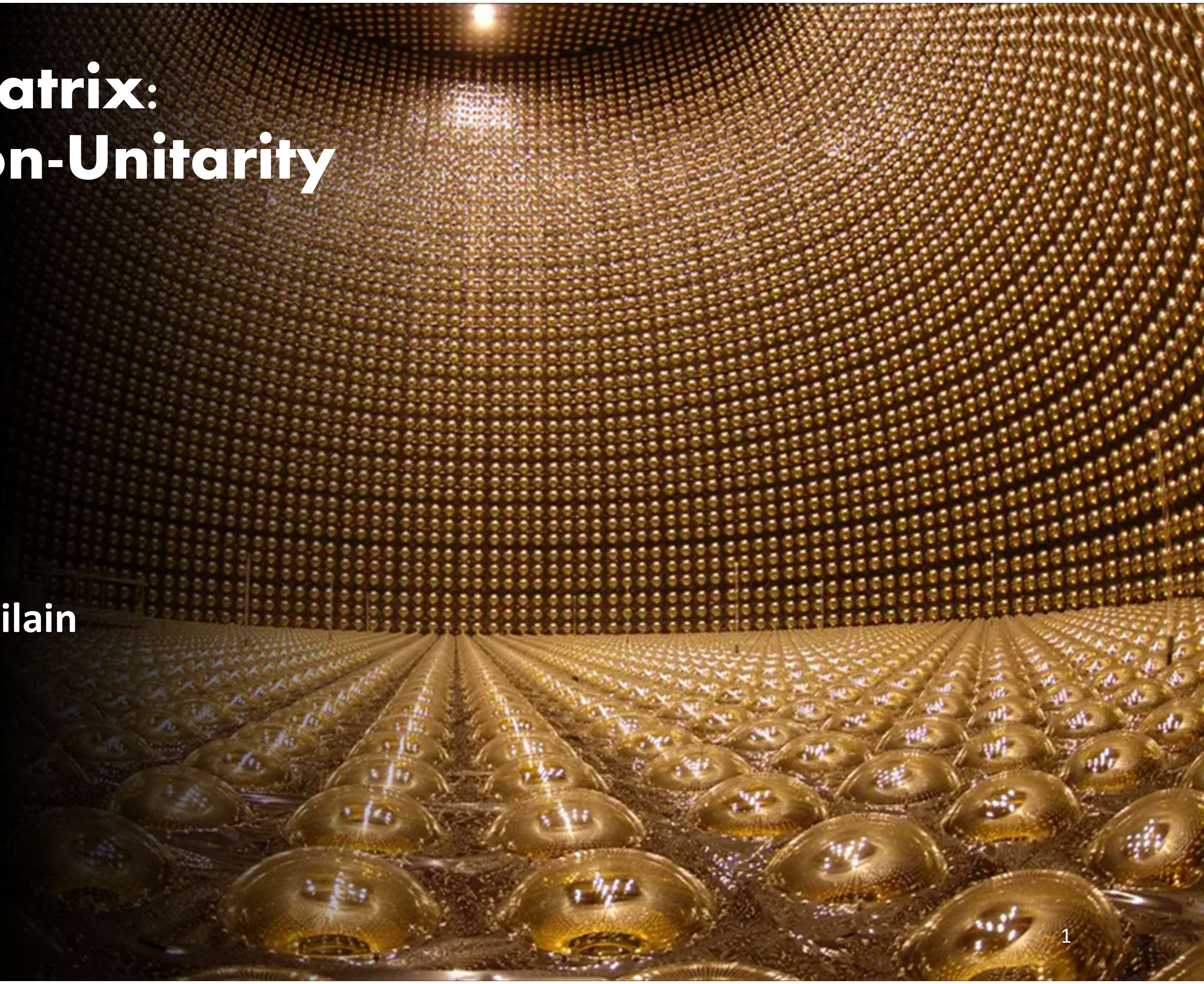
On the PMNS Matrix: Patterns and Non-Unitarity

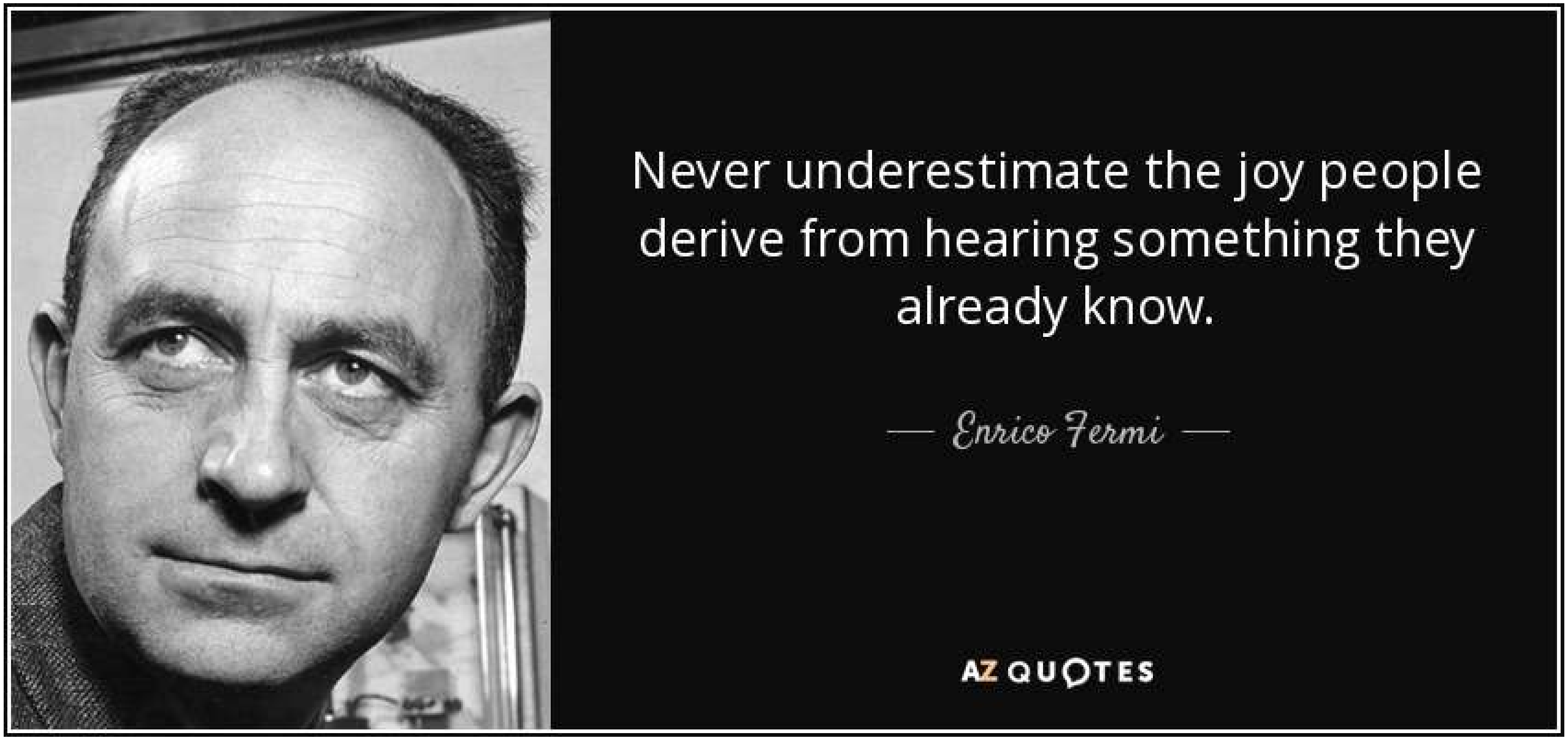


Aymane Maaitat
Ahmed Chahlaoui

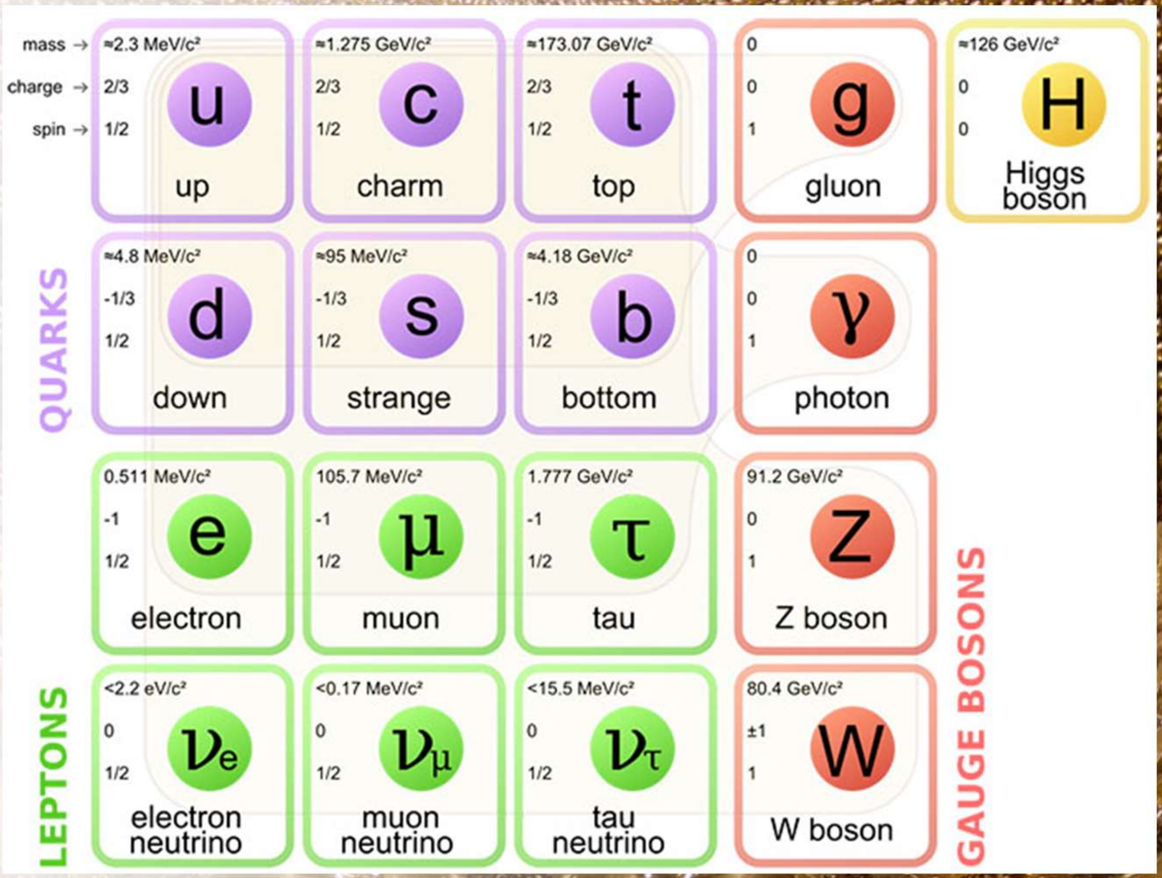
Supervised by Benjamin Quilain

Outline

1. ***Elements of neutrino oscillation formalism***
2. ***What we know***
3. ***Patterns of PMNS***
4. ***How to break unitarity***

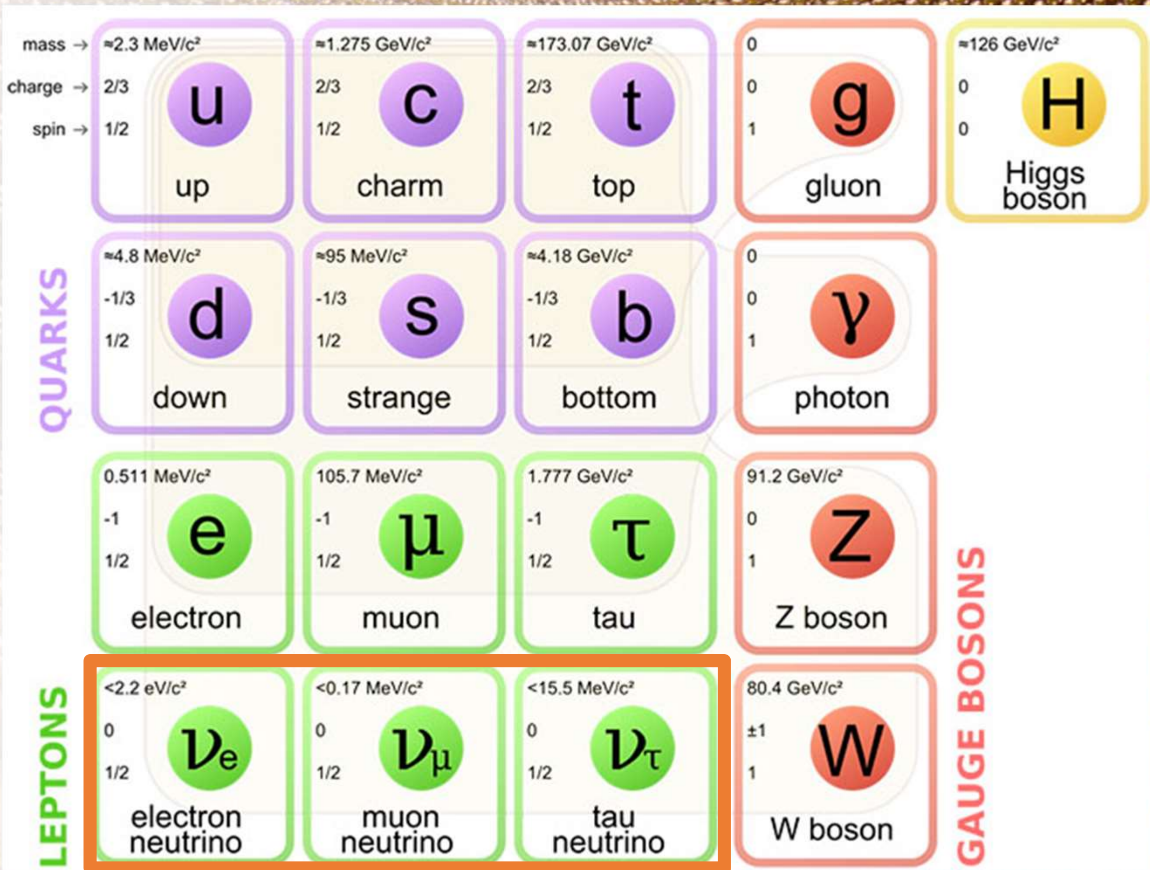

Never underestimate the joy people derive from hearing something they already know.
— Enrico Fermi —
AZ QUOTES

Neutrinos: a quick introduction



Neutrinos: a quick introduction

- ❑ Neutrinos are elementary particles.



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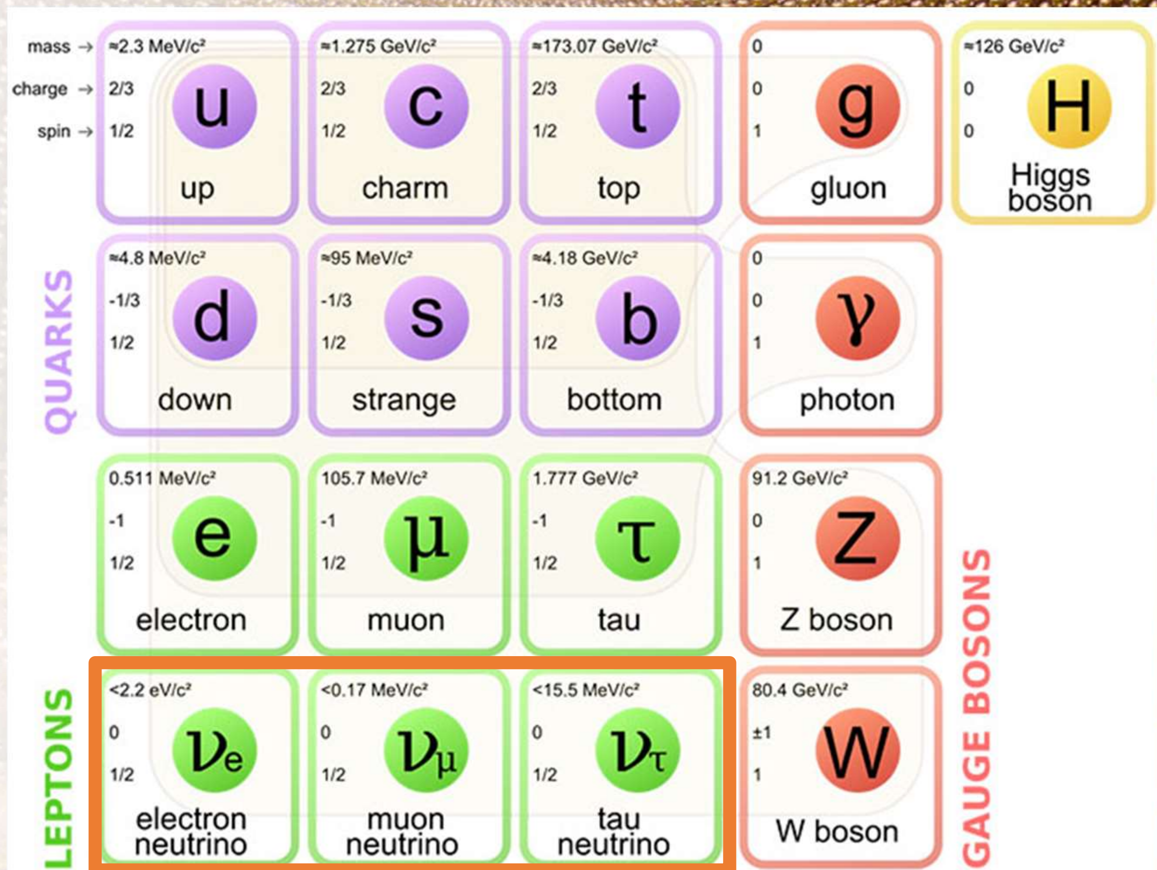

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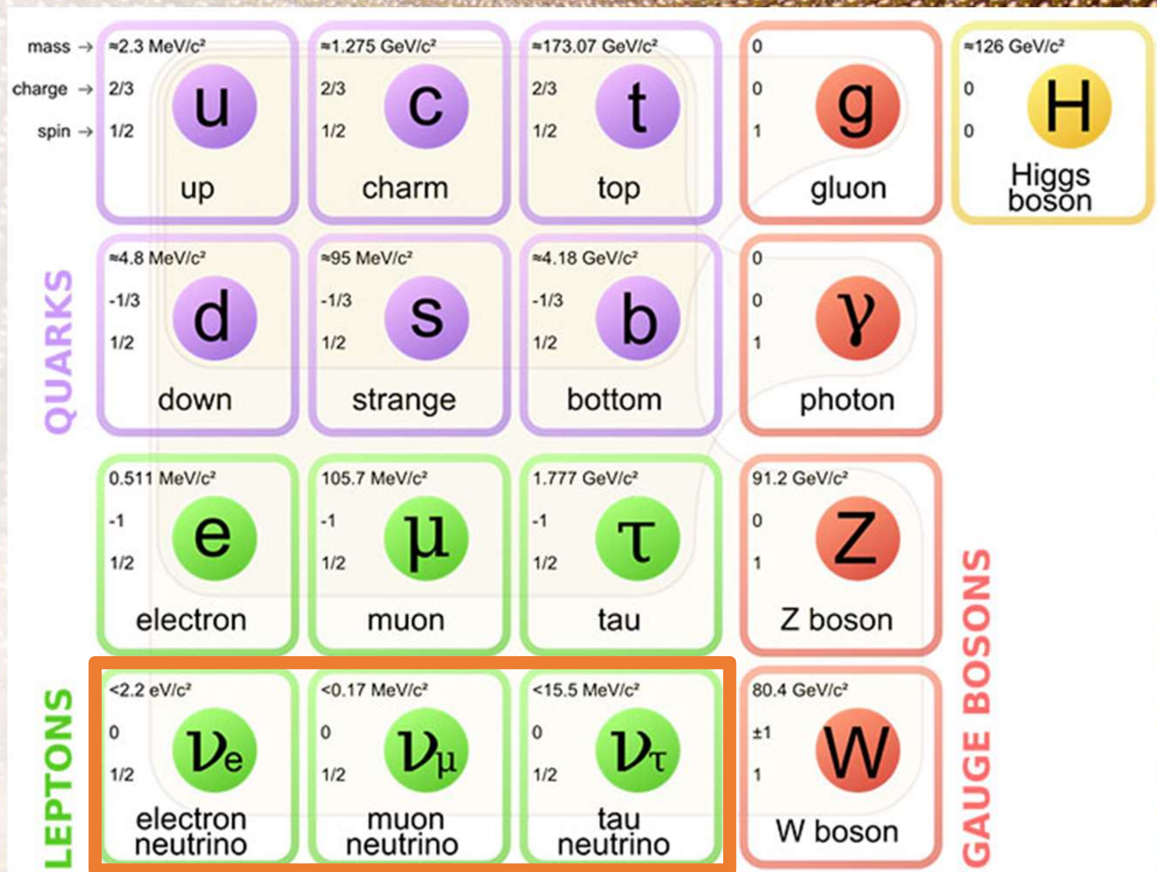

Neutrinos: a quick introduction

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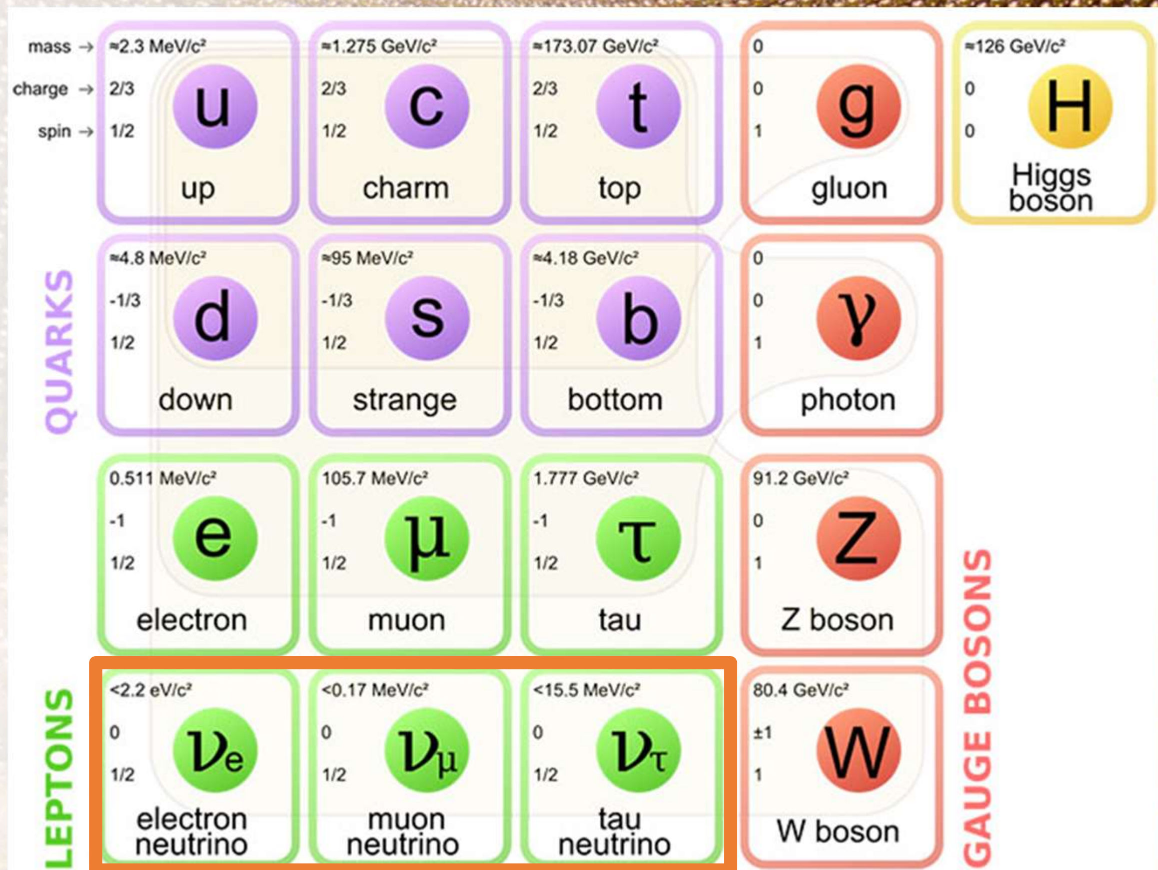
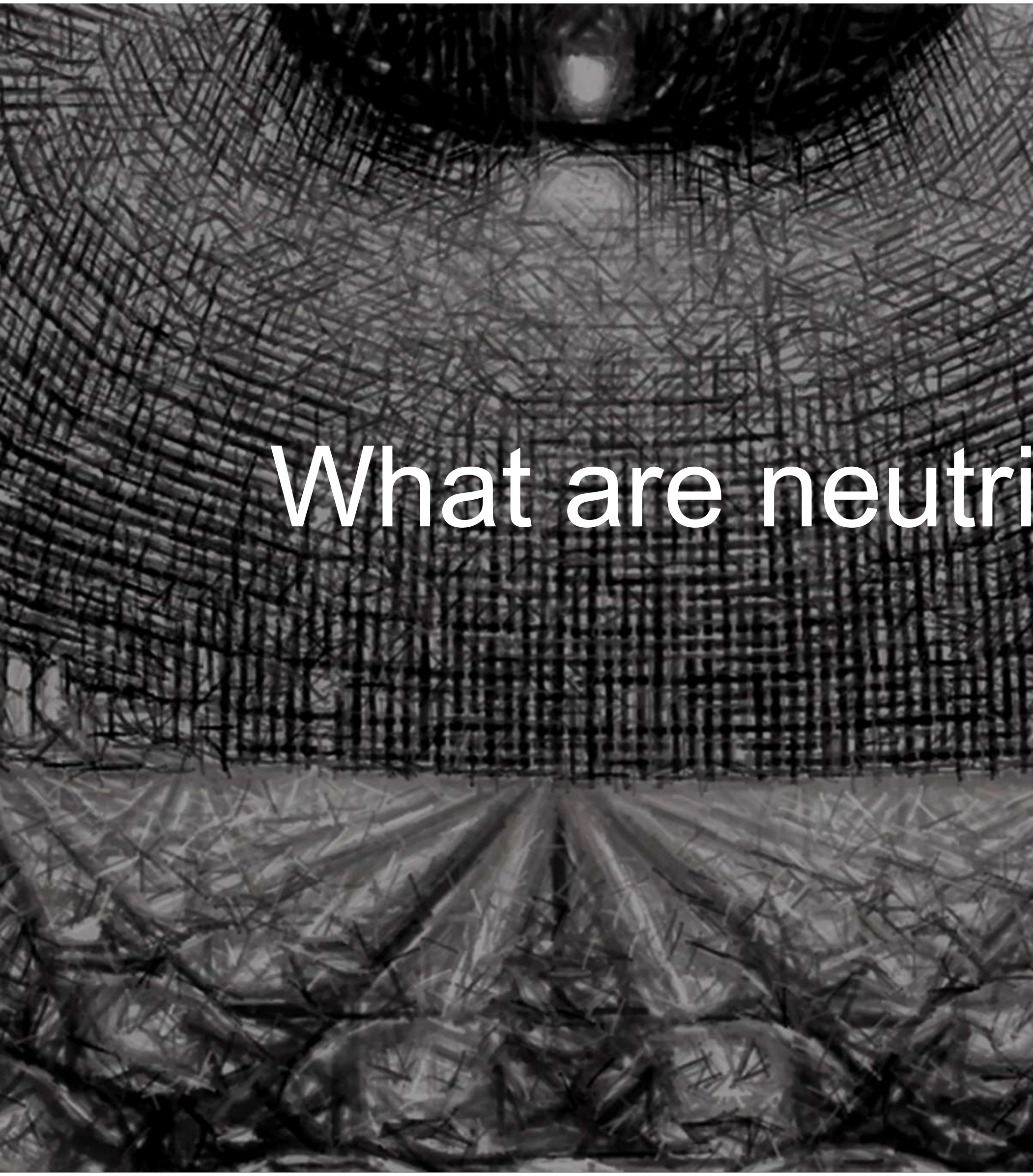
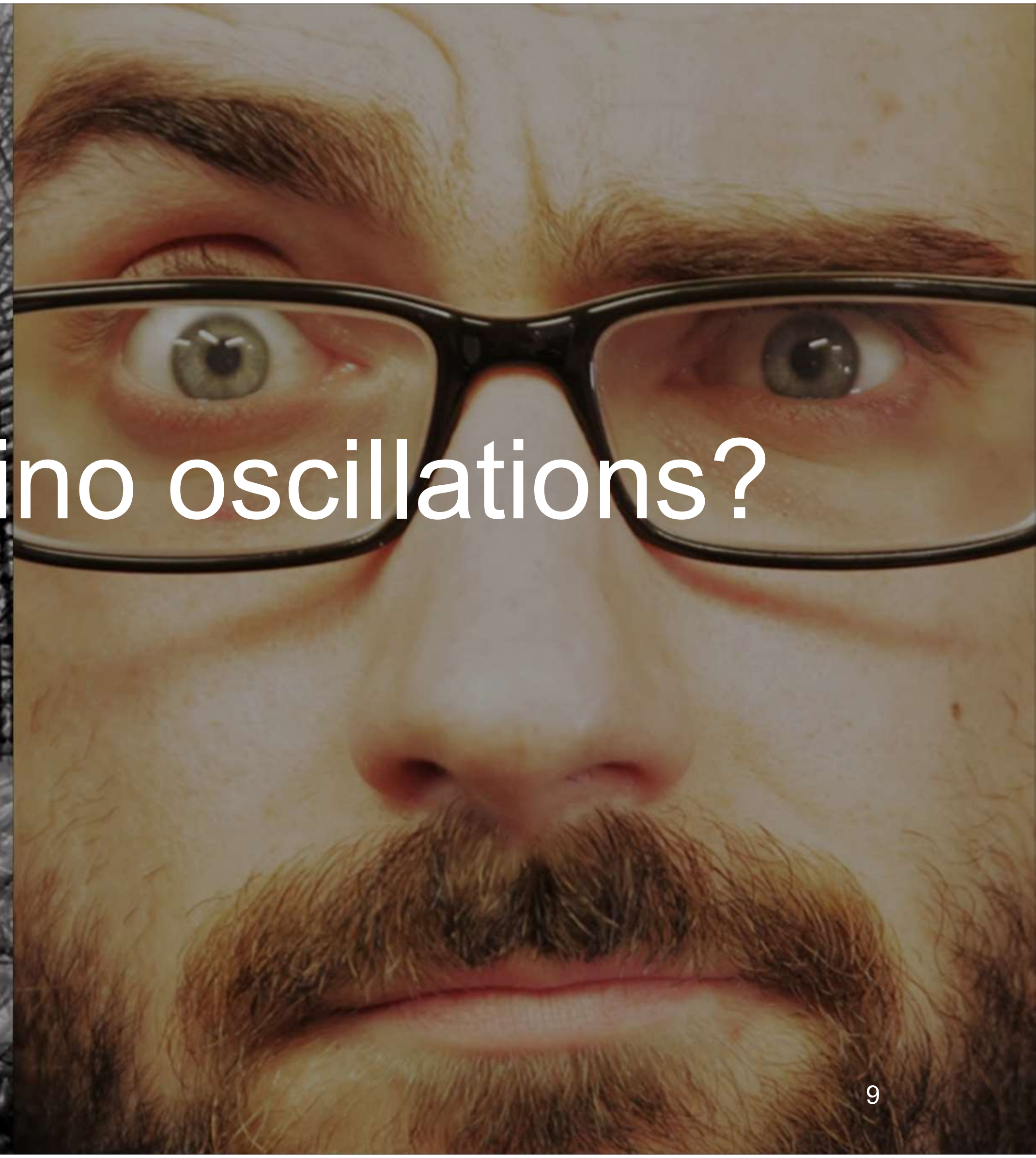
- ❑ The discovery of neutrino oscillations sheds light on their non zero mass.

- ❑ It is still not clear how, in their massive form, neutrinos can be inserted in the Standard Model.



What are neutrino oscillations?





Elements of Neutrino oscillations

- ❑ Mass states are different from flavour states
- ❑ Flavour states are not conserved by free hamiltonian evolution
- ❑ Oscillation is quantitatively described by a **unitary** matrix called the Pontecorvo–Maki–Nakagawa–Sakata matrix (PMNS matrix)

$$\begin{pmatrix} \nu_e(x) \\ \nu_\mu(x) \\ \nu_\tau(x) \end{pmatrix}_L = U \begin{pmatrix} \nu_1(x) \\ \nu_2(x) \\ \nu_3(x) \end{pmatrix}_L = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1(x) \\ \nu_2(x) \\ \nu_3(x) \end{pmatrix}_L$$

PMNS Matrix Unitary Parametrization

- In the most general case, PMNS is parametrized by 3 **independent** mixing angles and 6 **independent** complex phases.

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23}e^{i\phi_{23,a}} & s_{23}e^{i\phi_{23,b}} \\ 0 & -s_{23}e^{-i\phi_{23,b}} & c_{23}e^{-i\phi_{23,a}} \end{pmatrix} \begin{pmatrix} c_{13}e^{i\phi_{13,a}} & 0 & s_{13}e^{i\phi_{13,b}} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\phi_{13,b}} & 0 & c_{13}e^{-i\phi_{13,a}} \end{pmatrix} \begin{pmatrix} c_{12}e^{i\phi_{12,a}} & s_{12}e^{i\phi_{12,b}} & 0 \\ -s_{12}e^{-i\phi_{12,b}} & c_{12}e^{-i\phi_{12,a}} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

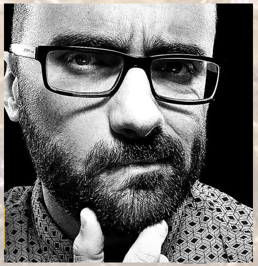
- We can rephase fields to absorb some **complex** phases which are not physical.

PMNS Matrix Unitary Parametrization

- Without going into the details, we can generally obtain this form, retaining **1 complex phase** and a matrix **P** which we ignore for the purposes of neutrino oscillations.

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{\mathrm{CP}}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{\mathrm{CP}}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} P$$

What do these parameters correspond to?



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Atmospheric neutrinos

$$\frac{L_{\text{baseline}}}{E_\nu}$$ Is fixed by the considered experiment, which determines the regime of oscillation.

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Atmospheric neutrinos

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Majorana Phase matrix

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Probability of neutrino oscillation

Given an initial flavor state alpha

$$|\nu(0)\rangle = |\nu_\alpha\rangle$$

The probability that we will observe a flavor state beta is

Extending the calculation we have

$$P(\nu_\alpha \to \nu_\beta)(t) = |\langle \nu_\beta | \nu(t) \rangle|^2$$

$$P(\nu_\alpha \to \nu_\beta)(L,E) = \underbrace{\delta_{\alpha\beta} - 4\sum_{i<j} \Re[U_{\alpha i} U^*_{\beta i} U^*_{\alpha j} U_{\beta j}] \sin^2\left(\frac{\Delta m^2_{ji} L}{4E}\right)}_{\text{CP-even}} \pm \underbrace{2\sum_{i<j} \Im[U_{\alpha i} U^*_{\beta i} U^*_{\alpha j} U_{\beta j}] \sin\left(\frac{\Delta m^2_{ji} L}{2E}\right)}_{\text{CP-odd}},$$

2. What we know about PMNS parameters

Goal of neutrino oscillation experiments: determine the oscillation parameters with high accuracy

Besides getting better precision on parameters, there are still mysteries to be unraveled

- δ_{CP}: CP-violation phase.
- Mass hierarchy: Normal Ordering (NO) or Inverted Ordering (IO)
- Octant degeneracy of θ_{23}

2. What we know about PMNS parameters

According to recent results from PDG:

Ordering	Param	bfp $\pm 1\sigma$
NO	$\theta_{12}/1^\circ$	33.41 ± 0.72
	$\theta_{23}/^\circ$	42.1 ± 1.0
	$\theta_{13}/^\circ$	8.58 ± 0.11
	$\delta_{CP}/^\circ$	$232 + 36 - 26$
	$\Delta m^2_{21}/10^{-5} eV^2$	7.41 ± 0.21
	$\Delta m^2_{32}/10^{-3} eV^2$	2.433 ± 0.026
IO	$\theta_{12}/^\circ$	33.41 ± 0.72
	$\theta_{23}/^\circ$	49.0 ± 1.2
	$\theta_{13}/^\circ$	8.57 ± 0.11
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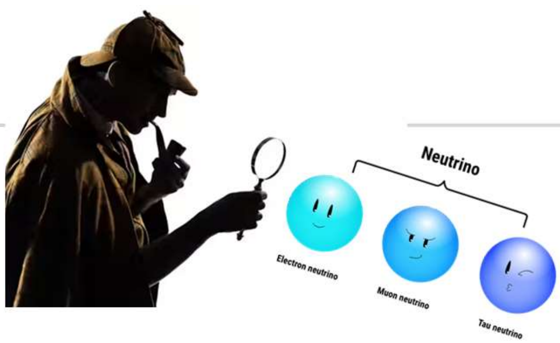
https://pdg.lbl.gov/2024/reviews/contents_sports.html

2. What we know about PMNS parameters

Speculation: Did Nature throw dice when choosing values of these parameters, or is there an underlying pattern?



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Do oscillation parameters follow a certain pattern?

3. Patterns of PMNS

Petcov et al. explored extending the standard group model by a discrete non-abelian group that has a 3-dimensional unitary irreducible representation.

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https://inspirehep.net/literature/1639463

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- ❑ Unify the three fermions generations
- ❑ Each **symmetry group** should produce a mixing matrix that we can compare to our PMNS

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3. Patterns of PMNS

Symmetry group	
$\mathcal{A}_4$	
$\mathcal{S}_4$	

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Symmetry group	**Test statistic**
$\mathcal{A}_4$	$T(\delta, \theta_{23}, \theta_{13}) = cos(\delta) sin(2\theta_{23}) sin(\theta_{13}) \sqrt{(2 - 3sin^2(\theta_{13}))} - cos(2\theta_{23}) cos(2\theta_{13})$
$\mathcal{S}_4$	$T(\delta, \theta_{23}, \theta_{13}) = \left(2\sqrt{2} \sin 2\theta_{23} \sin\theta_{13} \sqrt{1 - 3\sin^2\theta_{13}}\right) \cos(\delta) - ((-1 + 5\sin^2\theta_{13}) \cos 2\theta_{23})$

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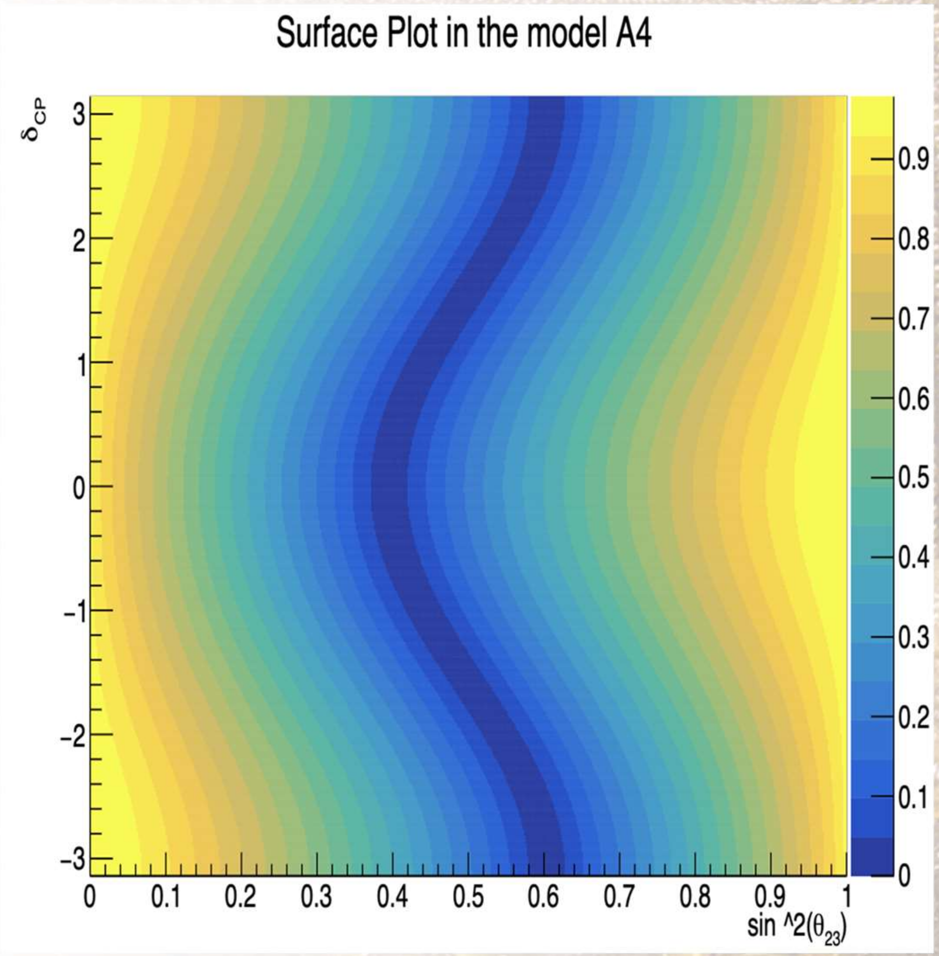
$\mathcal{S}_4$	$T(\delta,\theta_{23},\theta_{13}) = \left(2\sqrt{2}\sin 2\theta_{23}\sin\theta_{13}\sqrt{1-3\sin^2\theta_{13}}\right)\cos(\delta) - ((-1+5\sin^2\theta_{13})\cos 2\theta_{23})$

→ Each model predicts that its T-function is zero
→ The null hypothesis is T=0

Patterns of PMNS: The A4 model

- We use BF value of the remaining angle θ_{13}
- We plot T as a function of δ_{CP} and θ_{23}



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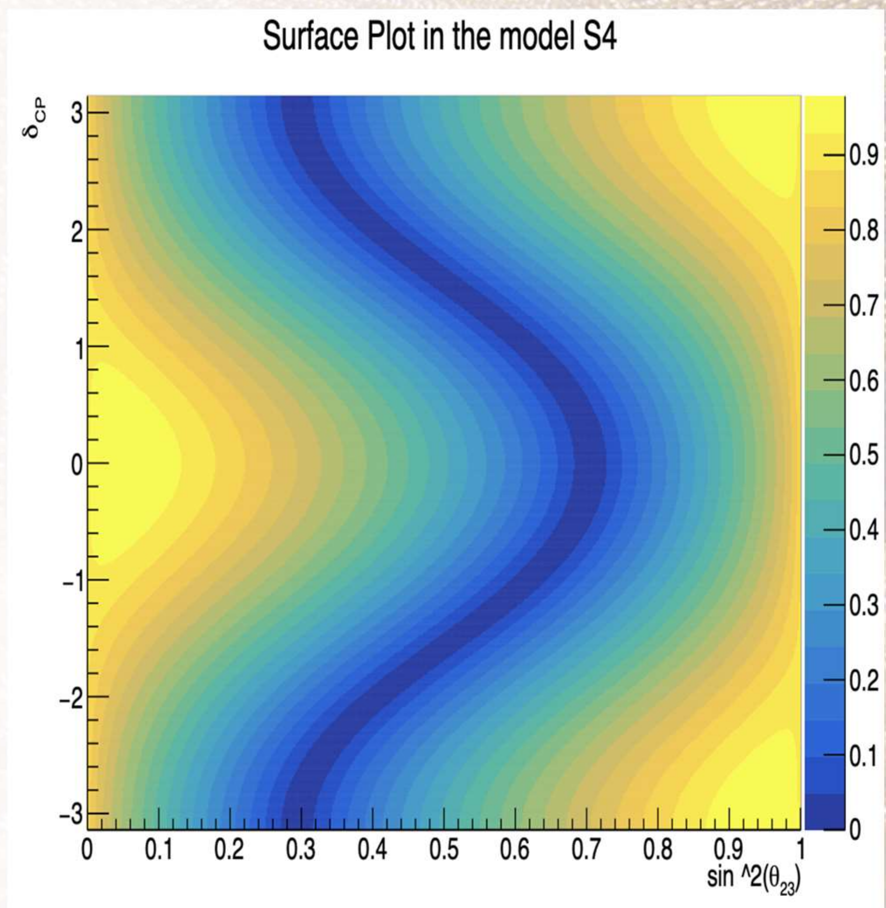
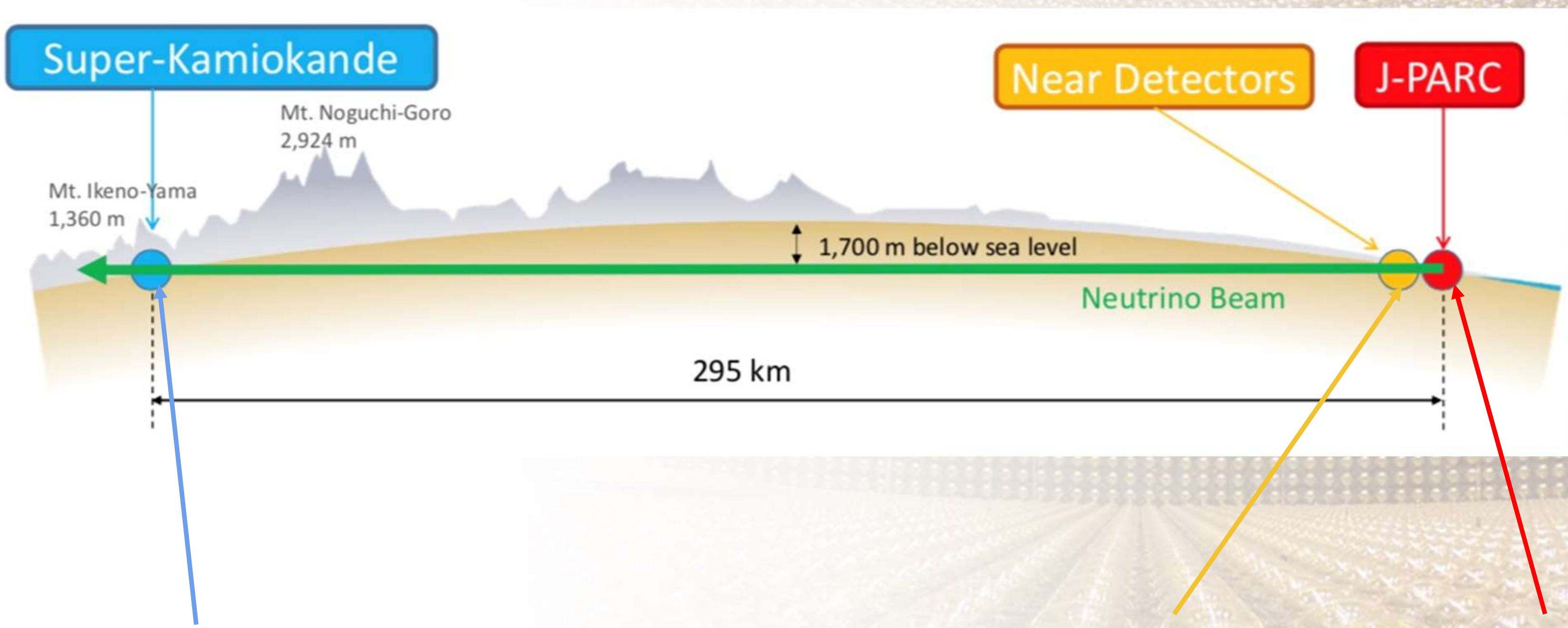


Testing the hypothesis using T2K data and P-Theta Framework



Detects cherenkov radiation produced by neutrino interactions with matter

Measures muon neutrinos count and estimate relative flux

Neutrino beam production

https://t2k-experiment.org/t2k/

Testing the hypothesis using T2K data and P-Theta Framework

P-theta: a framework to perform far detector fit and make inference on neutrino oscillation parameters.

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The **likelihood** is defined as:

$$\mathcal{L}\left(\left\{N_s^{\text{obs.}}, \boldsymbol{x}_s^{\text{obs.}}\right\}_{\forall s}, \boldsymbol{o}, \boldsymbol{f}\right) = \prod_{s \in \text{samples}} \left[\mathcal{L}_s\left(N_s^{\text{obs.}}, \boldsymbol{x}_s^{\text{obs.}}, \boldsymbol{o}, \boldsymbol{f}\right)\right] \times \mathcal{L}_{\text{syst.}}(\boldsymbol{f})$$

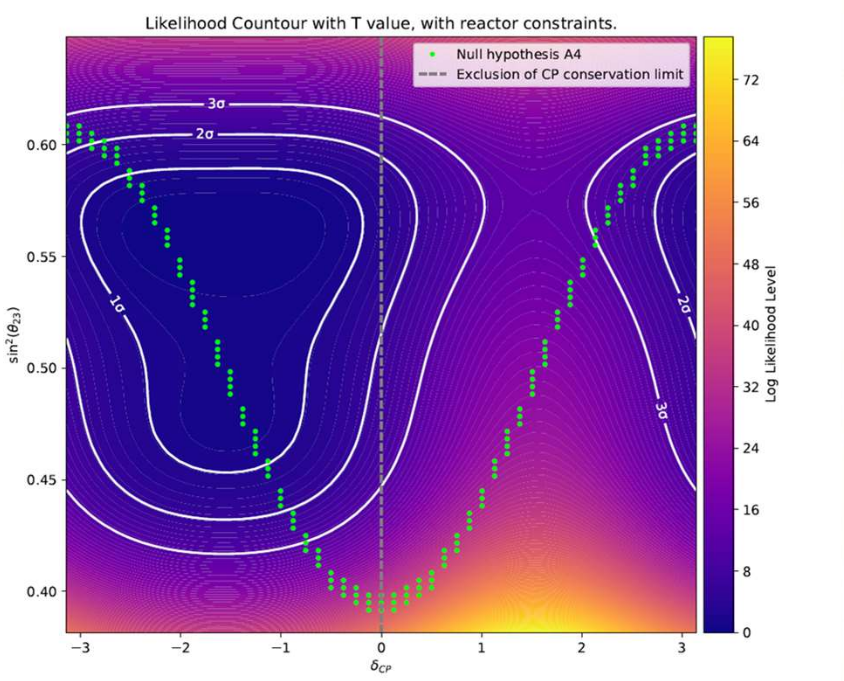
$$\ln \mathcal{L}_s\left(N_s^{\text{obs}}, \boldsymbol{x}_s^{\text{obs.}}, \boldsymbol{o}, \boldsymbol{f}\right) = \sum_{i \in \text{bins}} \left[\left(N_{s,i}^{\text{exp}} - N_{s,i}^{\text{obs}}\right) + N_{s,i}^{\text{obs}} \times \ln\left(N_{s,i}^{\text{obs}} / N_{s,i}^{\text{exp}}\right)\right]$$

$$\mathcal{L}_{\text{syst}} = \exp\left(-0.5 \sum_{i,j} v_i M_{ij} v_j\right)$$

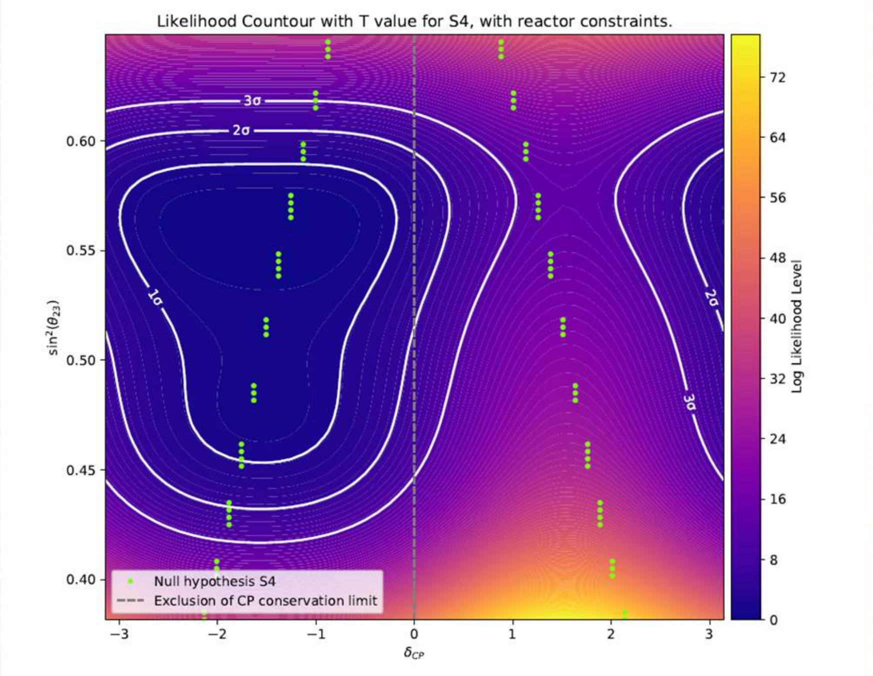
$N_{s,i}^{exp}/N_{s,i}^{obs}$ –number of expected/observed events is sample s in bin i
$\mathbf{o}/\mathbf{f}$ –vector of all oscillation/systematic parameters
$\boldsymbol{v}_i$ – the difference of one systematic parameter i from its central value
M_{ij} – is the element (i, j) of the inverted covariance matrix

Testing the null hypothesis using T2K data

A4 model

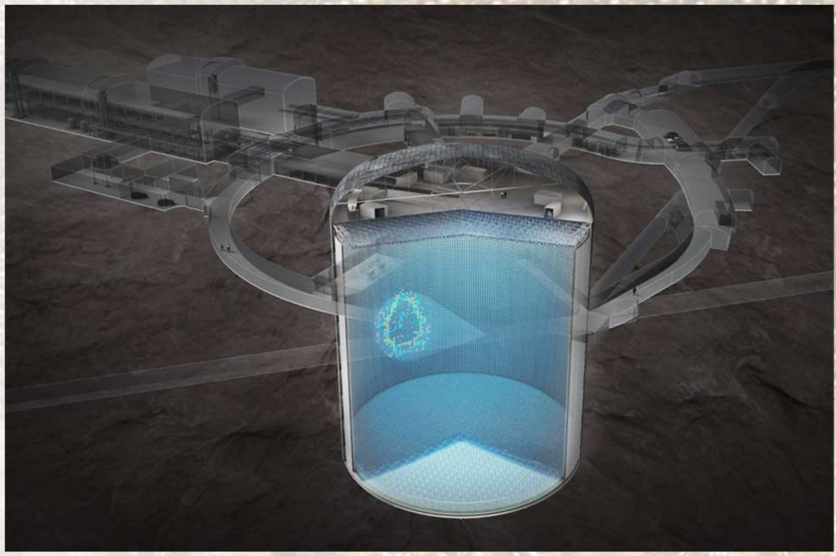


S4 model



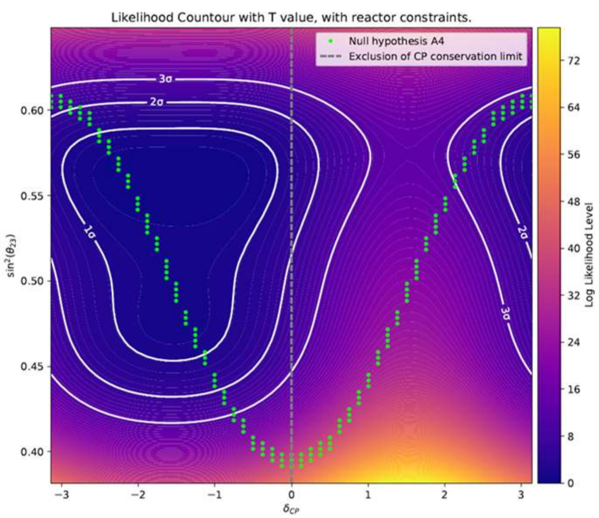
Testing the null hypothesis using HK data

Feature	Super-Kamiokande	Hyper-Kamiokande
Water Mass	50,000 tons (22,500 tons fiducial mass)	260,000 tons (190,000 tons fiducial mass)
Photomultiplier Tubes	11,146 tubes, 50cm diameter	About 40,000 tubes, 50cm diameter
Main and Expected Results	Discovery of neutrino oscillations, showing that neutrinos have mass	1. Discovery of CP violation differences between neutrino and antineutrino oscillations. 2. Advancement of neutrino astronomy. 3. Potential discovery of proton decay to evidence unification theories.



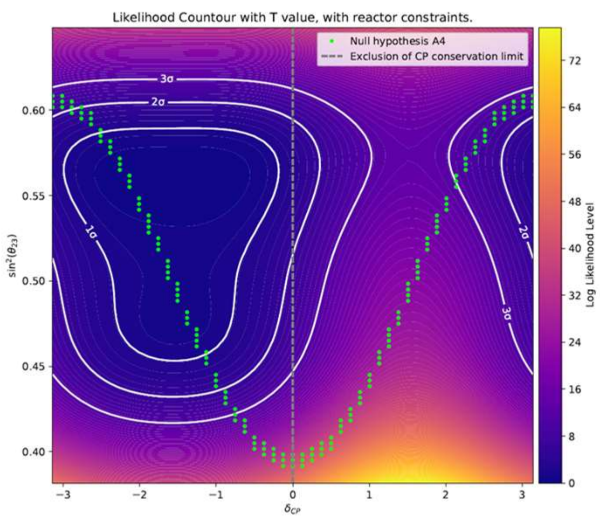
https://www-sk.icrr.u-tokyo.ac.jp/en/hk/about/outline/

Testing the null hypothesis using HK data



SK

Testing the null hypothesis using HK data



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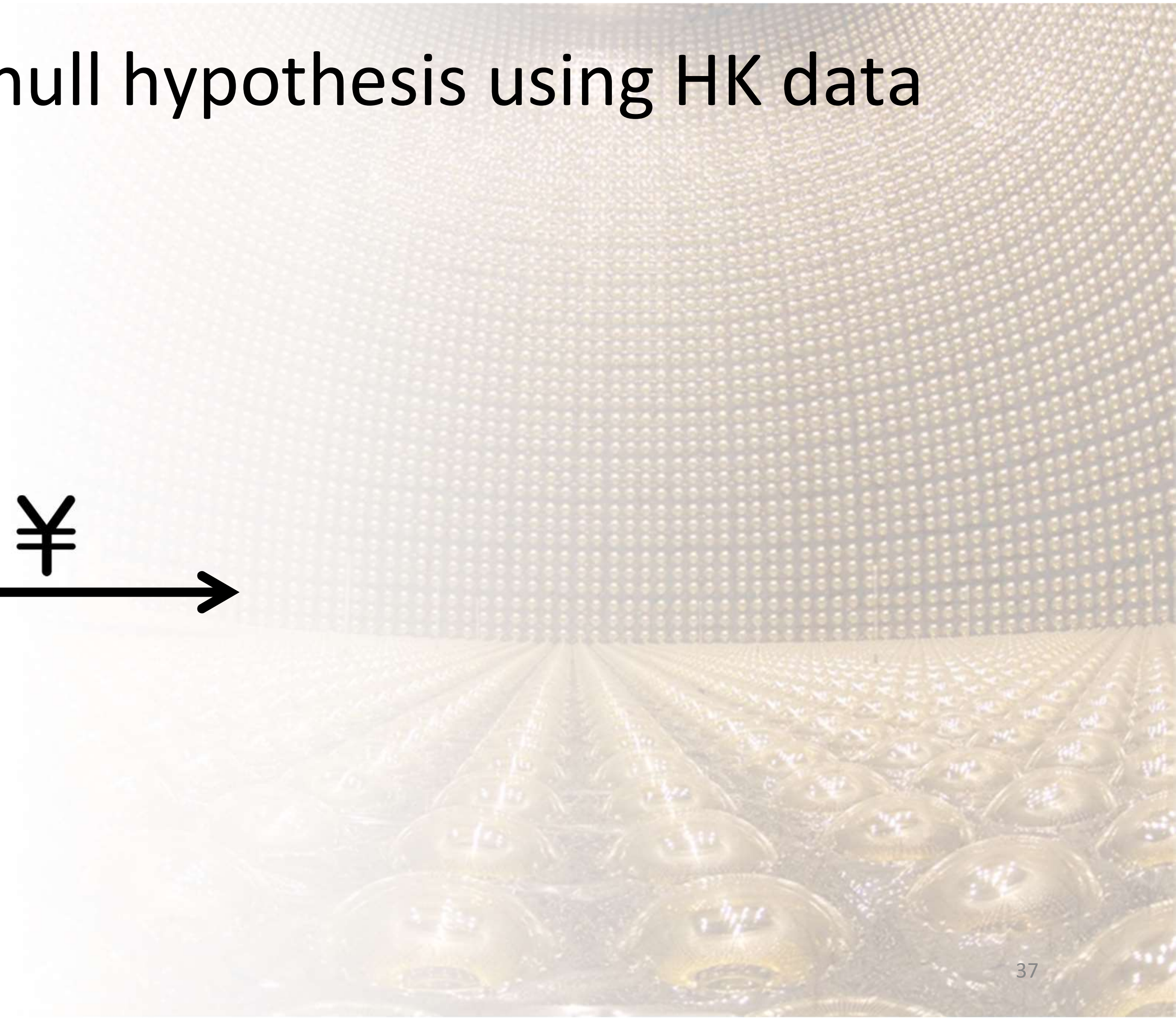

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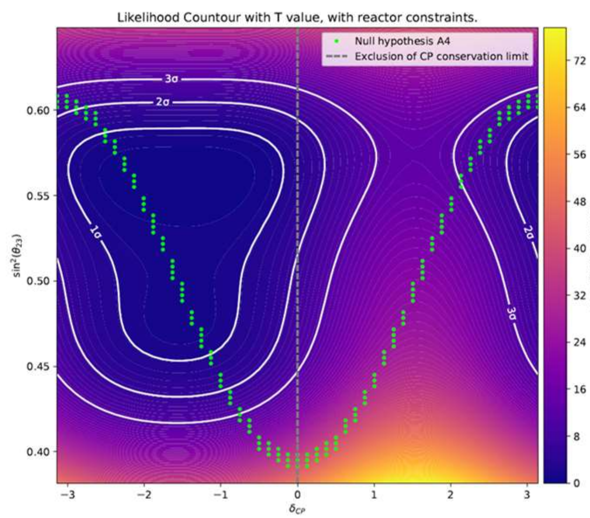

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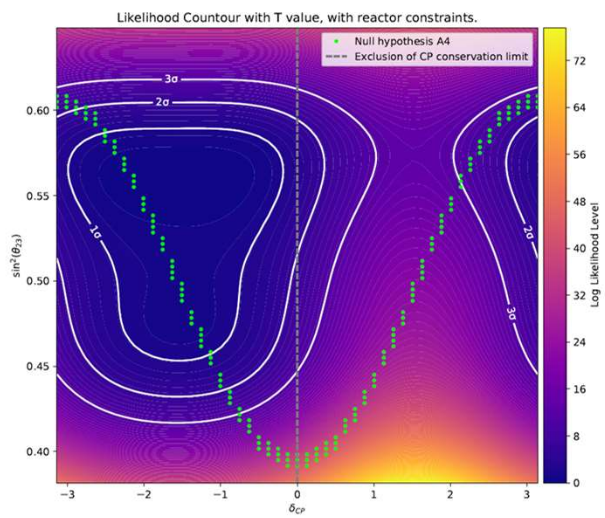

SK

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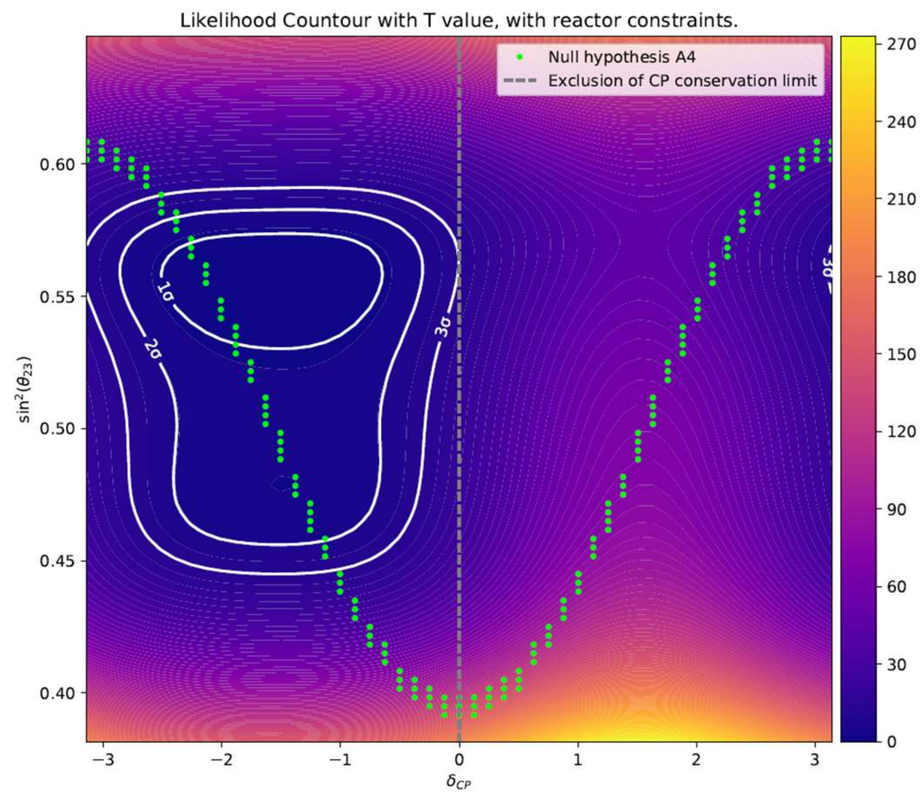

HK (<u>to be revisited</u>)

How to break unitarity?

Non-Unitarity and first principles:

If U is non-unitary, then without any constraints, the parameter space explodes.

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Doing the same trick of rephasing fields, we get:
9 amplitudes. (versus 3 for Uni.)
4 complex phases. (versus 1 for Uni.)

$$\begin{pmatrix} |U_{e1}| & |U_{e2}|\, e^{i\phi_{e_2}} & |U_{e_3}|\, e^{i\phi_{e3}} \\ |U_{\mu 1}| & |U_{\mu 2}| & |U_{\mu 3}| \\ |U_{\tau 1}| & |U_{\tau 2}|\, e^{i\phi_{\tau_2}} & |U_{\tau 3}|\, e^{i\phi_{\tau_3}} \end{pmatrix}$$

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"Conservation of probability" gives 6 upper bounds for the sum of the amplitudes.

Non-Unitarity parameterization(s)

As there is no universally agreed upon parametrization for non-unitarity, **we will spare you this technical part.**

For our part, we have developed a parametrization based on QR decomposition for its interpretability.

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_{CP}} & c_{13}c_{23} \end{pmatrix} T$$

What constraints on non-unitarity?

LFU-WMA bounds

Assuming Non-Unitarity of PMNS, *and other additional hypothesis, one can put bounds on the normalization coefficients from purely charged lepton decays!*

https://journals.aps.org/prd/abstract/10.1103/PhysRevD.109.055006

LFU-WMA bounds

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These bounds, which we dub, LFU-WMA put normalization of PMNS up to 1e-3.

Which, by simple inequalities, show that departures in matrix elements from unitarity can go no farther than 1e^-3.

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- Non-Unitarity adds a total of 9 free parameters to the unitary model.

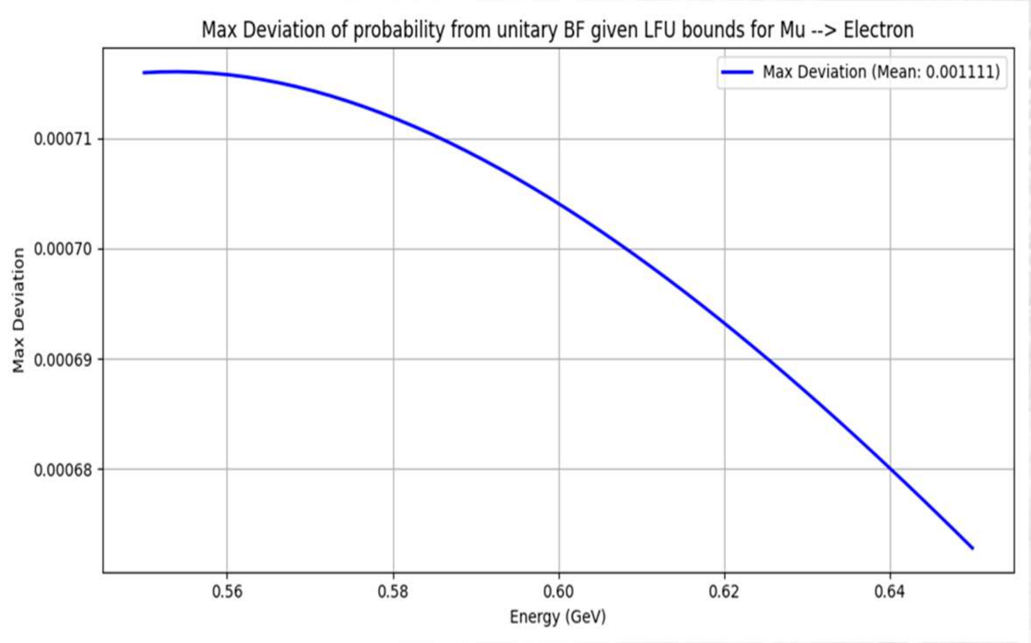
- The amplitudes are constrained by conservation of probability and Cauchy-Schwarz inequality

- An argument can be made that the normalization factors: $\sum_{\alpha} |U_{\alpha,i}|^2$ Are equal to 1 up to 1e-3, from lepton decays and weak mixing angle measurements.

We assume the last bounds hold in our consequent study.

Bounds on Non-Unitarity: recap

- Taking BF values and assuming unitarity, we compute maximum deviations.



Probability is modified by order of 10^-4 at T2K energy

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Thanks for your attention