MUSES — AN OVERVIEW

The nuclear phase diagram puzzle from neutron stars to heavy-ion collisions

IRL-NPA FRIB November 1st, 2024









MOTIVATING SCIENCE GOALS

- Where is the transition line at high density?
- Is there a critical point in the QCD phase diagram?
- What are the degrees of freedom in the vicinity of the phase transition?
- What are the phases of QCD at high density?
- How are heavy nuclei created and what is the site of the r-process?



WHAT HAPPENS AT FINITE DENSITIES?

From 1st principles calculation, lattice QCD only allows to compute thermodynamics at $\mu_B = 0$ (matter/antimatter=1).

- Expansion methods can be used, but still restricted to low density regions
- We need to merge the lattice QCD EoS with other effective theories
- Careful study of their respective range of validity

Could we try to merge them together to ensure full coverage of the phase diagram ?



Lattice QCD: S. Borsanyi, C. R. et al, PRL (2021) Interacting HRG: V. Vovchenko et al., PRL (2017) Liquid-gas, Nuclei: see e.g. Du et al. PRC (2019) Chiral EFT: see e.g. Holt, Kaiser, PRD (2017) Holography: R. Critelli, C. R. et al., PRD (2017)

Baryon Chemical Potential µ_p(MeV)

pQCD: Andersen et al., PRD (2002); Annala et al., Nat. Ph. (2020) Quarks: Ratti et al., PRD (2006), Dexheimer et al., PRC (2009); Baym et al., Astr. J. (2019) Quarkyonic: McLerran et al. (2007), Vovchenko et al. (2023) CSC: Alford et al., PLB (1998); Rapp et al., PRL (1998); S. Rossner, C. R. et al, PRD (2007).

MUSES — MODULAR UNIFIED SOLVER OF THE EQUATION OF STATE

"An open-source cyberinfrastructure fostering a community-driven ecosystem that provides key computational tools to promote, transform and support groundbreaking research in nuclear physics and astrophysics, computational relativistic fluid dynamics, gravitational-wave and computational astrophysics."

- Modular: while at low densities the equation of state is known from 1st principles, at high μ_B we will implement different models ("modules") that the user will be able to pick
- Unified: the different modules will be smoothly merged together to ensure maximal coverage of the phase diagram, while respecting established limiting cases (lattice, perturbative QCD, Chiral EFT...)



MUSES GOALS AND MILESTONES

• CyberInfrastructure of interoperating tools and services within a replicable and flexible deployment system

- Upgrade of existing calculation tools to modern programming languages
- Equation of State (EoS) package that combines all the EoS modules using smooth transition functions
- Observables & toolkit package to compute observables and provide tools to facilitate comparison with experiments
- Web-based tools and services that provide interactive interfaces to the calculation engine
- Job management system and a deployment system that can be reproduced in other computing environments







PARTICIPANTS

PI and co-PIs

- 1. Nicolas Yunes; University of Illinois at Urbana-Champaign; PI
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- 3. Jorge Noronha; University of Illinois at Urbana-Champaign; co-PI
- 4. Claudia Ratti; University of Houston; co-PI and spokesperson
- 5. Veronica Dexheimer; Kent State University; co-PI

6. Senior investigators

- 1. Roland Haas; National Center for Supercomputing Applications
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This is the list that appeared in the proposal, BUT: 1st MUSES collaboration paper was signed by 58 authors \rightarrow We are growing!

- 5. Reed Essick; Perimeter Institute
- 6. Rene Bellwied; University of Houston
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- 8. Michael Strickland; Kent State University
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(2024

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Living Rev.Rel

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(arXiv:2303

FIRST MUSES-WIDE PUBLICATION



Theoretical and Experimental Constraints for the Equation of State of Dense and Hot Matter

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PUBLIC RELEASE: NOV. 2024

DETAILS

1st set of modules will be released very soon (just a few more weeks to wait...)

- They will be publicly available & open-source
- We invite people to test these modules and give us feedback!
- The modules to be released will be marked by



during this talk





NASA Scientific Visualization Studio



EOS FOR NEUTRON STARS

NEUTRON STARS & MERGERS: DETAILS AND NEEDS

DETAILS

- Long lifetime
- Weak interaction must be considered
- Flavor is driven out of equilibrium temporarily (mergers)
- Electrically neutral for stability
 <ρ_Q>=0

NEEDS

- * Standard EoS: (p, s, ϵ , ρ_{B} , c_{s}^{2} , μ_{i} , Y_{i})
- T=0 EoS for mergers and neutron stars
- Finite-T EoS for mergers
- Lepton EoS
- Flavor equilibration tied to microscopic
 EOS models



MULTI-PHASE EQUATION OF STATE FOR NEUTRON STARS

Range: $T \simeq 0 \; \text{MeV}$; $\mu_{\scriptscriptstyle B} \! < 1600 \; \text{MeV} \; / \; n_B \! < \! \sim \! 10 n_s$



Outlooks:

- Add finite-T extension (T < 160 MeV)
- Add magnetic field
- Thermal meson interactions
- Study sensitivity of parameters

Chiral Mean Field (CMF) model

- Crossover at low density and first-order phase transition at high density
 - Based on non-linear sigma model with the addition of deconfined quarks and different parametrizations of vector meson self-interaction term
 - Reproduces nuclear & astrophysics constraints, and matches pQCD in relevant regimes



HADRONIC EQUATIONS OF STATE FOR NEUTRON STARS

Range: T \sim 0 MeV ; $n_B < \sim 2n_s$

\circ Chiral effective field theory (ChiEFT)

- Interacting nucleons and pions within chiral effective field theory
- Fitted to nucleon scattering data and boundstate potentials
- Can compute both symmetric and asymmetric EoS: $0 < Y_p < 0.5$

J. Holt & N. Kaiser, PRC (2017)

Outlooks:

• Add extension at finite-T (up to 30 MeV)

RELEASE

- Include a wider variety of ChiEFT potentials
- Provide uncertainty quantification



HADRONIC EQUATIONS OF STATE FOR NEUTRON STARS

Range: T ~ 0 MeV ; $10^{-12} < n_B < 2 \text{ fm}^3$

- \circ University of Tennessee in Knoxville (UTK) EoS
 - Includes nucleonic degrees of freedom based on a phenomenological fit of free energy density to
 - nuclear experiments
 - astronomical observations
 - + guided by many-body theory calculations
 - Defined for both symmetric and asymmetric matter for 0.01 $< Y_p < 0.5$

X. Du, A. Steiner, J. Holt, PRC (2019) X. Du, A. Steiner, J Holt, PRC 110 (2022)

Outlooks:

- Addition of finite T equations of state
- Extension to strangeness degrees of freedom
- Machine learned emulator







MADAI collaboration



EOS FOR HEAVY IONS

HEAVY ION COLLISIONS: DETAILS AND NEEDS

DETAILS

- System is described in terms of hydrodynamic simulations and/or microscopic transport
- System is not in finite-sized, short lifetime
- Strangeness neutrality <n_S>=0
 + locally conserved
- ♦ Charge: p vs. n in ions → $< n_Q > = 0.4 < n_B >$

NEEDS

- To take into account local fluctuations,
 4D Equations of State are needed
- ***** Free parameters: *T*, μ_B , μ_S , μ_Q
- * Thermodynamic variables (p, s, ϵ , n_{B} , c_{s}^{2})
- 1st and 2nd order derivatives of pressure with respect to chemical potentials
- Inclusion of critical point
- Transport coefficients

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EQUATION OF STATE FROM HOLOGRAPHY

Range: 30 MeV < T < 400 MeV $\mbox{ ; } \mu_B <$ 1100 MeV

- Use gauge/gravity correspondence to obtain QCD thermodynamics from an Einstein-Maxell-Dilaton Holographic model
- Fix the parameters to reproduce everything we know from the lattice
- Calculate equation of state at finite density, but only for finite μ_{B}
- Currently includes a critical point (T^{CEP} = $\sim 100 \text{ MeV}$; $\mu_B^{CEP} = \sim 600 \text{ MeV}$)

Outlooks:

• Extension to multiple conserved charges

<u>J. Grefa, C. Ratti et al., PRD (2021)</u> J. Grefa, C. Ratti et al. subm. To PRD (2023)







4D EQUATION OF STATE FROM L-QCD (BQS-EOS) RELEASE



Range: 30 MeV < T < 600 MeV $\$; $\ \mu_{B/Q/S} < 450$ MeV

Full Taylor expansion needed to study different $\mu_{B/Q/S}$ scenarios ٠

$$\frac{p(T,\mu_B,\mu_Q,\mu_S)}{T^4} = \sum_{i,j,k} \frac{1}{i!j!k!} \chi_{ijk}^{BQS} \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

Parametrized susceptibilities fitted on lattice QCD + HRG

$$\chi_{ijk}^{BQS}(T) = \frac{\sum_{n=0}^{9} a_n^{ijk} / t^n}{\sum_{n=0}^{9} b_n^{ijk} / t^n} + c_0^{ijk} \qquad \text{where } t = T/154 \,\text{MeV}$$

$$\chi_2^B(T)) = e^{-h_1/t' - h_2/t'} \times f_3 \left(1 + \tanh(f_4 t' + f_5)\right) \quad \text{where } t' = T/200 \,\text{MeV}$$

Coefficients are available from lattice-QCD up to global order 4 ($\mu/T < 2.5$)

S. Borsanyi, C. Ratti et al., JHEP (2018) J. Noronha-Hostler, C. Ratti et al., PRC (2019) A. Monnai et al., PRC (2019)



2D ISING-T.EX.S EQUATION OF STATE FROM L-QCD

Range: 30 MeV < T < 800 MeV $\$; $\ \mu_B < 700$ MeV

• Novel T'-Expansion Scheme (TExS) allows to extend lattice-based EoS up to $\mu_B/T\sim3.5$ (EoS available at $\mu_S=\mu_Q=0$)

<u>S. Borsanyi, C. Ratti et al., PRL (2021)</u> <u>S. Borsanyi, C. Ratti et al., PRD (2022)</u>

 Includes a parametrized critical point from 3D-Ising universality class, with free location along a transition line parametrized according to physical constraints

M. Kahangirwe, J.J., C. Ratti et al., PRD (2024)

Outlooks:

- Extend it to the case $<\!n_S\!\!>=\!0$ & $<\!n_Q\!\!>=\!0.4<\!\!n_B\!\!>$, relevant for HICs
- Provide adaptive grid to better resolve CP



4D-T.EX.S EQUATION OF STATE FROM L-QCD

Range: T $< 800 \; \text{MeV}$; $\mu_{B/Q/S} <$ 700 MeV

- Generalization of the previous 2D T'-Expansion Scheme to 3 conserved charges ٠ by projecting the "cartesian" (μ_B , μ_O , μ_S) coordinates to spherical ones
 - $\hat{\mu} = \sqrt{\hat{\mu}_B^2 + \hat{\mu}_Q^2 + \hat{\mu}_S^2} \qquad \qquad \hat{\mu}_B = \hat{\mu} \cdot \cos(\theta) \\ \hat{\mu}_Q = \hat{\mu} \cdot \sin(\theta) \cos(\varphi)$

 \rightarrow still a 2D-TExS expansion, along a constant μ/T line



 $\hat{\mu}_{S} = \hat{\mu} \cdot \sin(\theta) \sin(\phi)$





Vovchenko/Noronha-Hostler/Ratti's groups

HADRON RESONANCE GAS (HRG) MODEL

Range: $0 < T < 160 \; \text{MeV}$; $\mu_B < 930 \; \text{MeV}$

- HRG model provides a well-established and realistic EoS at low temperatures
- Ideal version is based on the assumption that an interacting gas of hadrons in the ground state can be well-approximated by an ideal gas of resonances
- At large density, we need to incorporate additional interactions (van Der Waals)
- Describes the liquid-gas phase transition

Goals:

- Implementing FIST into module for thermodynamics results
- Give flexibiliy on the particle list as input
- Extend hadronic spectrum to the most updated PDG list









OBSERVABLES FOR HEAVY-IONS



- Transport coefficients from Holographic module
 - Thermal conductivity
 - Baryon conductivity & diffusion
 - Shear & bulk viscosities
 - HQ drag force & Langevin diffusion coefficients
 - Jet quenching parameter

J. Grefa, C. Ratti et al., PRD (2022)

- Freeze-out physics
 - * T and $\mu_{\scriptscriptstyle B}$ at chemical freeze-out can be fitted from experimental data with HRG
 - will be incorporated from Thermal-FIST

V. Vovchenko et al., PRC (2016)

ADDITIONAL TOOLKIT FOR HEAVY-IONS

Susceptibilities & hadronic species contributions

- Susceptibilities from lattice QCD will be computable
- using HRG, one can study the breakdown of different hadron families:
 - > we will provide combinations for hadronic contributions to total pressure
 - > we will provide analogous relations for susceptibilities



o PDG21+ list

- Add up all resonances from PDG (from * to *****)
- Create decay channels through radiation for unknown higher energy baryons



Figure 5.11: $P_{1|1|1} = -\frac{1}{4}\chi_{13}^{SQ} + \frac{1}{4}\chi_{22}^{SQ} - \frac{1}{4}\chi_{112}^{BSQ} + \frac{1}{4}\chi_{121}^{BSQ}$

J. Salinas San Martin, R. Hirayama, et al. (2016)

TOOLKIT FOR NEUTRON STARS

\circ Lepton module

Outlooks:

٠

Takes in nuclear EoS

include neutrinos

include photons

 Compute the charged lepton densities necessary to ensure having charge neutrality

RELEASE

• Flavour equilibration

- β-equilibrium (by balancing rates)
- Given an EoS, computes:
 - Urca rates
 - $(n \rightarrow p + e + v_e / p + e \rightarrow n + v_e)$
 - Equilibrium charge fractions
 - Relaxation rates
 - Damping time
 - Susceptibilities
 - Bulk viscosity

M. Alford, A. Haber et al., Universe (2021) M. Alford, A. Haber et al., PRC (2024)

• Adapter modules for NS & mergers simulation tools

- Ensuring compatibility with CompStar Online Supernovae Equations of State ($\underline{CompOSE}$), a standard format, with the aim to provide thousands of 1D/2D/3D EoS tables for NS
- Ensuring compatibility with merger simulations



RELEASE

OBSERVABLES FOR NEUTRON STARS

• QLIMR module

Given an EoS, solves the Tolmann-Oppenheimer-Volkoff (TOV) equations

$$\frac{dP}{dr} = -\frac{G\epsilon(r)m(r)}{c^2r^2} \left[1 + \frac{P(r)}{\epsilon(r)}\right] \left[1 + \frac{4\pi r^3 P(r)}{m(r)c^2}\right] \left[1 - \frac{2Gm(r)}{c^2r}\right]^{-1}$$

and computes:

- Q: quadrupole moment Q of NS
- L: tidal Love number (tidal force deformability)
- I: moment of inertia
- > M: mass of NS (+ Δ m to correct for rotation)
- > R: radius of NS (+ Δ R to correct for HRG too)
- \succ Local function p(R), m(R)...





R. Kumar, V. Dexheimer et al., PRD 109 (2024)



Getting started

Read the Quick Start guide to setup your account and learn how to run workflows.

Ask for help and join the discussion on our community support forum.

What is the Calculation Engine?

The Calculation Engine (CE) is an application that lets you **run scientific calculations** as composable workflows, constructed from a growing library of MUSES modules. The service hosted at https://ce.musesframework.io provides the research community with scalable, high-performance computing resources to run intensive calculations. You can also run the CE yourself, either on a single machine using Docker Compose or on a Kubernetes cluster using our Helm chart!

What are MUSES modules?

Modules are the atomic processing units of a MUSES workflow. There are several types of modules, including those that calculate equations of state (EoS) and those that derive observable guantities from the EoS.

The equation of state modules include:

• Chiral EFT - chiral_eft (v0.9.0)



What are workflows?

MUSES workflows provide a way to orchestrate a custom execution of MUSES modules, allowing you to generate equations of state, process and synthesize data, and calculate observable quantities. Individual workflow executions are called jobs. which you can run concurrently on our performant compute nodes to generates reproducible results to download and analyze. Learn more about MUSES workflows here.



PUBLIC RELEASE



USING THE <u>MUSES CALCULATION ENGINE</u>

What is the Calculation Engine?

The Calculation Engine (CE) is an application that lets you **run scientific calculations as composable workflows**, constructed from a growing library of MUSES modules.

- Modules can be run on a dedicated cluster (access given via login)
- <u>Online documentation</u> on the use of the CE, the different modules, etc.
- Forum as a platform for users-developers exchanges about support, feedback...

😭 muses		
Topics •	categories ▶ tags ▶ Latest Unread (2) Top	Categories
My Posts		
# Groups	Category	Topics
i More		16
 Categories 	Welcome to the Calculation Engine (CE) support	1 unread
Cyberinfrastructure	category. This is where you can	
f Seminar Series	 	164
≅ All categories	The Cyberinfrastructure category is a private	1 unread
∽ Tags	category only visible to the members of the Collaborators group and the Cyberinfrastructure	
meeting_notes	group.	
cyberinfrastructure	🖴 Seminar Series	48
calculation_engine	General	87
tutorials	The General category is a catch-all for any	01
web_services	discussions that should be publicly visible and do not	
∷ All tags	fall into other more specific categories.	
 Messages 	≙ Collaboration	189
🛥 Inbox	This category is accessible only to official members of	
← cyberinfrastructure	LIE MUSES CONADOLATION.	
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WORKFLOWS IN MUSES

• Example of a typical workflow within MUSES, implying EoS generation + observable calculation



• More complex workflows can also be defined



CONCLUSIONS

- MUSES will provide a public-accessible framework with a single friendly-user interface, to compute EoSs and related observables from different approaches
- Public release planned for Nov. 2024: open access soon!
- \circ 3 modules for NS EoS
- \circ 3 modules for HI EoS
- $_{\odot}$ 2 toolkit modules / 1 observable modules
- Most modules have plans for improvement (→ 1.1.0)



Suggestions, feedback and new collaborators are welcome!

Backup slides

GW170817

Demonstrated the ability of mergers to advance nuclear physics



LIGO/Virgo PRL (2017) P.S. Cowperthwaite et al., Astrophys. J. Lett. (2017)

MID - 57982.529

ANATOMY OF A NEUTRON STAR MERGER





E.R. Most, C.A. Raithel, PRD 104 (2021)



NEUTRON STAR MERGER AND THE EOS



- Post-merger signal sensitive to order of the phase transition
- Next generation observatories will be able to detect it!
- Need to combine the nuclear physics input and simulations

E.R. Most, V. Dexheimer et al., PRL (2019)

ANATOMY OF A HEAVY-ION COLLISION





The ALICE experiment: A journey through QCD, CERN-EP-2022-227 (2022)

THERMODYNAMICS RELATIONS

 $dp = sdT + nd\mu$

$$n = \left(\frac{\partial p}{\partial \mu}\right)_T \qquad s = \left(\frac{\partial p}{\partial T}\right)_\mu$$

$$c_s^2 = \left(\frac{\partial p}{\partial \epsilon}\right)_{s/n} = \frac{n^2 \frac{\partial^2 p}{\partial T^2} - 2sn \frac{\partial^2 p}{\partial T \partial \mu} + s^2 \frac{\partial^2 p}{\partial \mu^2}}{(\epsilon + p) \left(\frac{\partial^2 p}{\partial T^2} \frac{\partial^2 p}{\partial \mu^2} - \frac{\partial^2 p}{\partial T \partial \mu} \frac{\partial^2 p}{\partial T \partial \mu}\right)}$$

$$c_{V} = \frac{T}{V} \left(\frac{\partial S}{\partial T}\right)_{V,N} = T \left(\frac{\partial s}{\partial T}\right)_{n} = \frac{T \left(\frac{\partial^{2} p}{\partial T^{2}} \frac{\partial^{2} p}{\partial \mu^{2}} - \frac{\partial^{2} p}{\partial T \partial \mu} \frac{\partial^{2} p}{\partial T \partial \mu}\right)}{\frac{\partial^{2} p}{\partial \mu^{2}}}$$

EQUATION OF STATE FROM HOLOGRAPHY

Range: 30 MeV < T < 400 MeV ; μ_{B} < 1100 MeV

String theory/Classical gravity

in 5D

... one can obtain the following thermodynamic quantities b

- using the UV behavior of the EMD fields
- fixing free parameters Λ , κ_5 and the functional form of $V(\phi)$ and $f(\phi)$ by matching with IQCD results at $\mu_{\rm B} = 0$

$$T = \frac{1}{4\pi\phi_A^{1/\nu}\sqrt{h_0^{far}}}\Lambda \qquad s = \frac{2\pi}{\kappa_5^2\phi_A^{3/\nu}}\Lambda^3$$

$$\Phi_0^{far} \qquad \Phi_0^{far}$$

 $\mu_B = \frac{\tau_0}{\phi_A^{1/\nu} \sqrt{h_0^{far}}} \Lambda \qquad \rho_B = -\frac{\tau_2}{\kappa_5^2 \phi_A^{3/\nu} \sqrt{h_0^{far}}} \Lambda^3$







(simplest action reproducing a realistic 4D QCD EFT)

in 4D



2D ISING-T.EX.S EQUATION OF STATE FROM L-QCD

Range: 30 MeV < T < 800 MeV $\mbox{ ; } \mu_{B/Q/S} < 700$ MeV



one can thus redefine temperature and use an alternative expansion scheme:

$$T'(T, \hat{\mu}_B) = T\left(1 + \kappa_2^{BB}(T)\hat{\mu}_B^2 + \kappa_4^{BB}(T)\hat{\mu}_B^4 + \mathcal{O}(\hat{\mu}_B^6)\right)$$

with alternative expansion coefficients κ , related to susceptibilities:

$$\kappa_{2}^{BB}(T) = \frac{1}{6T} \frac{\chi_{4}^{B}(T)}{\chi_{2}^{B'}(T)} \qquad \kappa_{4}^{BB}(T) = \frac{1}{360\chi_{2}^{B'}(T)^{3}} \left(3\chi_{2}^{B'}(T)^{2}\chi_{6}^{B}(T) - 5\chi_{2}^{B''}(T)\chi_{4}^{B}(T)^{2}\right)$$
M. Kahangirwe, J.J., C. Ratti et al., PRD (2024); grXiv:2402.08636

Empirical observation:

• all 1st order susceptibilities scale when defining a μ_B -dependent temperature $T'(T, \mu_B)$

scales like:

$$\frac{\chi_1^B(T,\hat{\mu}_B)}{\hat{\mu}_B} = \chi_2^B(T',0)$$
Main identity



2D ISING-T.EX.S EQUATION OF STATE FROM L-QCD $(\alpha$ -RELEASE Range: 30 MeV < T < 800 MeV ; $\mu_{B/Q/S}$ < 700 MeV

Implement scaling behavior of 3D-Ising model EoS:

• Define map from 3D-Ising model to QCD

• Estimate contribution to Taylor coefficients from 3D-Ising model critical point

• Reconstruct full baryon density $\frac{n_B^{full}(T,\mu_B)}{(\mu_B/T)T^3} = \chi^B_{2,lattice}(T'_{full},0)$

with

$$T'_{full}(T,\mu_B) = \underbrace{T'_{lattice}(T,\mu_B)}_{\text{lowest orders in } (\mu_B/T)} + \underbrace{T'_{crit}(T,\mu_B) - Taylor[T'_{crit}(T,\mu_B)]}_{\text{higher order in } (\mu_B/T)}$$

M. Kahangirwe, J.J., C. Ratti et al., PRD (2024); arXiv:2402.08636



4D-T.EX.S EQUATION OF STATE FROM L-QCD

Range: T $< 800 \; \text{MeV}$; $\mu_{B/Q/S} <$ 700 MeV

• Generalization of the previous 2D T'-Expansion Scheme to 3 conserved charges by projecting the "cartesian" (μ_B , μ_Q , μ_S) coordinates to spherical ones

$$\hat{\mu} = \sqrt{\hat{\mu}_B^2 + \hat{\mu}_Q^2 + \hat{\mu}_S^2} \qquad \qquad \hat{\mu}_B = \hat{\mu} \cdot \cos(\theta) \\ \hat{\mu}_Q = \hat{\mu} \cdot \sin(\theta) \cos(\varphi) \\ \hat{\mu}_S = \hat{\mu} \cdot \sin(\theta) \sin(\varphi)$$

 \rightarrow still a 2D-TExS expansion, along a constant μ/T line

• Calculate expansion coefficient λ_2 based on so-called "generalized susceptibilities" $X_{2/4}$ (linear combinations of lattice QCD susceptibilities) + their Stefan-Boltzmann limits

$$\lambda_{2}^{\theta,\varphi}(T) = \frac{1}{6T} \frac{1}{X_{2}^{\prime\,\theta,\varphi}(T)} \times \left(X_{4}^{\theta,\varphi}(T) - \frac{\overline{X}_{4}^{\theta,\varphi}(0)}{\overline{X}_{2}^{\theta,\varphi}(0)} X_{2}^{\theta,\varphi}(T) \right) \qquad X_{2}^{\theta,\varphi}(T) = c_{\theta}^{2} \cdot \chi_{2}^{B}(T) + s_{\theta}^{2} c_{\varphi}^{2} \cdot \chi_{2}^{Q}(T) + s_{\theta}^{2} s_{\varphi}^{2} \cdot \chi_{2}^{S}(T) + \dots + s_{\theta}^{2} \cdot \chi_{2}^{S}(T) +$$



4D-T.EX.S EQUATION OF STATE FROM L-QCD

Range: T $< 800 \; \text{MeV}$; $\mu_{B/Q/S} <$ 700 MeV

• Compute the "generalized charge density" X_1 along the projected line using the expanded temperature T' and the T.Ex.S main identity (modified to match with Stefan-Boltzmann limit at $T \rightarrow \infty$)

$$X_1^{\theta,\phi}(T,\hat{\mu}) = \frac{\overline{X}_1^{\theta,\phi}(\hat{\mu})}{\overline{X}_2^{\theta,\phi}(0)} \times X_2^{\theta,\phi}\left(T^{\prime\,\theta,\phi}(T,\hat{\mu}),0\right)$$

with
$$T^{\prime\,\theta,\phi}(T,\hat{\mu})=T\left(1+\lambda_2^{\theta,\phi}(T)\hat{\mu}_B^2\right)$$

• Obtain pressure by integrating X_1 , allowing then to compute all thermodynamics

$$P^{\boldsymbol{\theta},\boldsymbol{\varphi}}(T,\hat{\boldsymbol{\mu}}) = P(T,0) + \int_0^{\hat{\boldsymbol{\mu}}} X_1^{\boldsymbol{\theta},\boldsymbol{\varphi}}(T,\hat{\boldsymbol{\mu}}') d\hat{\boldsymbol{\mu}}'$$





LOW-LEVEL SERVICES

• The client-facing API will handle communication with client applications through a webpage

- Direct communication with the Batch and Provenance for storage
- Provenance will record all useful information: user activity, workflows executed, models evaluated, inputs/outputs, details of computational jobs (all only accessible internally)





TYPICAL MODULE DESIGN



Original status