The effects of the selection function on the dark siren measurement of the Hubble constant

CBPF Centro Brasileiro de Pesquisas Físicas — UNIDADE DE PESQUISA DO MCTI · CAPES · LabIA

Naomi N. Toribio [1] Viviane A. P. Alfradique [1] Clécio R. De Bom [1]

[1]Brazilian Center for Research in Physics, CBPF

Gravitational Waves and H_0 measurement

The standard siren method enables an independent measurement of the Hubble constant (H_0) using the luminosity distance (d_L) inferred from gravitational wave (GW) signals emitted by compact binary coalescences (CBCs) combined with redshift measurement. When no electromagnetic counterpart is observed, the source's redshift must be inferred through alternative methods. One possibility is a Bayesian approach that uses redshift information from all potential host galaxies — this is the statistical dark siren approach.

As the number of GW detections increases, the precision of H_0 measurement improves, making it crucial to understand and control systematic uncertainties in the dark siren method. One source of systematics may arise from incorrect assumptions about the true source population, which leads to an incorrect computation of the selection function — that describes the probability of a source at a certain position and with a given set of parameters being detected. This function depends on the detector conditions and our prior assumptions about the underlying population of compact binaries (e.g., mass and redshift distributions).

In this work, we use LIGO Skympap's `Bayestar` package [1] to simulate GW events and construct selection functions based on different population models. We then evaluate how much the detection probability estimated under different mass distributions deviates from each other by computing the $\mathcal{L}_1$ norm between the respective results. This comparison helps to identify how incorrect population assumptions lead to discrepancies in detectability, providing insight into the potential impact of population mismodeling in the selection functions for a dark siren measurement of H_0.

Bayesian Model and Selection Function

To infer the Hubble constant H_0 using dark sirens, we adopt a Bayesian framework. According to Bayes' theorem, the posterior for a given event is written as $p(H_0 \mid d_{\rm GW}) \propto p(H_0)\mathcal{L}(d_{\rm GW} \mid H_0)$, where $p(H_0)$ is the prior (assumed uniform in the range $[20, 140]\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$), and the likelihood is:

$$\mathcal{L}(d_{\rm GW} \mid H_0) \propto \frac{\int dz\, \mathcal{L}_{GW}\left(\hat{d}_L \mid d_L(z, H_0)\right) p_{\rm CBC}(z)}{\int dz\, P^{\rm GW}_{\rm det}(z, H_0) p_{\rm CBC}(z)} \quad (1)$$

The denominator defines the selection function $\beta(H_0)$, which accounts for detection probability. Here $P^{\rm GW}_{\rm det}(z, H_0)$ models the detectability of events, which depends on the detector sensitivity and the source's population model. It is estimated by averaging over a population of $N_{\rm GW}$ simulated sources that satisfy the detection conditions: $D_L < 1500$ Mpc, $A_{90\%} < 10\,\mathrm{deg}^2$, and $\mathrm{SNR} > 12$, with $x^i_{\rm GW}$ representing the GW events data (eq. 2).

$$P^{\rm GW}_{\rm det}(z, H_0) = \frac{1}{N_{\rm GW}} \sum_{i=1}^{N_{\rm GW}} P^{\rm GW}_{\rm det}(x^i_{\rm GW}, z, H_0) \quad (2)$$

The other component of $\beta(H_0)$, $p_{\rm CBC}(z)$, describes the CBC probability at redshift z, depending on both $p_{\rm rate}(z, x_{\rm gal})$, which represents the merger rate — here assumed to be proportional to the stellar mass (eq.3) — and $p_{\rm cat}(z, x_{\rm gal})$, which describes the probability of a galaxy being located at redshift z (eq. 4). The only galaxy data considered was stellar mass, so $x_{gal} = M_{*,gal}$, and we assume a complete galaxy catalog with precise redshift and stellar mass measurements.

$$p_{rate}(z^i_{gal}, M^i_{*,gal}) \propto M^i_{*,gal} \quad (3)$$

$$p_{\rm cat}(z) \approx \frac{1}{N_{\rm gal}} \sum_{i=1}^{N_{\rm gal}} \delta(z - z^i_{\rm gal})\delta(M_{*,gal} - M^i_{*,gal}) \quad (4)$$

Under these assumptions, $p_{\rm CBC}(z)$ simplifies to eq. 5.

$$p_{\rm CBC}(z) \propto \frac{1}{N_{\rm gal}} \sum_{i=1}^{N_{\rm gal}} p_{\rm rate}(z, M^i_{*,gal})\delta(z - z^i_{\rm gal}) \quad (5)$$

Finally, we assume there is no dependence with sky position from the detection probability and approximate the GW likelihood $\mathcal{L}_{\rm GW}$ as a Gaussian. With this simplifications, the likelihood becomes:

$$\mathcal{L}(d_{\rm GW} \mid H_0) \propto \frac{\sum_{i=1}^{N_{\rm gal}} \mathcal{L}_{\rm GW}(\hat{d}_L \mid d_L(z^i_{\rm gal}, H_0), \hat{\Omega}_i)}{\beta(H_0)} \text{ , with } \beta(H_0) = \sum_{i=1}^{N_{\rm gal}} P^{\rm GW}_{\rm det}(z^i_{\rm gal}, H_0)\, p_{\rm rate}(z^i_{\rm gal}, x^i_{\rm gal}) \quad (6)$$

where $\hat{\Omega}_i$ represents the solid angle of the HEALPIX pixel for the galaxy.

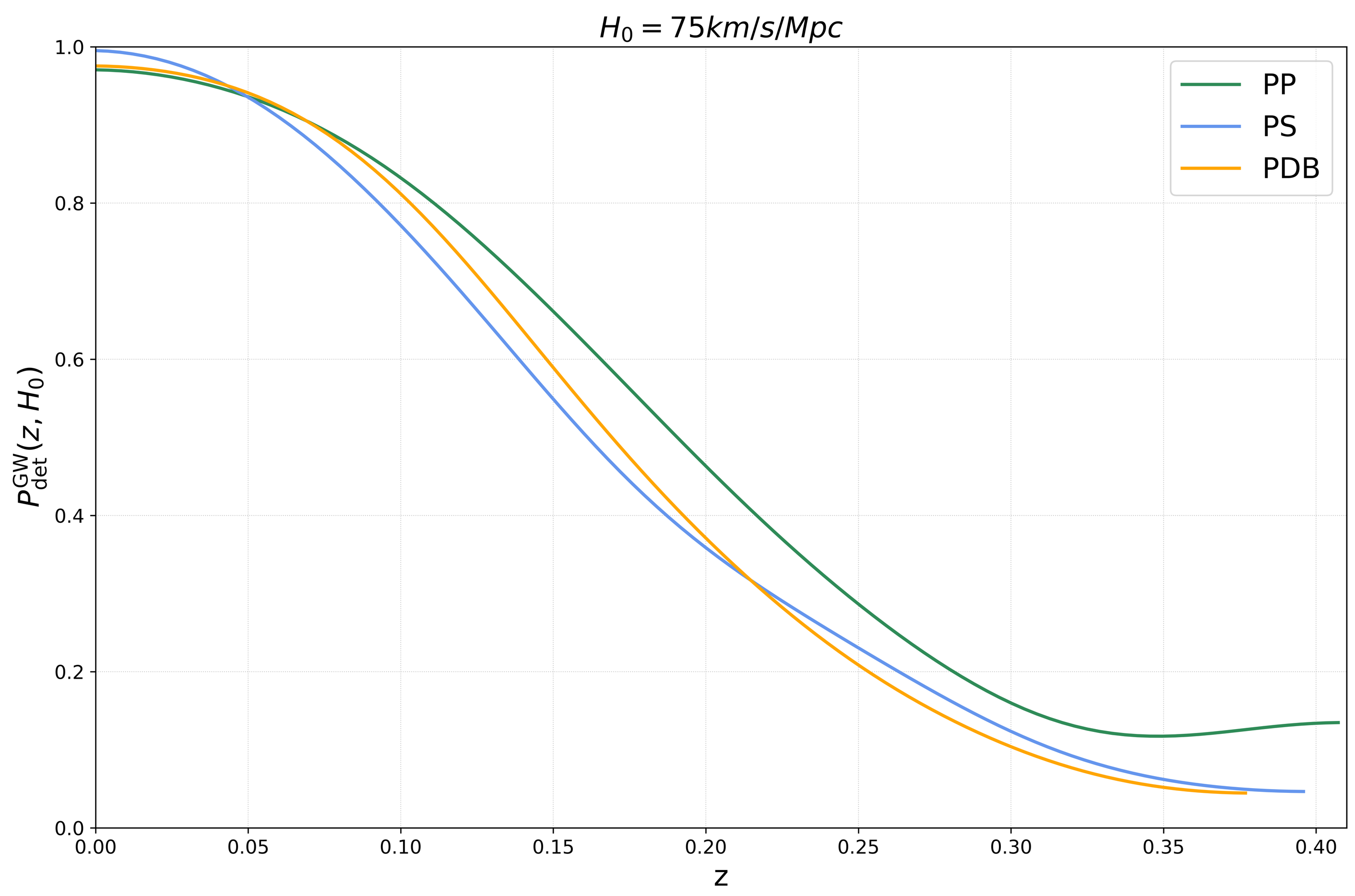
Mass Distributions

The binary black hole (BBH) mass distribution is modeled using the GWTC-3 population fits from Abbott et al. (2023) [2], considering three models: Power Law + Peak (PP) — a power law with a Gaussian peak; Power Law + Spline (PS) — a power law modified by a cubic spline; Power Law + Dip + Break (PDB) — a broken power law with suppression and tapering.

Our goal is to quantify how much the detection probability estimated using PS and PDB deviates from PP, wich is our current best fit. These discrepancies directly affect the selection function and can propagate to the H_0 inference.

Probability of GW detection: $P^{\rm GW}_{\rm det}(z, H_0)$

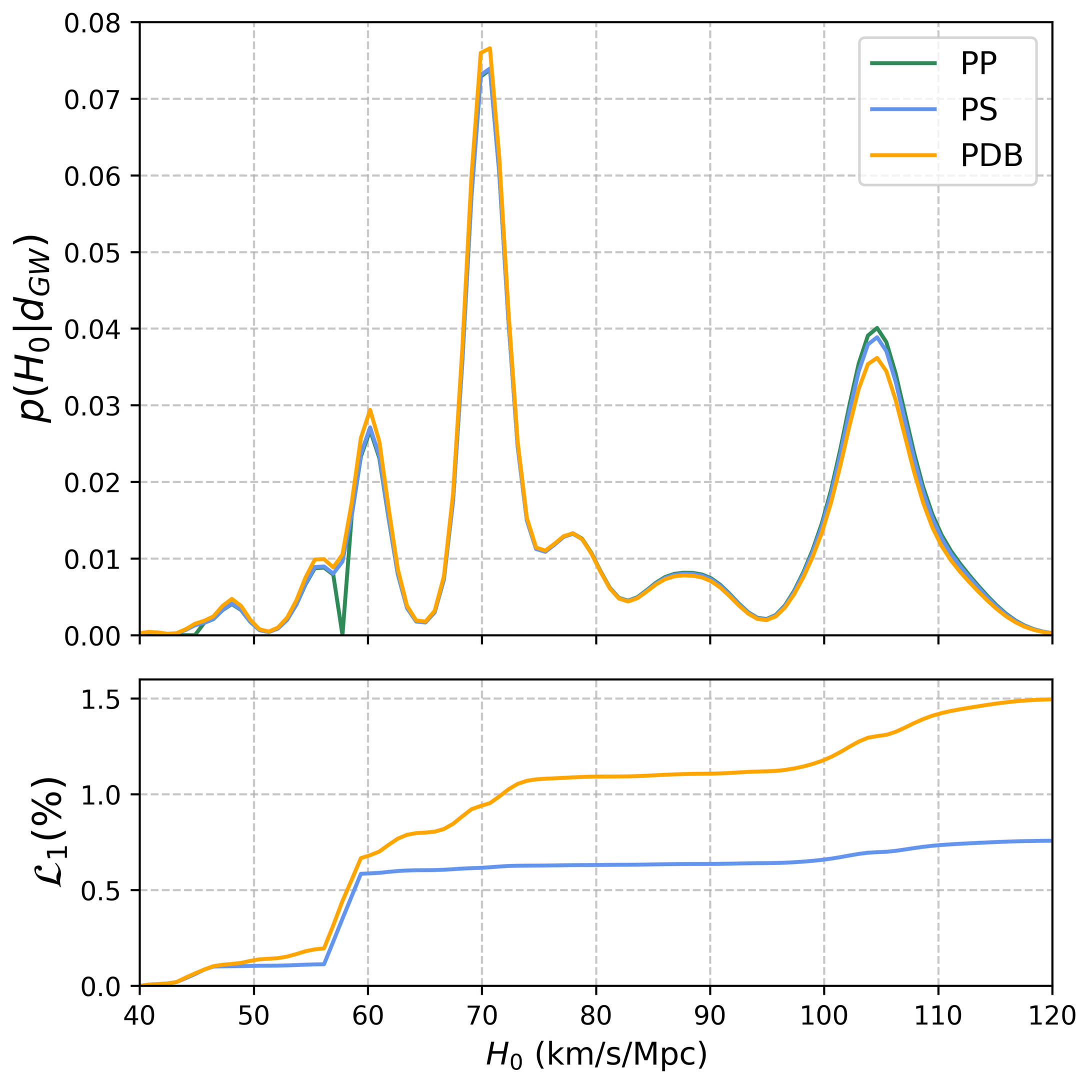
For each mass distribution model, the function $P^{\rm GW}_{\rm det}(z, H_0)$ was computed by comparing injected and detected events, enabling the computation of $\beta(H_0)$, as described by eq. 2.



Model comparison - Preliminary results

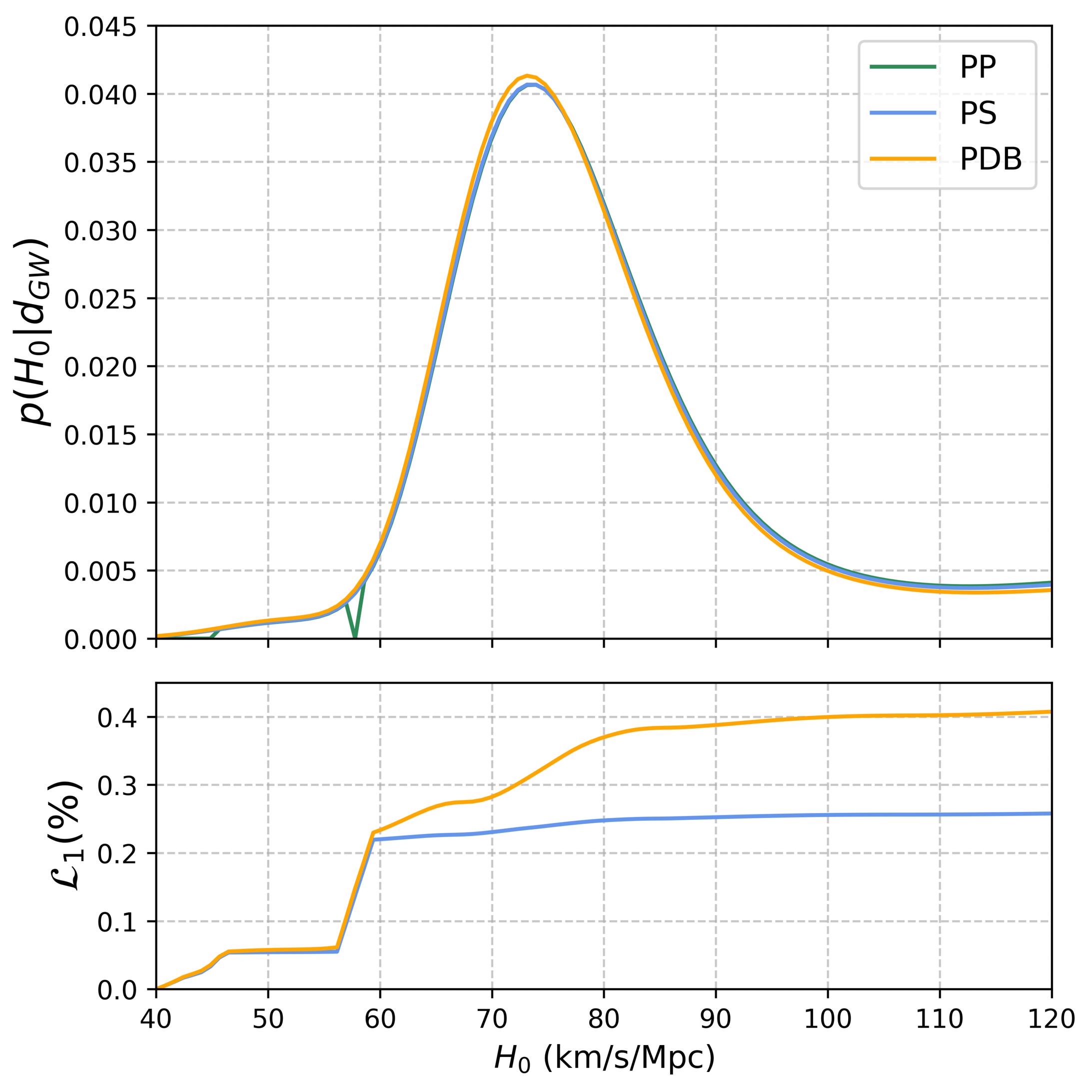
Event 1

- $z = 0.08$ • $SNR = 51$ • $d_L = 384.17Mpc$ • $\sigma_{d_L} = 13.61Mpc$ • $A_{90\%} = 1.1deg^2$



Event 2

- $z = 0.07$ • $SNR = 67$ • $d_L = 454.47Mpc$ • $\sigma_{d_L} = 53.60Mpc$ • $A_{90\%} = 1.0deg^2$



Conclusions and Perspectives

We applied three different mass distribution models (PP, PS, and PDB) to two simulated GW events and evaluated the deviation in detection probability, obtaining the cumulative absolute difference between PP distribution and PS and PDB distributions, kown as the $\mathcal{L}_1$ norm. Only a slight variation was observed, with the cumulative $\mathcal{L}_1$ under 2%.

For Event 1, there is no significative shif from the distribution, the only noticeable difference around the peak being 14% difference in height. The cumulative $\mathcal{L}_1$ norm shows differences around 0.6% (PP vs. PS) and 1.5% (PP vs. PDB).

Event 2 show a slight shif for both PS and PDB peaks, in compatison to PP, but no notable height difference. The cumulative $\mathcal{L}_1$ norm shows a total difference around 0.3% (PP vs. PS) and 0.4% (PP vs. PDB).

While Event 1 shows a slightly more pronounced mismodeling effect, the impact remains moderate. That said, this underscores the importance of accurate source population modeling. Further simulations are needed, including delayed merger rate scenarios to test alternative astrophysical assumptions.

References

[1] L. P. Singer and L. R. Price, "Rapid bayesian position reconstruction for gravitational-wave transients," *Physical Review D*, vol. 93, no. 2, p. 024013, 2016.

[2] R. Abbott, T. Abbott, F. Acernese, K. Ackley, C. Adams, N. Adhikari, R. Adhikari, V. Adya, C. Affeldt, D. Agarwal, *et al.*, "Population of merging compact binaries inferred using gravitational waves through gwtc-3," *Physical Review X*, vol. 13, no. 1, p. 011048, 2023.

[3] H.-Y. Chen, M. Fishbach, and D. E. Holz, "A two per cent hubble constant measurement from standard sirens within five years," *Nature*, vol. 562, no. 7728, pp. 545–547, 2018.

[4] P. Madau and M. Dickinson, "Cosmic star-formation history," *Annual Review of Astronomy and Astrophysics*, vol. 52, no. 1, pp. 415–486, 2014.

[5] V. Alfradique, C. R. Bom, A. Palmese, G. Teixeira, L. Santana-Silva, A. Drlica-Wagner, A. Riley, C. E. Martínez-Vázquez, D. Sand, G. S. Stringfellow, *et al.*, "A dark siren measurement of the hubble constant using gravitational wave events from the first three ligo/virgo observing runs and delve," *Monthly Notices of the Royal Astronomical Society*, vol. 528, no. 2, pp. 3249–3259, 2024.

[6] V. Alfradique, C. R. Bom, and T. Castro, "Systematic bias in dark siren statistical methods and its impact on hubble constant measurement," *arXiv preprint arXiv:2503.18887*, 2025.

[7] A. Palmese, C. R. Bom, S. Mucesh, and W. G. Hartley, "A standard siren measurement of the hubble constant using gravitational-wave events from the first three ligo/virgo observing runs and the desi legacy survey," *The Astrophysical Journal*, vol. 943, no. 1, p. 56, 2023.

[8] B. F. Schutz, *A First Course in GENERAL RELATIVITY*. 2nd ed., 2009.

naomi@cbpf.br