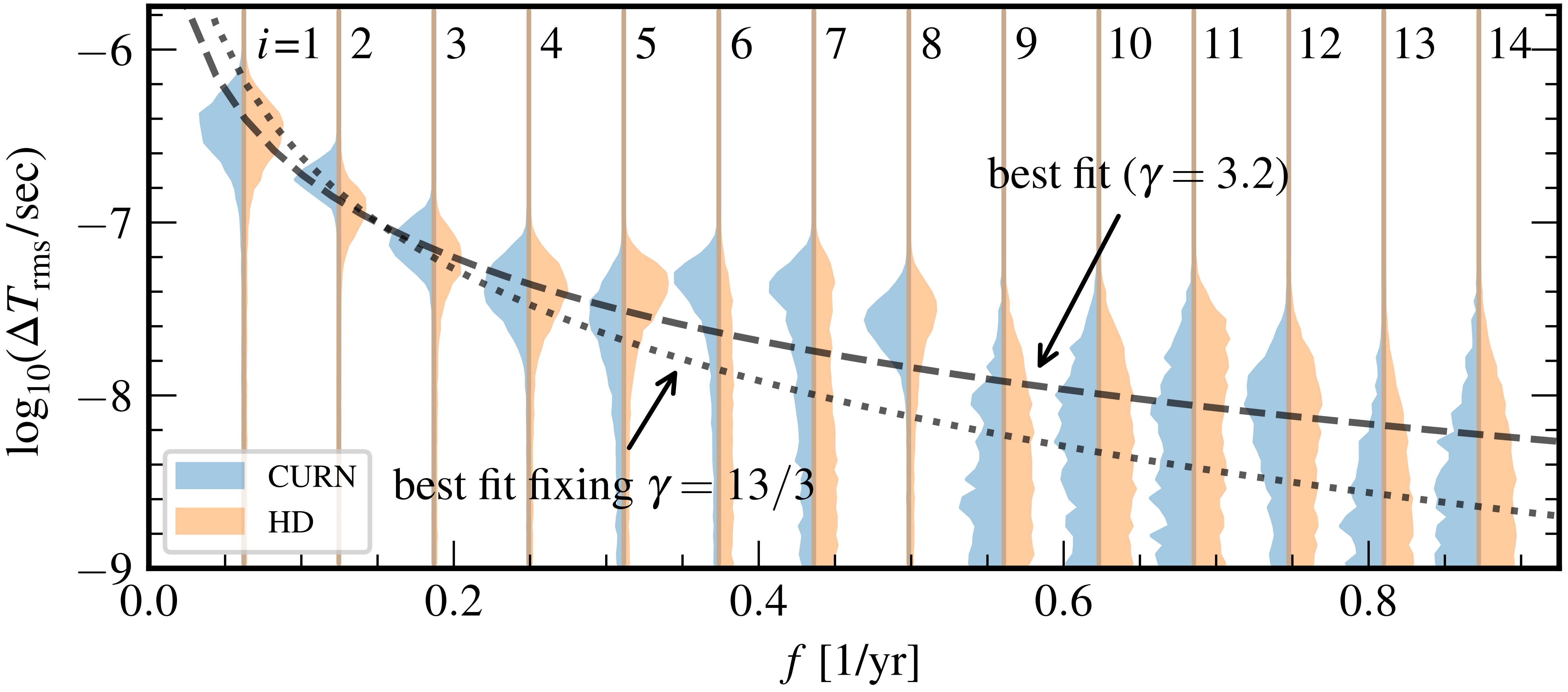
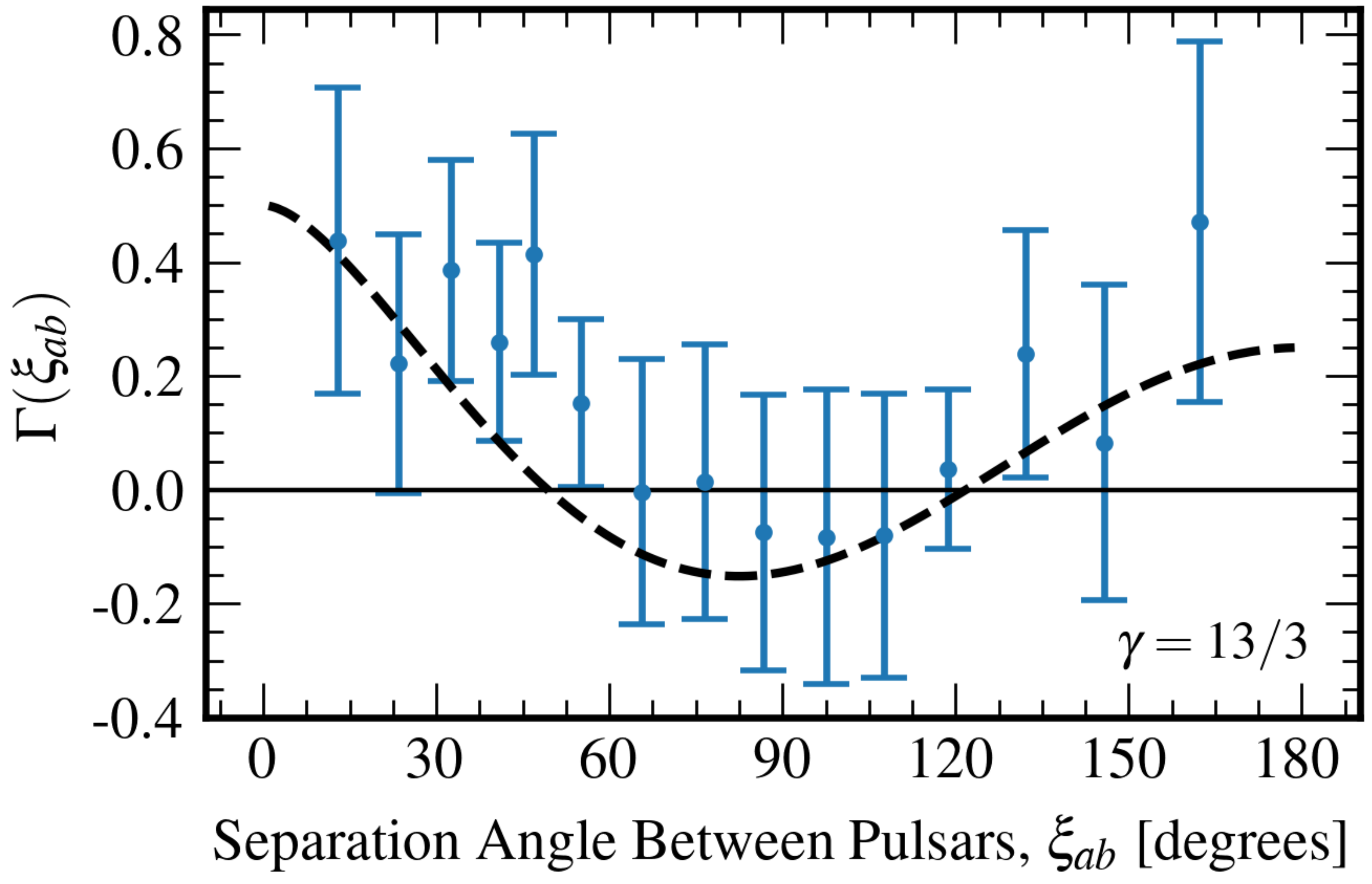
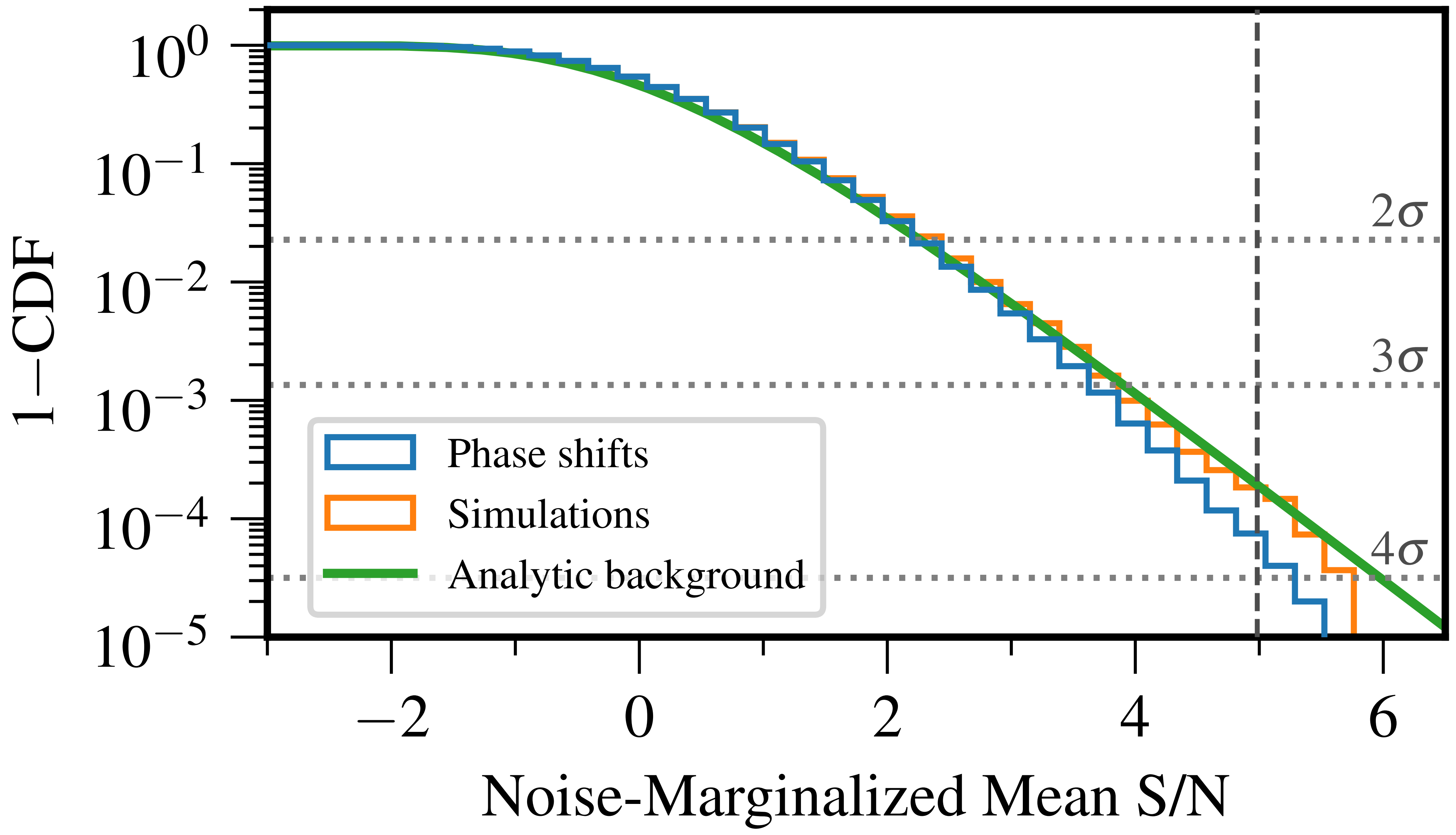
Part I. Theory and derivations



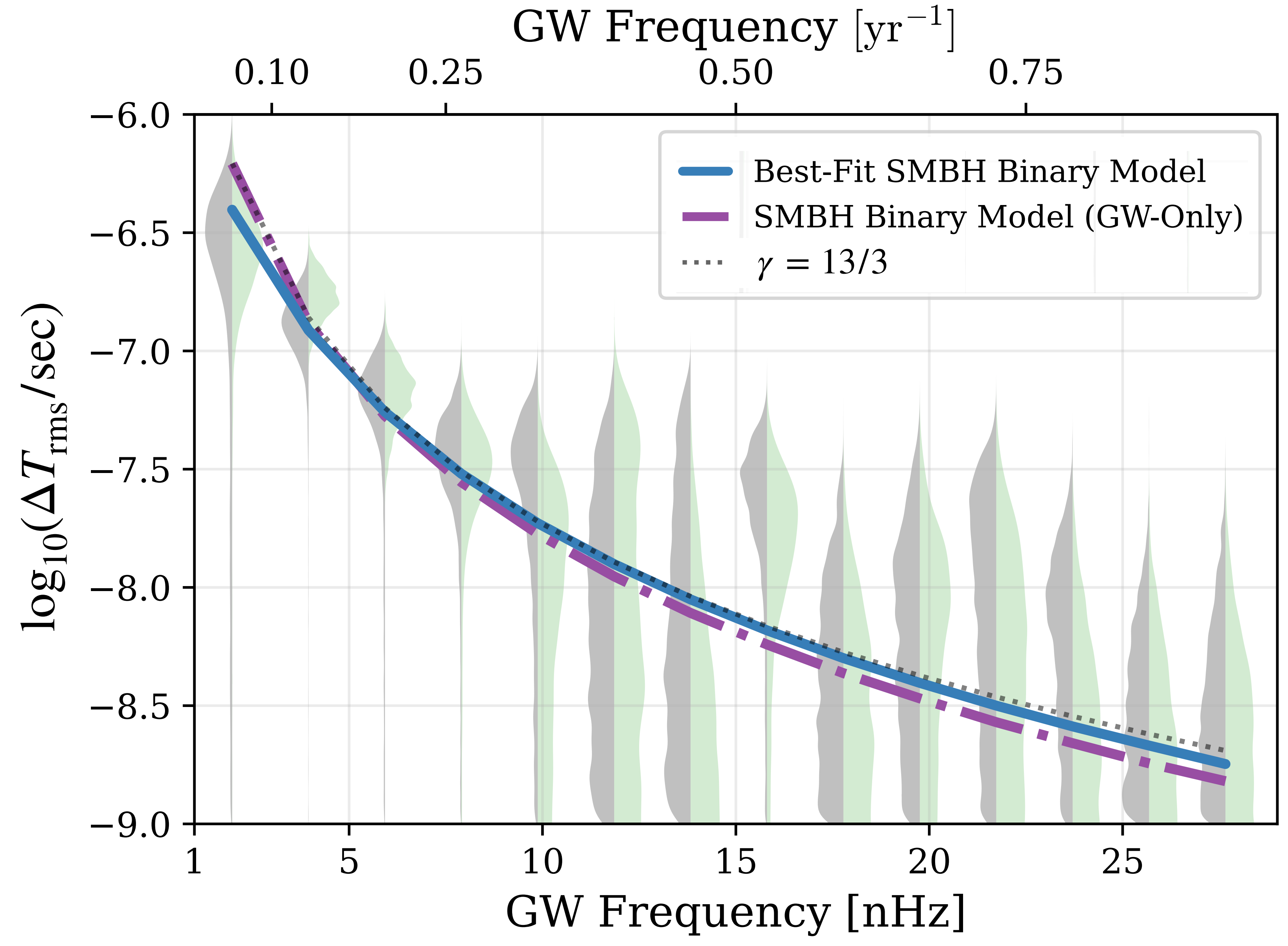
i=1
2
3
4
5
6
7
8
9
10
11
12
13
14
best fit (γ = 3.2)
best fit fixing γ = 13/3
CURN
HD
log10(ΔTrms/sec)
f [1/yr]



Γ(ξab)
γ = 13/3
Separation Angle Between Pulsars, ξab [degrees]



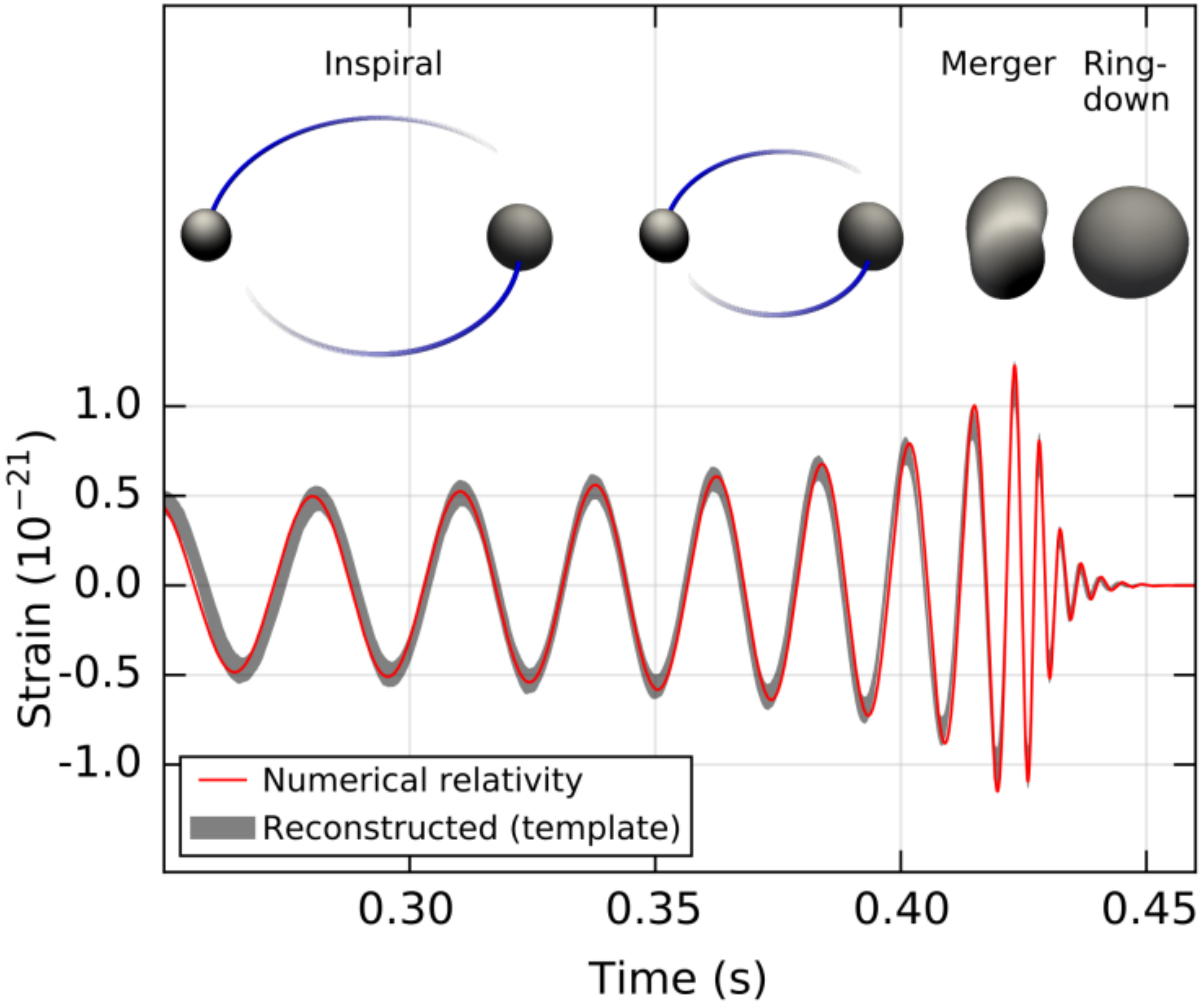
1−CDF
2σ
3σ
4σ
Phase shifts
Simulations
Analytic background
Noise-Marginalized Mean S/N

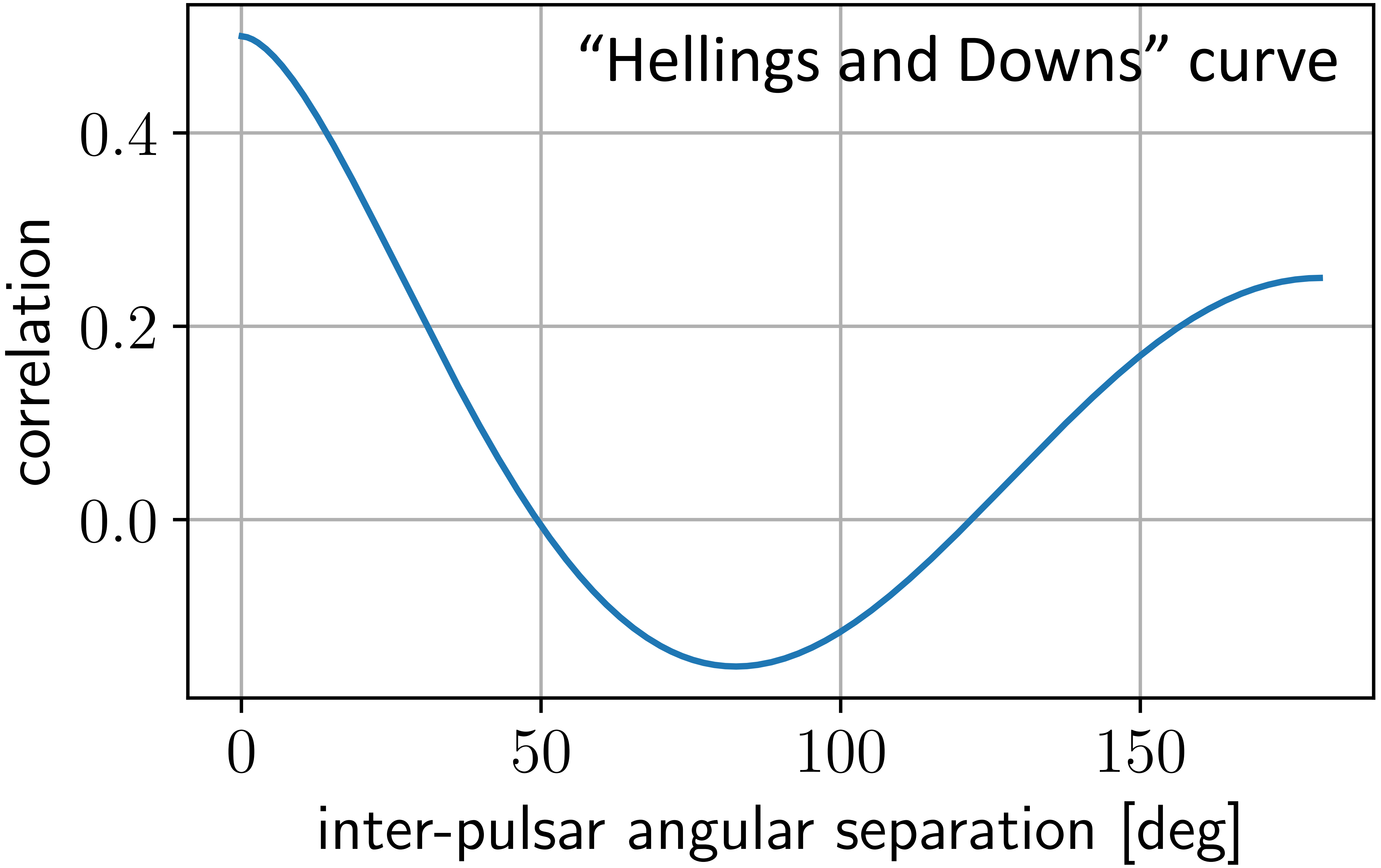


GW Frequency [yr−1]
Best-Fit SMBH Binary Model
SMBH Binary Model (GW-Only)
γ = 13/3
log10(ΔTrms/sec)
GW Frequency [nHz]

GW150914, etc	PTA observation
deterministic / transient signal	stochastic / persistent signal
waveforms & coincidence	power spectra & cross-correlations
single binary black hole merger	combined signal from a population of approx monochromatic inspiraling binaries
stellar mass black holes (1 - 100 solar masses)	supermassive black holes (10^9 solar masses)
audio frequencies (10's - 1000 Hz)	nanohertz frequencies (10^{-9} - 10^{-7} Hz) [periods: decades -> months]
laser interferometers with km-scale arms	galactic-scale detector using msec pulsars, with "arm" lengths ~100 - few x 1000 light-years
GW wavelength >> arm length	GW wavelength << arm length
"detection of …" (>5 sigma)	"evidence for …" (3-4 sigma)

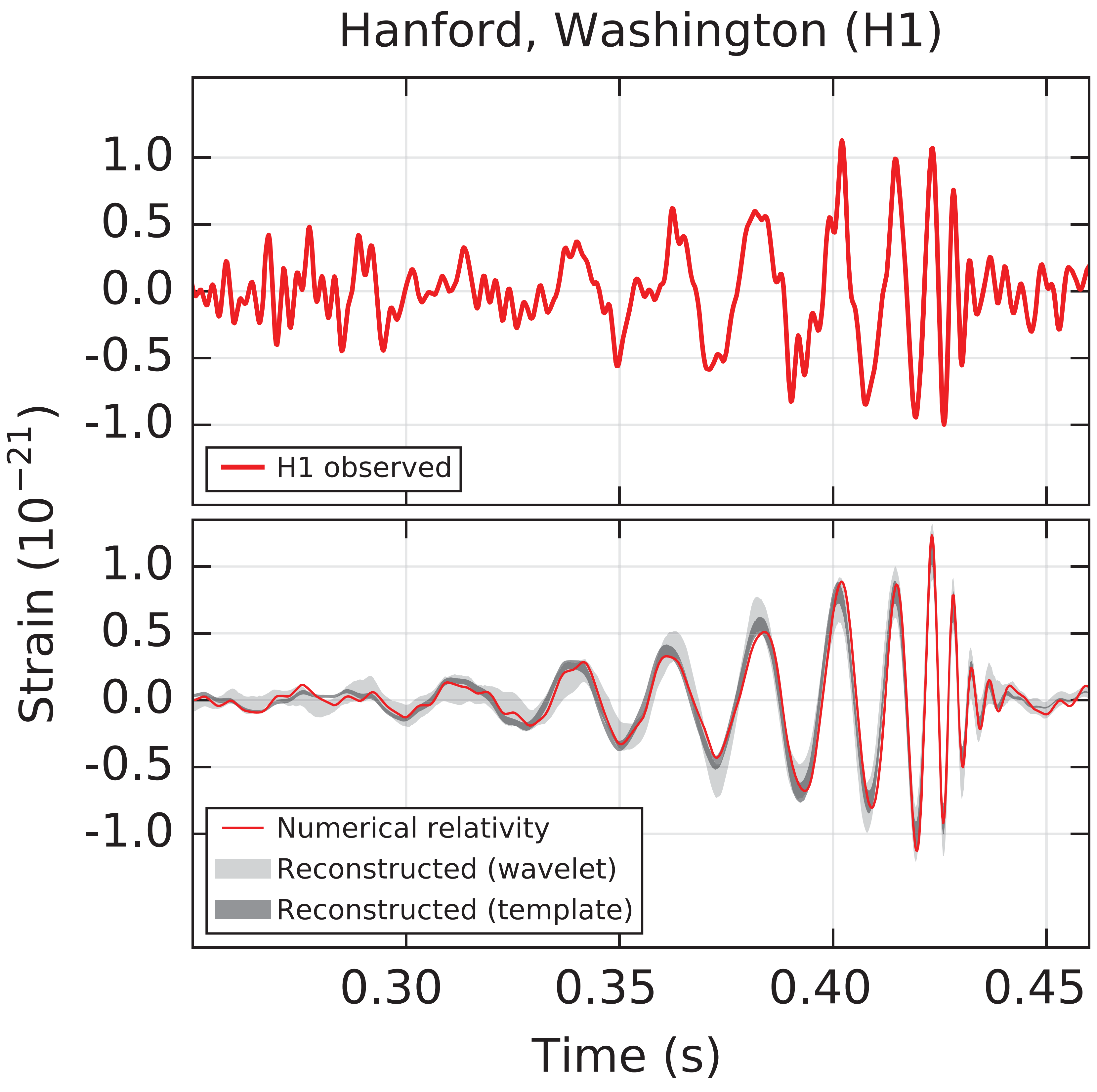
What plays the role of a binary "chirp" waveform for PTAs?

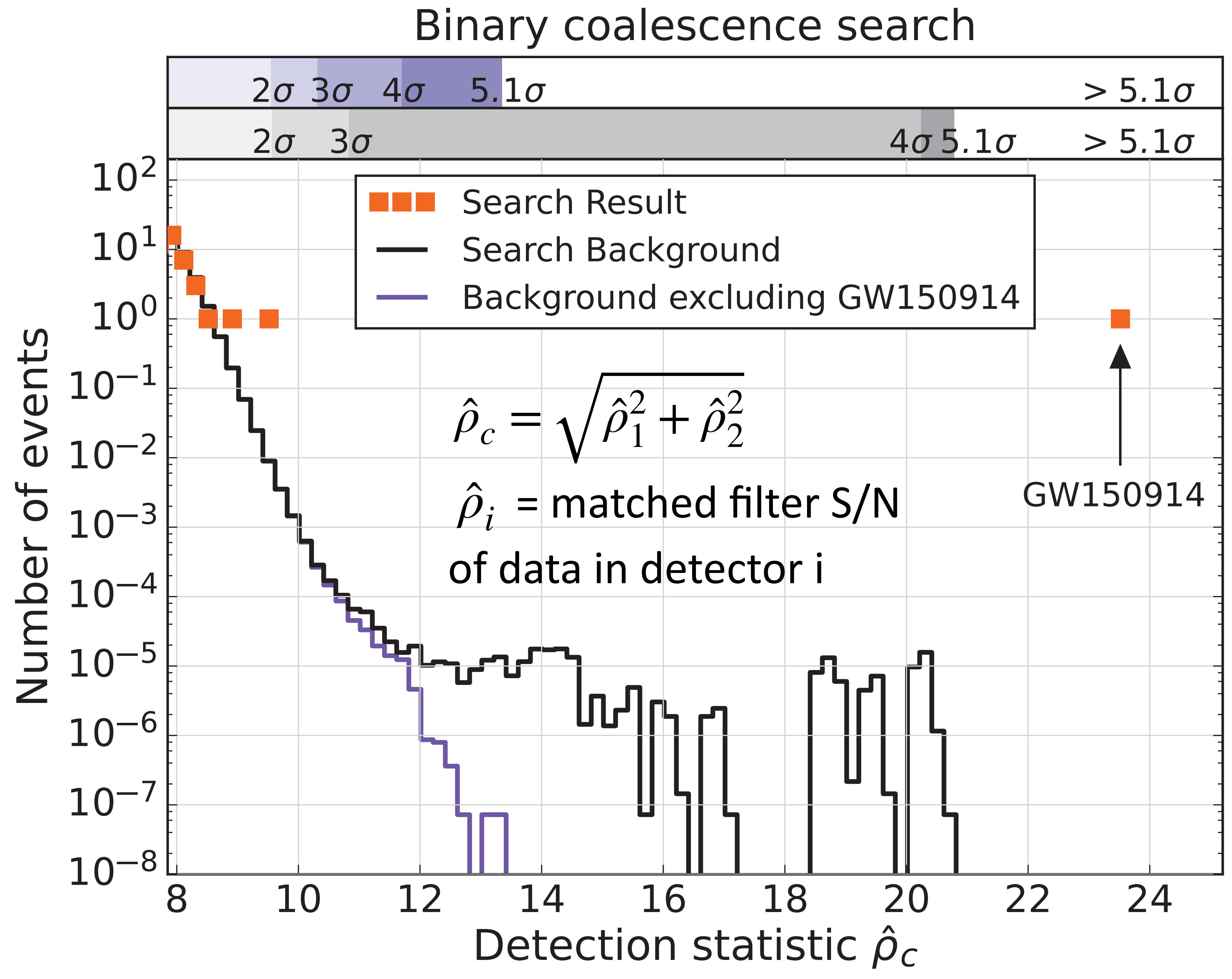




Why was GW150914 so convincing?

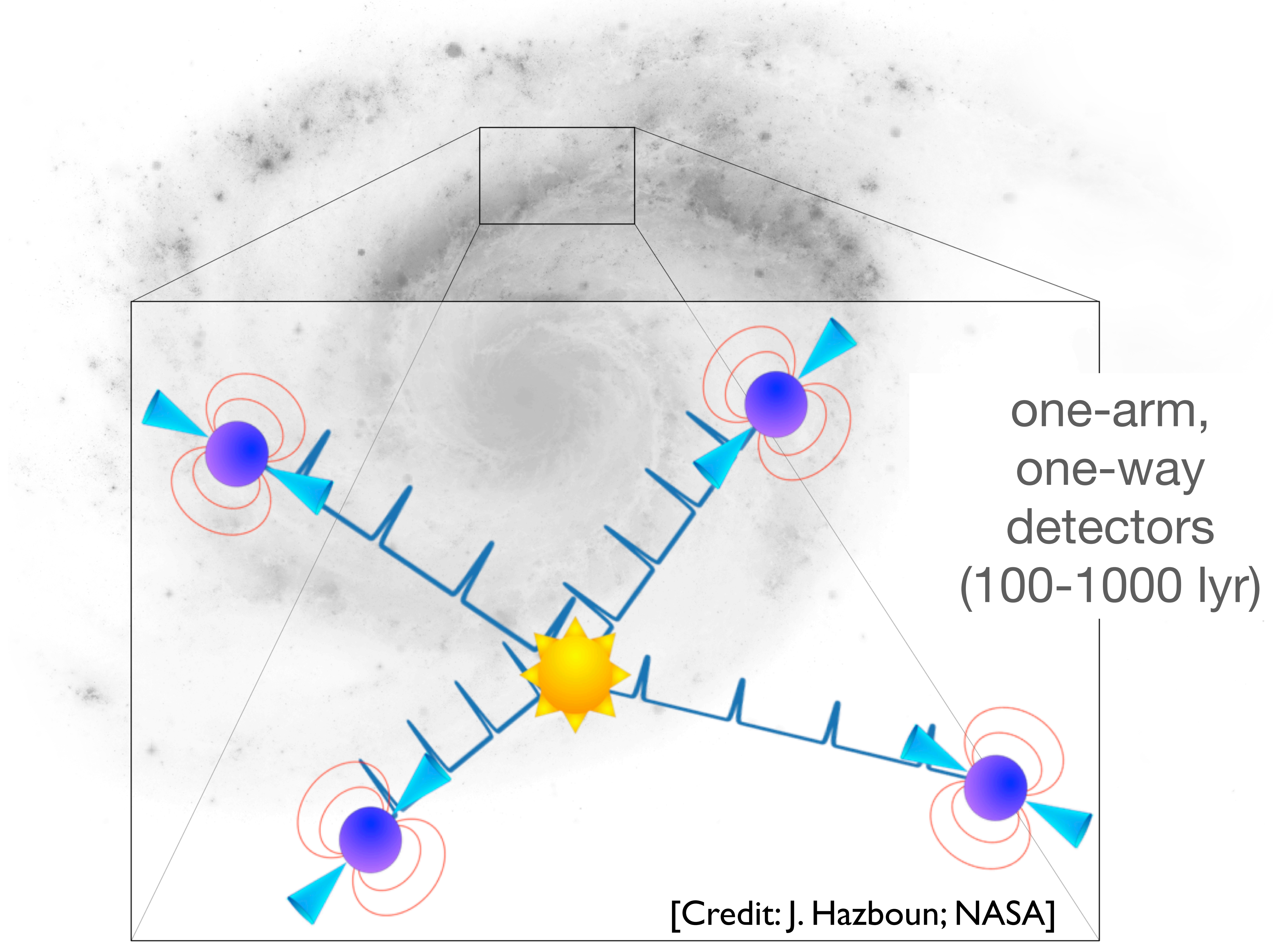
1. observed signal is consistent across detectors
2. observed signal agrees with predictions
3. observed signal is unlikely due to noise alone (< 1/5 million)

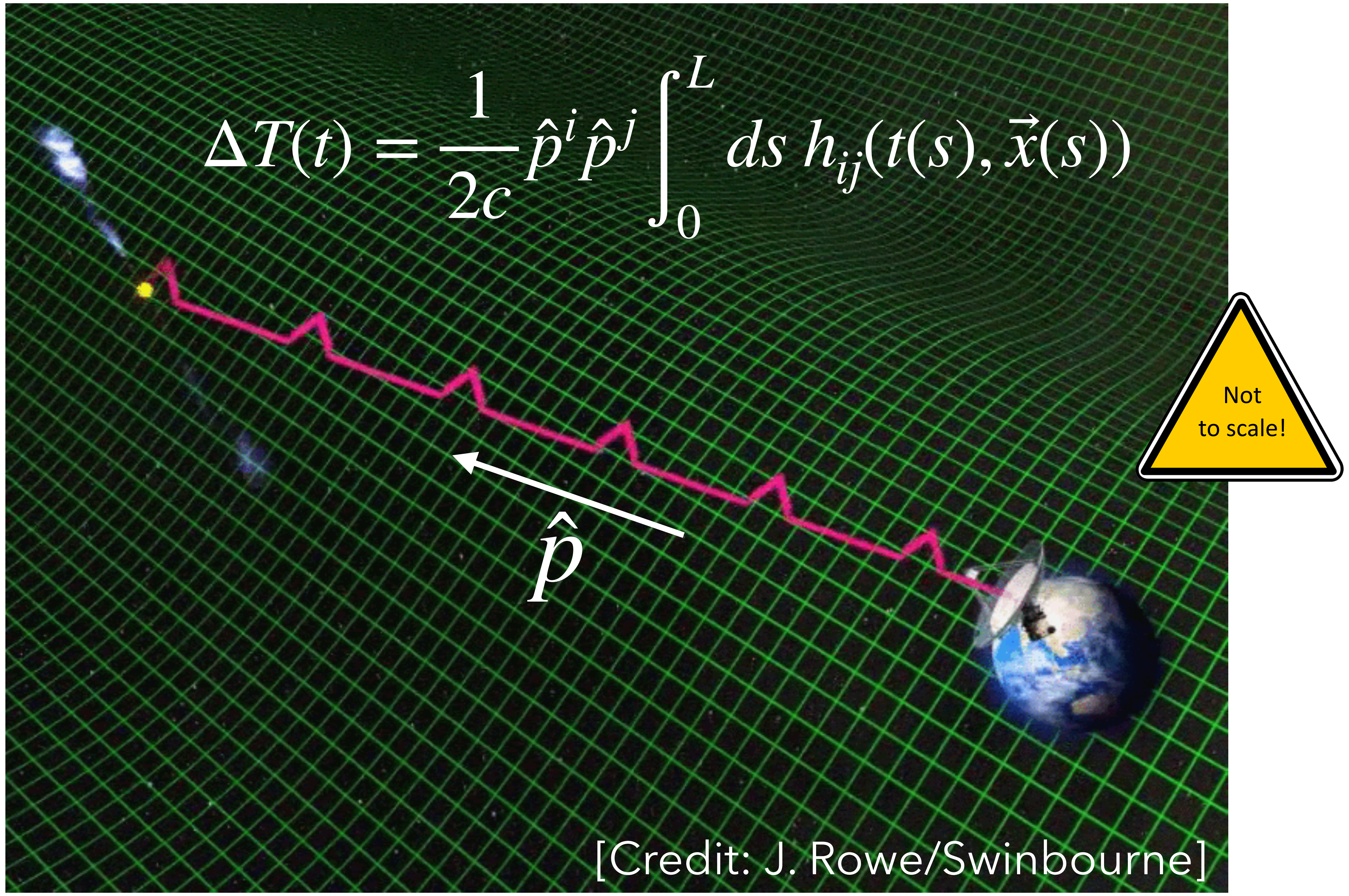




What is a pulsar timing array (PTA)?

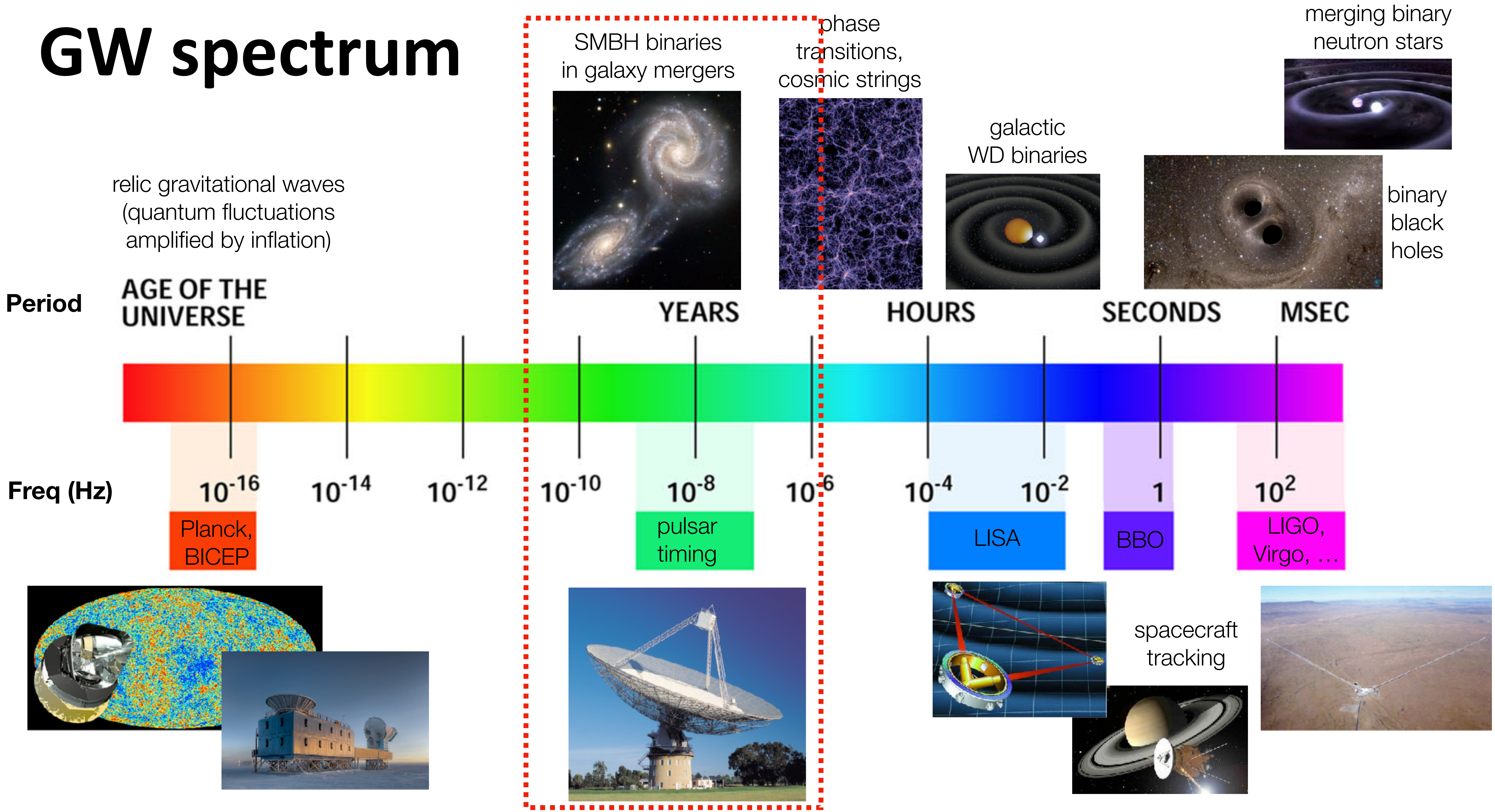
Galactic-scale GW detector





- GWs **perturb pulse arrival times** -> look for evidence of GWs in the **timing residuals**
- GW perturbations will be **correlated across pulsars** -> use this to **differentiate GWB from noise**

GW spectrum

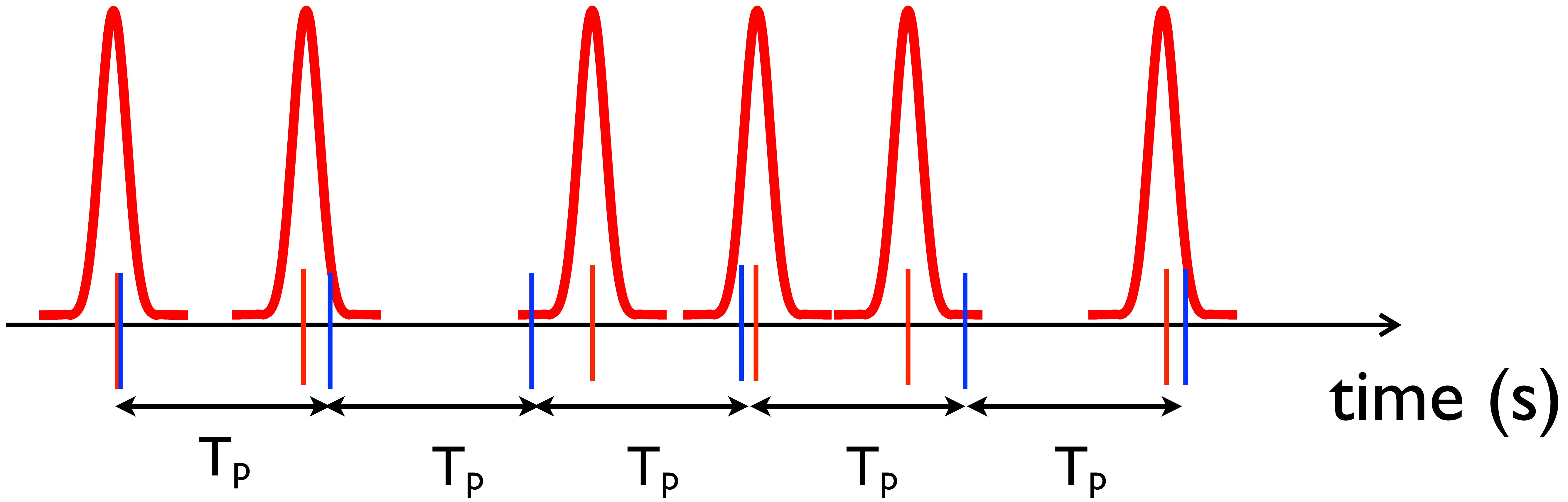


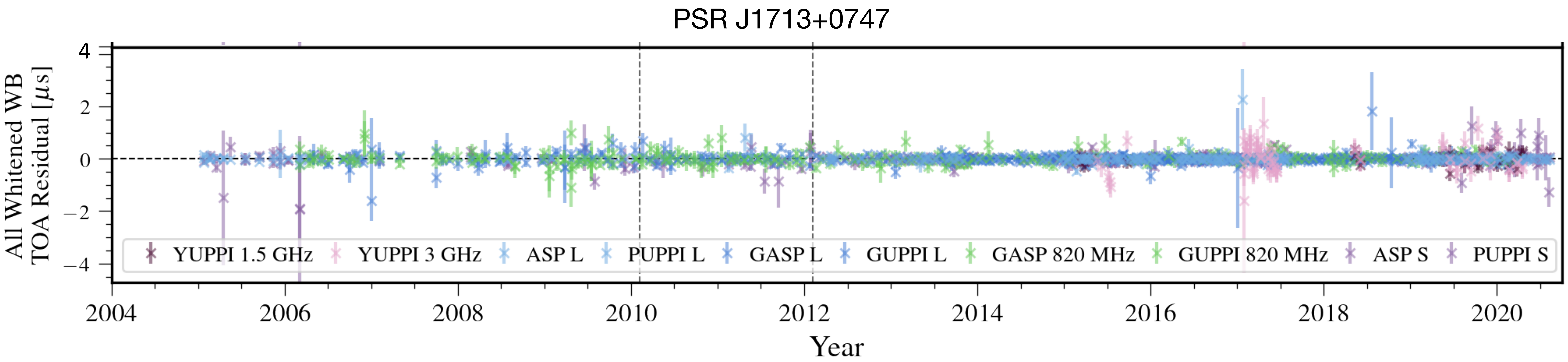
relic gravitational waves (quantum fluctuations amplified by inflation)
SMBH binaries in galaxy mergers
phase transitions, cosmic strings
galactic WD binaries
merging binary neutron stars
binary black holes
Period
AGE OF THE UNIVERSE
YEARS
HOURS
SECONDS
MSEC
Freq (Hz)
10^{-16}
10^{-14}
10^{-12}
10^{-10}
10^{-8}
10^{-6}
10^{-4}
10^{-2}
1
10^{2}
Planck, BICEP
pulsar timing
LISA
BBO
LIGO, Virgo, …
spacecraft tracking

What are the data for PTA analyses?

timing model: pulsar's spin period, period derivative, sky location, proper motion, …

$$\text{timing residual} = \text{observed arrival} - \text{predicted arrival}$$
$$= \text{unmodeled deterministic processes} + \text{noise sources} + \text{GW signals}$$





rms $\simeq$ 100 ns

$T_{\rm obs} \simeq 15$ yr

$h \simeq 10^{-16}$

How do GWs affect timing residuals?

- Perturbations to pulse arrival times:

$$\Delta T(t) = \frac{1}{2c}\hat{p}^i\hat{p}^j\int_0^L \mathrm{d}s\, h_{ij}(t(s),\vec{x}(s))$$

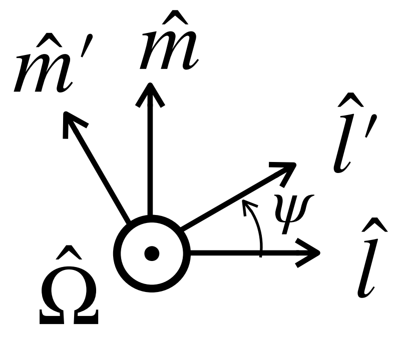
$$t(s) = t-(L-s)/c\,,\quad \vec{x}(s) = (L-s)\hat{p}$$

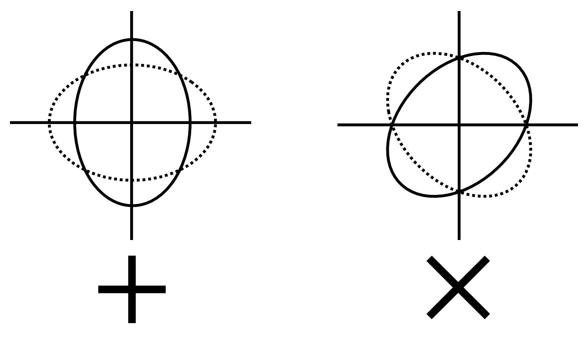
- Doppler shift (“redshift/blueshift”) of pulse frequency :

$$Z(t) \equiv \frac{\mathrm{d}\Delta T(t)}{\mathrm{d}t} = \frac{1}{2}\frac{\hat{p}^i\hat{p}^j}{1+\hat{\Omega}\cdot\hat{p}}\left[h_{ij}\left(t,\vec{0}\right) - h_{ij}\left(t-L/c,L\hat{p}\right)\right]$$

- In terms of polarizations $A = +\,,\times$:

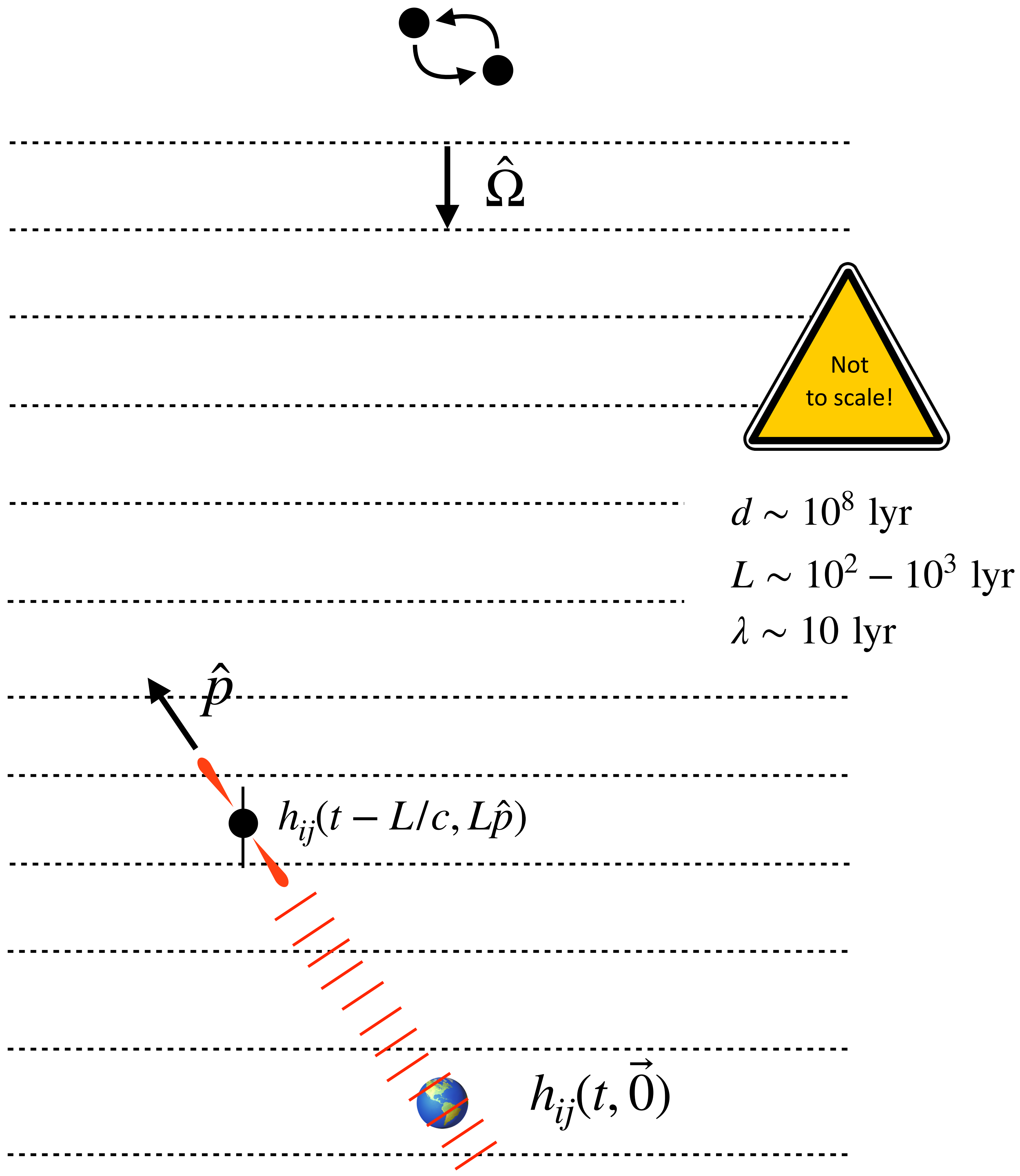
$$e^+_{ij}(\hat{\Omega}) = \hat{l}_i\hat{l}_j - \hat{m}_i\hat{m}_j$$
$$e^\times_{ij}(\hat{\Omega}) = \hat{l}_i\hat{m}_j + \hat{m}_i\hat{l}_j$$





$$Z(t) = \sum_{A=+,\times}\left[h^A(t) - h^A(t-L(1+\hat{\Omega}\cdot\hat{p})/c)\right]F^A(\hat{\Omega})$$

$$F^A(\hat{\Omega}) = \frac{1}{2}\frac{\hat{p}^i\hat{p}^j}{1+\hat{\Omega}\cdot\hat{p}}e^A_{ij}(\hat{\Omega}) \qquad \text{(antenna pattern)}$$



Can we detect GWs using data from a single pulsar?

Need to correlate data from multiple pulsars

- For expected correlations, can restrict to Earth-term contributions:

$$h_{ij}(t,\vec{0}) = h^{+}(t)\, e^{+}_{ij}(\hat{\Omega}) + h^{\times}(t)\, e^{\times}_{ij}(\hat{\Omega})$$

$$Z_a(t) = h^{+}(t)F^{+}_a(\hat{\Omega}) + h^{\times}(t)F^{\times}_a(\hat{\Omega}) \qquad Z_b(t) = h^{+}(t)F^{+}_b(\hat{\Omega}) + h^{\times}(t)F^{\times}_b(\hat{\Omega})$$

$$F^{A}_a(\hat{\Omega}) = \frac{1}{2}\frac{\hat{p}^i_a\hat{p}^j_a}{1+\hat{\Omega}\cdot\hat{p}_a}e^{A}_{ij}(\hat{\Omega}) \qquad F^{A}_b(\hat{\Omega}) = \frac{1}{2}\frac{\hat{p}^i_b\hat{p}^j_b}{1+\hat{\Omega}\cdot\hat{p}_b}e^{A}_{ij}(\hat{\Omega})$$

- Correlation is time-averaged product:

$$\rho_{ab} \equiv \overline{Z_a(t)Z_b(t)} \equiv \frac{1}{T}\int_0^T \mathrm{d}t\, Z_a(t)Z_b(t)$$

$$= \overline{(h^{+})^2}F^{+}_a(\hat{\Omega})F^{+}_b(\hat{\Omega}) + \overline{(h^{\times})^2}F^{\times}_a(\hat{\Omega})F^{\times}_b(\hat{\Omega}) + \overline{h^{+}h^{\times}}\left(F^{+}_a(\hat{\Omega})F^{\times}_b(\hat{\Omega}) + F^{\times}_a(\hat{\Omega})F^{+}_b(\hat{\Omega})\right)$$

($\overline{h^{+}h^{\times}} \to 0$)

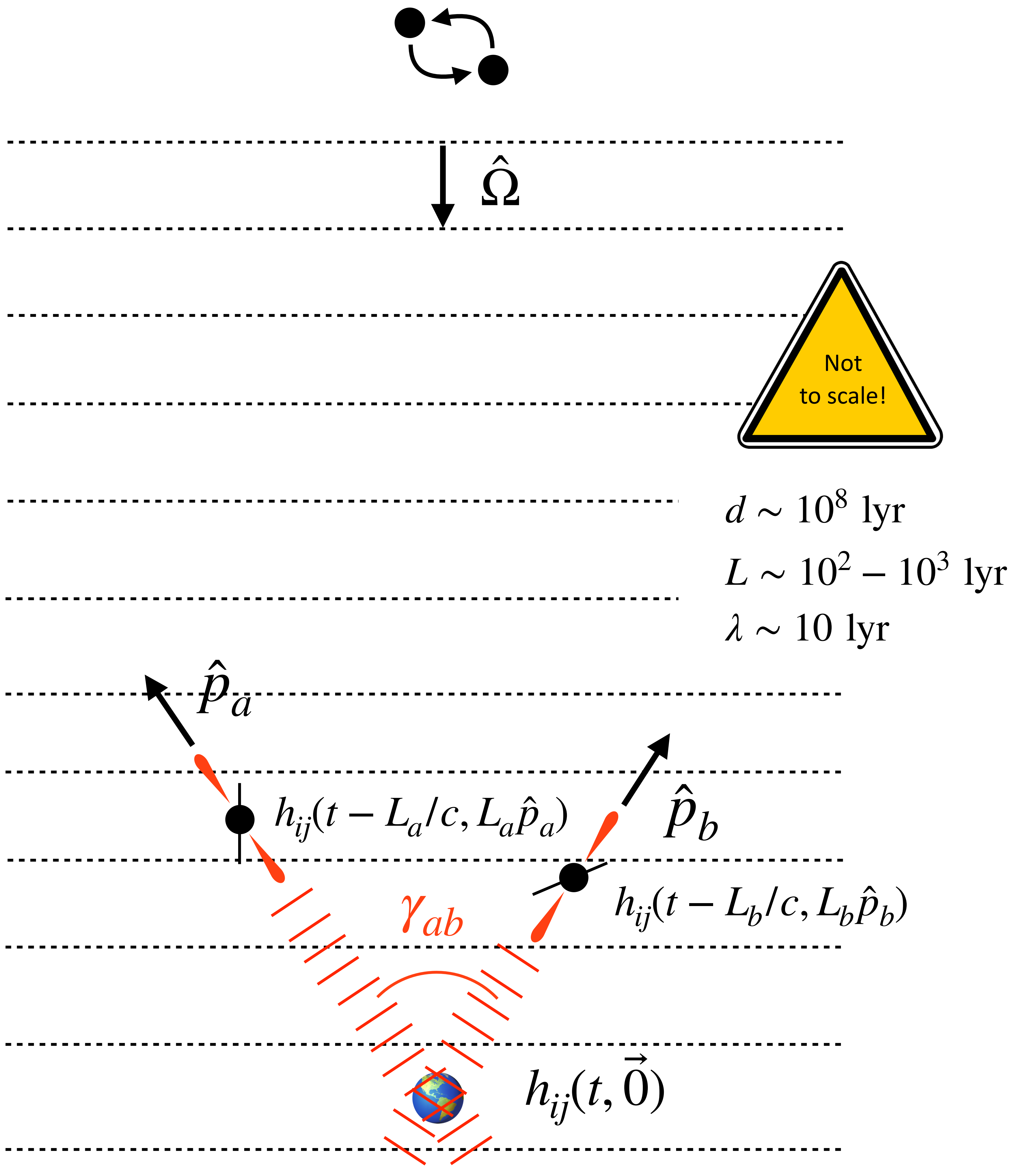
$$= F^{+}_a(\hat{\Omega})F^{+}_b(\hat{\Omega}) + F^{\times}_a(\hat{\Omega})F^{\times}_b(\hat{\Omega}) \quad \text{(unpolarized, unit amplitude)}$$

- **Hellings & Downs 1983**: fix pulsars; average over the GW source direction and polarization angle

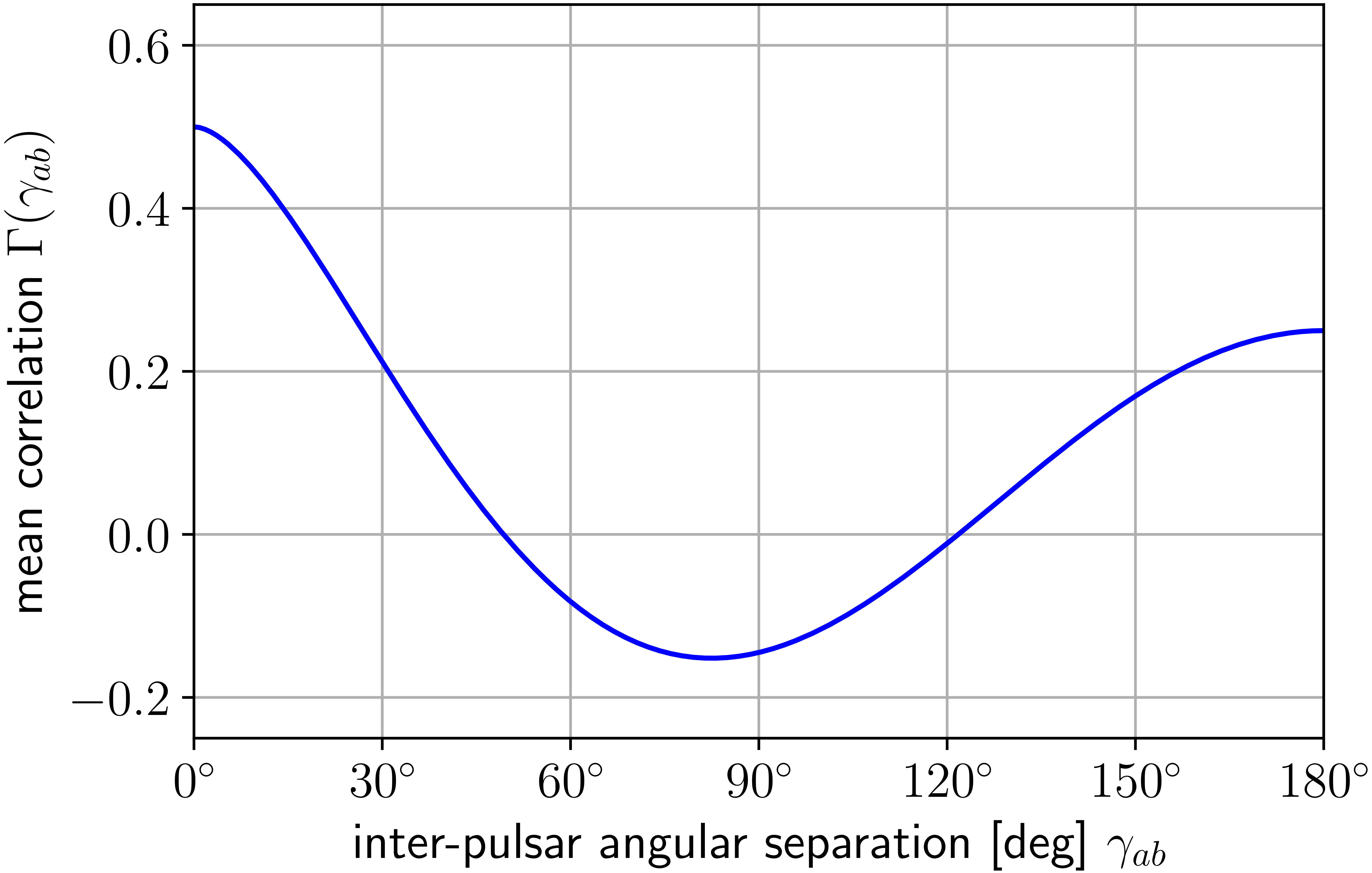
$\equiv$

Cornish & Sesana 2013: fix GW point source; average over all pulsar pairs separated by angle γ_{ab}

$$\langle\rho_{ab}\rangle_{\rm p} = \langle\rho_{ab}\rangle_{\rm s} = \frac{1}{2} - \frac{1}{4}\left(\frac{1-\cos\gamma_{ab}}{2}\right) + \frac{3}{2}\left(\frac{1-\cos\gamma_{ab}}{2}\right)\ln\left(\frac{1-\cos\gamma_{ab}}{2}\right) \equiv \Gamma(\gamma_{ab})$$

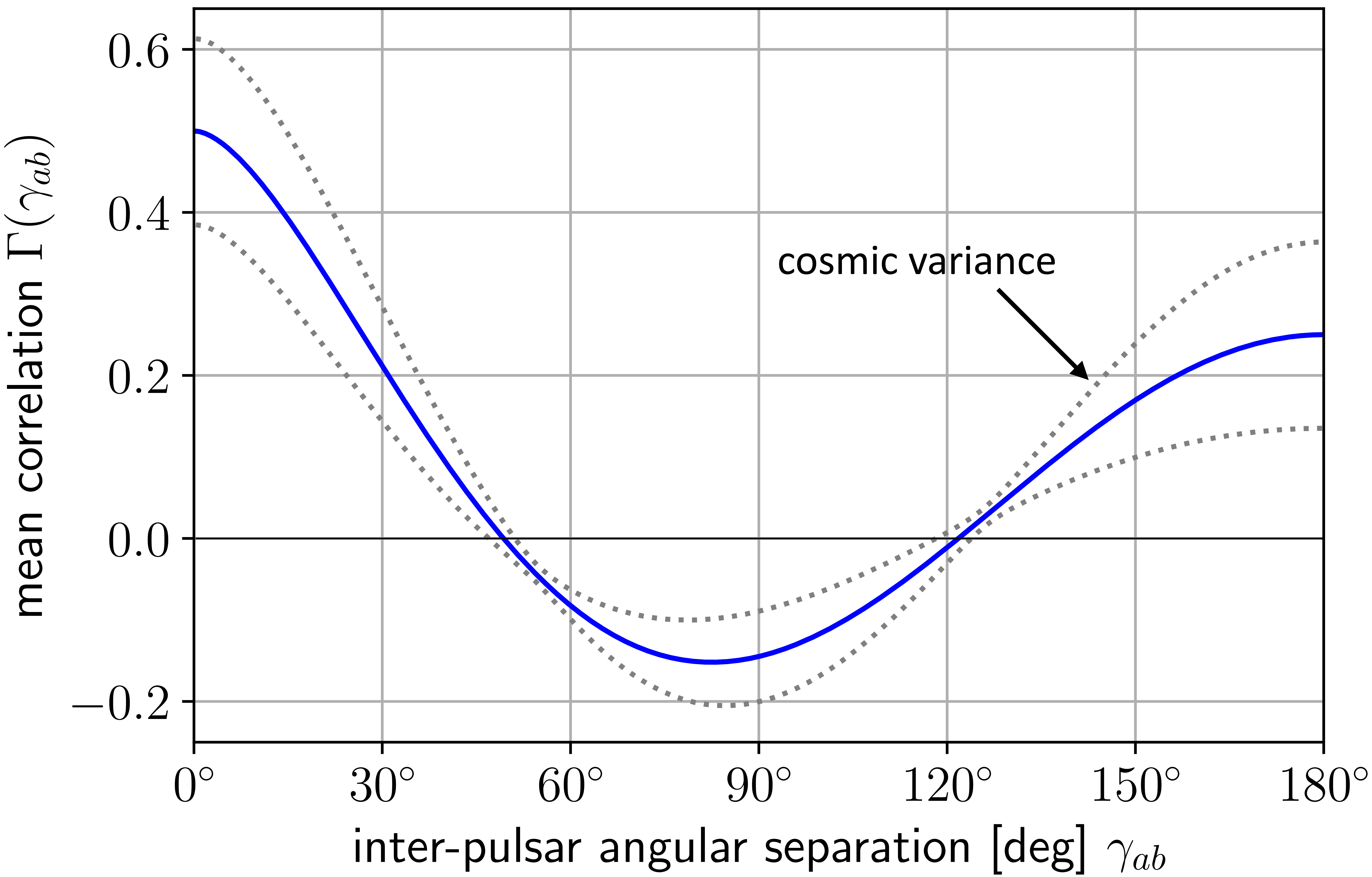


Hellings and Downs curve



Should we expect to recover the HD curve exactly?

Cosmic variance for interfering sources



Correlation calculation for interfering sources

- Two sources, same frequency (ignore polarization):

$$h_1(t) = A_1 \cos(2\pi f t + \phi_1), \quad h_2(t) = A_2 \cos(2\pi f t + \phi_2)$$

$$Z_a(t) = h_1(t) F_a(\hat{\Omega}_1) + h_2(t) F_a(\hat{\Omega}_2)$$

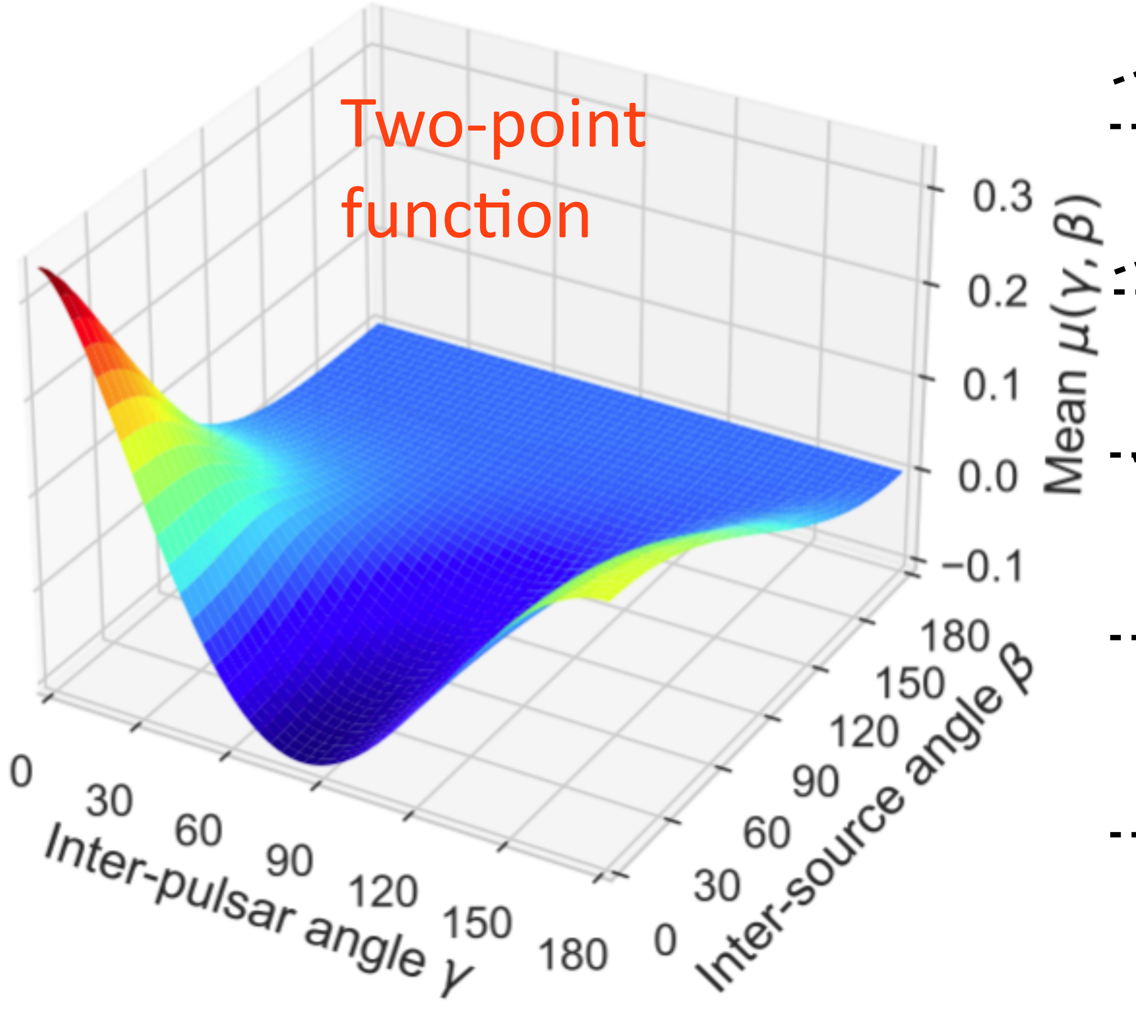
$$Z_b(t) = h_1(t) F_b(\hat{\Omega}_1) + h_2(t) F_b(\hat{\Omega}_2)$$

- Correlation:

$$\begin{aligned}\rho_{ab} &= \overline{Z_a(t) Z_b(t)} \\ &= \overline{h_1^2} F_a(\hat{\Omega}_1) F_b(\hat{\Omega}_1) + \overline{h_2^2} F_a(\hat{\Omega}_2) F_b(\hat{\Omega}_2) + \overline{h_1 h_2} \left(F_a(\hat{\Omega}_1) F_b(\hat{\Omega}_2) + F_a(\hat{\Omega}_2) F_b(\hat{\Omega}_1) \right) \\ &= \frac{1}{2} A_1^2 F_a(\hat{\Omega}_1) F_b(\hat{\Omega}_1) + \frac{1}{2} A_2^2 F_a(\hat{\Omega}_2) F_b(\hat{\Omega}_2) + \frac{1}{2} A_1 A_2 \cos(\phi_1 - \phi_2) \left(F_a(\hat{\Omega}_1) F_b(\hat{\Omega}_2) + F_a(\hat{\Omega}_2) F_b(\hat{\Omega}_1) \right)\end{aligned}$$

$$\langle \rho_{ab} \rangle_{\rm p} = \frac{1}{2} \sum_j A_j^2 \Gamma(\gamma_{ab}) + \frac{1}{2} \sum_{j \neq k} A_j A_k \cos(\phi_j - \phi_k) \mu(\gamma_{ab}, \beta_{jk})$$

$$\mu(\gamma_{ab}, \beta_{jk}) \equiv \langle F_a^+(\hat{\Omega}_j) F_b^+(\hat{\Omega}_k) + F_a^\times(\hat{\Omega}_j) F_b^\times(\hat{\Omega}_k) \rangle_{\rm p}$$

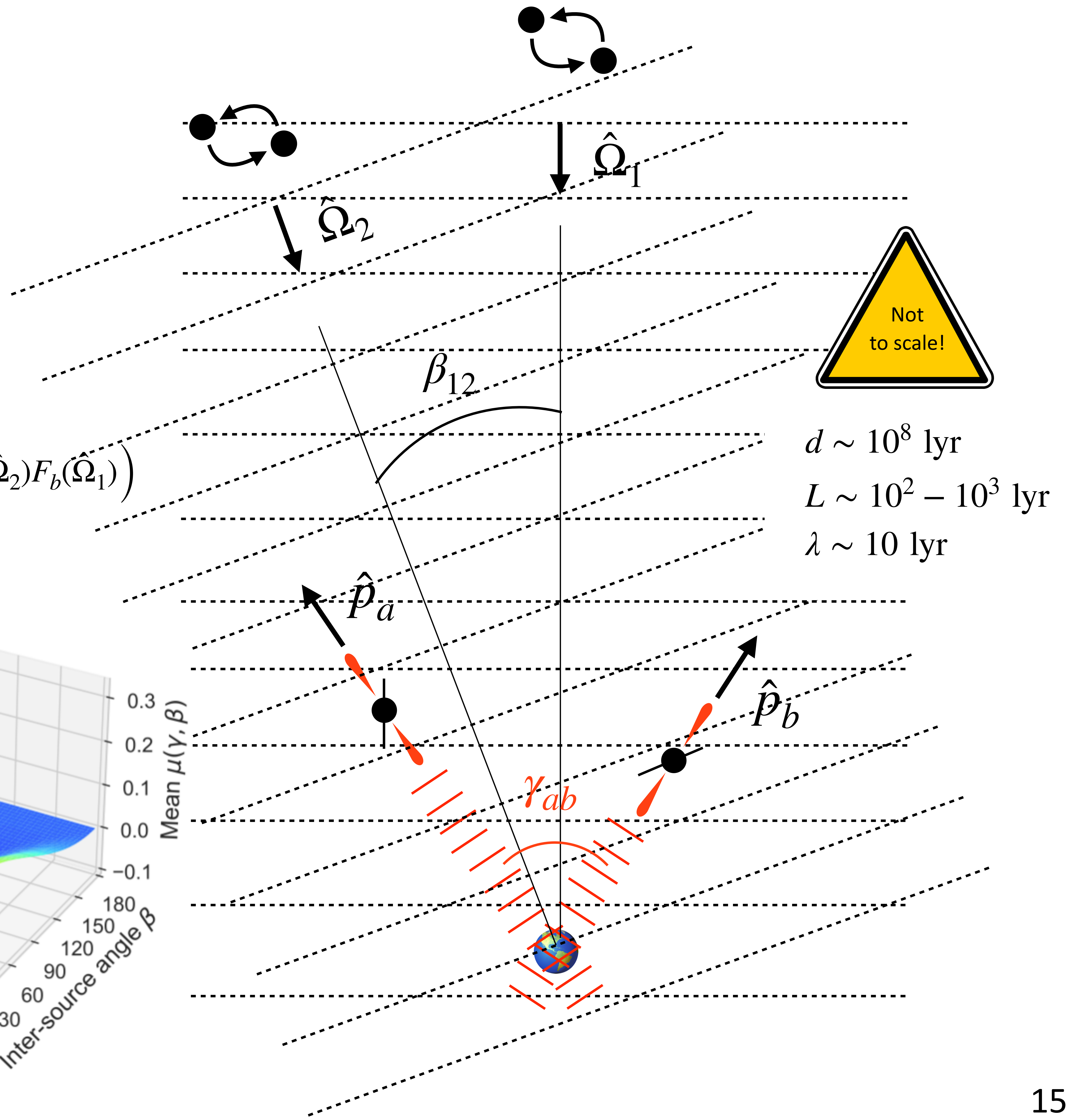


- Cosmic variance:

$$\mu_{ab} \equiv \Gamma(\gamma_{ab}) + \frac{1}{N} \sum_{j \neq k} \cos(\phi_j - \phi_k) \mu(\gamma_{ab}, \beta_{jk}) \quad \text{(unit amplitude)}$$

$$\langle \mu_{ab} \rangle_{\rm s} = \Gamma(\gamma_{ab}), \quad \sigma^2_{\rm cosmic}(\gamma_{ab}) = \langle \mu_{ab}^2 \rangle_{\rm s} - \langle \mu_{ab} \rangle_{\rm s}^2$$

$$\sigma^2_{\rm cosmic}(\gamma) = \frac{1}{4} \int_0^\pi {\rm d}\beta \, \sin\beta \, \mu^2(\gamma, \beta)$$



How does one analyze PTA data to search for GWs?

Bayesian model comparison

Pulsar 1		Pulsar 2		Pulsar N
Timing Model Measurement noise Intrinsic pulsar noise	+	Timing Model Measurement noise Intrinsic pulsar noise	+ … +	Timing Model Measurement noise Intrinsic pulsar noise

+

Common stochastic process (potentially correlated across pulsars)

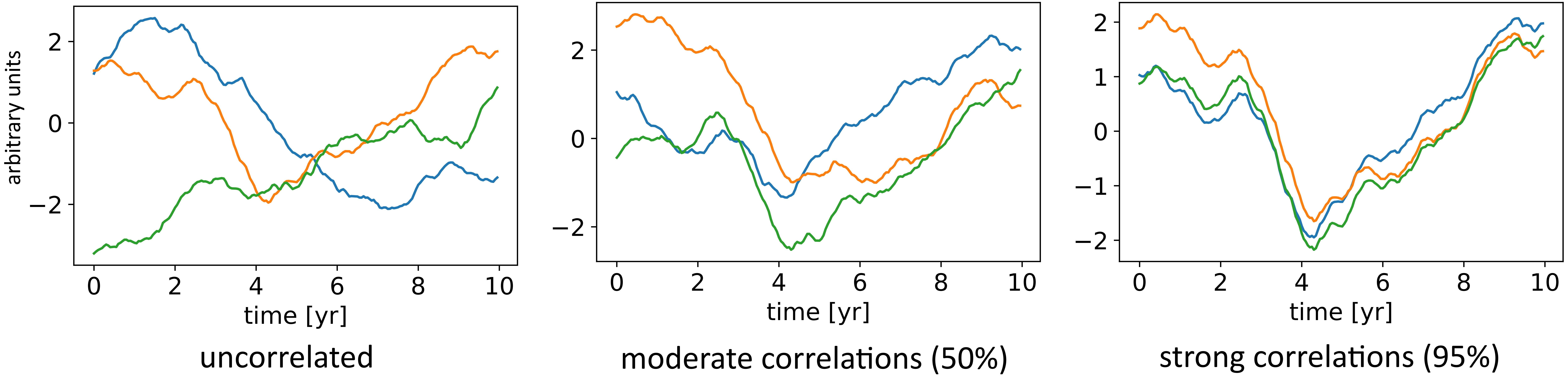
Individual power spectra:

$$\varphi_a(f) = \frac{A_a^2}{12\pi^2}\frac{1}{T_{\rm obs}}\left(\frac{f}{f_{\rm ref}}\right)^{-\gamma_a} f_{\rm ref}^{-3}$$

Cross power:

common process

$$C_{ab}(f) = \chi_{ab}\Phi(f) + \delta_{ab}\varphi_b(f)$$



Frequentist detection statistic

- Form general linear combination of inter-pulsar correlations:

$$S \equiv \sum_{a<b} \rho_{ab} w_{ab} \quad \text{where} \quad \rho_{ab} = \overline{Z_a(t) Z_b(t)} \quad \text{with} \quad \langle \rho_{ab} \rangle = A^2_{\rm gw}\, \Gamma_{ab}, \quad \langle \rho_{ab} \rangle_0 = 0$$

- Determine weights so they maximize $\langle S \rangle / N$, where

$N^2 \equiv \langle S^2 \rangle_0 - \langle S \rangle_0^2$ (variance of S in absence of spatial correlations)

- This leads to:

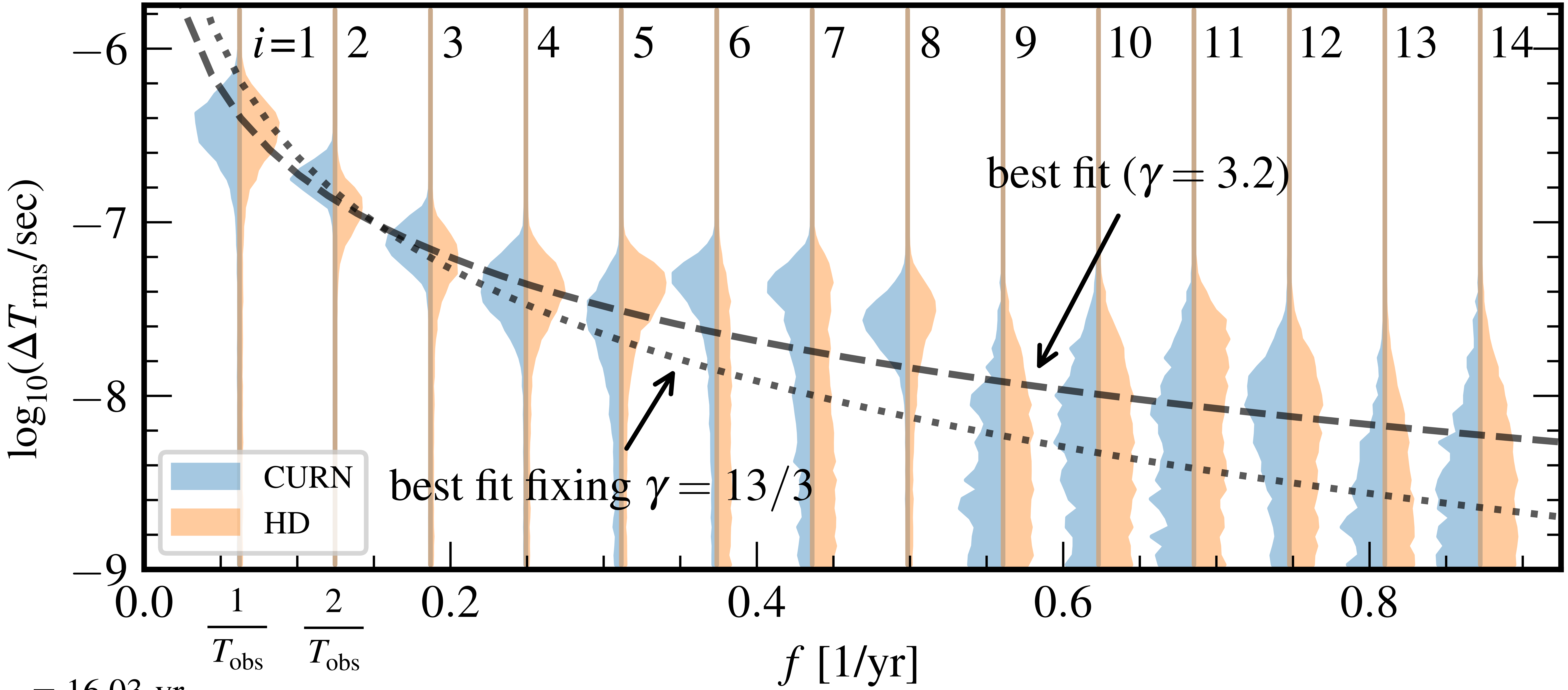
$$w_{ab} = \frac{\Gamma_{ab}/\sigma^2_{ab,0}}{\sqrt{\sum_{c<d} \Gamma^2_{cd}/\sigma^2_{cd,0}}} \quad \text{where } \sigma^2_{ab,0} = \langle \rho^2_{ab} \rangle_0 \text{ with } w_{ab} \text{ normalized so } N^2 = 1$$

- The detections statistic S has the interpretation of a signal-to-noise ratio:

$$S = \frac{\sum_{a<b} \rho_{ab} \Gamma_{ab}/\sigma^2_{ab,0}}{\sqrt{\sum_{c<d} \Gamma^2_{cd}/\sigma^2_{cd,0}}} \equiv \mathrm{S/N}$$

Part II. Plots from NANOGrav 15-yr papers

NANOGrav's observed common power spectrum



$T_{obs} = 16.03$ yr

$\gamma = 13/3$ (binary inspiral)

$A = 2.4^{+0.7}_{-0.6} \times 10^{-15}$

(strain amplitude at $f_{ref} = 1/\text{yr}$)

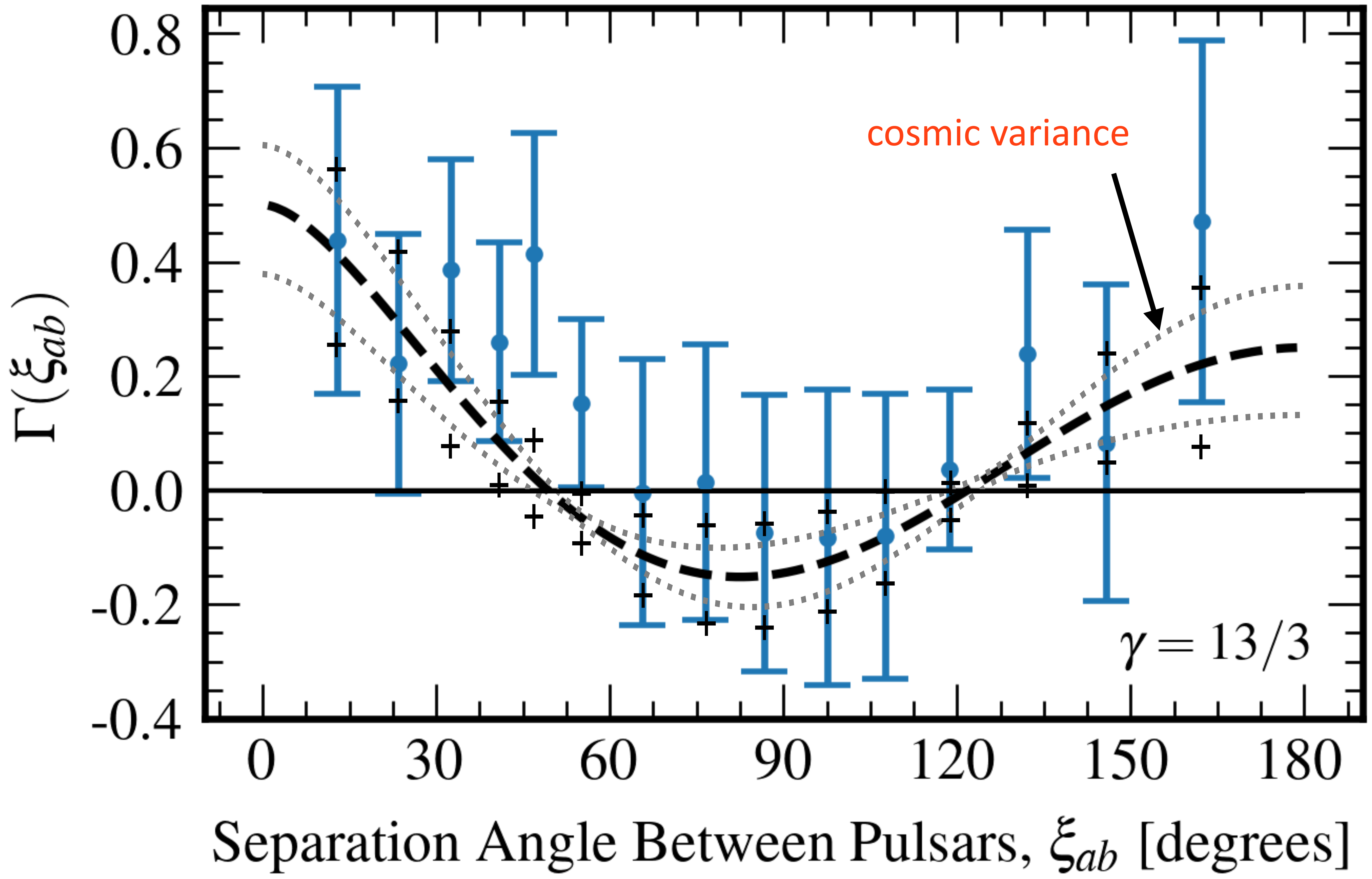
$$\Omega_{GW}(f) \equiv \frac{1}{\rho_c}\frac{d\rho_{GW}}{d\ln f}$$

$$= 7.4 \times 10^{-9}\left(\frac{f}{f_{ref}}\right)^{2/3}$$

$\Omega_{GW} = 8.4 \times 10^{-9}$

- consistent with predictions from SMBH binaries (and many other source models)

NANOGrav's observed correlations



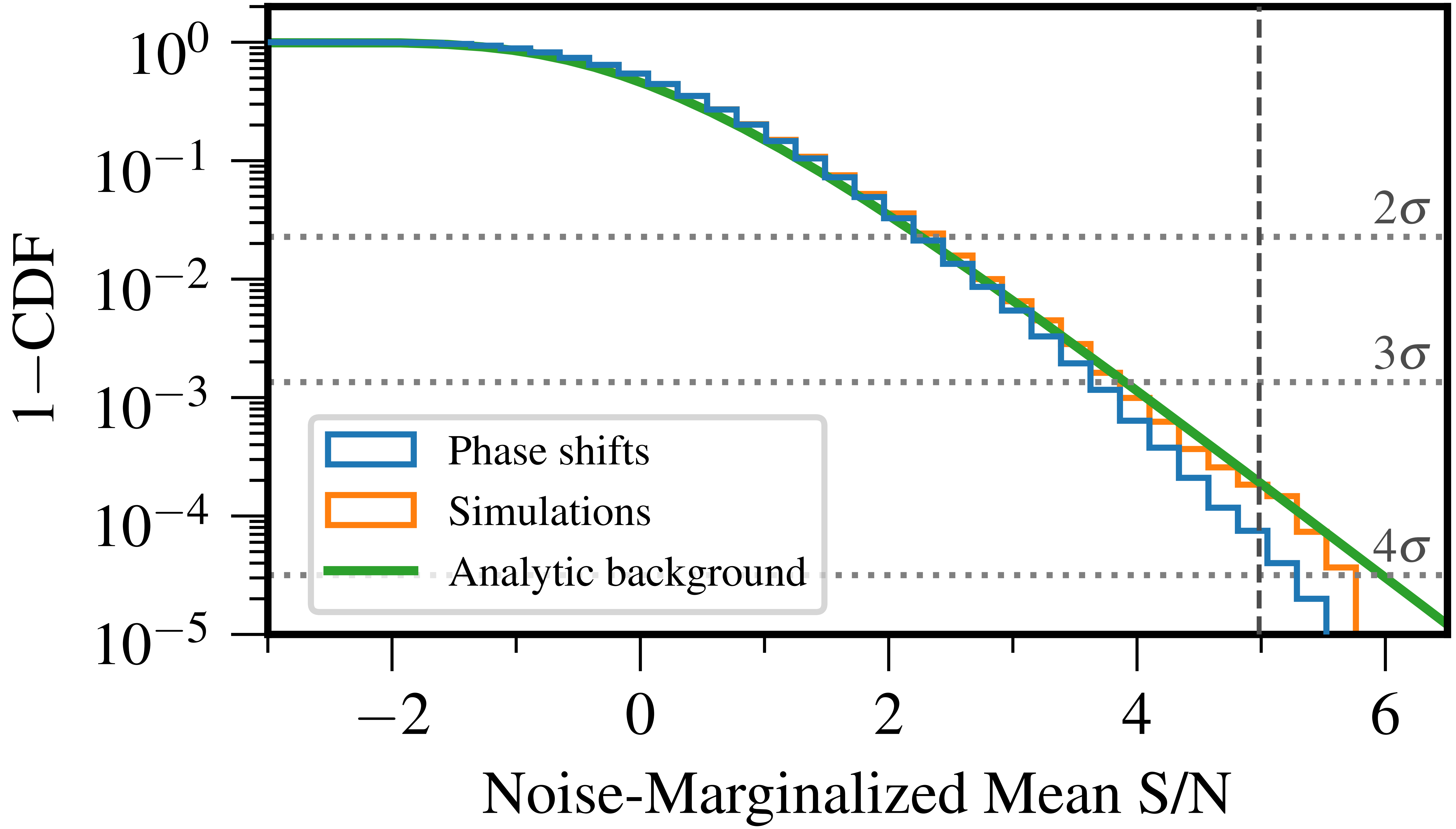
$$\frac{67(67-1)}{2} = 2211 \text{ distinct pairs}$$

$$\frac{2211}{15} \approx 150 \text{ pairs per bin}$$

- weighted averages of measured correlations ρ_{ab} in each bin
- includes contributions from GW-induced covariances $C_{ab,cd} \equiv \langle \rho_{ab}\rho_{cd} \rangle - \langle \rho_{ab} \rangle \langle \rho_{cd} \rangle$

-correlations follow the pattern expected for a GW backgound

NANOGrav's detection confidence

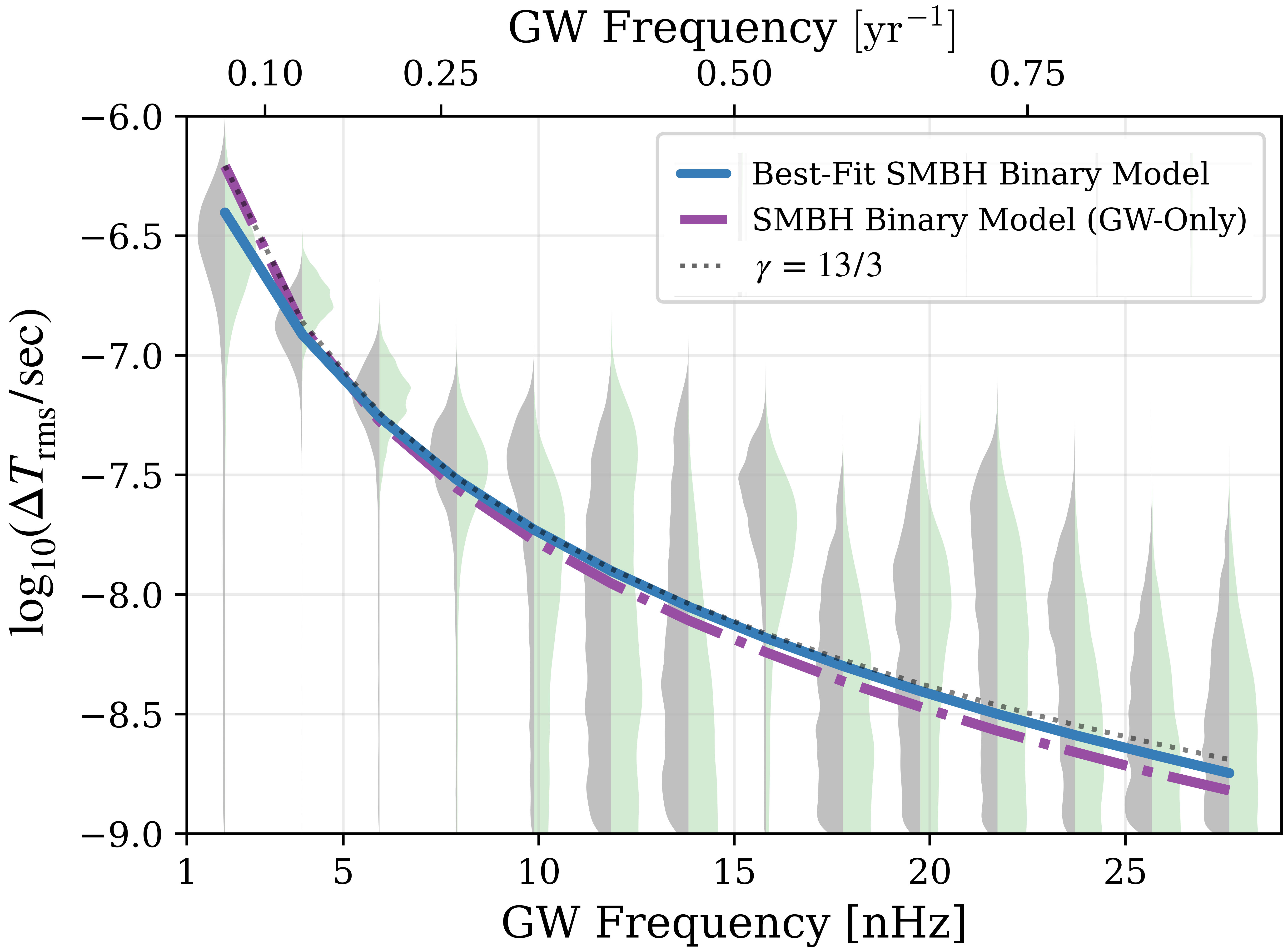


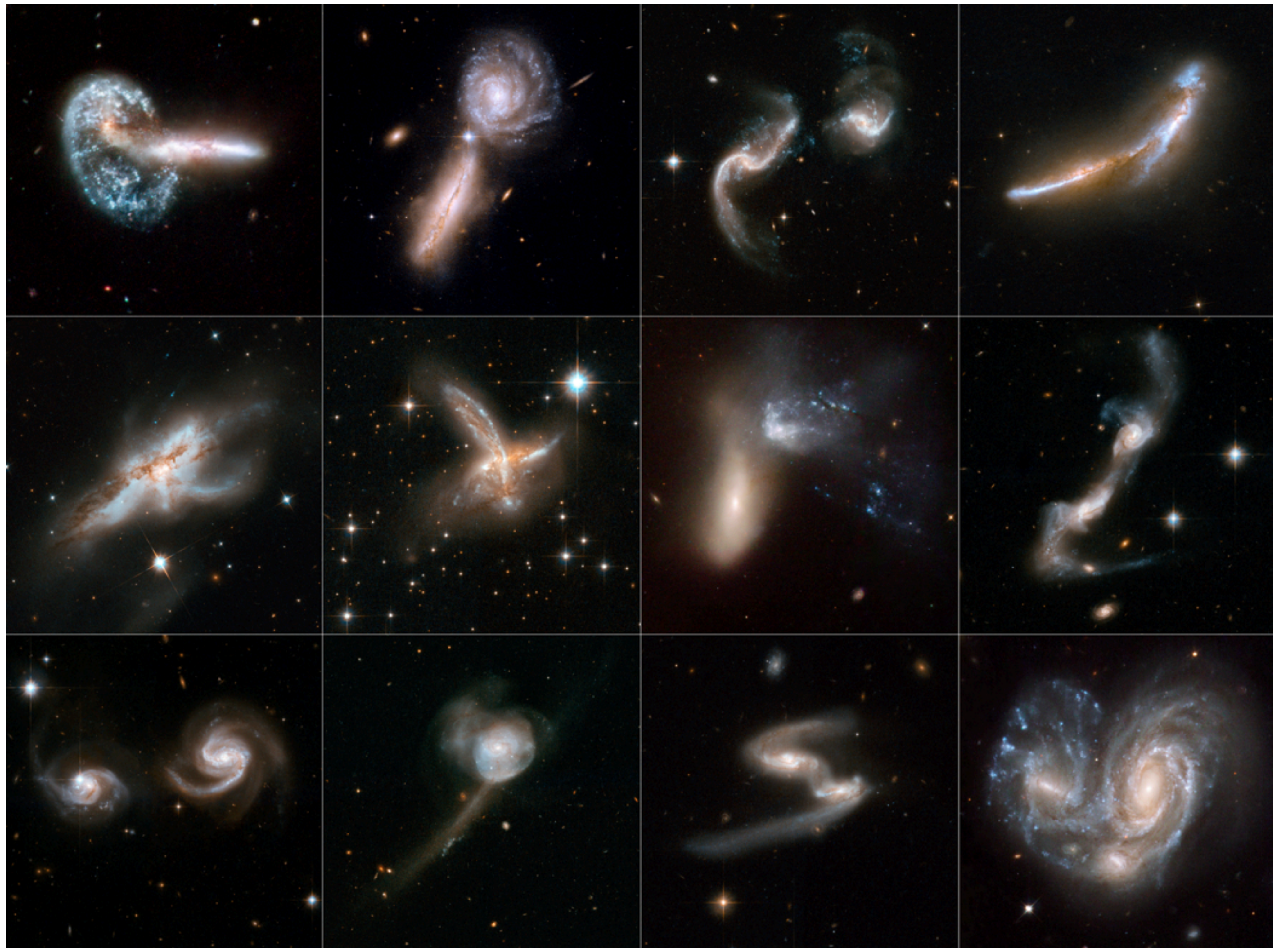
$$\mathrm{S/N} = \frac{\sum_{a<b} \rho_{ab}\Gamma_{ab}/\sigma^2_{ab,0}}{\sqrt{\sum_{c<d} \Gamma^2_{cd}/\sigma^2_{cd,0}}}$$

- inner product of measured and expected correlations ("**matched filter" statistic**)
- null distribution has zero mean, unit variance; but is not Gaussian

-unlikely due to noise alone (prob $\approx 1/10{,}000$) —> "evidence for"

Possible astrophysical interpretation





pairs of inspiraling supermassive black holes
(masses $\sim 10^9 M_\odot$; millions of such binaries)

- **environmental interactions remove GW power at low freqs, better fitting data**

Summary

The NANOGrav 15-year Data Set: Evidence for a Gravitational-Wave Background

The NANOGrav Collaboration

ABSTRACT

We report multiple lines of evidence for a stochastic signal that is correlated among 67 pulsars from the 15-year pulsar-timing data set collected by the North American Nanohertz Observatory for Gravitational Waves. The correlations follow the Hellings–Downs pattern expected for a stochastic gravitational-wave background. The presence of such a gravitational-wave background with a power-law–spectrum is favored over a model with only independent pulsar noises with a Bayes factor in excess of 10^{14}, and this same model is favored over an uncorrelated common power-law–spectrum model with Bayes factors of 200–1000, depending on spectral modeling choices. We have built a statistical background distribution for these latter Bayes factors using a method that removes inter-pulsar correlations from our data set, finding $p = 10^{-3}$ (approx. 3σ) for the observed Bayes factors in the null no-correlation scenario. A frequentist test statistic built directly as a weighted sum of inter-pulsar correlations yields $p = 5 \times 10^{-5}$–1.9×10^{-4} (approx. 3.5–4σ). Assuming a fiducial $f^{-2/3}$ characteristic-strain spectrum, as appropriate for an ensemble of binary supermassive black-hole inspirals, the strain amplitude is $2.4^{+0.7}_{-0.6} \times 10^{-15}$ (median + 90% credible interval) at a reference frequency of 1 yr^{-1}. The inferred gravitational-wave background amplitude and spectrum are consistent with astrophysical expectations for a signal from a population of supermassive black-hole binaries, although more exotic cosmological and astrophysical sources cannot be excluded. The observation of Hellings–Downs correlations points to the gravitational-wave origin of this signal.

stochastic signal, correlated among 67 pulsars

follows Hellings and Downs pattern expected for a stochastic gravitational-wave background

approx $3.5 - 4\ \sigma$

$f^{-2/3}$ characteristic-strain spectrum,

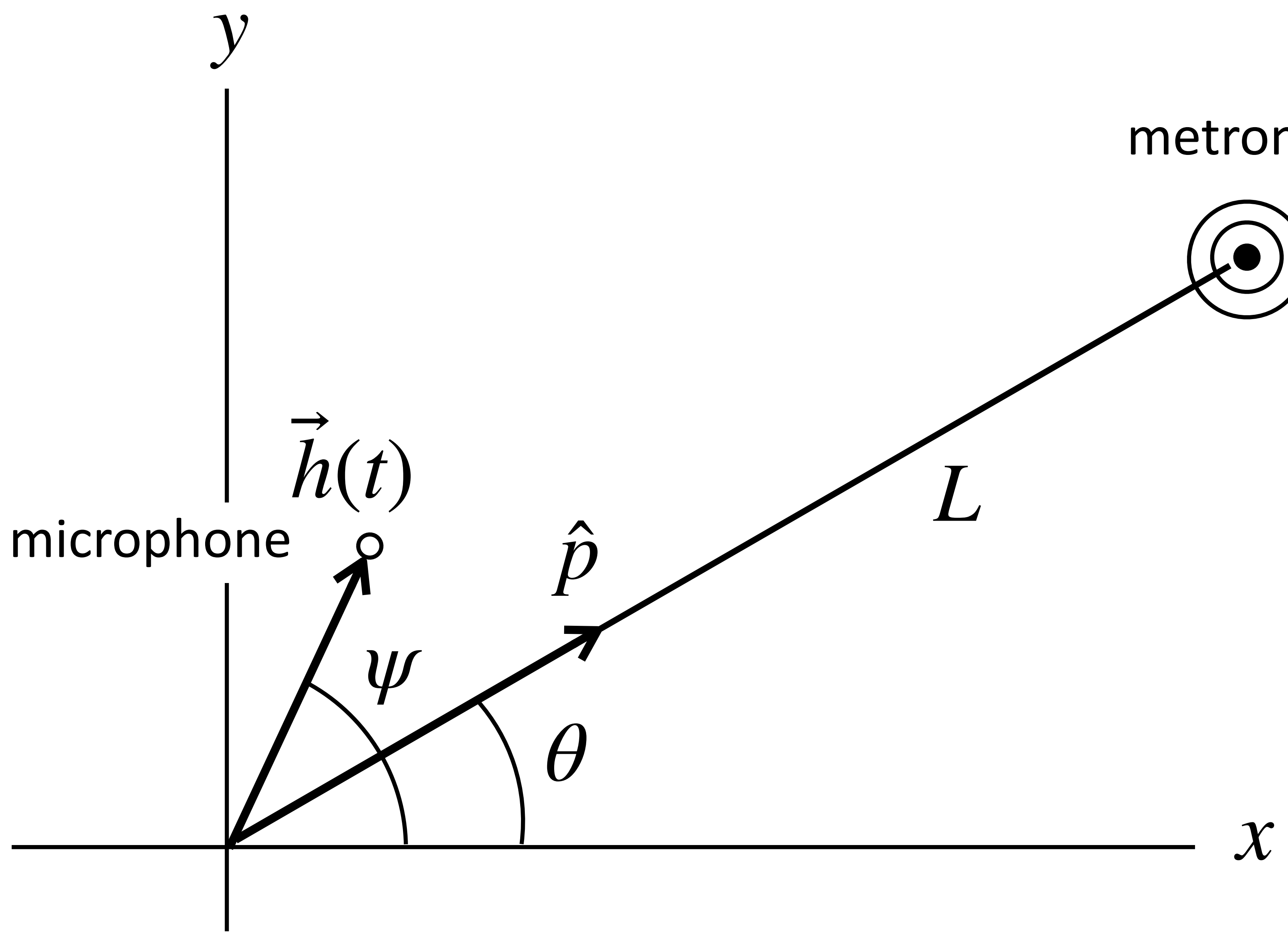
strain amplitude 2.4×10^{-15} at $f_{\mathrm{ref}} = 1/\mathrm{yr}$

population of supermassive black-hole binaries, …
more exotic cosmological cannot be excluded

Metronome timing array

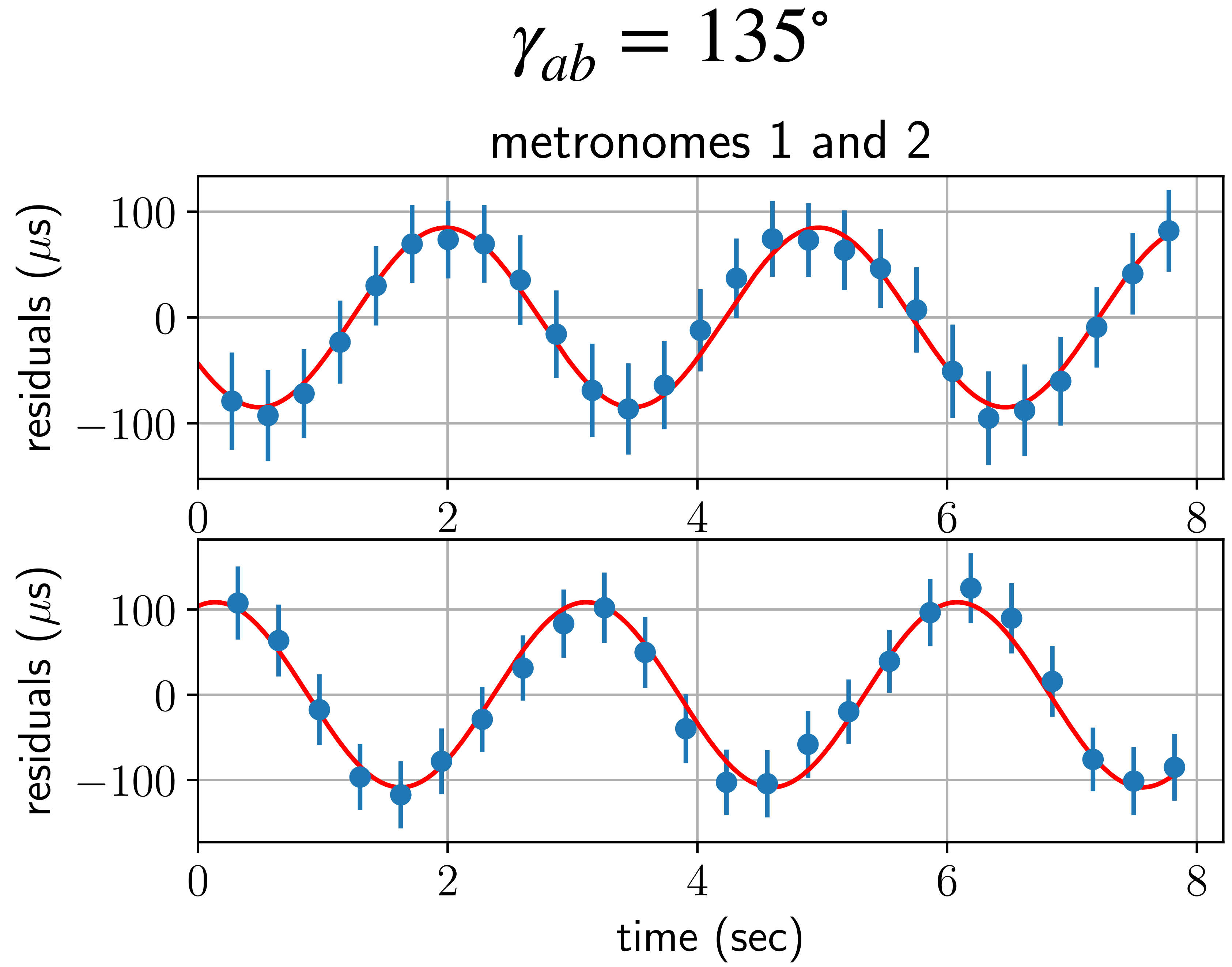
[AJP: Lam et al, 2018]

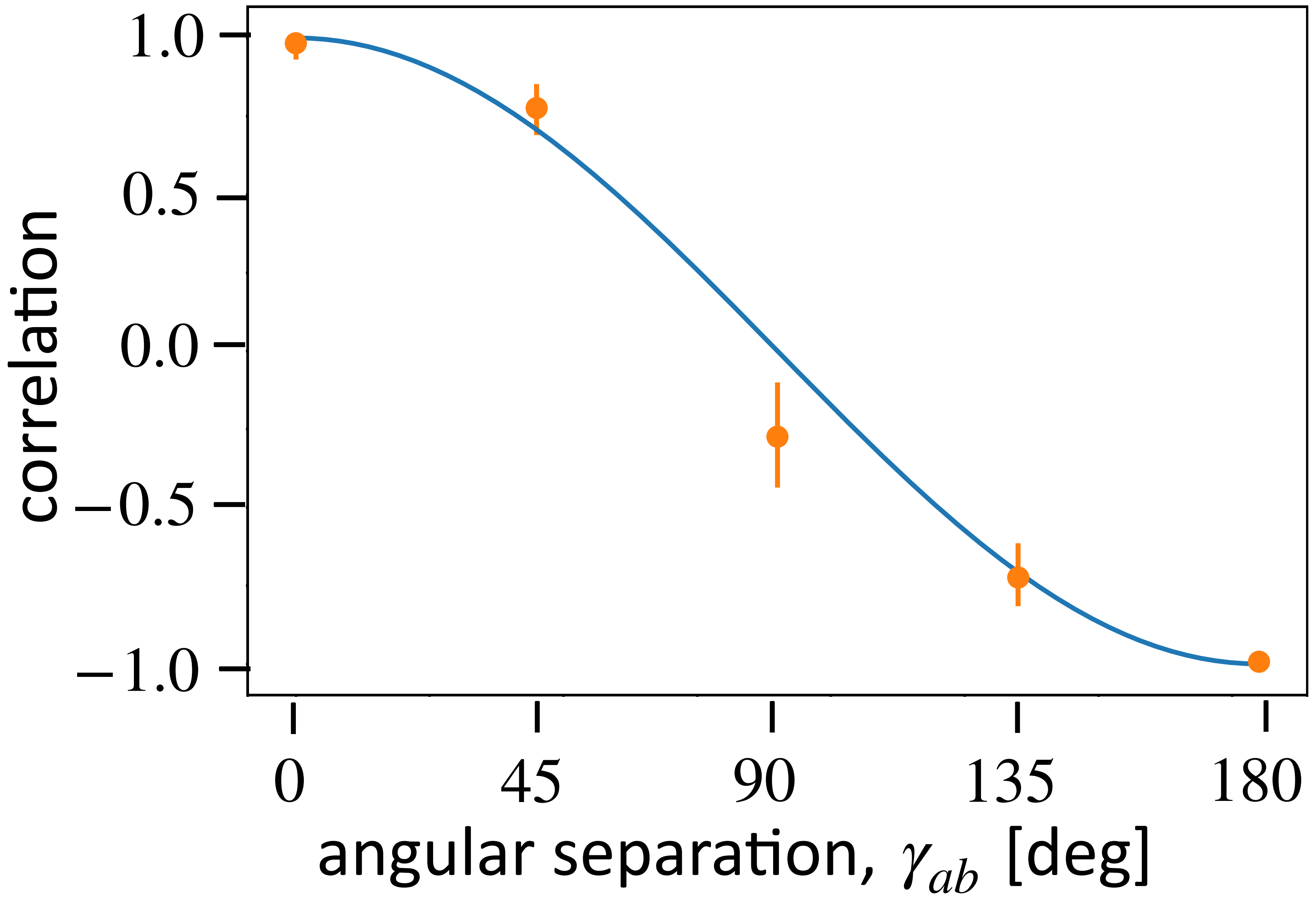
https://github.com/josephromano/pta-demo



$$\Delta T(t) = \frac{\Delta L(t)}{c_s} \simeq -\frac{\hat{p}\cdot\vec{h}(t)}{c_s}$$

Unif circular motion: $\Delta T_{a,b}(t) = -\frac{A}{c_s}\cos(2\pi f_0 t + \phi_0 - \theta_{a,b})$

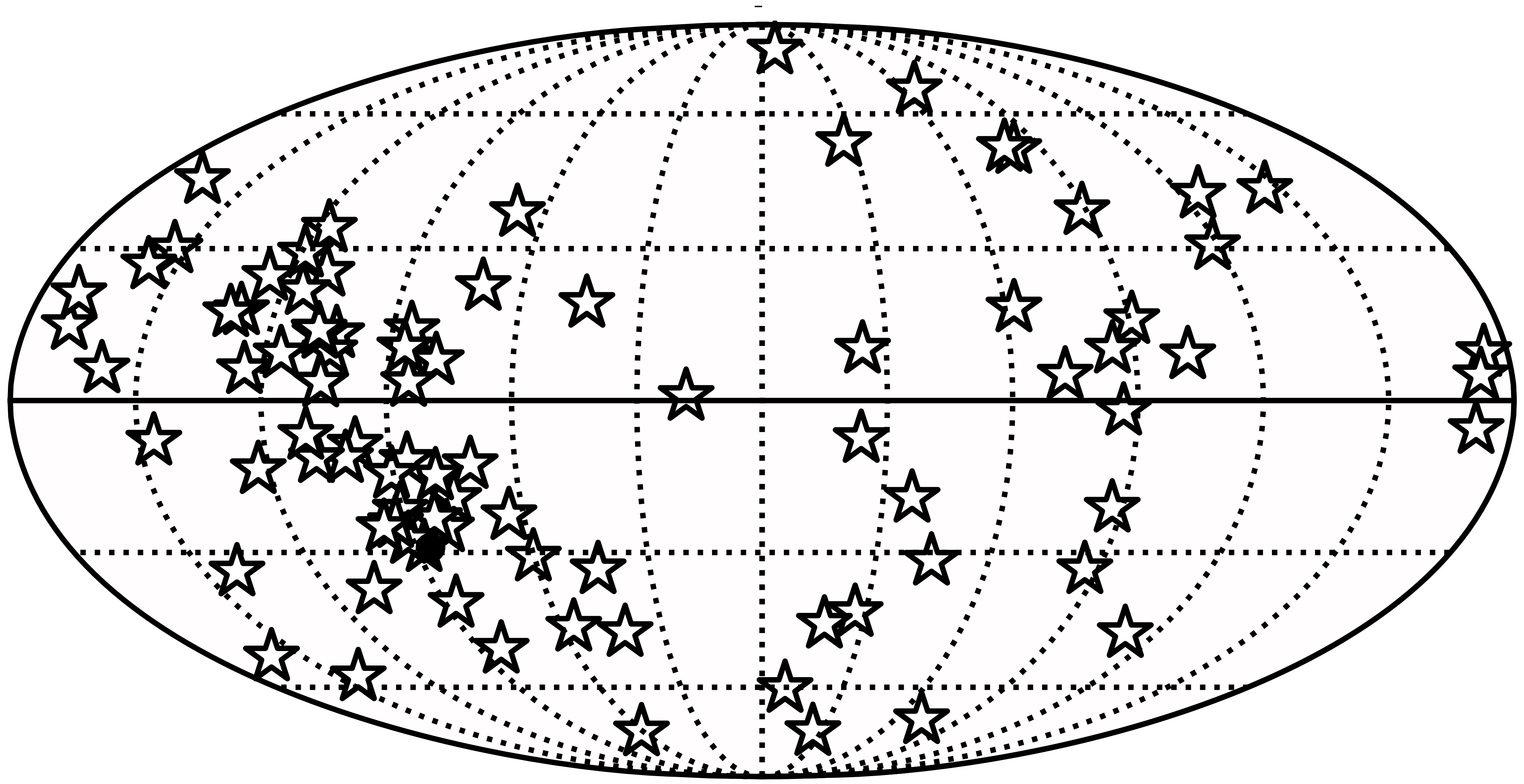




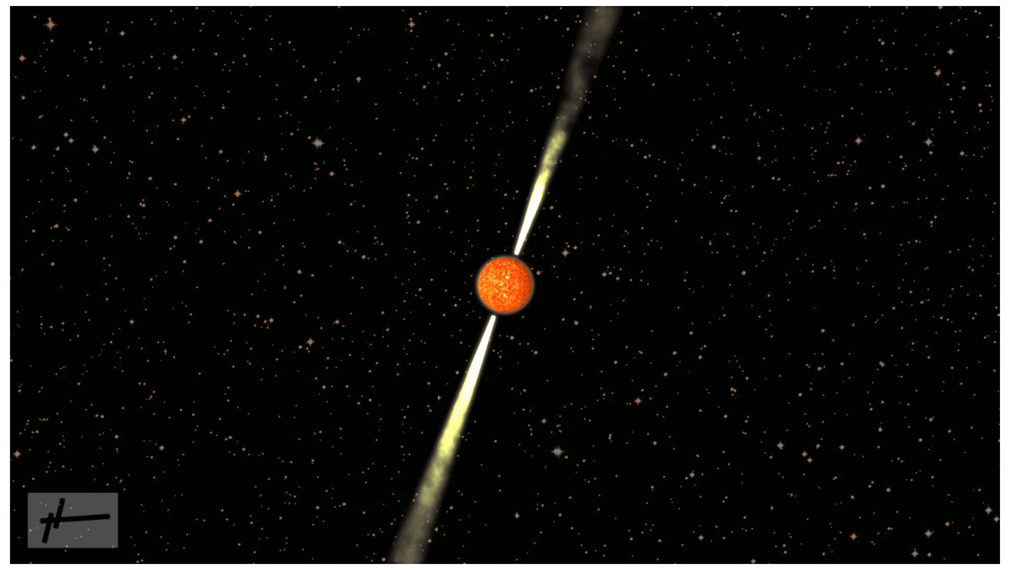
extra slides

NANOGrav + EPTA + PPTA + InPTA (Indian Pulsar Timing Array) = IPTA (International Pulsar Timing Array)

sky map of 88 IPTA millisecond pulsars

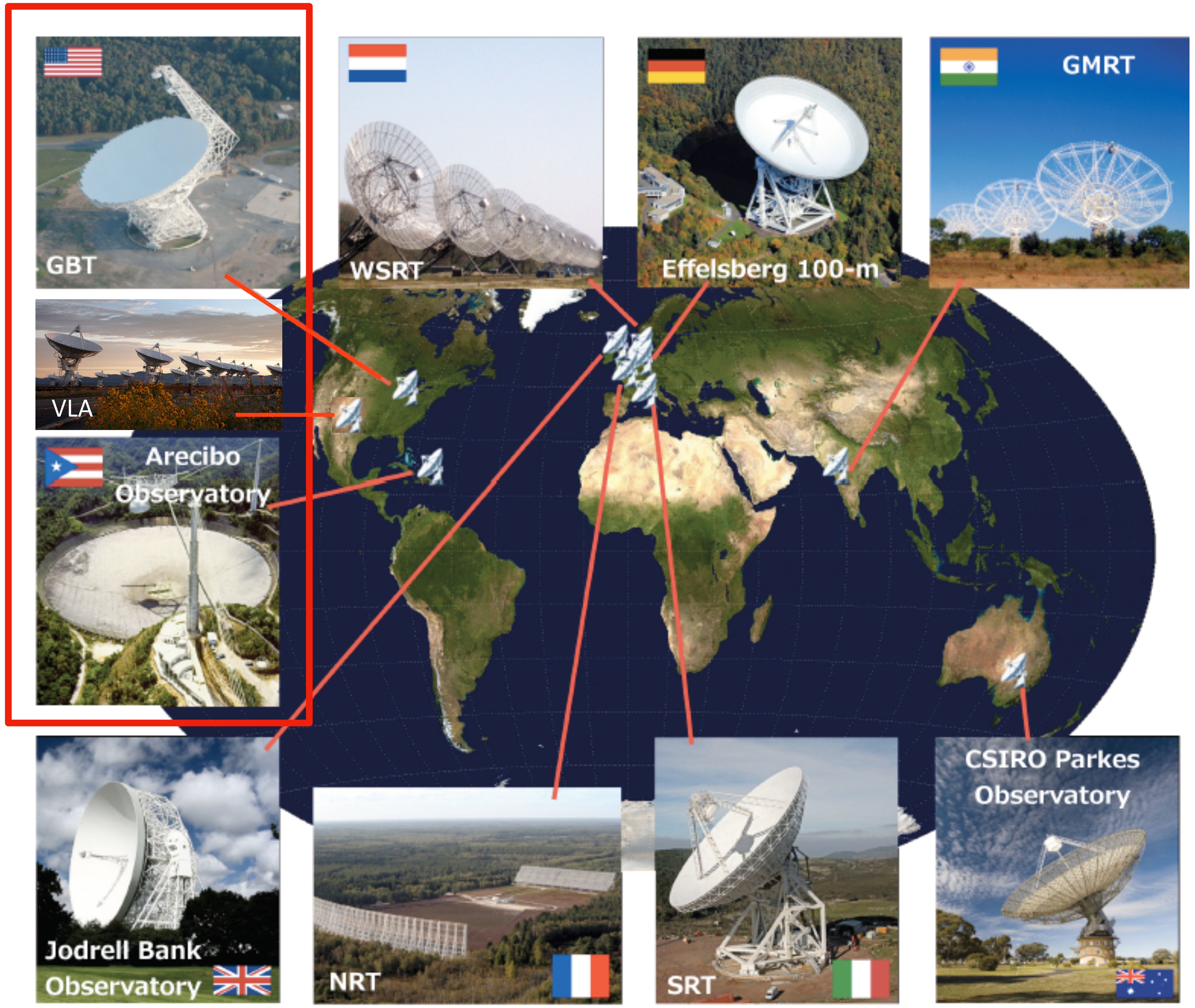


Rapidly rotating neutron star; strong magnetic field; narrow beam of radiation



Nature's most precise clocks!

$(\Delta T_p / T_p < 10^{-14})$



Optimal binned HD estimator

- Form general linear combination of pulsar pairs within each angular bin (labeled by j) with $\gamma_j = \mathrm{avg}(\gamma_{ab})$ in the bin:

$$\hat{\Gamma}_j \equiv \sum_{ab\in j} \rho_{ab} w_{ab} \quad \text{where} \quad \rho_{ab} = \overline{Z_a(t) Z_b(t)} \quad \text{with} \quad \langle \rho_{ab} \rangle = A_{\rm gw}^2 \Gamma_{ab}$$

- Determine weights such that:

  1. $\langle \hat{\Gamma}_j \rangle = \Gamma(\gamma_j)$ (unbiased)
  2. $\sigma_j^2 \equiv \langle \hat{\Gamma}_j^2 \rangle - \langle \hat{\Gamma}_j \rangle^2$ is minimized

- These lead to

$$w_{ab} = \frac{\Gamma(\gamma_j)}{A_{\rm gw}^2} \frac{\sum_{cd\in j} C^{-1}_{ab,cd} \Gamma_{cd}}{\sum_{ef\in j} \sum_{gh\in j} \Gamma_{ef} C^{-1}_{ef,gh} \Gamma_{gh}} \quad \text{where} \quad C_{ab,cd} \equiv \langle \rho_{ab} \rho_{cd} \rangle - \langle \rho_{ab} \rangle \langle \rho_{cd} \rangle$$

- Optimal binned estimator to the binned HD correlation:

$$\hat{\Gamma}_j = \frac{\Gamma(\gamma_j)}{A_{\rm gw}^2} \frac{\sum_{ab\in j} \sum_{cd\in j} \rho_{ab} C^{-1}_{ab,cd} \Gamma_{cd}}{\sum_{ef\in j} \sum_{gh\in j} \Gamma_{ef} C^{-1}_{ef,gh} \Gamma_{gh}} \quad \text{with} \quad \sigma_j^2 = \frac{\Gamma^2(\gamma_j)}{A_{\rm gw}^4} \frac{1}{\sum_{ef\in j} \sum_{gh\in j} \Gamma_{ef} C^{-1}_{ef,gh} \Gamma_{gh}}$$

- Optimal binned estimator is used to test for consistency with GWB model and includes GW-induced covariances between pulsar pairs; it is not a detection statistic