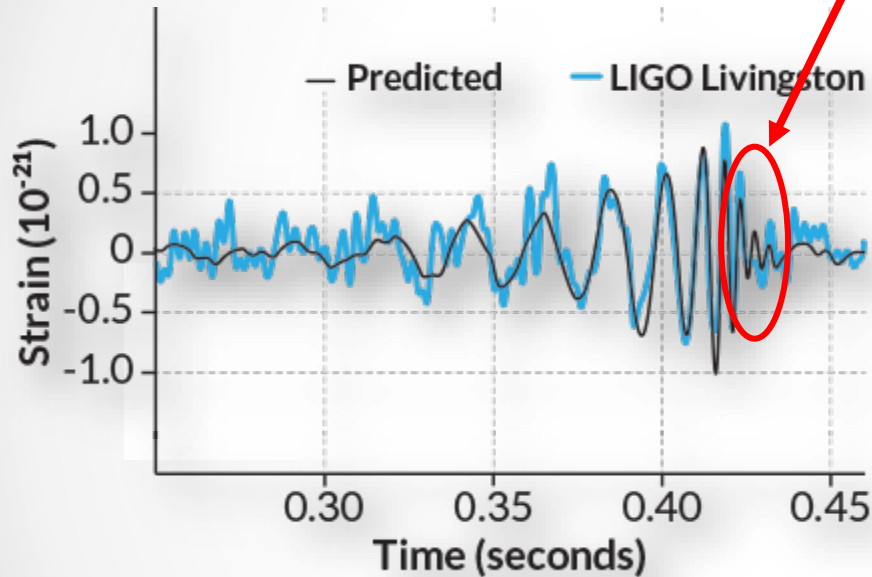
<https://www.ligo.org/detections.php>

Tidal effects: modification of the gravitational waveform wrt a GR point-particle calculation, due to the star deformation just before merging

Extraction of the tidal effects



Gieg et al., Particle 2, 23 (201)



$$\delta\psi_{\text{tidal}} = \frac{3}{128\eta x^{5/2}} \left[\left(-\frac{39}{2} \tilde{\Lambda} \right) x^5 \right]$$

Quadrupole
moment

Companion
tidal field

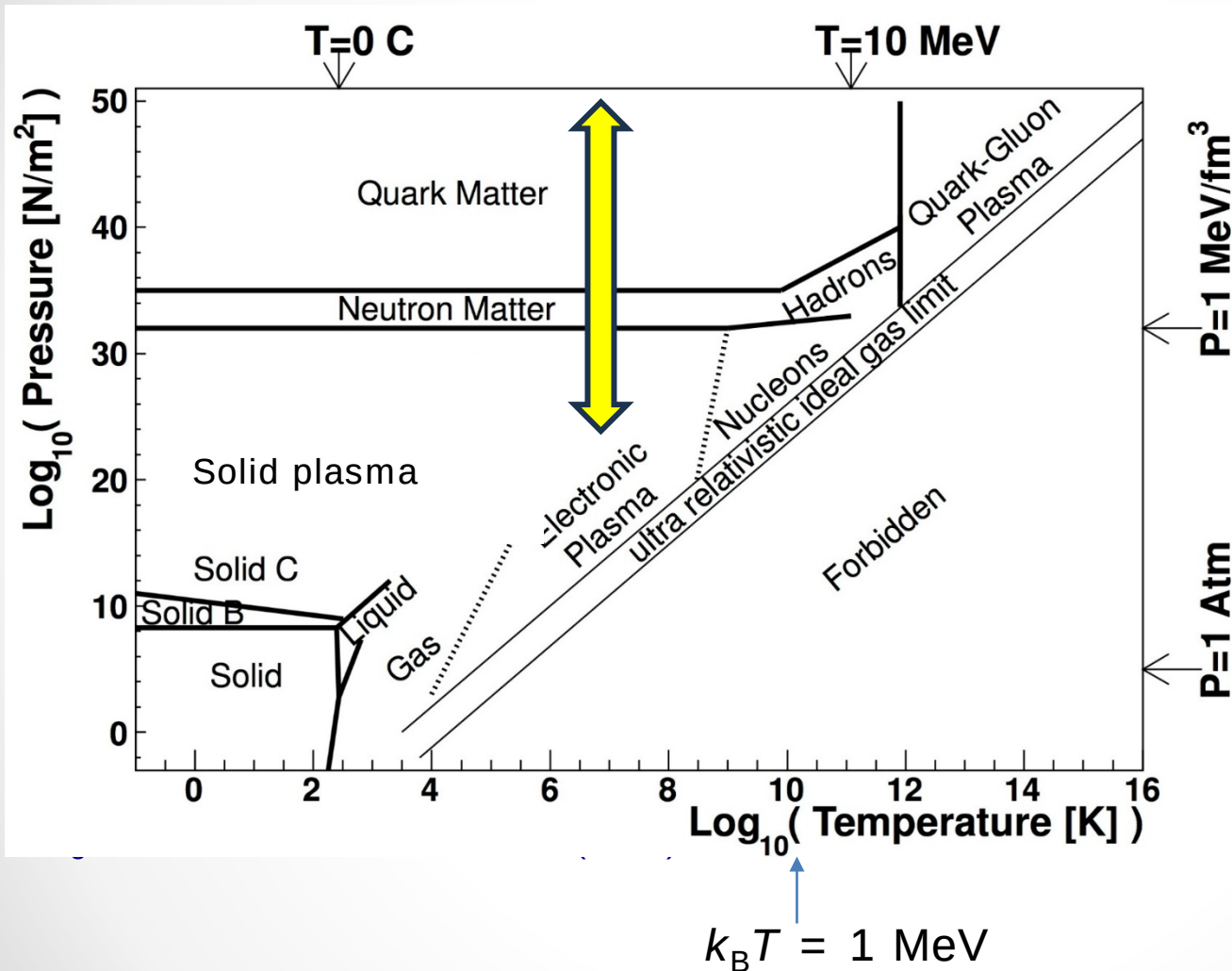
$$\bar{Q}^{(i)} = -\Lambda_i \bar{\mathcal{E}}^{(j)}$$

LIGO
Livingston

<https://www.ligo.org/detections.php>

Tidal effects: modification of the gravitational waveform wrt a GR point-particle calculation, due to the star deformation just before merging

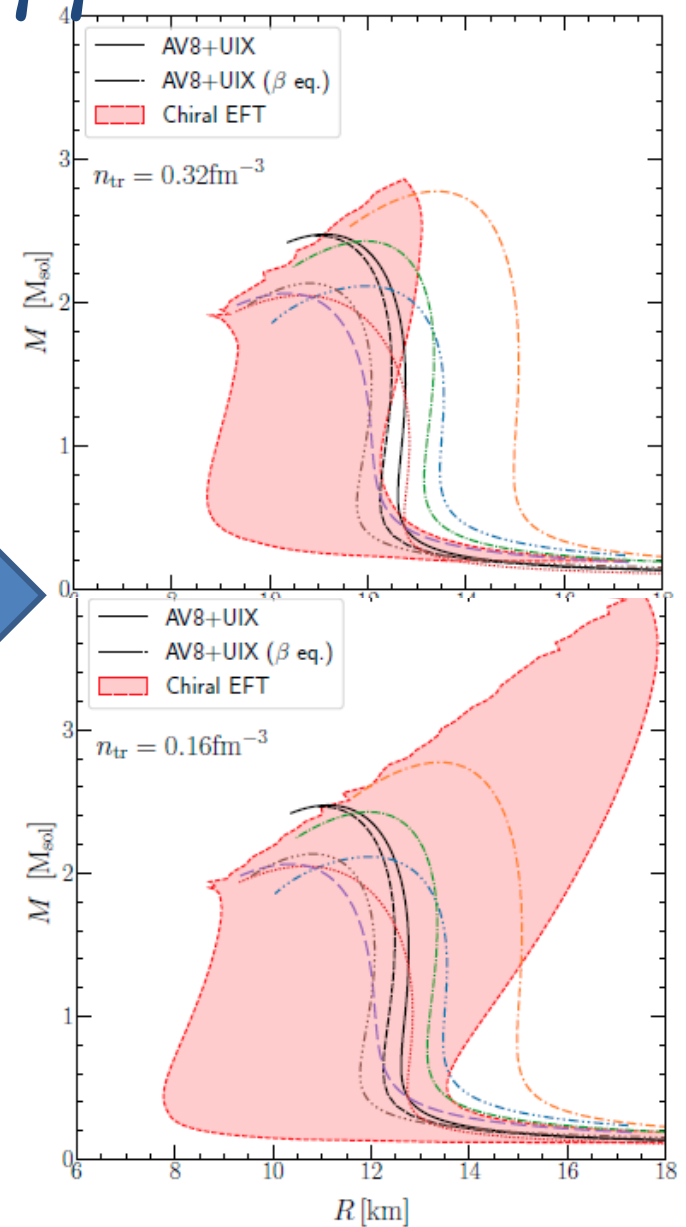
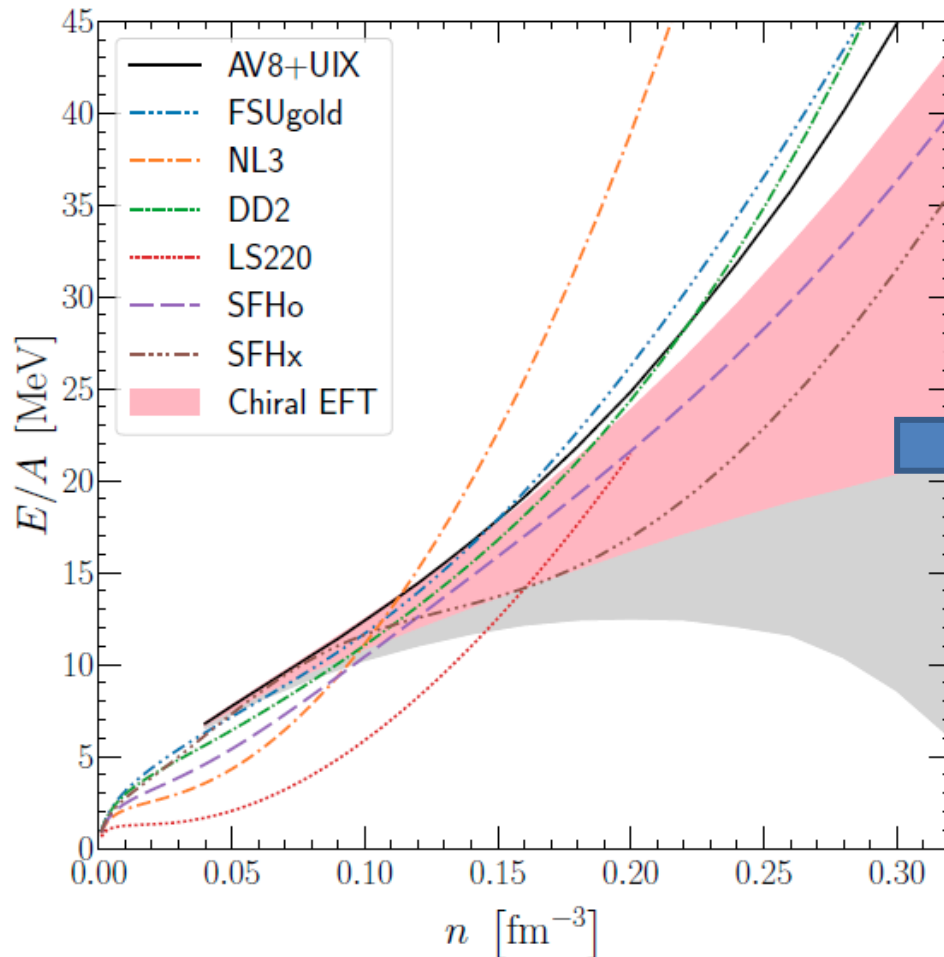
The phases of matter



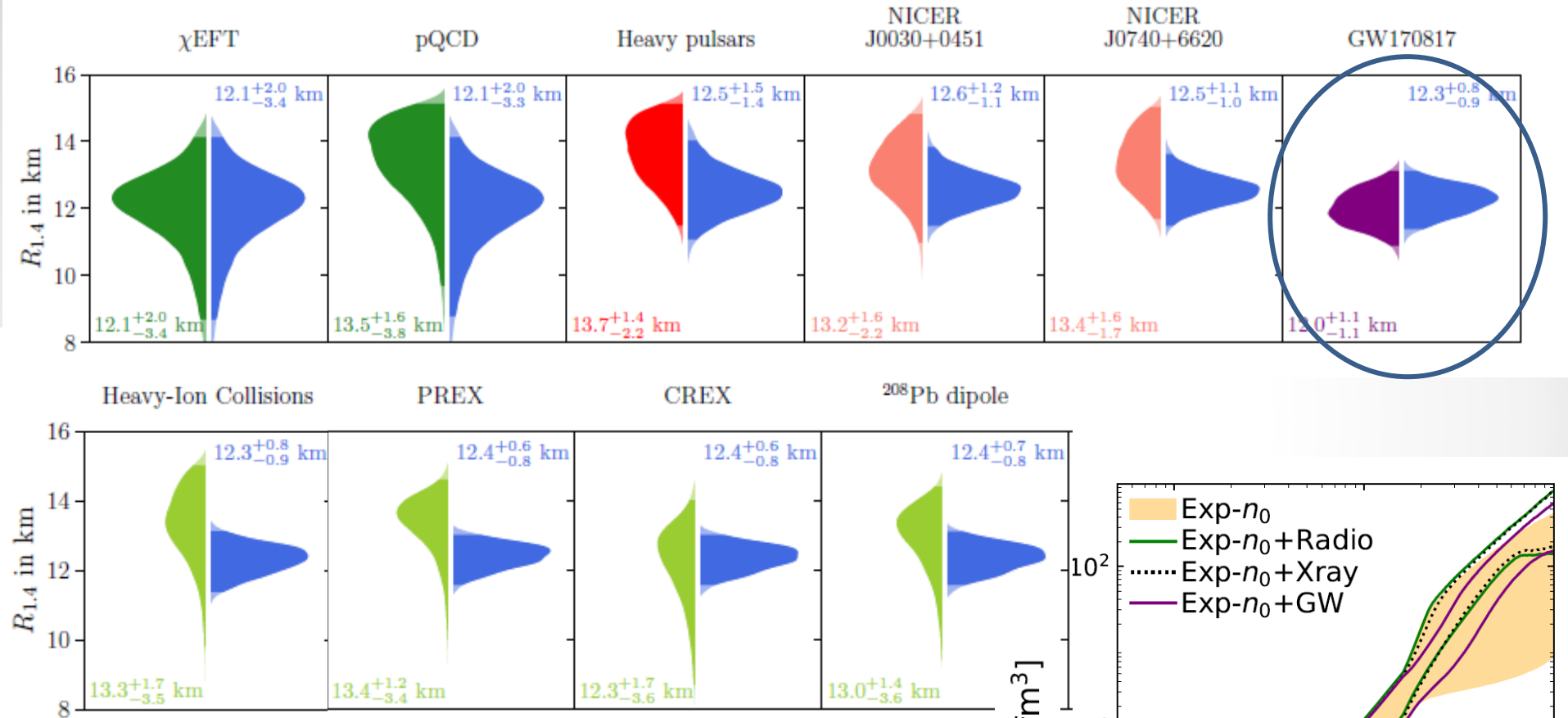
Where can ultra-dense matter be found in nature?

Nucleonic « ab-initio » approach

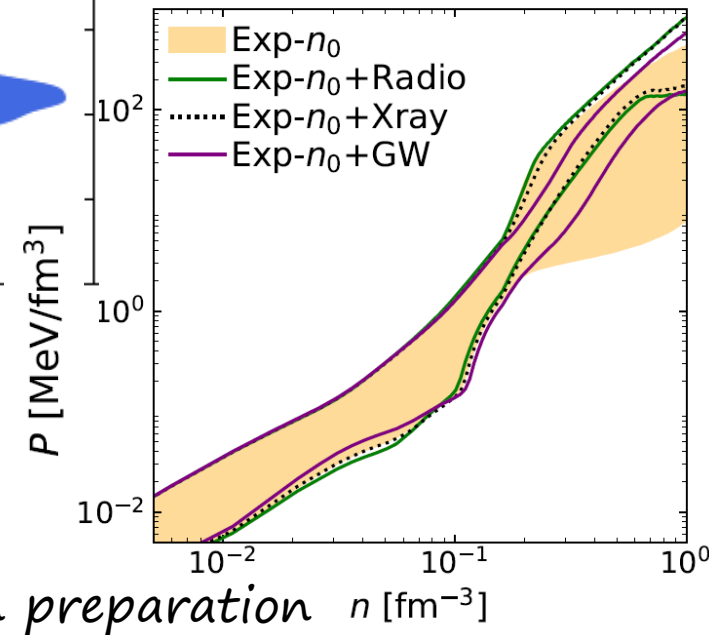
Tews, Carlson, Gandolfi, Reddy 2018



Who determines what?



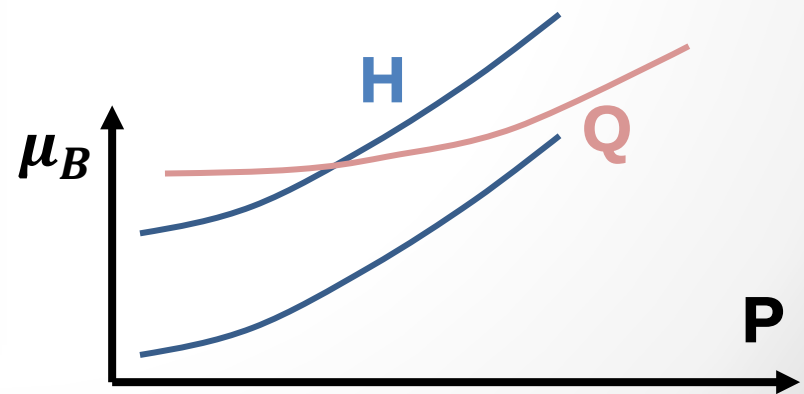
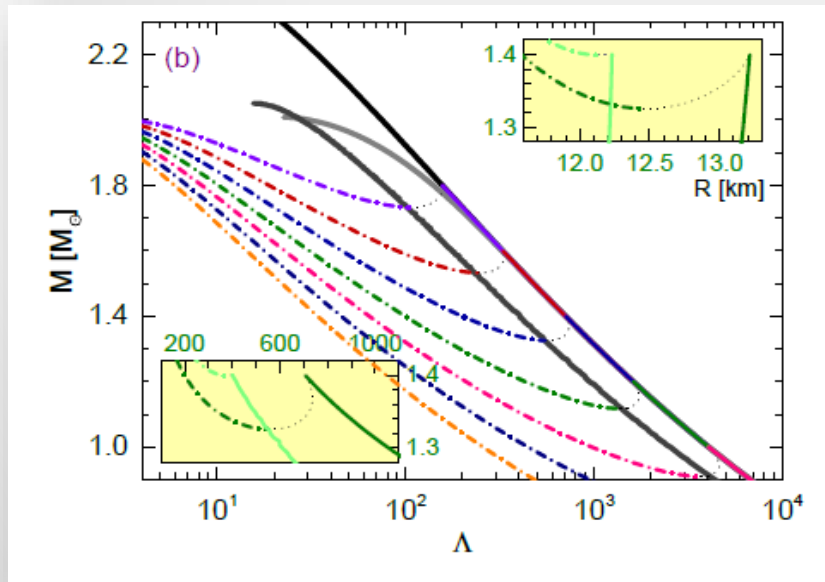
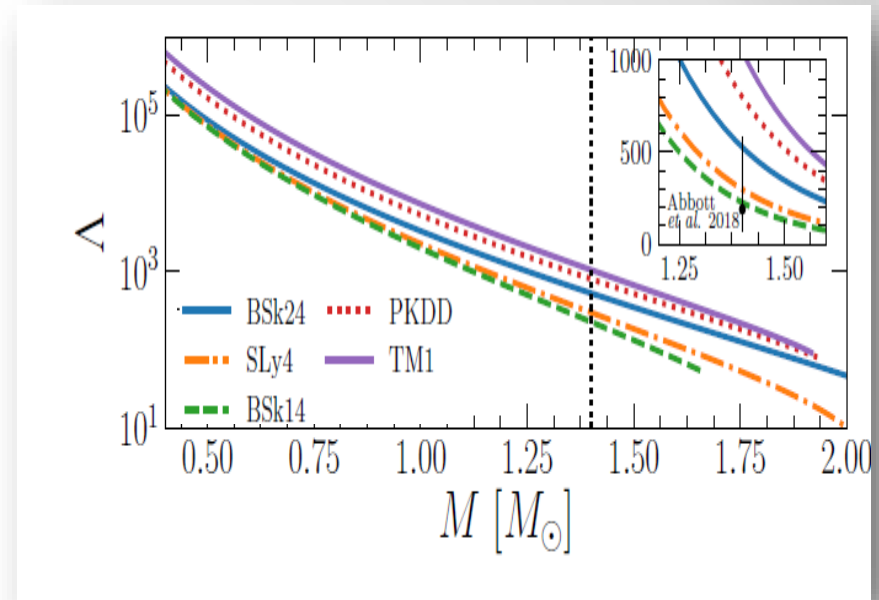
H.Koehn et al, PRX 2025



L.Suleiman et al, in preparation n [fm⁻³]

Problem II : no ab-initio theory of dense matter

- Hybrid stars: a first order phase transition between a nucleonic (RMF, Skyrme..) and a quark ((p)NJL, Bag, CSS...) EoS
- Huge uncertainties on both the Q and the H side!!



J.J.Li, A.Sedrakian, M.Alford,
PRD101 (2020) 063022