Superfluid fraction in the inner crust of neutron stars

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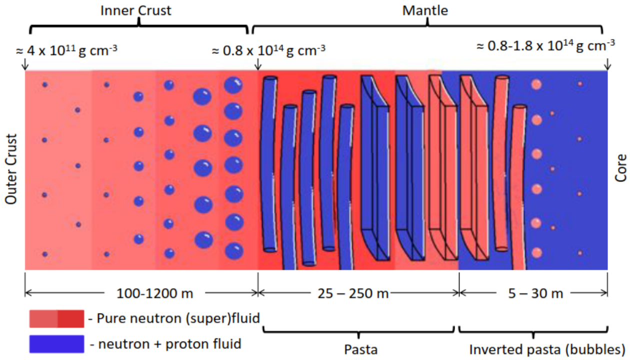
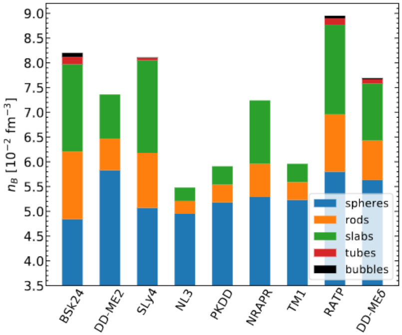
Outline

- Introduction: inner crust of neutron stars and its superfluid fraction
- Formalism: Hartree-Fock-Bogoliubov and Andreev-Bashkin two fluid model
- Results: a whole lasagna and some spaghetti

Inner crust of neutron stars...

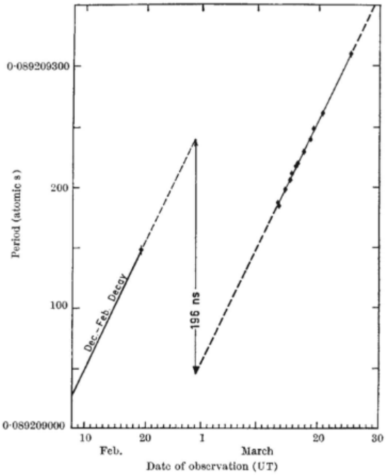




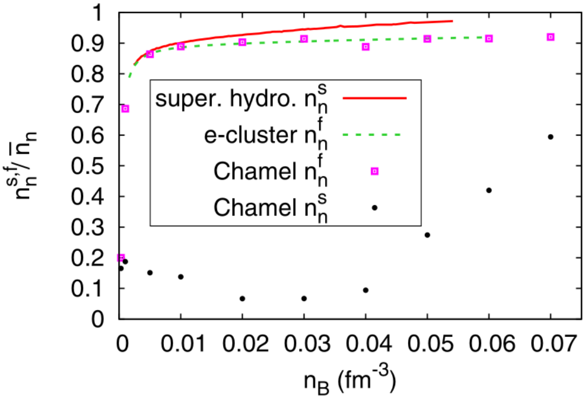
W.G. Newton et al,
Sym.En.,In.Crust,Gl.Mod. (2011)

H. Dinh Thi et al,
A&A 654, A114 (2021)

...and its superfluid fraction



Radhakrishnan & Manchester,
Nature 222, 228-229 (1969)



Martin & Urban,
Phys. Rev. C 94, 065801 (2016)

Hartree-Fock-Bogoliubov in the pasta phases

$$\begin{pmatrix} h-\mu & -\Delta \\ -\Delta^\dagger & -\bar{h}+\mu \end{pmatrix} \begin{pmatrix} U_\alpha^\star \\ -V_\alpha \end{pmatrix} = E_\alpha \begin{pmatrix} U_\alpha^\star \\ -V_\alpha \end{pmatrix}$$

in momentum space only integer indices, diagonal in the Bloch and parallel momenta

$$h_{kk'} = \left(\frac{\hbar^2}{2m^*}\right)_{kk'} k \cdot k' + U_{kk'} - \hbar\, k \cdot v\, \delta_{kk'} \tag{1}$$

$$U_{kk'} = -\sum_{pp'} V_{kpk'p'} \rho_{p'p} \rightarrow \text{Skyrme potential} \tag{2}$$

$$\Delta_{kk'} = -\sum_{pp'} V_{kk'p'p} \kappa_{p'p} \rightarrow \text{separable interaction} \tag{3}$$

Andreev-Bashkin formalism

Our setup gets us access to densities, currents and other relevant quantities.
Then with the formalism due to Andreev and Bashkin we can compute the superfluid fraction

$$\vec{\rho}_n = (\rho_n - \rho_S)\vec{v} + \rho_S \vec{V}_n \quad ; \quad \vec{\rho}_p = \rho_p \vec{v}$$

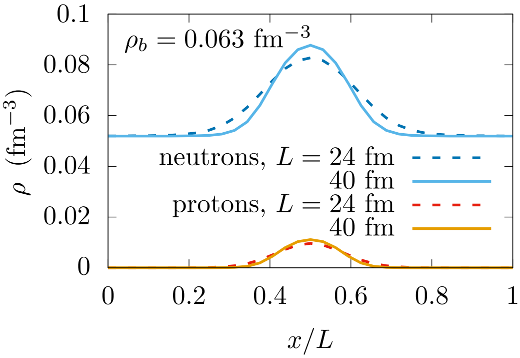
This relation has to be understood in an average sense ($\phi =$ phase of the gap: $\Delta = |\Delta| e^{i\phi}$)

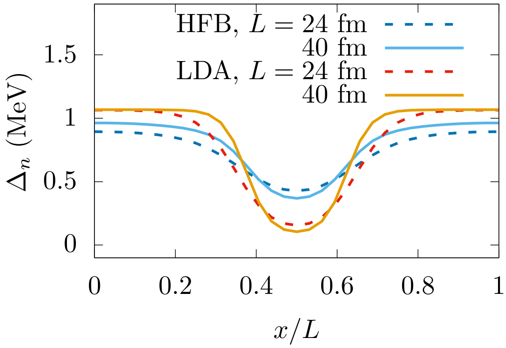
$$\vec{V}_n = \int_V \frac{d^3x}{V} \frac{\hbar}{2m} \vec{\nabla}\phi$$

$$(V_n)_x = \iint \frac{dydz}{V} \frac{\hbar}{2m} \Big(\phi(L_x, y, z) - \phi(0, y, z) \Big)$$

Since our quantities are periodic the average superfluid velocity is zero, thus we are working in the reference frame in which the superfluid component carries no momentum

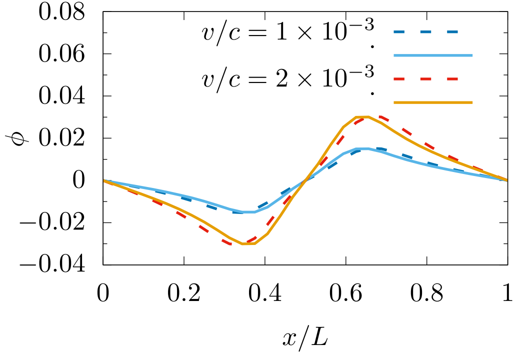
Lasagna statics

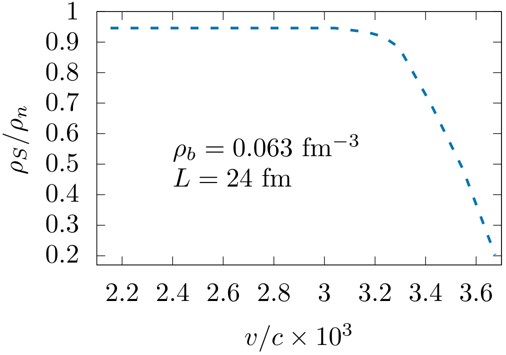




Almirante & Urban, Phys. Rev. C 109, 045805 (2024)

Lasagna dynamics

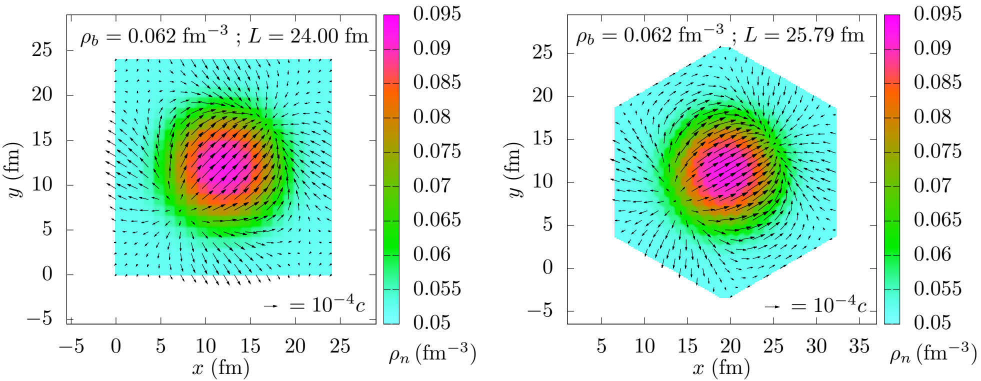




Almirante & Urban, Phys. Rev. C 109, 045805 (2024)

Spaghetti

$$\rho_b = 0.062 \text{ fm}^{-3} \quad ; \quad \rho_S/\rho_n = 0.94$$



Almirante & Urban, arXiv:2407.14574 (2024)

Results summary and comparison

Spaghetti

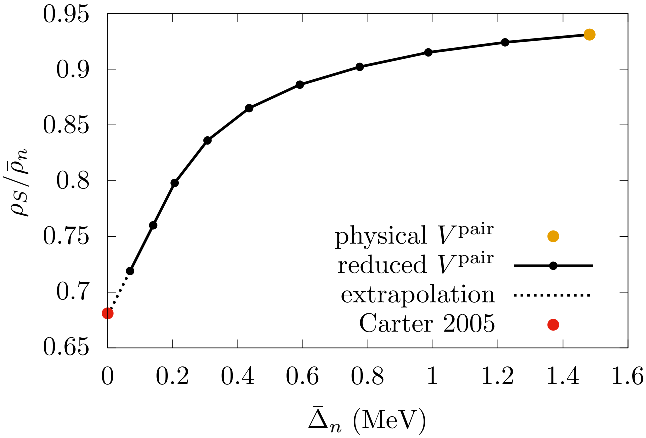
μ_n (MeV)	L (fm)	ρ_b (fm^{-3})	ρ_S/ρ_n (our%)	ρ_S/ρ_n (Chamel%)
12	24	0.0619	94.5	75
	28	0.0617	95.7	
12.5	24	0.0670	95.4	82
	28	0.0668	96.7	

Lasagna

μ_n (MeV)	L (fm)	ρ_b (fm^{-3})	ρ_S/ρ_n (our%)	ρ_S/ρ_n (Chamel%)
13	20	0.0723	96.3	93
	24	0.0720	96.2	
13.5	20	0.0768	97.2	94
	24	0.0766	97.1	

Bands effects VS pairing gap

Normal band theory should be valid in the weak-coupling limit (pairing gap $\ll$ Fermi energy)



$\bar{\rho}_n = 0.059$ fm^{-3} ; $L = 27.17$ fm

Summary and Outlook

- 1D: $\rho_S/\bar{\rho}_n \simeq 97\%$ No surprise.
- 2D: $\rho_S/\bar{\rho}_n \simeq 95\%$ News! Normal band theory underestimates the superfluid fraction
- 3D: $\rho_S/\bar{\rho}_n \simeq ???\%$ Crystal is the most extended layer thus this really matters

Thanks for your attention!