Frozen and β-equilibrated f and p modes of cold neutron stars: metamodel predictions

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Non-radial oscillations

The different modes are classified by the **restoring force**

They can be excited in different scenarios: supernovae, merger, magnetar flares and glitches

Their evaluation requires to solve perturbation equations in general relativity

Various **quasi-universal relations** between static properties and the frequencies have been developed

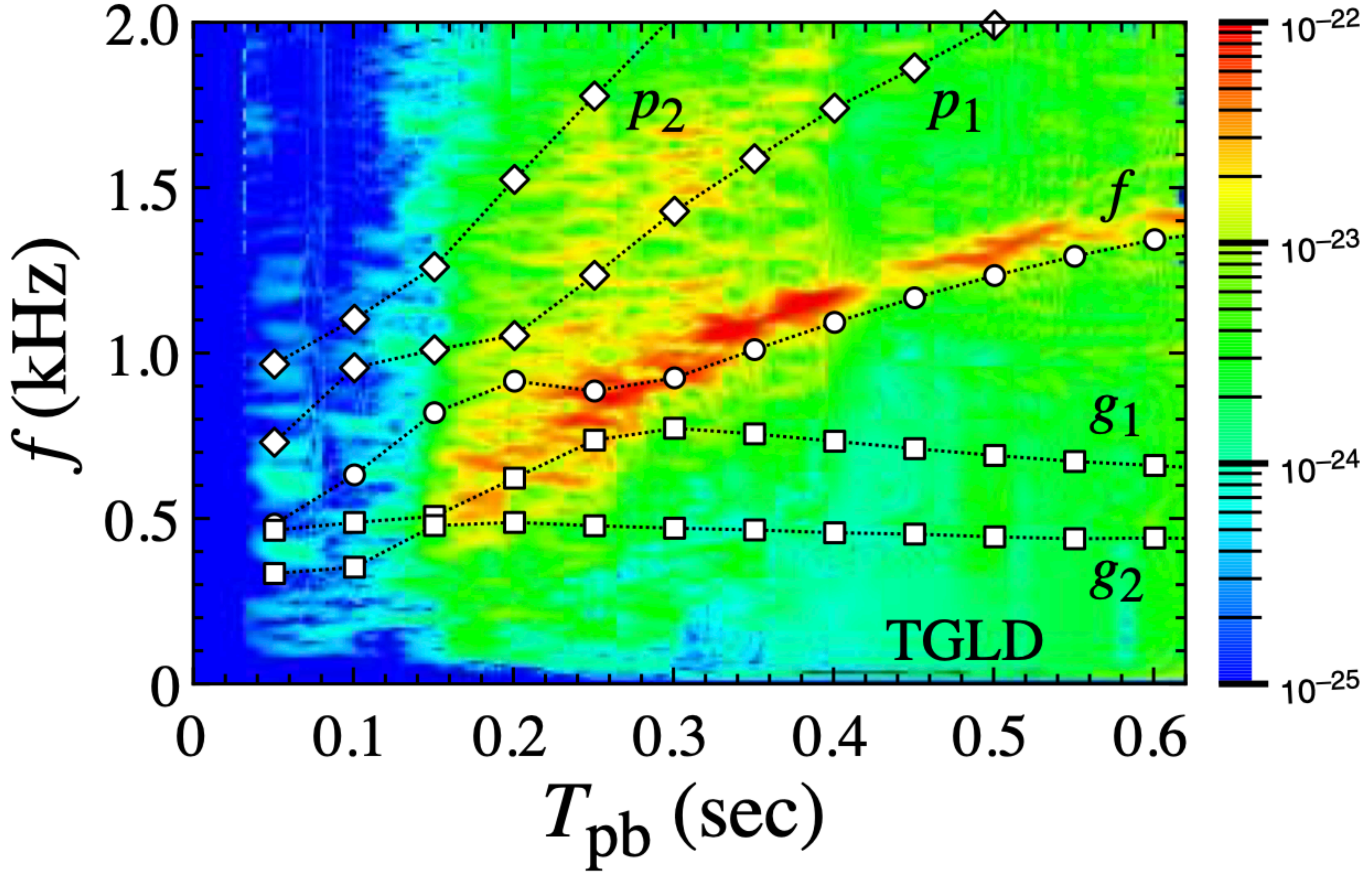
Goal of the work:

- Evaluate the **f-mode** (fundamental) and the first **p-mode** (pressure) frequencies
- Test the difference between **frozen** and **equilibrium** composition regime
- Obtain the **posterior distribution** of the frequencies with updated observations
- Test several **quasi-universal relations**

Astrophysical scenario

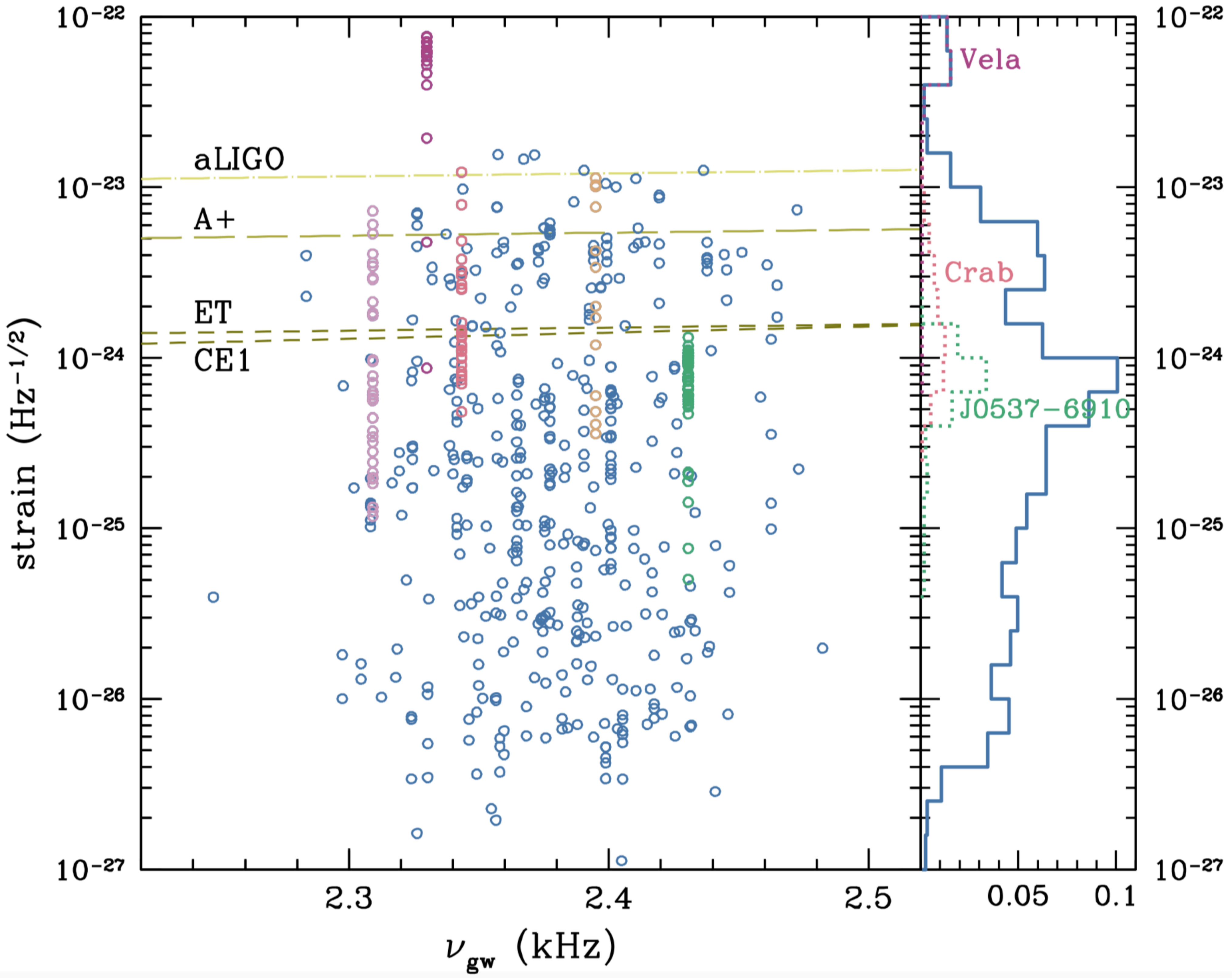
Comparison between **numerical simulations** and **eigenvalue obtained** frequencies for PNS

[Phys. Rev. D 104, 123009]



Strain prediction for the **f-mode excited by glitches**

[Phys. Rev. D 101, 103009]



Nucleonic Meta-Model

Presented in [PRC 97, 025805 (2018)]

Dense matter properties can't be derived from first principle or tested by terrestrial experiment

Many different approach have been developed for the accurate prediction of the **neutron stars equation of states**

The Meta-Model is a **flexible functional** able to reproduce existing effective nucleonic models and interpolate between them.

Features:

Based on a Taylor expansion around saturation density

Characterized by 13 parameters

It models directly the energy functional $e(n_n, n_p)$

It makes possible to explore the EOS space

Both astrophysical and nuclear constraints are accessible

Bayesian Inference

The aim is the **meta model's posterior** compatible with astrophysical constraints, nuclear physics experimental data, and current theoretical estimates from chiral field theory.

$$\mathscr{M} : \mathbf{X} \rightarrow \{\epsilon(n_B), P(n_B), \delta(n_B), v_\beta(n_B), v_{FR}(n_B), \ldots\}$$

$$\mathscr{L}(\mathbf{X}) = \prod_j \mathscr{L}_j(\mathbf{X}) = \prod_j p\left(D_j | \mathscr{M}(\mathbf{X})\right)$$

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Previous constraints taken into account

- AME2016 masses
- χ_{EFT} information on nuclear energy
- Maximum observed mass
- Tidal deformability from GW170817 event detected by Ligo/Virgo collaboration
- Mass and radius measurements from NICER

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Previous constraints taken into account

- AME2016 masses — **Prior**
- χ_{EFT} information on nuclear energy — **LD**
- Maximum observed mass
- Tidal deformability from GW170817 event detected by Ligo/Virgo collaboration
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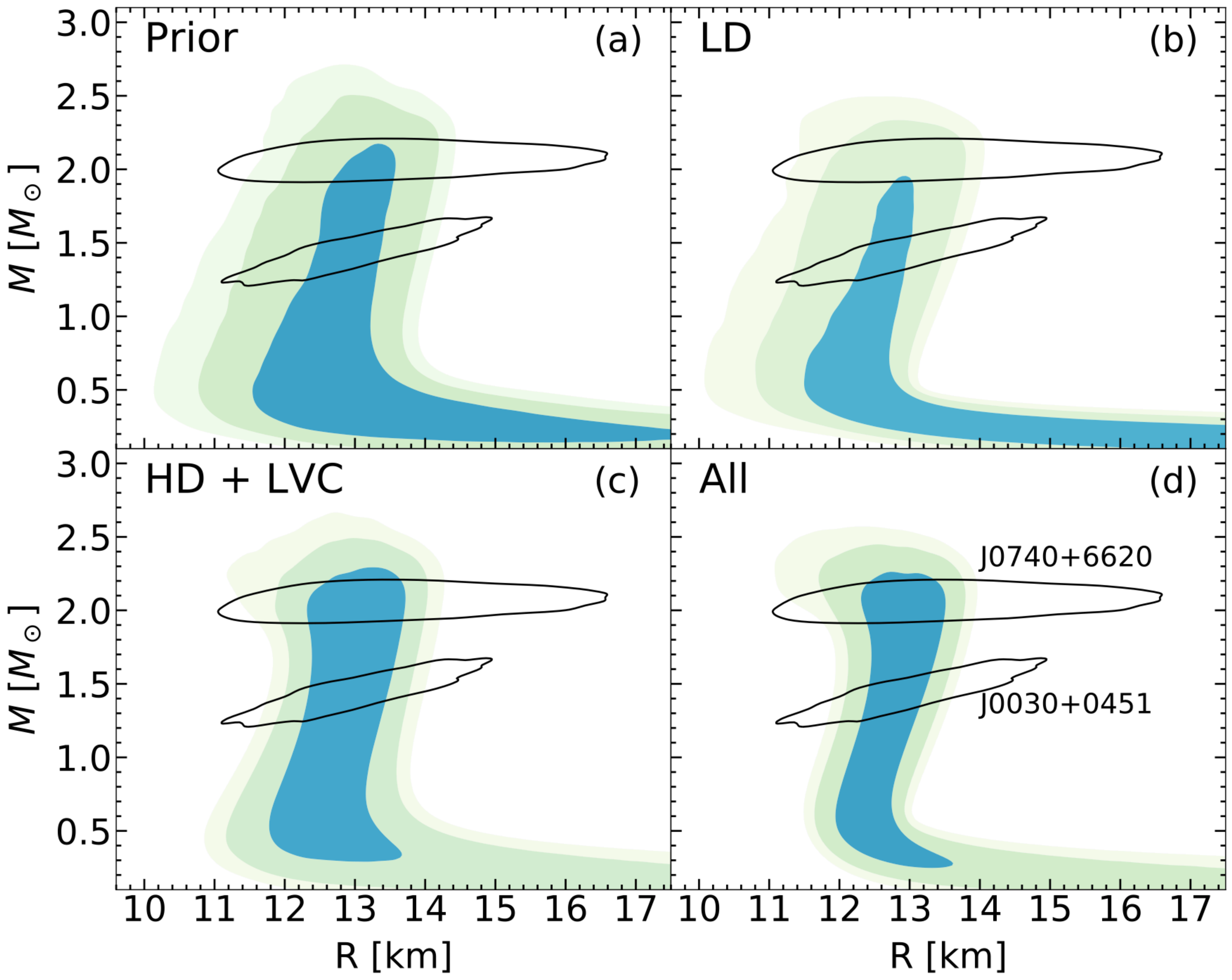
(Maximum observed mass and tidal deformability from GW170817: **HD + LVC**)

Dinh Thi et al [Universe 2021, 7(10), 373]



Updated Constraints

We implemented the **newest available data** in the Bayesian inference and **used a Metropolis algorithm** to improve the statistics

Informed prior using the χ_{EFT} band[1] of pure nuclear matter energy **sampled with a Metropolis-Hastings**

At this stage we have 10^9 models

[1] Huth et al, 2021, Phys. Rev. C, 103, 025803

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We extract 10^5 models that pass through the (updated) remaining filter:

- AME2020 masses
- Maximum observed mass **reached with causal speed of sounds**
- Tidal deformability from GW170817 event detected by Ligo/Virgo collaboration
- NICER+XMN M-R measurements of PSRJ0030, **PSRJ0347** and PSRJ0740

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Cowling approximations

For numerical efficiency we have decide to work within the Cowling approximations

It consists of fixing the background metric on which the fluid will oscillate

The frequencies are real because there's no dissipation due to GW emissions or other mechanisms

The equations takes the form

$$W' = \left(\frac{dP}{d\epsilon}\right)^{-1} \left[\omega^2 r^2 V + \Phi' W\right] - l(l+1)e^{\Lambda} V$$

$$V' = 2\Phi' V - e^{\Lambda}\frac{W}{r^2}$$

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By imposing the **boundary condition** $\Delta P = 0$ at the stellar surface it is obtained

$$\omega^2 R^2 e^{\Lambda - 2\Phi} V + \Phi' W = 0$$

This let us obtain the frequency as the omega for which this BC is satisfied

Frozen vs β-equilibrated regime

We tested the impact of assuming two
opposite idealized limits on the frequency

equilibrium regime

Each perturbed fluid element has time to re-equilibrate the composition

$$v_\beta^2 = \frac{dP}{d\epsilon} = \frac{dP(n,\delta(n))/dn}{d\epsilon(n,\delta(n))/dn}$$

Used in the studies that assumes agnostic barotropic or β-equilibrated EoS

frozen regime

Each perturbed fluid element remain at fixed composition

$$v_{FR}^2 = \left.\frac{\partial P}{\partial \epsilon}\right|_\delta = \frac{\partial P(n,\delta(n))/\partial n}{\partial \epsilon(n,\delta(n))/\partial n}$$

This is the maximal speed of information in a reacting mixture[1]:
For stable and causal matter

$$0 < v_\beta < v_{FR} < 1$$

[1] Camelio G. et al Phys. Rev. D, 107, 103031

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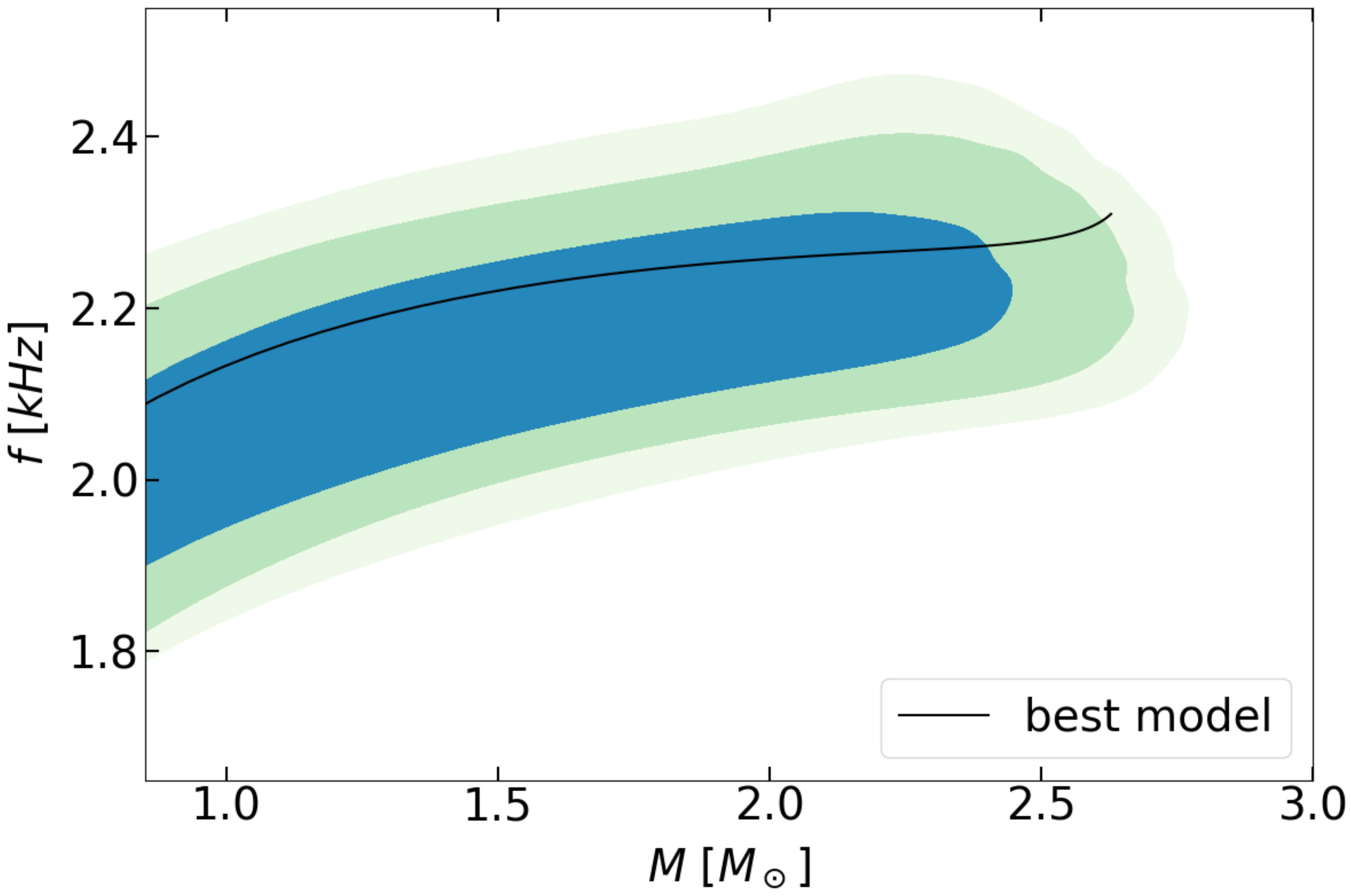
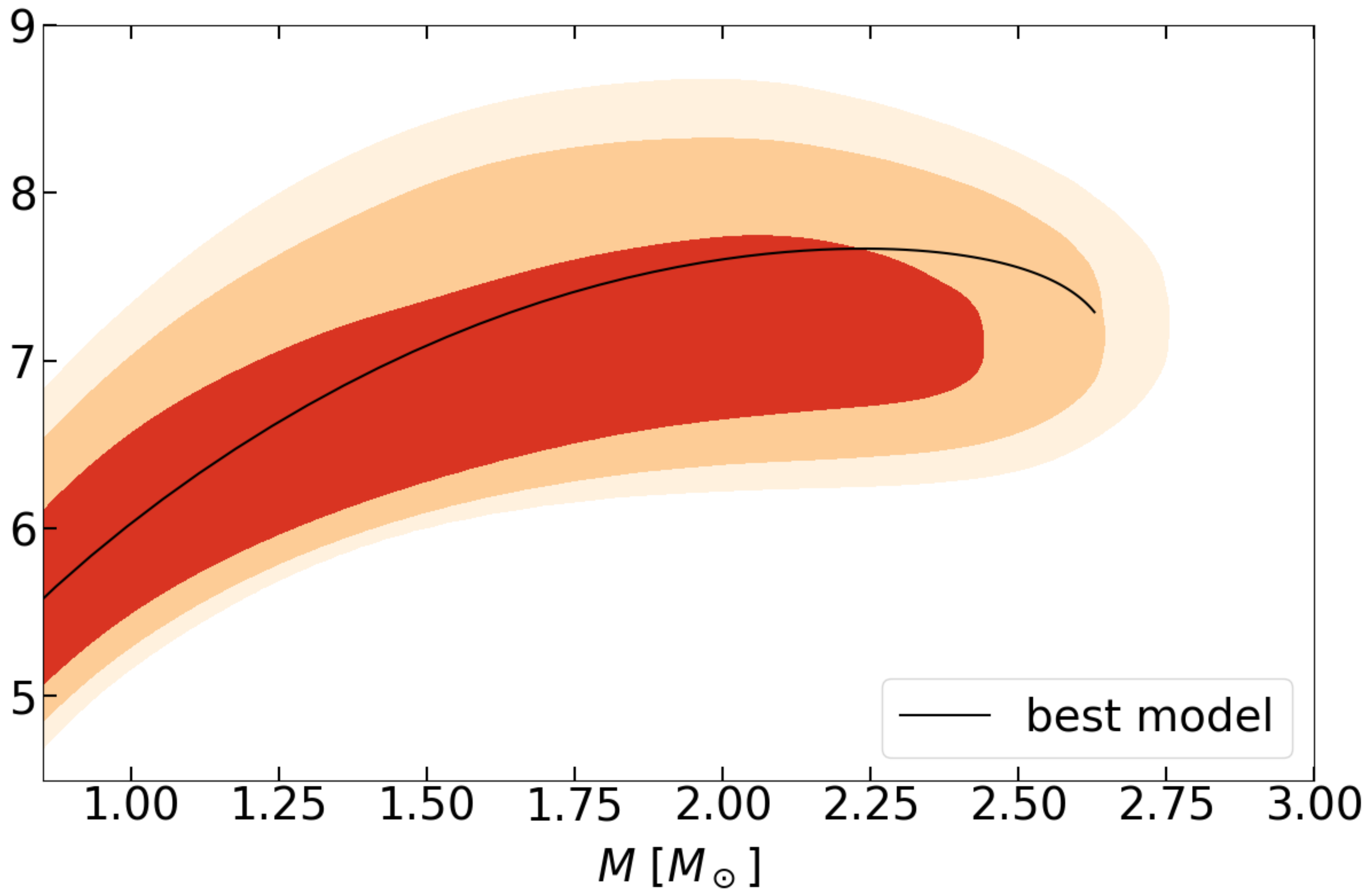
Cowling Frequencies Posteriors

$f - mode$ [https://doi.org/10.48550/arXiv.2410.08008] $p_1 - mode$



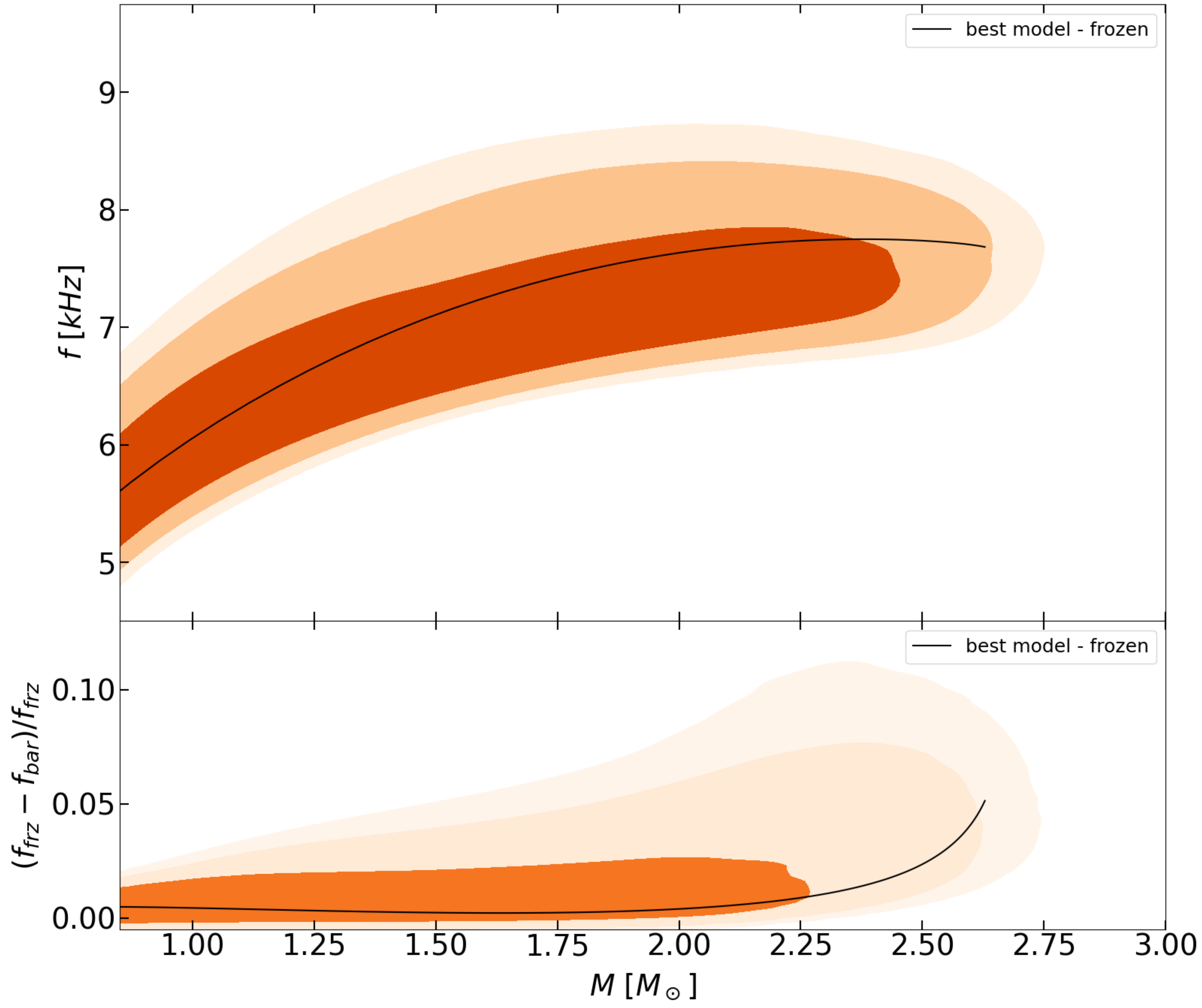


Frozen vs β-equilibrated frequencies

f-**mode** differences < 0.5%

p_1-**mode**: difference < 5% except for $M \rightarrow M_{max}$ where they reach up to 10%

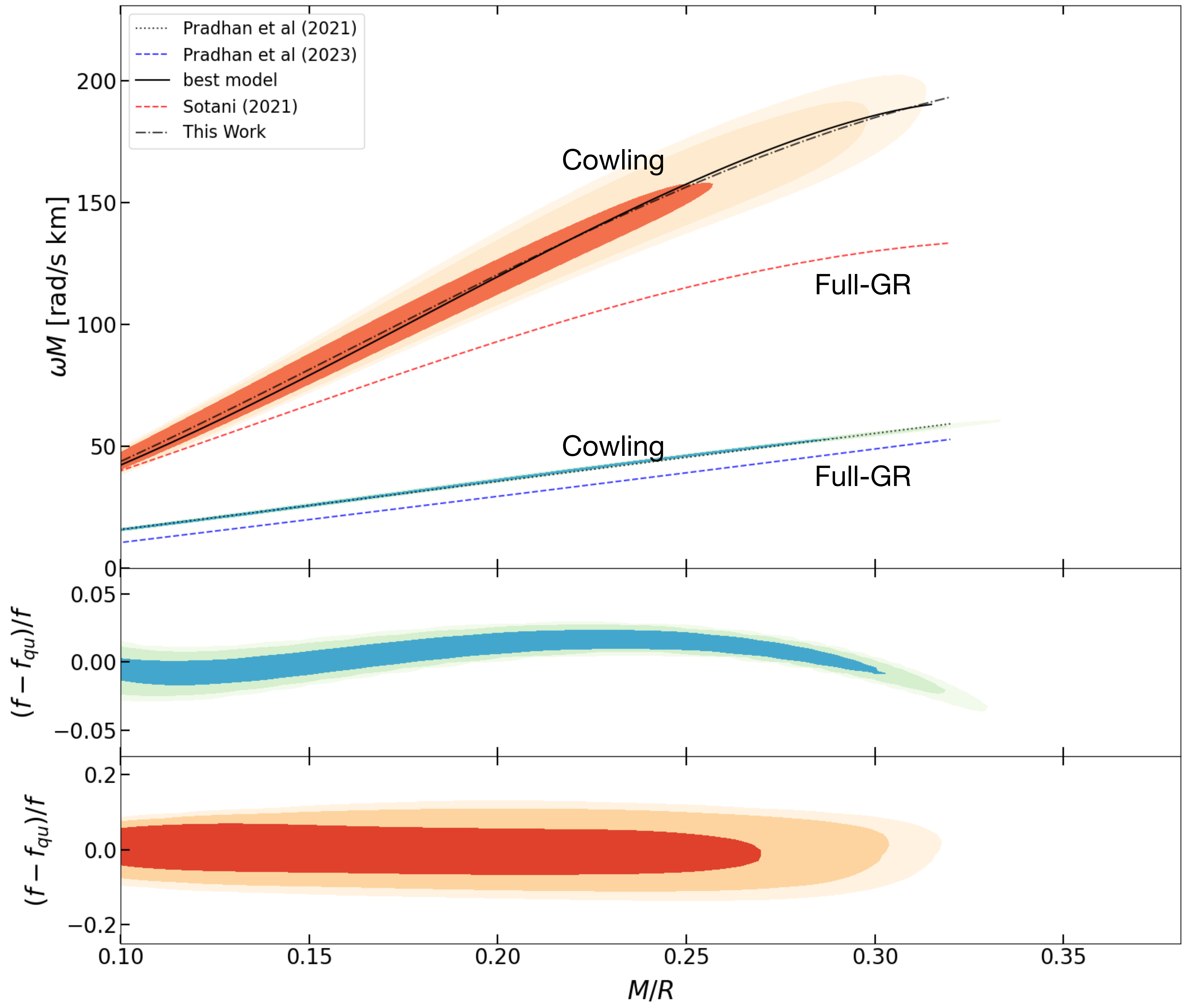
Since the differences are small, if v_{FR} is not available, it is possible to use the barotropic frequencies with very little errors.



Quasi-universal relation test

$$\omega M = a_0 + a_1 \frac{M}{R} + a_2 \left(\frac{M}{R}\right)^2 + a_3 \left(\frac{M}{R}\right)^3$$

[Phys. Rev. C 103, 035810] [Phys. Rev. D 103, 123015]



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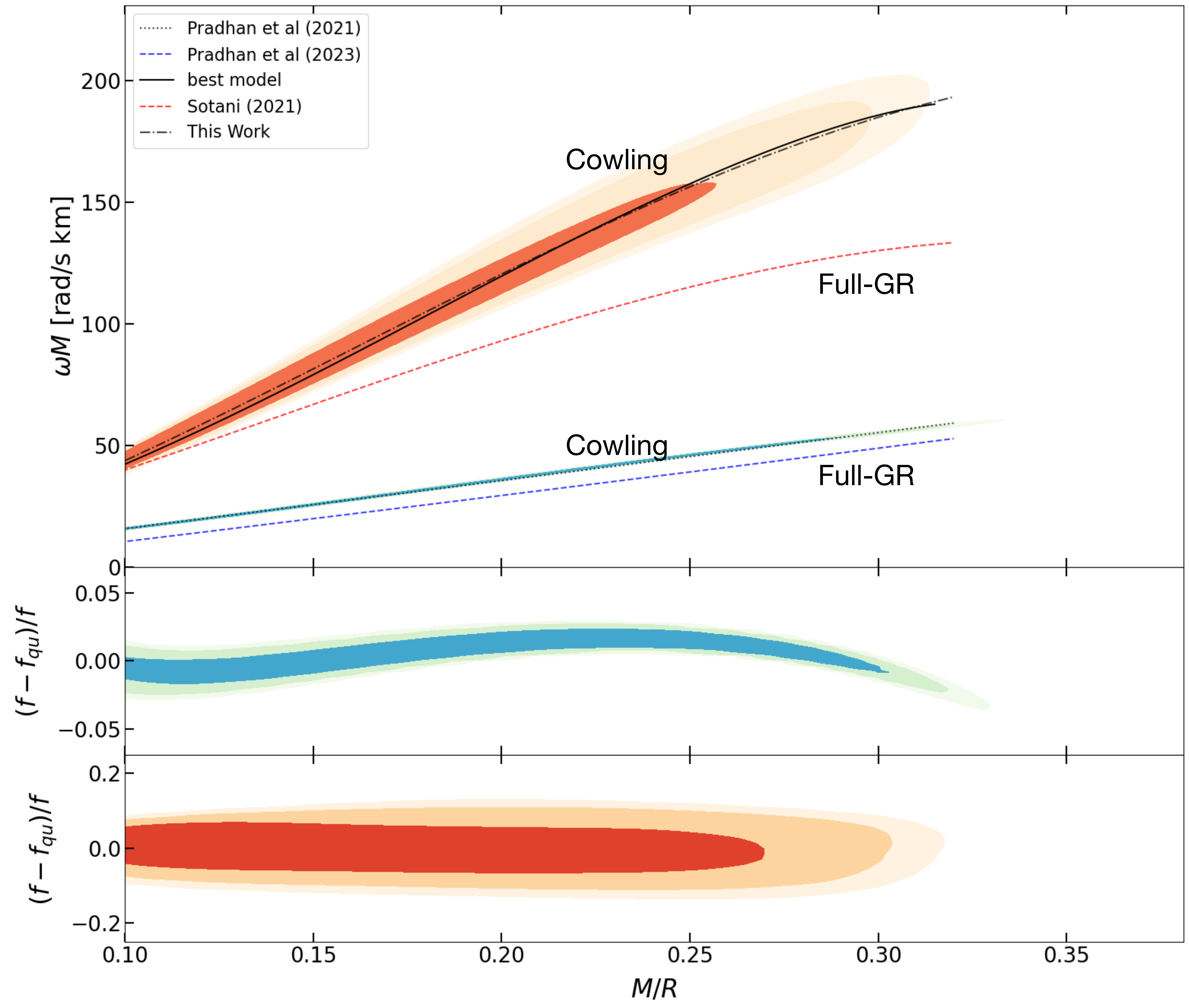
f-mode
accuracy > 95%

p_1-mode
accuracy > 90%



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f-mode
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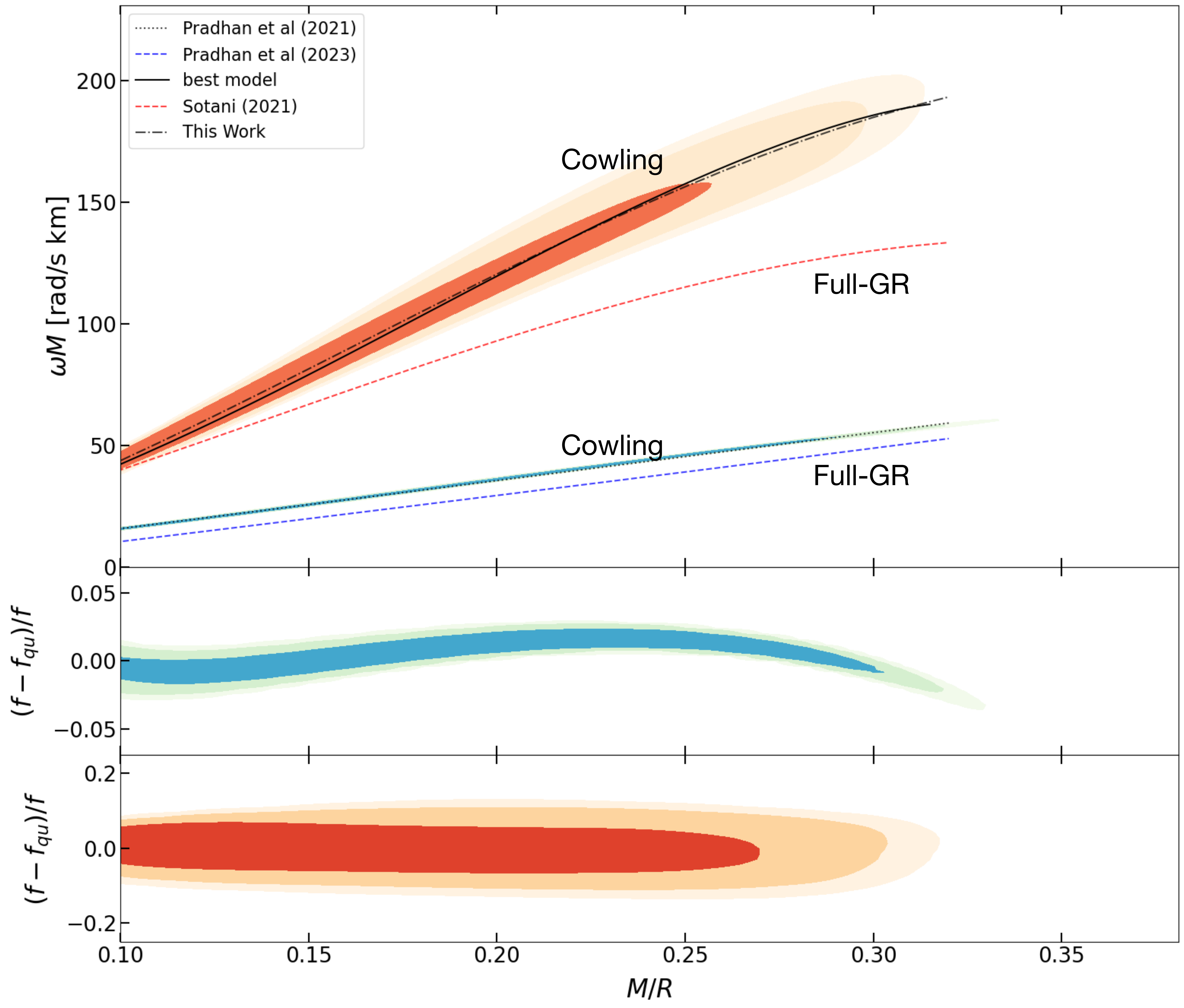
Every different set of EoS gave compatible fit

It is a **Quasi-universal** relation

It is possible to obtain the frequencies by evaluating only the M-R relation

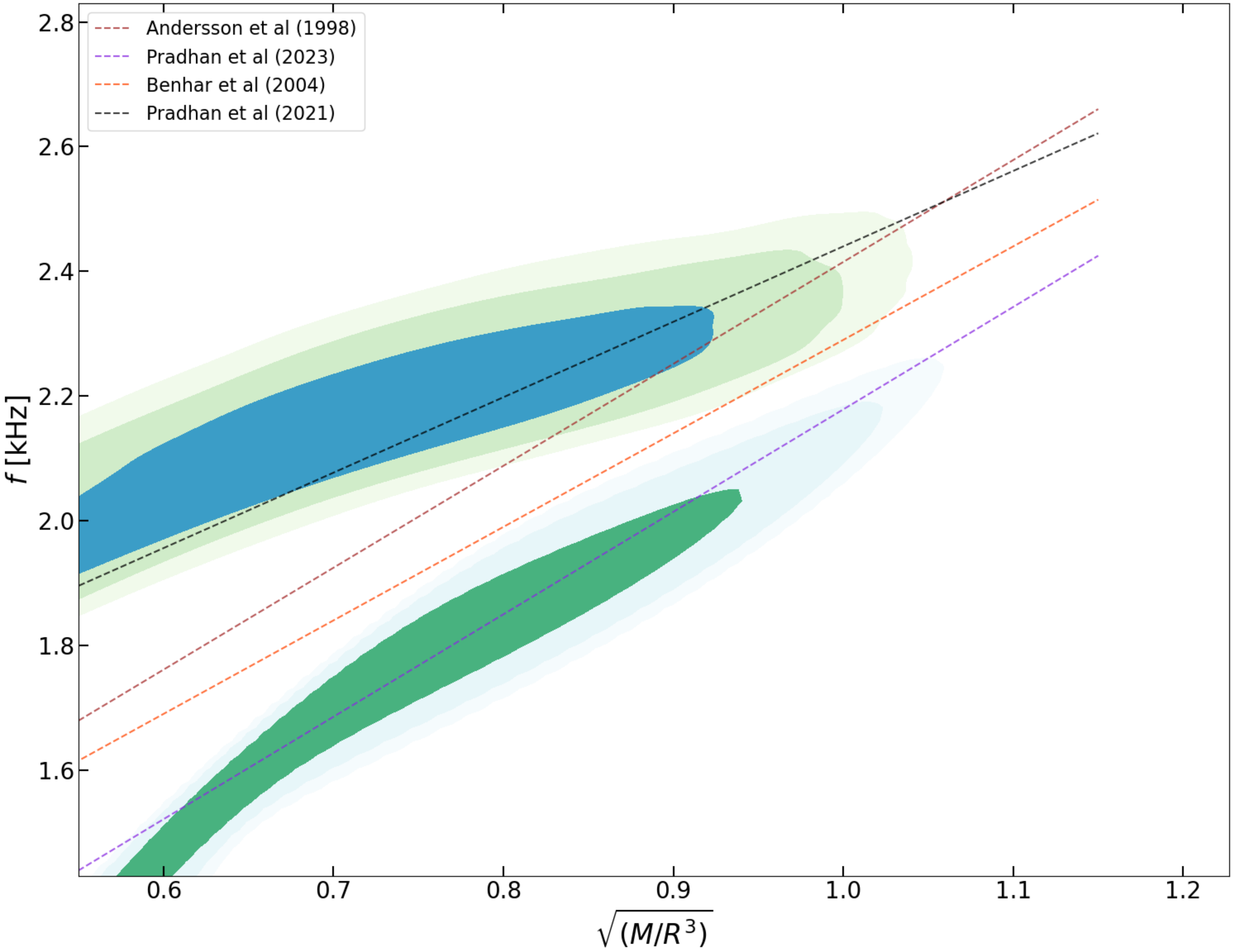


Quasi-universal relation test

$$f = a + b\sqrt{\frac{\bar{M}}{\bar{R}^3}}$$

[Phys. Rev. C 103, 035810]

Proposed for the *f*-mode



Quasi-universal relation test

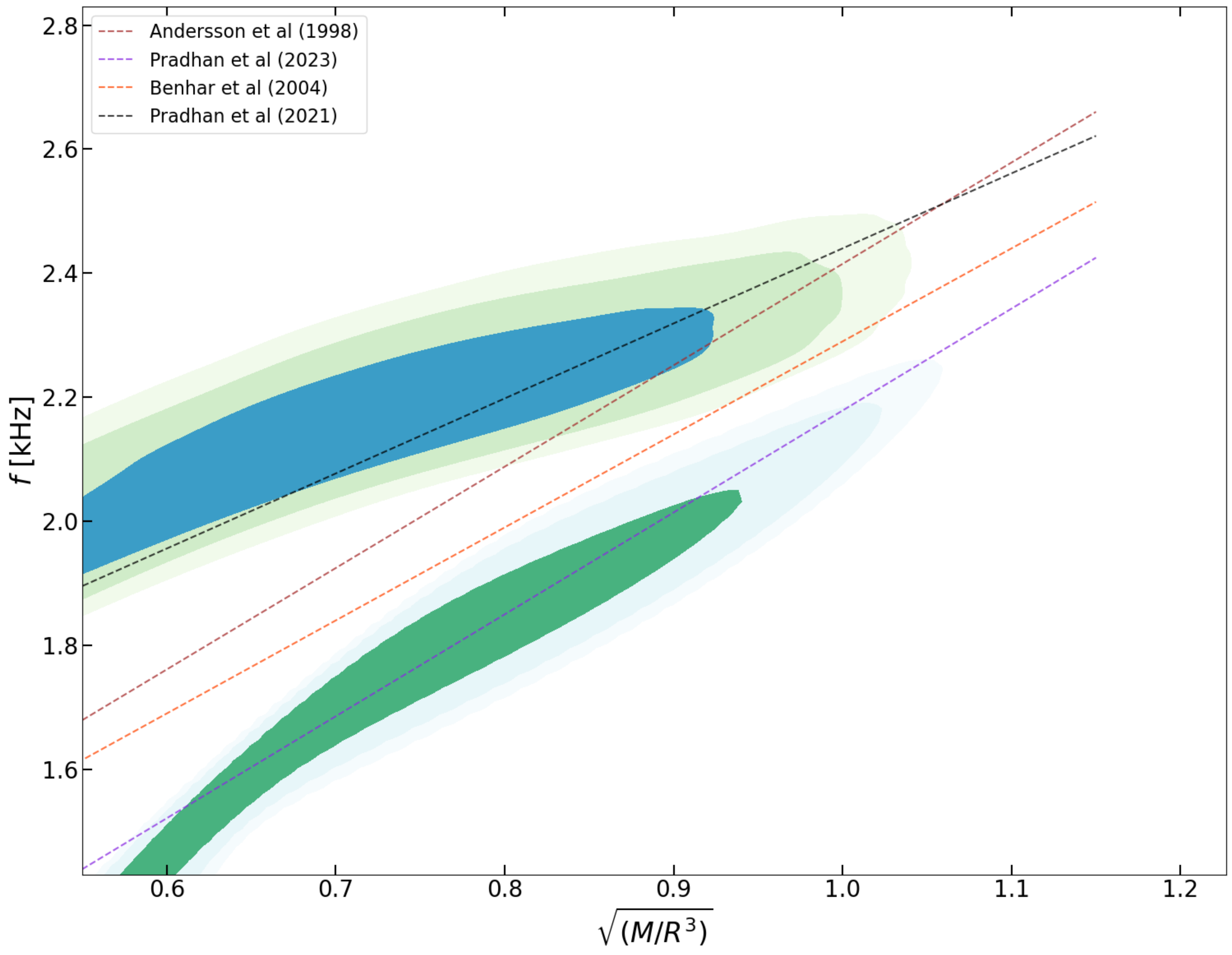
$$f = a + b\sqrt{\frac{\bar{M}}{\bar{R}^3}}$$

[Phys. Rev. C 103, 035810]

Proposed for the *f*-mode

Different set of EoS give
different fit

It is **not** a **Quasi-universal**
relation



Synthetic Full-GR Frequencies

To obtain a synthetic Full-GR prediction we use the prescription:

$$f_{GR}(M) = \frac{1 + \Delta(M)}{2\pi M} U_{GR}(M)$$

$$\Delta(M) = \frac{M\,\omega_C(M) - U_C(M)}{U_C(M)}$$

where $U_{GR,C}$ is the quasi-universal obtained in full-GR or in cowling

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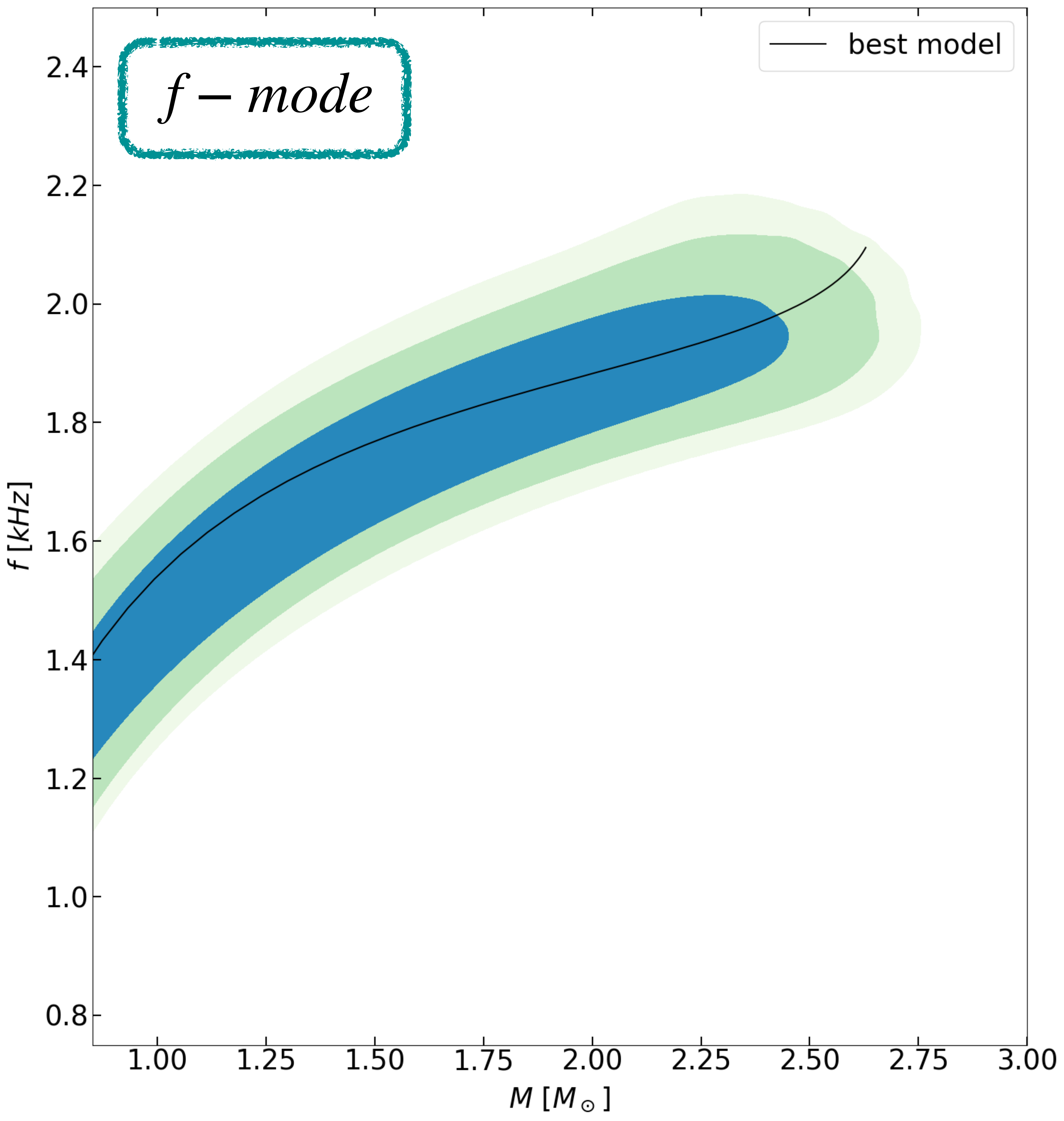
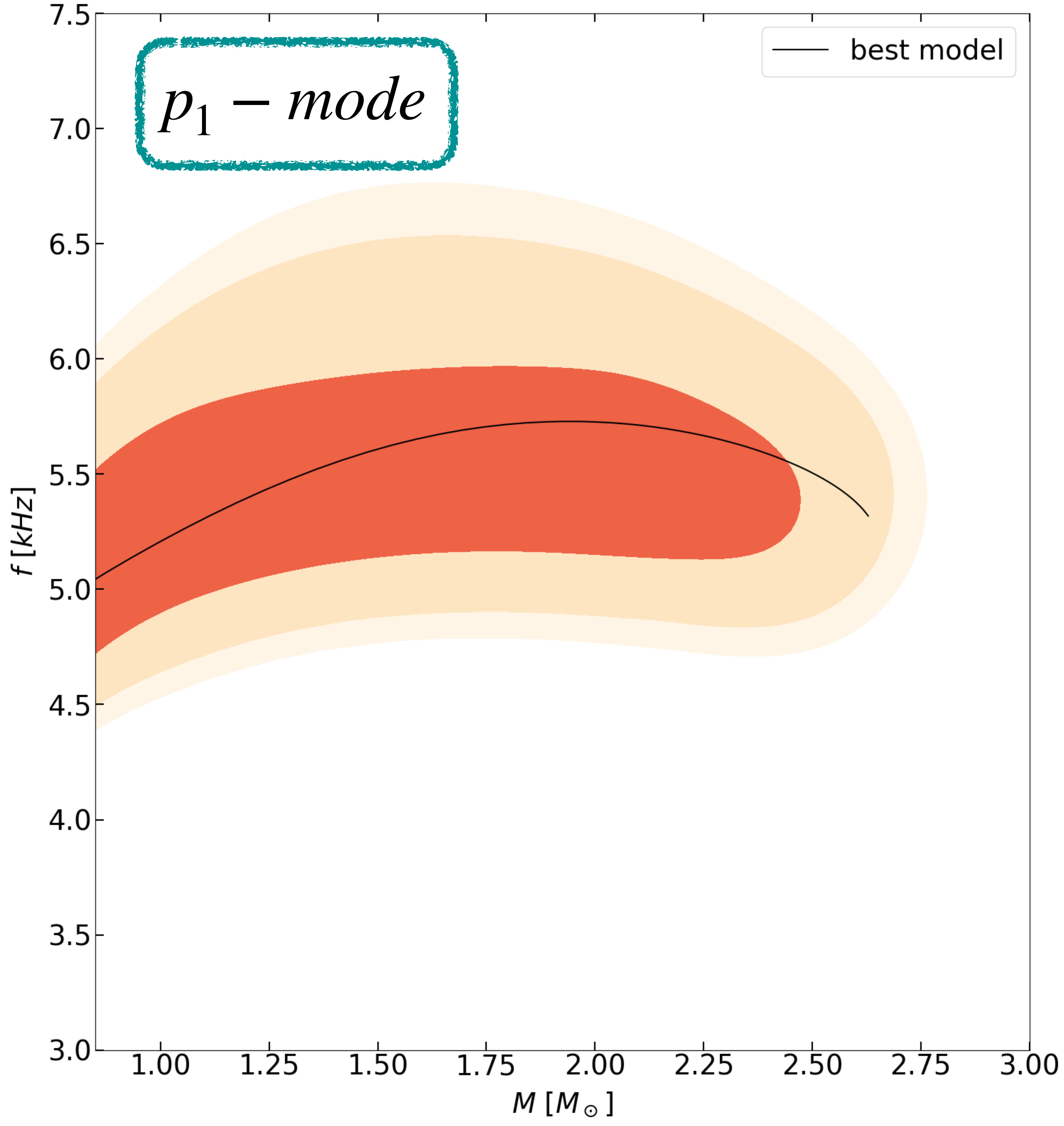
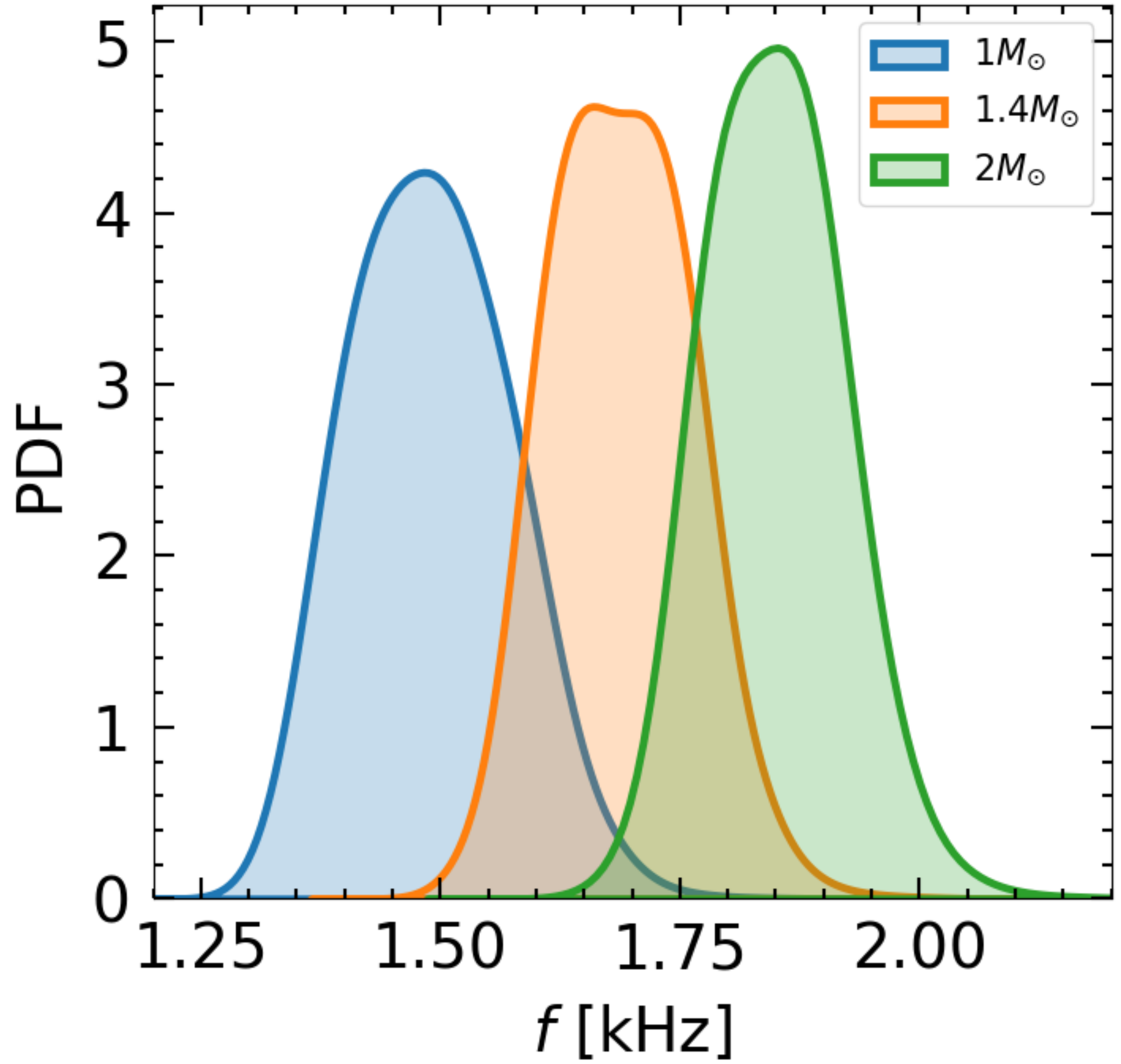




Synthetic Full-GR Frequencies

$f - mode$

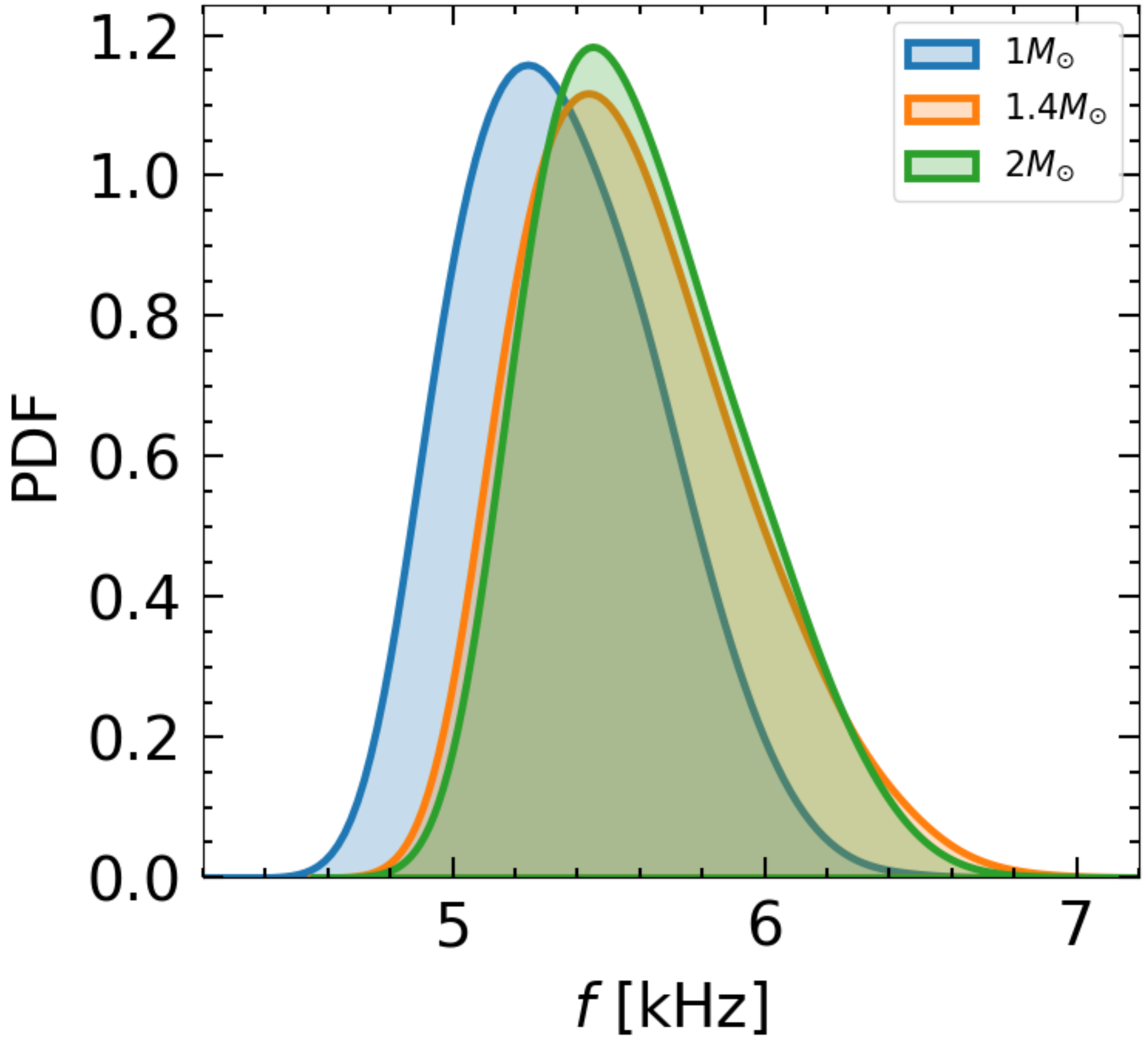


The split is larger for the f-mode than the p_1-mode

It is possible to constrain the mass from an f-mode observation

This information is lost for the p_1-mode

$p_1 - mode$



Conclusion

[https://doi.org/10.48550/arXiv.2410.08008]

f and p_1 modes **do not depend** much on the "barotropic" or "frozen" character of the speed of sound. → Studies that assume phenomenological models for the barotropic EOS are accurate within a few percent.

The compactness quasi-universal accuracy is greater than 90% → It can be used to obtain the frequencies without solving the perturbation equations: **Useful for bayesian studies**

We obtained a prediction for the full-GR frequencies → It is possible to put a **constrain on the mass of the star** from an **f-mode observation** but not from a p_1-mode one