Equation of state and pairing in dilute neutron matter

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Outline

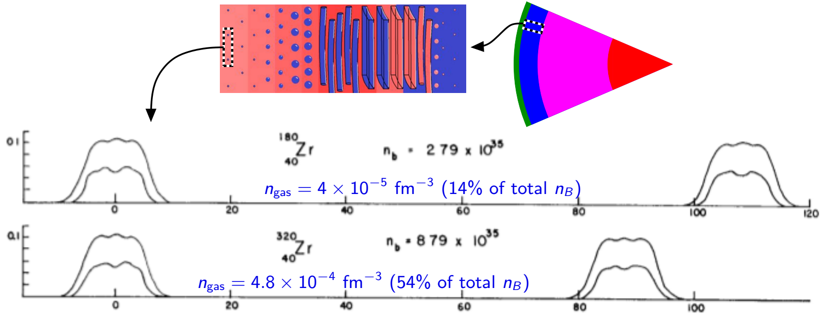
- Dilute neutron matter
- Low-momentum interactions
- Hartree-Fock-Bogoliubov with perturbative corrections
- Effect of induced 3-body force
- Pairing with screening corrections
- Summary and outlook

References:

Screening with $V_{\text{low-}k}$: S. Ramanan & MU, PRC 98, 024314 (2018)
Screening with RPA: MU & S. Ramanan, PRC 101, 035803 (2020)
Pairing in neutron matter: S. Ramanan & MU, EPJ ST 230, 567 (2021)
BMBPT in cold atoms: MU & S. Ramanan, PRA 103, 063306 (2021)
EoS with $V_{\text{low-}k}$: V. Palaniappan, S. Ramanan & MU, PRC 107, 025804 (2023)
effect of EoS on crust composition: S.K. Gupta & MU, PRC 109, 045806 (2024)
SRG and induced 3-body force: V. Palaniappan, S. Ramanan & MU, in preparation

What is “dilute” neutron matter?

- Upper layers of the inner crust (close to neutron-drip density $\sim 2.5 \times 10^{-4}$ fm^{-3})



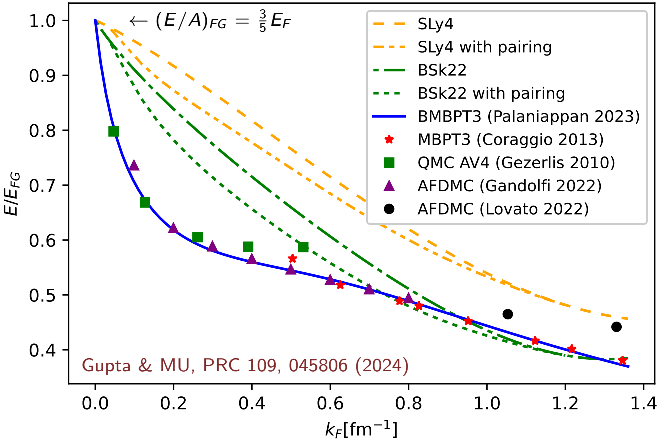
[Negele and Vautherin, NPA 207 (1973); similar results by Baldo et al., PRC 76 (2007)]

- In spite of its “low” density (still $\rho \gtrsim 10^{11}$ g/cm^3), the neutron gas is relevant because it occupies a much larger volume than the clusters

- Deeper in the crust: n_{gas} increases up to $\sim n_0/2 = 0.08$ fm^{-3}

$\to$ Fermi momentum $k_F = (3\pi^2 n)^{1/3} \sim 0.1 \ldots 1.3$ fm^{-1}

Ab-initio EoS vs. phenomenological energy functionals



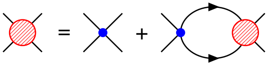
- For $n \to 0$: $\frac{E}{E_{FG}} = 1 - \frac{10}{9\pi} k_F a + \cdots$ ($a \approx -18$ fm $=$ s-wave scattering length)
- Finite range ($r_{\mathrm{eff}} \approx 2.5$ fm) and higher partial waves are also important
- Above $k_F \sim 1\ \mathrm{fm}^{-3}$: effects of 3-body force (two pion exchange)

Low-momentum interactions

Example: contact interaction $V(k,k') = g$

- Scattering length a for coupling constant $g < 0$ and cutoff Λ $\quad (\epsilon_k = \frac{k^2}{2m})$

$$\frac{4\pi a}{m} = g + g \int^{\Lambda} \frac{d^3k}{(2\pi)^3} \frac{1}{-2\epsilon_k} \frac{4\pi a}{m}$$



$\longrightarrow$ Relationship between g and Λ: $\quad \dfrac{1}{g} = \dfrac{m}{4\pi a} - \dfrac{m\Lambda}{2\pi^2}$

$V_{\mathrm{low}-k}$ interactions

- Matrix elements $V(k,k') = 0$ for k or $k' > \Lambda$
- Not only a, but all phase shifts $\delta(k)$ for $k < \Lambda$ are independent of Λ

Similarity renormalization group (SRG)

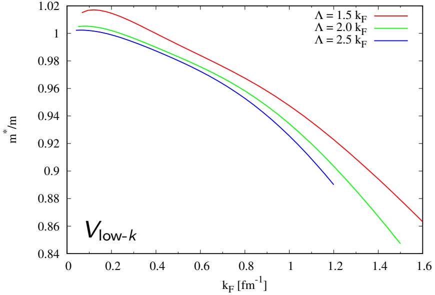
- Unitary transformation: phase shifts unchanged at all energies
- For $k, k' \gg \Lambda$: only diagonal matrix elements survive $\longrightarrow$ decoupling
- In principle, one can compute "induced" 3- (and higher-) body forces that are generated by the transformation when decreasing the cutoff

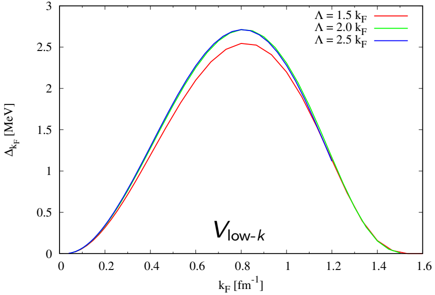
Hartree-Fock-Bogoliubov (HFB)

- Hard core of "realistic" potentials requires resummations, and nuclei are not bound in HF(B) approximation
- Soft interactions ($V_{\text{low-}k}$, SRG) much better suited for perturbative methods
- HFB with perturbative corrections can give good results for open-shell nuclei [e.g., Tichai et al. 2019]
- Scale Λ with k_F effectively resumming ladders in U_k
- gap Δ_k and mean field U_k:

$$\Delta_k = -\int \frac{d^3p}{(2\pi)^3}\, V(k,p)\, u_p v_p$$

$$U_k = \int \frac{d^3p}{(2\pi)^3} V\left(\frac{\mathbf{p}-\mathbf{k}}{2}, \frac{\mathbf{p}-\mathbf{k}}{2}\right) v_p^2$$





Bogoliubov Many-Body Perturbation Theory (BMBPT)

- Express $\hat{K} = \hat{H} - \mu\hat{N}$ in terms of quasiparticle operators

$$\beta_{\mathbf{k}\uparrow} = u_k\, a_{\mathbf{k}\uparrow} - v_k\, a^\dagger_{-\mathbf{k}\downarrow}\,, \qquad \beta_{\mathbf{k}\downarrow} = u_k\, a_{\mathbf{k}\downarrow} + v_k\, a^\dagger_{-\mathbf{k}\uparrow}$$

$$\hat{K} = \mathcal{E}_{\mathrm{HFB}} + \sum_{\mathbf{k}\sigma} E_k\, \beta^\dagger_{\mathbf{k}\sigma}\beta_{\mathbf{k}\sigma} + :\hat{V}:$$

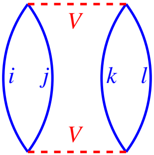
$$:\hat{V}: = V_{04}\,\beta\beta\beta\beta + V_{13}\,\beta^\dagger\beta\beta\beta + V_{22}\,\beta^\dagger\beta^\dagger\beta\beta + V_{31}\,\beta^\dagger\beta^\dagger\beta^\dagger\beta + V_{40}\,\beta^\dagger\beta^\dagger\beta^\dagger\beta^\dagger$$

- BMBPT: treat $:\hat{V}:$ as a perturbation

- Example: leading correction to ground-state energy is 2nd order

$$\mathcal{E}_2 = -\frac{1}{4!}\sum_{ijkl}\frac{|\langle ijkl|\hat{V}_{40}|\mathrm{HFB}\rangle|^2}{E_i + E_j + E_k + E_l}$$

with $|ijkl\rangle = \beta^\dagger_i\beta^\dagger_j\beta^\dagger_k\beta^\dagger_l|\mathrm{HFB}\rangle$ (4-quasiparticle state)

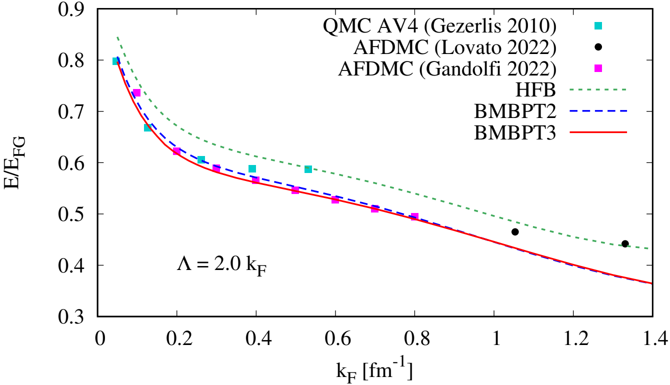


- Very large number of terms at higher orders $\longrightarrow$ Mathematica

- Summation over intermediate quasiparticle states $\longrightarrow$ Monte-Carlo integration

HFB+BMBPT results with $V_{\text{low-}k}$

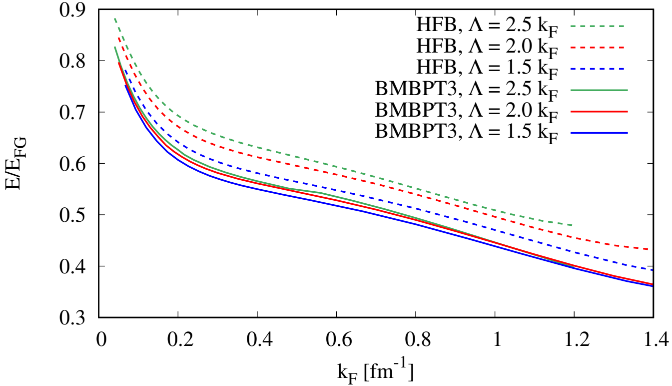
- The correct low-density limit requires $V \to \frac{4\pi a}{m}$, i.e., $\Lambda \to 0$
  $\to$ density dependent cutoff $\Lambda \simeq 1.5 - 3k_F$



- With $V_{\text{low-}k}$ ($\Lambda = 2k_F$), BMBPT seems to converge rapidly
- Good agreement with QMC results at low densities
- Energies too low at high densities: missing (bare) 3BF?

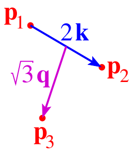
Cutoff dependence of $V_{\text{low-}k}$ results

- Physical results should be independent of the ratio Λ/k_F

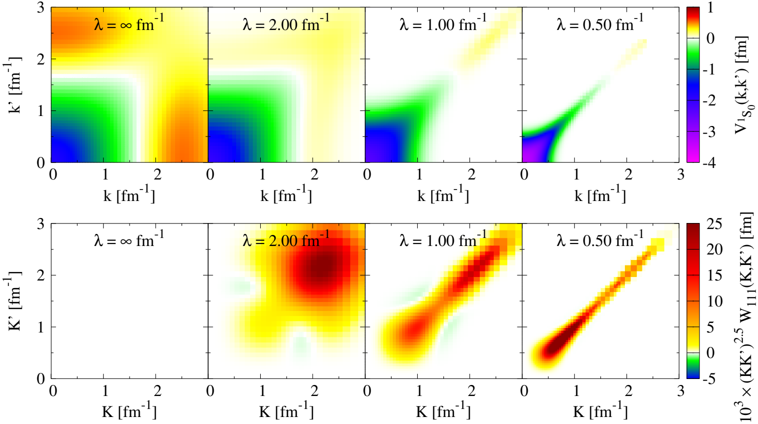


- Varying Λ/k_F in a reasonable range, we see that the BMBPT results show much less cutoff dependence than the HFB results
- The residual cutoff dependence indicates the necessity of including higher orders of BMBPT or induced many-body forces

NN and induced $3N$ SRG matrix elements

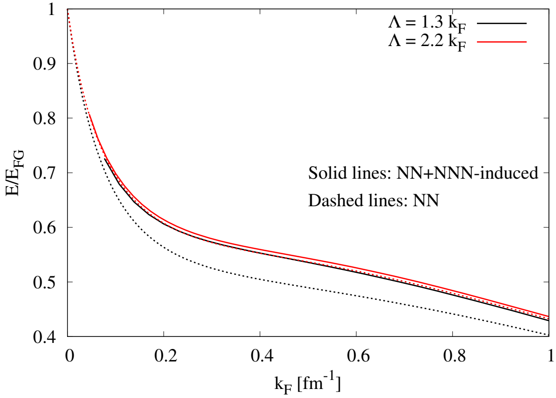


- Starting point: V_{NN} = chiral N4LO potential, $V_{3N} = 0$
- Basis of hyperspherical harmonics for the 3-body space
  $k = K\cos\alpha\,,\ q = K\sin\alpha\,,\ \mathcal{Y}_{L\ell_1 m_1 \ell_2 m_2} = Y_{\ell_1 m_1}(\hat{\mathbf{k}})\,Y_{\ell_2 m_2}(\hat{\mathbf{q}})\,\mathcal{P}_L^{\ell_2 \ell_1}(\alpha)$
- So far, only 3-body force induced by $^1S_0$ 2-body interaction:



Cutoff dependence of SRG results

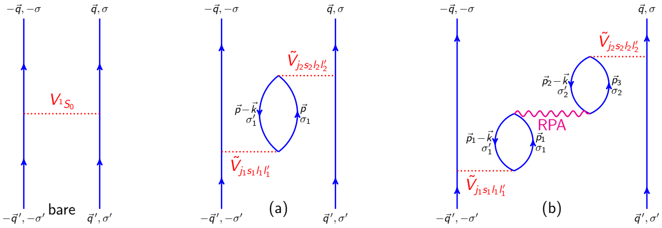
- Since induced $3N$ force is weak, we include it perturbatively $\mathcal{E}_{3N} \approx \langle \mathrm{HFB} | V_{3N} | \mathrm{HFB} \rangle$



- BMBPT3(NN) with SRG has stronger cutoff dependence than with $V_{\mathrm{low}\text{-}k}$
- Cutoff dependence almost cancelled by the contribution of the induced $3N$ force (maybe perturbative treatment not sufficient for the lowest cutoffs)

$^1S_0$ pairing: screening corrections (medium polarization)

- Screening can significantly reduce the BCS (or HFB) pairing gap
- Diagrams (analogous to screening of Coulomb interaction):



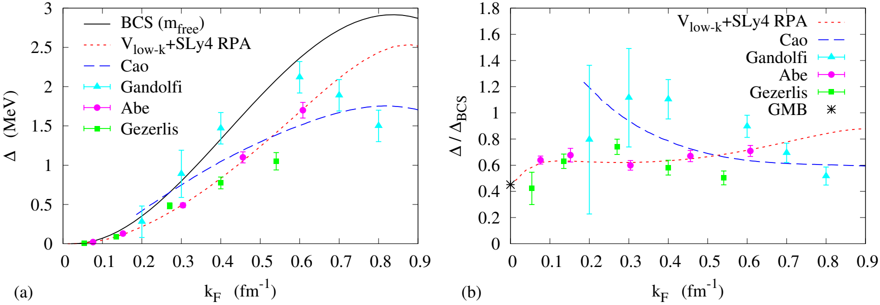
($\tilde{V}$ = anti-symmetrized matrix element)

(1) RPA bubble summation

(2) Treat (a)+(b) (in static approx.) as correction δV to V in the gap equation

- (1) and (2) are non-perturbative, i.e., unlikely to work well in BMBPT
- Here: use $V_{\text{low-}k}$ with $\Lambda = 2.5 k_F$ but take m^* and particle-hole interaction from a phenomenological Skyrme functional → to be improved...

Pairing in the low-density limit

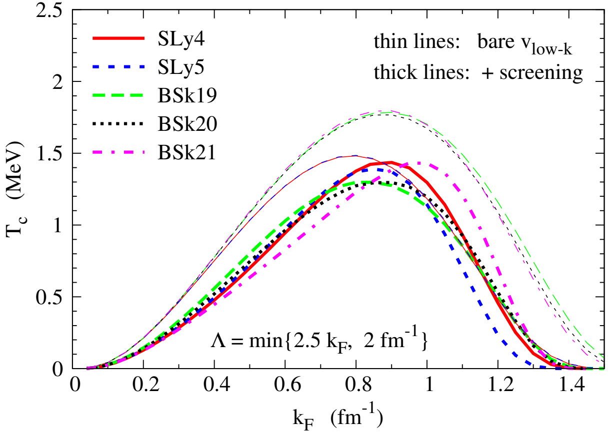


- $V_{low\text{-}k}$+ Skyrme RPA calculation can reproduce results from QMC (Quantum Monte Carlo) and GMB [Gor'kov & Melik-Barkhudarov (1961)]
- Some problems of the calculation by Cao, Lombardo & Schuck (2006):
  - No screening at low density? → Contradiction to GMB result
  - Some arguable approximations (3p1h matrix elements averaged over the Fermi sphere, RPA in lowest-order Landau approximation, . . .)
  - "Babu-Brown theory" which artificially reduces the Landau parameters (suppressing the liquid-gas instability in symmetric matter)

Dependence on the Skyrme parametrization

- Compare results using different SLy and BSk functionals for m^* and the RPA:

$$T_c = \frac{\Delta_{T=0}}{1.76}$$



- Sizeable differences at BCS level (w/o screening) because of different m^*
- Results with screening are surprisingly close to each other
- Density where the ${}^1S_0$ gap disappears is uncertain

Summary

- Dilute neutron matter with densities $10^{-4}\ \mathrm{fm}^{-3} \lesssim n \lesssim 0.08\ \mathrm{fm}^{-3}$ relevant for the inner crust ($0.14\ \mathrm{fm}^{-1} \lesssim k_F \lesssim 1.33\ \mathrm{fm}^{-1}$)
- Low-momentum potentials with $\Lambda \propto k_F$ are a powerful method, avoiding the necessity to resum ladder diagrams
- EoS and pairing gap well under control up to $n \approx 0.03\ \mathrm{fm}^{-3}$ ($k_F \approx 1\ \mathrm{fm}^{-1}$)
- Pairing beyond that density **very** sensitive to details of screening

Outlook

- Include genuine 3-body force (2-pion exchange) for EoS at higher densities
- Screening should be computed from the NN (and $3N$) interactions within BMBPT (with appropriate resummations if needed)
- Extension to finite proton fraction (neutron-star core): 3-body force is crucial