

# Pulsar Timing Arrays and Gravitational Waves

Gilles Theureau,  
LPC2E/CNRS and Observatoire de Paris

# Context

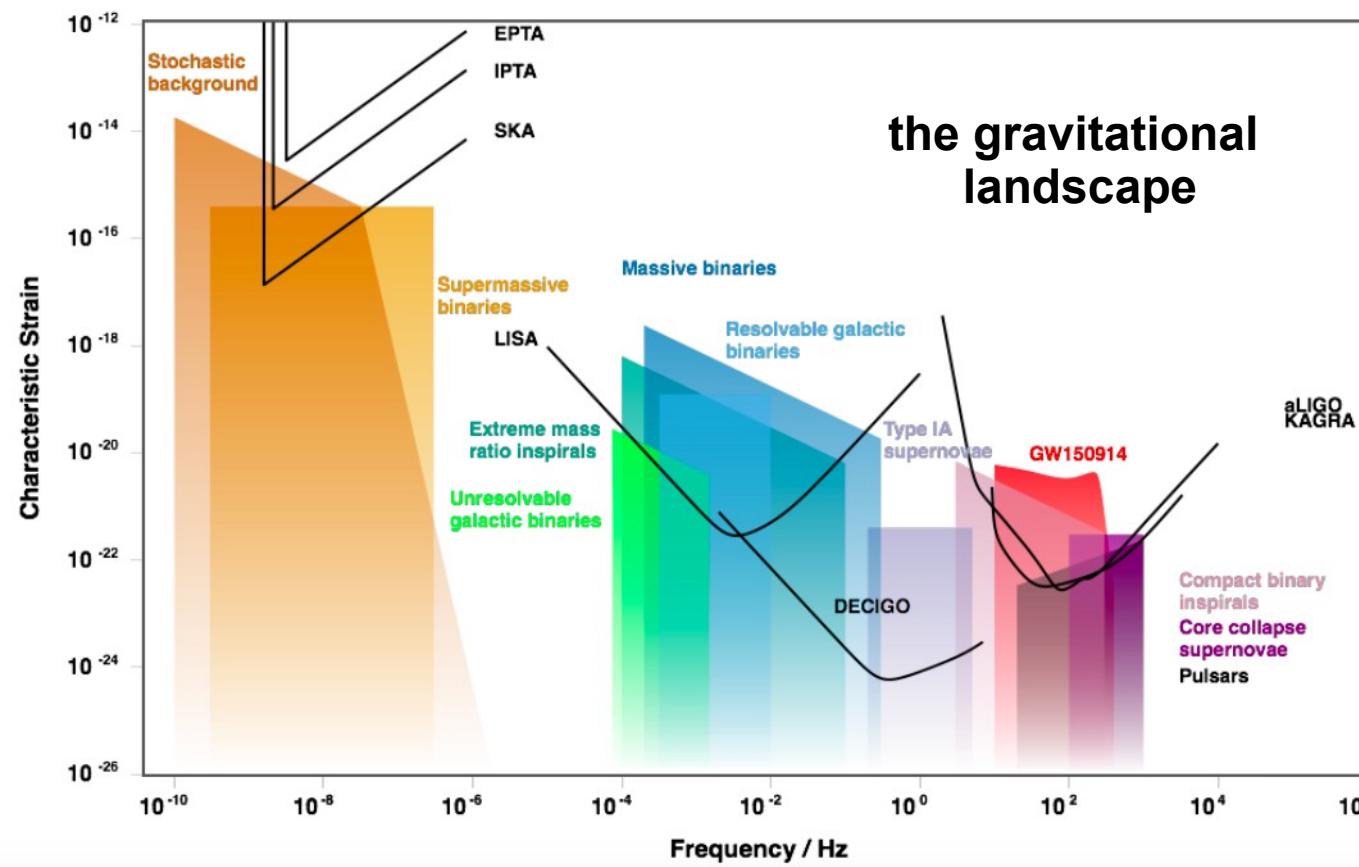
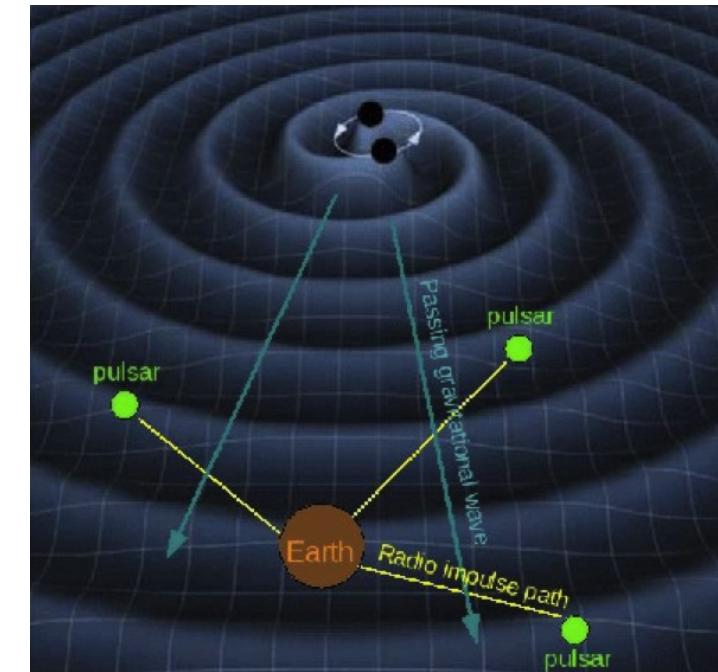
NANOGrav : Arzoumanian et al, December 2020

PPTA : Goncharov et al, August 2021

EPTA : Chen et al, December 2021

IPTA : Antoniadis et al, March 2022

## « On the Evidence for a Common-spectrum Process in the Search for the nHz Gravitational-wave Background »



### *The nanoHertz domain*

- Super Massive Black Hole Binaries (SMBHB)
- Cosmic string loops
- Relics of inflation
  - (e.g. quantum fluctuations of the gravitational field in the early universe, amplified by an inflationary phase)
- First-order phase transition
  - (e.g. due to MHD turbulence induced by primordial mag field)

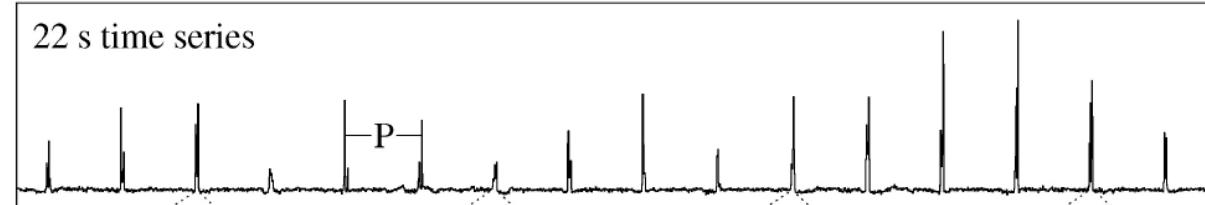
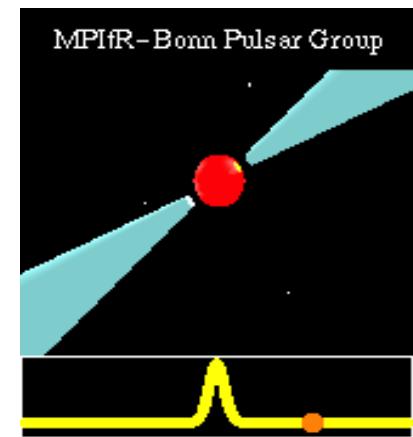
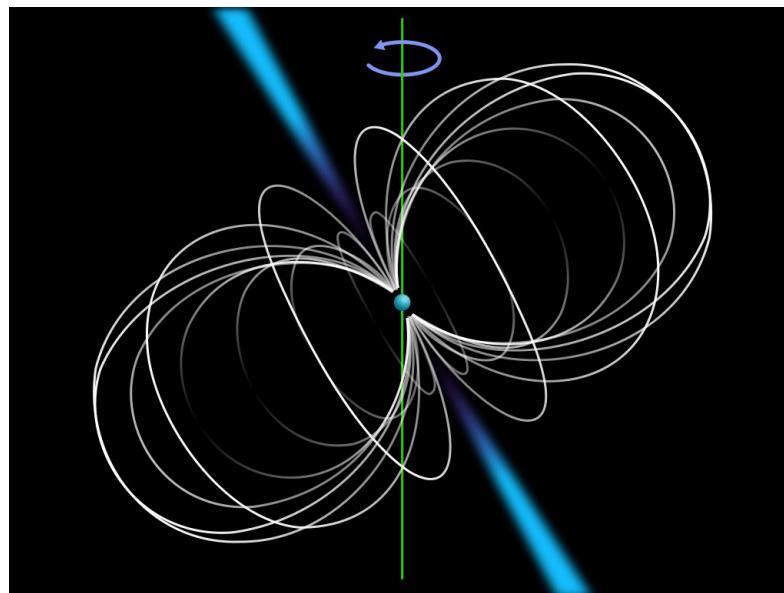
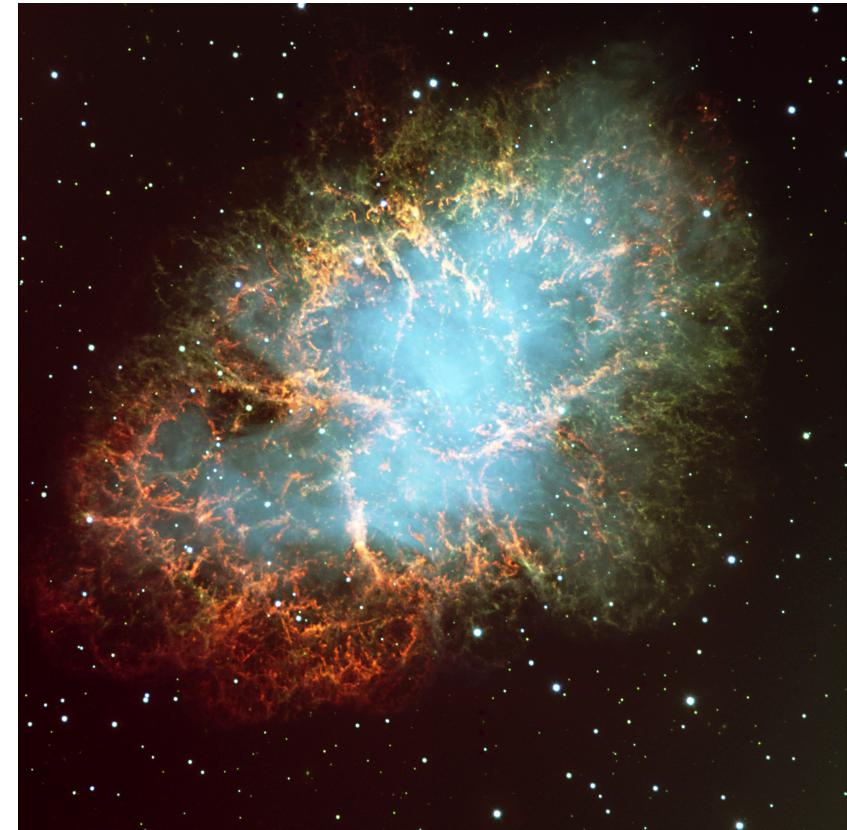
# Pulsars = fastly rotating neutron stars

Supernova explosion of a massive star ( $> 9 M_{\text{sun}}$ )

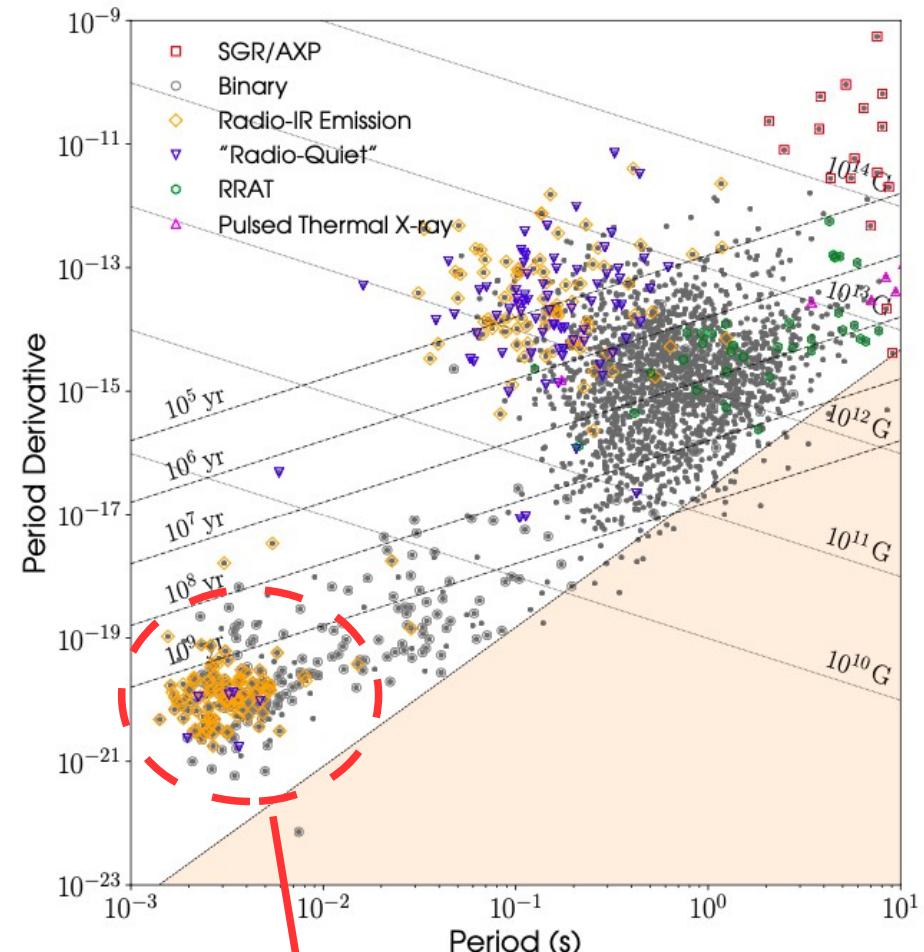
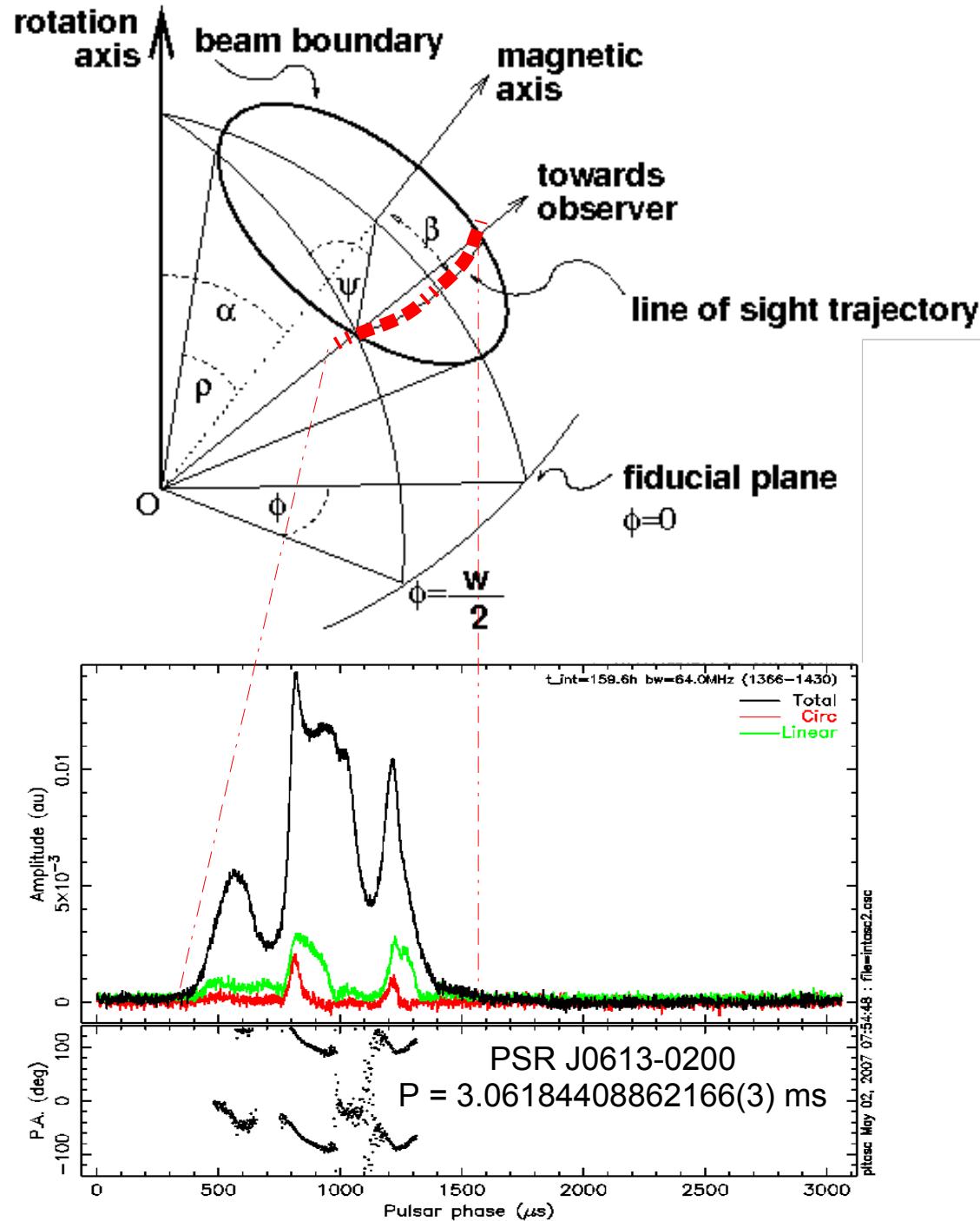
Core collapse in a neutron star of  $1.3\text{-}2.2 M_{\text{sun}}$

Huge magnetic field:  $10^8 \text{ - } 10^{14}$  Gauss

Rotation periods: 0.001-10 seconds



# Pulsars = fastly rotating neutron stars

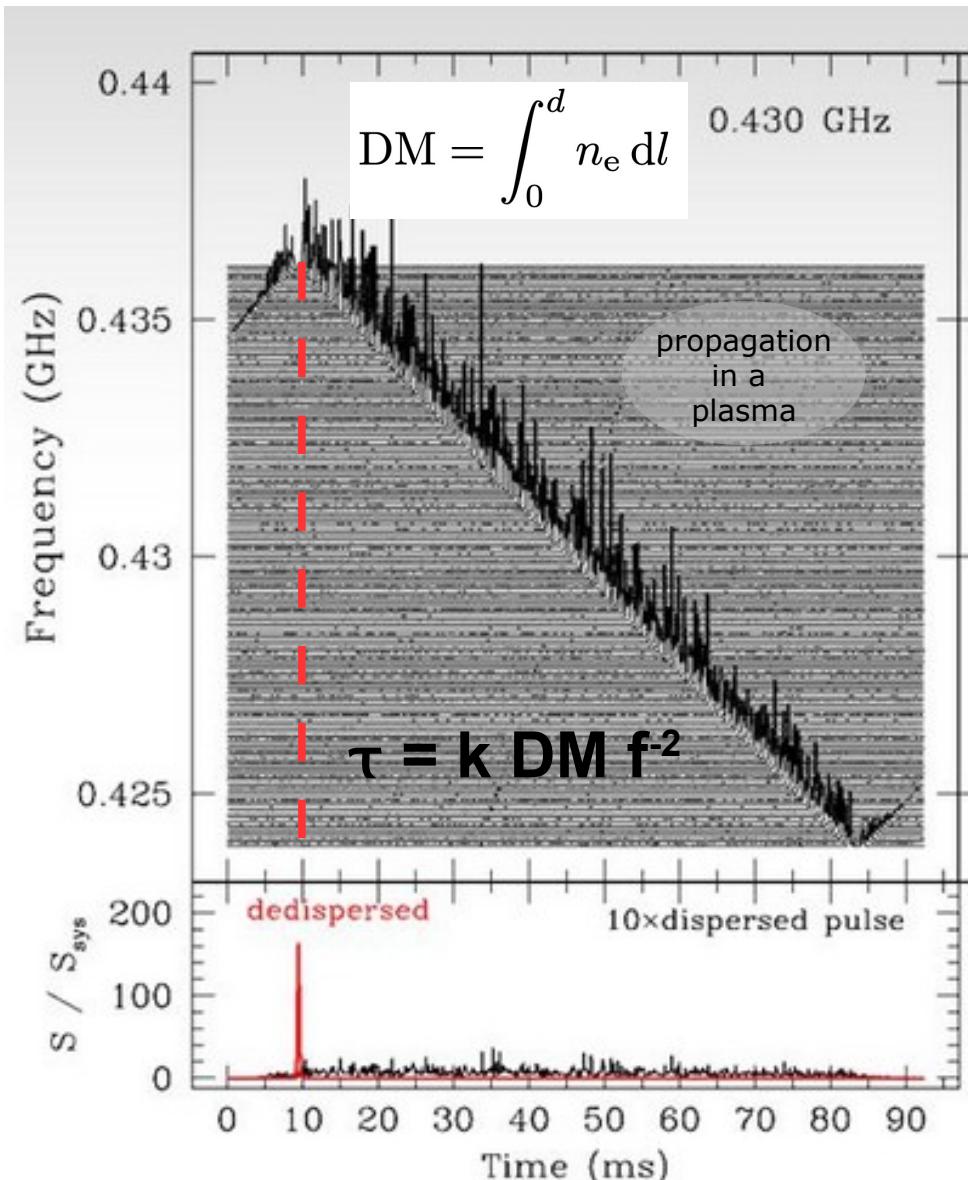


Millisecond pulsar population  
very stable rotation < 1  $\mu$ s rms over years

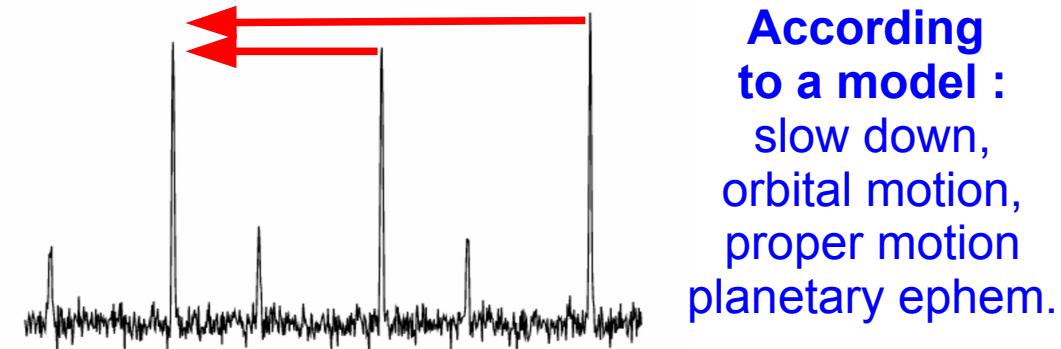
# The art of timing

## I – the de-dispersion problem

The lowest frequencies are delayed

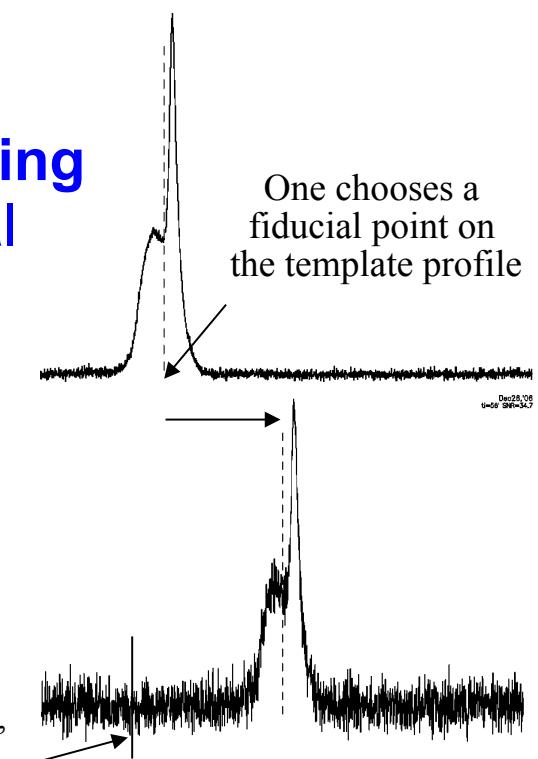


## II- phase folding with rotation



## III – Time stamping (Time of arrival computation)

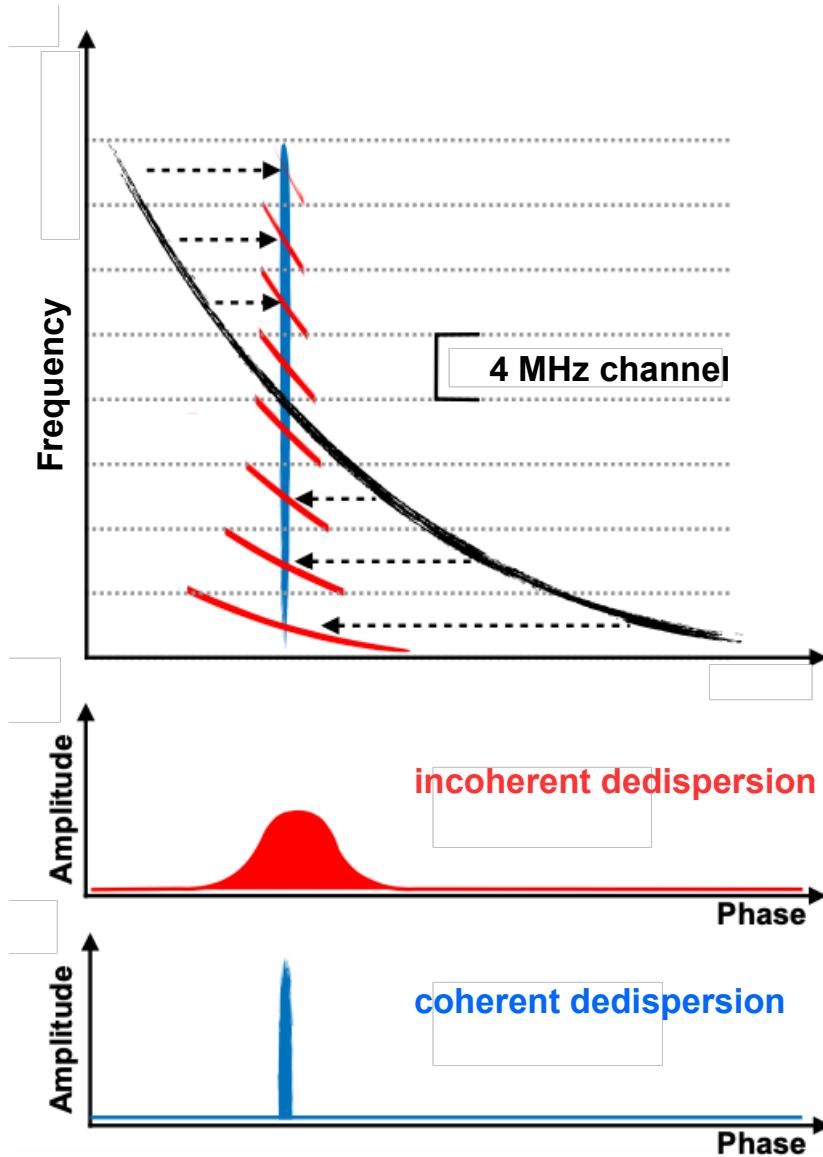
« TOA »



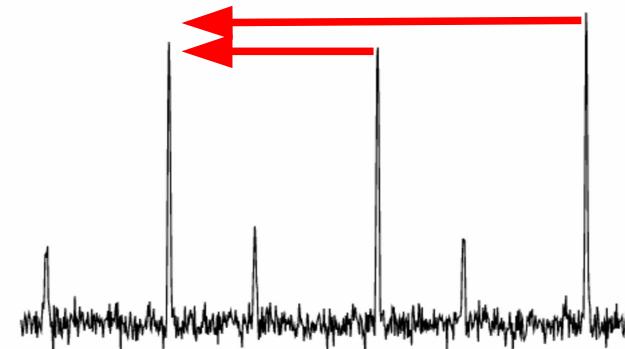
# The art of timing

## I – the de-dispersion problem

The lowest frequencies are delayed



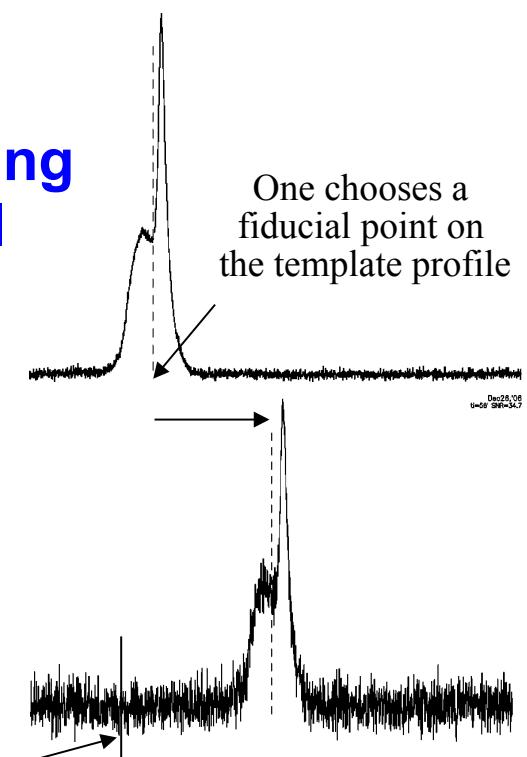
## II- phase folding with rotation



According to a model :  
slow down,  
orbital motion,  
proper motion  
planetary ephem.

## III – Time stamping (Time of arrival computation)

« TOA »



Position of the first data sample,  
corresponding to start  
of observation

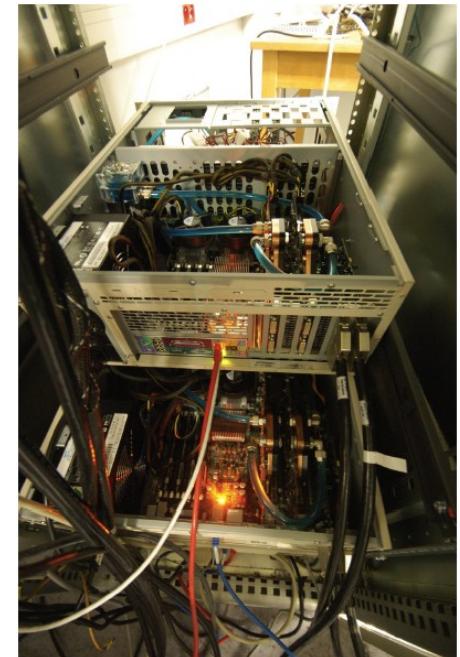
## Looking for extreme timing precision

the timing uncertainty can go down  
to 10-20 ns for some pulsars.

$$\sigma_{\text{TOA}} \propto \frac{w}{S_{\text{PSR}}} \frac{T_{\text{sys}}}{A} \frac{1}{\sqrt{BT}}$$

**Weak fluxes ~mJy (1 Jy =  $10^{-26}$  W/m<sup>2</sup>)**

- **requires wide band pass in frequency**
- **requires a large radio telescope**



**Current instrumentation in Nançay:**  
Coherent dedispersion over 512 MHz  
**4 PCs / 8 GPUs** (16 Gb / s flux)

## NRT : Nançay decimetric Radio Telescope

7000 m<sup>2</sup> ~ 94 m circular dish

1.1- 3.5 GHz



# The International Pulsar Timing Array

Effelsberg



Jodrell



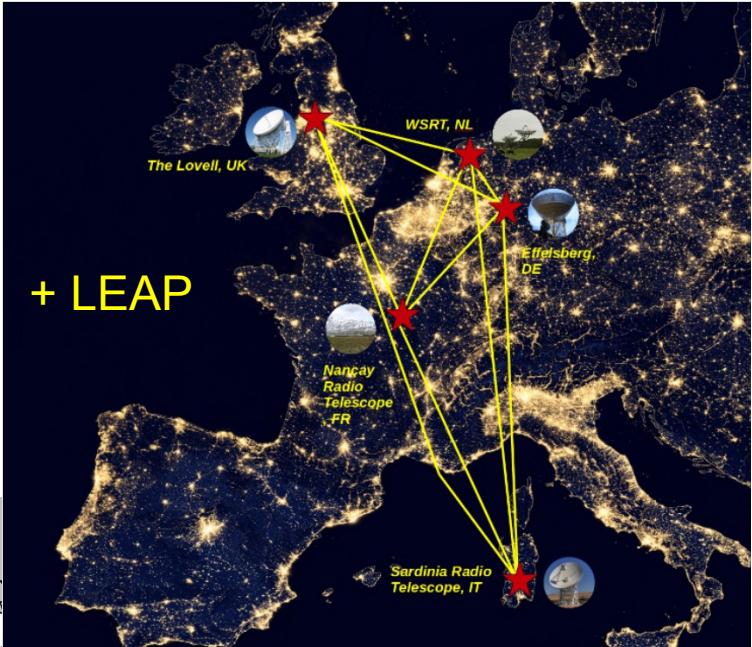
Westerbork



NRT



SRT



Green Bank



CHIME



VLA



Arecibo



EPTA  
(5 radio telescopes)



NANOGrav



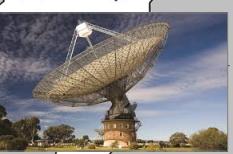
MeerKAT



InPTA



FAST



Parkes

PPTA

# Pulsar Timing Arrays : principles

The Earth and the distant pulsar  
are considered as free masses  
whose position responds to changes  
in the metric of space-time

→ *The passage of a gravitational wave  
disturbs the metric and produces fluctuations  
in the arrival times of the pulses*



© Tonia Klein

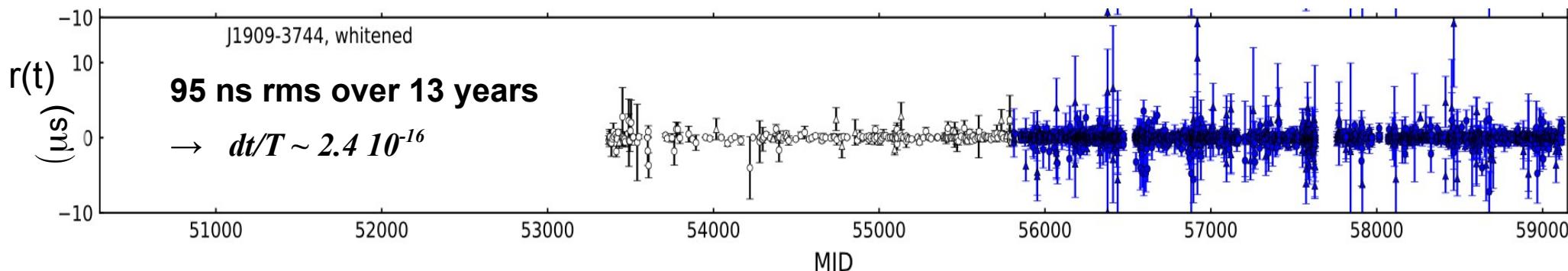
With timing uncertainties  $dt$  ( $\sim 100$  ns) and observation time spans  $T$  ( $\sim 25$  years)  
→ PTA are sensitive to *amplitudes*  $\sim dt/T$  and to *frequencies*  $f \sim 1/T$

$$\text{Sensitivity} \sim 100 \ 10^{-9} / 25 \times 3 \ 10^7$$

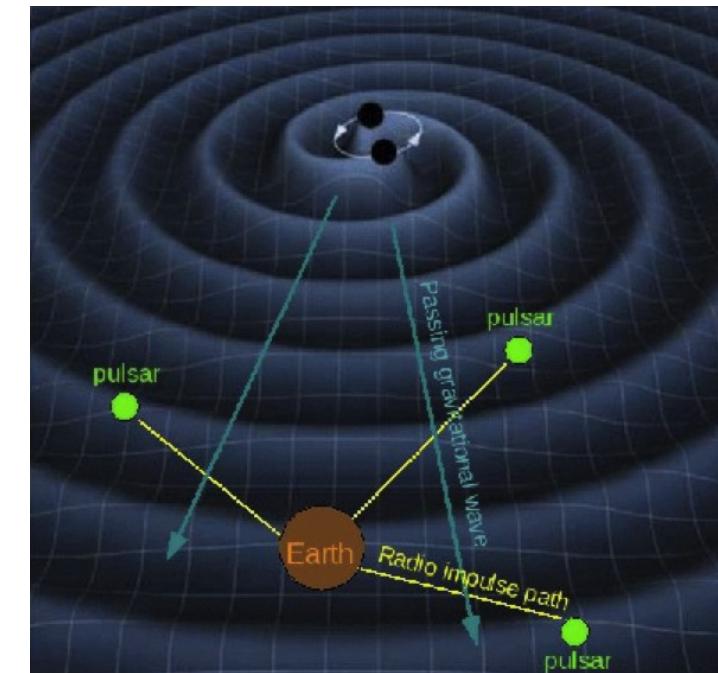
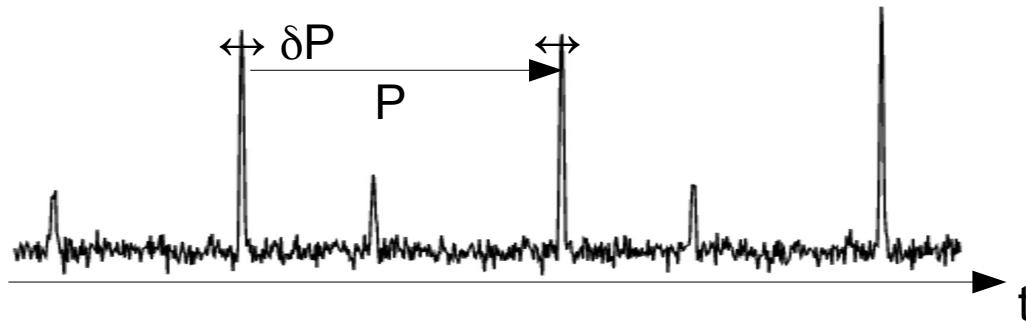
$$\rightarrow A \sim 1.3 \ 10^{-16}$$

Frequency domain (25 years - 1 week)

$$\rightarrow 10^{-9} - 10^{-6} \text{ Hz}$$



# Pulsar Timing Arrays : principles



## Analysis of time residuals

$$r(t) = \int_0^t \frac{\delta\nu}{\nu}(t') dt'$$

pulsar-Earth distance

$$\frac{\delta\nu}{\nu}(t) = \frac{1}{2} \frac{\hat{n}^i \hat{n}^j}{1 + \hat{n} \cdot \hat{k}} \left( h_{ij}(t - \overline{L}(1 + \hat{k} \cdot \hat{n})) - h_{ij}(t) \right)$$

dir pulsar
wave amplitude at the pulsar
wave amplitude at the Earth

dir GW source

# Pulsar Timing Arrays : principles

## 1) Describe the pulsar rotation in a reference frame co- moving with the pulsar

$$\nu(t) = \nu_0 + \dot{\nu}_0(t - t_0) + \frac{1}{2}\ddot{\nu}_0(t - t_0)^2 + \dots$$

The observed parameters  $\nu$  and  $\dot{\nu}$  are associated with the physical processes causing pulsars to spin down

## 2) Timing model

$$t_{SSB} = \overbrace{t_{topo} + t_{corr}}^{\text{clock}} - \delta D/f_{obs}^2 + \overbrace{\Delta_{R\odot} + \Delta_\pi + \Delta_{S\odot} + \Delta_{E\odot}}^{\text{Solar System}} + \overbrace{\Delta_R + \Delta_S + \Delta_E + \Delta_A}^{\text{binary system}}$$

$\tau^{\text{TM}}$

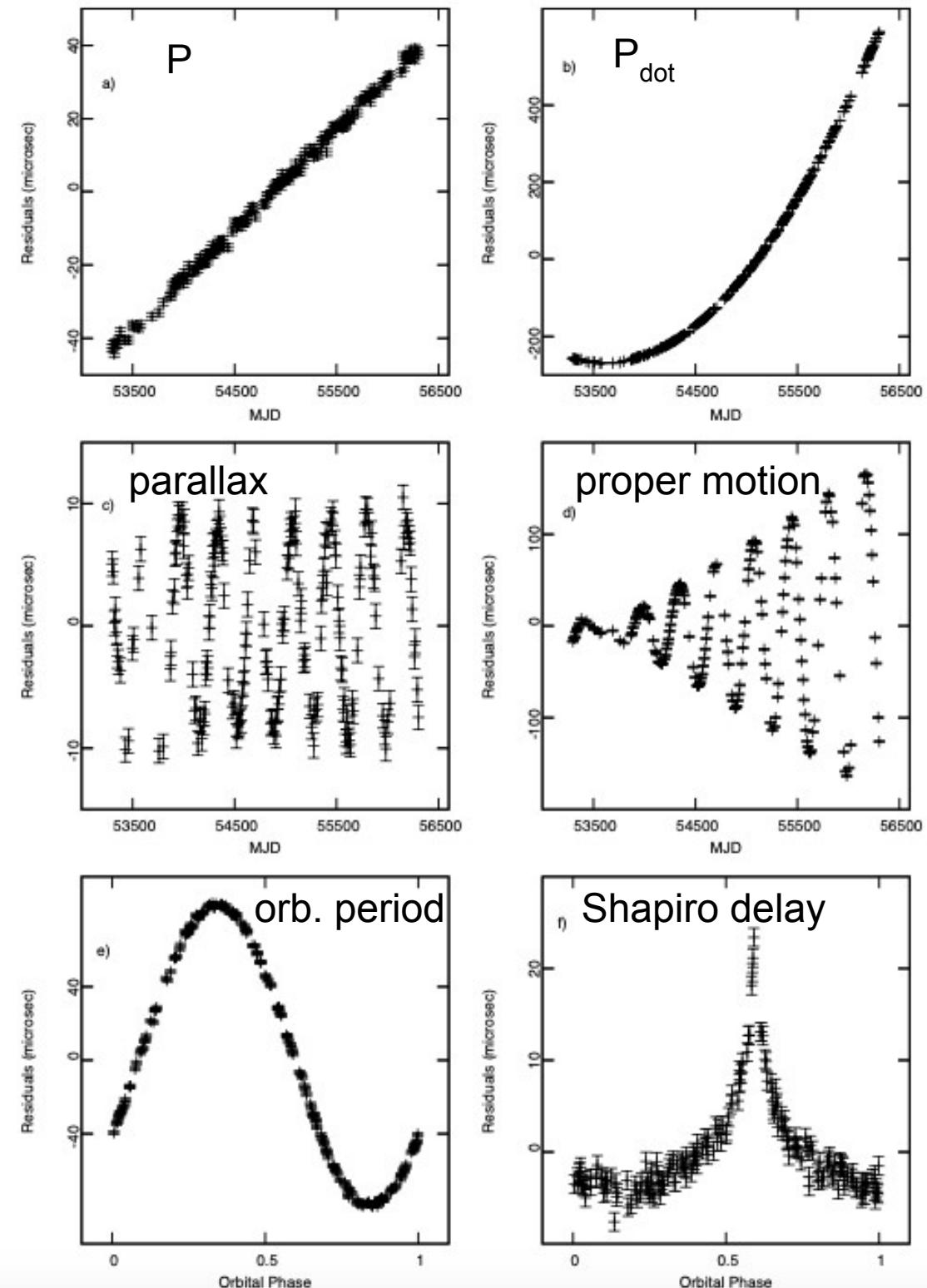
Solar System  
Römer, parallax, Shapiro  
and Einstein delays

binary system  
Römer, Shapiro, Einstein  
and Aberration delays

# Pulsar Timing Arrays : principles

## 2) Timing model

(examples)



# Pulsar Timing Arrays : principles

## 1) Describe the pulsar rotation in a reference frame co-moving with the pulsar

$$\nu(t) = \nu_0 + \dot{\nu}_0(t - t_0) + \frac{1}{2}\ddot{\nu}_0(t - t_0)^2 + \dots$$

The observed parameters  $\nu$  and  $\dot{\nu}$  are associated with the physical processes causing pulsars to spin down

## 2) Timing model

$$t_{SSB} = \overbrace{t_{topo} + t_{corr}}^{\text{clock}} - \delta D/f_{obs}^2 + \underbrace{\Delta_{R\odot} + \Delta_\pi + \Delta_{S\odot} + \Delta_{E\odot}}_{\substack{\text{Solar System} \\ \text{Römer, parallax, Shapiro} \\ \text{and Einstein delays}}} + \underbrace{\Delta_R + \Delta_S + \Delta_E + \Delta_A}_{\substack{\text{binary system} \\ \text{Römer, Shapiro, Einstein} \\ \text{and Aberration delays}}}$$

## 3) Full noise model

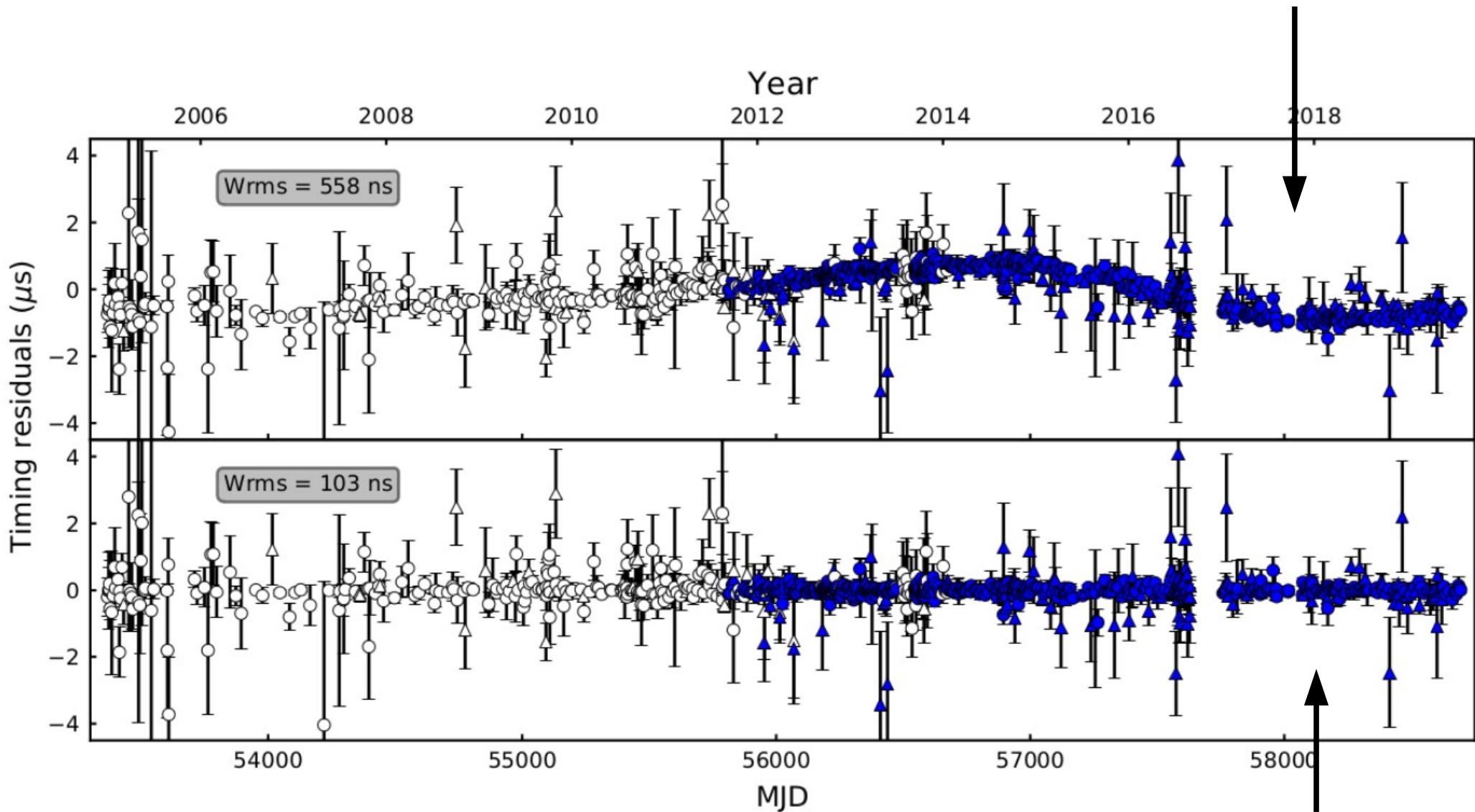
$$\text{observed TOA} = \tau^{\text{TM}} + \tau^{\text{WN}} + \tau^{\text{SN}} + \tau^{\text{DM}} + \tau^{\text{CN}} + \tau^{\text{GW}}$$

Timing Model (deterministic)	meas. (white) noise	pulsar spin (red) noise	DM (red) noise	Clock Ephem. Astroph. (red) noise	GWB (red) noise
---------------------------------	---------------------------	-------------------------------	----------------------	--	-----------------------

# Pulsar Timing Arrays : principles

Example : PSR J1909-3744

including timing model



including timing model + noise model

## Analysis of foregrounds: characterisation and separation of the noise components

**« White noises » (un-correlated noise)**  $\hat{\sigma}^2 = (\sigma \cdot \text{EFAC})^2 + \text{EQUAD}^2$

Instrumental → telescope gain stability, pass band, backend used

Astrophysical → 'pulse jitter' (statistics of variations in pulsar magnetosphere)

**« Red noises » (correlated noise)**  $S \propto A^2 f^{-\gamma}$

$\tau^{\text{DM}}$  Variations in the Dispersion Measure → changes « e- » content along line of sight  
(chromatic : multi-frequency measurements)

$\tau^{\text{SN}}$  Intrinsic rotation noise → perturbation from small bodies disc ?  
variations in radiated energy ? series of micro-glitches ?

Clock variations → clock-telescope link → TAI → TT-BIPM

$\tau^{\text{CN}}$  Solar System ephemerides → position of SS barycentre → links to INPOP, JPL

Galactic motion of the Sun → LSR

$\tau^{\text{GW}}$  Gravitational waves → indiv. sources, stochastic background, « bursts » events

## Pulsar Timing Arrays : principles

Pulse jitter

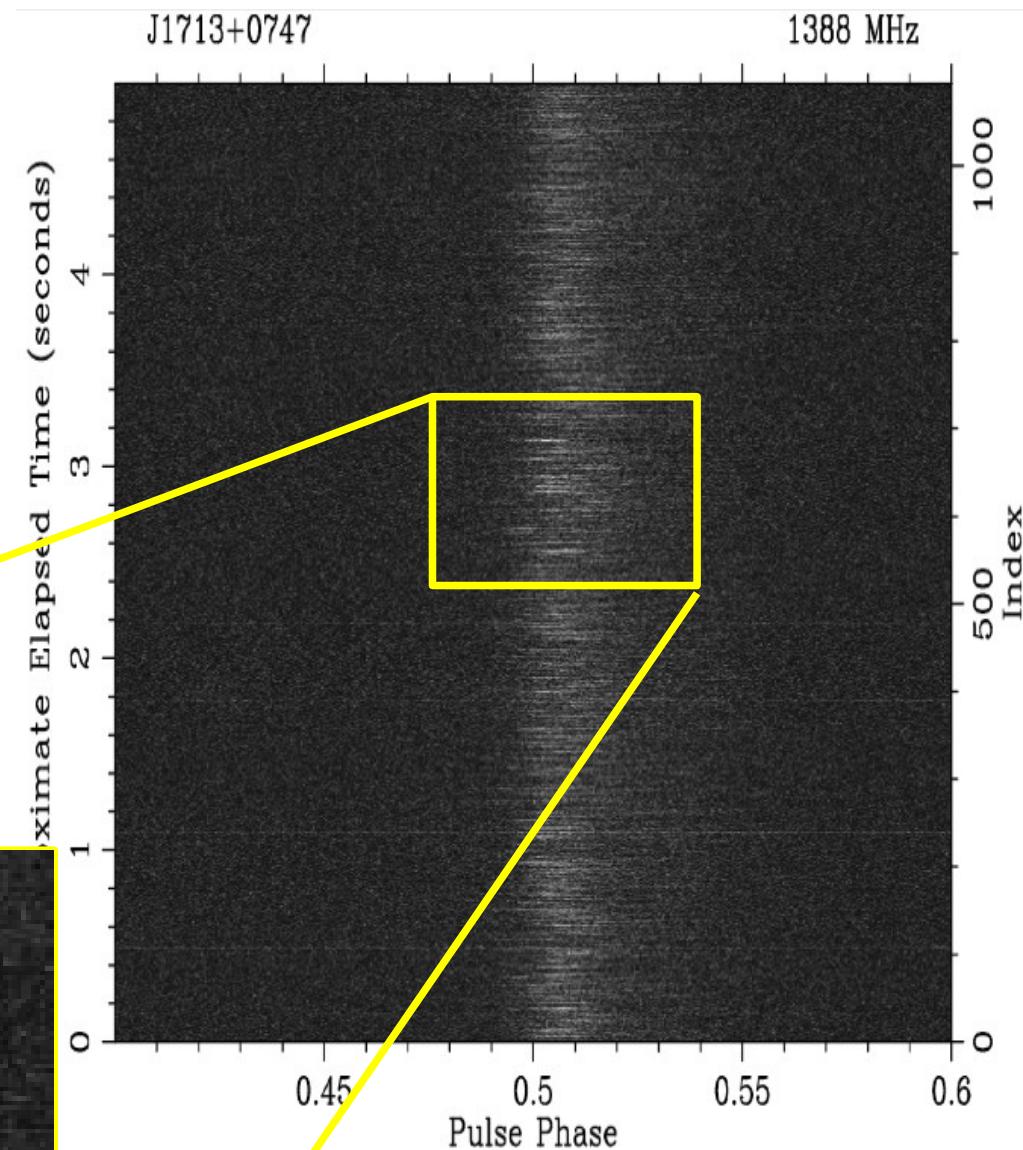
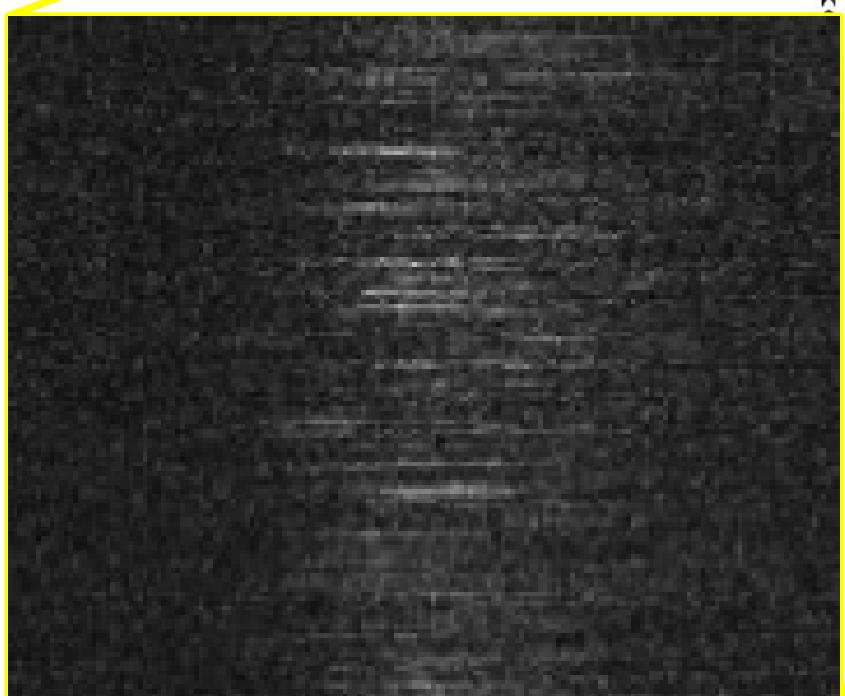


**PSR B1919+21**  
**P = 1.3 s**

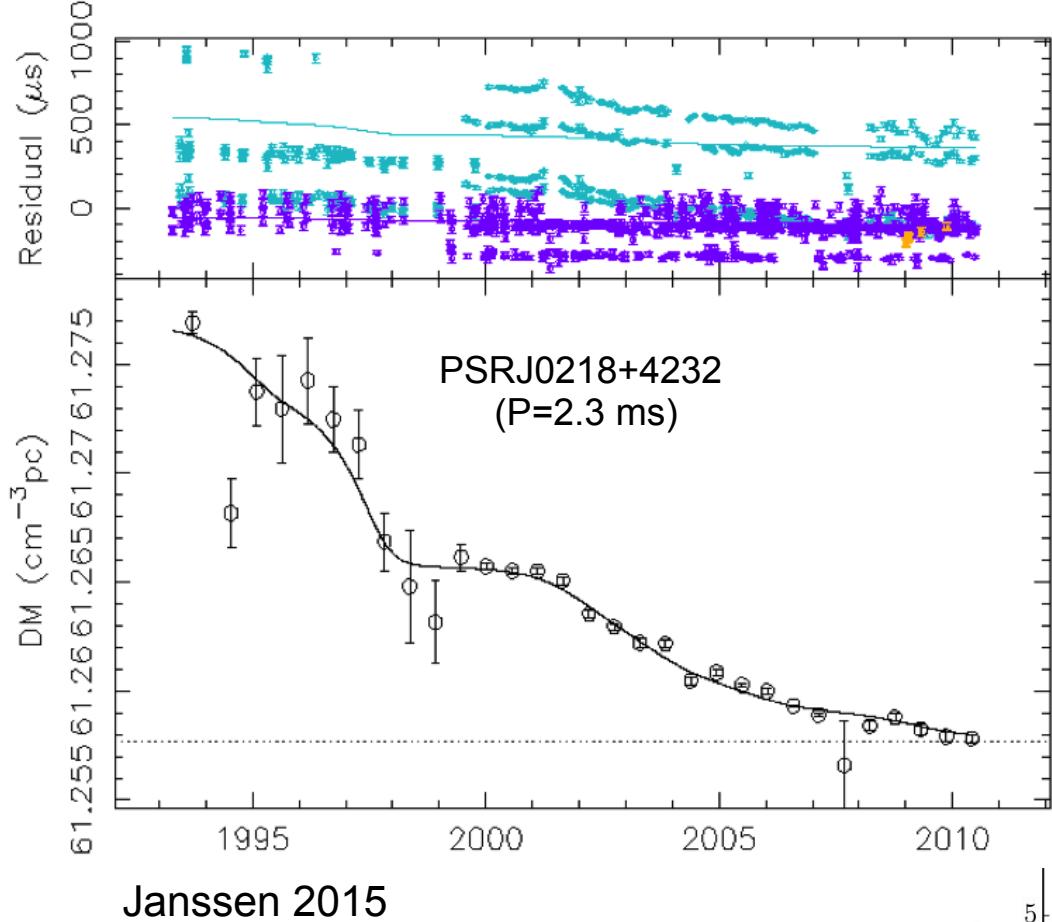
# Pulsar Timing Arrays : principles

## Pulse jitter

PSRJ1713+0747  
 $P = 4.57 \text{ ms}$   
LEAP Observations  
'pulse to pulse' variations  
(Bassa et al 2015)  
1% in phase  $\leftrightarrow \sim 100 \text{ ns}$  over 1 h



# Red noise : dispersion noise or chromatic noise



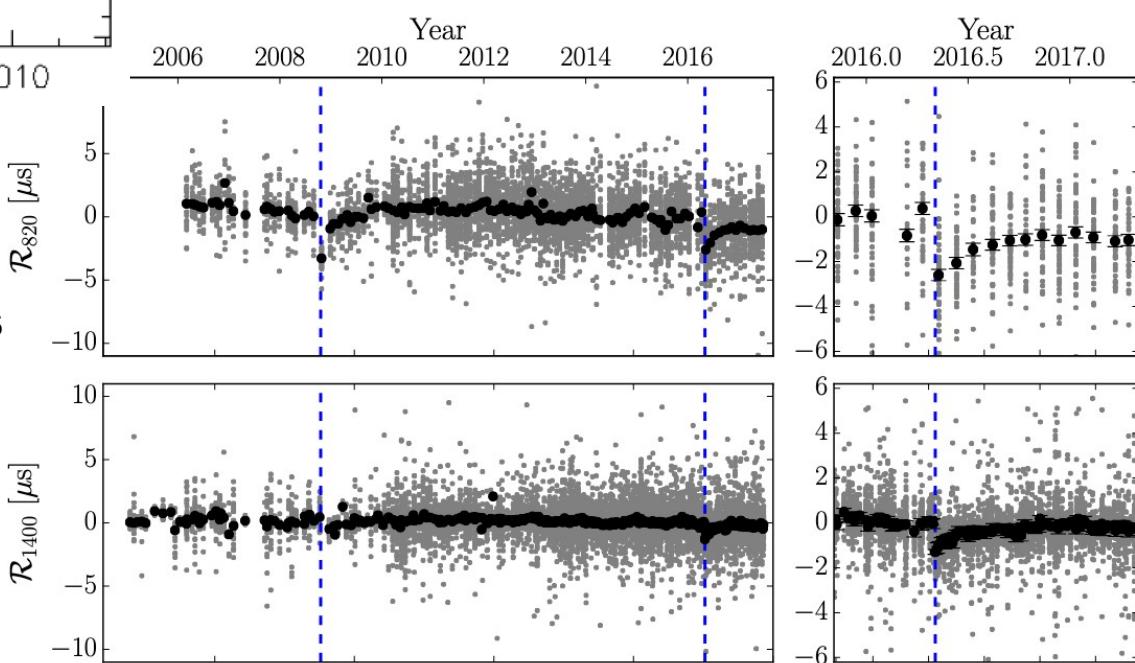
= effects of interstellar medium

◀ Secular variation of the Dispersion Measure  
(due to relative proper motion)

DM events: lense effect due to a plasma  
bubble along the line of sight

INTERSTELLAR MEDIUM EVENTS IN PSR J1713+0747

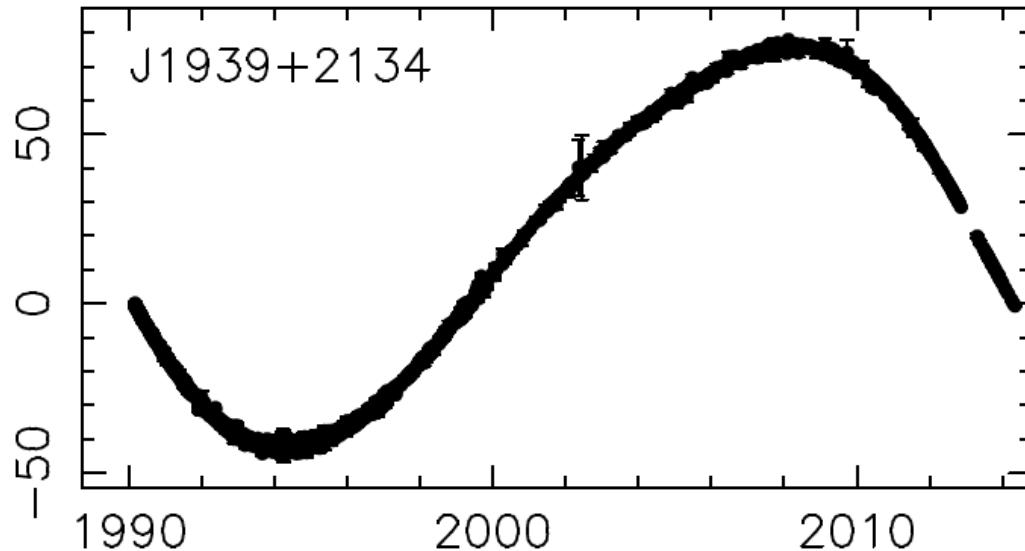
Lam et al 2018



requires multi-wavelength observations  
e.g. 500 MHz, 1400 MHz, 2.5 GHz

# Red noise : spin noise

$P=1.55 \text{ ms}$  rms  $\sim 34.5 \mu\text{s}$   $\langle \text{unc.} \rangle \sim 60 \text{ ns}$

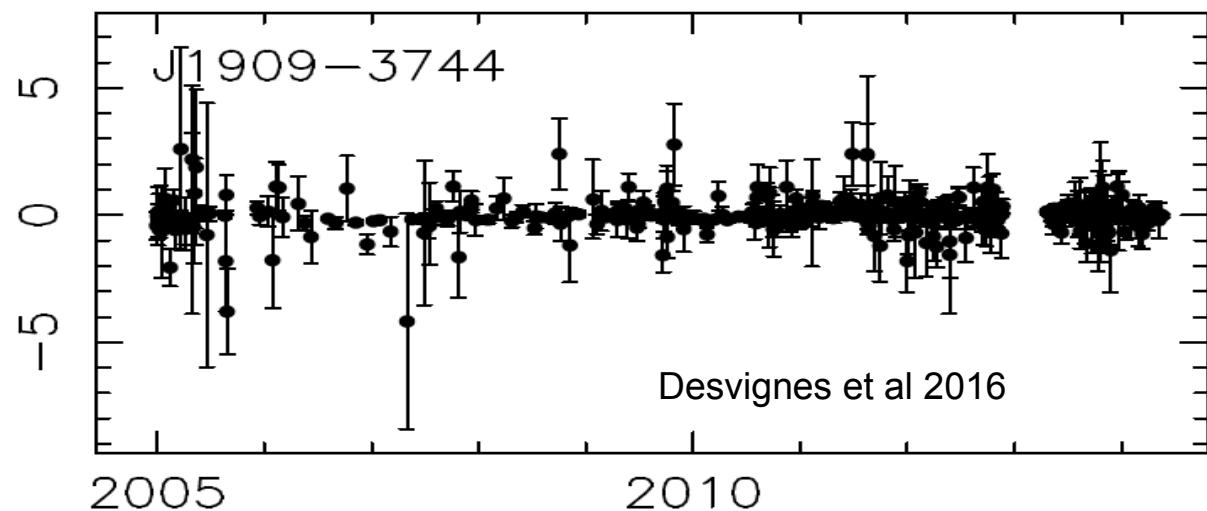


$P=2.9 \text{ ms}$  rms  $\sim 0.092 \mu\text{s}$   $\langle \text{INC}\% \rangle \sim 60 \text{ ns}$

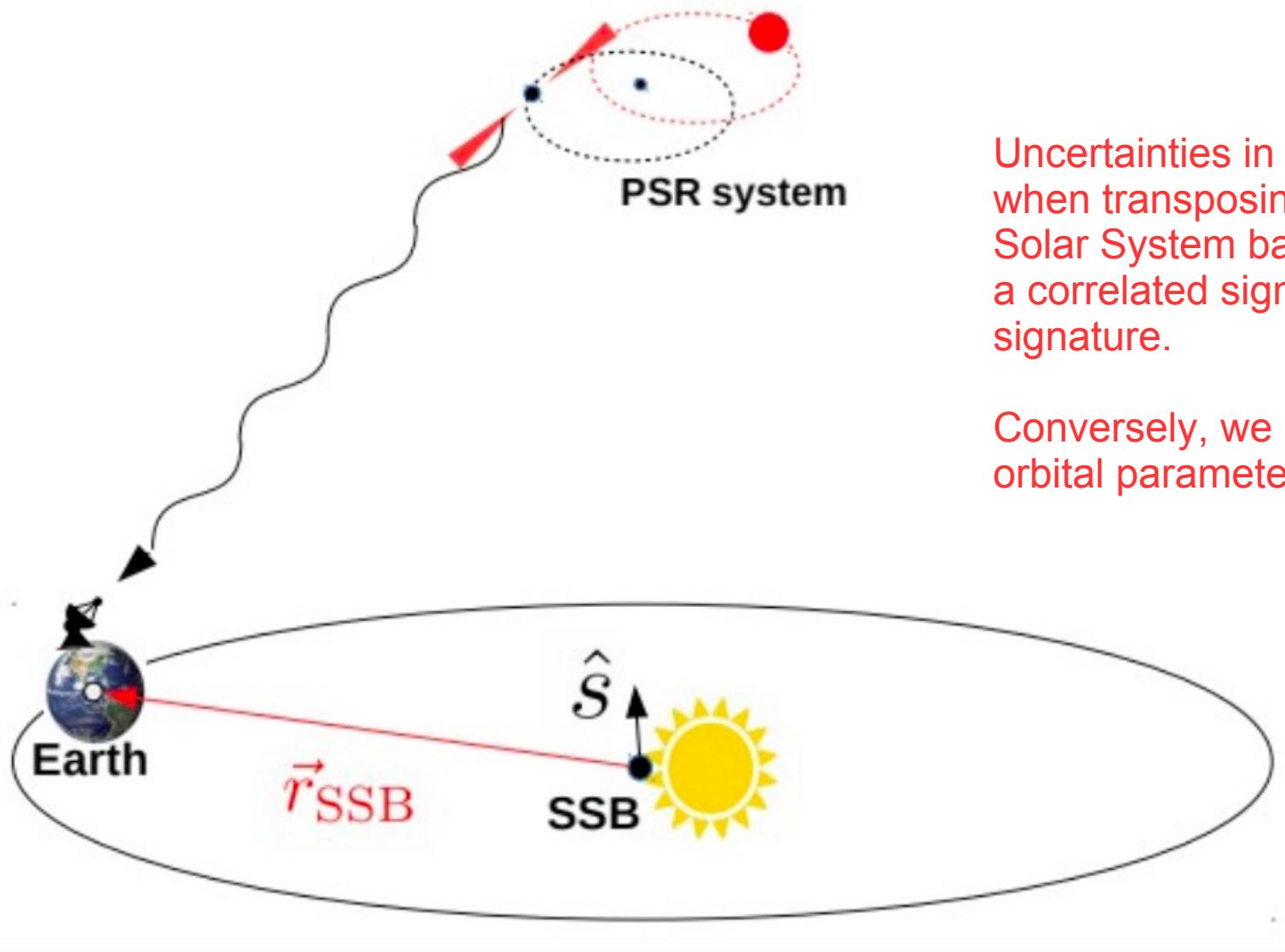
Small bodies disc perturbation ?

$E_{\dot{d}ot}$  variations?

Series of micro-glitches ?



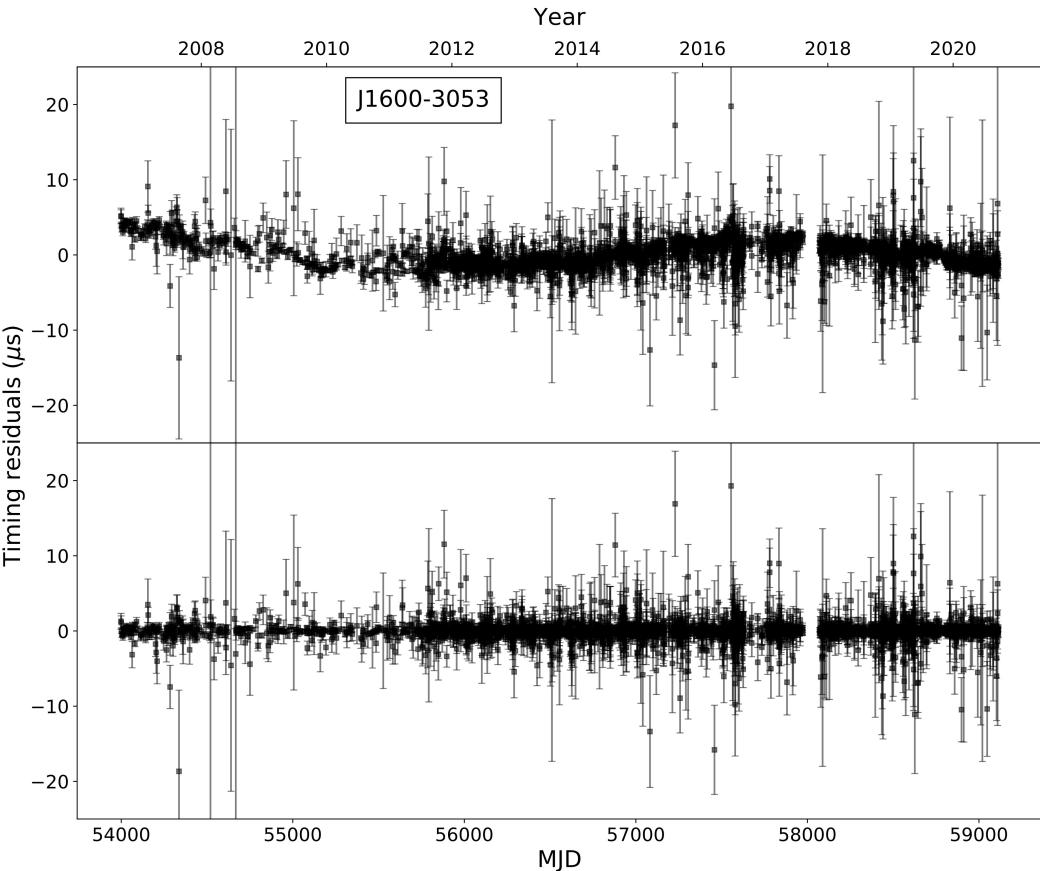
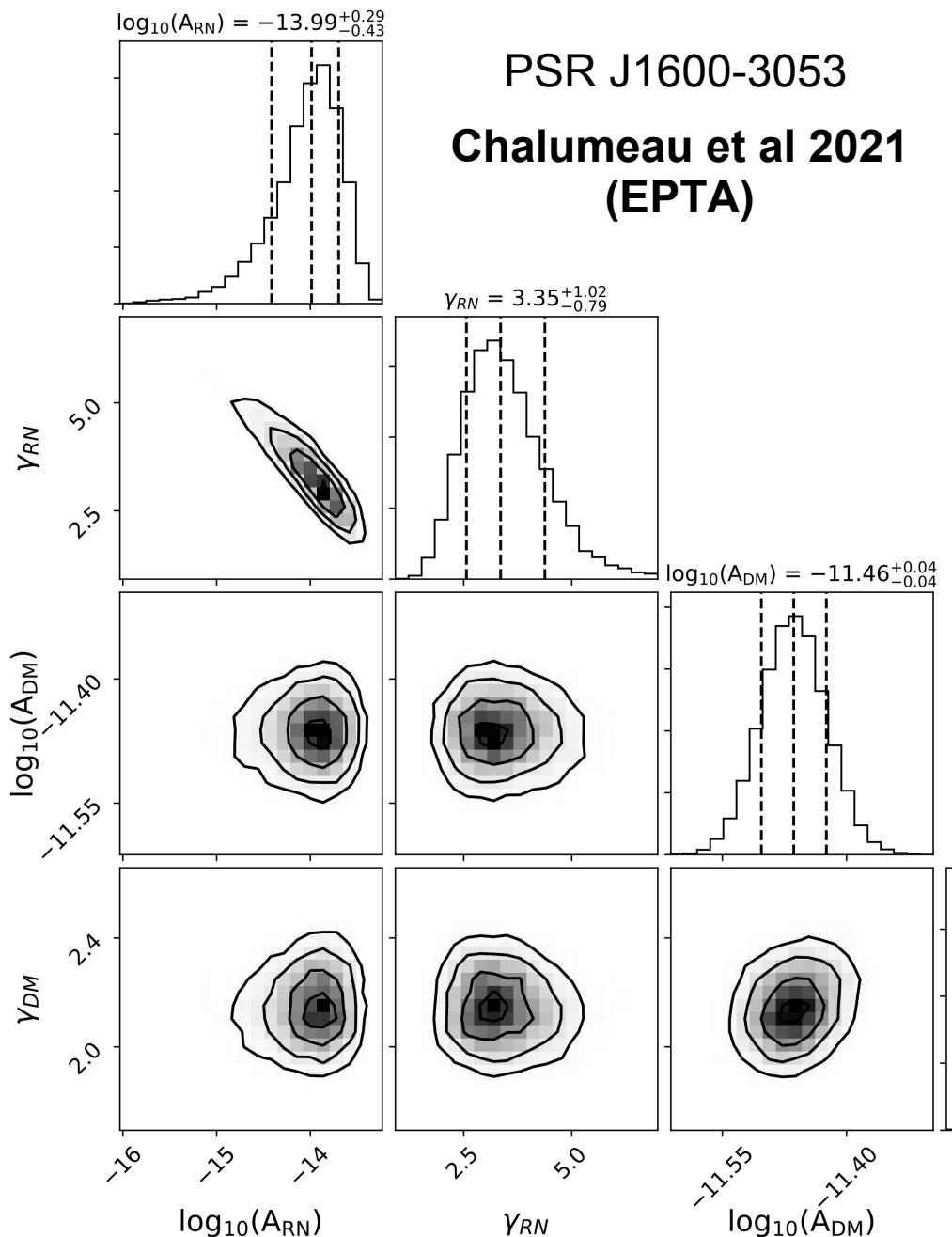
## Red noise : Impact of planetary ephemerides



Uncertainties in the Römer delay when transposing to the Solar System barycentre induce a correlated signal with a dipole signature.

Conversely, we are sensitive to the orbital parameters of the planets!

# Red noise : individual pulsar models



- **Spin noise**
  - **DM chromatic noise**
  - Scattering noise
  - Band noise
  - System noise
- +
- Nb of freq bins  
to characterise each

# Pulsar Timing Arrays : principles

## 1) Describe the pulsar rotation in a reference frame co-moving with the pulsar

$$\nu(t) = \nu_0 + \dot{\nu}_0(t - t_0) + \frac{1}{2}\ddot{\nu}_0(t - t_0)^2 + \dots$$

The observed parameters  $\nu$  and  $\dot{\nu}$  are associated with the physical processes causing pulsars to spin down

## 2) Timing model

$$t_{SSB} = \overbrace{t_{topo} + t_{corr}}^{\text{clock}} - \delta D/f_{obs}^2 + \underbrace{\Delta_{R\odot} + \Delta_\pi + \Delta_{S\odot} + \Delta_{E\odot}}_{\text{Solar System Römer, parallax, Shapiro and Einstein delays}} + \underbrace{\Delta_R + \Delta_S + \Delta_E + \Delta_A}_{\text{binary system Römer, Shapiro, Einstein and Aberration delays}}$$

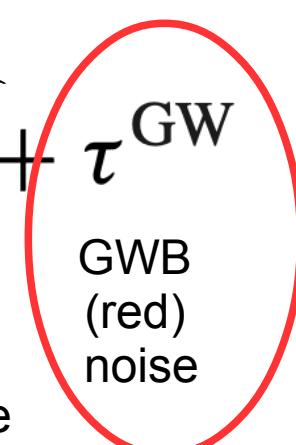
$$\tau^{\text{TM}}$$

## 3) Full noise model

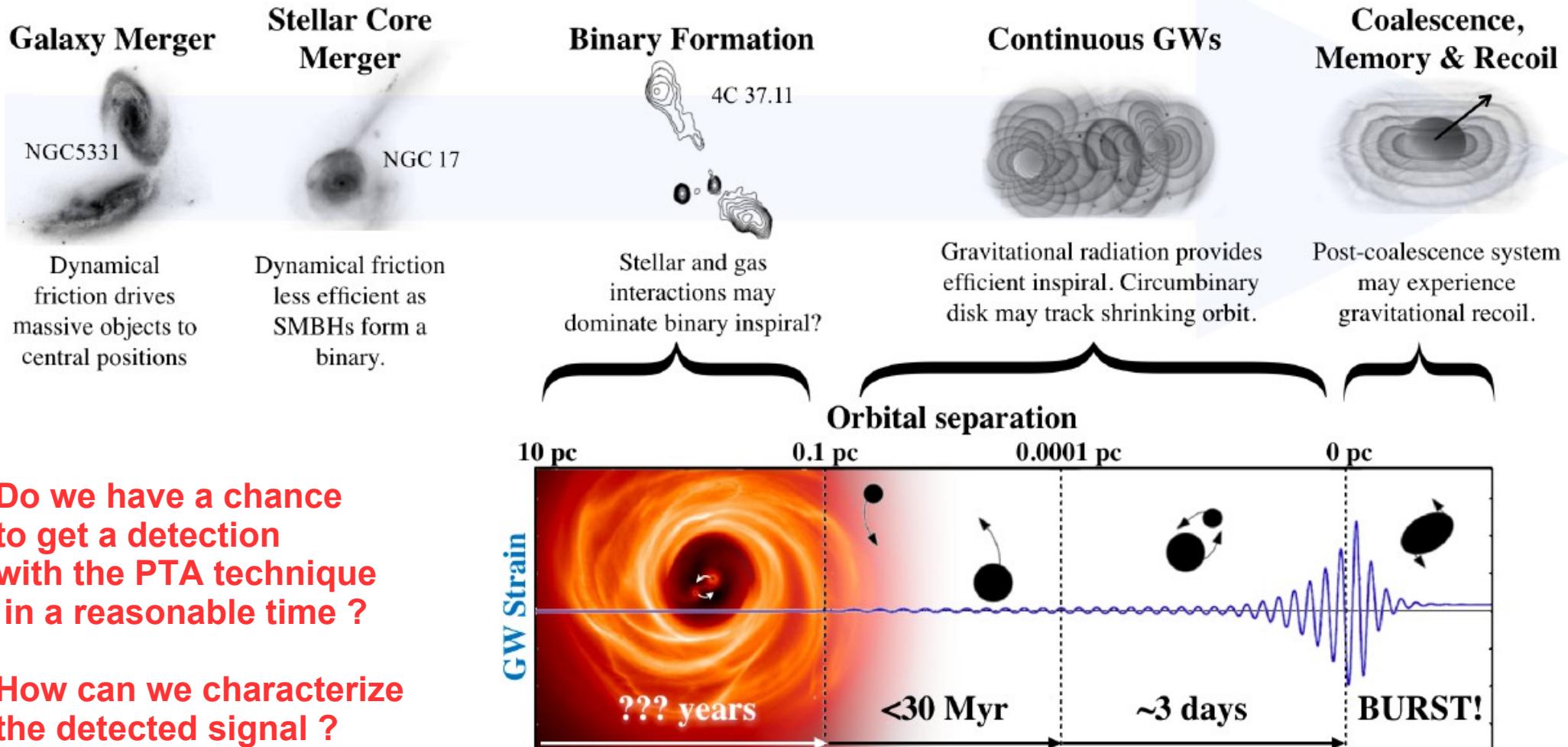
$$\text{observed TOA} = \tau^{\text{TM}} + \tau^{\text{WN}} + \tau^{\text{SN}} + \tau^{\text{DM}} + \tau^{\text{CN}} + \tau^{\text{GW}}$$

Timing Model (deterministic)	meas. (white) noise	pulsar spin (red) noise	DM (red) noise	Clock Ephem. Astroph. (red) noise	GWB (red) noise
------------------------------	---------------------	-------------------------	----------------	-----------------------------------	-----------------

Noise model



# The life cycle of supermassive binary black holes



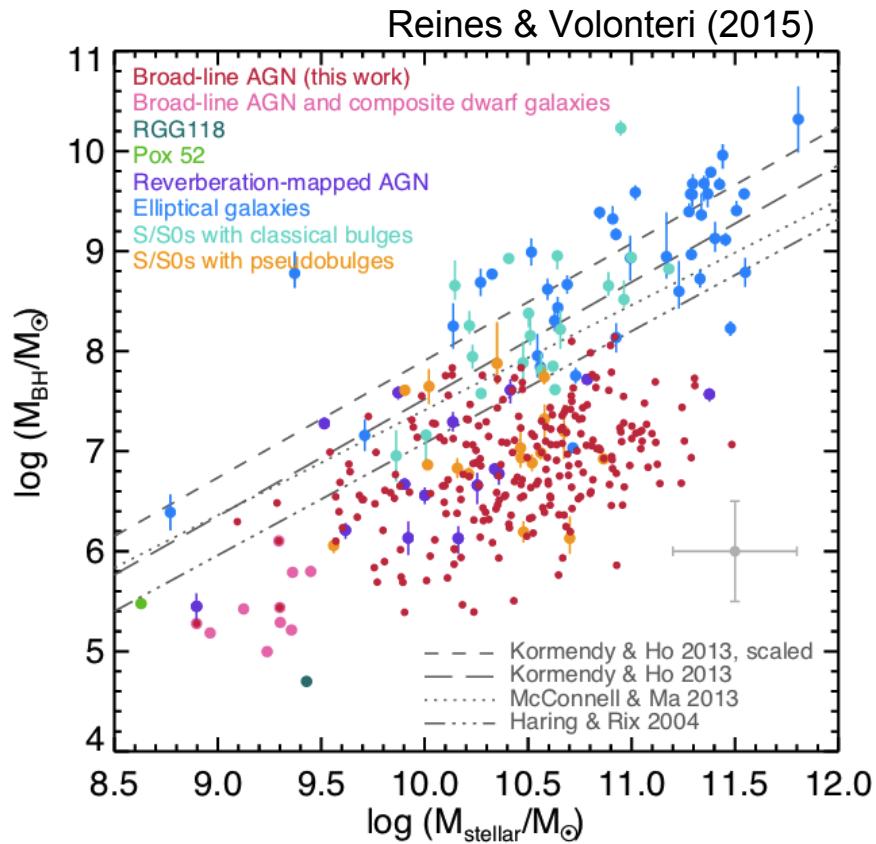
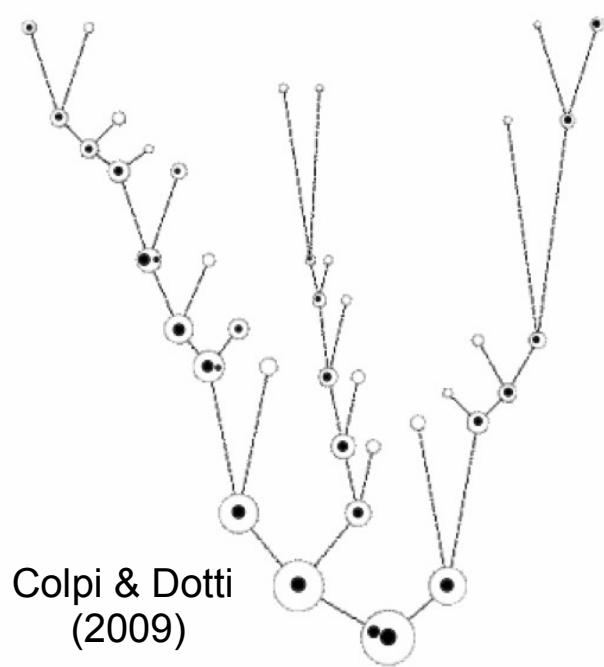
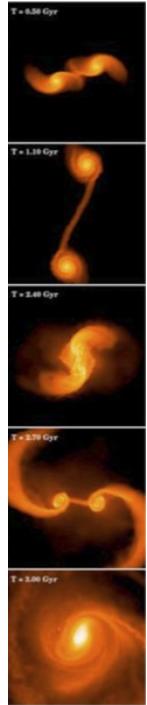
Do we have a chance  
to get a detection  
with the PTA technique  
in a reasonable time ?

How can we characterize  
the detected signal ?

monochromatic  
PTA regime

Burke-Spoliar 2018

# Population synthesis ingredients



Merger trees from cosmological N-body simulations (Illustris, TNG, EAGLE, Horizon-AGN, SIMBA ...)

Bulge to BH mass ratio from galaxies dynamical studies

Add dynamical friction with stars and gas to migrate the BHs towards the center

Three body interaction with stars from the loss cone region (when binary orbital velocity > stars)

# Population synthesis ingredients

## Last parsec problem:

the BH pair empties its environment and stops losing energy.

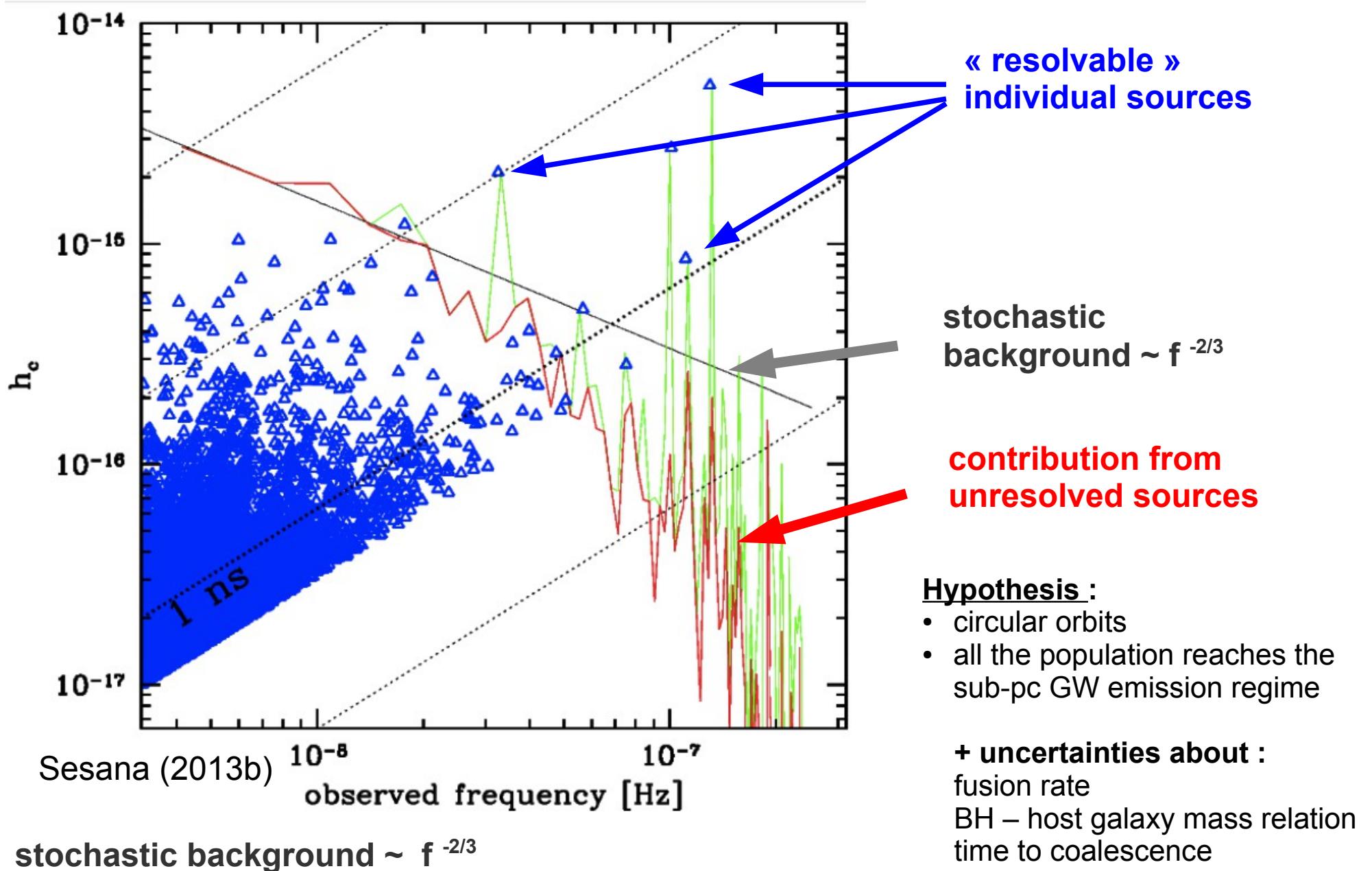


do most of the pairs reach the gravitational regime within a Hubble time ?

## A few answers:

- massive BH triplets (Bonetti et al 2018),
- triaxial potential/density of the nuclei refilling the loss-cone (Vasiliev et al 2015),
- circumbinary accretion disk (Tang et al 2017)
- Continuous accretion of clumpy cold gas on to the nucleus (Goicovic et al 2018)
- a large population of stalled binaries at low frequencies (Dvorkin&Barausse 2017)

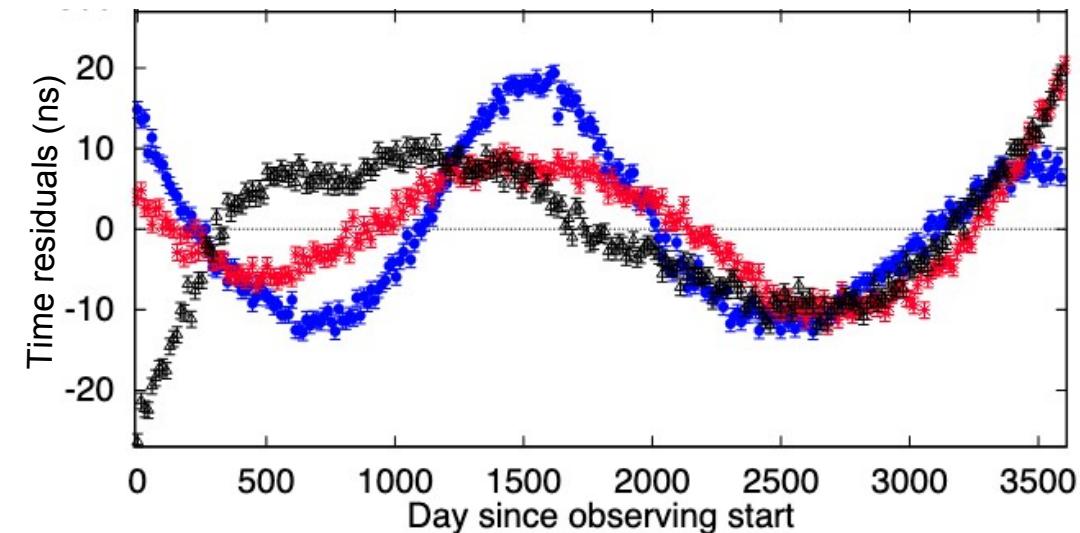
## Population of SMBH : contribution from background & individual sources



$$h_c^2(f) = \int_0^\infty dz \int_0^\infty d\mathcal{M} \frac{d^3 N}{dz d\mathcal{M} d\ln f_r} h^2(f_r) \rightarrow h_c(f) = A \left( \frac{f}{\text{yr}^{-1}} \right)^{-2/3} \quad (\text{Phinney 2001})$$

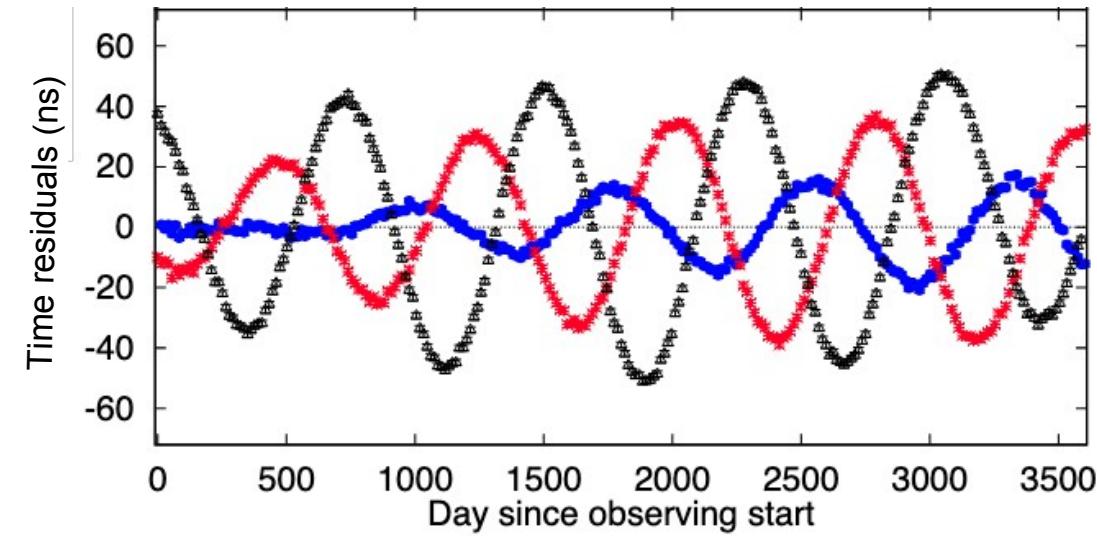
# Pulsar Timing Arrays : principles

GW induced timing residuals (simulated data)  
from Burke-Spolaor (2015)



(a) Gravitational Wave Background

(a) a GWB with  $h_c = 10^{-15}$  and  $\alpha = -2/3$

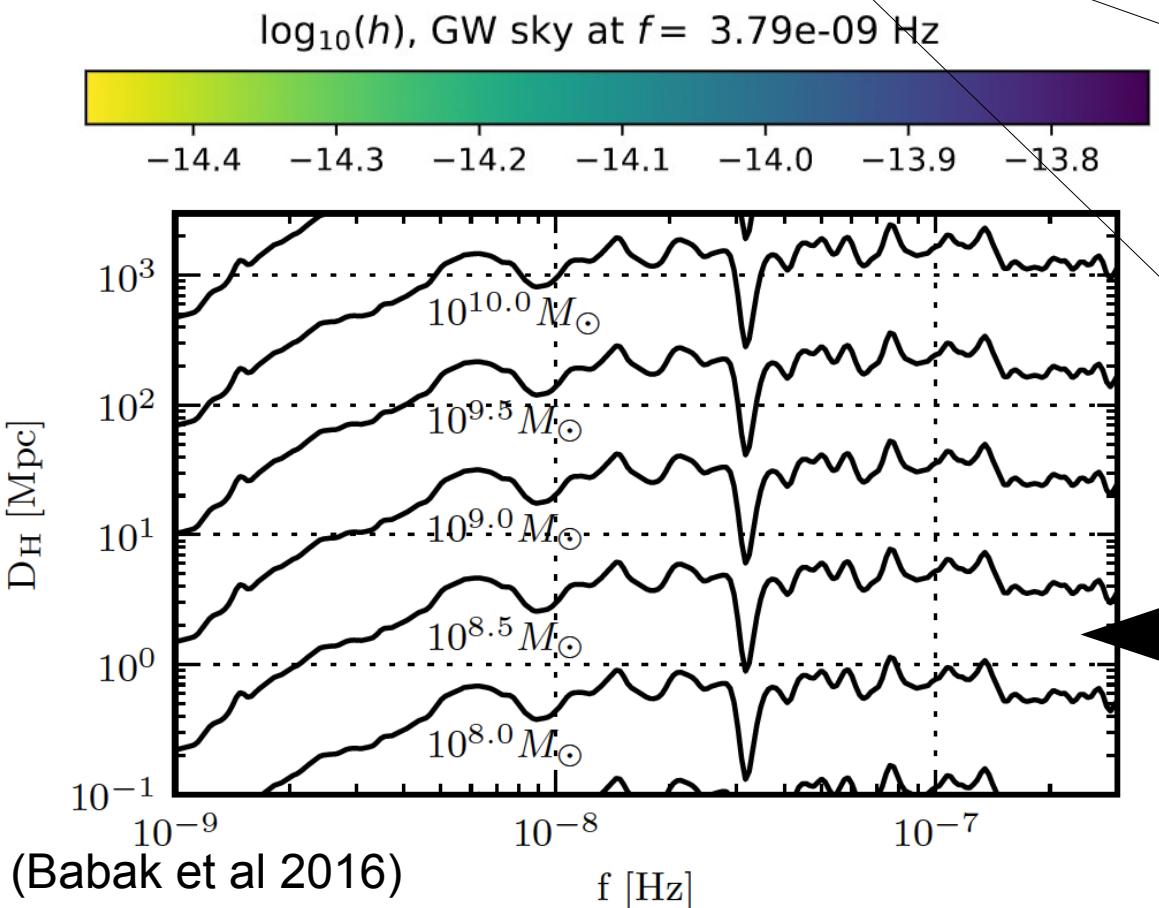
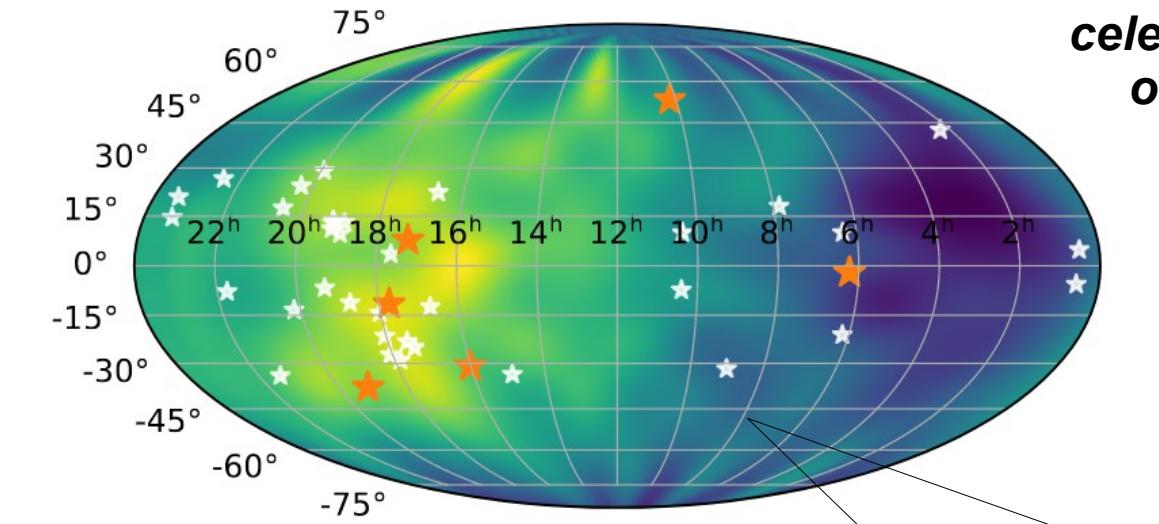


(b) Continuous Wave

(b) a continuous wave  
(injected in the same sky location)  
from an equal-mass  $10^9 M_\odot$  BSMBH  
at redshift  $z = 0.01$ .

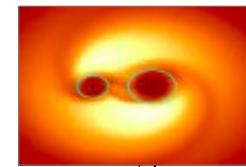
distortion from a perfect sinusoid is caused  
by the lower-frequency pulsar term

# Searching for individual sources

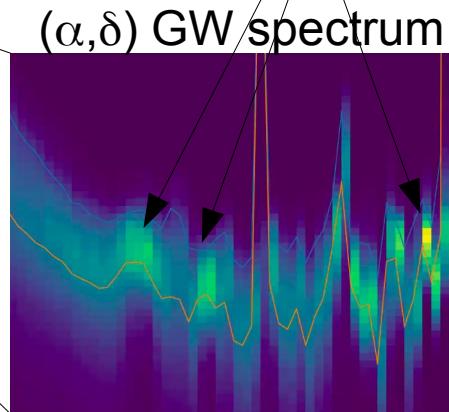


*The sensitivity of the pulsar array depends on the position on the celestial sphere and on the distribution of pulsar pair angular separations*

Mingarelli et al 2017  
Sky sensitivity map  
from EPTA-2015 at 3.8 nHz



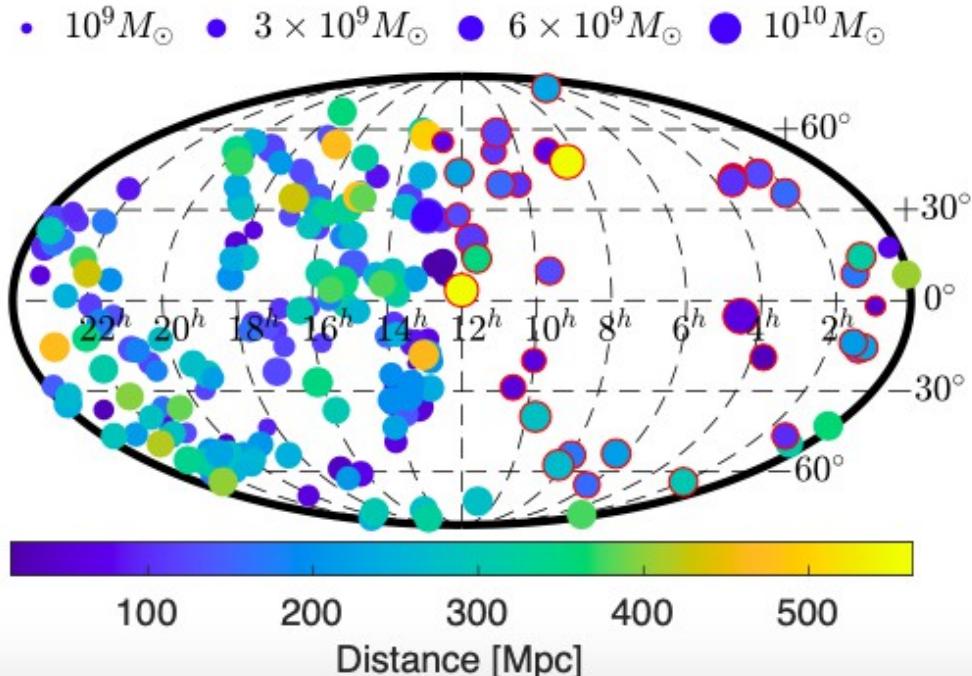
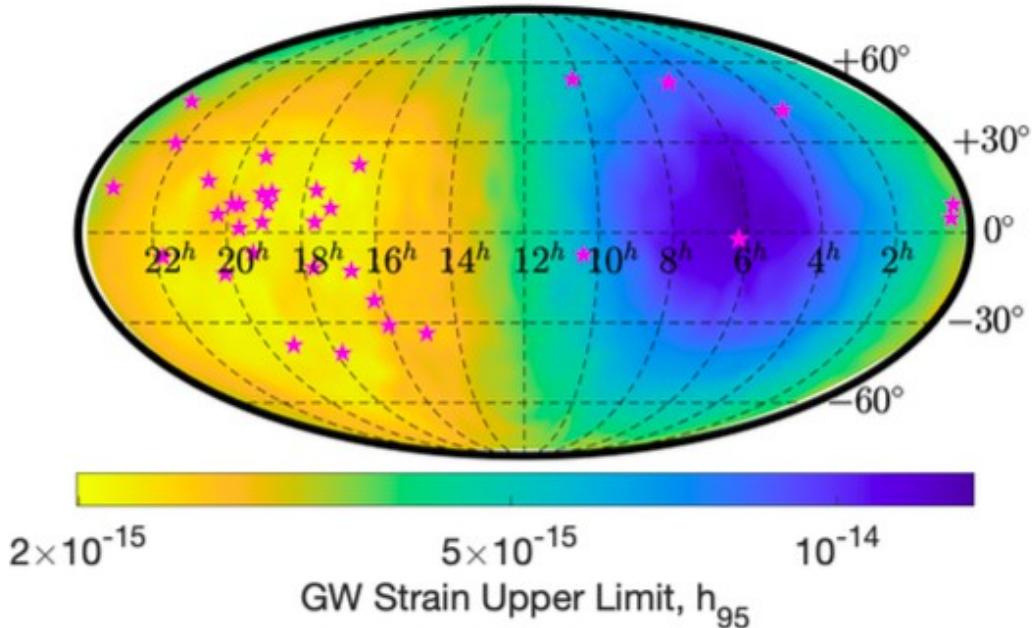
individual  
Sources ?



EPTA-2015 (Babak et al 2016) :  
 $h c < 1.1 \times 10^{-14}$  at 10 nHz

On can exclude the presence of  
a SMBHB with a « chirp mass »  
 $\mathcal{M}_c > 10^9 M_\odot$  up to 25Mpc  
 $\mathcal{M}_c > 3 \cdot 10^9 M_\odot$  up to 200 Mpc

# Searching for individual sources



*The sensitivity of the pulsar array depends on the position on the celestial sphere and on the distribution of pulsar pair angular separations*

**Arzoumanian et al 2021**  
**Sky sensitivity map**  
**from NANOGrav-11yr at 8 nHz**  
 $h_c < 7.3 \times 10^{-15}$

No equal mass SMBHB  
with chirp mass  $M > 1.6 \times 10^9 M_\odot$   
in the Virgo Cluster  
(Aggarwal et al 2019)

**Place constraints on putative SMBHBs in nearby massive galaxies (Arzoumanian et al 2021)**

44,000 galaxies in the local universe (up to redshift 0.05) and populated them with hypothetical binaries

216 galaxies with dynamical mass within NANOGrav's sensitivity volume

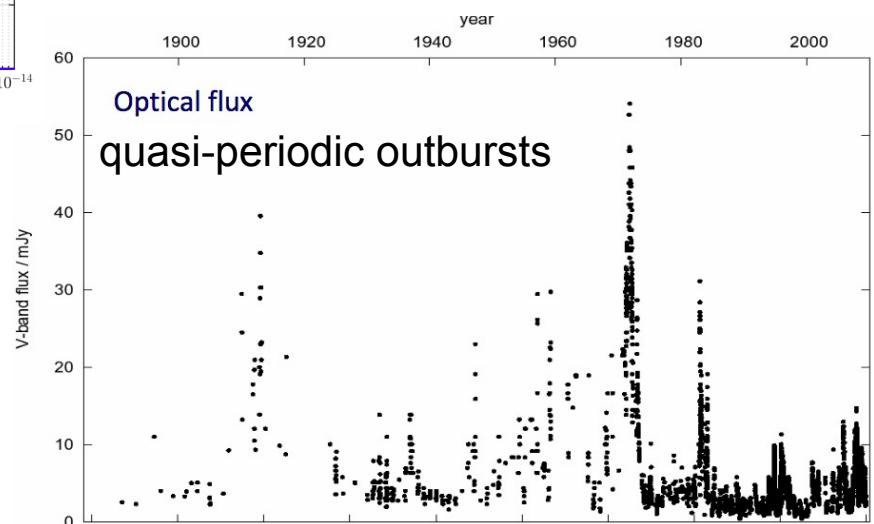
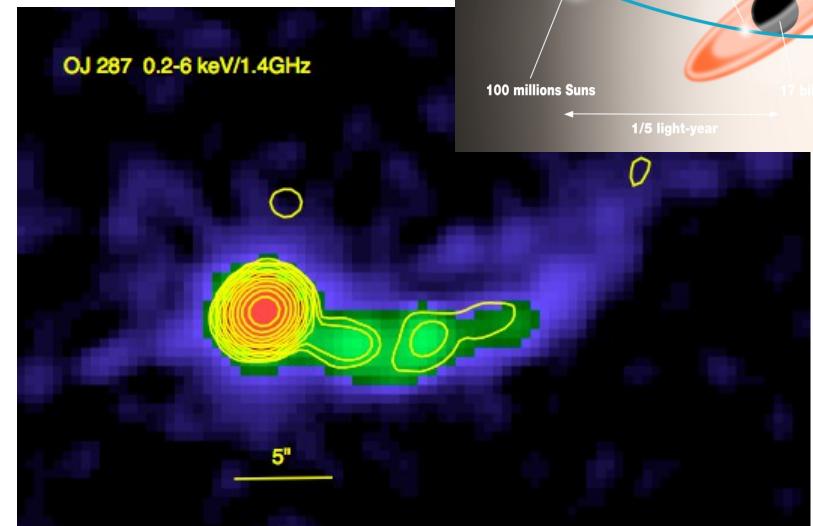
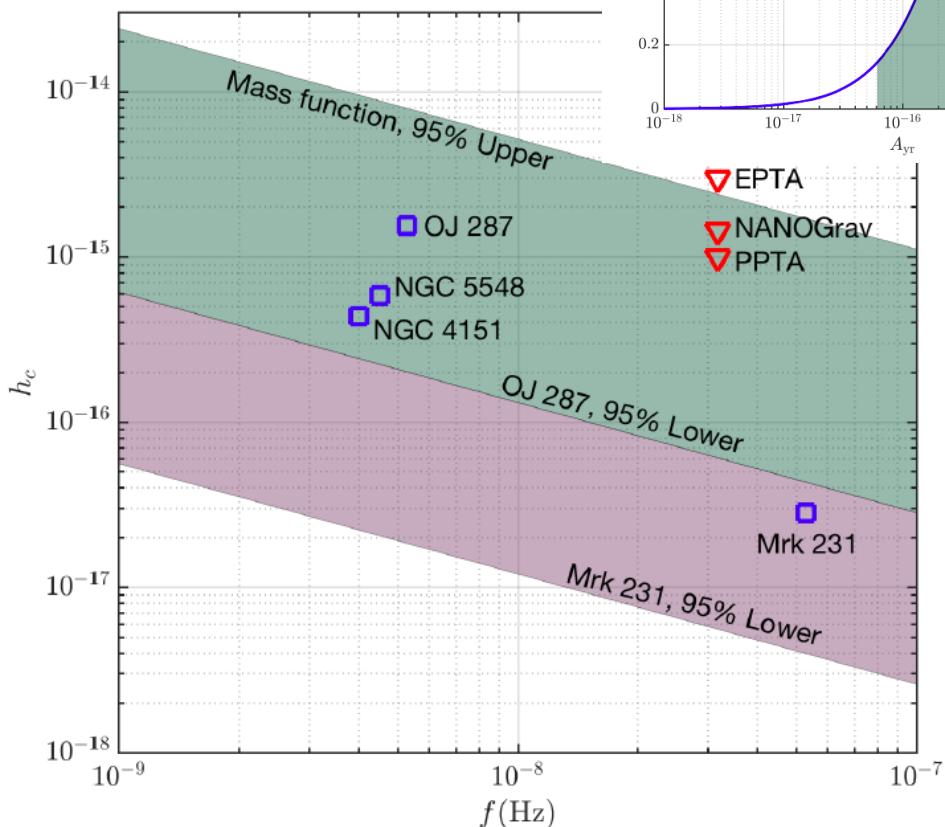
+ constraints on their chirp mass and mass ratio

# OJ 287 as a SMBH binary candidate

(discovered in 1988 by Sillanpää et al)

also NGC5548, NGC4151, Mrk231

Zhu et al 2018 computed the probability distribution of the gravitational wave background amplitude assuming those objects are true SMBHBs



**OJ 287**  
~12 yrs period  
 $18 \times 10^9$  solar masses  
0.663 eccentricity  
 $z = 0.3$

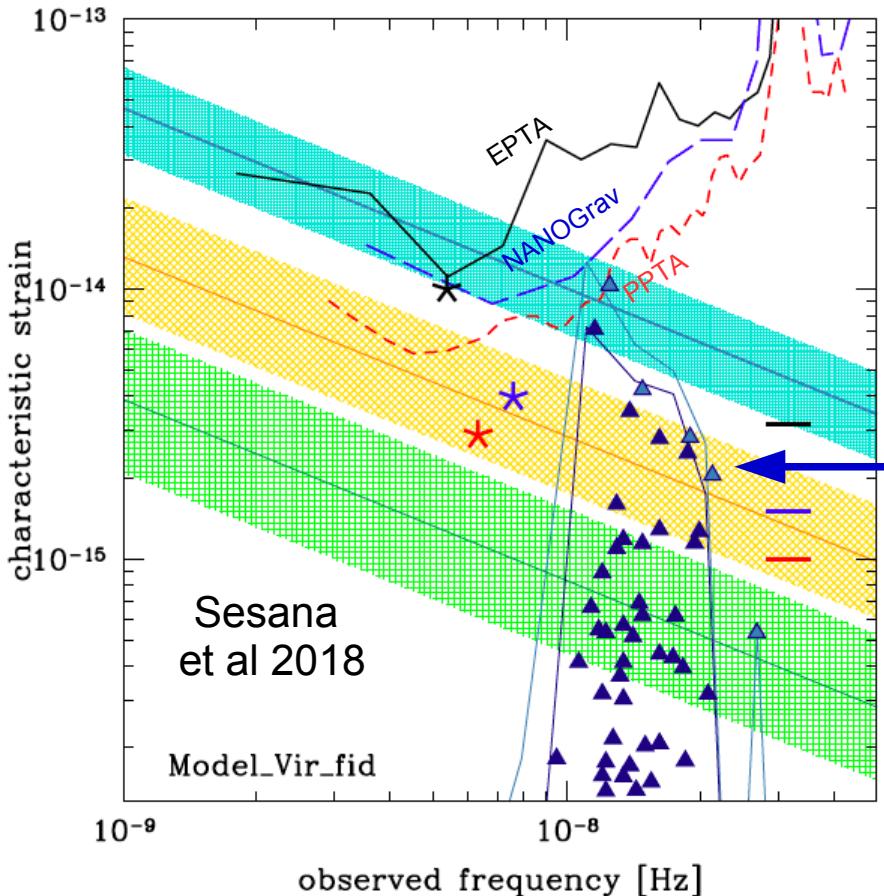
Cipriani et al 2016  
Valtonen et al 2008

# Periodic variability of quasars and AGNs (Graham et al 2015, Sesana et al 2018)

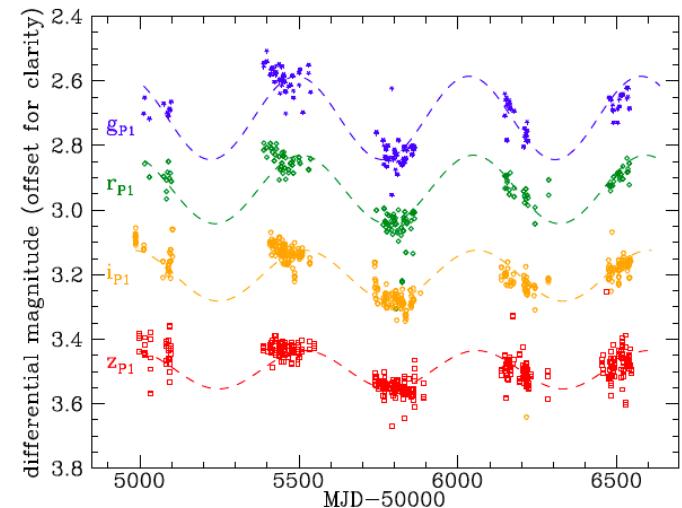
Binarity induces periodic material streaming from the cavity edge onto the binary, and hence luminosity periodicity, better detected in X-ray and UV

Catalina Real-time Transient Survey (CRTS – 250,000 QSOs)  
 → 111 periodic sources ( $P < 6$  yrs ; Graham et al 2015)

Palomar Transient Factory (OTF – 35,000 QSOs)  
 → 33 periodic sources (Charisi et al 2016)



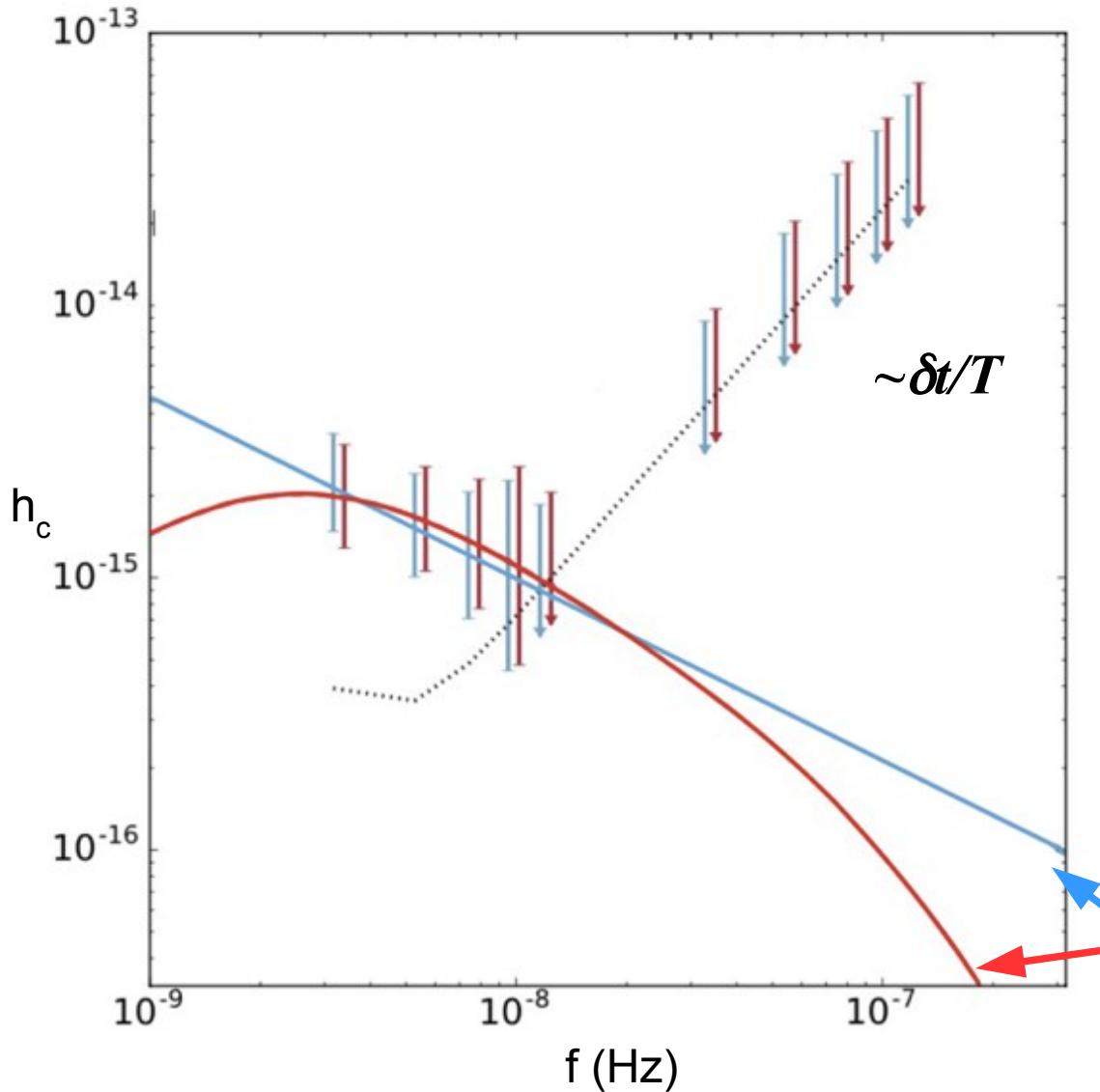
Liu et al 2015 (Catalina)  
**PSO J334.2028+01.4075**  
 $P_{\text{obs}} = 542$  days  
 $\log(M_{\text{BH}}) = 9.97$   
 $Q = 0.05-0.25$



Sesana et al 2018 :  
 compute SMBHB merger rate  
 for periodic sources  
 and construct the expected  
 gravitational wave background

Periodic QSO candidates in tension with PTA results and population models (mainly high  $z > 1.3$ )

# The stochastic gravitational wave background (SGWB): detection vs upper limit

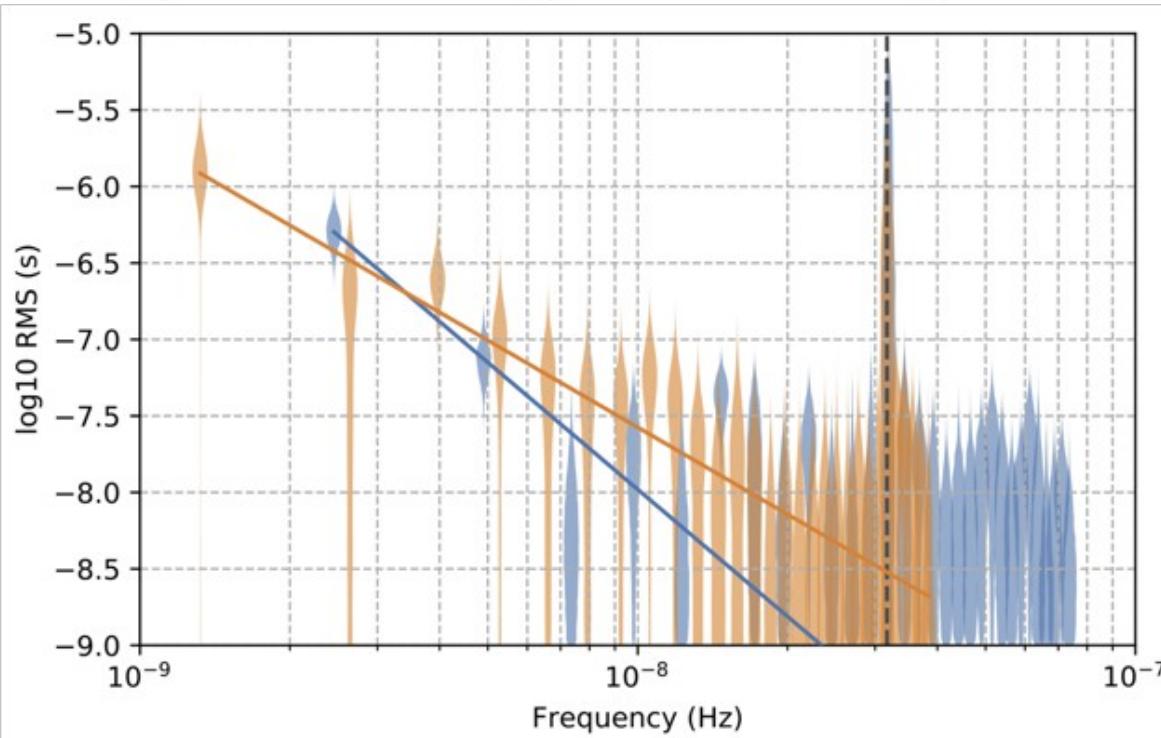
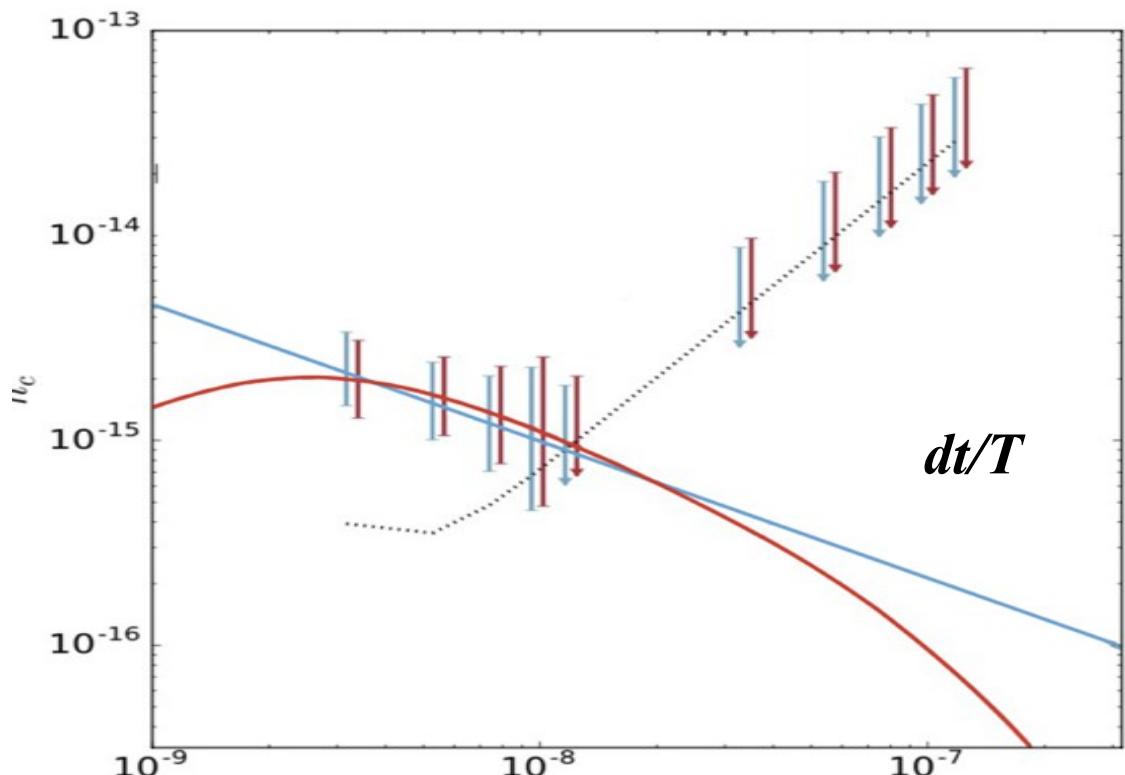


$$h_c(f) = A \left( \frac{f}{\text{yr}^{-1}} \right)^{-2/3}$$

Expected spectrum for a population  
of super massive black hole binaries

purely circular orbits, isolated pairs

including eccentricity, stellar hardening, ...



### Bayes factor diagnostic (EPTA):

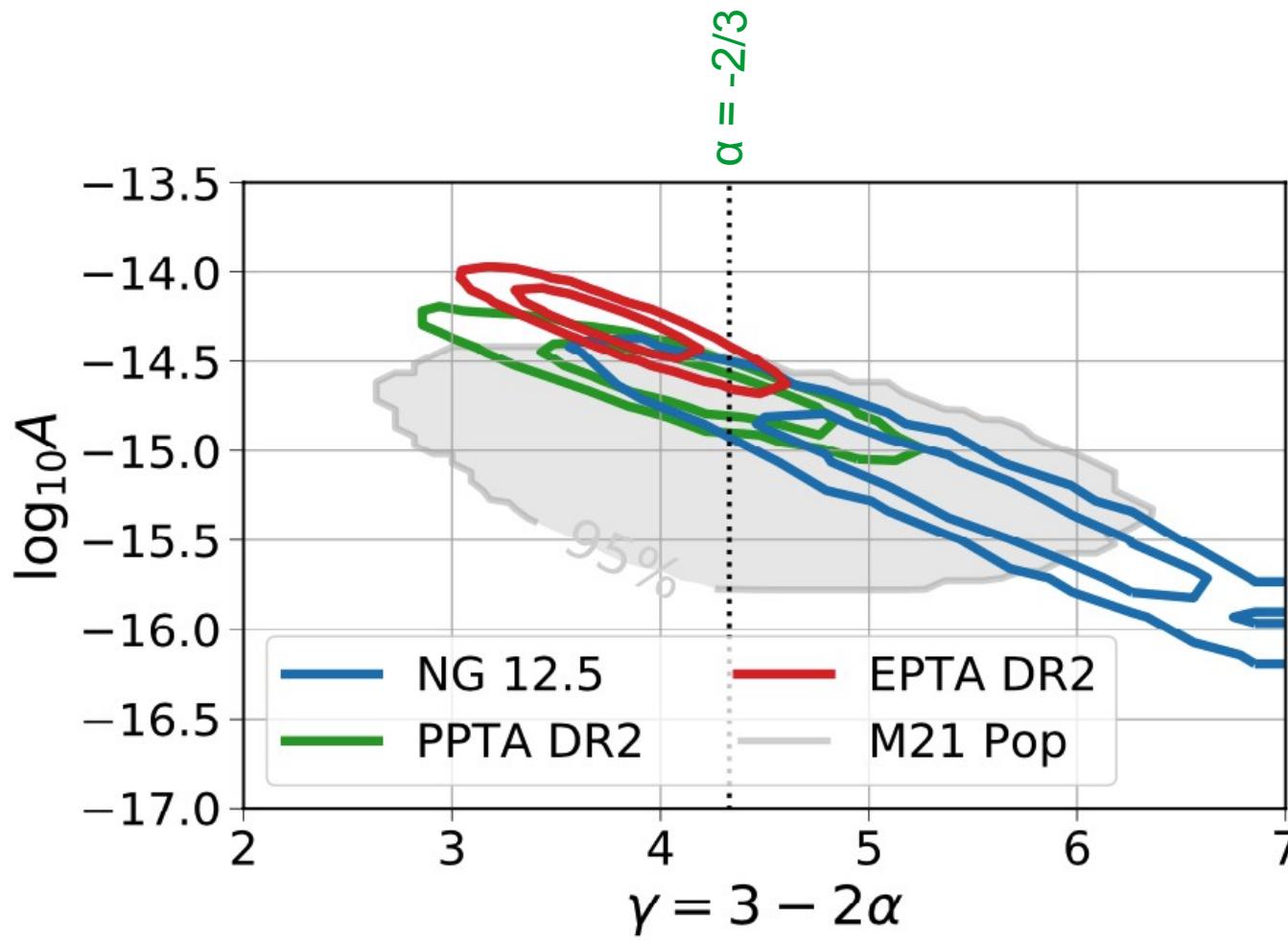
ID	Model	lg10BF	
		ENTERPRISE	FORTYTWO
0	PSRN	0	0
1	PSRN + CURN	$3.1 \pm 0.05$	$3.6 \pm 0.1$
2	PSRN + GWB	$2.73 \pm 0.03$	$3.2 \pm 0.1$
3	PSRN + CLK	$0.62 \pm 0.03$	$0.8 \pm 0.1$
4	PSRN + EPH	$2.06 \pm 0.04$	$2.1 \pm 0.1$
5	PSRN + CURN + GWB	$2.89 \pm 0.03$	$3.7 \pm 0.3$
6	PSRN + CURN + CLK	$3.06 \pm 0.03$	$3.4 \pm 0.1$
7	PSRN + CURN + EPH	$2.99 \pm 0.03$	$3.4 \pm 0.2$

**A first detection ?**

EPTA result :  
6 « best » pulsars, 14-25 years  
(Chen et al 2021a)

NANOGrav result :  
47 pulsars, 12.5 years  
(Arzoumanian et al 2020)

# The PTA common red noise signal vs SMBHB population models



Antoniadis et al 2022

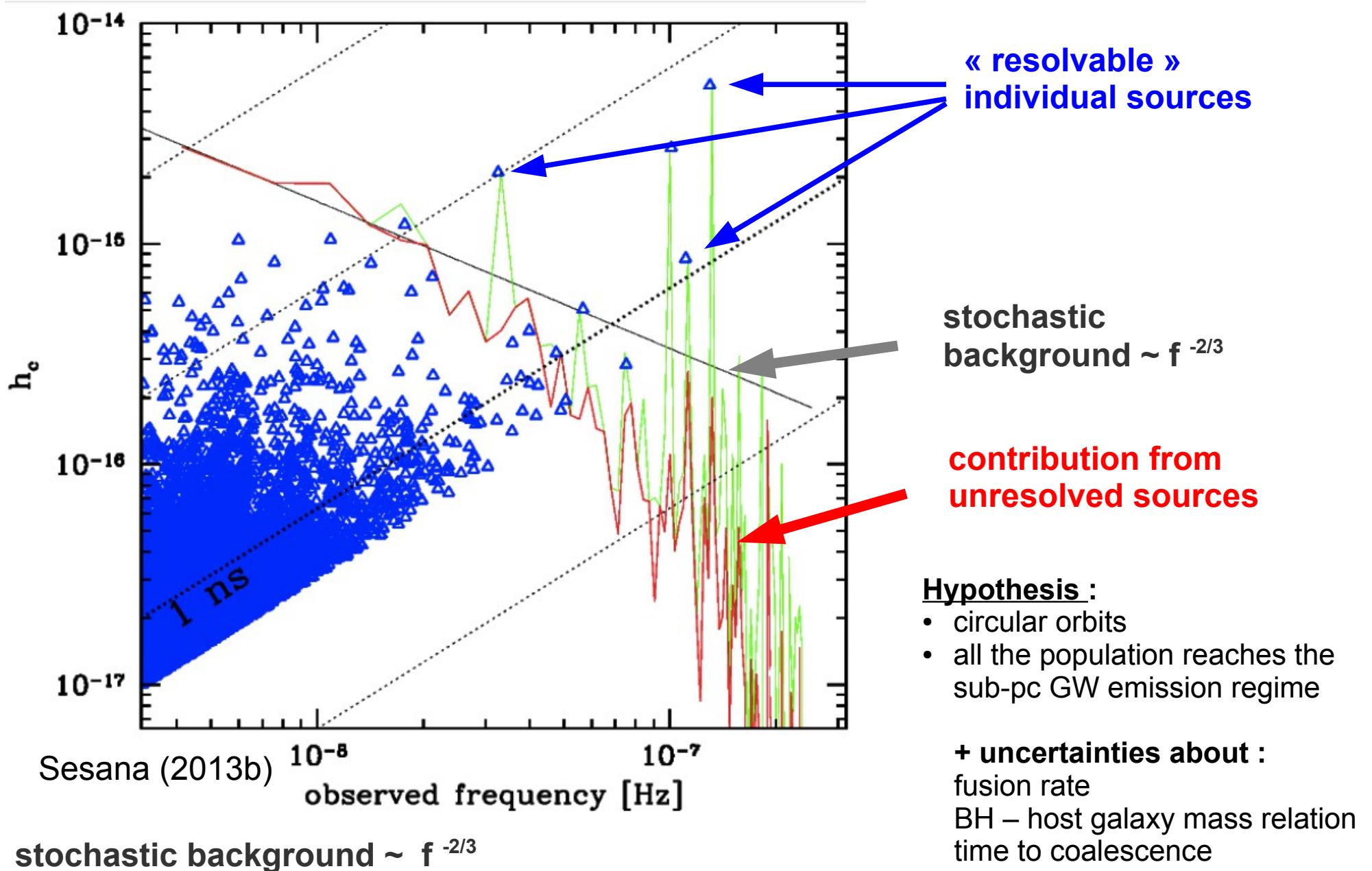
high merger rate densities  
short merger timescales  
high normalization  
for BH-bulge mass relation

Arzoumanian et al 2020  
Chen et al 2021  
Goncharov et al 2021

Comparing with the predictions of astrophysical models (Middleton et al 2021)

$$h_c(f) = A \left( \frac{f}{\text{yr}^{-1}} \right)^{-2/3}$$

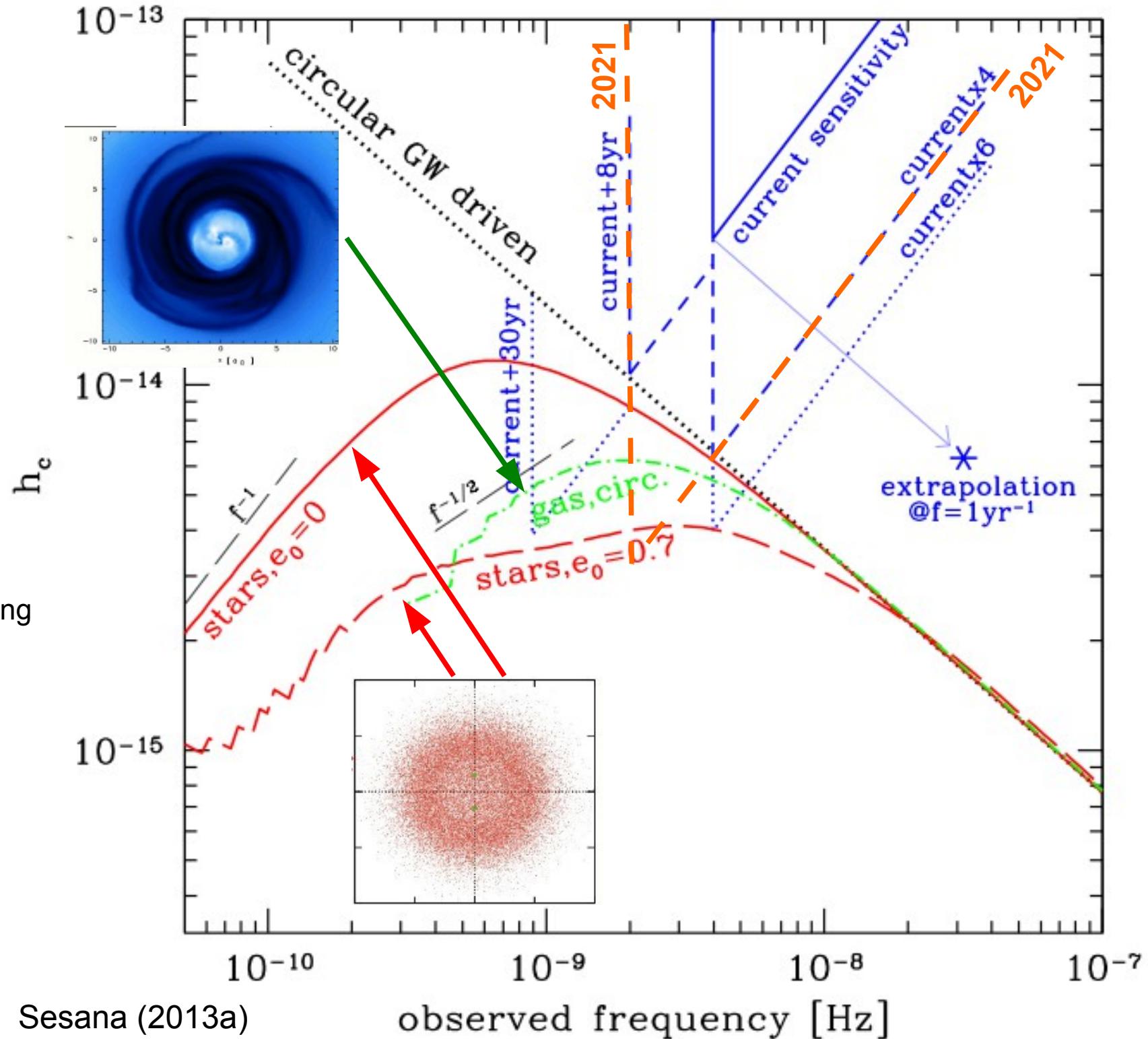
## Population of SMBH : contribution from background & individual sources



$$h_c^2(f) = \int_0^\infty dz \int_0^\infty d\mathcal{M} \frac{d^3 N}{dz d\mathcal{M} d\ln f_r} h^2(f_r) \rightarrow h_c(f) = A \left( \frac{f}{\text{yr}^{-1}} \right)^{-2/3} \quad (\text{Phinney 2001})$$

## A more realistic scenario :

- + eccentricity
- + interactions with stars and gas
- + spin/orbit coupling



# Constraints on astrophysical models

Chen et al 2019

EPTA – population synthesis

parameter	description	standard	extended
$\Phi_0$	GSMF norm	$-2.8 \pm 0.3$	$-2.8 \pm 0.3$
$\Phi_I$	GSMF norm redshift evolution	$-0.25 \pm 0.22$	$-0.25 \pm 0.22$
$\log_{10}M_0$	GSMF scaling mass	$11.25 \pm 0.2$	$11.25 \pm 0.2$
$\alpha_0$	GSMF mass slope	$-1.25 \pm 0.17$	$-1.25 \pm 0.17$
$\alpha_I$	GSMF mass slope redshift evolution	$0 \pm 0.15$	$0 \pm 0.15$
$f_0$	pair fraction norm	[0.02,0.03]	[0.01,0.05]
$\alpha_f$	pair fraction mass slope	[-0.2,0.2]	[-0.5,0.5]
$\beta_f$	pair fraction redshift slope	[0.6,1]	[0,2]
$\gamma_f$	pair fraction mass ratio slope	[-0.2,0.2]	[-0.2,0.2]
$\tau_0$	merger time norm	[0.1,2]	[0.1,10]
$\alpha_\tau$	merger time mass slope	[-0.2,0.2]	[-0.5,0.5]
$\beta_\tau$	merger time redshift slope	[-2,1]	[-3,1]
$\gamma_\tau$	merger time mass ratio slope	[-0.2,0.2]	[-0.2,0.2]
$\log_{10}M_*$	$M_{\text{bulge}} - M_{\text{BH}}$ relation norm	$8.17 \pm 0.33$	$8.17 \pm 0.33$
$\alpha_*$	$M_{\text{bulge}} - M_{\text{BH}}$ relation slope	$1 \pm 0.1$	$1 \pm 0.1$
$e$	$M_{\text{bulge}} - M_{\text{BH}}$ relation scatter	[0.3,0.5]	[0.2,0.5]
$e_0$	binary eccentricity	[0.01,0.99]	[0.01,0.99]
$\log_{10}\zeta_0$	stellar density factor	[-2,2]	[-2,2]

# Constraints on astrophysical models

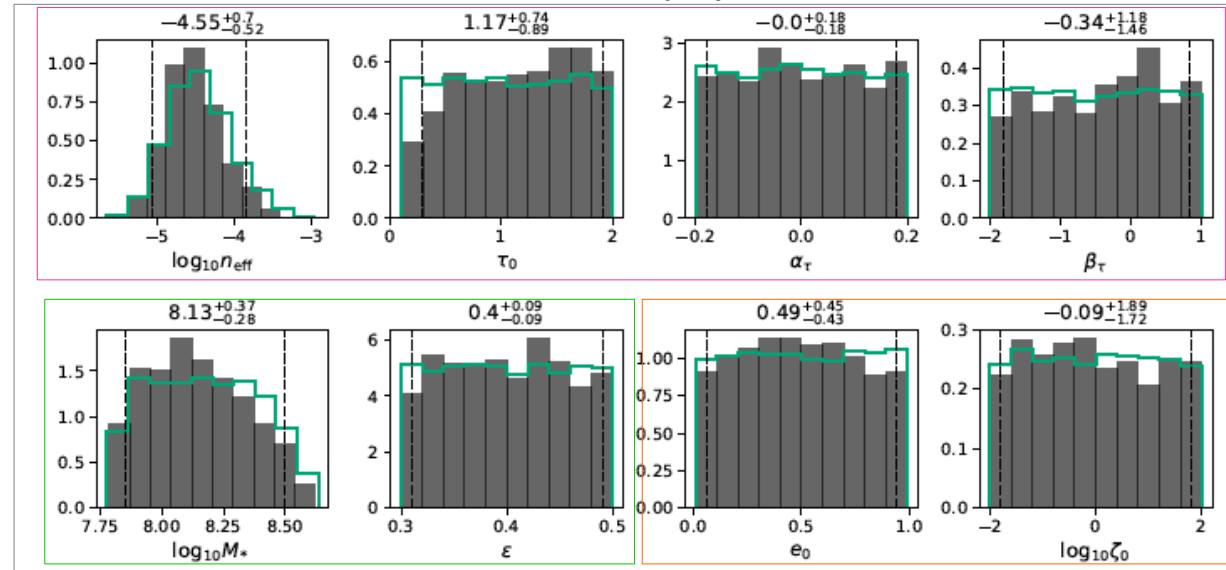
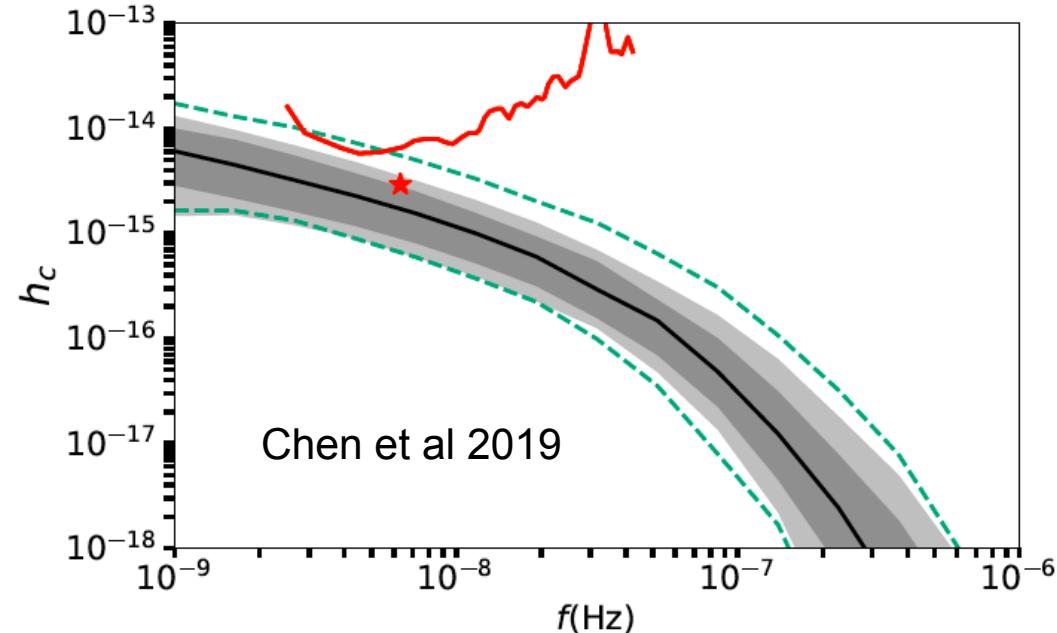
Chen et al 2019

EPTA – population synthesis

parameter	description	standard	extended
$\Phi_0$	GSMF norm	$-2.8 \pm 0.3$	$-2.8 \pm 0.3$
$\Phi_I$	GSMF norm redshift evolution	$-0.25 \pm 0.22$	$-0.25 \pm 0.22$
$\log_{10}M_0$	Galaxy stellar mass function		
$\alpha_0$	GSMF scaling mass	$11.25 \pm 0.2$	$11.25 \pm 0.2$
$\alpha_I$	GSMF mass slope	$-1.25 \pm 0.17$	$-1.25 \pm 0.17$
$\alpha_I$	GSMF mass slope redshift evolution	$0 \pm 0.15$	$0 \pm 0.15$
$f_0$	pair fraction norm	[0.02,0.03]	[0.01,0.05]
$\alpha_f$	pair fraction mass slope	[-0.2,0.2]	[-0.5,0.5]
$\beta_f$	Pair fraction		
$\beta_f$	pair fraction redshift slope	[0.6,1]	[0,2]
$\gamma_f$	pair fraction mass ratio slope	[-0.2,0.2]	[-0.2,0.2]
$\tau_0$	merger time norm	[0.1,2]	[0.1,10]
$\alpha_\tau$	merger time mass slope	[-0.2,0.2]	[-0.5,0.5]
$\beta_\tau$	Merger timescale		
$\beta_\tau$	merger time redshift slope	[-2,1]	[-3,1]
$\gamma_\tau$	merger time mass ratio slope	[-0.2,0.2]	[-0.2,0.2]
$\log_{10}M_*$	$M_{\text{bulge}} - M_{\text{BH}}$ relation norm	$8.17 \pm 0.33$	$8.17 \pm 0.33$
$\alpha_*$	$M_{\text{bulge}} - M_{\text{BH}}$ relation slope	$1 \pm 0.1$	$1 \pm 0.1$
$\epsilon$	$M_{\text{bulge}} - M_{\text{BH}}$ relation scatter	$M_{\text{bulge}} - M_{\text{BH}}$	$M_{\text{bulge}} - M_{\text{BH}}$
$e_0$	$M_{\text{bulge}} - M_{\text{BH}}$ relation	[0.3,0.5]	[0.2,0.5]
$\log_{10}\zeta_0$	Eccentricity and stellar density	[0.01,0.99]	[0.01,0.99]
$\log_{10}\zeta_0$	stellar density factor	[-2,2]	[-2,2]

## Case of no-detection

Strain limit :  $A(f = \text{yr}^{-1}) = 1 \times 10^{-15}$



# Constraints on astrophysical models

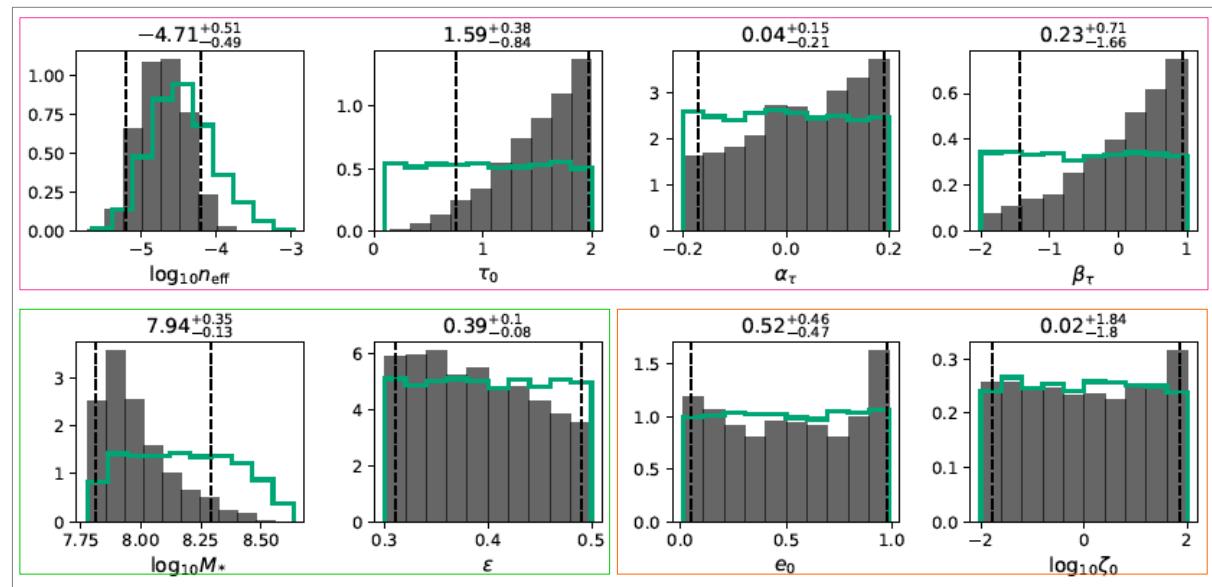
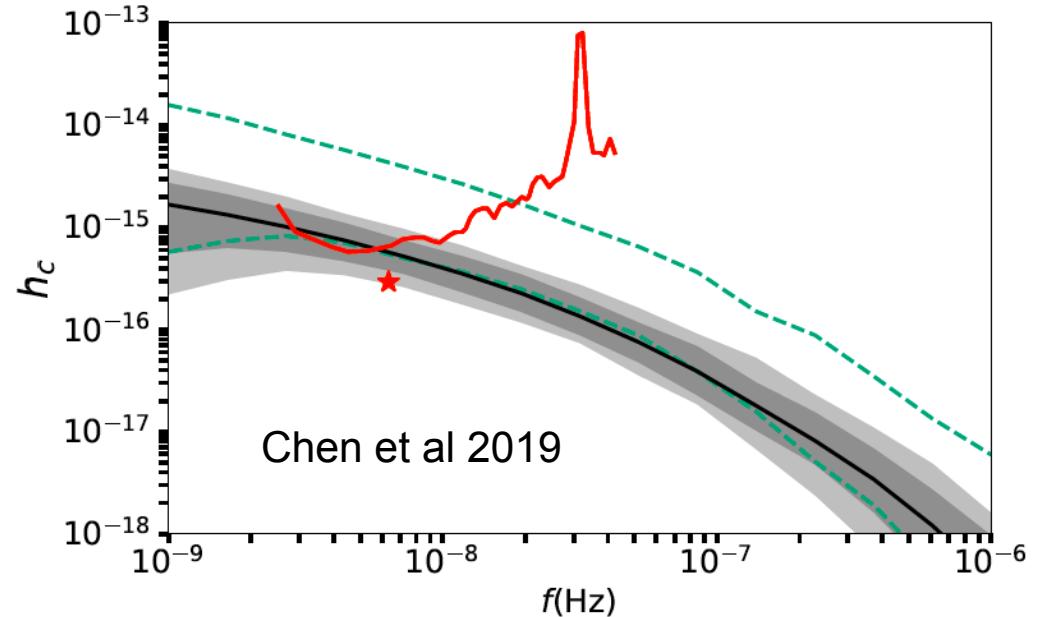
Chen et al 2019

EPTA – population synthesis

parameter	description	standard	extended
$\Phi_0$	GSMF norm	$-2.8 \pm 0.3$	$-2.8 \pm 0.3$
$\Phi_I$	GSMF norm redshift evolution	$-0.25 \pm 0.22$	$-0.25 \pm 0.22$
$\log_{10}M_0$	GSMF scaling mass	$11.25 \pm 0.2$	$11.25 \pm 0.2$
$\alpha_0$	GSMF mass slope	$-1.25 \pm 0.17$	$-1.25 \pm 0.17$
$\alpha_I$	GSMF mass slope redshift evolution	$0 \pm 0.15$	$0 \pm 0.15$
$f_0$	pair fraction norm	[0.02,0.03]	[0.01,0.05]
$\alpha_f$	pair fraction mass slope	[-0.2,0.2]	[-0.5,0.5]
$\beta_f$	pair fraction redshift slope	[0.6,1]	[0,2]
$\gamma_f$	pair fraction mass ratio slope	[-0.2,0.2]	[-0.2,0.2]
$\tau_0$	merger time norm	[0.1,2]	[0.1,10]
$\alpha_\tau$	merger time mass slope	[-0.2,0.2]	[-0.5,0.5]
$\beta_\tau$	merger time redshift slope	[-2,1]	[-3,1]
$\gamma_\tau$	merger time mass ratio slope	[-0.2,0.2]	[-0.2,0.2]
$\log_{10}M_*$	$M_{\text{bulge}} - M_{\text{BH}}$ relation norm	$8.17 \pm 0.33$	$8.17 \pm 0.33$
$\alpha_*$	$M_{\text{bulge}} - M_{\text{BH}}$ relation slope	$1 \pm 0.1$	$1 \pm 0.1$
$\epsilon$	$M_{\text{bulge}} - M_{\text{BH}}$ relation scatter	$[0.3,0.5]$	$[0.2,0.5]$
$e_0$	binary eccentricity	[0.01,0.99]	[0.01,0.99]
$\log_{10}\zeta_0$	stellar density factor	[-2,2]	[-2,2]

## Case of no-detection

Strain limit :  $A(f = \text{yr}^{-1}) = 1 \times 10^{-16}$



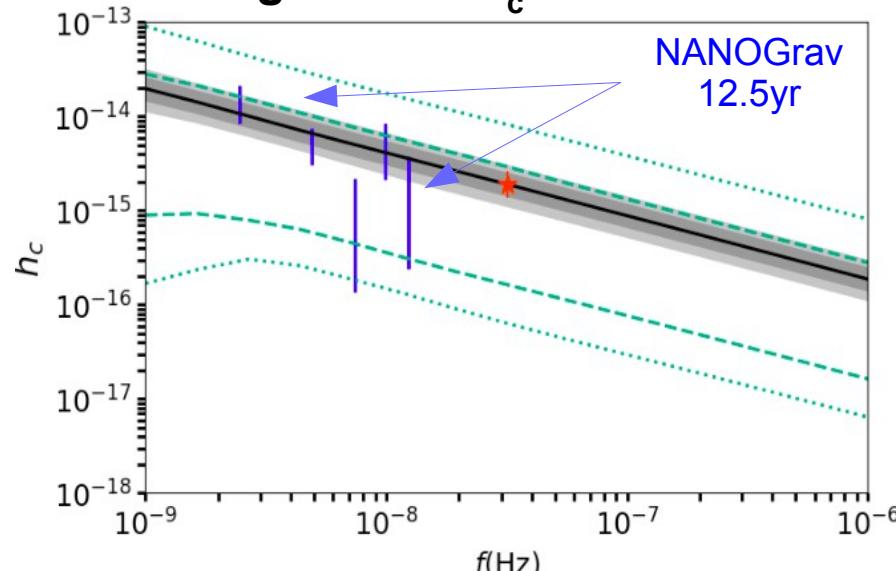
# Constraints on astrophysical models

Chen et al 2019

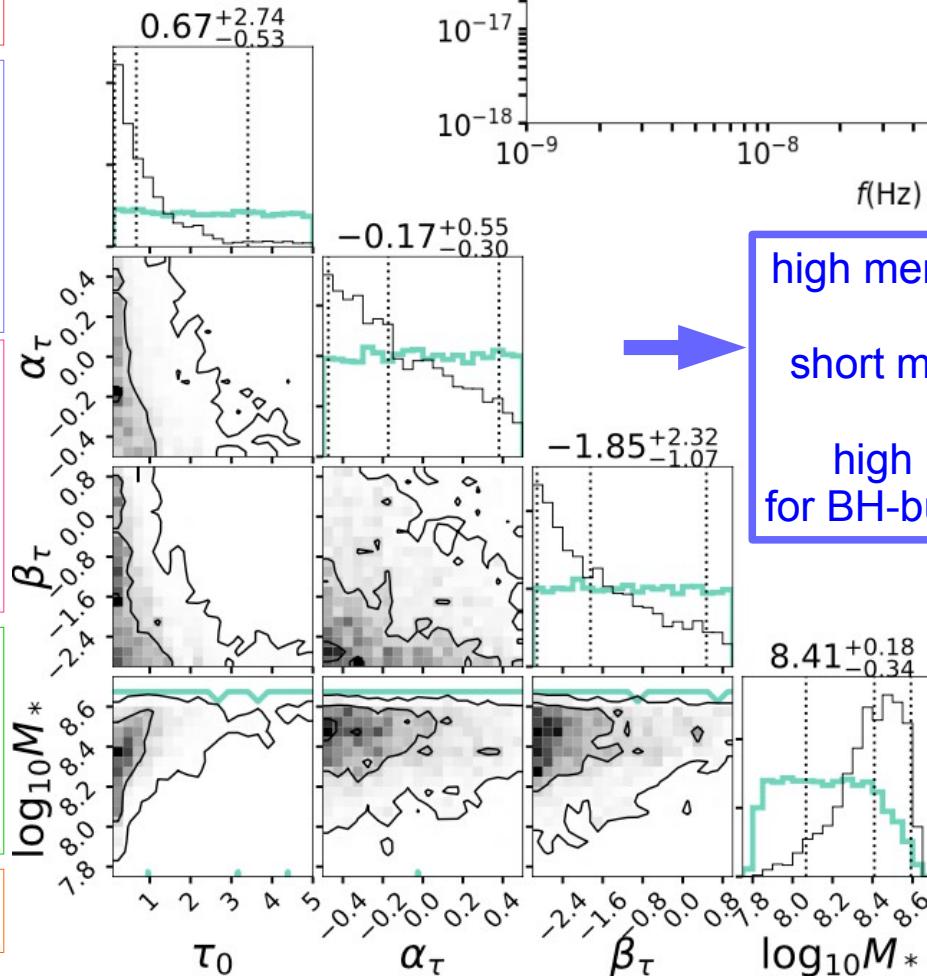
EPTA – population synthesis

parameter	description	standard	extended
$\Phi_0$	GSMF norm	$-2.8 \pm 0.3$	$-2.8 \pm 0.3$
$\Phi_I$	GSMF norm redshift evolution	$-0.25 \pm 0.22$	$-0.25 \pm 0.22$
$\log_{10}M_0$	Galaxy stellar mass function		
$\alpha_0$	GSMF scaling mass	$11.25 \pm 0.2$	$11.25 \pm 0.2$
$\alpha_I$	GSMF mass slope	$-1.25 \pm 0.17$	$-1.25 \pm 0.17$
	GSMF mass slope red-shift evolution	$0 \pm 0.15$	$0 \pm 0.15$
$f_0$	pair fraction norm	[0.02,0.03]	[0.01,0.05]
$\alpha_f$	pair fraction mass slope	[-0.2,0.2]	[-0.5,0.5]
$\beta_f$	Pair fraction	pair fraction redshift slope	[0.6,1]
$\gamma_f$	pair fraction mass ratio slope	[-0.2,0.2]	[-0.2,0.2]
$\tau_0$	merger time norm	[0.1,2]	[0.1,10]
$\alpha_\tau$	merger time mass slope	[-0.2,0.2]	[-0.5,0.5]
$\beta_\tau$	Merger timescale	merger time redshift slope	[-2,1]
$\gamma_\tau$	merger time mass ratio slope	[-0.2,0.2]	[-0.2,0.2]
$\log_{10}M_*$	$M_{\text{bulge}} - M_{\text{BH}}$ relation norm	$8.17 \pm 0.33$	$8.17 \pm 0.33$
$\alpha_*$	$M_{\text{bulge}} - M_{\text{BH}}$ relation slope	$1 \pm 0.1$	$1 \pm 0.1$
$\epsilon$	$M_{\text{bulge}} - M_{\text{BH}}$ relation scatter	$M_{\text{bulge}} - M_{\text{BH}}$	[0.3,0.5]
$e_0$	binary eccentricity	[0.01,0.99]	[0.01,0.99]
$\log_{10}\zeta_0$	stellar density factor	[-2,2]	[-2,2]
Eccentricity and stellar density			

case of a common red signal with  $h_c \sim 2 \cdot 10^{-15}$  ?



Middleton  
et al 2021



high merger rate densities  
short merger timescales  
high normalization  
for BH-bulge mass relation

# An essential diagnostic : the spatial correlation of the signal

The gravitational signal is contained in the covariance matrix C of the arrival time residuals r(t)

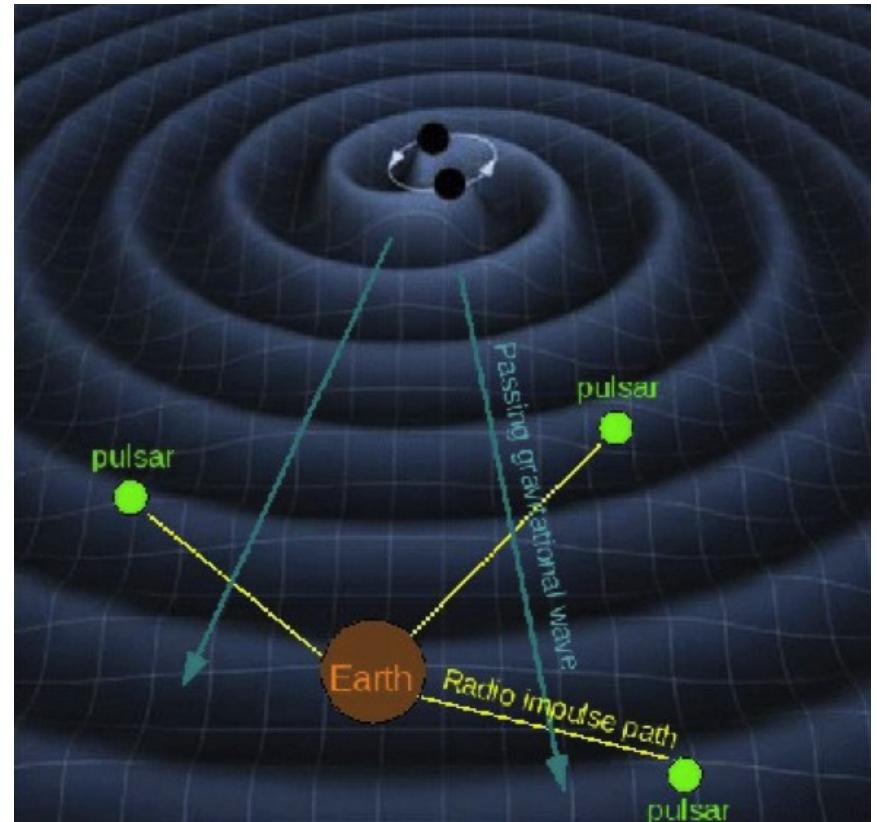
It is decomposed into a sum of « noises » whose spectrum is described by a power law

$$S \propto A^2 f^{-\gamma}$$

**the covariance matrix C** depends both on the amplitude of the signal as a function of its sky position and on the «antenna pattern »

$$C \sim \underbrace{\Gamma_{ab} \rho_i \delta_{ij}}_{\text{GW}} + \underbrace{\epsilon_i \delta_{ij}}_{\text{clock/eph.}} + \underbrace{\eta_i \delta_{ab} \delta_{ij}}_{\text{astro}\varphi} + \underbrace{\kappa_{ai} \delta_{ab} \delta_{ij}}_{\text{indiv. rot./disp.}}$$

$$\Gamma_{ab} = \frac{3}{8\pi} (1 + \delta_{ab}) \int_{S^2} d\hat{\Omega} P(\hat{\Omega}) \sum_q F_a^q(\hat{\Omega}) F_b^q(\hat{\Omega})$$



# An essential diagnostic : the spatial correlation of the signal

The gravitational signal is contained in the covariance matrix C of the arrival time residuals r(t)

It is decomposed into a sum of « noises » whose spectrum is described by a power law

$$S \propto A^2 f^{-\gamma}$$

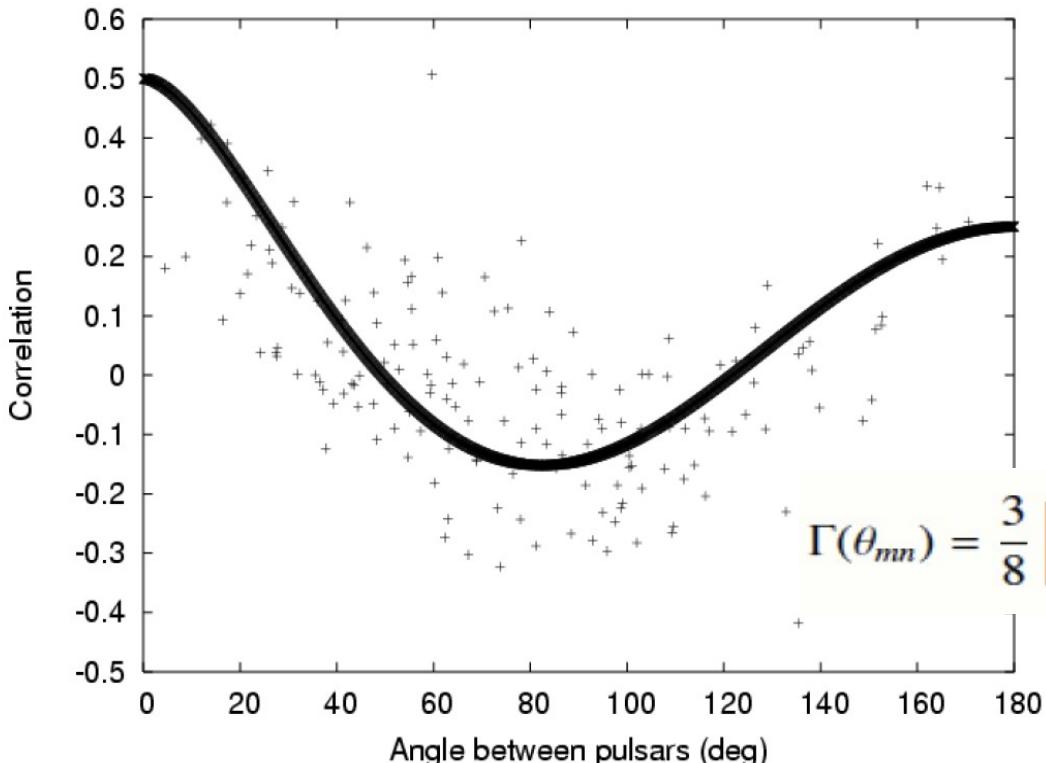
**the covariance matrix C** depends both on the amplitude of the signal as a function of its sky position and on the «antenna pattern»

$$C \sim \underbrace{\Gamma_{ab} \rho_i \delta_{ij}}_{\text{GW}} + \underbrace{\epsilon_i \delta_{ij}}_{\text{clock/eph.}} + \underbrace{\eta_i \delta_{ab} \delta_{ij}}_{\text{astro}\varphi} + \underbrace{\kappa_{ai} \delta_{ab} \delta_{ij}}_{\text{indiv. rot./disp.}}$$

$$\Gamma_{ab} = \frac{3}{8\pi} (1 + \delta_{ab}) \int_{S^2} d\hat{\Omega} P(\hat{\Omega}) \sum_q F_a^q(\hat{\Omega}) F_b^q(\hat{\Omega})$$

→ **Earth term: the stochastic signal is spatially correlated between all pulsars**

**as a function of their angular separation**



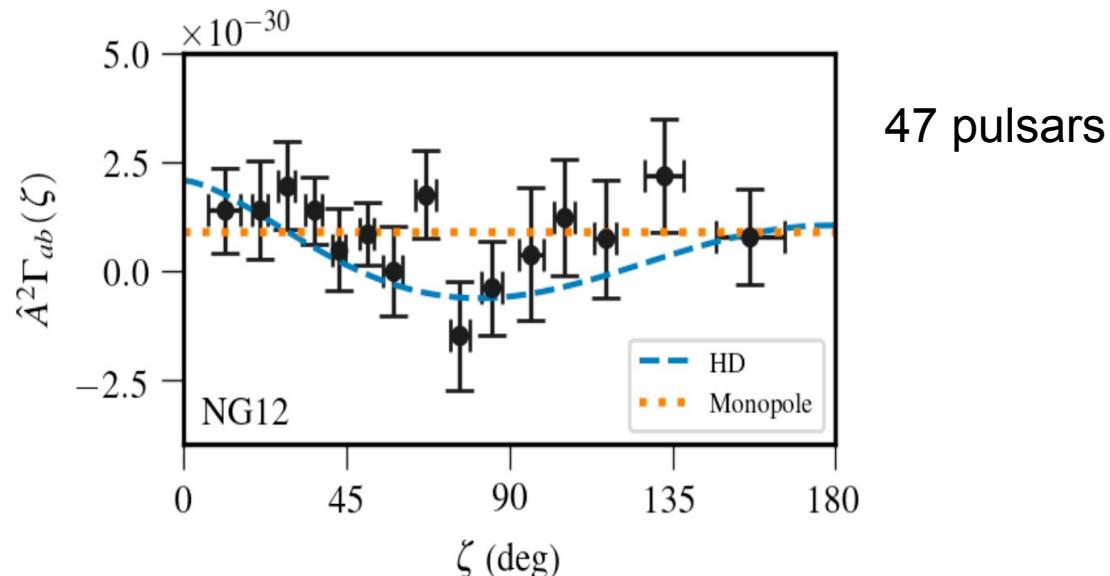
Cf Hellings & Downs 1983

**solution for an isotropic background :**

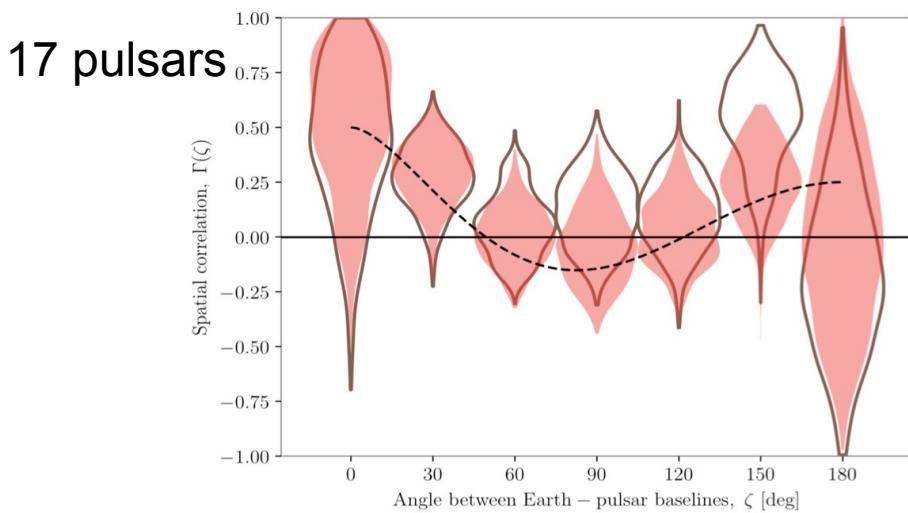
$$\Gamma(\theta_{mn}) = \frac{3}{8} \left[ 1 + \frac{\cos \theta_{mn}}{3} + 4(1 - \cos \theta_{mn}) \ln \left( \sin \frac{\theta_{mn}}{2} \right) \right] (1 + \delta_{mn})$$

# An essential diagnostic : the spatial correlation of the signal

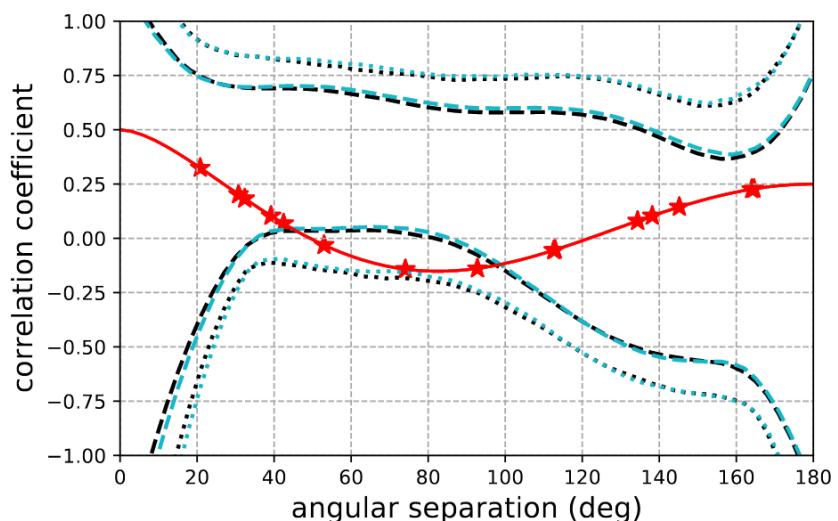
Not detected yet!



NANOGrav ; Arzoumanian et al 2020

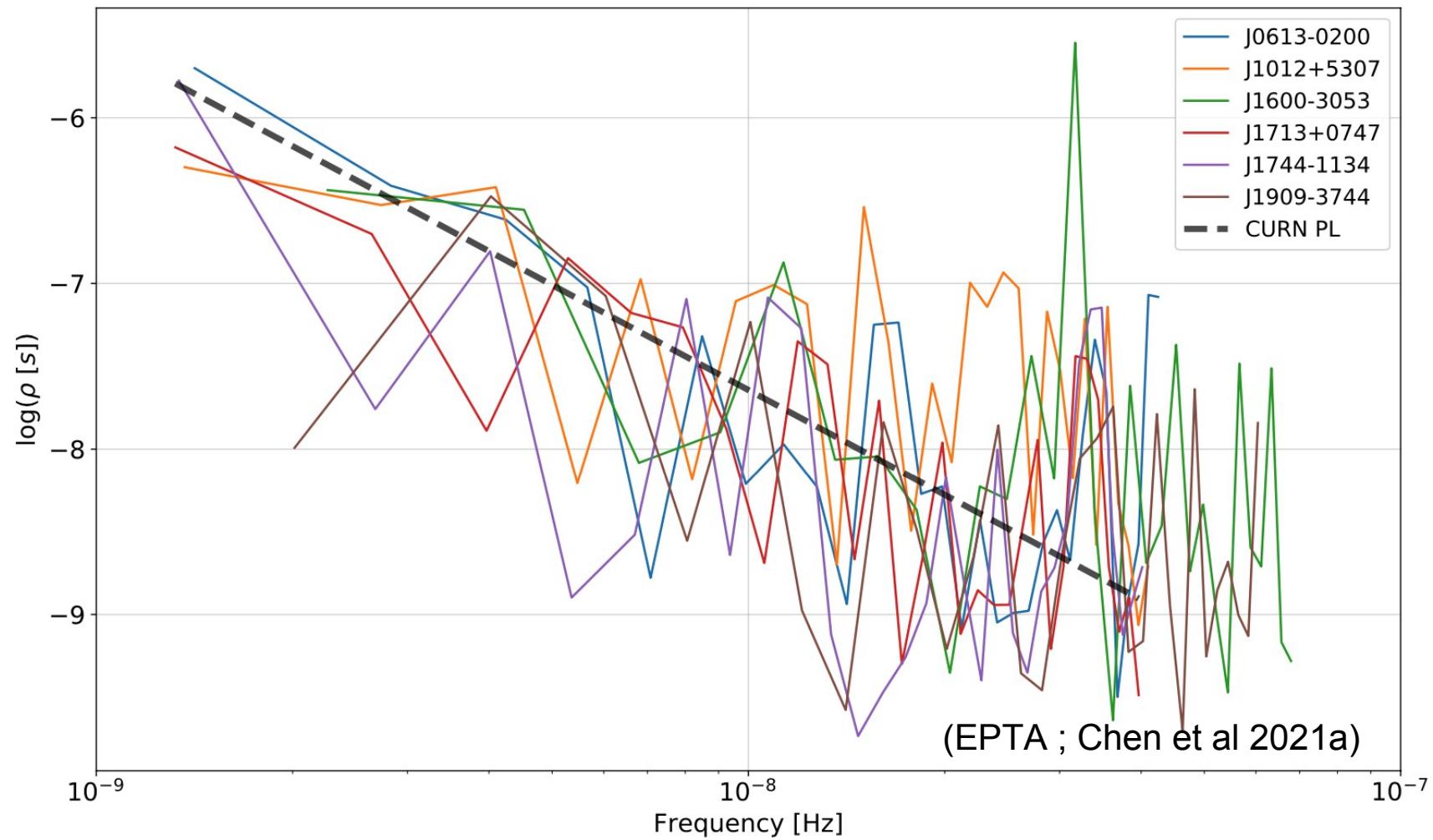


PPTA ; Goncharov et al 2021



EPTA – 6 best ; Chen et al 2021

# We cannot distinguish yet a common uncorrelated red noise of some astrophysical origin from a GW stochastic background

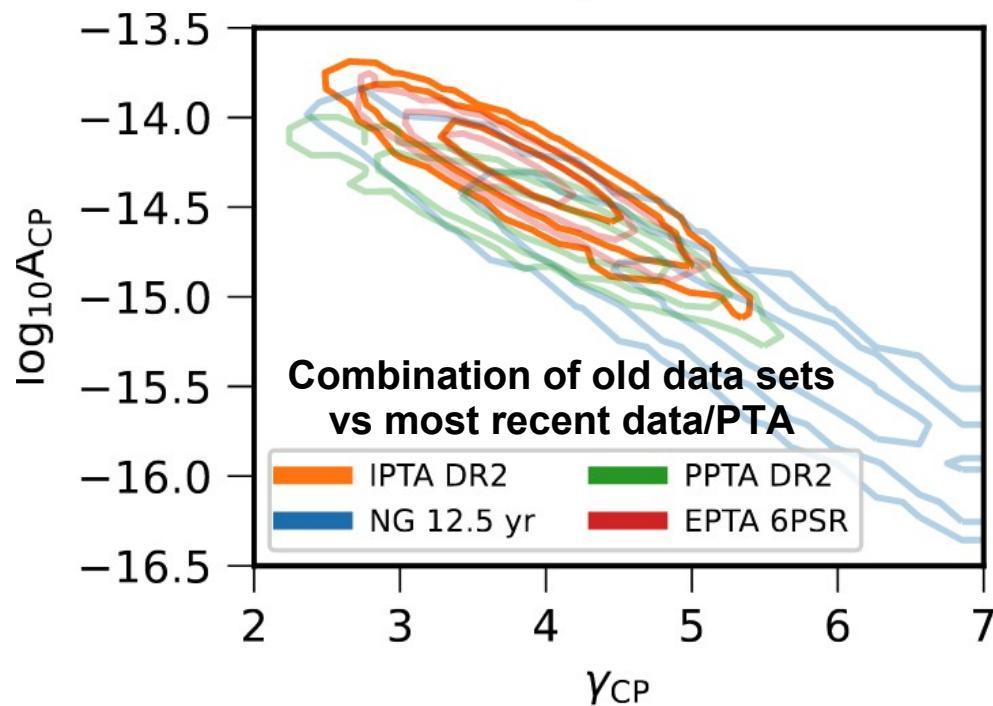
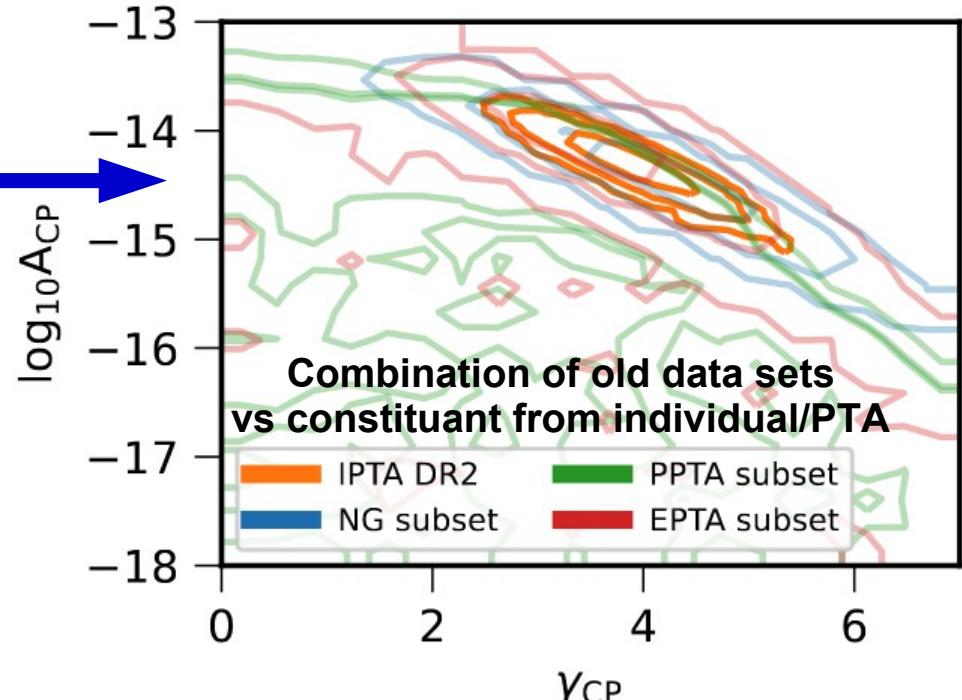


Comparing individual pulsar noise models to inferred Common Uncorrelated Red Noise

## Follow-up results

1) confirmation using IPTA-DR2  
= combination of « old » data  
(EPTA 15 years, NANOGrav 9 years, PPTA 6 years)

Antoniadis et al 2022



# Follow-up results

## 1) confirmation using IPTA-DR2 = combination of « old » data

(EPTA 15 years, NANOGrav 9 years, PPTA 6 years)

## 2) « 3+ paper agreement » (late 2022)

- building extended data sets
- issue = clear detection of the spatial correlation of the signal

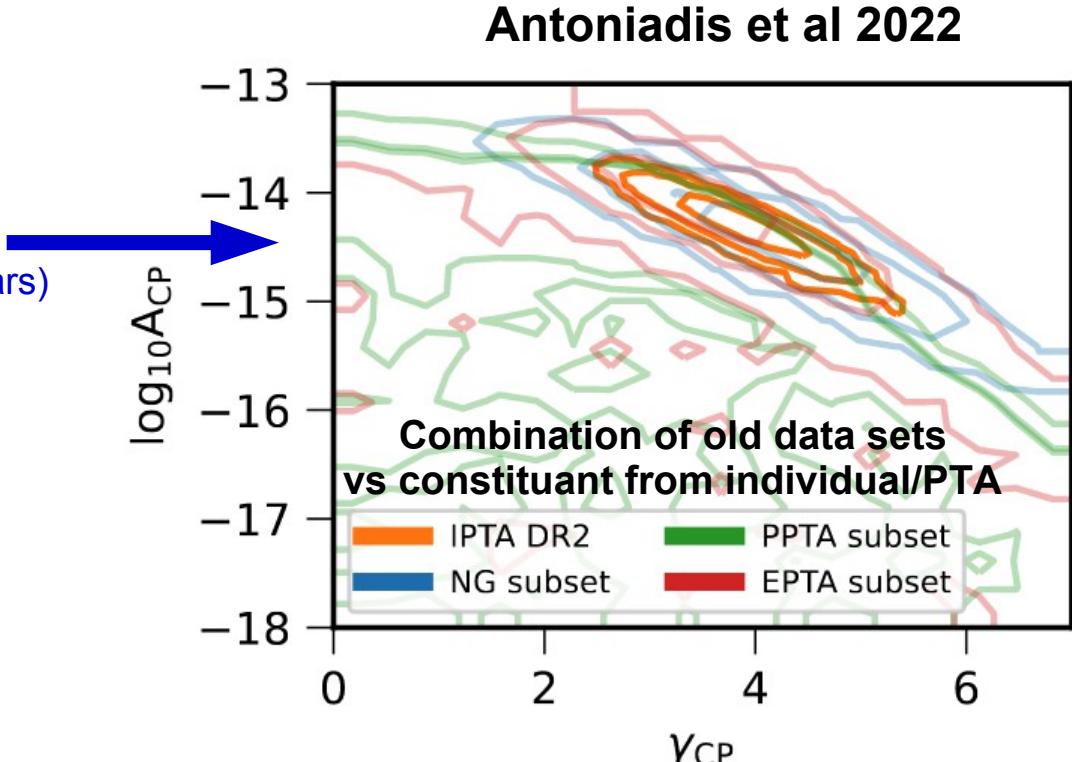
|

- a common detection check list

EPTA (+InPTA)    6 → 26 pulsars, T = 25 years

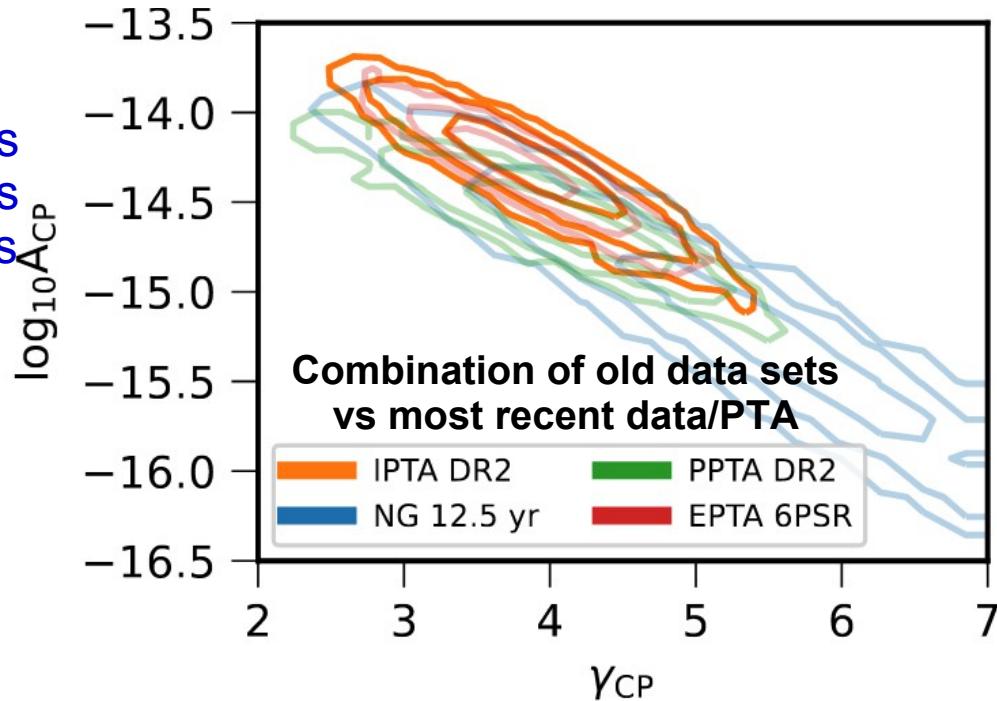
PPTA                17 → 26 pulsars, T = 25 years

NANOGrav          47 → 60 pulsars, T = 15 years

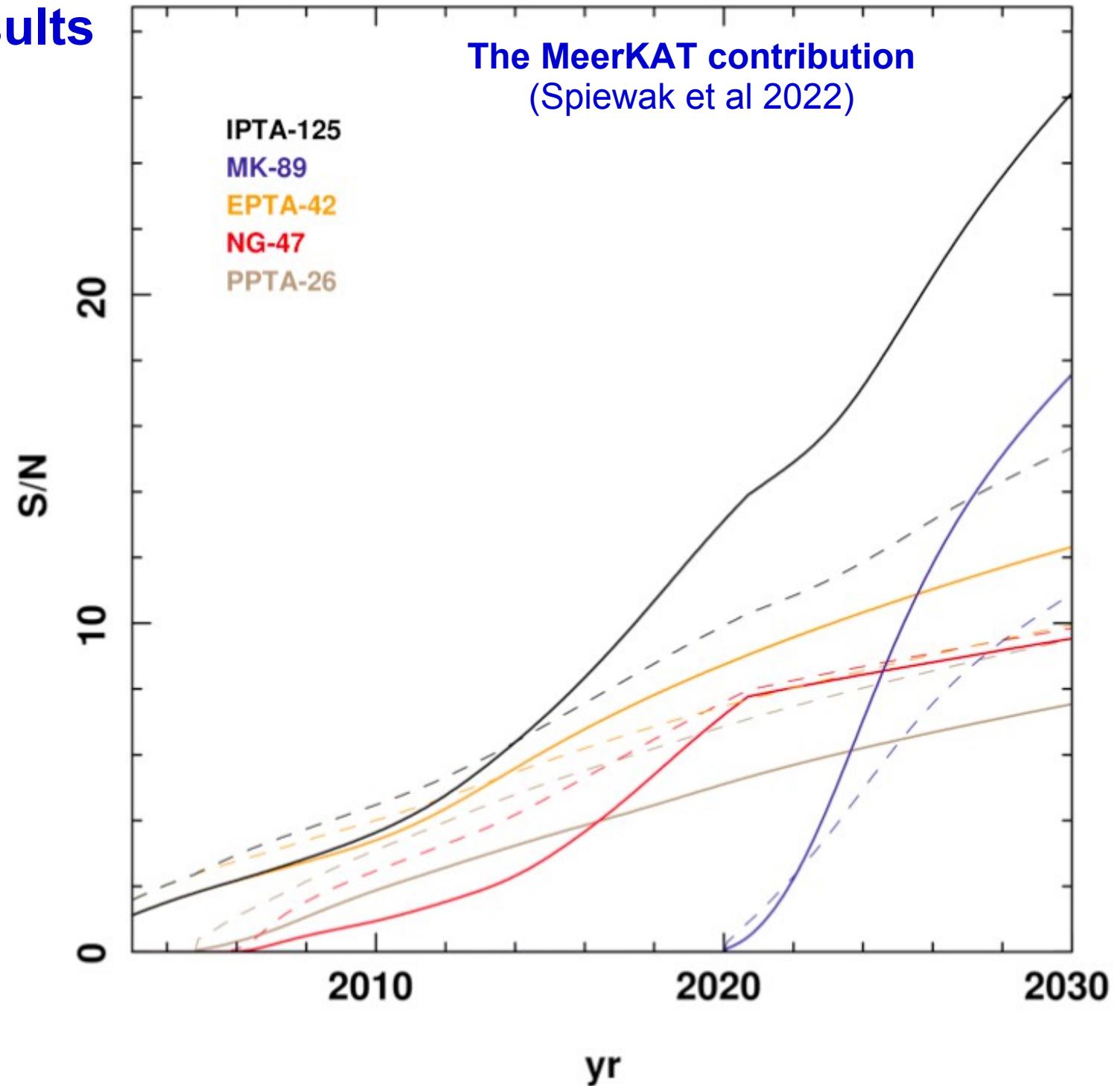


## 3) IPTA DR3 release (late 2023?)

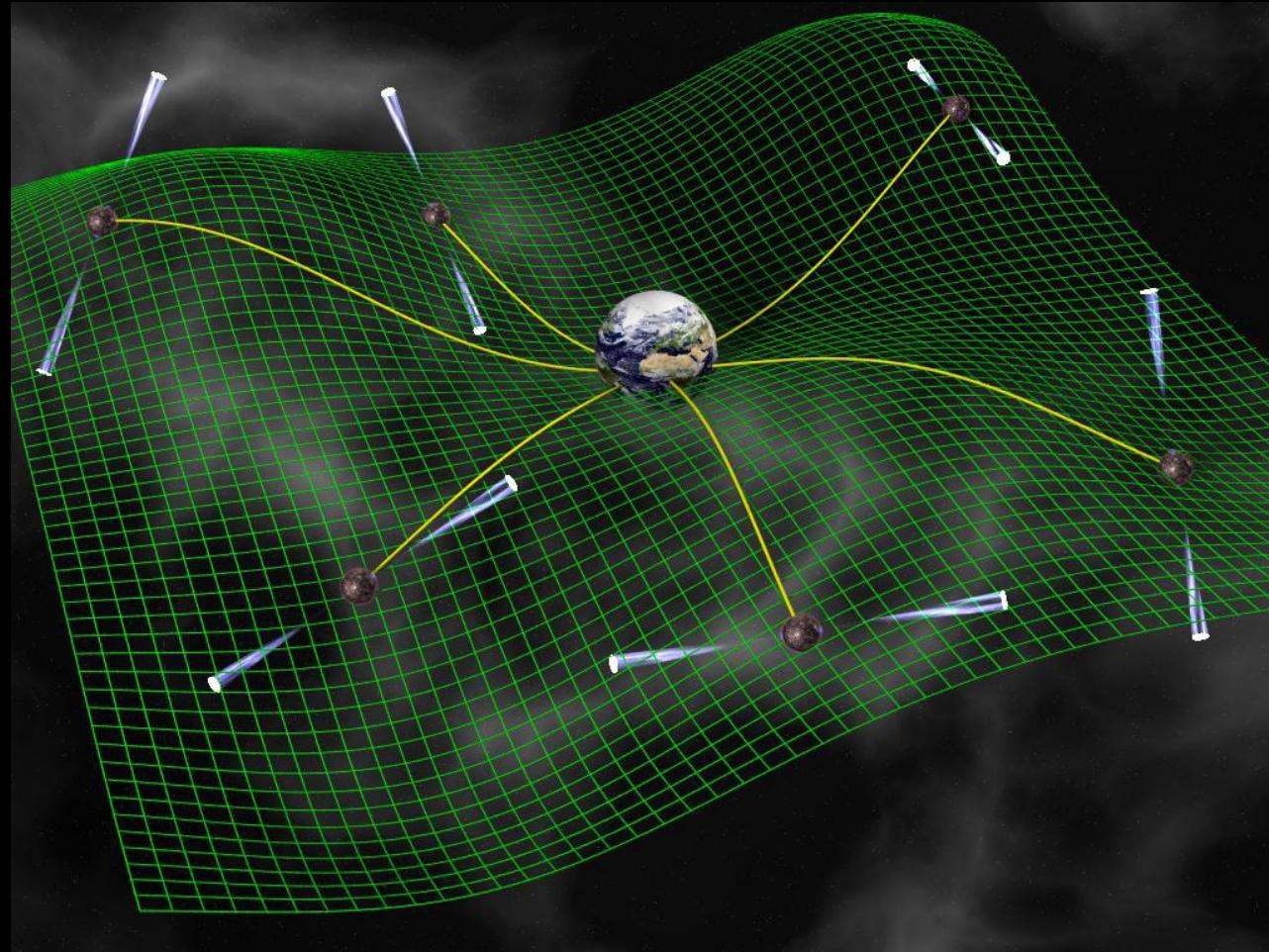
= combination of the three new data sets  
+ 3 years MeerKAT + 3 years FAST



## Follow-up results



# Work in progress...



Expect interesting results in the coming year