



MANITOU  
summer school

# Ground based GW detectors

Walid Chaibi

# French-Italian-(Dutch) ground based Gravitational wave detector

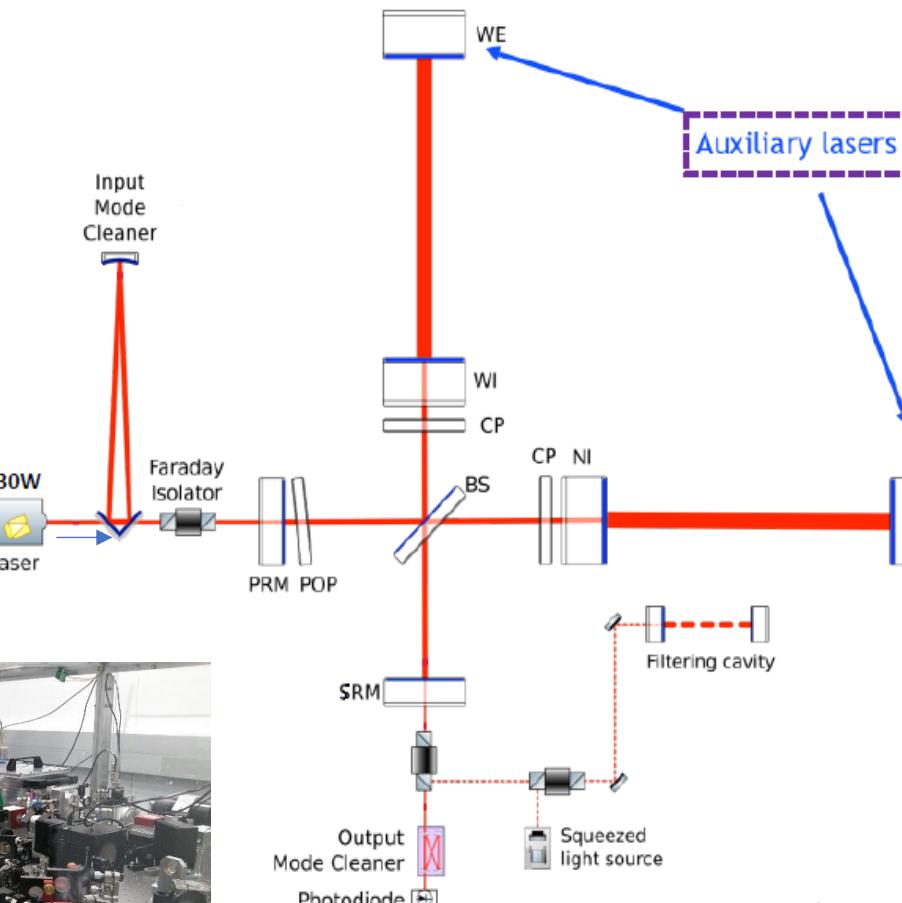


Cascina-Pisa-Italy

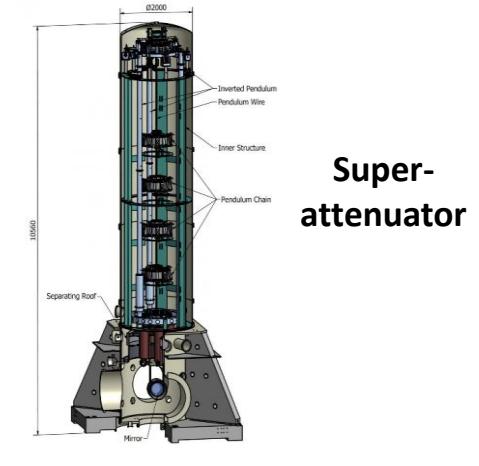


High power laser

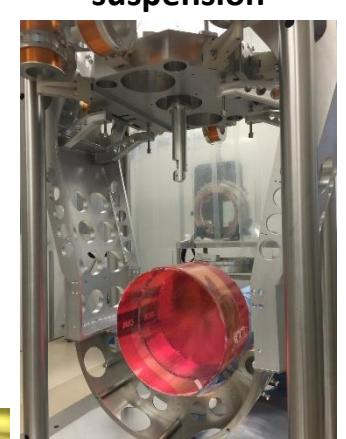
- O4 : 45W
- O5 : 80-150W
- Post-O5 : 450W
- E-T : 700W



Squeezed light source



Super-attenuator



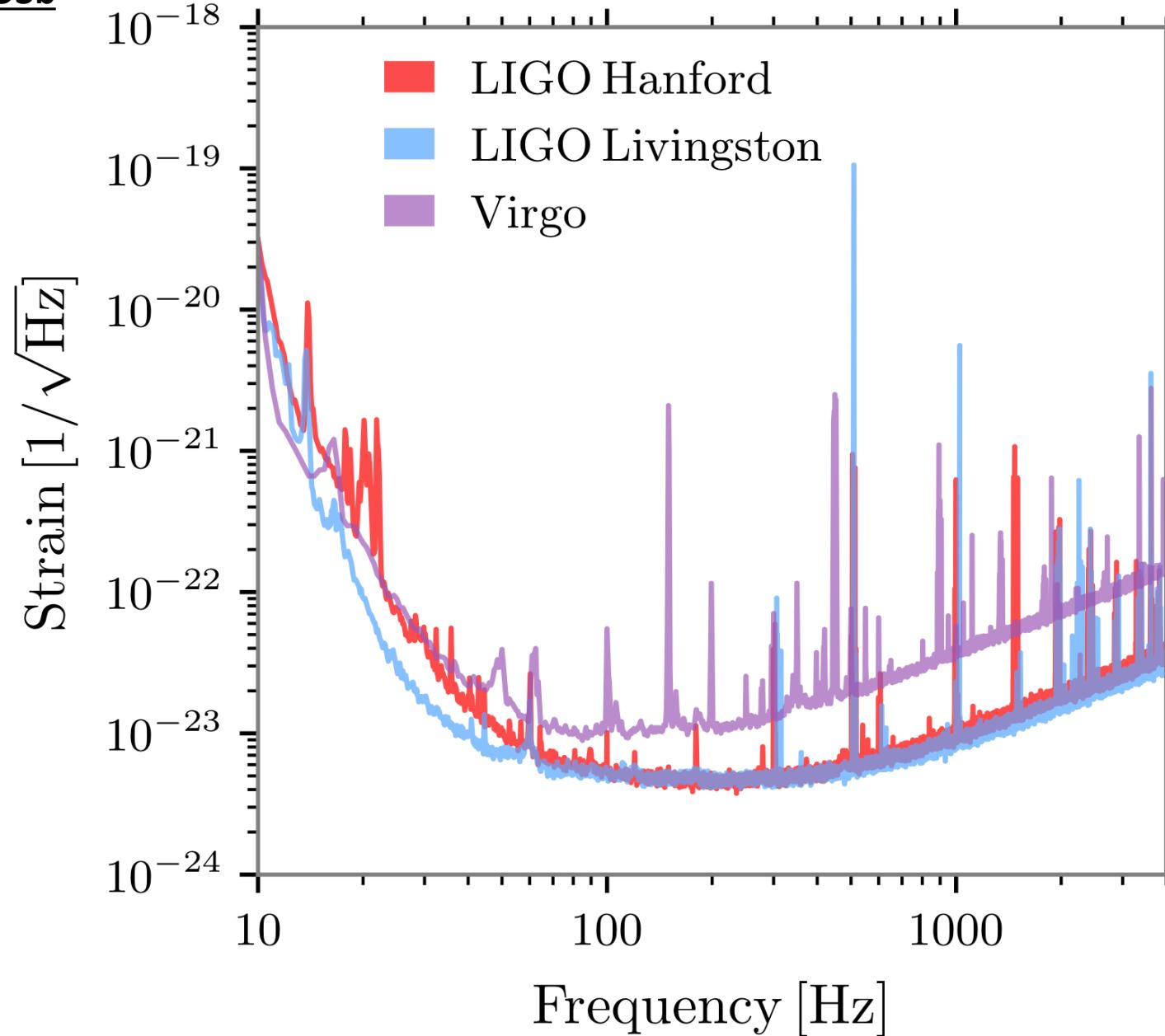
Monolithic suspension



Silicate bonding

# Sensitivity curves

Observing run: O3b



**Optical detection principle : interferometry**

**Sensitivity enhancement : The Fabry Perot cavity**

**Stable recycling cavities**

**Towards low frequency : Atom interferometry**

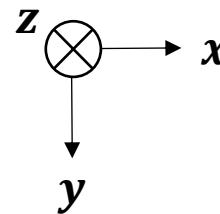
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# *GW, plane wave*



(+) polarization



(x) polarization



Space time metric in the TT gauge

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$\eta_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Minkowski

$$h_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_x & 0 \\ 0 & h_x & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

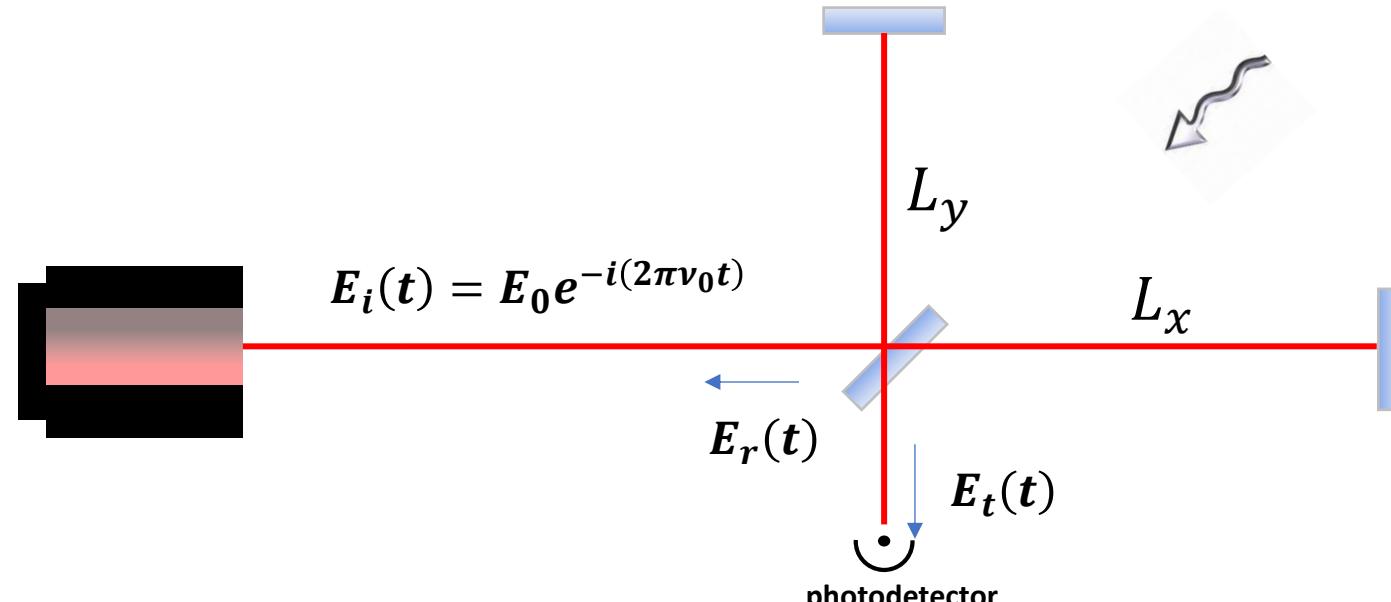
GW

Light follows the geodesic

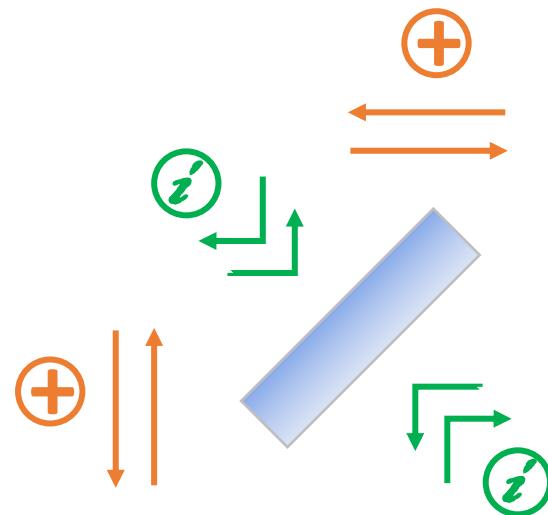
$$g_{\mu\nu} dx^\mu dx^\nu = 0$$

$$dx^\mu = (cdt, dx, dy, dz)$$

# Michelson interferometer

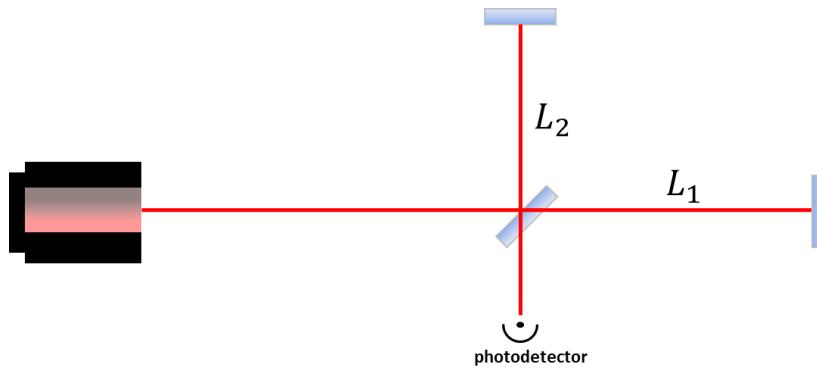


**Beam splitter 50/50**



$$E_t(t) = -\frac{1}{2} E_0 e^{-i2\pi\nu_0 t} (e^{i\phi_x} + e^{i\phi_y})$$
$$E_r(t) = \frac{i}{2} E_0 e^{-i2\pi\nu_0 t} (e^{i\phi_x} - e^{i\phi_y})$$

# Michelson interferometer : Dark fringe



$$P_t(t) \propto |E_t(t)|^2 = P_0 \times \cos^2 \left( \Delta\phi + \phi_h \cos \left( \Omega \left( t - \frac{\bar{L}}{c} \right) \right) \right)$$

Signal  $\propto \frac{Lh(t)}{\lambda}$ : increase the arm length of the interferometer

Dark fringe  $\Delta L \simeq 0, P_t \propto \left( \frac{L \times h}{\lambda} \right)^2$ : need an offset  $\Delta\phi$

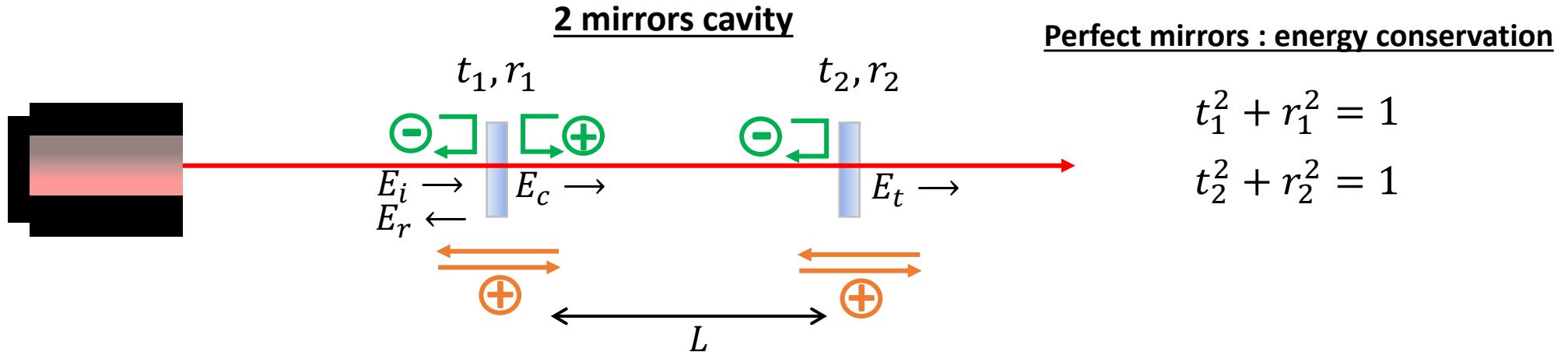
**Optical detection principle : interferometry**

**Sensitivity enhancement : The Fabry Perot cavity**

**Stable recycling cavities**

**Towards low frequency : Atom interferometry**

# Enhancement of the interferometer sensitivity: Optical Cavities



## Cavity equation

$$\left\{ \begin{array}{l} E_c(t) = t_1 E_i(t) - r_2 r_1 E_c \left( t - \frac{2L}{c} \right) \\ E_t(t) = t_2 E_c \left( t - \frac{L}{c} \right) \\ E_r(t) = -r_1 E_i(t) - r_2 t_1 E_c \left( t - \frac{L}{c} \right) \\ E_{(i,r,c,t)}(t) = E_{(i,c,r,t)0}(t) e^{-i2\pi\nu_0 t} \end{array} \right.$$

Steady state

$$E_{(i,c,r,t)0} \cancel{\rightarrow}$$



$$\left\{ \begin{array}{l} E_{c0} = t_1 E_{i0} - r_2 r_1 e^{i\frac{4\pi\nu_0 L}{c}} E_{c0} \\ E_{t0} = t_2 e^{i\frac{2\pi\nu_0 L}{c}} E_{c0} \\ E_{r0} = -r_1 E_{i0} - r_2 t_1 E_{c0} e^{i\frac{4\pi\nu_0 L}{c}} \end{array} \right.$$

# Intracavity wave

$$\phi = \frac{4\pi\nu_0 L}{c} : \text{Round trip propagation phase}$$

$$E_{c0} = \frac{t_1}{1+r_2r_1e^{i\phi}} E_{i0} = \Sigma(\phi) E_{i0}$$

$\Sigma(\phi)$ : Enhancement factor

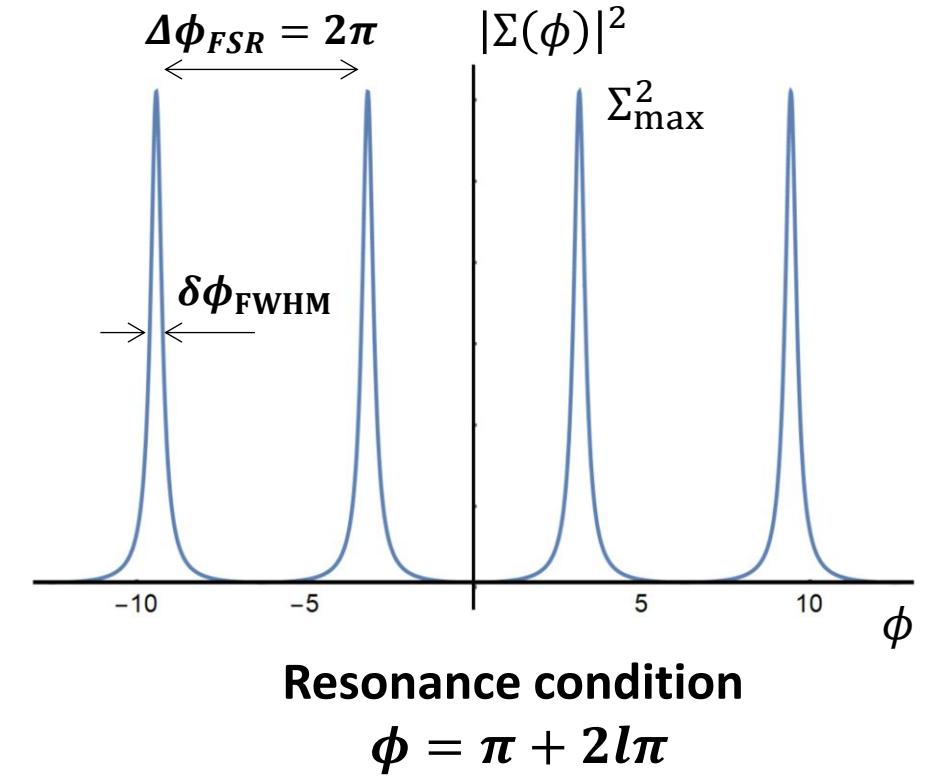
$$|\Sigma(\phi)|^2 = \frac{\Sigma_{\max}^2}{1 + \frac{4F^2}{\pi^2} \cos^2\left(\frac{\phi}{2}\right)}$$

$$\Sigma_{\max}^2 = \frac{t_1^2}{(1 - r_2r_1)^2} \underset{r_1=r_2}{\simeq} \frac{1}{t_1^2} \gg 1$$

$$F = \frac{\pi\sqrt{r_1r_2}}{1-r_2r_1} \underset{r_1=r_2}{\simeq} \frac{\pi}{t_1^2} \gg 1, F = 10 \rightarrow 10^6$$

$$\Delta\phi_{FSR} = 2\pi ; \Delta\nu_{FSR} = \frac{c}{2L} ; \Delta L_{FSR} = \frac{\lambda}{2}$$

$$\delta\phi_{FWHM} = \frac{2\pi}{F} ; \delta\nu_{FWHM} = \frac{\Delta\nu_{FSR}}{F} ; \delta L_{FWHM} = \frac{\lambda}{2F}$$



# Arm Cavity reflectivity

$$\varrho(\phi_0) = \frac{E_r}{E_i}$$

Specific configuration :  $r_2 = 1 ; t_2 = 0$

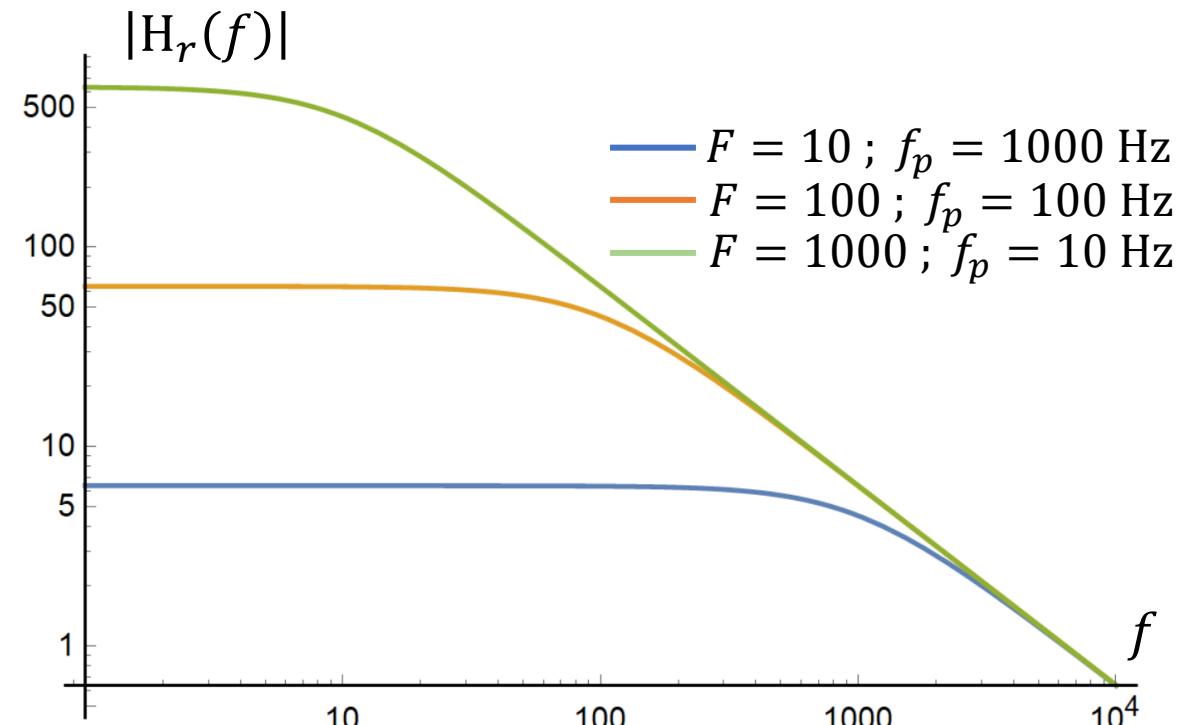
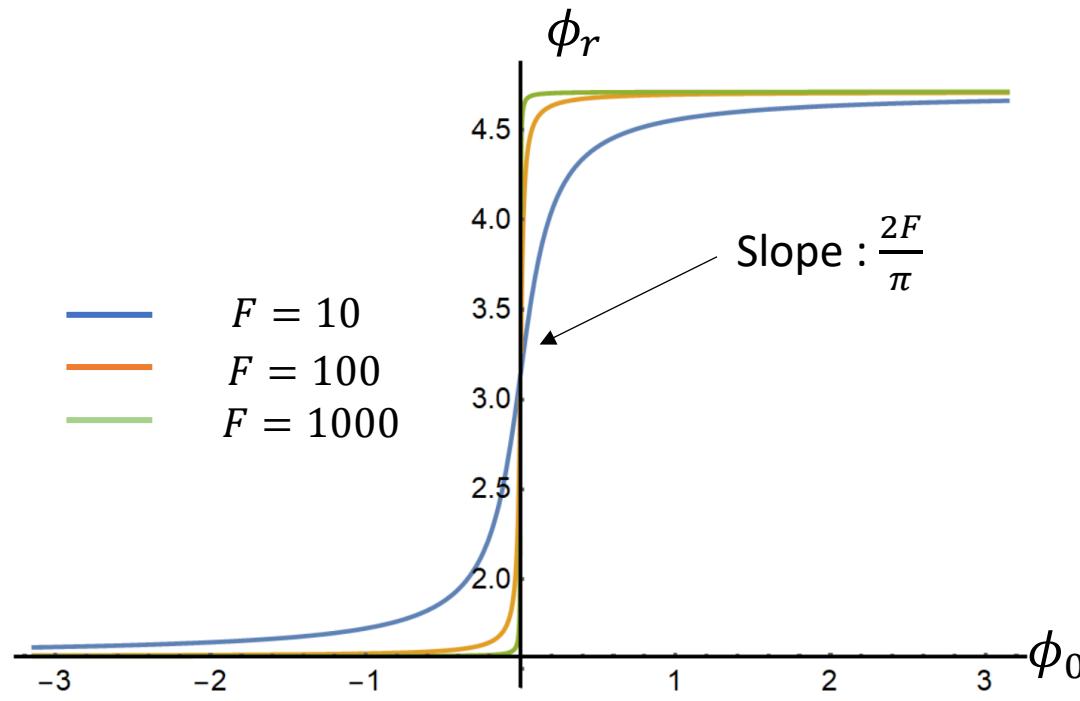
Phase without cavity

$$\tau = \frac{E_t}{E_i}; |\varrho(\phi_0)|^2 = 1$$

$$\phi_r = \text{Arg}(\varrho(\phi_0)) = \pi + \tan^{-1}\left(\frac{2F}{\pi}\phi_0\right)$$

$$H_r(f) = \frac{\phi_r}{\phi_0}(f) \simeq \frac{2F/\pi}{1+i\frac{f}{f_p}}; f_p = \frac{\delta\nu_{FWHM}}{2}$$

cavity pole  
1<sup>st</sup> order filter



**Optical detection principle : interferometry**

**Sensitivity enhancement : The Fabry Perot cavity**

**Stable recycling cavities**

**Towards low frequency : Atom interferometry**

# Gaussian beams : solution of the paraxial equation

Fundamental mode

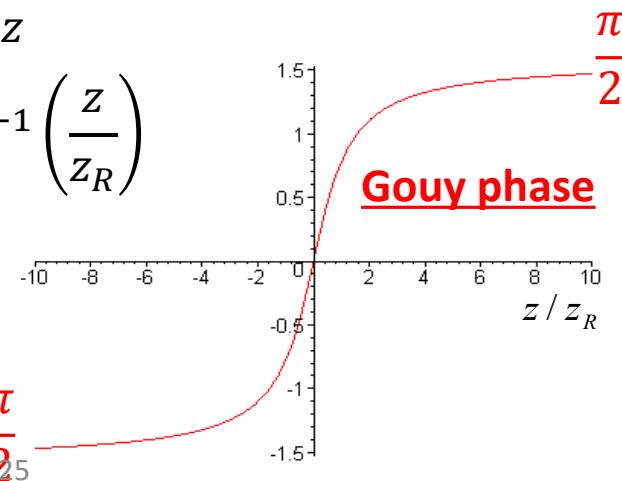
$$E(x, y, z, t) = E_0 \underbrace{\frac{1}{w(z)}}_{\text{Beam radius}} \times e^{i \left( \frac{2\pi}{\lambda} z + \psi_G(z) \right)} \times e^{-i \underbrace{\frac{2\pi}{\lambda} \frac{x^2 + y^2}{2R(z)}}_{\text{Spherical wave front}}} \times e^{-\frac{x^2 + y^2}{w^2(z)}} \quad \text{Gaussian profile}$$

$$z_R = \frac{\pi w_0^2}{\lambda} : \text{Rayleigh range}$$

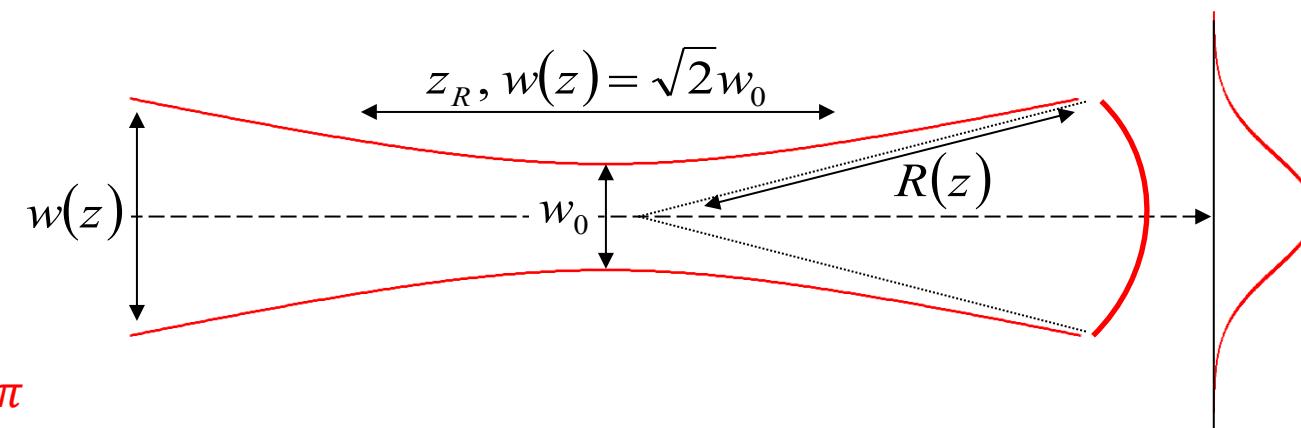
$$w(z) = w_0 \sqrt{1 + \frac{z^2}{z_R^2}}$$

$$R(z) = z + \frac{z_R^2}{z}$$

$$\psi_G(z) = \tan^{-1} \left( \frac{z}{z_R} \right)$$



09/07/2025



Resonance condition

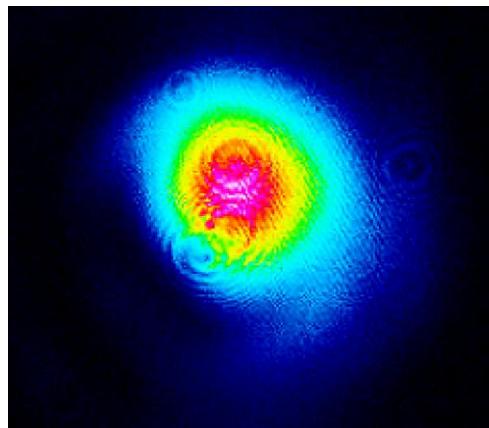
$$2\phi_{res} + 2\Delta\psi_G = \pi + 2l\pi$$

# High order modes

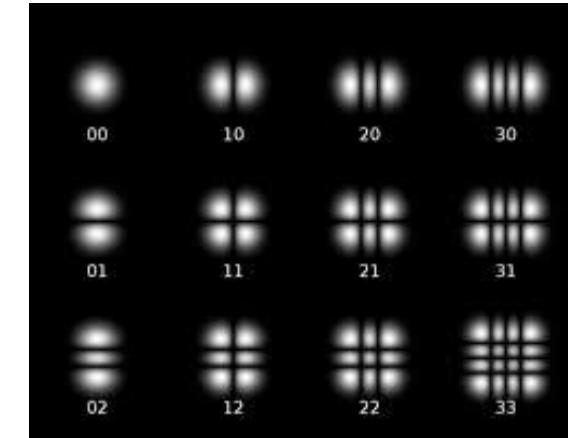
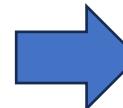
Gaussian mode : fundamental mode of a set of high order modes ; example : Hermite-Gauss modes

$$E_{n,m}(x, y, z, t) = E_{n,m,0} \frac{1}{w(z)} \times e^{i\left(\frac{2\pi}{\lambda}z + (n+m+1)\psi_G(z)\right)} \times H_m\left(\frac{\sqrt{2}x}{w(2)}\right) H_n\left(\frac{\sqrt{2}y}{w(2)}\right) e^{-i\frac{2\pi}{\lambda}\frac{x^2+y^2}{2R(z)}} \times e^{-\frac{x^2+y^2}{w^2(z)}}$$

$H_j$ : Hermite polynomial of order  $j$

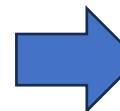


decomposition

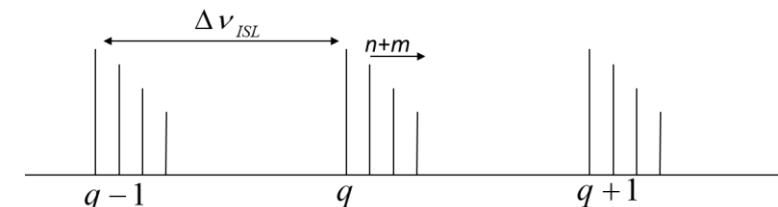


**Resonance condition**

$$\phi_{n,m,res} + 2(m+n+1)\Delta\psi_G = \pi + 2l\pi$$



**depends on the mode order**

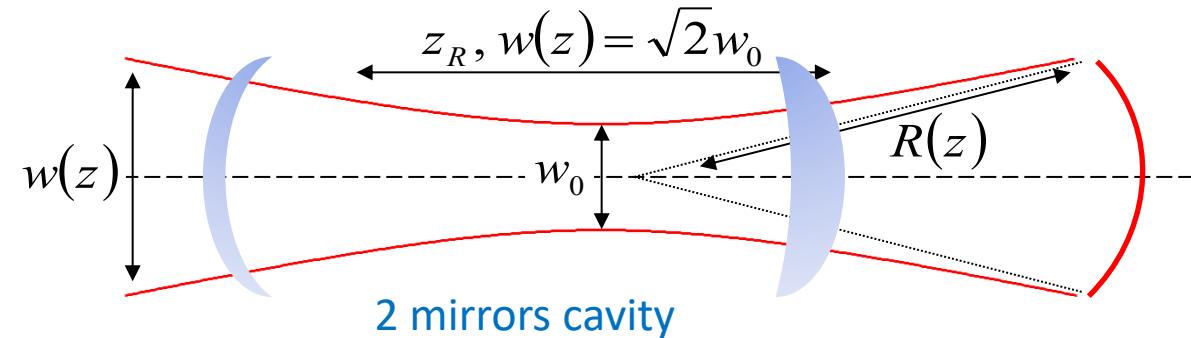


# Optical cavity

Spherical wavefront



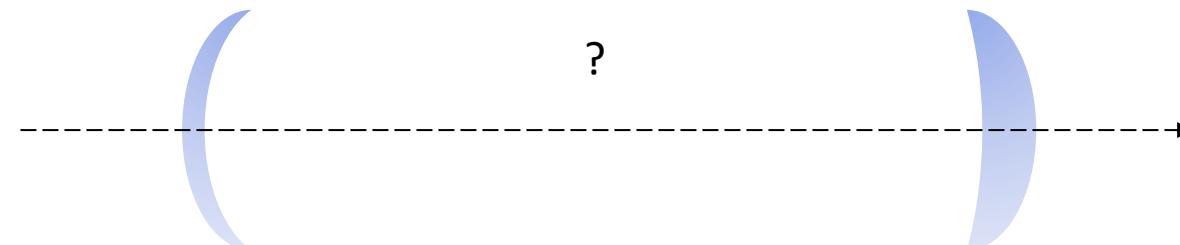
Spherical mirrors



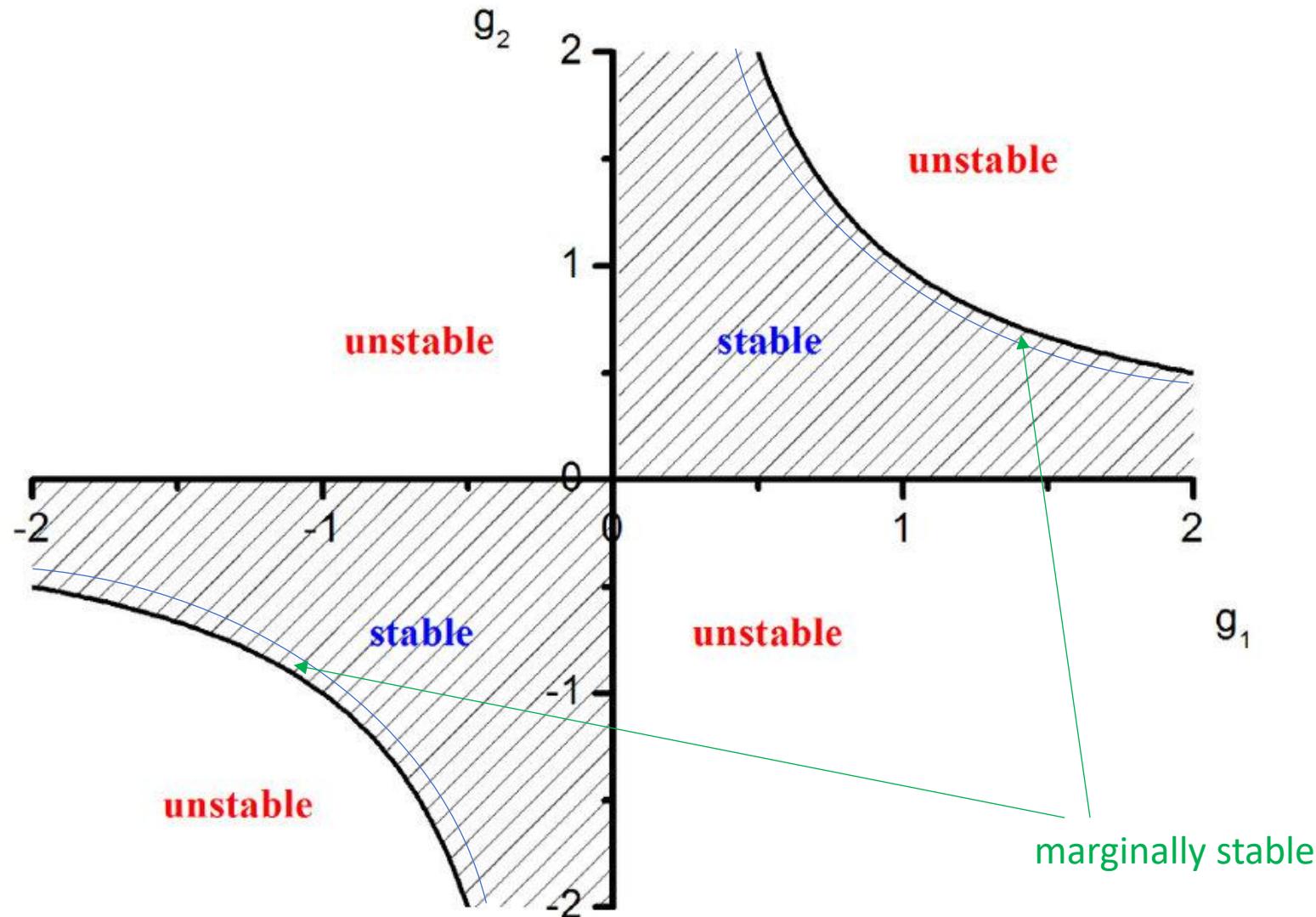
We set two mirrors



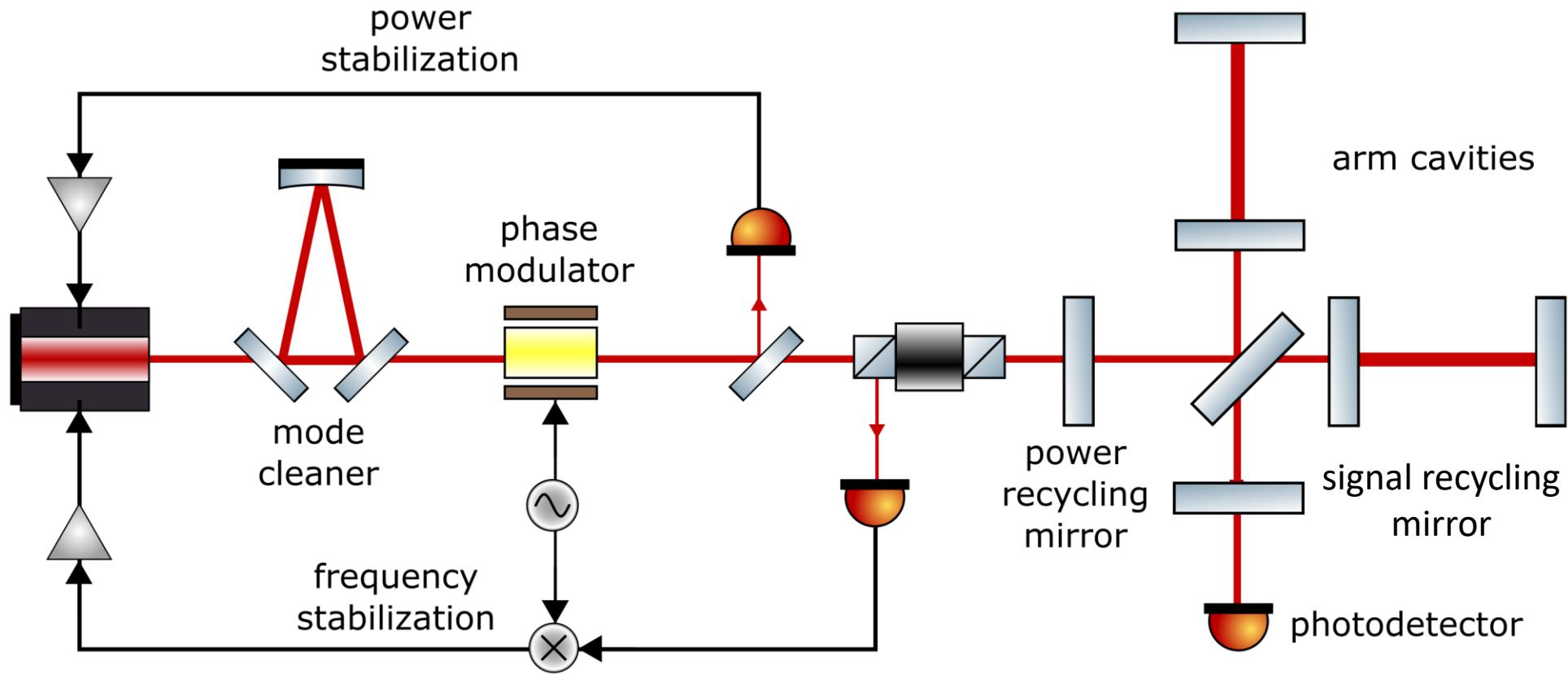
Which Gaussian beam can fit inside the cavity (mode of the cavity)? Is there a solution? Unicity?



# Stability diagram

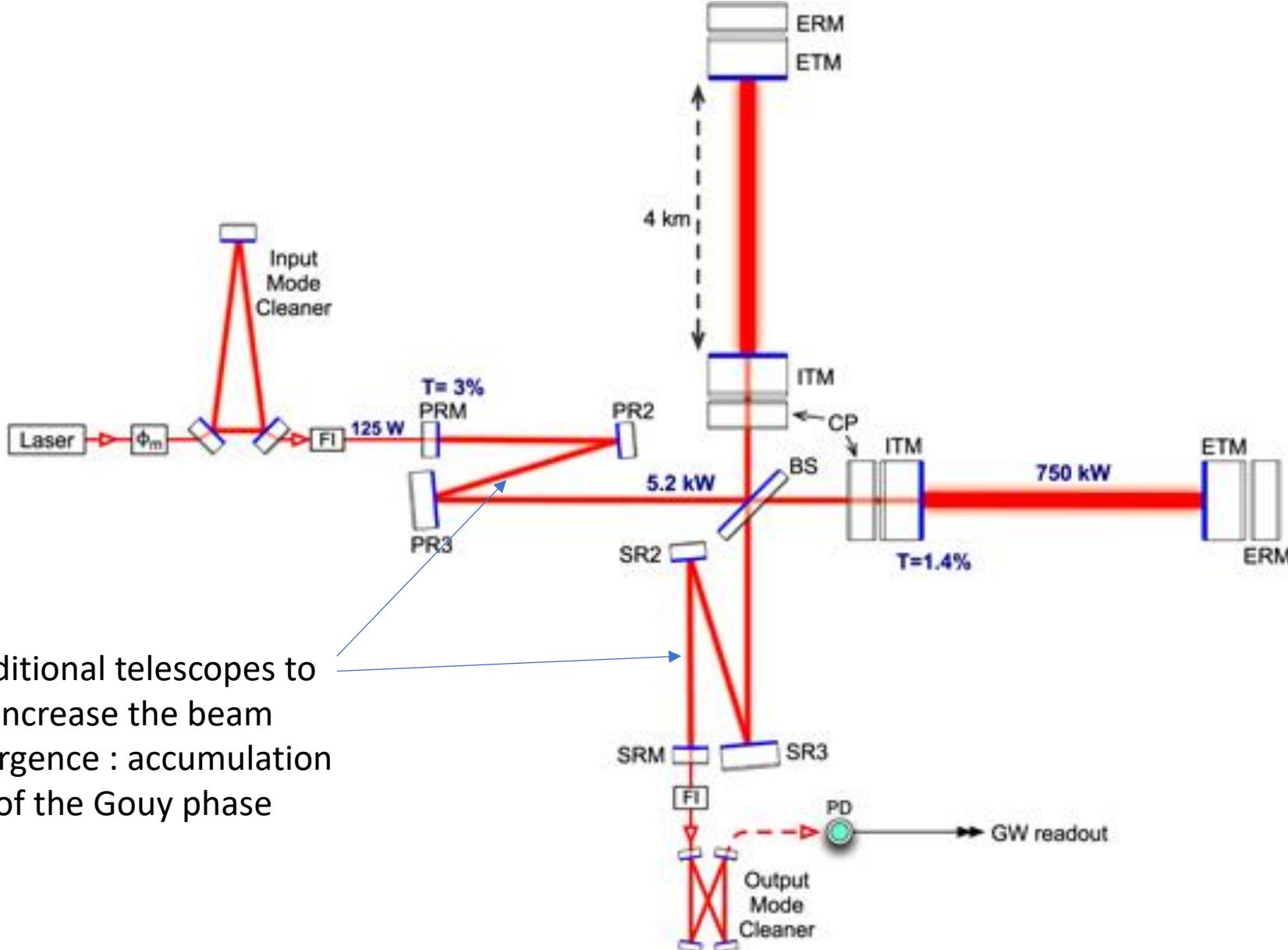


# Marginally stable recycling cavities



$$P_0 = 25 \text{ W} ; G = 50 ; P_{PD} = 1 \text{ mW} ; F = 400 ; h = 2 \times 10^{-23} \text{ Hz}^{-1/2} @ 100 \text{ Hz}$$

# stable recycling cavities : LIGO



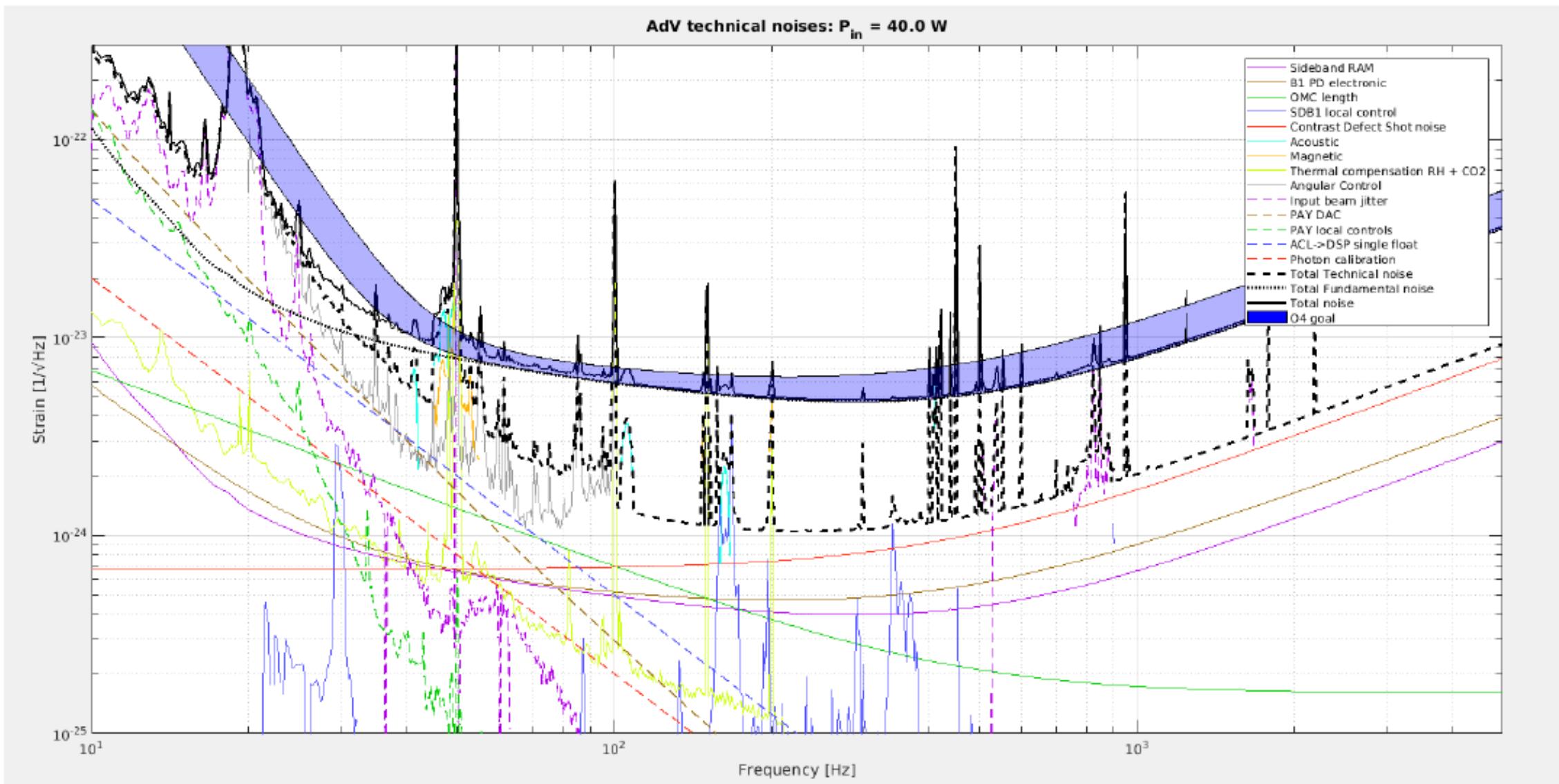
**Optical detection principle : interferometry**

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**Towards low frequency : Atom interferometry**

# A lot of noise



Thermal compensation system

Controls

High power laser

Electronics

Vacuum

Parametric instabilities

Newtonian noise

Squeezing, dependent and independent

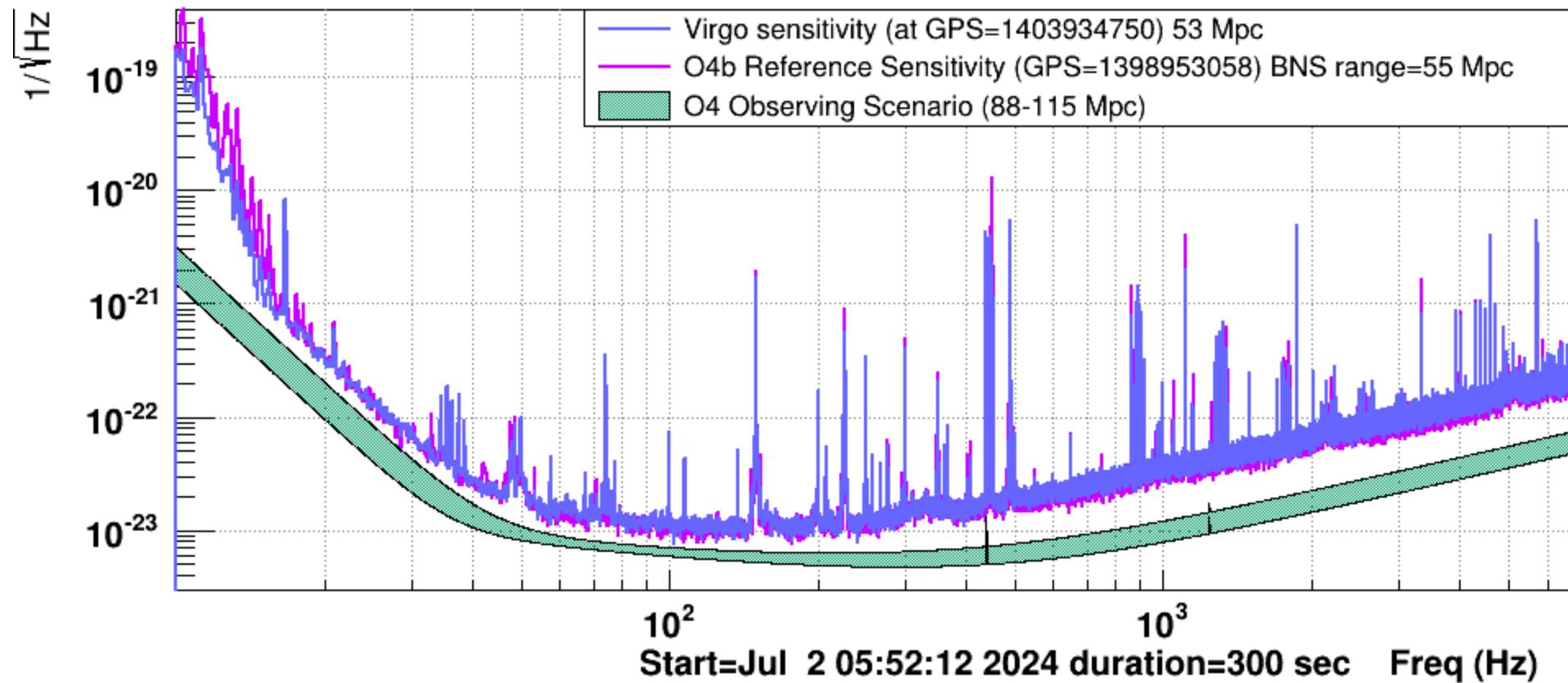
Calibration

Coatings

Simulations

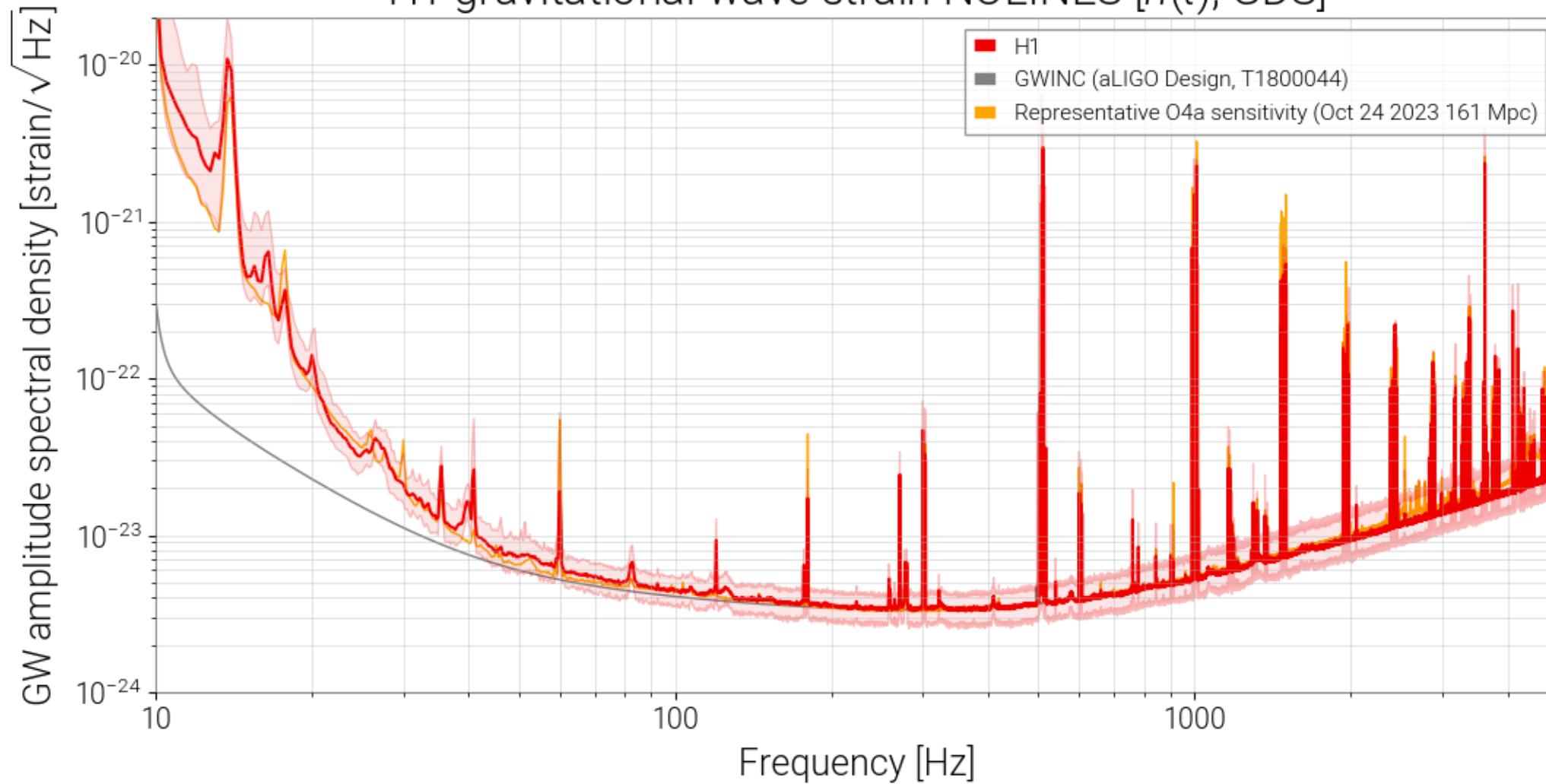
....and R&D

## Last Sensitivity (Tue Jul 2 05:52:12 2024 UTC)



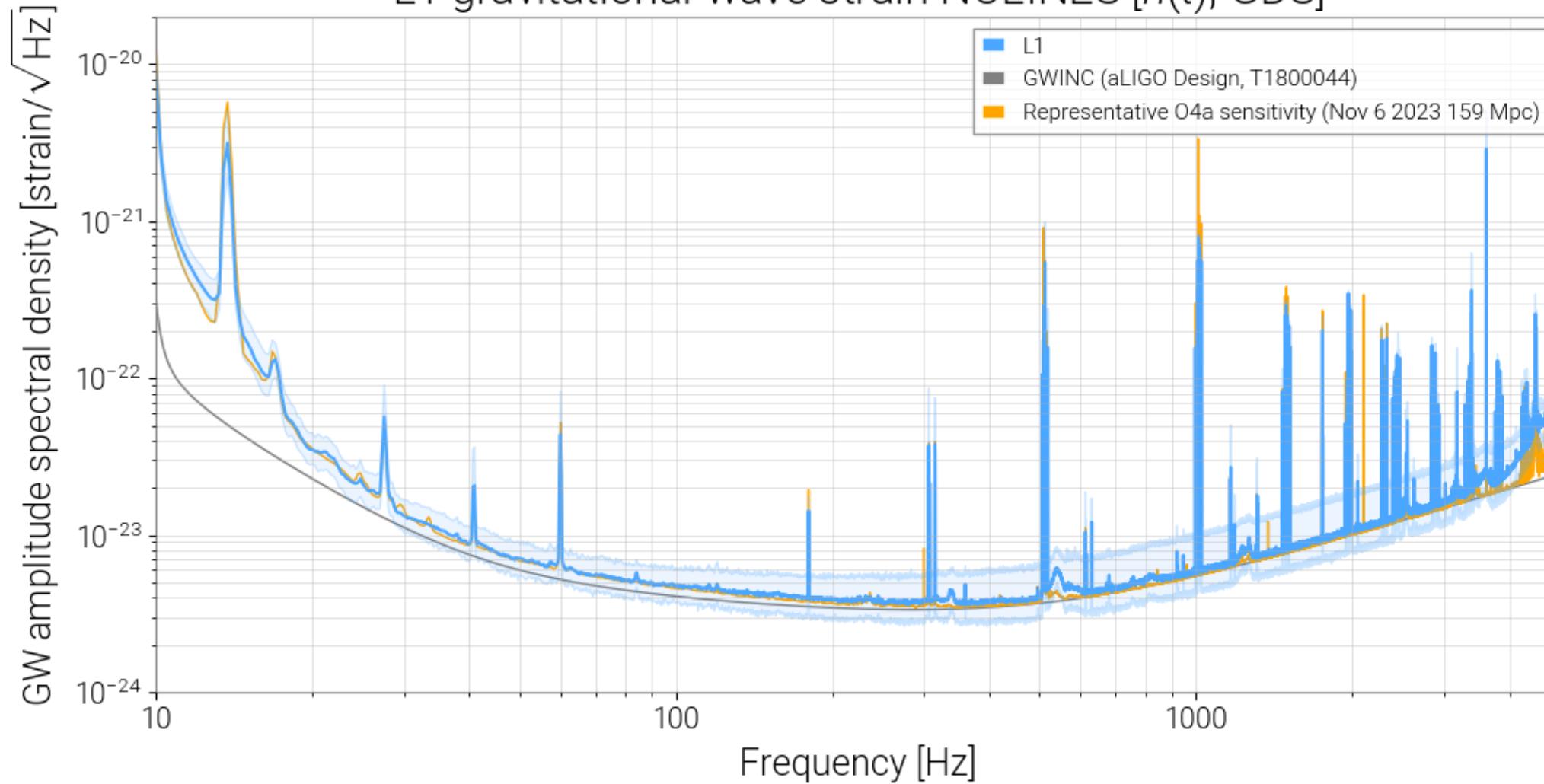
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## H1 gravitational-wave strain NOLINES [ $h(t)$ , GDS]

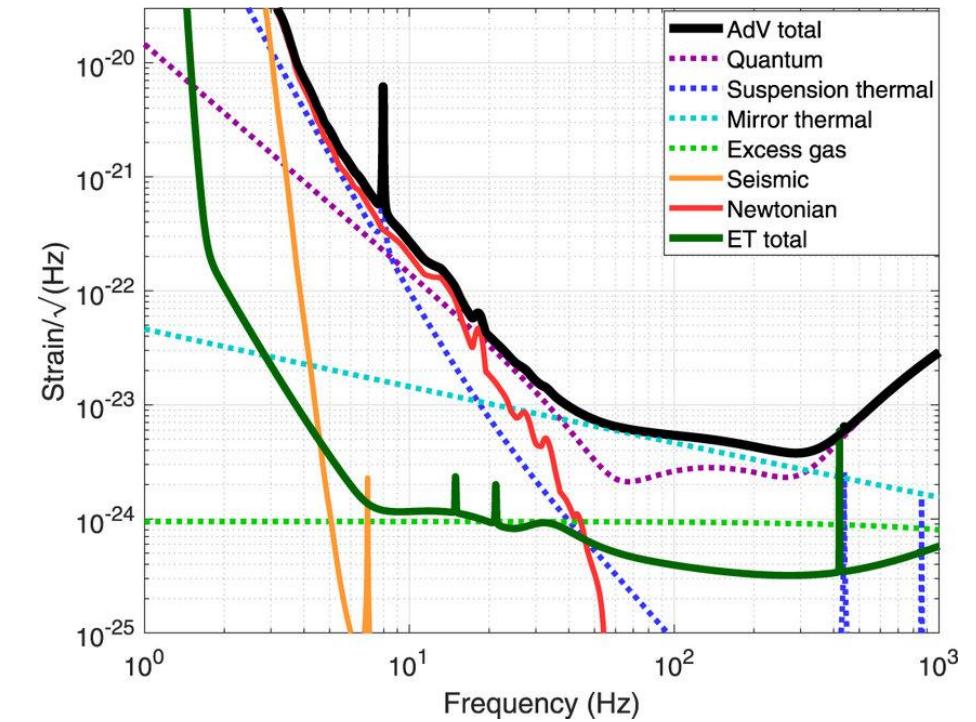
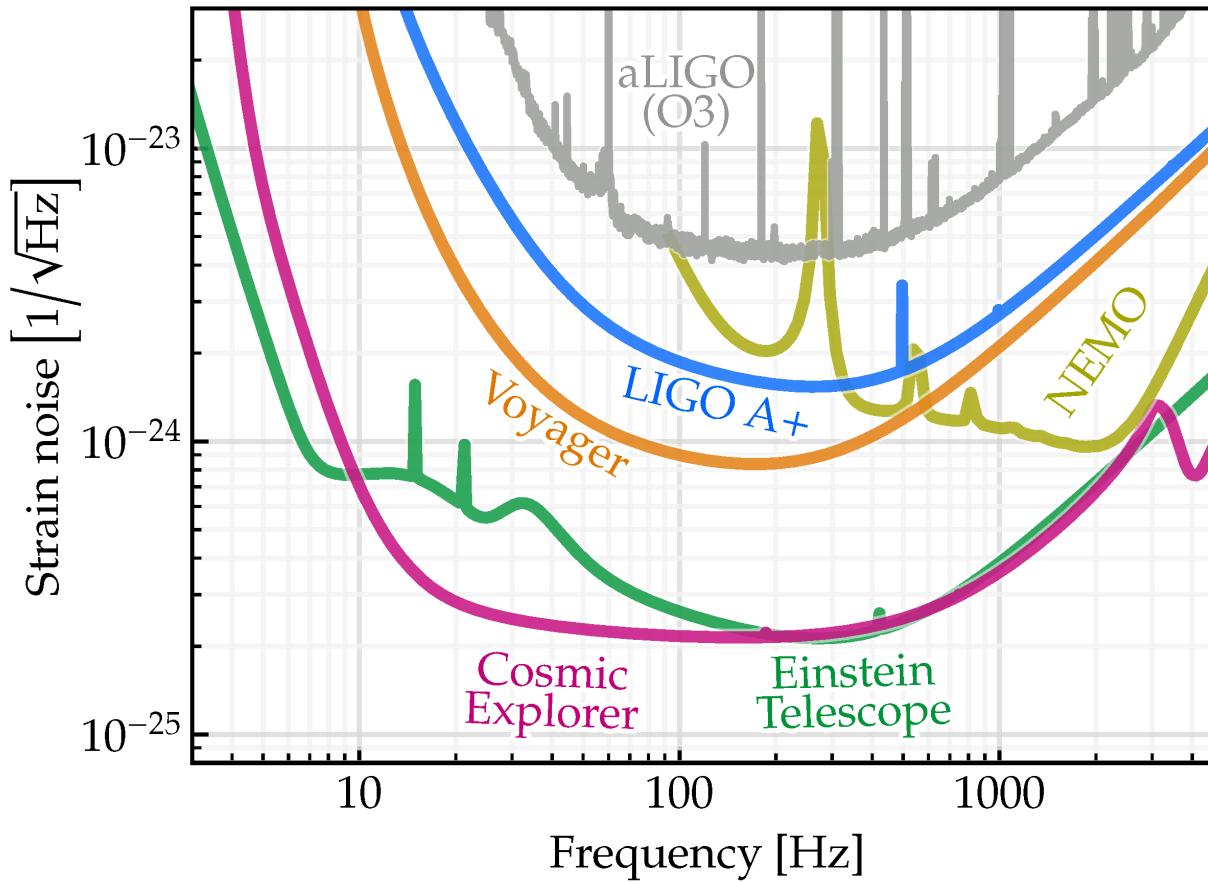


[1403913618-1404000018, state: Locked]

## L1 gravitational-wave strain NOLINES [ $h(t)$ , GDS]

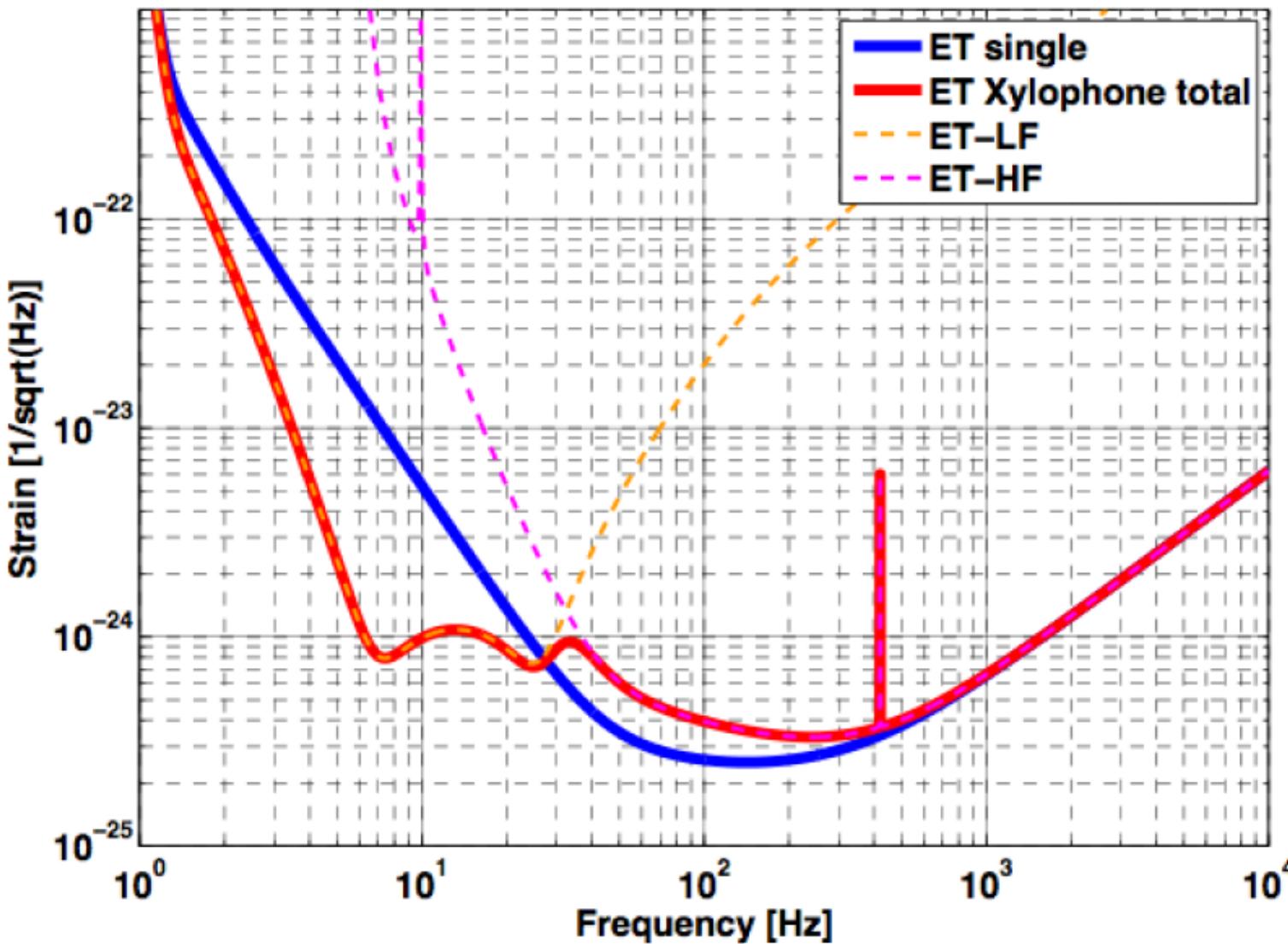


## Third generation detectors



→ Towards low frequency : technological issues

# Third generation detectors : The Einstein Telescope



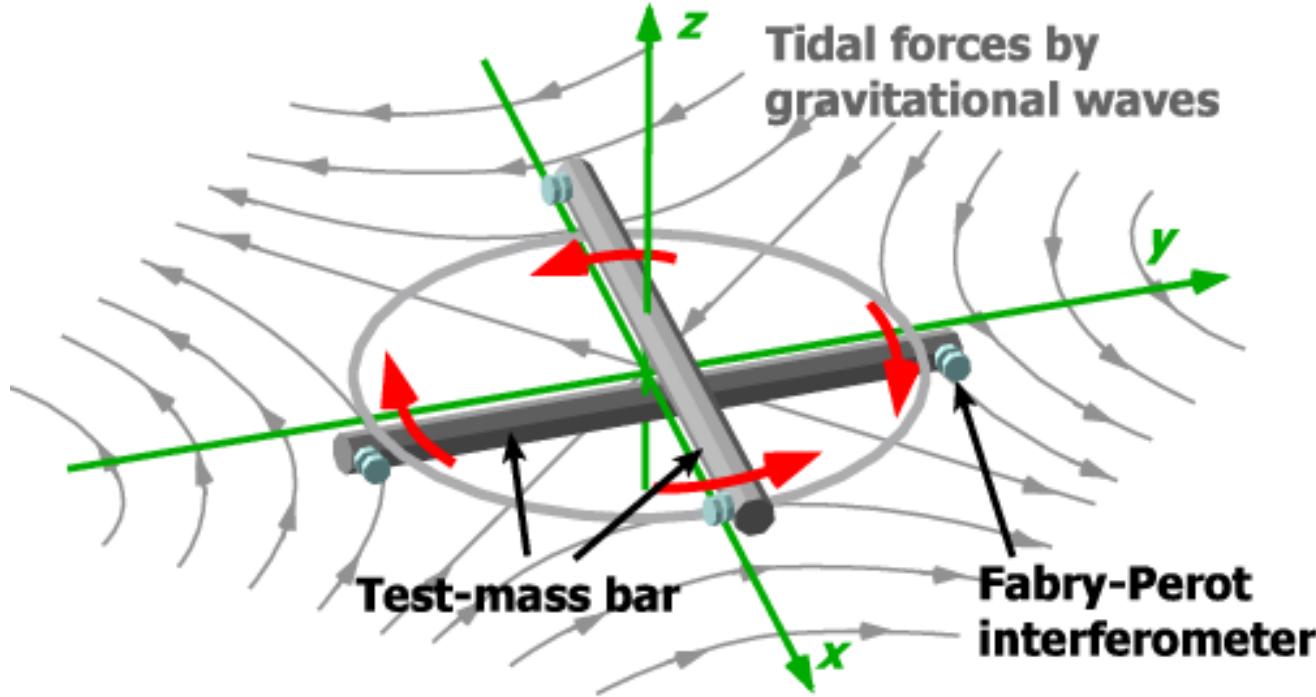
ET-HF :

- Same wavelength : 1064nm
- High power : 700W
- Squeezing

ET-LF :

- Low frequency super-attenuators
- Newtonian noise subtraction
- Silicon mirrors @ 10K, low power 1550nm laser
- Squeezing

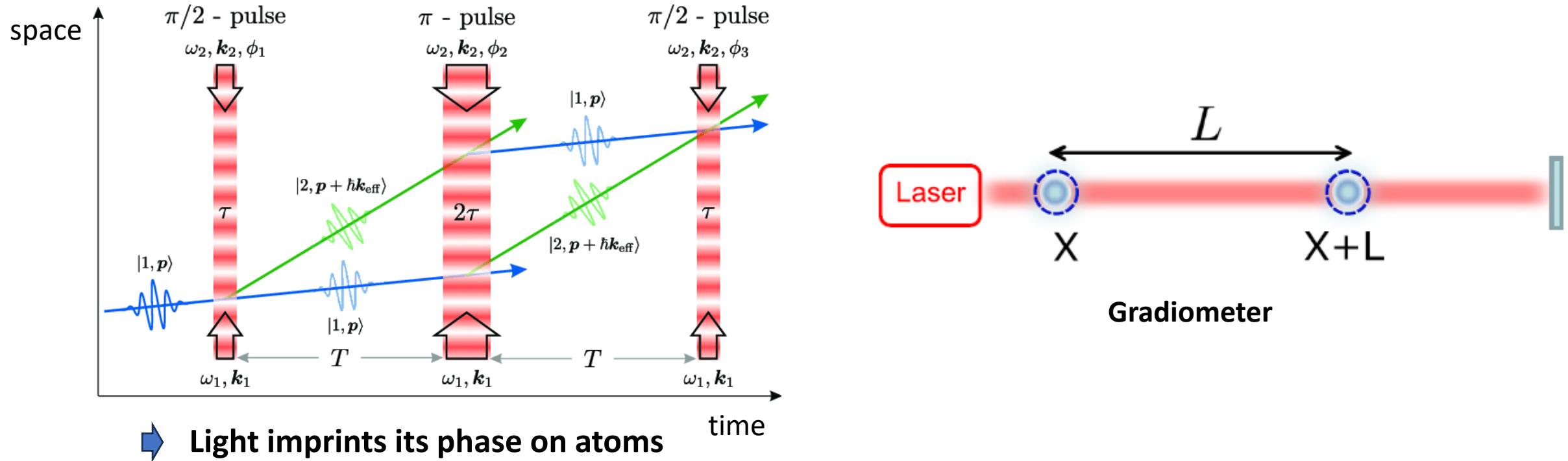
## Low frequency detectors : Torsion bars



- Torsion pendulum : very low frequency
- Technically limited by the size of the bars

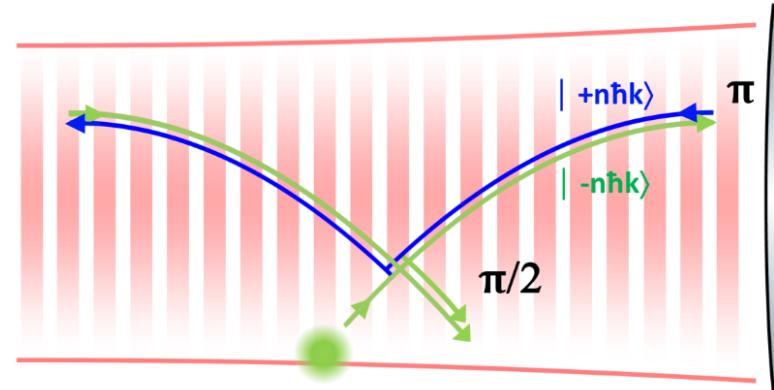
# Low frequency detectors : Atom interferometers

## Free falling atoms

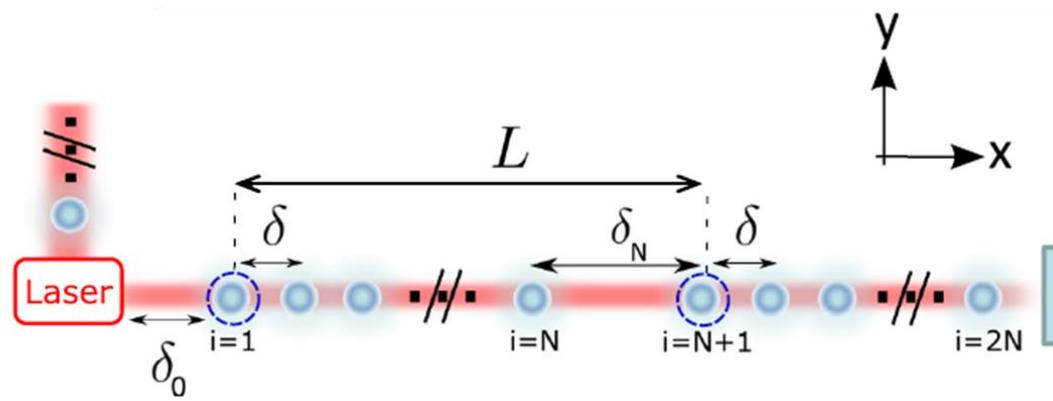


Light imprints its phase on atoms

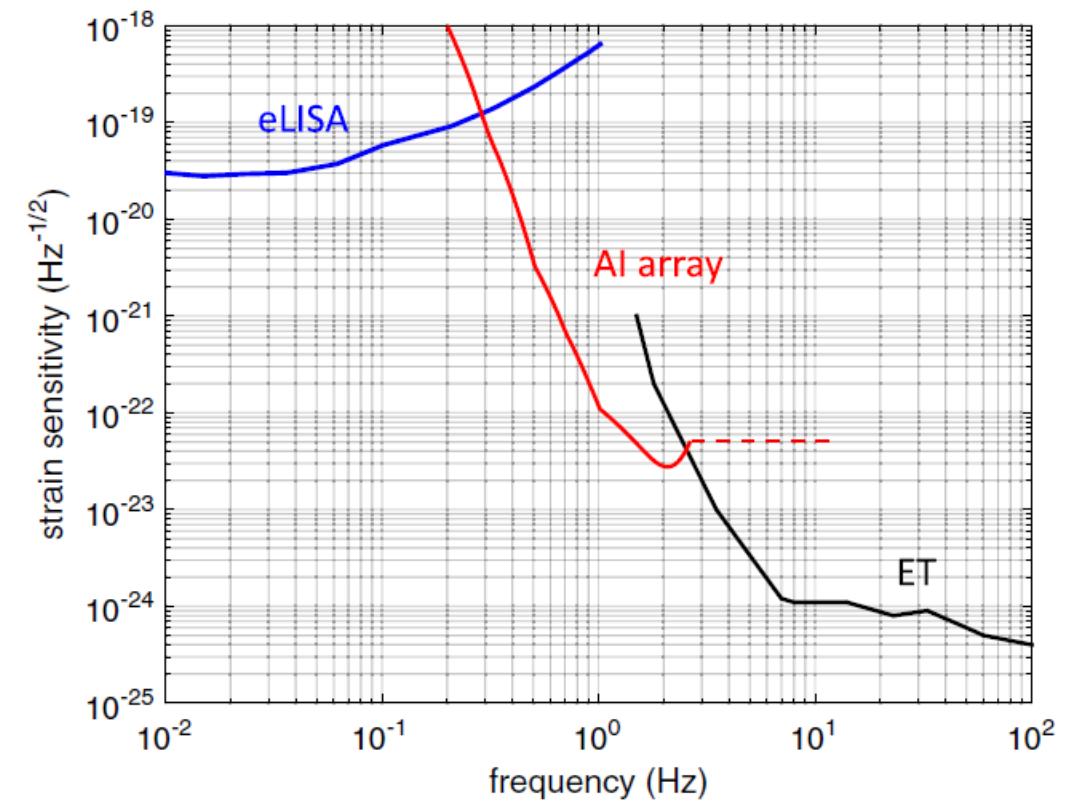
Bragg configuration



# Low frequency detectors : Atom interferometers



→ Multi-gradiometer configuration :  
Cancel out Newtonian noise



→ Fill in the gap 0.1Hz-10Hz

Multi diffraction, squeezing, but need more atoms...

# Thank you!