Nuclear equation of state from nuclear experiments and neutron stars observations

IRL NPA workshop on Dense Matter EoS, FRIB, East Lansing, Michigan

Pietro Klausner

31/10/2024

Collaborators

Gianluca **Colò** (University of Milano)

Xavier **Roca-Maza** (University of Milano & University of Barcelona)

Enrico **Vigezzi** (I.N.F.N.)

Francesca **Gulminelli** (University of Normandie-Caen & L.P.C. Caen)

Anthea **Fantina** (GANIL)

Marco **Antonelli** (L.P.C. Caen)

Structure of the presentation

Nuclear equation of state from nuclear experiments and neutron stars observations

- First Part: constraints on EoS from nuclear experiments[1]

- Bayesian inference
- Skyrme Interaction

- Second Part: constraints on EoS from Neutron Stars observations

- Second Bayesian inference

[1] https://arxiv.org/abs/2410.18598

Structure of the presentation

Nuclear equation of state from nuclear experiments and neutron stars observations

- First Part: constraints on EoS from nuclear experiments[1]

- Bayesian inference
- Skyrme Interaction

- Second Part: constraints on EoS from Neutron Stars observations

- Second Bayesian inference

[1] https://arxiv.org/abs/2410.18598

Parameters of the model

Parameters

ρ_0, E_0, K_0, J, L	Nuclear matter parameters
G_0, G_1	Surface term parameters
W_0	Spin-orbit parameter
$m_0^*/m, m_1^*/m$	Effective masses

0 = isoscalar; 1 = isovector

Parameters of the model

Parameters

ρ_0, E_0, K_0, J, L — Nuclear matter parameters

G_0, G_1 — Surface term parameters

W_0 — Spin-orbit parameter

$m_0^*/m, m_1^*/m$ — Effective masses

0 = isoscalar; 1 = isovector

1-to-1 correspondence with usual Skyrme parameters[1]!

[1] L.-W. Chen et al. Phys. Rev. C 80, 014322 (2009)

Parameters of the model

Parameters

ρ_0, E_0, K_0, J, L	Nuclear matter parameters
G_0, G_1	Surface term parameters
W_0	Spin-orbit parameter
$m_0^*/m, m_1^*/m$	Effective masses

0 = isoscalar; 1 = isovector

1-to-1 correspondence with usual Skyrme parameters[1]!

Prior distribution

Par.	Units	Lower limit	Upper limit
ρ_0	[fm^{-3}]	0.150	0.175
E_0	[MeV]	-16.50	-15.50
K_0	[MeV]	180.00	260.00
J	[MeV]	24.00	40.00
L	[MeV]	-20.00	120.00
G_0	[MeV fm^5]	90.00	170.00
G_1	[MeV fm^5]	-90.00	70.00
W_0	[MeV fm^5]	60.00	190.00
m_0^*/m		0.70	1.10
m_1^*/m		0.60	0.90

[1] L.-W. Chen et al. Phys. Rev. C 80, 014322 (2009)

Observable chosen for the fit

“hfbcs-qrpa[1]” code to compute observables from parameters

[1]G. Colò, X. Roca-Maza, arXiv:2102.06562v1 [nucl-th]

Observable chosen for the fit

Ground-state properties			
	$B.E.$ [MeV]	R_{ch} [fm]	ΔE_{SO} [MeV]
^{208}Pb	1636.4 ± 2.0*	5.50 ± 0.05*	2.02 ± 0.50*
^{48}Ca	416.0 ± 2.0*	3.48 ± 0.05*	1.72 ± 0.50*
^{40}Ca	342.1 ± 2.0*	3.48 ± 0.05*	-
^{56}Ni	484.0 ± 2.0*	-	-
^{68}Ni	590.4 ± 2.0*	-	-
^{100}Sn	825.2 ± 2.0*	-	-
^{132}Sn	1102.8 ± 2.0*	4.71 ± 0.05*	-
^{90}Zr	783.9 ± 2.0*	4.27 ± 0.05*	-

"hfbcs-qrpa[1]" code to compute observables from parameters

$B.E.$: Binding Energy

R_{ch} : Charge radius

ΔE_{SO} : Spin-orbit splitting

[1]G. Colò, X. Roca-Maza, arXiv:2102.06562v1 [nucl-th]

Observable chosen for the fit

Ground-state properties			
	$B.E.$ [MeV]	R_{ch} [fm]	ΔE_{SO} [MeV]
^{208}Pb	1636.4 ± 2.0*	5.50 ± 0.05*	2.02 ± 0.50*
^{48}Ca	416.0 ± 2.0*	3.48 ± 0.05*	1.72 ± 0.50*
^{40}Ca	342.1 ± 2.0*	3.48 ± 0.05*	-
^{56}Ni	484.0 ± 2.0*	-	-
^{68}Ni	590.4 ± 2.0*	-	-
^{100}Sn	825.2 ± 2.0*	-	-
^{132}Sn	1102.8 ± 2.0*	4.71 ± 0.05*	-
^{90}Zr	783.9 ± 2.0*	4.27 ± 0.05*	-

"hfbcs-qrpa[1]" code to compute observables from parameters

$B.E.$: Binding Energy

R_{ch} : Charge radius

ΔE_{SO} : Spin-orbit splitting

* Theoretical error

[1]G. Colò, X. Roca-Maza, arXiv:2102.06562v1 [nucl-th]

Observable chosen for the fit

Ground-state properties			
	$B.E.$ [MeV]	R_{ch} [fm]	ΔE_{SO} [MeV]
^{208}Pb	$1636.4 \pm 2.0^*$	$5.50 \pm 0.05^*$	$2.02 \pm 0.50^*$
^{48}Ca	$416.0 \pm 2.0^*$	$3.48 \pm 0.05^*$	$1.72 \pm 0.50^*$
^{40}Ca	$342.1 \pm 2.0^*$	$3.48 \pm 0.05^*$	-
^{56}Ni	$484.0 \pm 2.0^*$	-	-
^{68}Ni	$590.4 \pm 2.0^*$	-	-
^{100}Sn	$825.2 \pm 2.0^*$	-	-
^{132}Sn	$1102.8 \pm 2.0^*$	$4.71 \pm 0.05^*$	-
^{90}Zr	$783.9 \pm 2.0^*$	$4.27 \pm 0.05^*$	-

Isoscalar resonances		
	E^{IS}_{GMR} [MeV]	E^{IS}_{GQR} [MeV]
^{208}Pb	$13.5 \pm 0.5^*$	$10.9 \pm 0.5^*$
^{90}Zr	$17.7 \pm 0.5^*$	-

"hfbcs-qrpa[1]" code to compute observables from parameters

$B.E.$: Binding Energy

R_{ch} : Charge radius

ΔE_{SO} : Spin-orbit splitting

E^{IS}_{GMR} : IsoScalar Giant monopole resonance excitation energy (constrained)

E^{IS}_{GQR} : IsoScalar Giant quadrupole resonance excitation energy (centroid)

* Theoretical error

[1]G. Colò, X. Roca-Maza, arXiv:2102.06562v1 [nucl-th]

Observable chosen for the fit

Ground-state properties			
	$B.E.$ [MeV]	R_{ch} [fm]	ΔE_{SO} [MeV]
^{208}Pb	1636.4 ± 2.0*	5.50 ± 0.05*	2.02 ± 0.50*
^{48}Ca	416.0 ± 2.0*	3.48 ± 0.05*	1.72 ± 0.50*
^{40}Ca	342.1 ± 2.0*	3.48 ± 0.05*	-
^{56}Ni	484.0 ± 2.0*	-	-
^{68}Ni	590.4 ± 2.0*	-	-
^{100}Sn	825.2 ± 2.0*	-	-
^{132}Sn	1102.8 ± 2.0*	4.71 ± 0.05*	-
^{90}Zr	783.9 ± 2.0*	4.27 ± 0.05*	-

Isoscalar resonances		
	E^{IS}_{GMR} [MeV]	E^{IS}_{GQR} [MeV]
^{208}Pb	13.5 ± 0.5*	10.9 ± 0.5*
^{90}Zr	17.7 ± 0.5*	-

Isovector properties			
	α_D [fm^3]	$m(1)$ [MeV fm^2]	A_{PV} (ppb)
^{208}Pb	19.60 ± 0.60	961 ± 22	550 ± 18
^{48}Ca	2.07 ± 0.22	-	2668 ± 113

"hfbcs-qrpa[1]" code to compute observables from parameters

$B.E.$: Binding Energy

R_{ch} : Charge radius

ΔE_{SO} : Spin-orbit splitting

E^{IS}_{GMR} : IsoScalar Giant monopole resonance excitation energy (constrained)

E^{IS}_{GQR} : IsoScalar Giant quadrupole resonance excitation energy (centroid)

α_D: Nuclear polarizability

$m(1)$: EWSR of IVGDR

A_{PV}: Parity violating asymmetry

* Theoretical error

[1]G. Colò, X. Roca-Maza, arXiv:2102.06562v1 [nucl-th]

The need for emulation

→ Computing all the observables → ~ 2 hours!

The need for emulation

→ Computing all the observables → ~ 2 hours!

→ Bayesian inference (Sampling: Metropolis-Hastings algorithm) → 10^{6-7} model evaluations!

The need for emulation

→ Computing all the observables → ~ 2 hours!

→ Bayesian inference (Sampling: Metropolis-Hastings algorithm) → 10^{6-7} model evaluations!

→ 2 h. x 10’000’000 points…
Just too much time

The need for emulation

→ Computing all the observables → ~ 2 hours!

→ Bayesian inference (Sampling: Metropolis-Hastings algorithm) → 10^{6-7} model evaluations!

→ 2 h. x 10'000'000 points…
Just too much time

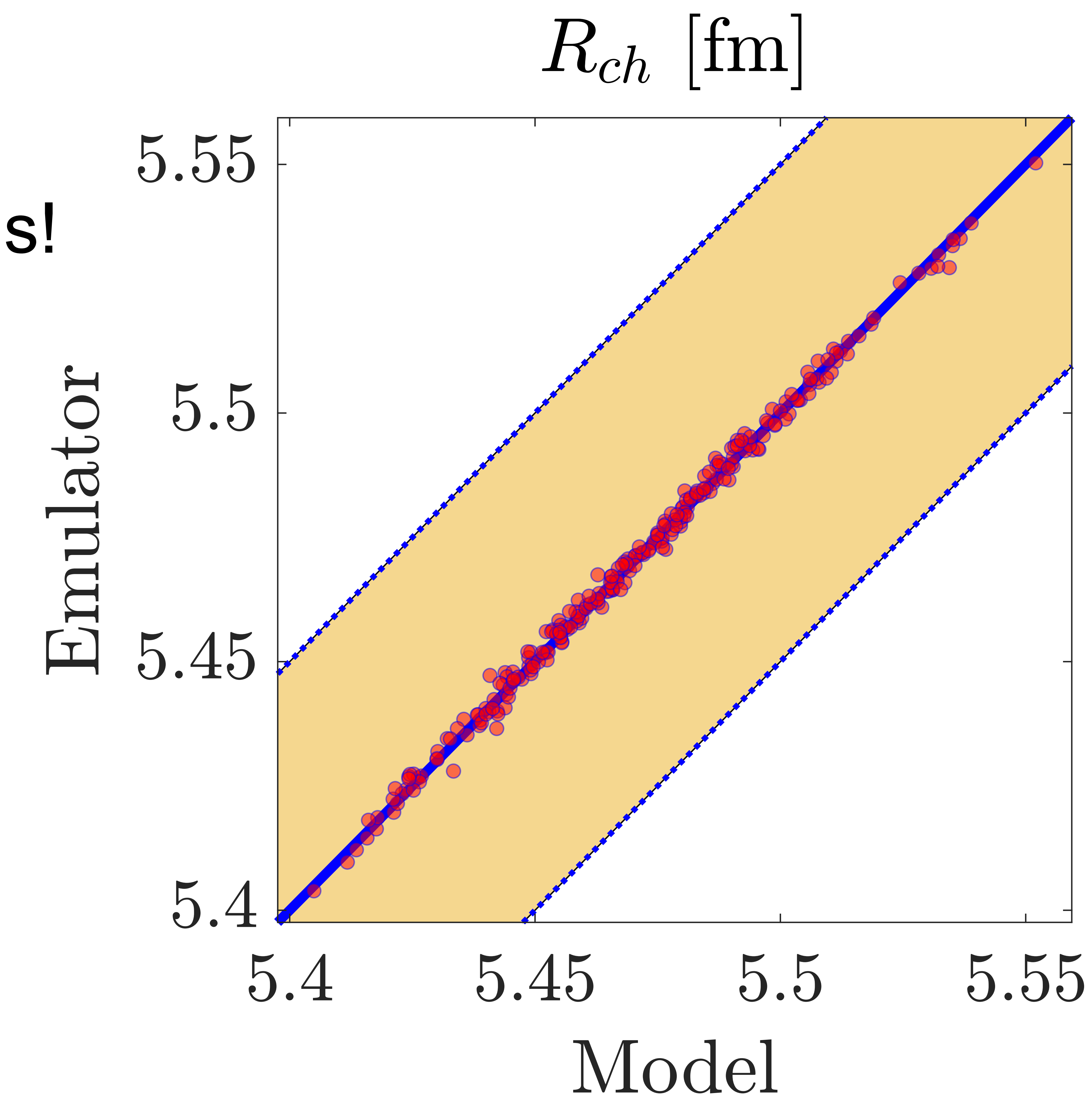
MADAI package[1]
(Emulator for Bayesian inference)

[1]https://madai.phy.duke.edu/

The need for emulation

→ Computing all the observables → ~ 2 hours!

→ Bayesian inference (Sampling: Metropolis-Hastings algorithm) → 10^{6-7} model evaluations!



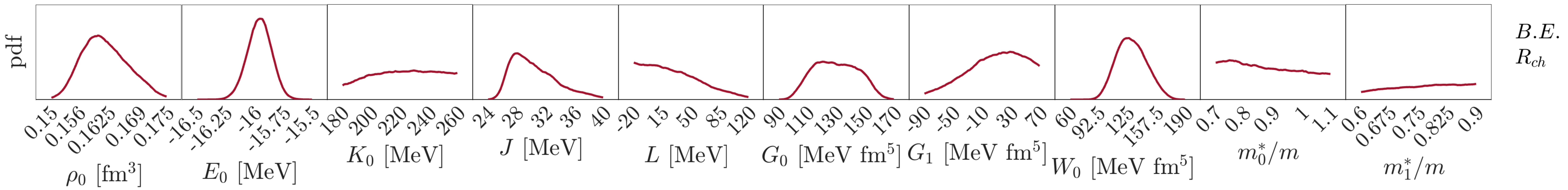
→ 2 h. x 10’000’000 points…
Just too much time

MADAI package[1]
(Emulator for Bayesian inference)

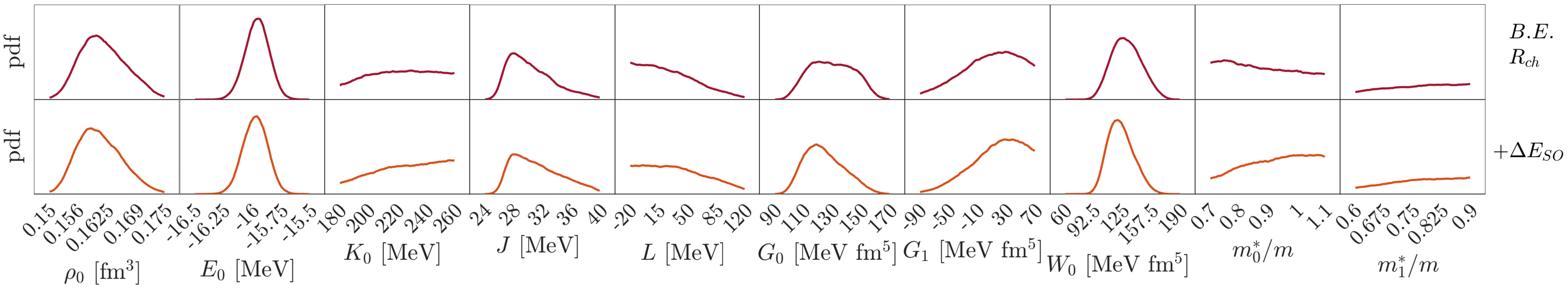
[1]https://madai.phy.duke.edu/

Progressive marginalized posteriors

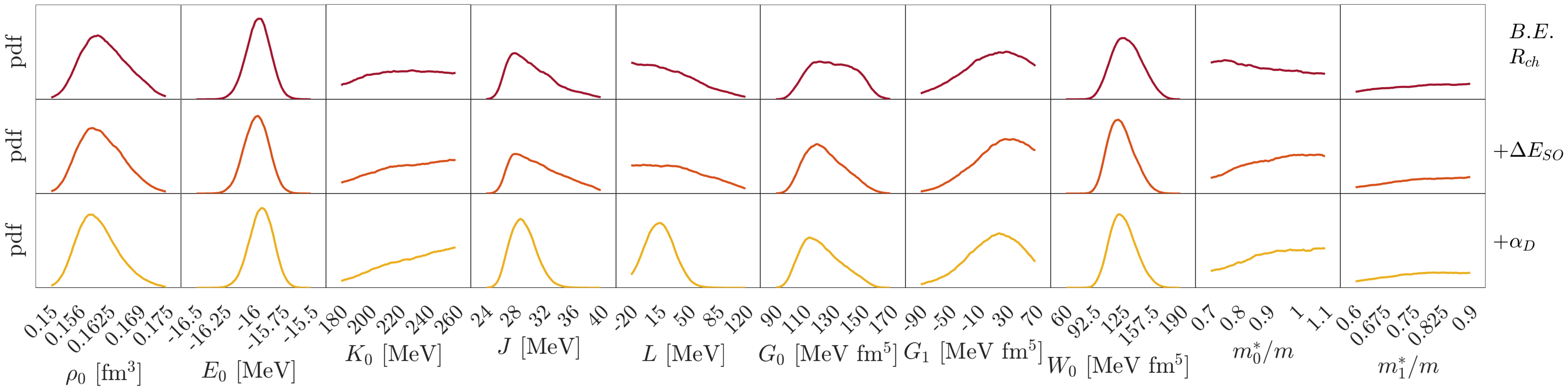
Progressive marginalized posteriors



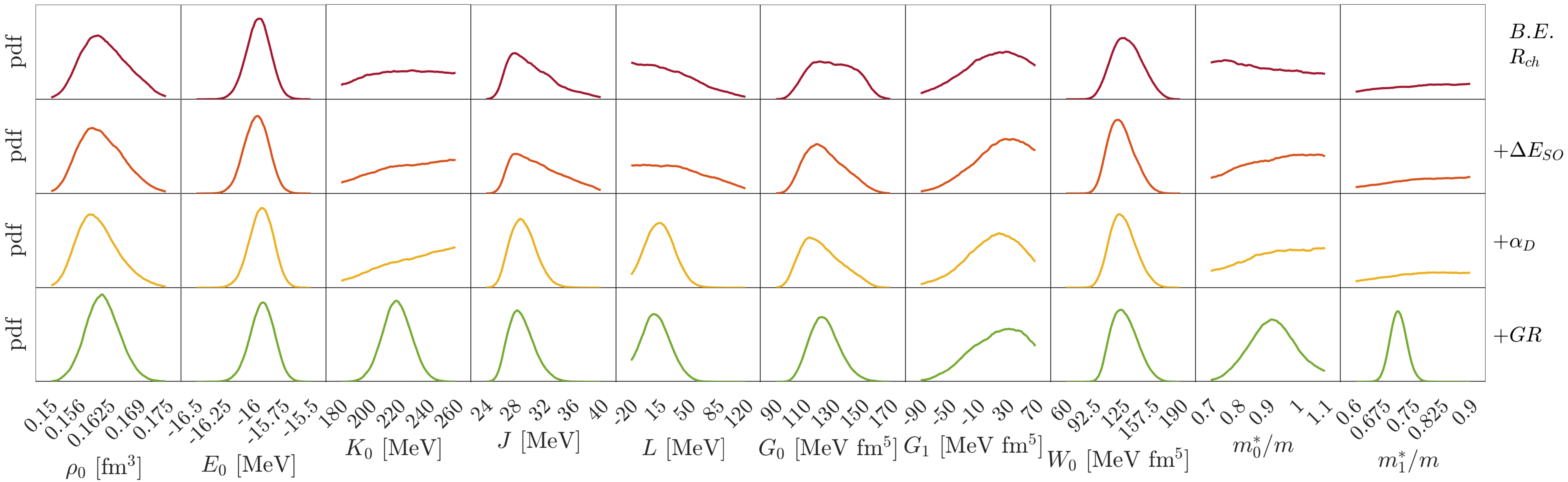
Progressive marginalized posteriors



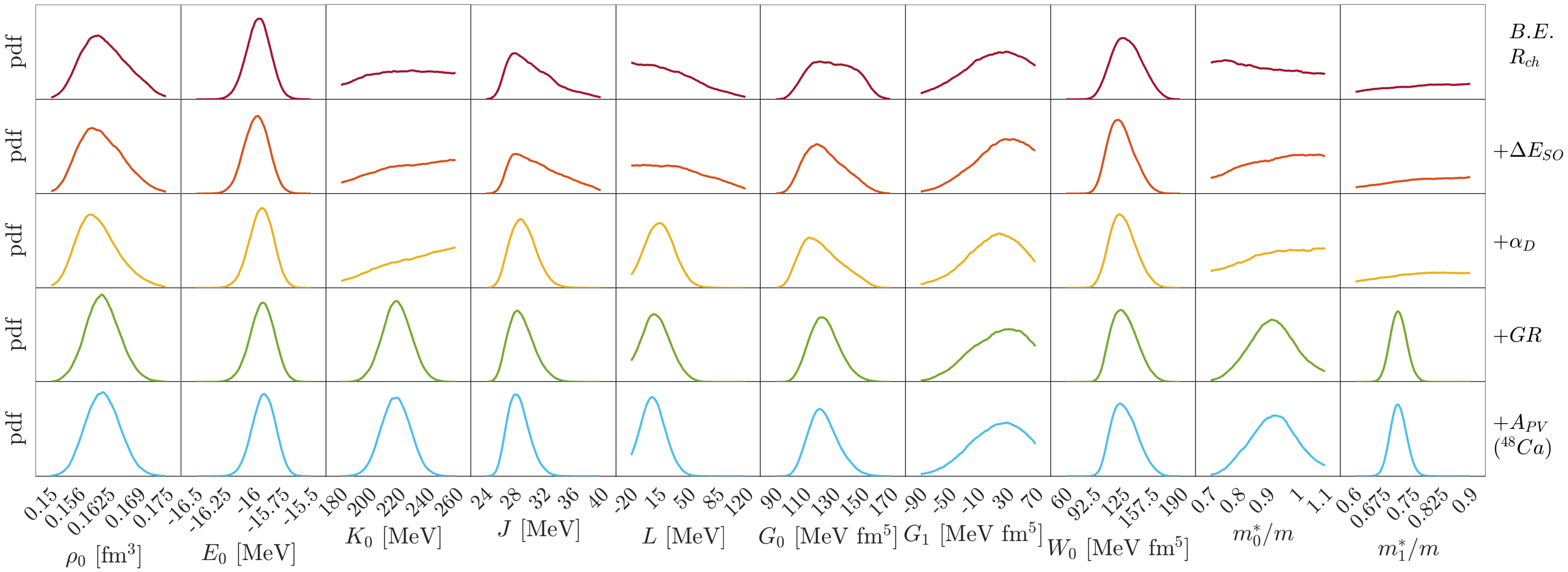
Progressive marginalized posteriors



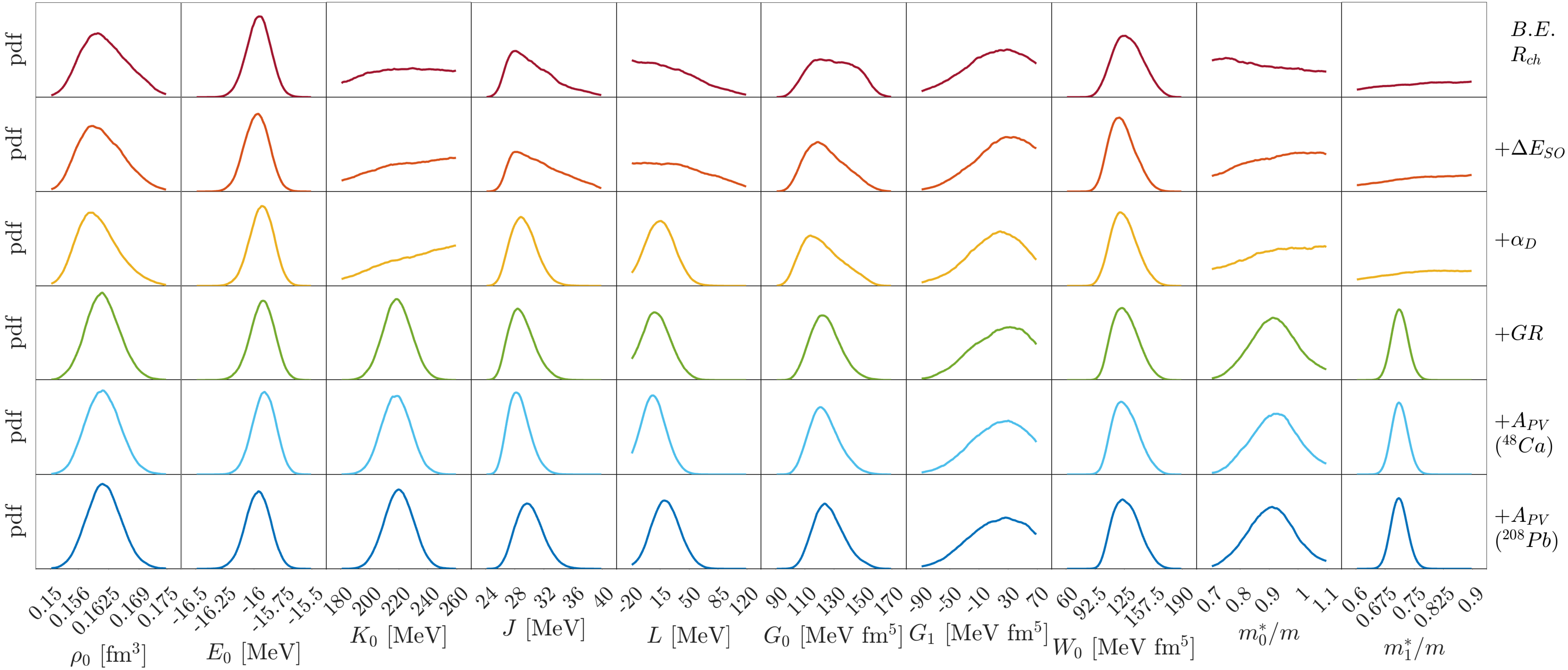
Progressive marginalized posteriors



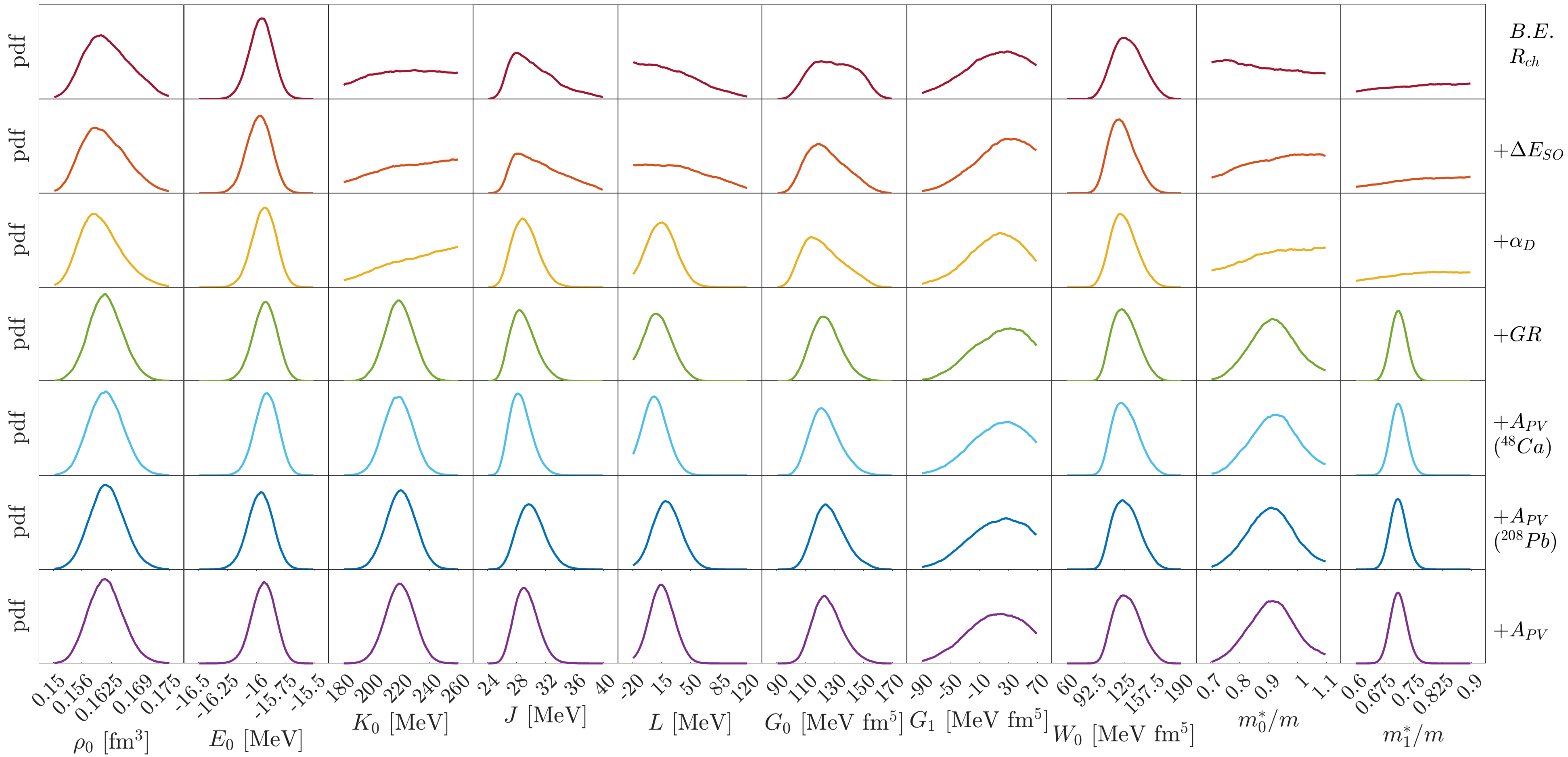
Progressive marginalized posteriors



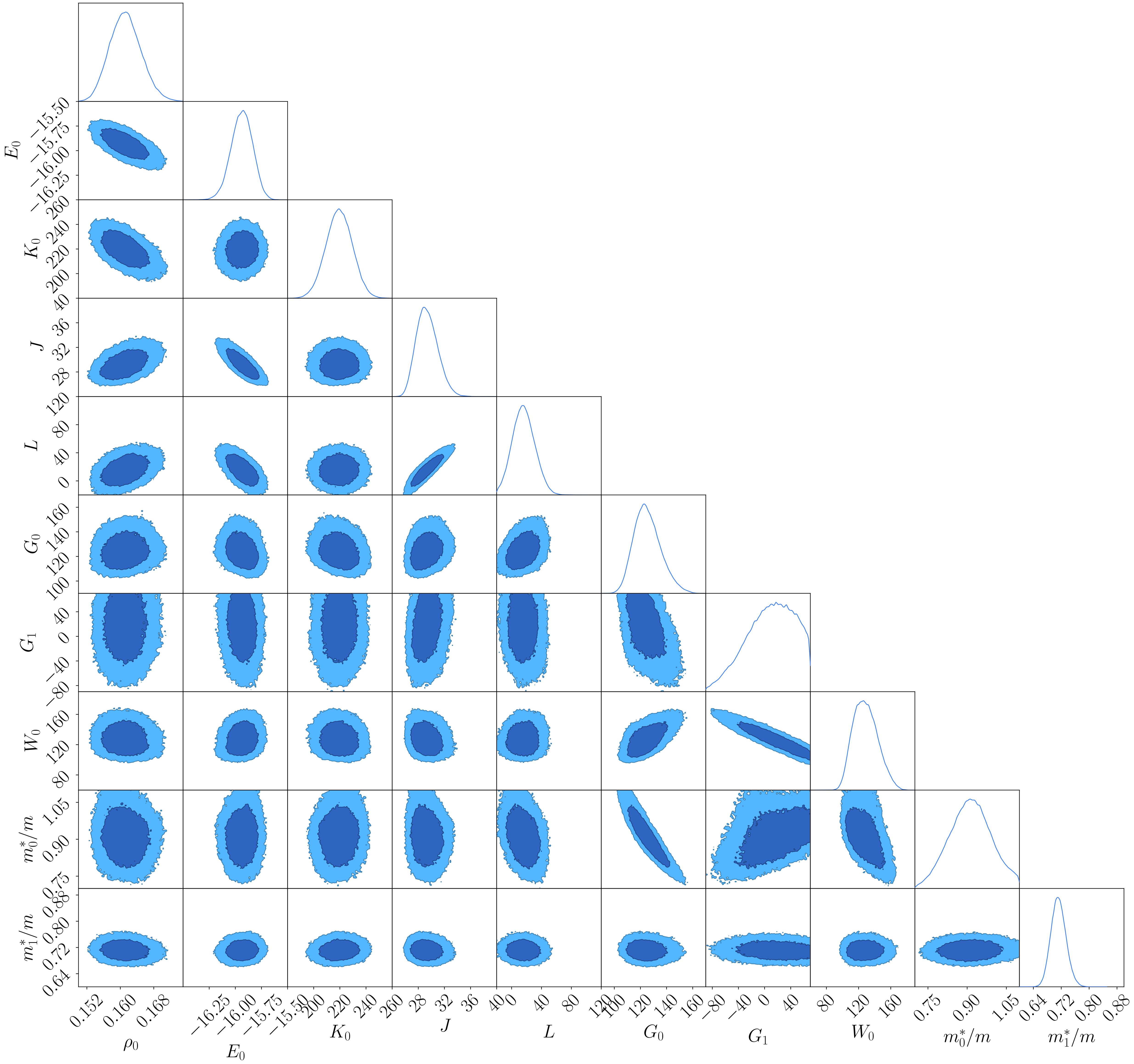
Progressive marginalized posteriors



Progressive marginalized posteriors



Corner plot and mean values



Parameter		μ	σ
ρ_0	[fm^3]	0.161	0.004
E_0	[MeV]	-15.938	0.102
K_0	[MeV]	219.483	10.007
J	[MeV]	29.378	1.626
L	[MeV]	16.136	14.732
G_0	[MeV fm^5]	125.470	10.210
G_1	[MeV fm^5]	9.439	35.735
W_0	[MeV fm^5]	128.719	14.848
m_0^*/m		0.913	0.079
m_1^*/m		0.712	0.021

Posterior observables means and uncertainties

$|\mu_{exp} - \mu_{theo}|$ in units of $\sigma_c = \sqrt{\sigma_{exp}^2 + \sigma_{theo}^2}$

(orange): $[1,2)\ \sigma_c$

(red): $[2,\infty)\ \sigma_c$

Inference

	Ground-state properties		
	$B.E.$ [MeV]	R_{ch} [fm]	ΔE_{SO} [MeV]
^{208}Pb	1636 ± 1.8	5.49 ± 0.03	2.34 ± 0.16 (orange)
^{48}Ca	417 ± 1.2	3.51 ± 0.02	1.92 ± 0.20
^{40}Ca	342 ± 1.6	3.50 ± 0.02	-
^{56}Ni	482 ± 1.4 (orange)	-	-
^{68}Ni	590 ± 1.0	-	-
^{100}Sn	826 ± 1.6	-	-
^{132}Sn	1103 ± 1.7	4.71 ± 0.03	-
^{90}Zr	784 ± 1.3	4.27 ± 0.02	-

Experiment

	Ground-state properties		
	$B.E.$[1] [MeV]	R_{ch}[2] [fm]	ΔE_{SO}[3] [MeV]
^{208}Pb	$1636.4 \pm 1 \times 10^{-3}$	5.50 ± 0.001	1.96 ± 0.05
^{48}Ca	$416.0 \pm 2 \times 10^{-5}$	3.48 ± 0.002	1.72 ± 0.05
^{40}Ca	$342.1 \pm 4 \times 10^{-5}$	3.48 ± 0.002	-
^{56}Ni	$484.0 \pm 1 \times 10^{-3}$	-	-
^{68}Ni	$590.4 \pm 4 \times 10^{-4}$	-	-
^{100}Sn	825.2 ± 0.25	-	-
^{132}Sn	$1102.8 \pm 1 \times 10^{-3}$	4.71 ± 0.002	-
^{90}Zr	$783.9 \pm 1 \times 10^{-4}$	4.27 ± 0.001	-

[1]Meng Wang *et al* 2021 *Chinese Phys. C* **45** 030003

[2]Angeli et al., Atomic Data and Nuclear Data Tables 99 (2013)

[3]Zalewski et al., Phys. Rev. C **77**, 024316 (2008)

Posterior observables means and uncertainties

$|\mu_{exp} - \mu_{theo}|$ in units of $\sigma_c = \sqrt{\sigma_{exp}^2 + \sigma_{theo}^2}$

: $[1,2)\ \sigma_c$

: $[2,\infty)\ \sigma_c$

Inference

Isoscalar resonances		
	$E^{\mathrm{IS}}_{\mathrm{GMR}}$ [MeV]	$E^{\mathrm{IS}}_{\mathrm{GQR}}$ [MeV]
^{208}Pb	13.5 ± 0.3	10.8 ± 0.4
^{90}Zr	17.8 ± 0.4	-

Experiment

Isoscalar resonances		
	$E^{\mathrm{IS}}_{\mathrm{GMR}}$ [1,2] [MeV]	$E^{\mathrm{IS}}_{\mathrm{GQR}}$ [3] [MeV]
^{208}Pb	13.5 ± 0.1	10.9 ± 0.3
^{90}Zr	17.7 ± 0.07	-

[1]D. Patel et al., Physics Letters B 726 (2013)

[2]Y.K. Gupta et al., Physics Letters B 760 (2016)

[3]Youngblood et al., Phys. Rev. C 69, 034315 (2004)

Posterior observables means and uncertainties

$|\mu_{exp} - \mu_{theo}|$ in units of $\sigma_c = \sqrt{\sigma_{exp}^2 + \sigma_{theo}^2}$

(orange) : $[1,2)\ \sigma_c$

(red) : $[2,\infty)\ \sigma_c$

Inference

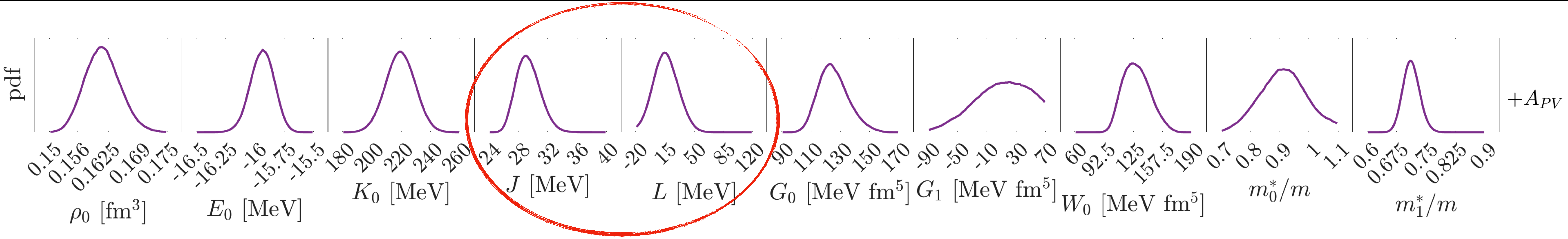
Isovector properties			
	α_D [fm^3]	$m(1)$ [MeV fm^2]	A_{PV} [p.p.b.]
^{208}Pb	19.5 ± 0.5	958 ± 22	589 ± 5
^{48}Ca	2.30 ± 0.08	-	2591 ± 54

Experiment

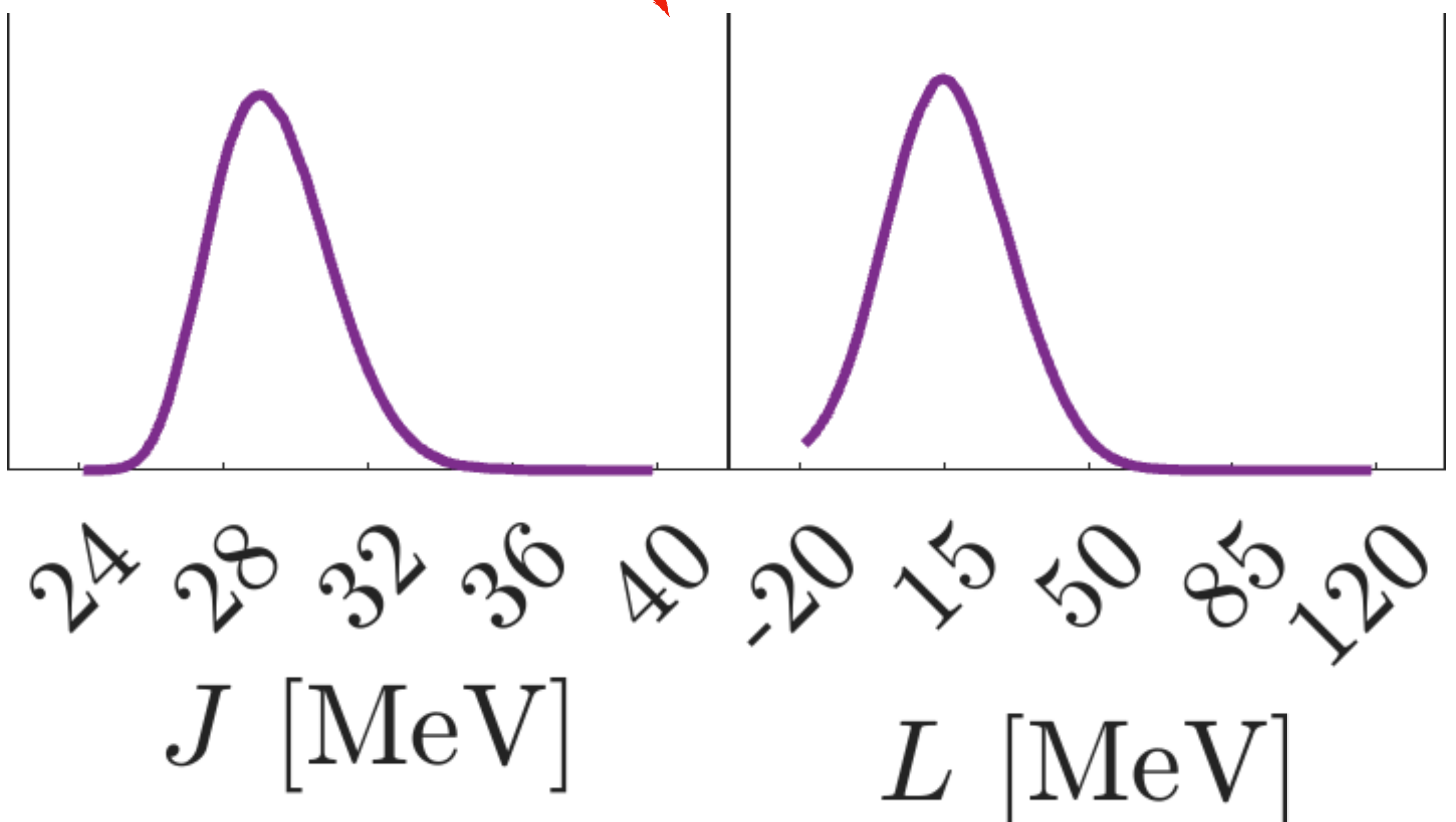
Isovector properties			
	α_{D} [fm^3]	$m(1)$ [MeV fm^2]	A_{PV} (ppb)
^{208}Pb	19.60 ± 0.60	961 ± 22	550 ± 18
^{48}Ca	2.07 ± 0.22	-	2668 ± 113

[1]Tamii et al., PRL 107, 062502 (2011)
[2]Birkhan et al., PRL 118, 252501 (2017)
[3]S. GORIELY et al., Phys. Rev. C 102, 064309 (2020)
[4]PREX Collaboration, Phys. Rev. Lett. 126, 172502 (2021)
[5]CREX Collaboration, Phys. Rev. Lett. 129, 042501 (2022)

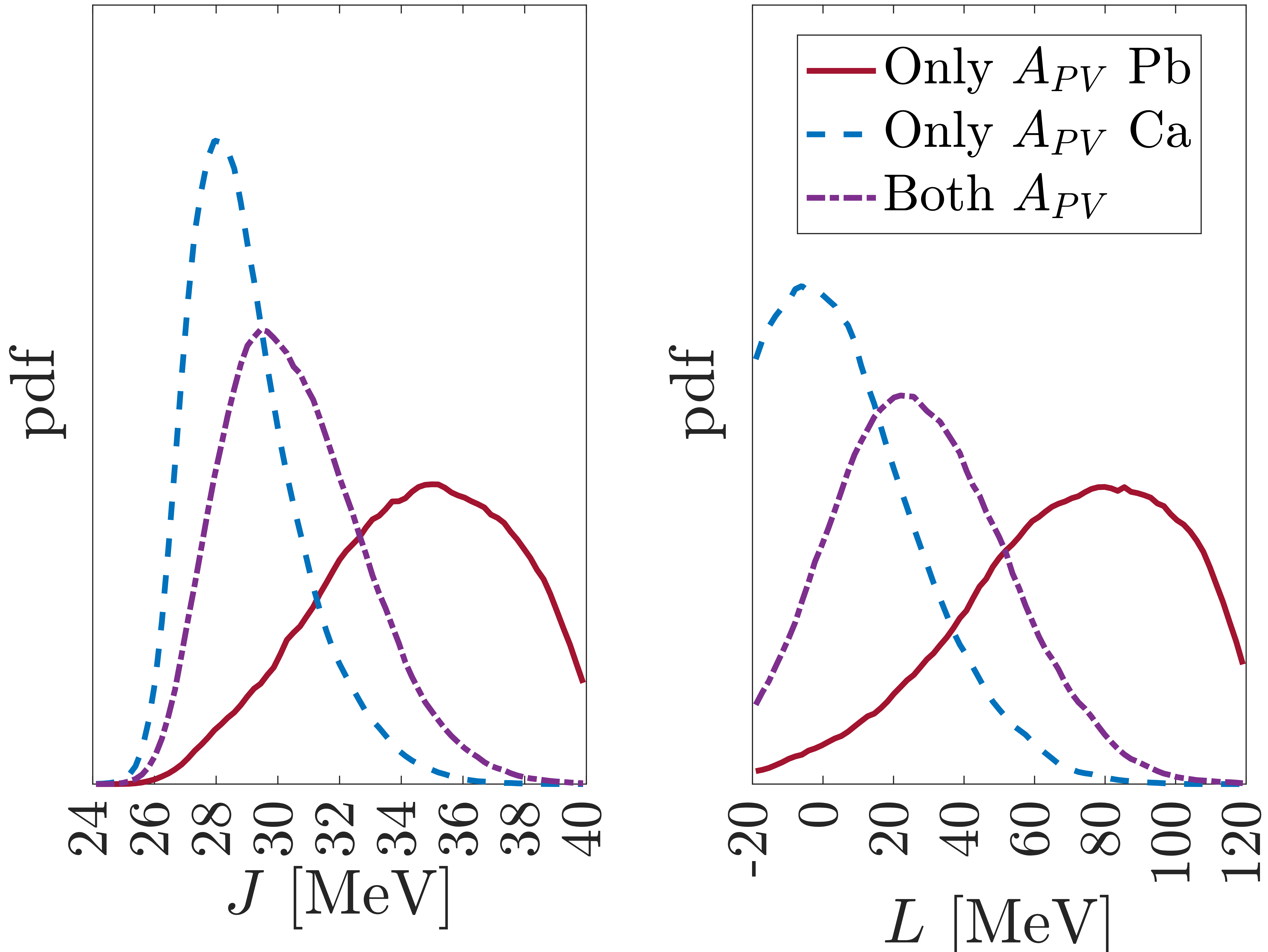
Why is L so small?



the eLephant in the room



Effect of A_{PV} without α_D



Sensitivity analysis: J fixed

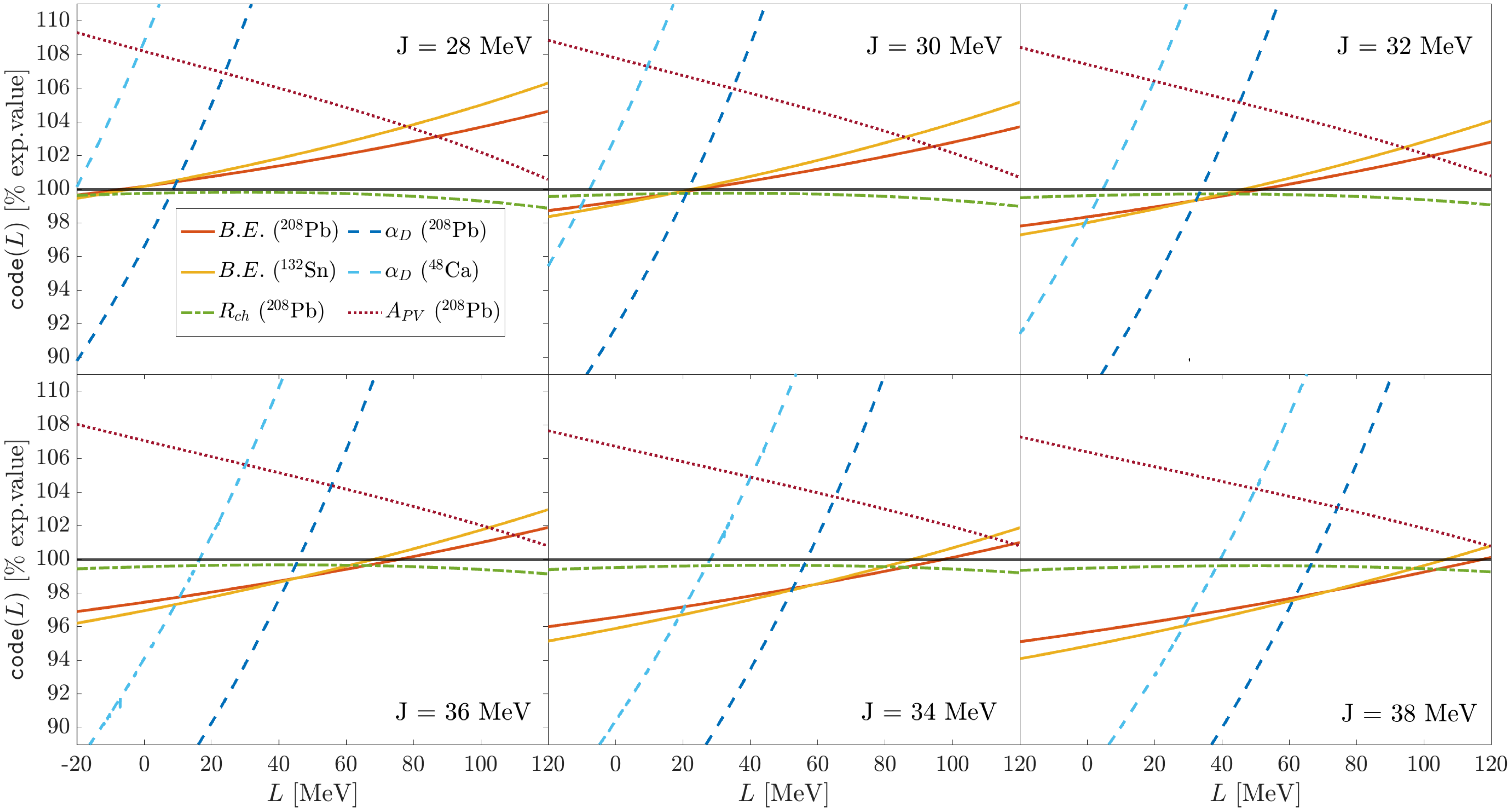
Training grids

- L only free parameter
- J fixed to (28,…,38) MeV
- Other parameters fixed at best log(Likelihood) values

Sensitivity analysis: J fixed

Training grids

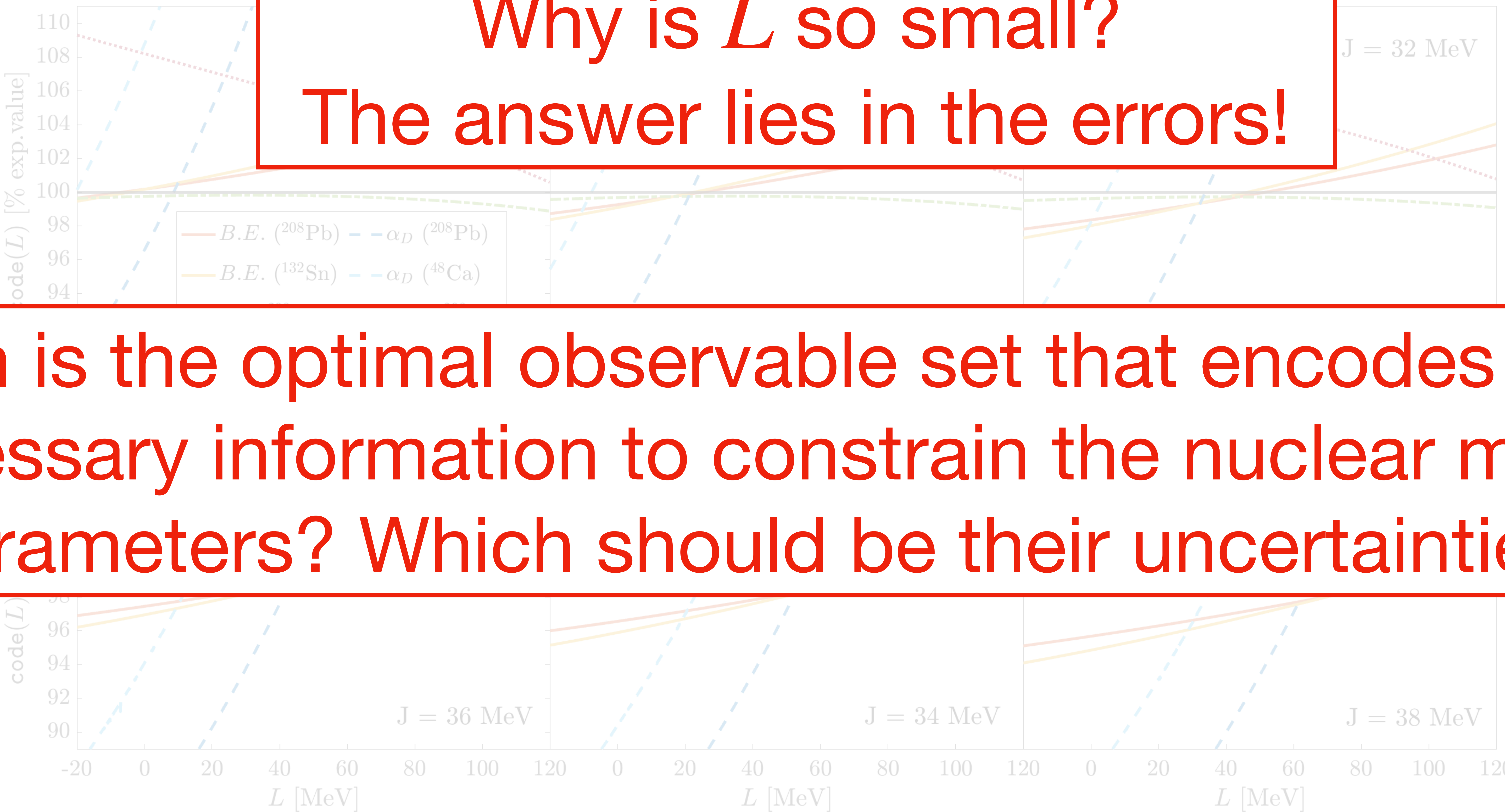
- L only free parameter
- J fixed to (28,…,38) MeV
- Other parameters fixed at best log(Likelihood) values



Sensitivity analysis: J fixed

Training grids

- L only free parameter
- J fixed to (28,…,38) MeV
-



Why is L so small?

The answer lies in the errors!

Which is the optimal observable set that encodes all the necessary information to constrain the nuclear matter parameters? Which should be their uncertainties?

Structure of the presentation

Nuclear equation of state from nuclear experiments and neutron stars observations

- First Part: constraints on EoS from nuclear experiments[1]

- Bayesian inference
- Skyrme Interaction

- Second Part: constraints on EoS from Neutron Stars observations

- Second Bayesian inference

[1]https://arxiv.org/abs/2410.18598

Meta-Model nuclear equation of state

Meta-Model (M.M.): Taylor expansion of the nuclear equation of state around saturation[1]

[1]Margueron et al., Phys. Rev. C **97**, 025805 (2018)

Meta-Model nuclear equation of state

Meta-Model (M.M.): Taylor expansion of the nuclear equation of state around saturation[1]

M.M.

[1]Margueron et al., Phys. Rev. C **97**, 025805 (2018)

Meta-Model nuclear equation of state

Meta-Model (M.M.): Taylor expansion of the nuclear equation of state around saturation[1]

M.M.

↓

Neutron star EoS[2]

[1]Margueron et al., Phys. Rev. C **97**, 025805 (2018)

[2]Developed by T. Carreau and later by H. Dinh-Tin

Meta-Model nuclear equation of state

Meta-Model (M.M.): Taylor expansion of the nuclear equation of state around saturation[1]

M.M.

↓

Neutron star EoS[2]

Crust computed consistently in extended Thomas Fermi

[1]Margueron et al., Phys. Rev. C **97**, 025805 (2018)

[2]Developed by T. Carreau and later by H. Dinh-Tin

Meta-Model nuclear equation of state

Meta-Model (M.M.): Taylor expansion of the nuclear equation of state around saturation[1]

M.M.

↓

Neutron star EoS[2]

↓

Neutron star observables!

Crust computed consistently in extended Thomas Fermi

[1]Margueron et al., Phys. Rev. C **97**, 025805 (2018)

[2]Developed by T. Carreau and later by H. Dinh-Tin

Second Bayesian inference: Parameters & Constraints

Parameters and prior distribution:

$\rho_0, E_0, K_0, J, L, m_0^*/m, m_1^*/m$

K_{sym}

$Q_0, Z_0, Q_{sym}, Z_{sym}$

Second Bayesian inference: Parameters & Constraints

Parameters and prior distribution:

$\rho_0, E_0, K_0, J, L, m_0^*/m, m_1^*/m$ Previous Posterior distribution

K_{sym}

$Q_0, Z_0, Q_{sym}, Z_{sym}$

Second Bayesian inference: Parameters & Constraints

Parameters and prior distribution:

$\rho_0, E_0, K_0, J, L, m_0^*/m, m_1^*/m$

K_{sym}

$Q_0, Z_0, Q_{sym}, Z_{sym}$

Previous Posterior distribution

$K_{sym} = K_{sym}(\rho_0, E_0, K_0, \ldots)$

Second Bayesian inference: Parameters & Constraints

Parameters and prior distribution:

$\rho_0, E_0, K_0, J, L, m_0^*/m, m_1^*/m$ Previous Posterior distribution

K_{sym} $K_{sym} = K_{sym}(\rho_0, E_0, K_0, \ldots) \longrightarrow$ Not a free parameter!

$Q_0, Z_0, Q_{sym}, Z_{sym}$

Second Bayesian inference: Parameters & Constraints

Parameters and prior distribution:

$\rho_0, E_0, K_0, J, L, m_0^*/m, m_1^*/m$ Previous Posterior distribution

K_{sym} $K_{sym} = K_{sym}(\rho_0, E_0, K_0, \ldots) \longrightarrow$ Not a free parameter!

$Q_0, Z_0, Q_{sym}, Z_{sym}$ Uniform distribution

Second Bayesian inference: Parameters & Constraints

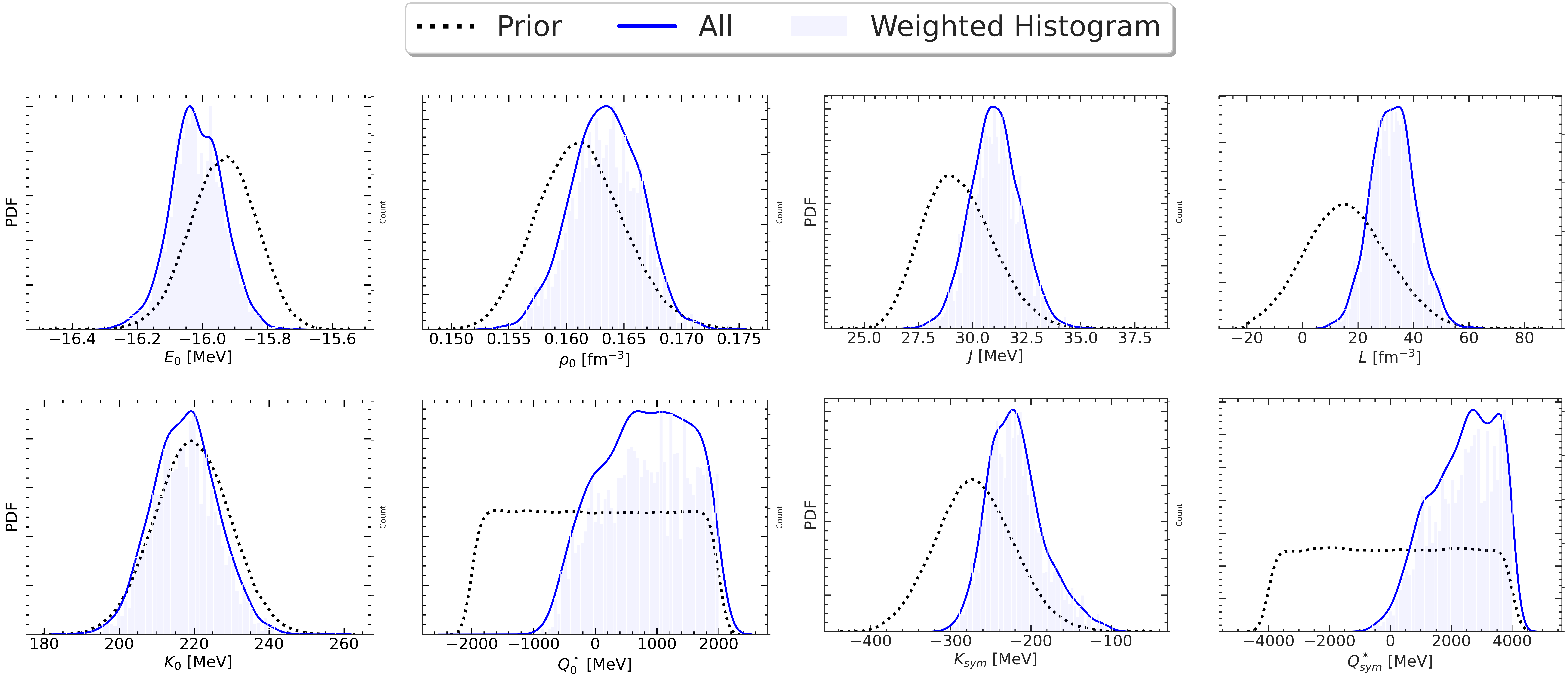
Parameters and prior distribution:

$\rho_0, E_0, K_0, J, L, m_0^*/m, m_1^*/m$	Previous Posterior distribution
K_{sym}	$K_{sym} = K_{sym}(\rho_0, E_0, K_0, \ldots) \longrightarrow$ Not a free parameter!
$Q_0, Z_0, Q_{sym}, Z_{sym}$	Uniform distribution

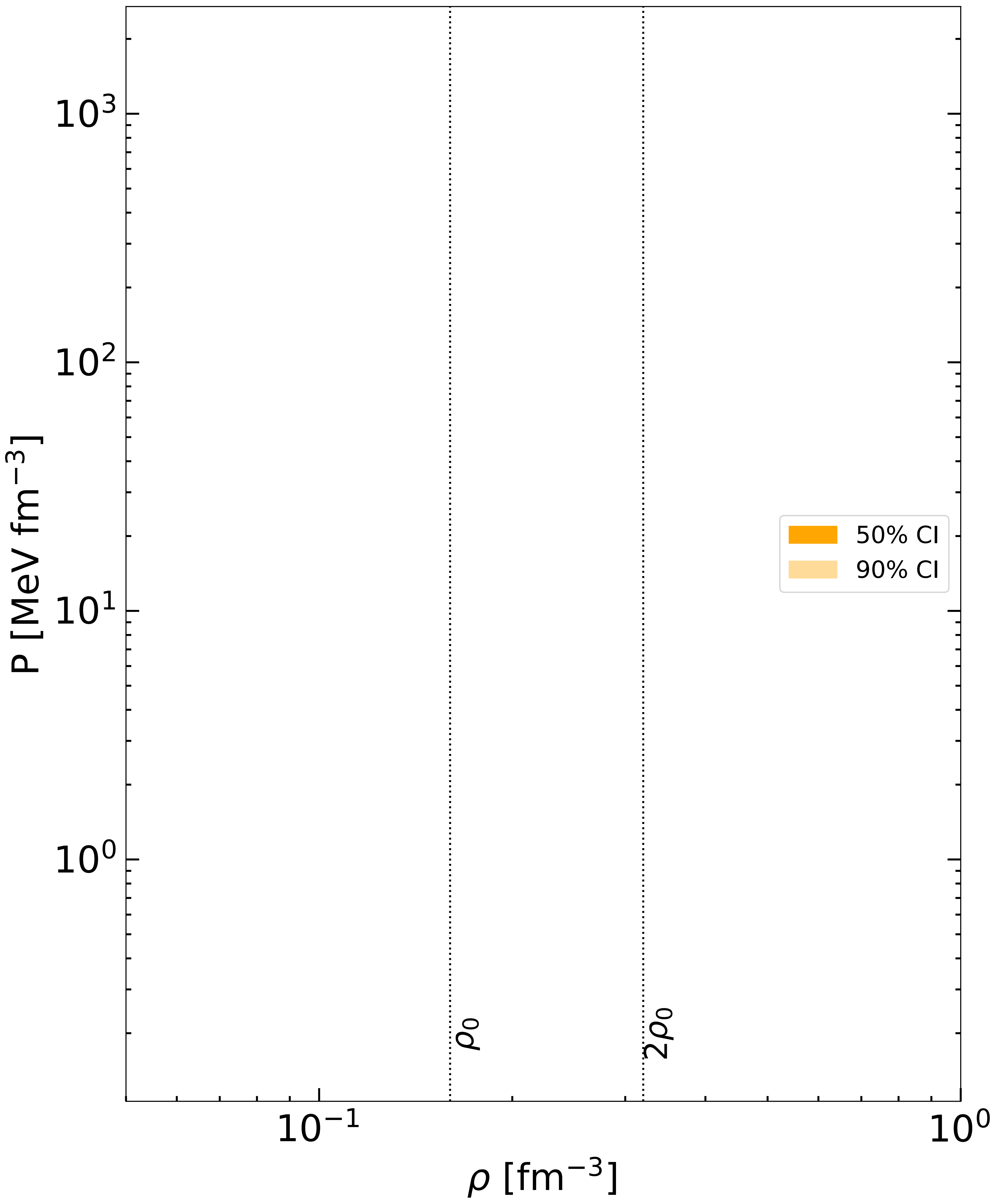
Observational constraints:

- Maximum observed mass of Neutron Star;
- Ligo-Virgo-Collaboration tidal deformability results;
- NICER mission simultaneous mass-radius measurements
- Ab-initio computations of neutron matter at low density

Marginalized posteriors



Equation of State



Conclusions

- Bayesian statistical analysis on nuclear matter parameters with nuclear experiments :

Conclusions

- Bayesian statistical analysis on nuclear matter parameters with nuclear experiments :
  - Skyrme ansatz

Conclusions

- Bayesian statistical analysis on nuclear matter parameters with nuclear experiments :
  - Skyrme ansatz
  - Fit with experimental observables of different types (ground state, giant resonances,…)

Conclusions

- Bayesian statistical analysis on nuclear matter parameters with nuclear experiments :
  - Skyrme ansatz
  - Fit with experimental observables of different types (ground state, giant resonances,…)
  - Result: a robust posterior distribution of the (nuclear matter) parameters

Conclusions

- Bayesian statistical analysis on nuclear matter parameters with nuclear experiments :
  - Skyrme ansatz
  - Fit with experimental observables of different types (ground state, giant resonances,…)
  - Result: a robust posterior distribution of the (nuclear matter) parameters
  - Indications for crucial dependancies on the (often theoretical) uncertainties of the observables

Conclusions

- Bayesian statistical analysis on nuclear matter parameters with nuclear experiments :
  - Skyrme ansatz
  - Fit with experimental observables of different types (ground state, giant resonances,…)
  - Result: a robust posterior distribution of the (nuclear matter) parameters
  - Indications for crucial dependancies on the (often theoretical) uncertainties of the observables
  - Our protocol could describe the observables we chose; the only tension is with A_{PV} of ^{208}Pb

Conclusions

- Bayesian statistical analysis on nuclear matter parameters with nuclear experiments :
  - Skyrme ansatz
  - Fit with experimental observables of different types (ground state, giant resonances,…)
  - Result: a robust posterior distribution of the (nuclear matter) parameters
  - Indications for crucial dependancies on the (often theoretical) uncertainties of the observables
  - Our protocol could describe the observables we chose; the only tension is with A_{PV} of ^{208}Pb

- Bayesian statistical analysis on nuclear matter parameters with neutron star observations:

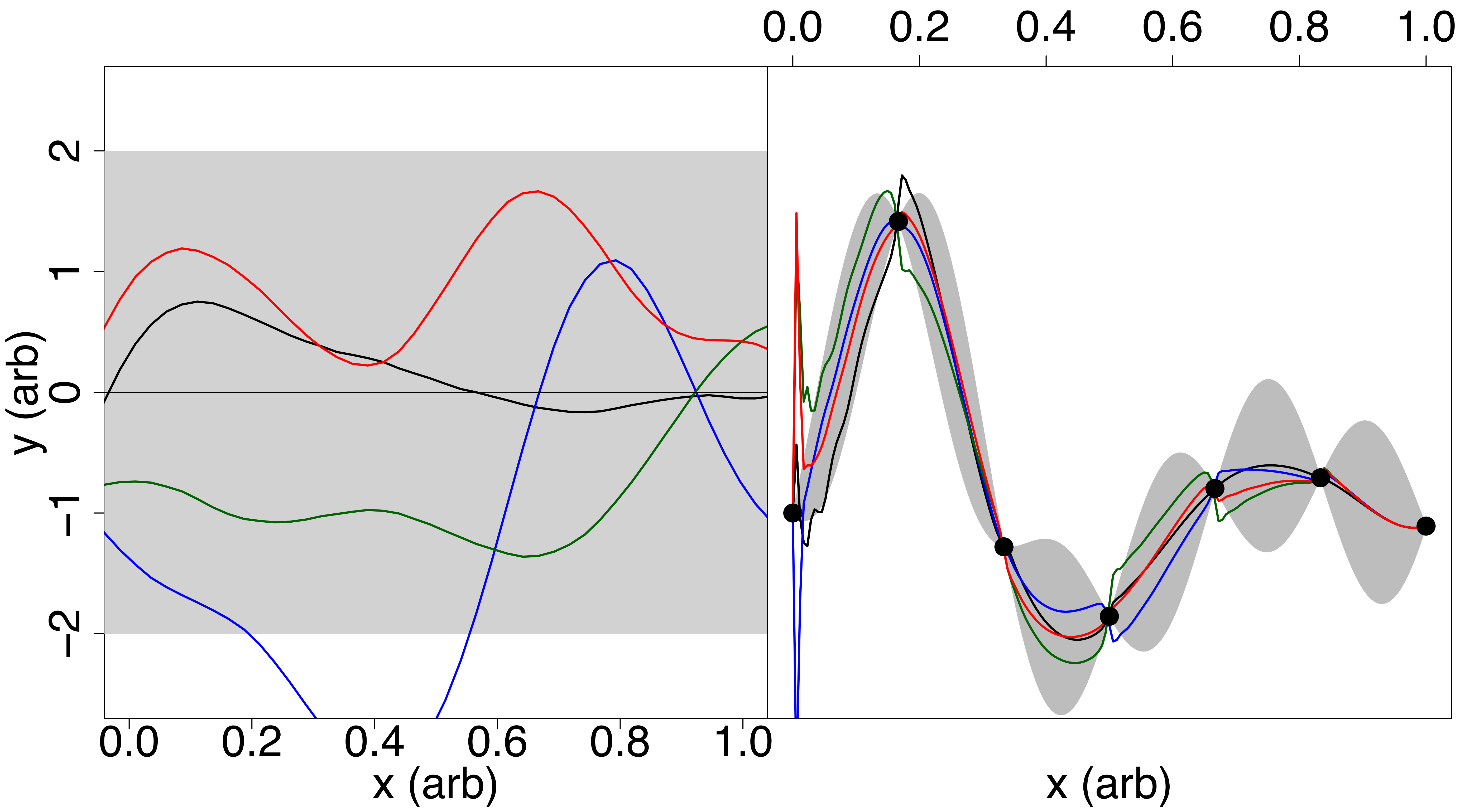
Conclusions

- Bayesian statistical analysis on nuclear matter parameters with nuclear experiments :
  - Skyrme ansatz
  - Fit with experimental observables of different types (ground state, giant resonances,…)
  - Result: a robust posterior distribution of the (nuclear matter) parameters
  - Indications for crucial dependancies on the (often theoretical) uncertainties of the observables
  - Our protocol could describe the observables we chose; the only tension is with A_{PV} of ^{208}Pb

- Bayesian statistical analysis on nuclear matter parameters with neutron star observations:
  - Final distribution of parameters informed by both nuclear physics and neutron star observations!

Thank you for your attention!

Gaussian process (GP) emulator

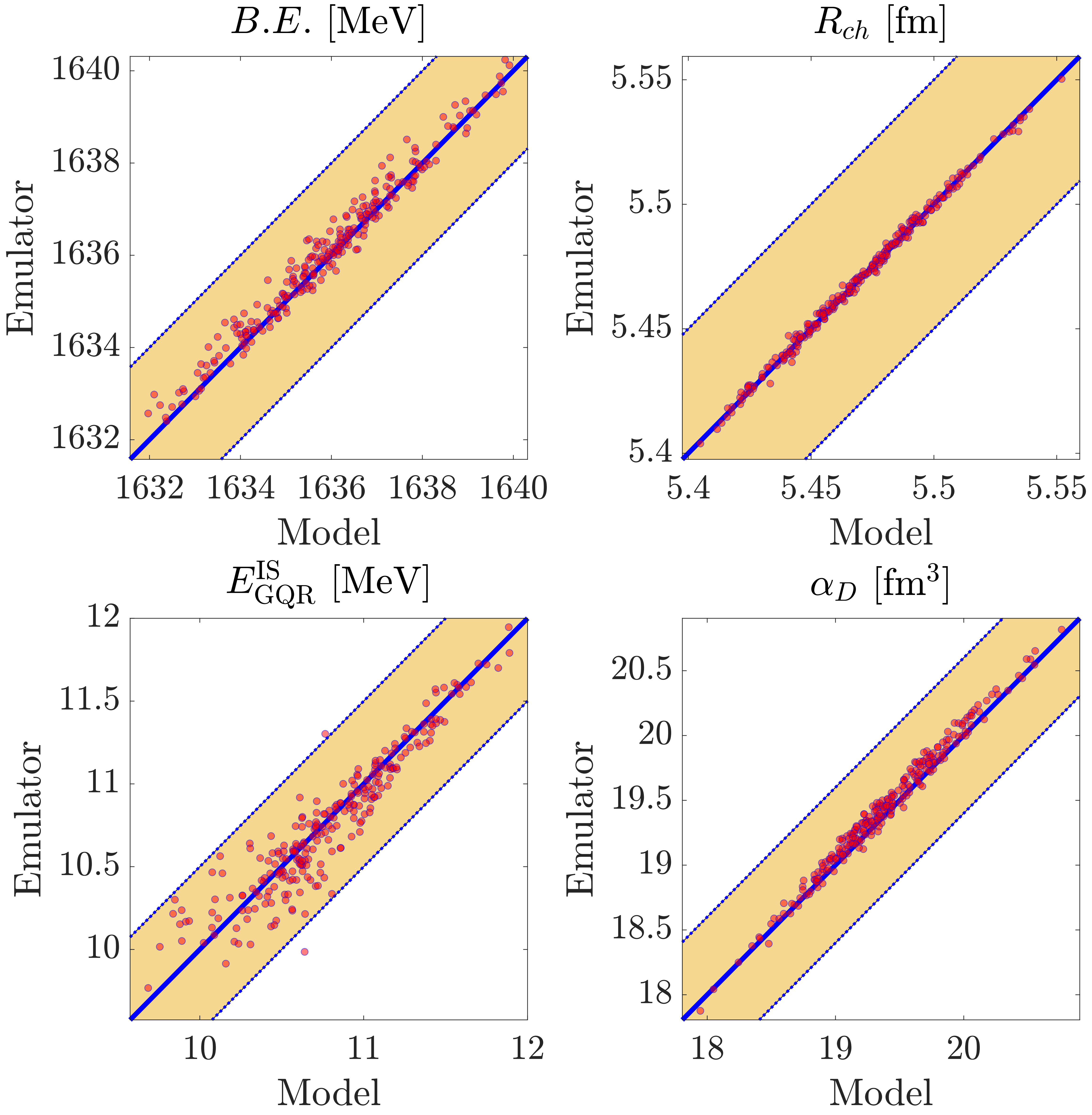


From MADAI user manual

The MADAI package:

- was built for GP applied to bayesian inference
- given the parameters prior distributions, it automatically builds the grid
- it does a MCMC to estimate the posterior distribution
- it extracts parameters sample following the posteriors

Validation

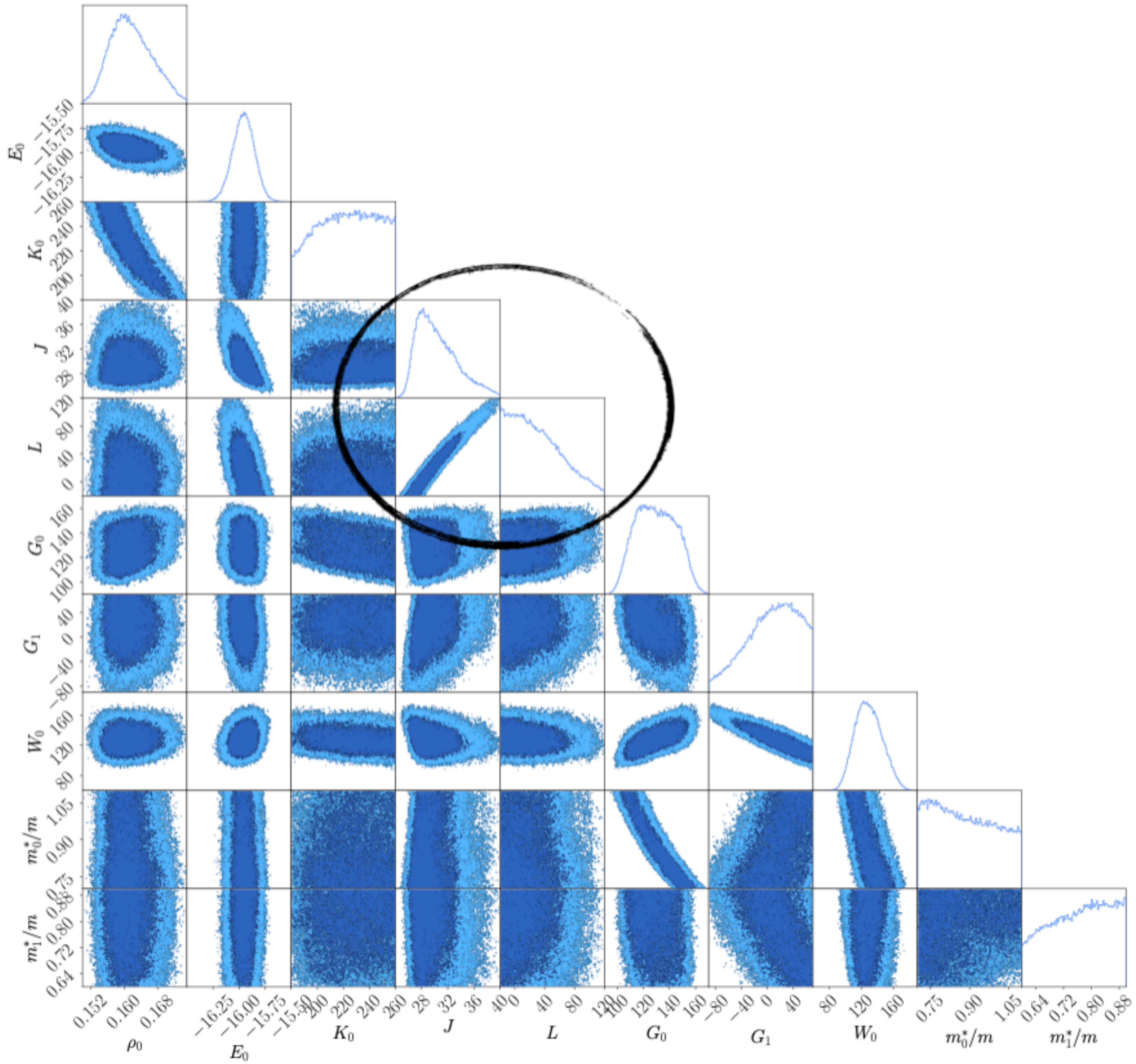


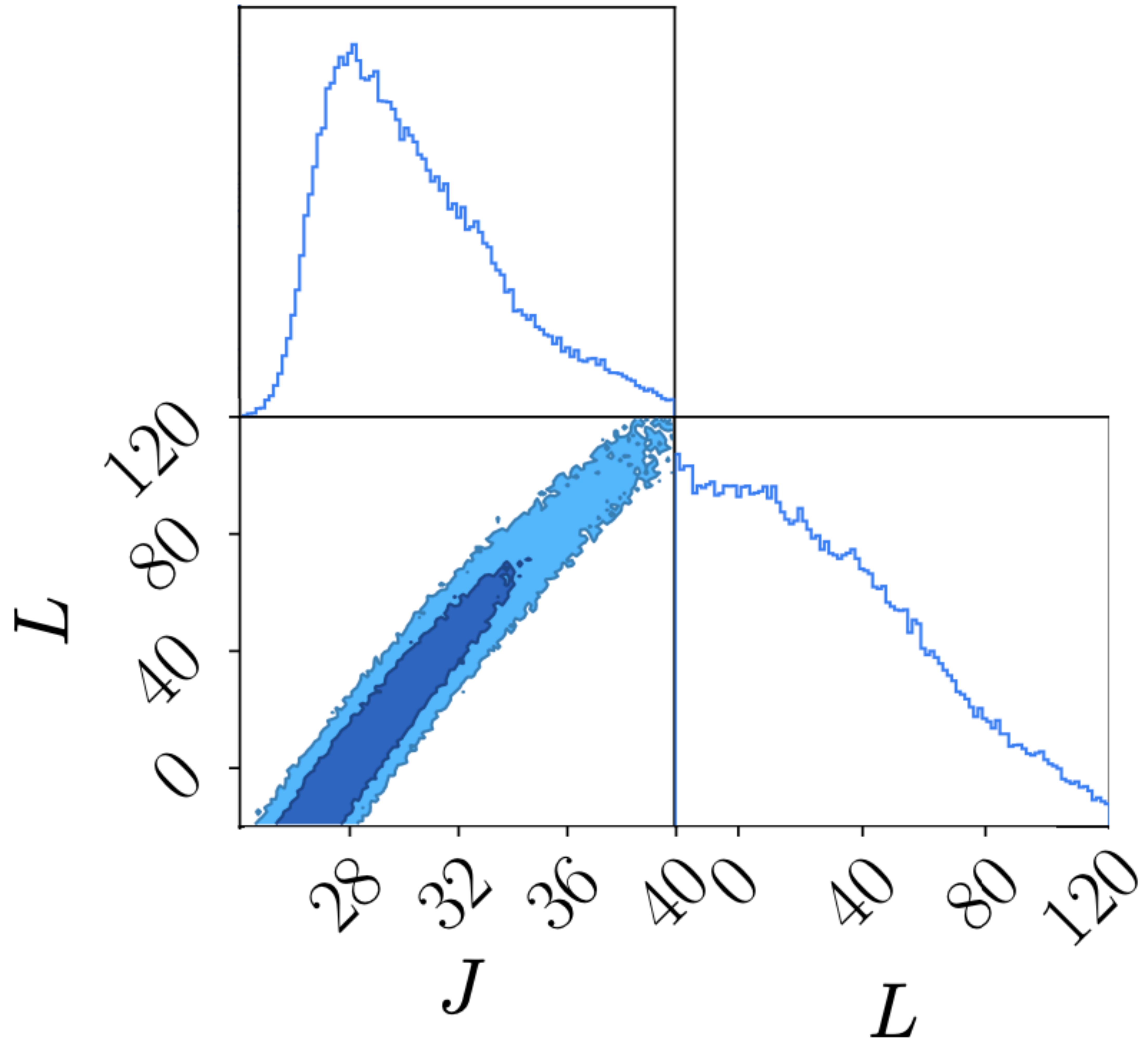
Ground-state properties						
	Discrepancy			Corr. coefficient		
	$B.E.$	$R_{\rm ch}$	$\Delta E_{\rm SO}$	$B.E.$	$R_{\rm ch}$	$\Delta E_{\rm SO}$
^{208}Pb	0 %	0 %	0 %	0.993	1.000	0.997
^{48}Ca	0 %	0 %	0 %	0.998	0.999	0.998
^{40}Ca	0 %	0 %	-	0.999	0.999	-
^{56}Ni	0 %	-	-	0.996	-	-
^{68}Ni	0 %	-	-	0.994	-	-
^{100}Sn	0 %	-	-	0.994	-	-
^{132}Sn	0 %	0 %	-	0.992	1.000	-
^{90}Zr	0 %	0 %	-	0.996	1.000	-

Isoscalar resonances				
	Discrepancy		Corr. coefficient	
	$E^{\rm IS}_{\rm GMR}$	$E^{\rm IS}_{\rm GQR}$	$E^{\rm IS}_{\rm GMR}$	$E^{\rm IS}_{\rm GQR}$
^{208}Pb	0 %	1.0 %	1.000	0.904
^{90}Zr	0 %	-	1.000	-

Isovector properties						
	Discrepancy			Corr. coefficient		
	α_D	$m(1)$	A_{PV}	α_D	$m(1)$	A_{PV}
^{208}Pb	0 %	0 %	0 %	0.988	0.9999	0.998
^{48}Ca	0 %	-	0 %	0.990	-	0.9992

$B.E., R_{ch}$ only corner plot

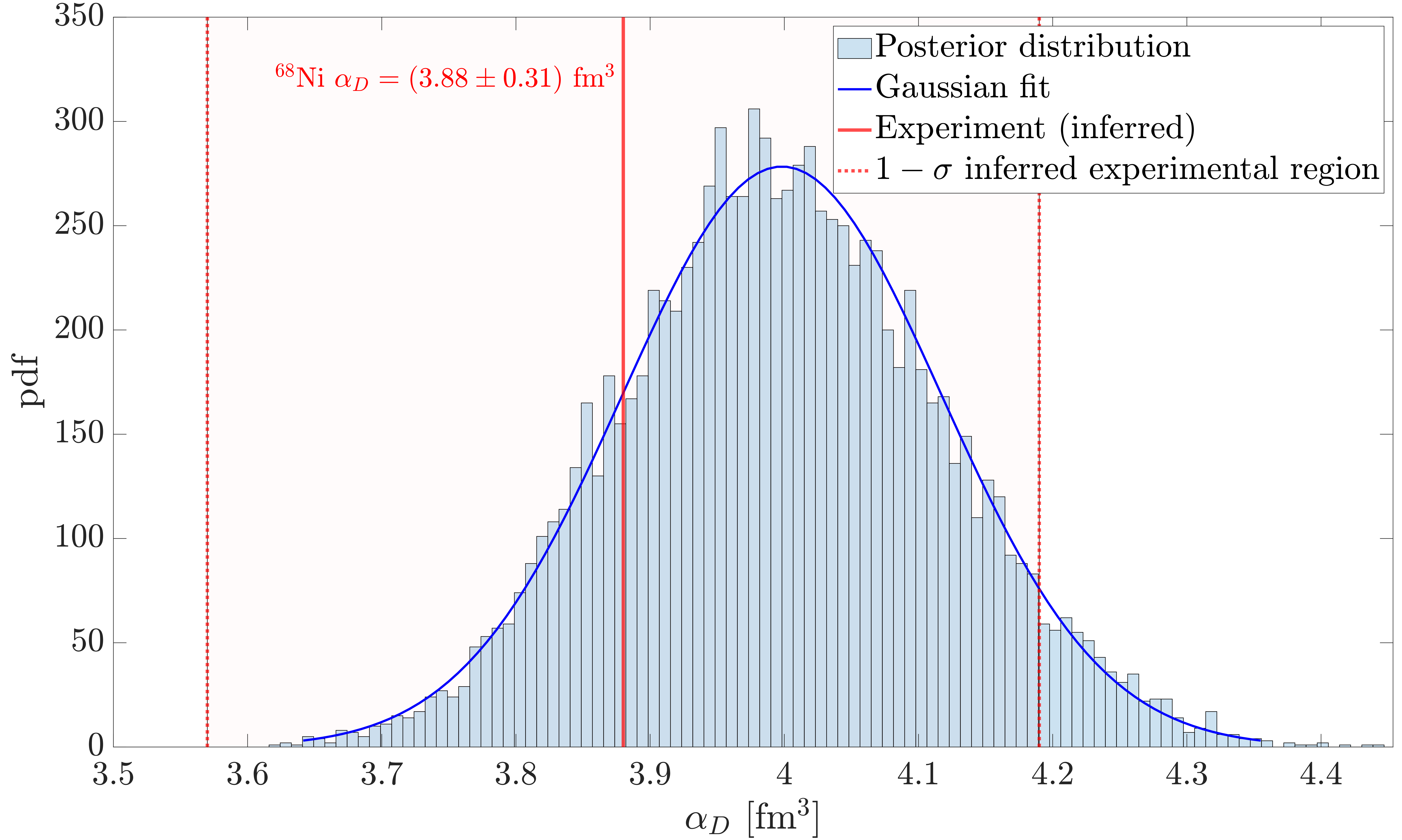




Sensitivity analysis: J fixed

Posterior distributions

^{68}Ni α_D posterior distribution



B5

Crust core properties; crust radius