From heavy ion collisions at intermediate energies to the nuclear equation of state in symmetric nuclear matter

Rohit Kumar

IRL – NPA, CNRS & MSU, FRIB, East Lansing, Michigan, USA

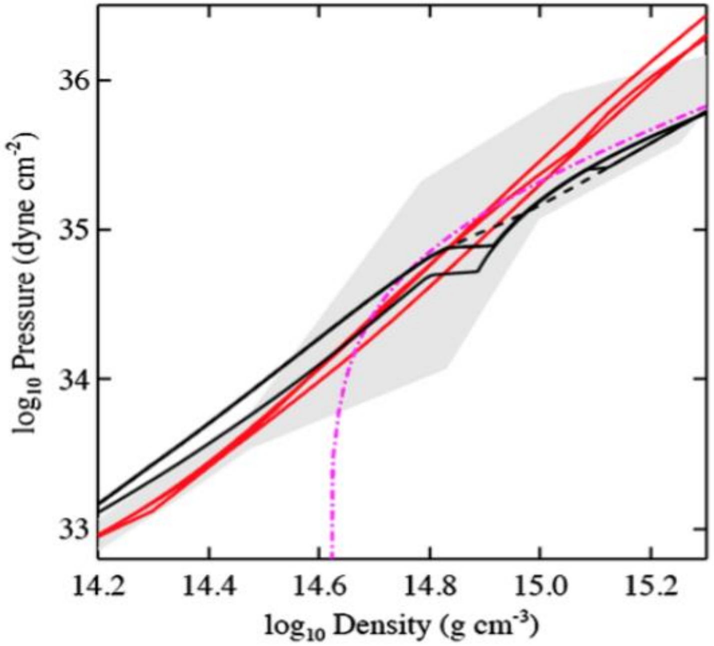
Dense matter equation of state from theory and experiments
IRL-NPA, FRIB 2024

Outline

- Introduction
- Heavy-ion collisions to neutron stars
- Transport model
- Bayesian framework
- Results
- Comparison with other available results
- Summary

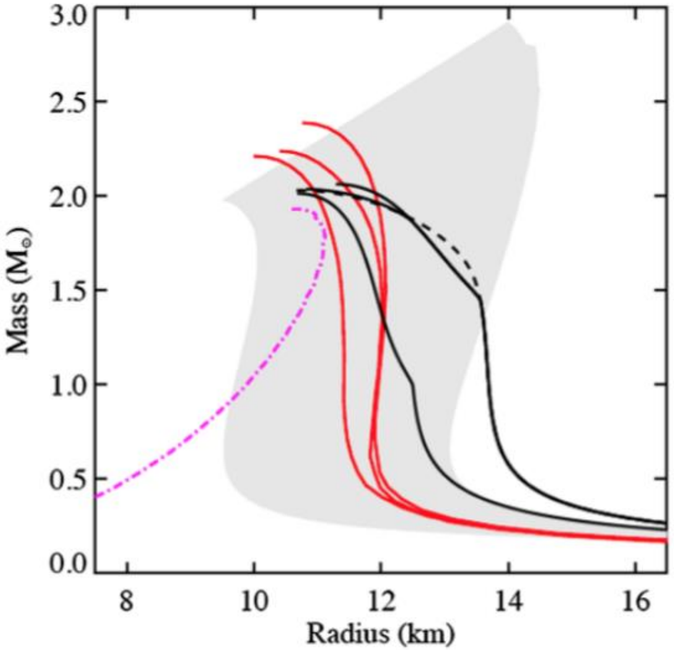
Nuclear Physics ⟷ Astrophysics

Equation of State



Stellar Structure Equations

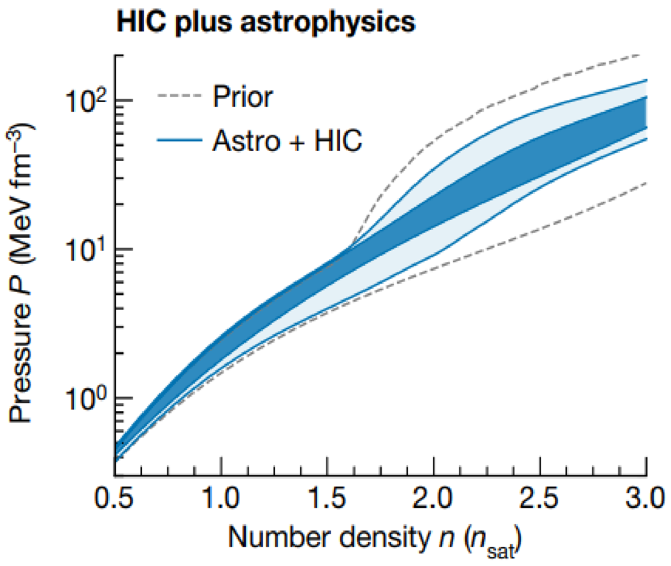
Mass – Radius



Credit: Anna Watts

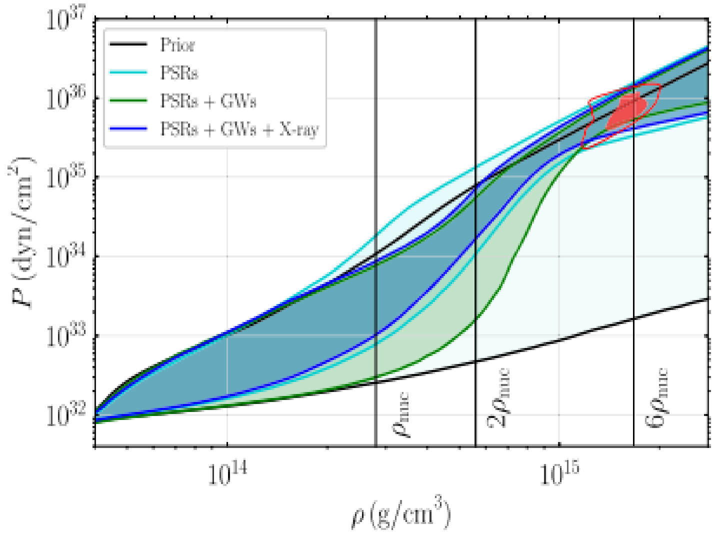
- Equation of state having a unique density- Pressure relationship maps to a unique mass-Radius Relationship
- Each of the different EoS corresponds to the different Mass-Radius relationship

Constraining equation of state



Priors: Chiral Effective Field Theory (ChEFT) (upto $1.5n_0$) Constraints:

- HIC: FOPI, AGS, ASY-EOS
- Astro: GW170817+kilonova, GW190425, NICER (2 NS), M_{max}



Priors: Non-parametric EoS Constraints:

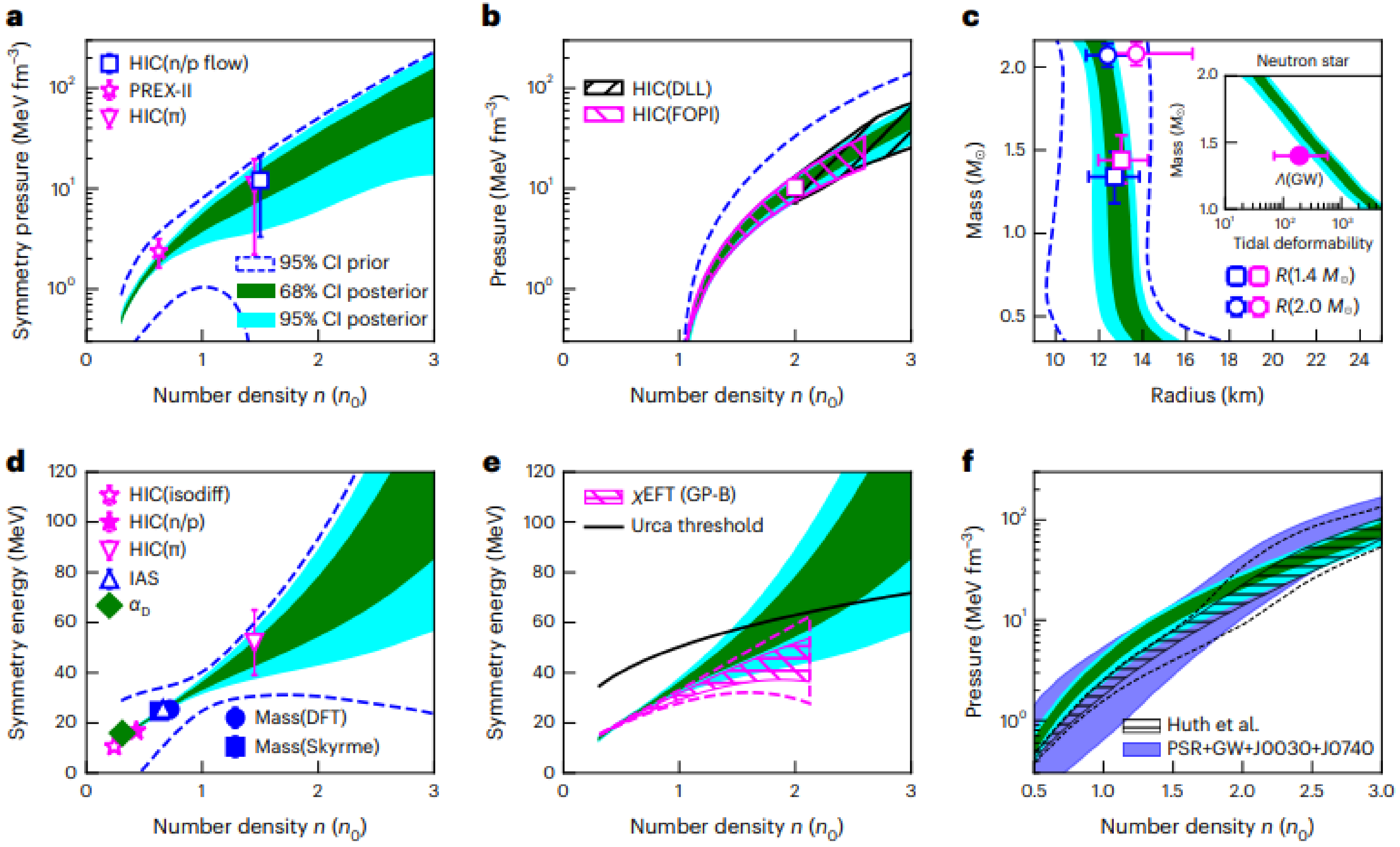
- Deformability of neutron star (GW170817)
- Mass-Radius of neutron stars (NICER) +X-ray timing

Huth, Pang et al., Nature 606, 279 (2022)

Legred et al., PRD 104, 063003 (2021)

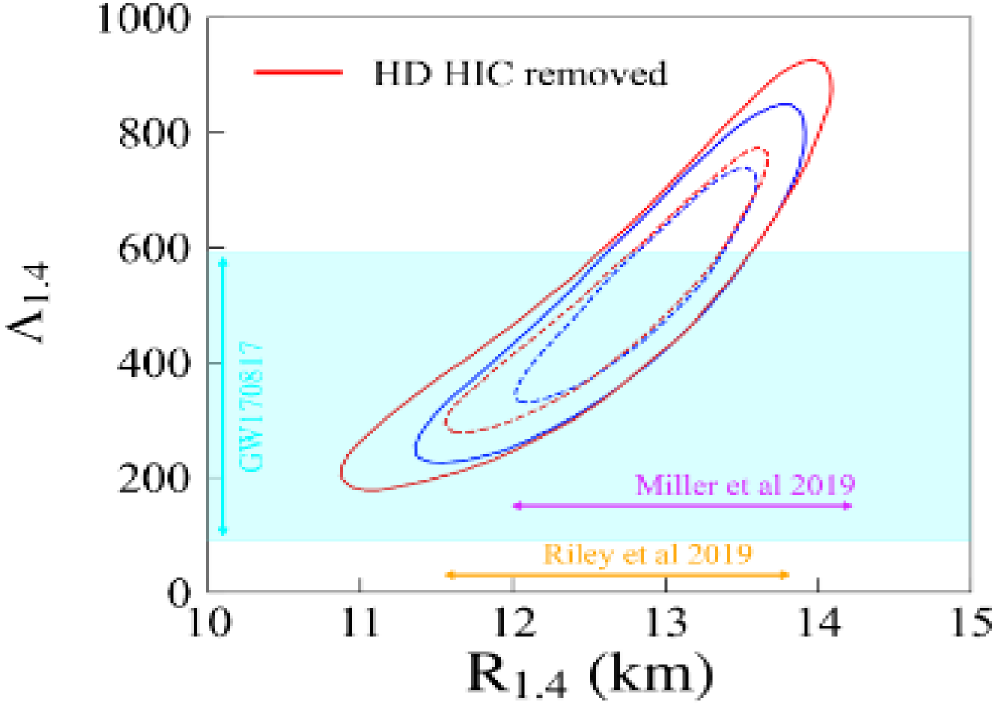
HIC constraints at work

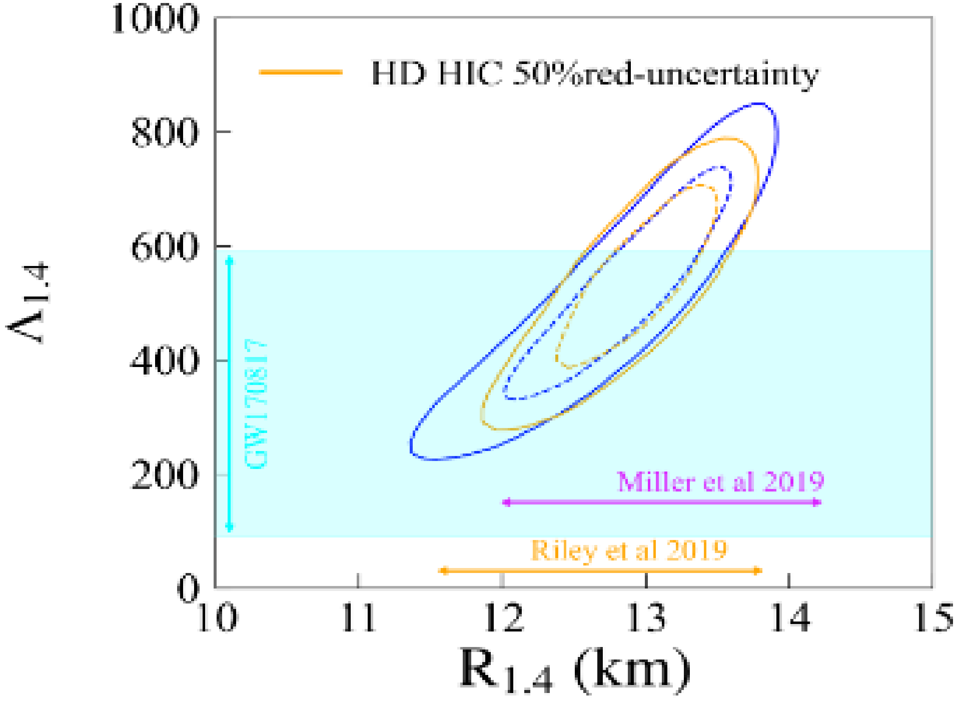
Priors: Meta modelling EoS
Constraints:
- Nuclear structure studies
- HIC: FOPI, AGS, ASY-EOS, NSCL
- Astro: GW170817, NICER (2 NS), M_{max}



Nature Astronomy 8, 328-336 (2024)

Sensitivity analysis





HD HIC removed: Remove all data from heavy-ion collisions

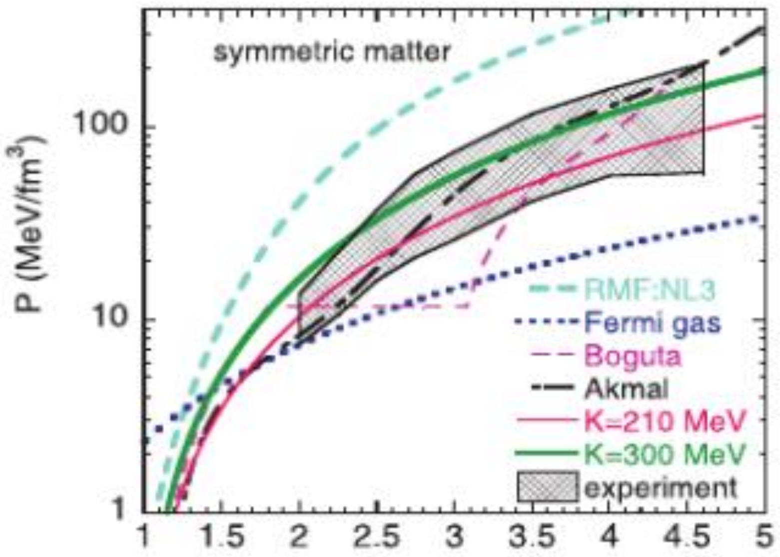
HD HIC removed: Reduce the uncertainty for HIC data by 50%

Flow data analysis

- Au + Au collision data in the energy range of 0.15 to 10 GeV/nucleon
- Transverse and elliptical flow are studied
- Flow data exclude very repulsive and very soft equations of state

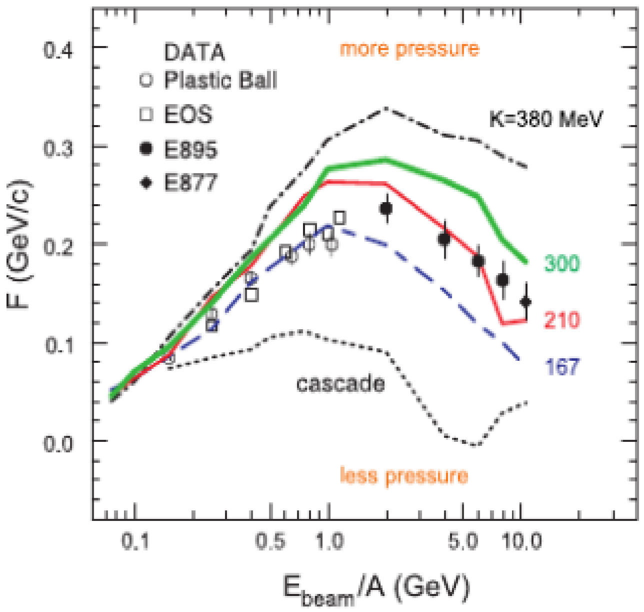
Danielewicz, Lacey, Lynch, Science 298, 1592 (2002)

$2 < n/n_0 < 4.6$

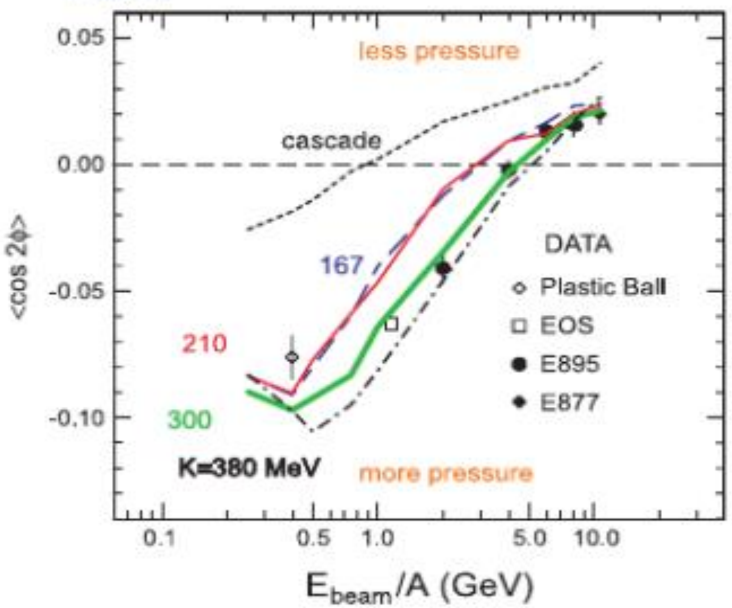


n_{max}/n_0

n_{max}/n_0



n_{max}/n_0

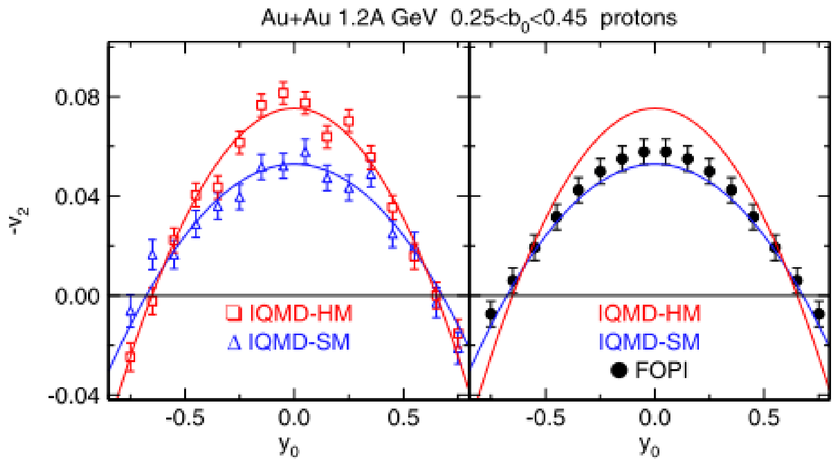


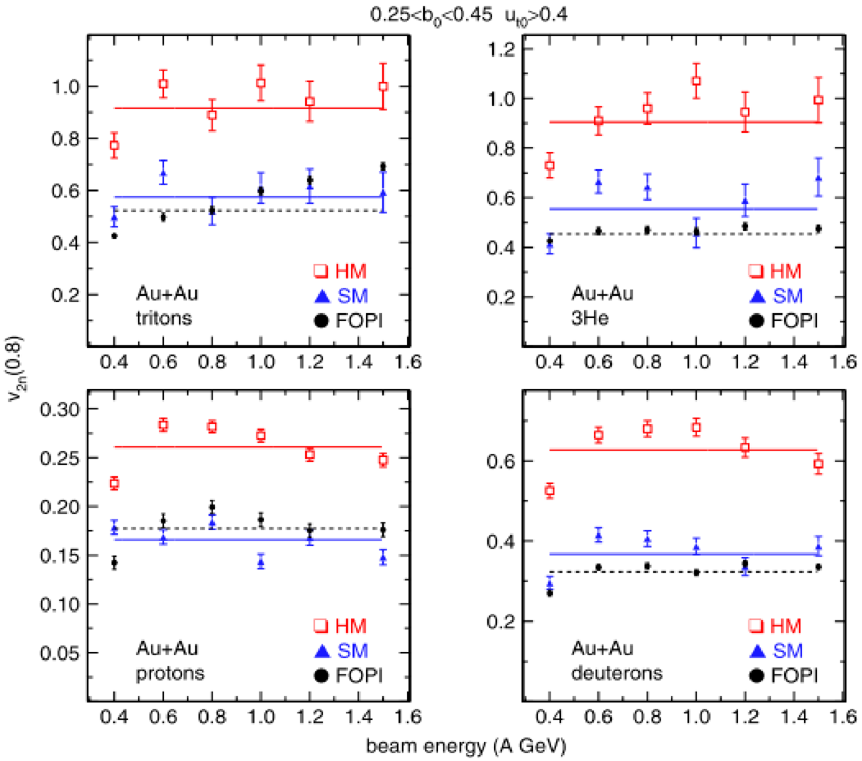
At low incident energies: The compressed matter expands while the spectator Matter is still present and blocks the in-plane emission (shadowing) v_2 <0

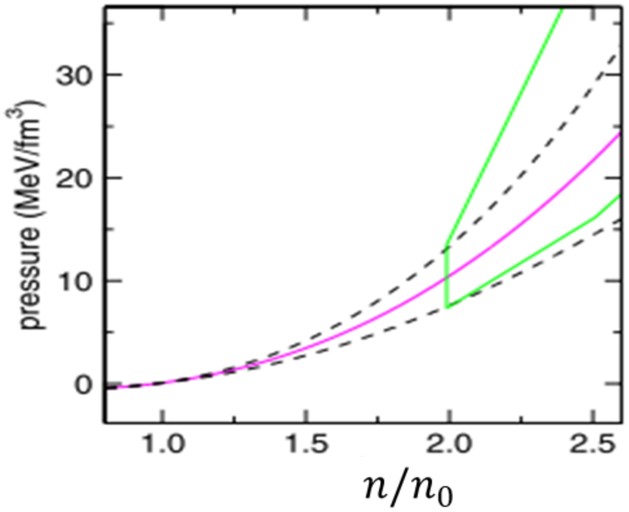
At higher incident energies: expansion takes place after the spectator matter has passed the compressed zone v_2 >0

K_{sat}=167-200 MeV (soft) from transverse flow
K_{sat} = 300 MeV (semi-stiff) from elliptical flow

- FOPI experiments of Au+Au collisions at 0.4 to 1.5 GeV/nucleon
- Elliptic flow of protons and heavier isotopes







$$v_2(y_0) = v_{20} + v_{22} \cdot y_0^2$$

$$v_{2n} = |v_{20}| + |v_{22}|$$

$1 < n/n_0 < 2.6$

SM: K_{sat} = 200 MeV

HM: K_{sat} = 380 MeV

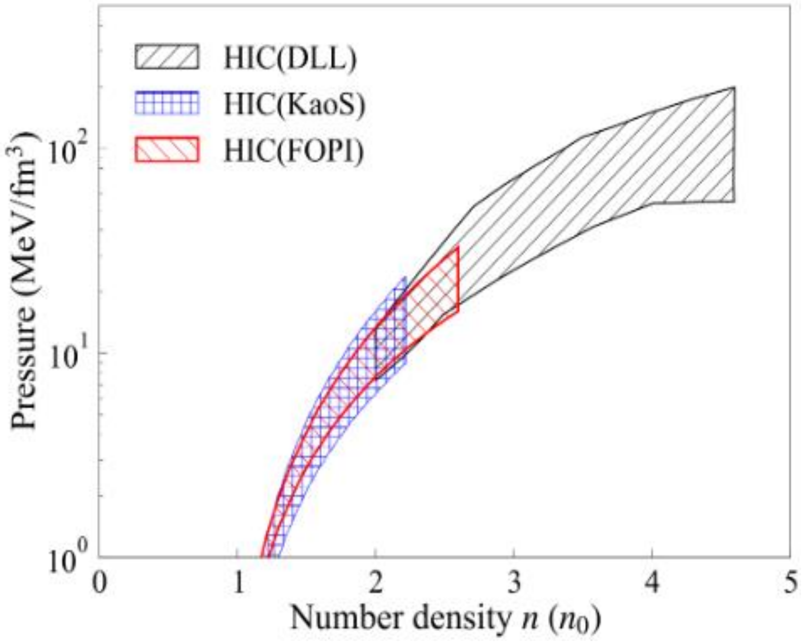
Derivation of the effective K using the elliptic flow observable v_{2n}.

	v_{2n} SM	v_{2n} HM	v_{2n} FOPI	HM/SM	HM–SM	FOPI–SM	ΔK (MeV)
^{1}H	0.1658 ± 0.0030	0.2609 ± 0.0027	0.1774 ± 0.0028	1.57 ± 0.07	0.0951	+0.0116	$+22 \pm 8$
^{2}H	0.3676 ± 0.0080	0.6274 ± 0.0087	0.3237 ± 0.0029	1.71 ± 0.08	0.2598	−0.0439	-30 ± 8
^{3}H	0.5740 ± 0.0214	0.9161 ± 0.0252	0.5223 ± 0.0048	1.60 ± 0.08	0.3421	−0.0517	-27 ± 17
^{3}He	0.5540 ± 0.0217	0.9048 ± 0.0265	0.4537 ± 0.0050	1.63 ± 0.16	0.3501	−0.1010	-52 ± 18

Linear extrapolation/interpolation assumption was used to draw final results.

Le Fevre, Leifels, Reisdorf, Aichelin, Hartnack, Nuclear Physics A 945, 112 (2016)

HIC constraints: Symmetric nuclear matter



EOS parametrized by Ksat+ momentum dependence

- Au + Au collision data in the energy range of 0.15 to 10 GeV/nucleon
- Transverse and elliptical flow were studied
- Flow data exclude very repulsive and very soft equations of state
- Model used: pBUU

Danielewicz, Lacey, Lynch, Science 298, 1592 (2002)

- $^{197}Au+^{197}Au$ & $^{12}C+^{12}C$ at < 1.5 GeV/u
- Study of subthreshold kaon production (KaoS-collaboration)
- Model used: QMD; kaon potentials, momentum dependence

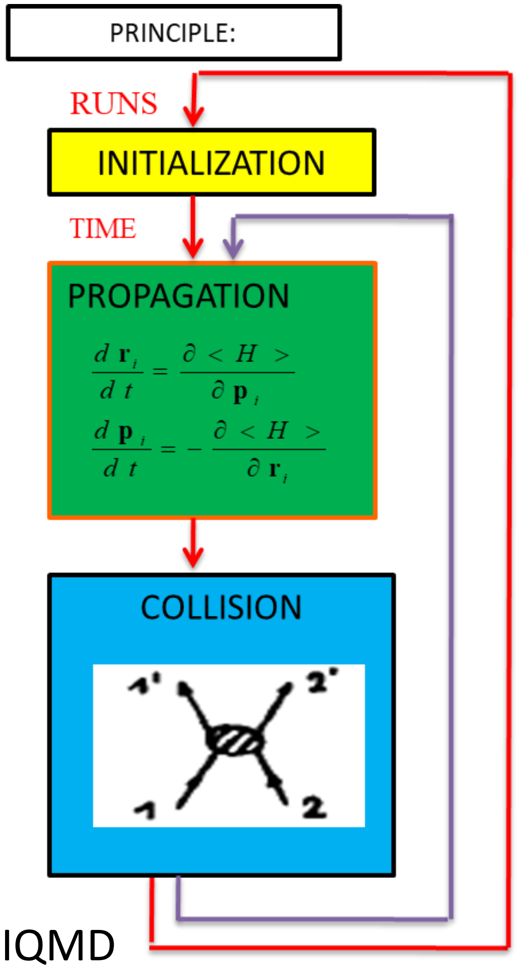
C. Fuchs et al., Prog. Part. Nucl. Phys. 53, 113–124 (2004)
Lynch et al., 2007

- FOPI experiments of $^{197}Au+^{197}Au$ collisions at 0.4 to 1.5 GeV/u
- Study of elliptic flow of protons and heavier isotopes
- Model used: IQMD model (FOPI collaboration)

Le Fevre, Leifels, Reisdorf, Aichelin, Hartnack, Nuclear Physics A 945, 112 (2016)

Isospin quantum molecular dynamics model

Isospin Quantum Molecular Dynamics (IQMD): Molecular Dynamics (MD) approach + n-body correlations + Equation of State (EOS) + Important Quantum Features (Pauli Principle, Stochastic Scattering, Particle Production)



- Nucleons of P/T are initialized
- Nucleons propagate under the mean field
- Nucleons scatter if they come too close

Model details in
J. Aichelin, Physics Reports 202 (1991) 233—360
Ch. Hartnack et al., Eur. Phys. J. A 1, 151–169 (1998)

Extensive comparisons with data in
W. Reisdorf et al., Nucl. Phys.A 848, 366-427 (2010)
W. Reisdorf et al., Nuclear Physics A 876 (2012) 1–60

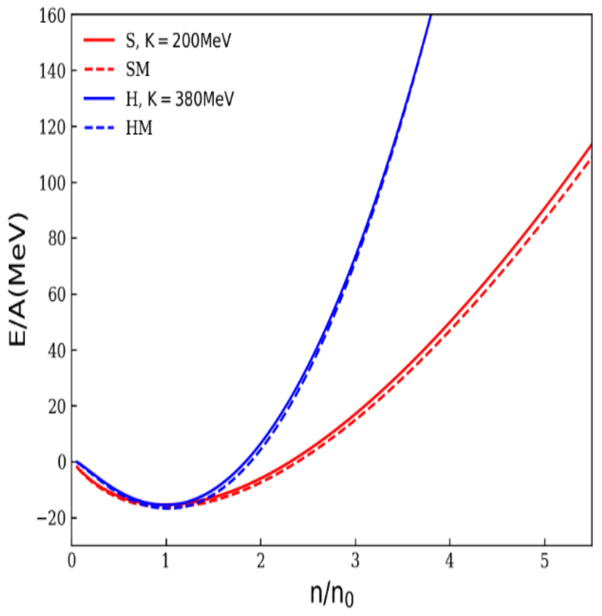
Symmetric nuclear matter:

$$\frac{E}{A}(n,0) = \frac{3}{5}E_F\left(\frac{n}{n_o}\right)^{2/3} + \frac{\alpha}{2}\left(\frac{n}{n_o}\right) + \frac{\beta}{\gamma+1}\left(\frac{n}{n_o}\right)^{\gamma} + E_{mdi}$$

- The parameters α, β are fixed by choice of n and E/A at n_o
- γ can be varied to choose value of K_{sat}

Parameters of the different parametrizations of the EoS used in the IQMD model.

	α (MeV)	β (MeV)	γ	δ (MeV)	$\varepsilon\left(\frac{c^2}{GeV^2}\right)$	K (MeV)
S	−356	303	1.17	–	–	200
SM	−390	320	1.14	1.57	500	200
H	−124	71	2.00	–	–	376
HM	−130	59	2.09	1.57	500	376



Introducing flexible EoS in transport model

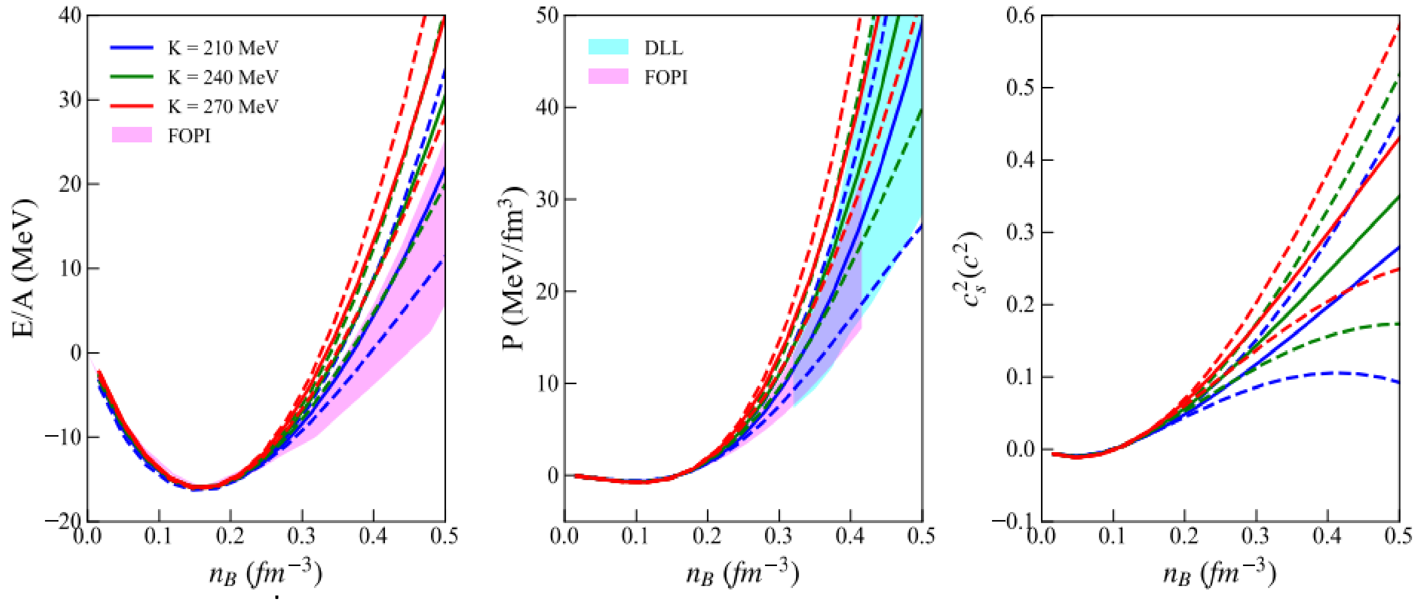
Symmetric nuclear matter:

$$\frac{E}{A}(n,0) = \frac{3}{5}E_F\left(\frac{n}{n_o}\right)^{2/3} + \frac{\alpha}{2}\left(\frac{n}{n_o}\right) + \frac{\beta_1}{\gamma_1+1}\left(\frac{n}{n_o}\right)^{\gamma_1} + \frac{\beta_2}{\gamma_2+1}\left(\frac{n}{n_o}\right)^{\gamma_2} + E_{mdi}$$

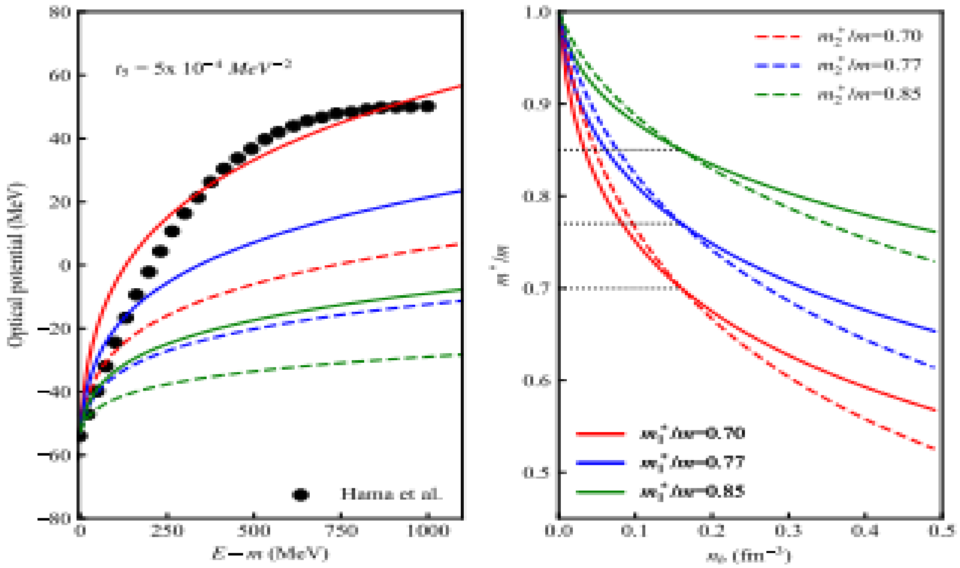
- Two of these parameters are fixed by choice of n and E/A at n_o
- Other parameters can be varied to choose value of K_{sat} and Q_{sat}

$n_0 = 0.16\ fm^{-3}, E_{sat} = -16.0\ MeV$
$K_{sat} = 240 \pm 30\ MeV$
$m^*/m = 0.75 \pm 0.1$

PRC 85, 035201 (2012)
PRC 97, 025805 (2018)



K_{sat} = 210, 240, 270 MeV
Q_{sat} = -650 to -100 MeV
m*/m = 0.70, 0.77, 0.85
Free cross-sections



Clusterization:
Identify clusters in the freeze out state
Coordinate-space distance and momentum space correlations as a criterion

Bayesian analysis

$$\textbf{Bayes' Theorem}: P(EoS|data) \propto P(EoS) \prod_i \mathcal{L}(i^{th}\ Data\ point)$$

Posterior: probability of EoS given flow data

Prior: Uniform prior within design range

Likelihood: probability of data given EoS

Likelihood and uncertainty Quantification:

Likelihood= $\exp\left(-\frac{1}{2}\chi_{k,E}\right)$ $\chi_{k,E} = \frac{1}{N_{\rm dof}}\hat{\Delta}_{k,E}\hat{\Sigma}^{-2}_{k,E}\hat{\Delta}_{k,E}$

$$\hat{\Delta}_{k,E}(i,i) = O_i(\mathrm{data},E) - O_i(\mathrm{model},k,E)\,,$$
$$\hat{\Sigma}^2_{k,E}(i,i) = \sigma_i(\mathrm{data},E)^2 + \sigma_i(\mathrm{model},k,E)^2$$

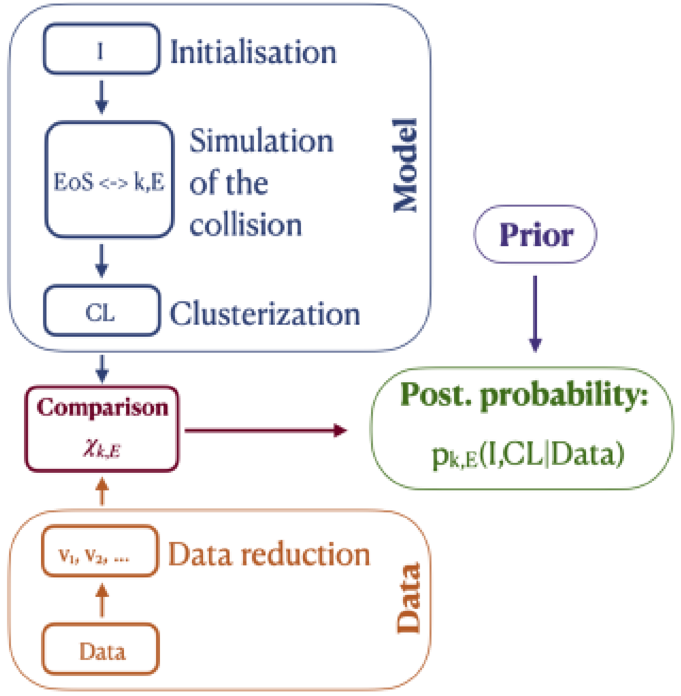
$O_i\ (data, E)$ → experimental data as a function of rapidity

$O_i\ (model, k, E)$ → kth model results as a function of rapidity

N_{dof} → Number of data points – Number of parameters

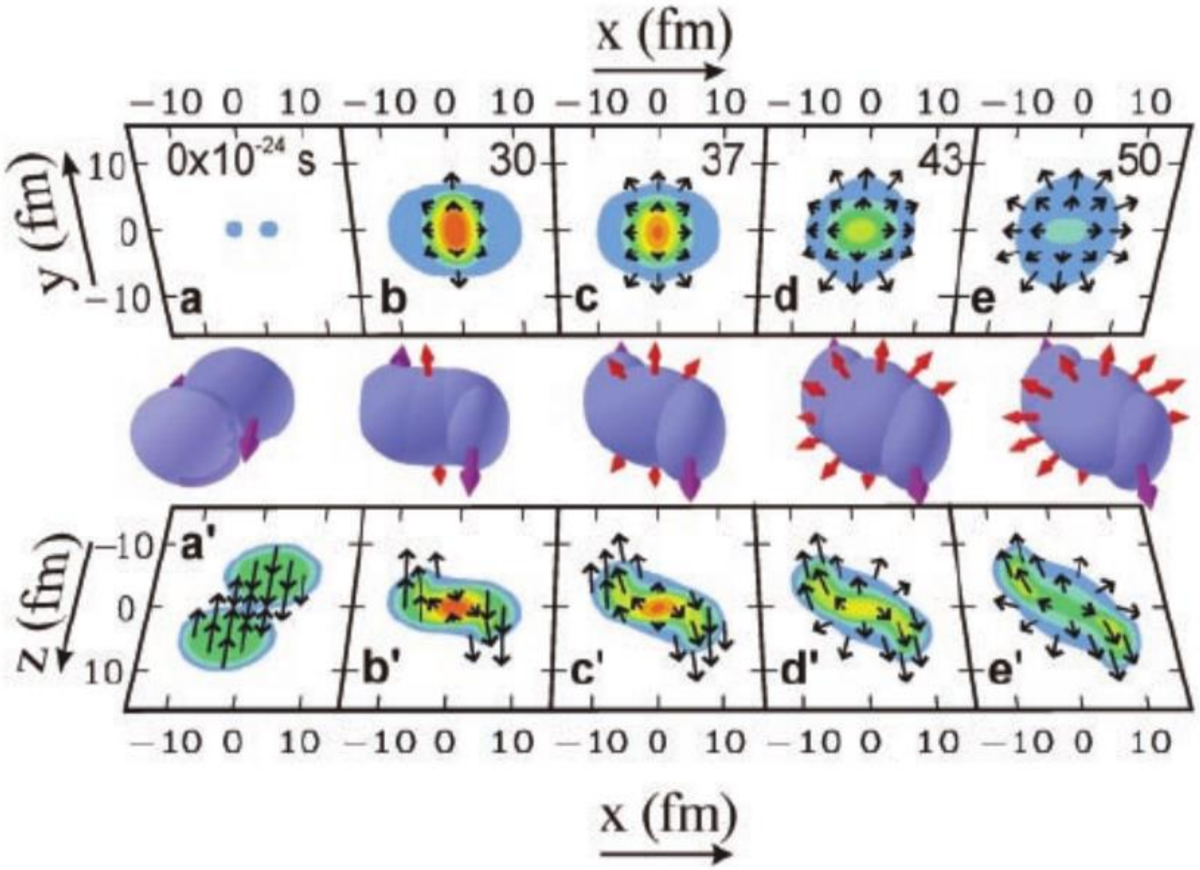
Experimental data: W. Reisdorf et al., NPA 876 (2012) 1–60 FOPI Collaboration

- Colliding nuclei: Au + Au @ b_0 = 0.25 – 0.45, u_{t0} >0.4
- Projectile energies: 400, 600, 800, 1000, 1200, 1500 MeV/nucleon
- Observables: Rapidity dependent of v_1, v_2 of protons, deuterons, tritons, ^{3}He and alpha
- Rapidity range: -0.5 to 0.5
- Number of rapidity bins = 10
- Total of 480 data points



Framework

Flow



Danielewicz, Lacey, Lynch, Science 298, 1592 (2002)

Various flows of nucleons/clusters which are obtained from a Fourier expansion of azimuthal distribution of nucleons/clusters:

$$\frac{dN}{d\phi}(y, p_t) \propto 1 + 2v_1 cos\phi + 2v_2 cos2\phi + ...$$

y = rapidity

p_t = transverse momentum

Nuclear transverse flow is given by first harmonic coefficient,

$$v_1 = < cos\phi > = < \frac{p_x}{p_t} > \qquad p_t = \sqrt{p_x^2 + p_y^2}$$

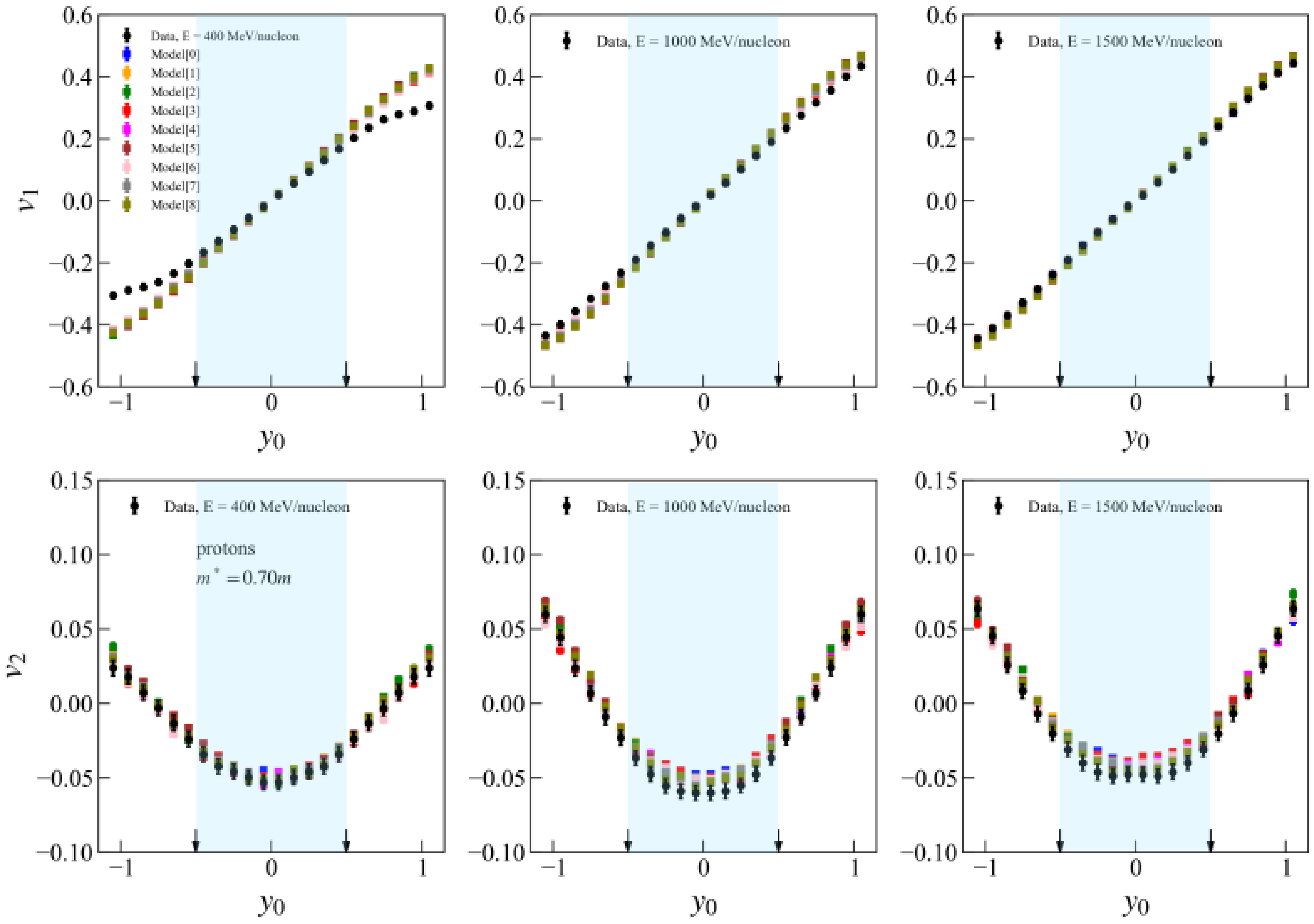
Nuclear elliptic flow is given by second harmonic coefficient,

$$v_2 = < cos2\phi > = < \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} >$$

Flow is powerful observable to determine the nuclear EoS, for symmetric nuclear matter as well as symmetry energy

W. Reisdorf et al., NPA 876 (2012) 1–60 FOPI Collaboration

Model comparison with data at different incident energies



W. Reisdorf et al., NPA 876 (2012) 1–60

Au + Au @ b_0 = 0.25 – 0.45, u_{t0} >0.4

E = 400, 1000, 1500 AMeV

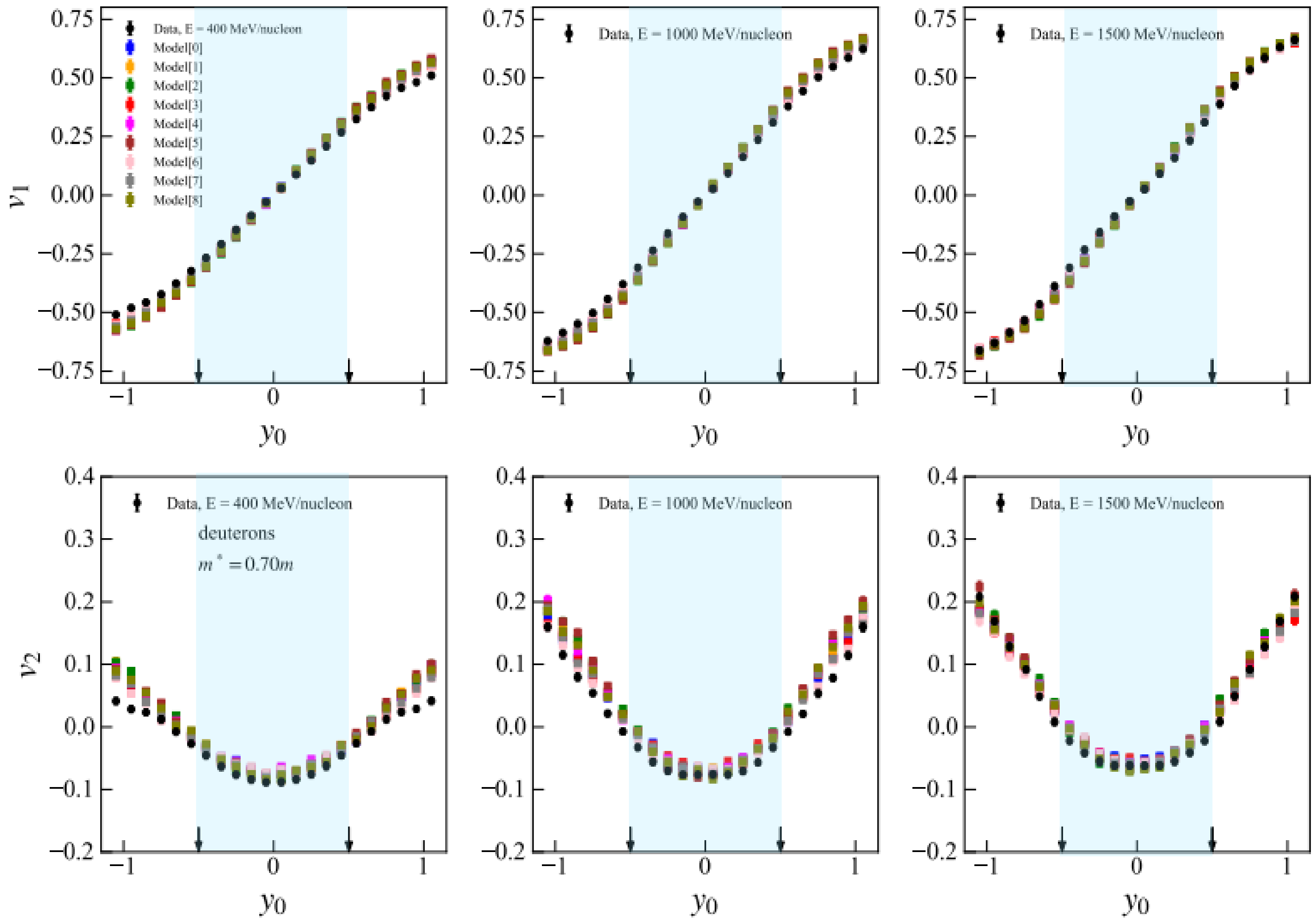
Black symbols: Experimental data

v_1 and v_2 as a function of rapidity

Protons

Similar results @ 600, 800, 1200 AMeV

Model comparison with data at different incident energies



W. Reisdorf et al., NPA 876 (2012) 1–60

Au + Au @ b_0 = 0.25 – 0.45, u_{t0} >0.4

E = 400, 1000, 1500 AMeV

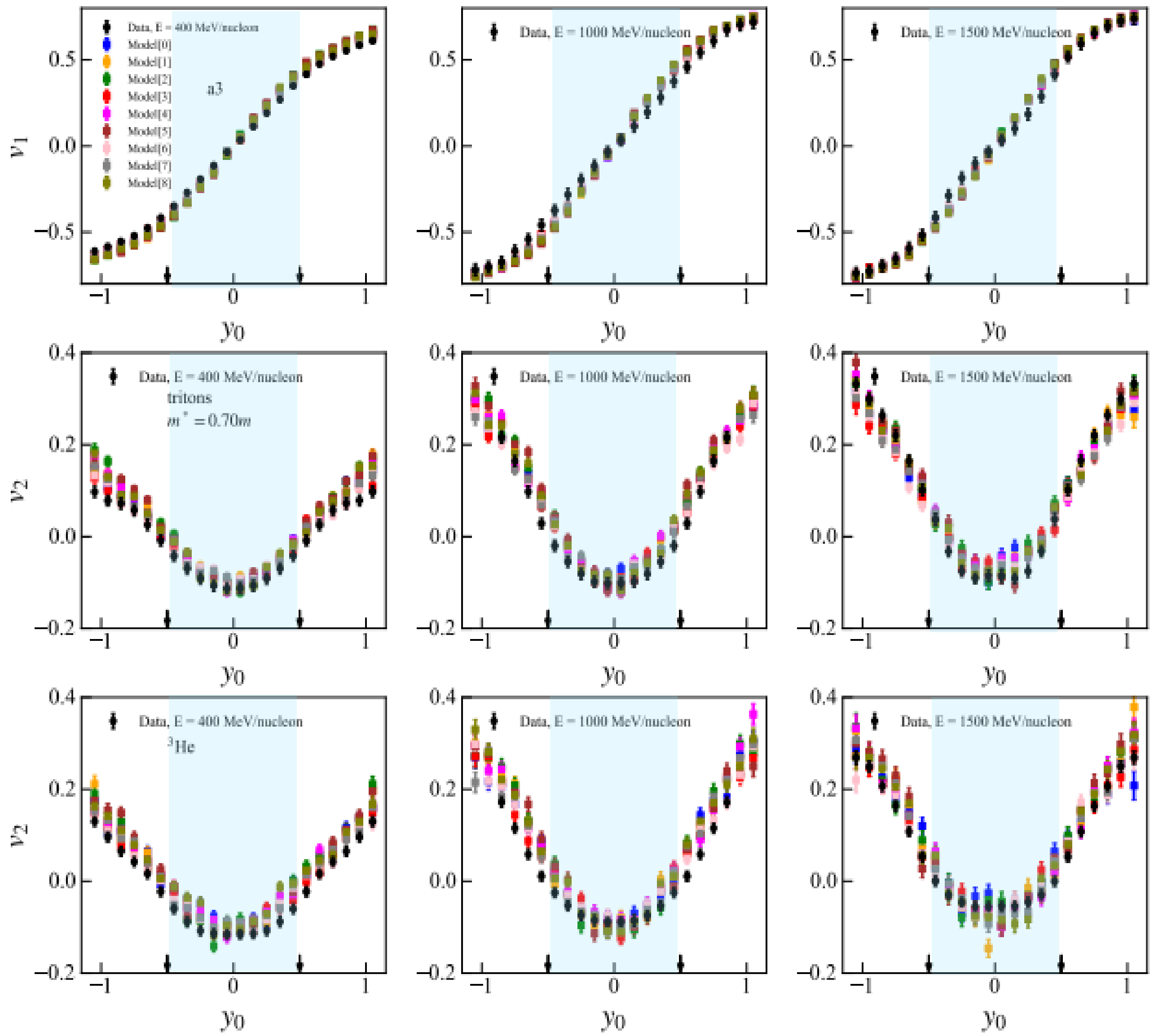
Black symbols: Experimental data

v_1 and v_2 as a function of rapidity

deuterons

Similar results @ 600, 800, 1200 AMeV

Model comparison with data at different incident energies



W. Reisdorf et al., NPA 876 (2012) 1–60

Au + Au @ b_0 = 0.25 – 0.45, u_{t0} >0.4

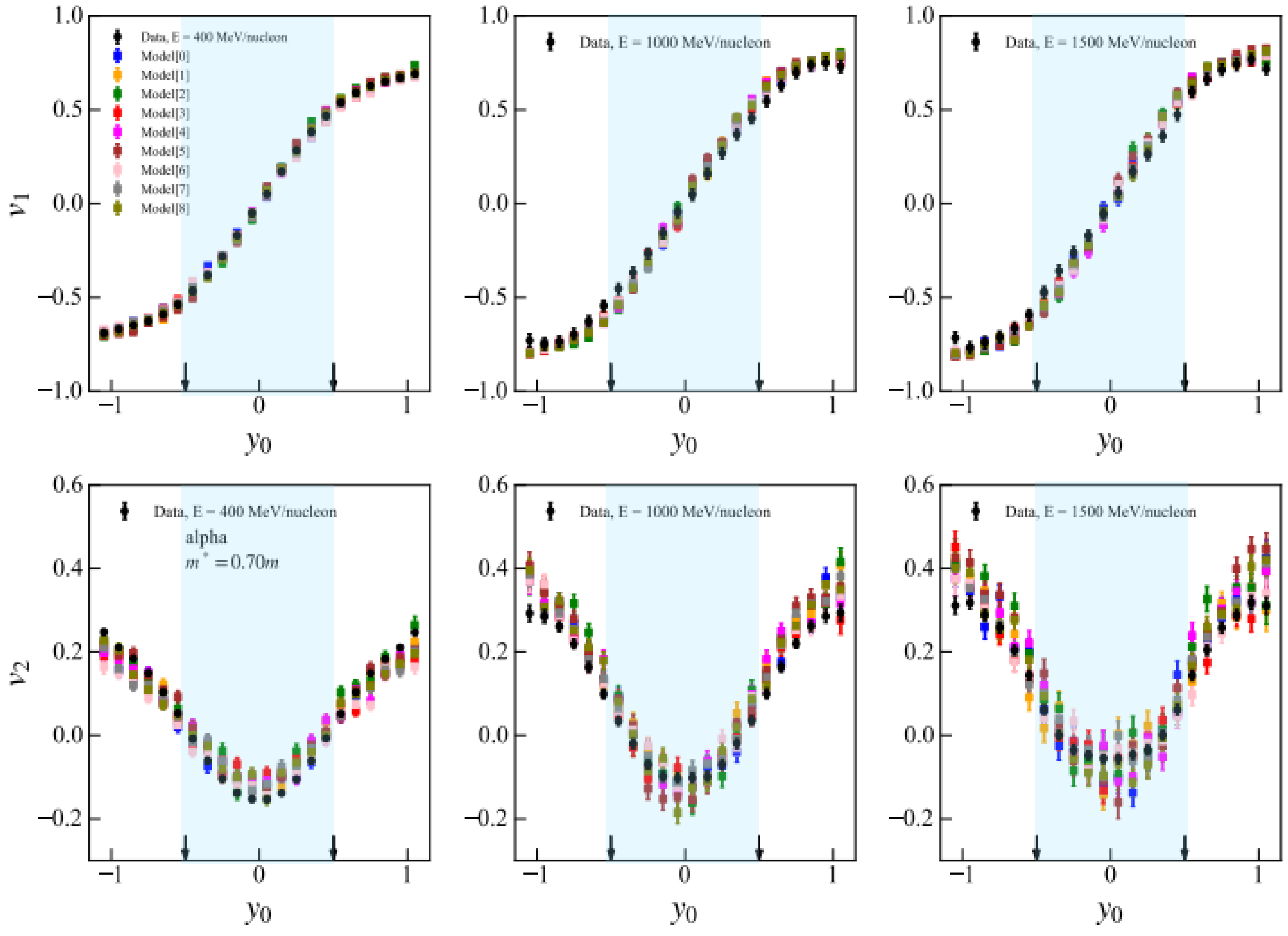
E = 400, 1000, 1500 AMeV

Black symbols: Experimental data

v_1 and v_2 as a function of rapidity

Tritons
^{3}He

Model comparison with data at different incident energies



W. Reisdorf et al., NPA 876 (2012) 1–60

Au + Au @ b_0 = 0.25 – 0.45, u_{t0} >0.4

E = 400, 1000, 1500 AMeV

Black symbols: Experimental data

v_1 and v_2 as a function of rapidity

alpha

Similar results @ 600, 800, 1200 AMeV

Combining the EoS models that explains the data at different energies

W. Reisdorf et al., NPA 876 (2012) 1–60

Au + Au @ b_0 = 0.25 – 0.45, u_{t0} >0.4

E = 400, 600, 800, 1000, 1200, 1500 AMeV

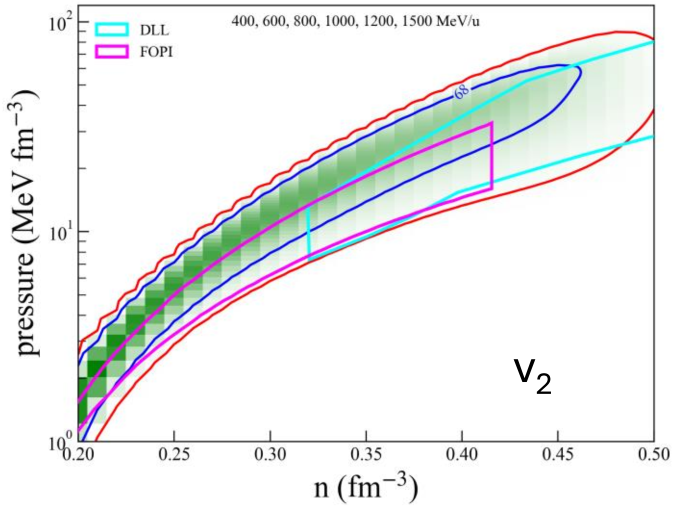
Blue lines: 68% CI
Red lines: 95% CI

v_2 data:

$$K_{sat} = 250^{+31}_{-24}\ MeV$$
$$Q_{sat} = -296^{+216}_{-180}\ MeV$$

v_1+v_2 data:

$$K_{sat} = 230^{+24}_{-31}\ MeV$$
$$Q_{sat} = -332^{+198}_{-198}\ MeV$$

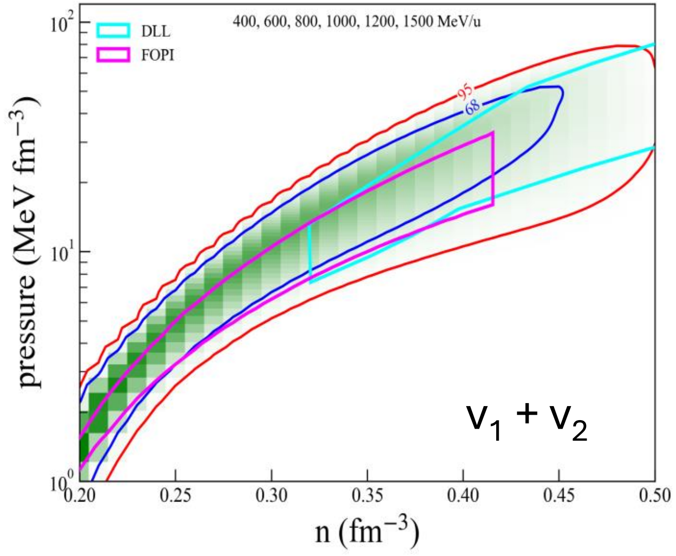


Comparing models with different effective masses: m*/m = 0.70 is favored

Bayes' factor:

$$BF_{12} = \frac{P(\text{data}|\text{model 1})}{P(\text{data}|\text{model 2})}$$

Data	model 1 m^*/m	model 2 m^*/m	flow coefficients v_2	flow coefficients $v_1 + v_2$
p+d+t+^{3}He+^{4}He	0.70	0.77	1.87	1.45
p+d+t+^{3}He+^{4}He	0.70	0.85	15.09	12.92
p+d+t+^{3}He+^{4}He	0.77	0.85	8.07	8.85



Comparison with previous results

W. Reisdorf et al., NPA 876 (2012) 1–60

Au + Au @ b_0 = 0.25 – 0.45, u_{t0} >0.4

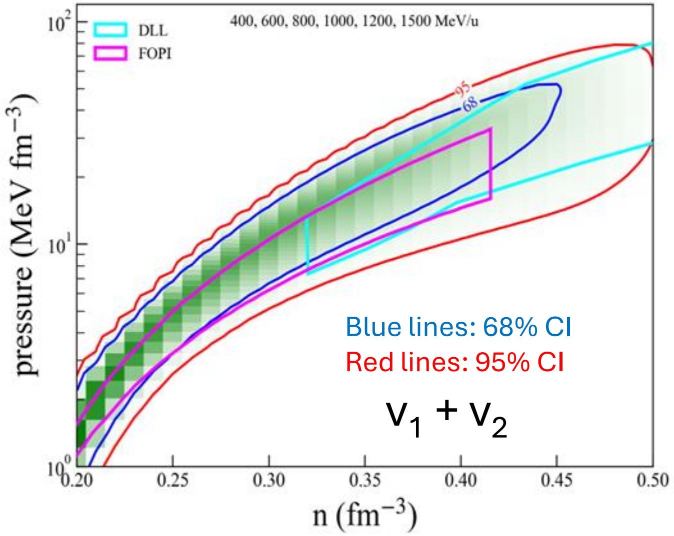
E = 400, 600, 800, 1000, 1200, 1500 AMeV

Using v_1+v_2 data:

$K_{sat} = 230^{+24}_{-31}\ MeV$

$Q_{sat} = -332^{+198}_{-198}\ MeV$

Using v_1 and v_2 data as a function of rapidity

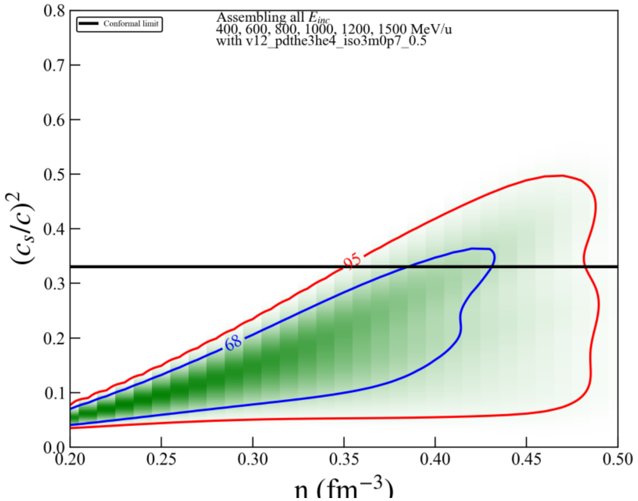


Results from previous analysis:

IQMD → $K_{sat} = 190^{+30}_{-30}\ MeV$ Using v_{2n} observable (p, d, t, 3He) 0.4 – 1.5 AGeV
Free cross-sections
A. Le Fèvre et al., NPA945(2016)112-133

UrQMD → $K_{sat} = 220^{+40}_{-40}\ MeV$ Using v_{2n} observable (p, d) 0.4 – 1.0 AGeV
In-medium cross-sections
Y. Wang et al., PLB-778(2018)207-212

dcQMD → $K_{sat} = 230^{+9}_{-11}\ MeV$ Using v_1, v_2 and stopping data, includes data at lower incident energies also
In-medium cross-sections,
Neutron-proton mass splitting
M.D. Cozma, arXiv:2407.16411v2

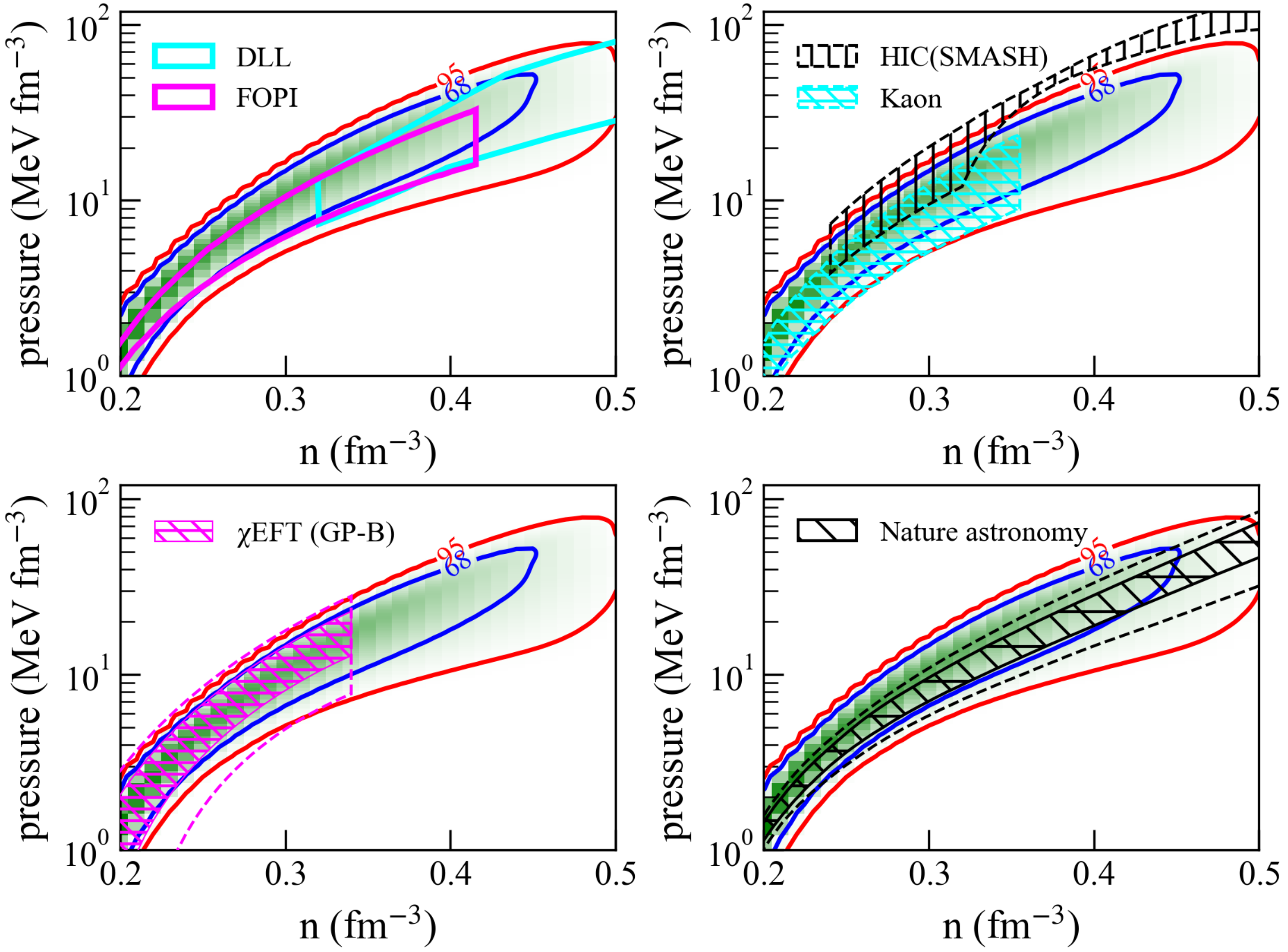


Note: All analysis used different set/sub-set of data

Observable v_{2n}:
$v_{2n} = |v_{20}| + |v_{22}|$
$v_2(y_0) = v_{20} + v_{22}y_0^2$: y_0 : -0.8 to 0.8

Using v_{2n} (but in Bayesian way) in present analysis: $K_{sat} = 242^{+24}_{-22}\ MeV$; $Q_{sat} = -512^{+144}_{-198}\ MeV$

Comparisons with other available results



DLL: Danielewicz, Lacey, Lynch, Science 298, 1592 (2002)
FOPI:Le Fevre, Leifels, Reisdorf, Aichelin, Hartnack, Nuclear Physics A 945, 112 (2016)

Kaon: C. Fuchs et al., Prog. Part. Nucl. Phys. 53, 113–124 (2004); Lynch et al., 2007

Sorensen and Koch, Phys. Rev. C 104, 034904 (2021)
Oliinychenko et al., Phys. Rev. C 108, 034908 (2023)

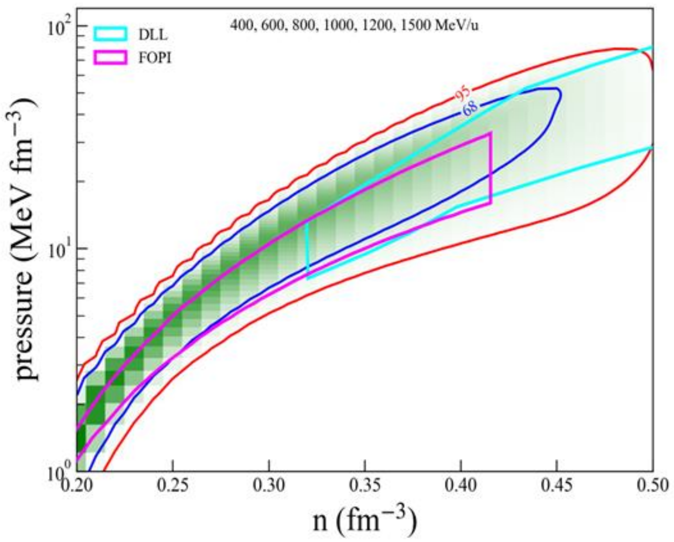
Drischler et al., PRL 125, 202702 (2020)
PRC 102, 054315 (2020)
PRL 122, 042501 (2019)

Tsang et al., Nature Astronomy 8, 328-336 (2024)

Blue lines: 68% CI
Red lines: 95% CI

Summary

- E_{sat} and n_0 are fixed using experimental studies.
- Limit our parameter range for K_{sat} using the Giant monopole resonance studies.
- Derived new more flexible equation of state for which we can vary high density part for fixed K_{sat} values
- We used v_1 and v_2 data for protons, deuterons, tritons, ^{3}He and alpha as a functic of rapidity in Bayesian framework (Constraints on K_{sat}, Q_{sat} and m*/m)



$$K_{sat} = 230^{+24}_{-31}\ MeV$$
$$Q_{sat} = -332^{+198}_{-198}\ MeV$$

- Model dependency need to be explored using this data
- Other factors such as Pauli blocking, in-medium cross-sections and initialization need to be explored

Work in collaboration with:

- Prof. Jerome Margueron, International Research Laboratory for Nuclear Physics and Nuclear Astrophysics East Lansing, Michigan, USA
- Prof. Pawel Danielewicz, FRIB, Michigan State University, East Lansing, Michigan, USA
- Prof. Betty Tsang, FRIB, Michigan State University, East Lansing, Michigan, USA
- Prof. William Lynch, FRIB, Michigan State University, East Lansing, Michigan, USA

Thanks to Transport Model Evaluation Project (TMEP) Collaborators for many discussions on the work

H. Wolter, D. Cozma, B. Tsang, Pawel Danielewicz, Y. Zhang, M. Colonna, Akira Ono, A. Le-Fevre, Zhen Zhang, Cheming Ko and others

I appreciate for your time and welcome any suggestions. Thank you!