Estimating the Hubble constant from the mock GW data of Einstein Telescope

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Founded in 1825

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Abstract

The Hubble constant is a crucial cosmological parameter that is a measure of the rate of change of the cosmic scale factor per unit cosmic scale factor i.e. $\dot{a}/a$. There is a considerable discrepancy between the measurements of the Hubble constant from standard candle observations and those from cosmic microwave background (CMB) observations. Data from gravitational wave (GW) events can provide an independent constraint on the Hubble constant. Higher the number of events, the stronger is the constraint. A tight constraint is expected to be achieved in the era of the third generation detectors such as the Einstein Telescope (ET). Without relying on any electromagnetic observation, one can either use the double black hole (BH) merger or the double neutron star (NS) merger detections to break the mass-redshift degeneracy. We present a method of estimating the Hubble parameter using ET mock data for NS-NS events and discuss the challenges. We assume flat cosmology in our analysis.

Einstein Telescope

- ET is a proposed Gen III gravitational wave detector
- Tenfold better sensitivity than present Gen II detectors
- GW bandwidth: 1 Hz – 10 kHz (LIGO: 10 – 1000 Hz)
- Located underground at a depth of 100–300 m to reduce noise
- Equilateral triangle configuration with arm-length 10 km
- 2-band xylophone design with 6 interferometers
- Low Frequency (1 – 40 Hz); High Frequency (40 Hz – 10 kHz)
- Sensitive to GW from all directions without any blind spot
- Can generate null streams useful to eliminate glitches

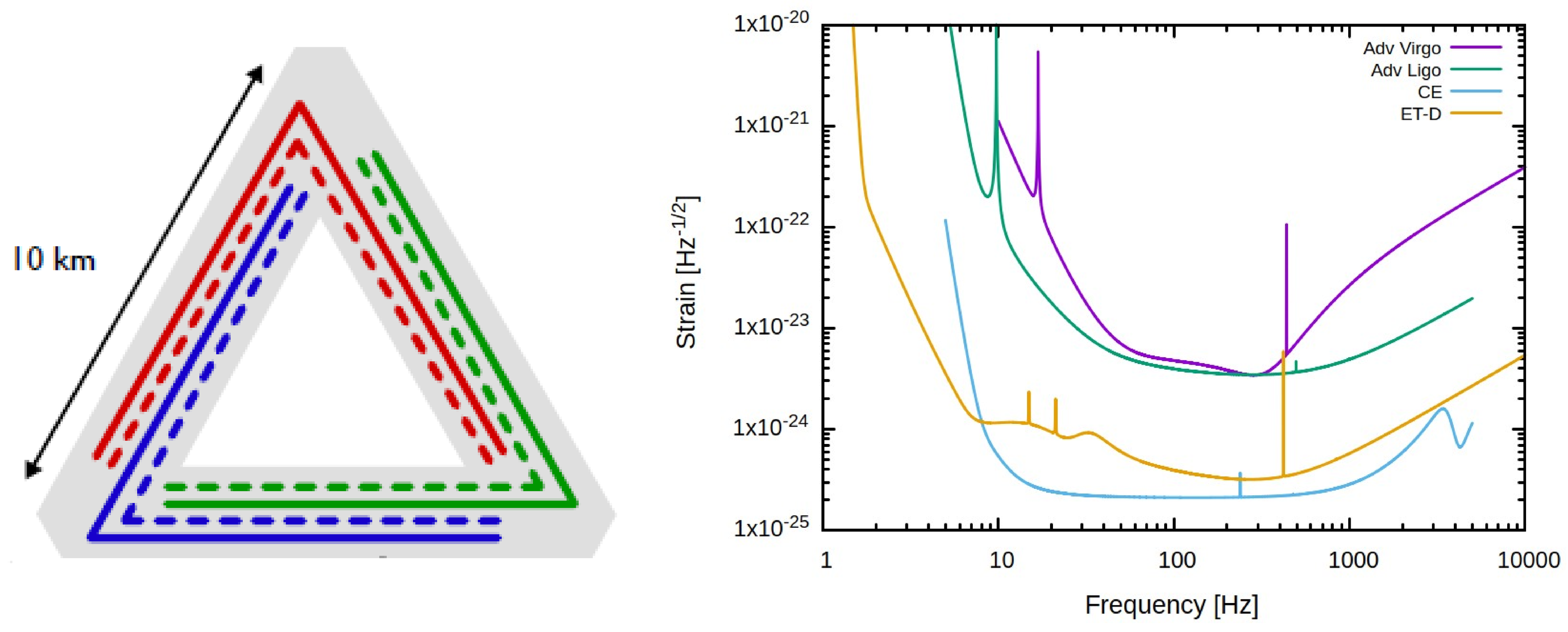

Figure 1: **(a)** ET-D design **(b)** Amplitude spectral density of ET

- Detect BH-BH mergers upto $z \sim 20$ @ 10^5–10^6 events/year
- Detect NS-NS mergers upto $z \sim 3$ @ 10^4–10^5 events/year
- H_0 measurement to 1% uncertainty in 1 year of ET (Zhang et al. 2020, You et al. 2021)

Gravitational Wave Astronomy

- The strain (amplitude), h, in the interferometer arm of length, L, of a GW detector is given by

$$h(t) = \Delta L/L = \text{constant} \times \frac{\mathcal{M}^{5/3}}{d_L} f^{2/3}\,\Theta \cos\Phi$$

where $\mathcal{M} = \dfrac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}}$ is called the chirp mass of a binary.

- Due to the cosmological redshift of the incoming GW frequency, what we measure is the redshifted chirp mass, $\mathcal{M}_z = (1+z)\mathcal{M}$.
- Since a measured chirp mass can correspond to many chirp mass and redshift values, we get **mass-redshift degeneracy**.
- In the absence of an EM counterpart of the GW event, one can lift the degeneracy with the population method.

ΛCDM cosmological model

- The current standard model of the universe is called the Λ-Cold Dark Matter (ΛCDM).
- It is characterized by the parameters: H_0, Ω_m, Ω_Λ, Ω_rad, Ω_k, w
- Hubble constant, H_0, quantifies the expansion rate of universe; ~70 km/s/Mpc.
- By construction, $\Omega_\text{m} + \Omega_\text{rad} + \Omega_k + \Omega_\Lambda = 1$. In the minimal six-parameter model: $\Omega_\text{rad} \sim 0$, $\Omega_k = 0$ (flat), $w = -1$ so that $\Omega_\Lambda + \Omega_\text{m} = 1$

Hubble Tension

- The $\sim 3-5\sigma$ discrepancy in the H_0 using late universe souces (e.g. Cepheids, SNe Ia) and early universe sources (e.g. CMB) is called the **Hubble tension**.
- GW standard siren measurements using ET can solve the discrepancy in future.

Mock data

- We generate a NS-NS binary merger population using the binary evolution code `StarTrack` (Belczynski et al. 2008, 2020).
- These NS binaries are analyzed using ET's design sensitivity (Singh et al. 2021, 2022).
- The ones which exceed the detection threshold (SNR_eff > 8 and at least one SNR_i > 3 for $i \in [1,2,3]$) are identified as events.
- Using a cosmological model, the luminosity distance is measured for each detected event from the observable quantities.

Analysis

Given data: $P(\mathcal{M})$, $P(\mathcal{M}_z)$, $P(d_L)$, $\Omega_\text{m} = 0.3$, $\Omega_\Lambda = 1 - \Omega_\text{m} = 0.7$, $w = -1$

$$\because \mathcal{M}_z = \mathcal{M}(1+z) \implies z = \frac{\mathcal{M}_z}{\mathcal{M}} - 1$$

Probability distribution of z:

$$P(z) = \int d\mathcal{M}_z\, P(\mathcal{M}_z) \int d\mathcal{M}\, P(\mathcal{M})\, \delta\left(z - \left(\frac{\mathcal{M}_z}{\mathcal{M}} - 1\right)\right)$$

$$\because H_0 \equiv H_0(z, d_L) = \frac{c}{d_L}(1+z)\int_0^z \frac{dz'}{\sqrt{\Omega_\text{m}(1+z')^3 + (1-\Omega_\text{m})}}$$

Probability distribution of H_0:

$$P(H_0) = \int dz\, P(z) \int dd_L\, P(d_L)\, \delta\left(H_0 - H_0(z, d_L)\right)$$

Results and Conclusion

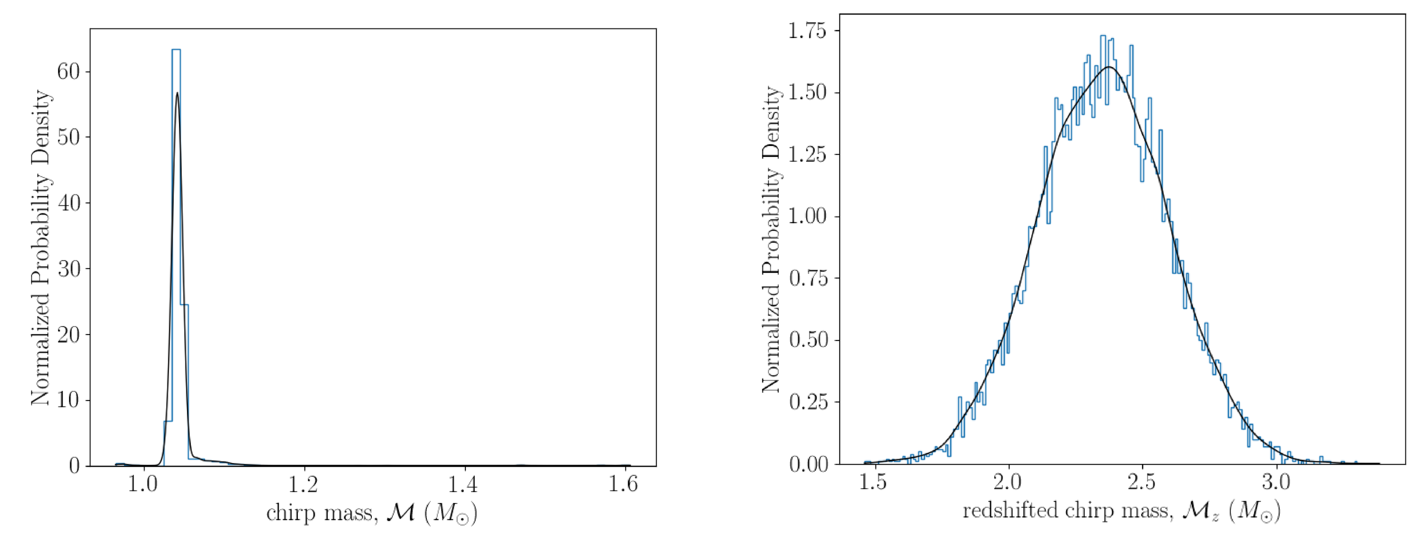

Figure 2: $P(\mathcal{M})$ used in the study. Here, $\mathcal{M}_{min} = 0.96\, M_\odot$ and $\mathcal{M}_{max} = 1.60\, M_\odot$. Binsize: 0.01 $M_\odot$. **(b)** $P(\mathcal{M}_z)$ for one of the events. Binsize: 0.01 $M_\odot$. This $P(\mathcal{M}_z)$ is to be used as input together with $P(\mathcal{M})$ to determine $P(z)$ for this specific event.

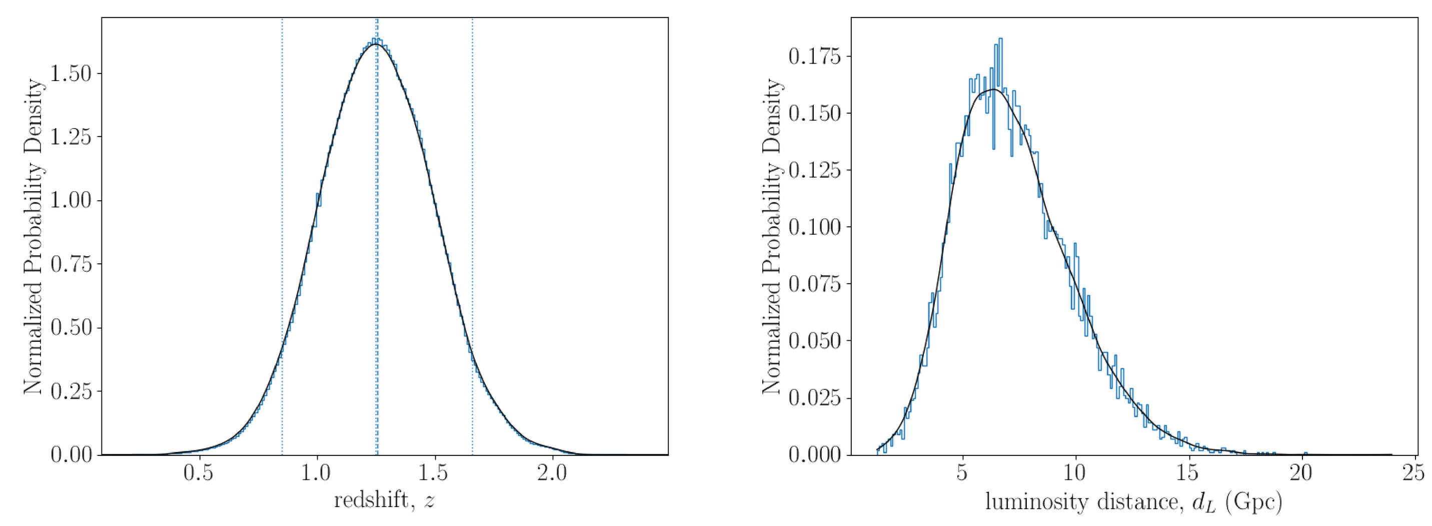

Figure 3: $P(z)$ for the same event. Middle dotted line shows the median of the distribution. Dashed line shows the injected value. Binsize: 0.01. **(b)** $P(d_L)$ for the same event. Binsize: 0.1 Gpc. This $P(d_L)$ is to be used as input together with $P(z)$ to determine $P(H_0)$ for this specific event.

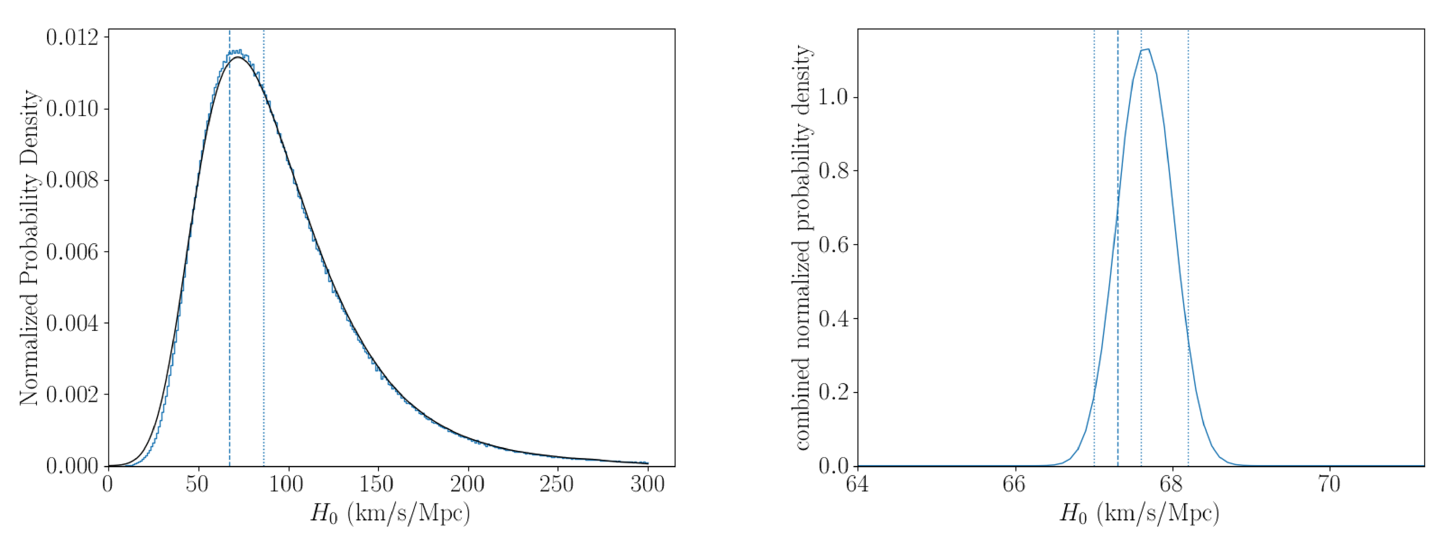

Figure 4: $P(H_0)$ for the same event. Dotted line shows the median of the distribution. **(b)** Combined $P(H_0)$ with stepsize 0.1 km/s/Mpc. Dashed line shows the injected value of 67.3 km/s/Mpc.

The estimate derived is $H_0 = 67.6 \pm 0.6$ km/s/Mpc. The uncertainty falls inversely as $\sim 1/\sqrt{N}$ with number of events, and becomes $< 1\%$ for > 5000 events.