THE DOs AND DON'Ts OF HIGH FREQUENCY GRAVITATIONAL WAVE DETECTION

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Work in progress with S. Ellis (U. Geneva)

$$\omega_g$$


ω_g
nHz
μHz
10 Hz
kHz
PULSAR TIMING
LIGO-VIRGO-KAGRA

$$\omega_g$$

$$\lambda(T_*) < \frac{1}{H(T_*)}$$ Causality

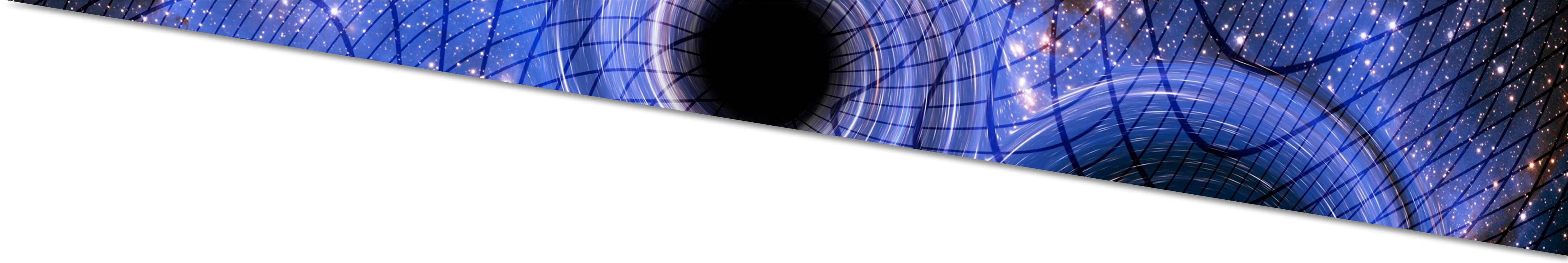
$$\omega_g$$

$$\lambda(T_*) < \frac{1}{H(T_*)}$$ Causality

$$\omega_0(T_*) = \omega(T_*)\frac{a(T_*)}{a_0} \gtrsim \boxed{100\ \mathrm{MHz}} \left(\frac{T_*}{10^{15}\mathrm{GeV}}\right) \left(\frac{g_*(T_*)}{100}\right)^{1/6}$$

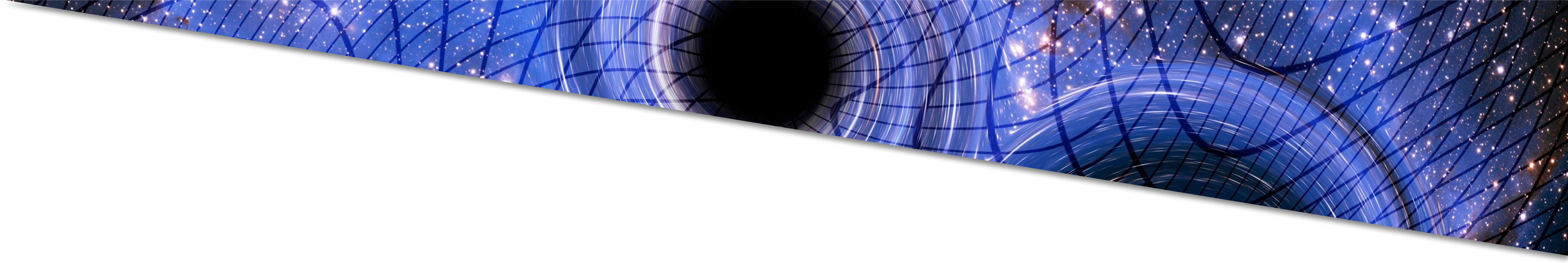

DETECTORS

NATURAL UNITS



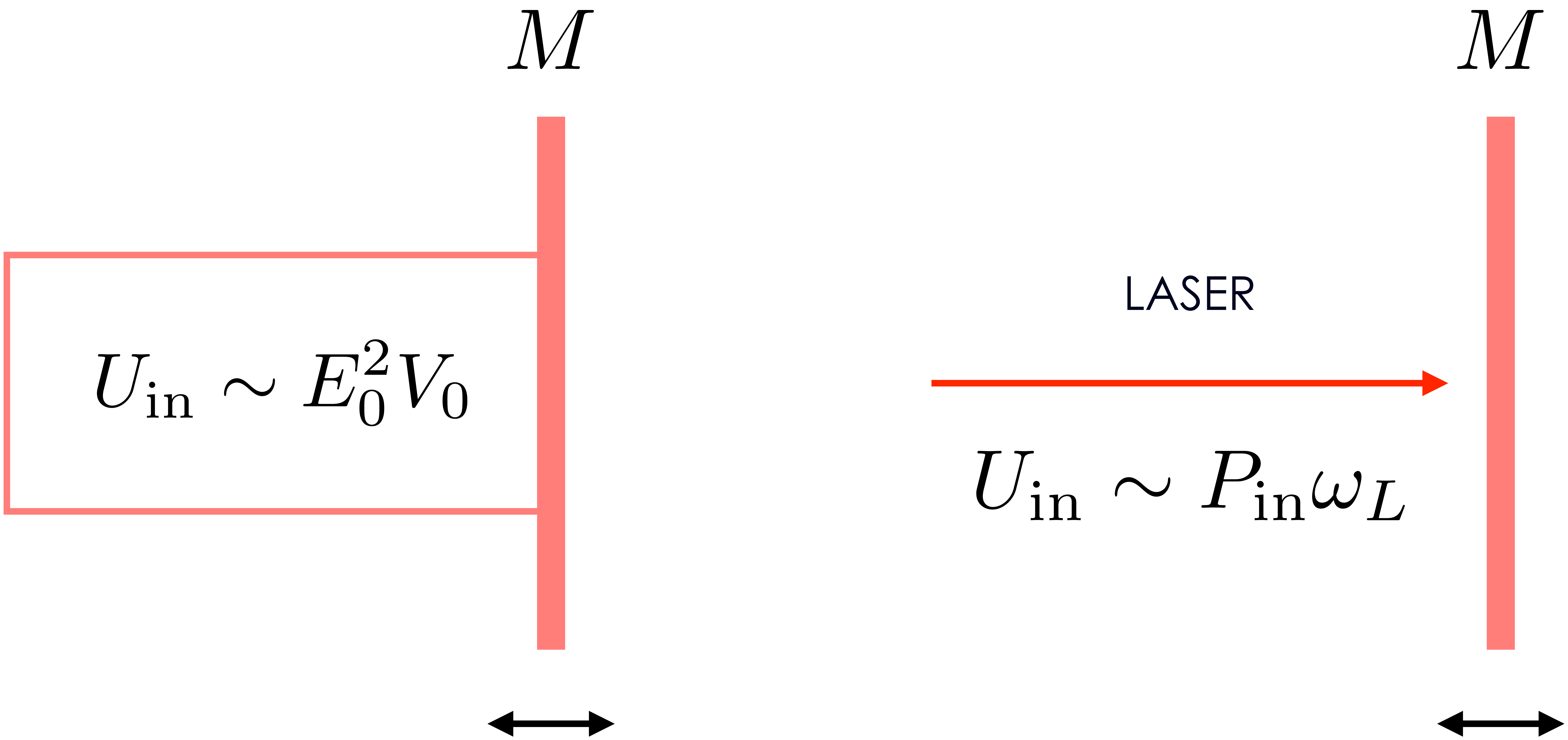
$$\hbar = c = 1$$

DETECTOR I



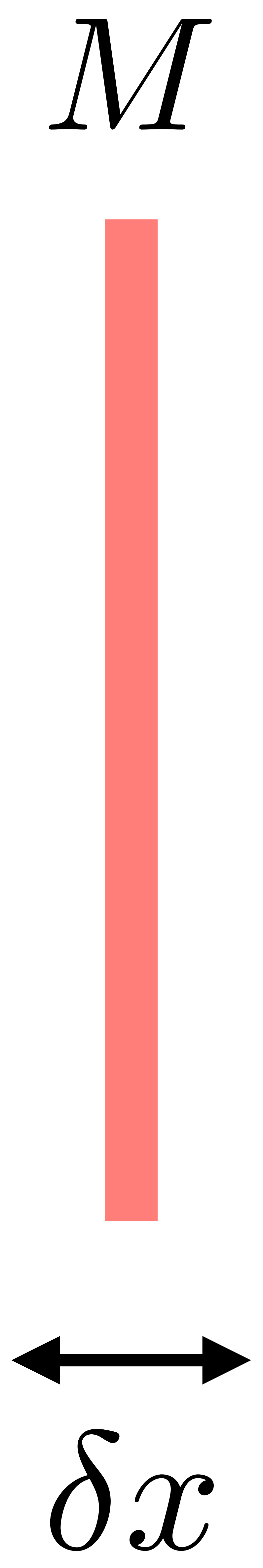
$$U_{\mathrm{in}} \sim E_0^2 V_0$$

DETECTOR II



DETECTOR II

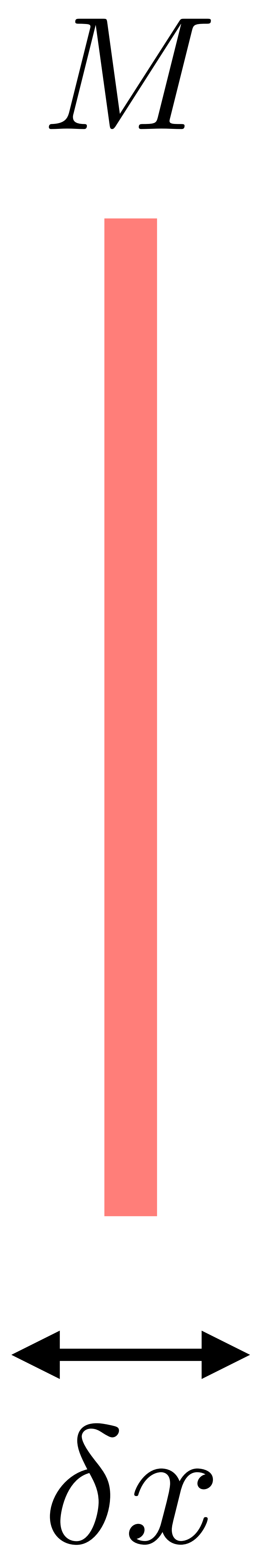
$$M\ddot{\delta x} = M\ddot{h}x + F_{\text{ext}}$$



DETECTOR II

$$M \to \infty$$

$$M\ddot{\delta x} = M\ddot{h}x + F_{\text{ext}}$$

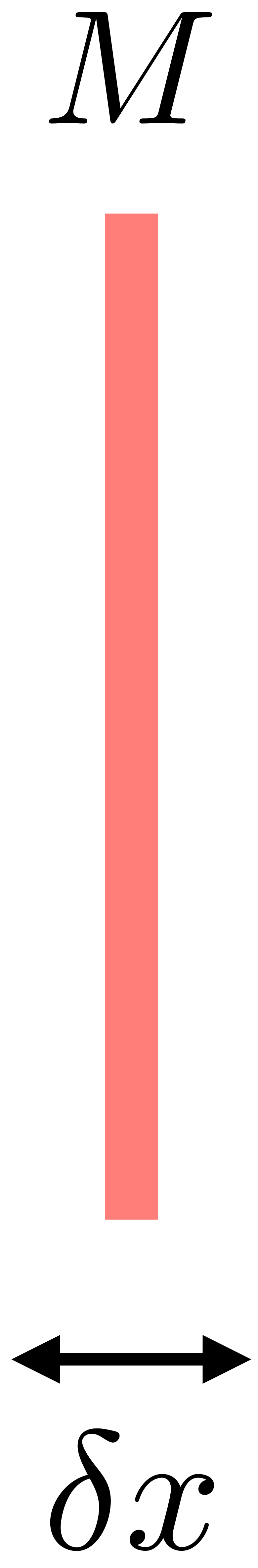


DETECTOR II

$$M \to \infty$$

$$M\ddot{\delta x} = M\ddot{h}x + F_{\mathrm{ext}}$$

$$\ddot{\delta x}_{\mathrm{sig}} \sim \ddot{h}x \sim \mathrm{const}$$



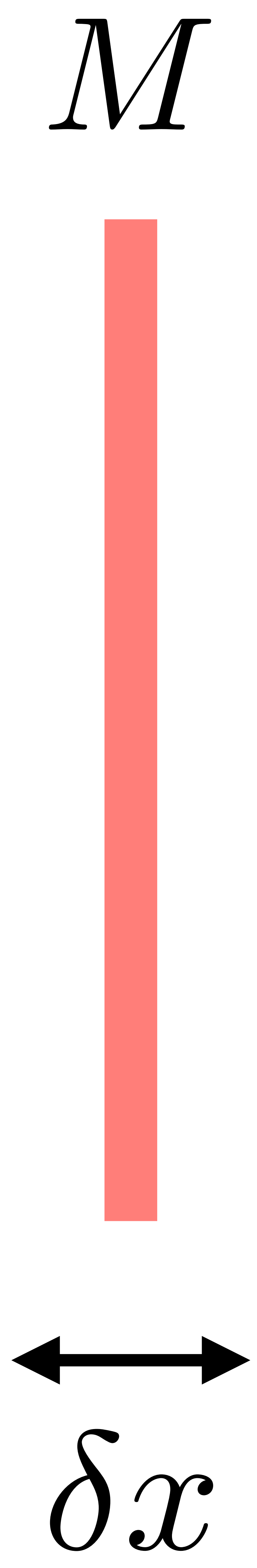
DETECTOR II

$$M \to \infty$$

$$M\ddot{\delta x} = M\ddot{h}x + F_{\mathrm{ext}}$$

$$\ddot{\delta x}_{\mathrm{sig}} \sim \ddot{h}x \sim \mathrm{const}$$

$$\ddot{\delta x}_{\mathrm{noise}} \sim \frac{F_{\mathrm{ext}}}{M} \to 0$$



THE BEST POSSIBLE SENSITIVITY

IN THE FOLLOWING I WILL ALWAYS CONSIDER

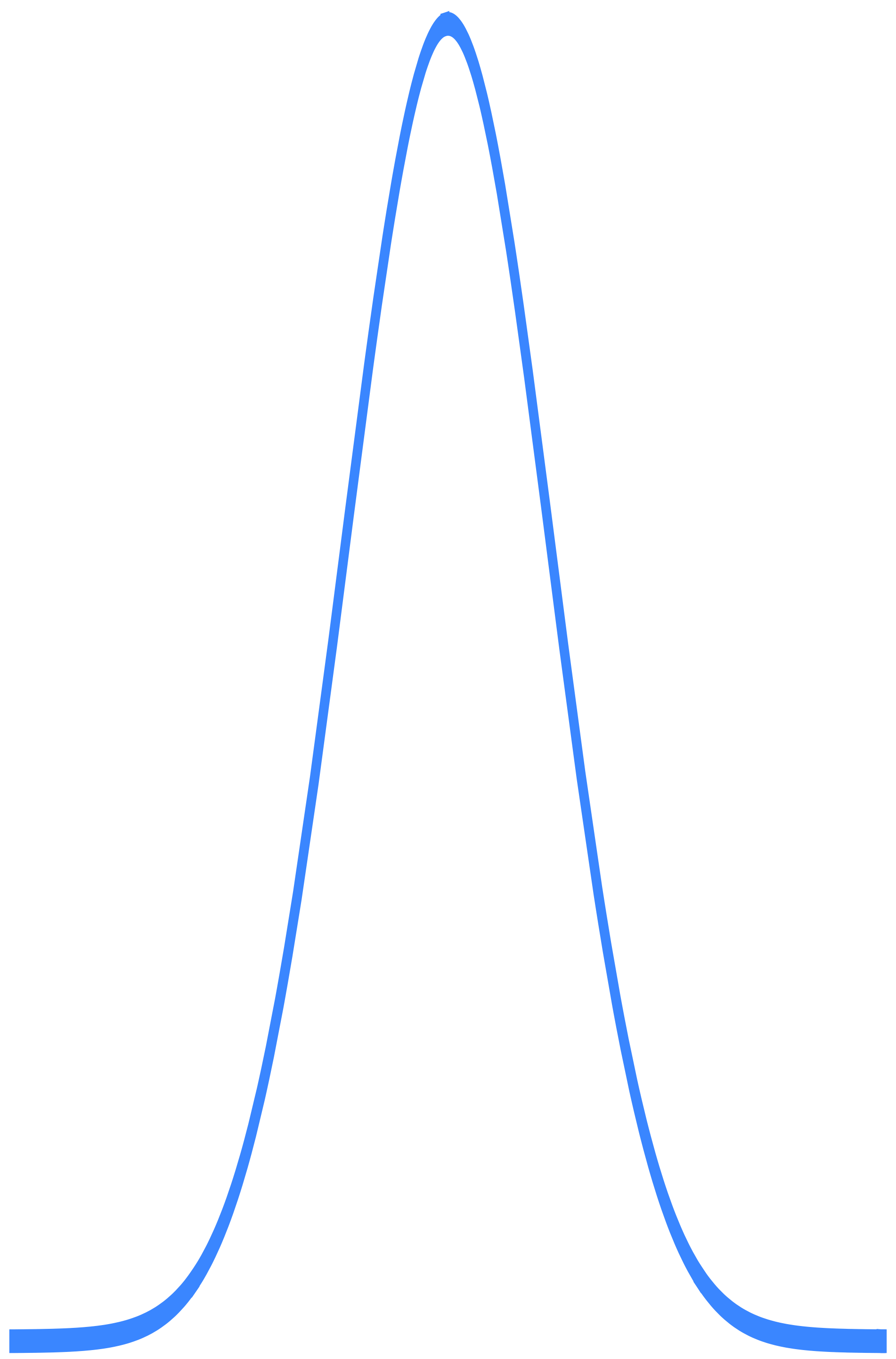
$$M \to \infty$$

$$\ddot{\delta x}_{\mathrm{noise}} \sim \frac{F_{\mathrm{ext}}}{M} \to 0$$

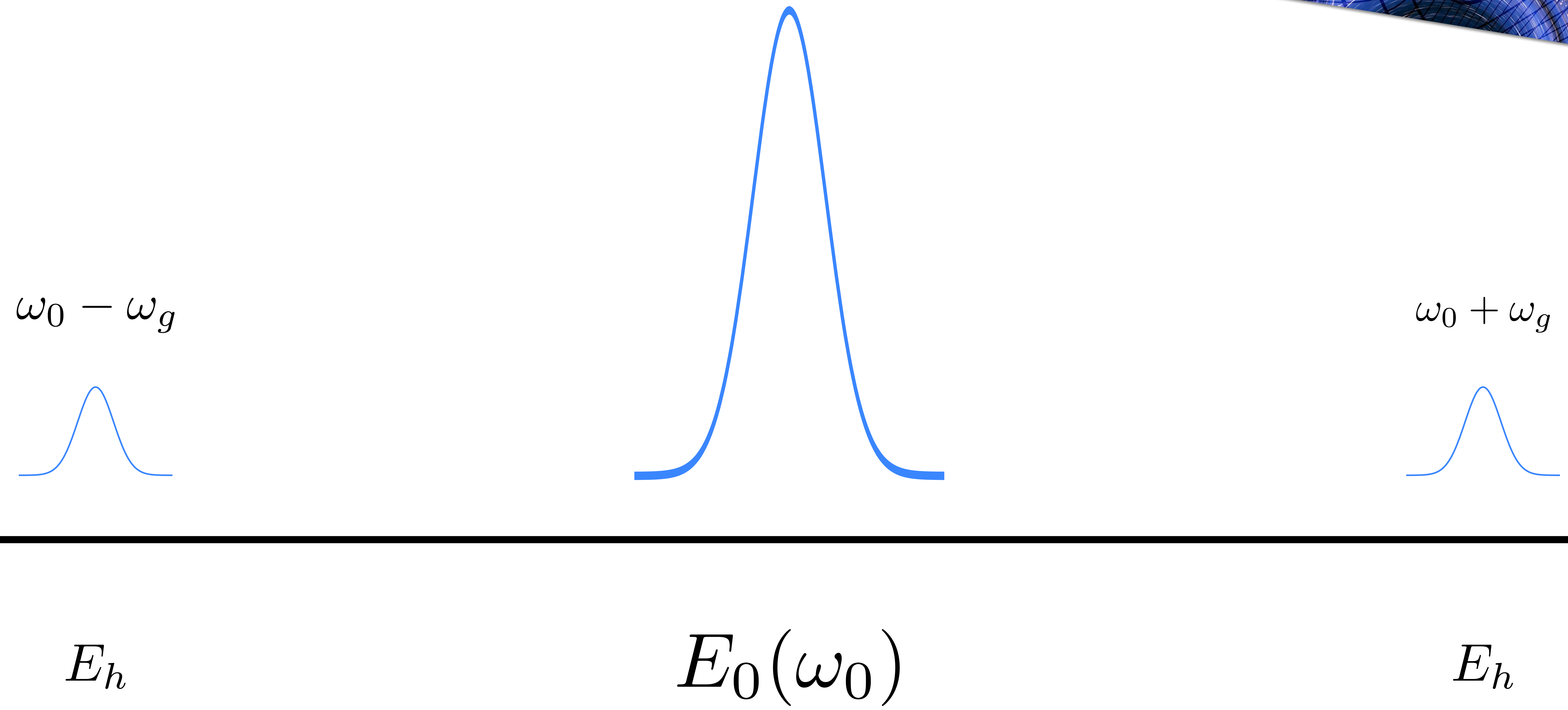
THE BEST POSSIBLE SENSITIVITY

In this limit I can ignore noise from the test mass and focus on the signal (and noise) photons that I can detect

DETECTOR I → $$U_{\rm in} \sim E_0^2 V_0$$ ← DETECTOR II



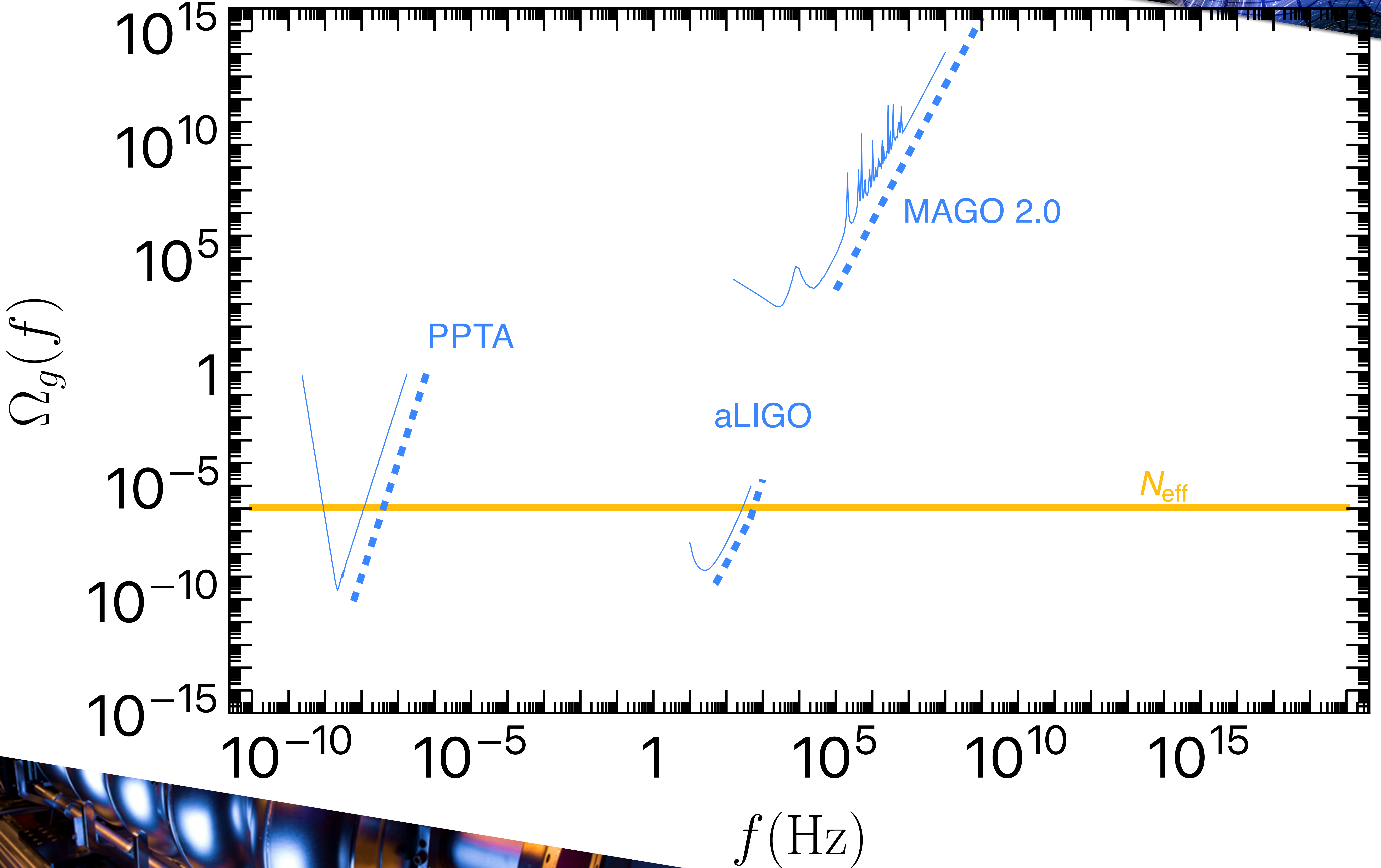
$$E_0(\omega_0)$$



$\omega_0 - \omega_g$
$\omega_0 + \omega_g$
E_h
$E_0(\omega_0)$
E_h

$$U_{\text{in}} \sim E_0^2 V_0$$

$$\text{———} \; \mathcal{T}(\omega) \; \text{———}$$



10^{15}
10^{10}
10^{5}
1
10^{-5}
10^{-10}
10^{-15}
$\Omega_g(f)$
MAGO 2.0
PPTA
aLIGO
N_{eff}
10^{-10}
10^{-5}
1
10^{5}
10^{10}
10^{15}
$f\,(\text{Hz})$


SIGNALS

$$U_{\text{sig}} \sim U_{\text{in}} \times \begin{cases} (h\mathcal{T})^2 \\ h\mathcal{T} \end{cases}$$

$$U_{\text{in}} \sim E_0^2 V_0$$

CASE I: QUADRATIC SIGNALS

$$\langle E_h(t)E_0(t)\rangle \propto \langle \tilde{E}_h(\omega)\tilde{E}_0(\omega)\rangle = 0$$

$$\langle E_h(t) E_0(t) \rangle \propto \langle \tilde{E}_h(\omega) \tilde{E}_0(\omega) \rangle = 0$$


$\Delta\omega_d$
Detector Bandwidth
$\tilde{E}_0(\omega)$
$\tilde{E}_h(\omega)$

In absence of a signal,
the detector is empty (classically)

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the detector is empty (classically)

$$P_{\text{min}} \simeq \frac{2\pi\omega}{t_{\text{int}}}$$

But we need at least one photon generated by the signal

QUADRATIC SIGNAL

$$P_{\mathrm{sig}} \lesssim \underline{h^2 U_{\mathrm{in}}} \, \omega_s \mathcal{T}^2(\omega_s)$$

Signal Energy

QUADRATIC SIGNAL

$$P_{\mathrm{sig}} \lesssim h^2 U_{\mathrm{in}} \underline{\omega_s} \mathcal{T}^2(\omega_s)$$

Maximum power
from Poynting's theorem

QUADRATIC SIGNAL

$$\frac{P_{\rm sig}}{P_{\rm min}} \simeq 1 \quad \longrightarrow \quad h_{\rm min} \gtrsim \sqrt{\frac{2\pi}{U_{\rm in} t_{\rm int}} \frac{1}{\mathcal{T}}}$$

ENERGY DENSITY

$$\Omega_g^{\min}(\omega) \simeq \frac{\omega^3 h_{\min}^2 t_{\text{int}}}{3H_0^2}$$

ENERGY DENSITY

$$\Omega_g^{\text{min}}(\omega) \simeq \frac{\omega^3 h_{\text{min}}^2 t_{\text{int}}}{3H_0^2}$$

$$U_{\text{in}}^{\text{BBN}} = 10^{12}\ J\ \left(\frac{\omega}{2\pi \times 80\text{kHz}}\right)\left(\frac{10^{-6}}{\Omega_g}\right)\left(\frac{10^{12}}{\mathcal{T}^2 \frac{\omega}{\Delta\omega}}\right)$$

ENERGY DENSITY

$$\Omega_g^{\min}(\omega) \simeq \frac{\omega^3 h_{\min}^2 t_{\text{int}}}{3H_0^2}$$

$$U_{\text{in}}^{\text{BBN}} \simeq U_{\text{ITER}}$$

$$U_{\text{in}}^{\text{BBN}} = 10^{12}\, J \left(\frac{\omega}{2\pi \times 80\text{kHz}}\right)\left(\frac{10^{-6}}{\Omega_g}\right)\left(\frac{10^{12}}{\mathcal{T}^2 \frac{\omega}{\Delta\omega}}\right)$$

CASE II: LINEAR SIGNALS

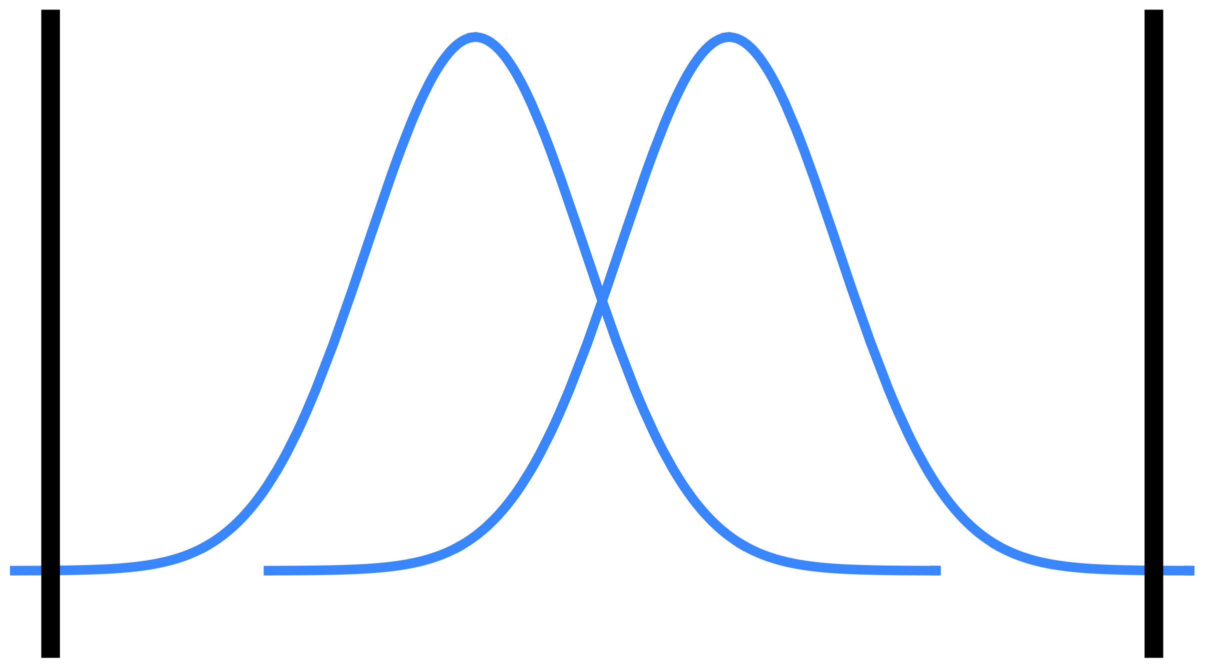
$$\langle E_h(t) E_0(t) \rangle \propto \langle \tilde{E}_h(\omega) \tilde{E}_0 \omega) \rangle \neq 0$$

$\tilde{E}_0(\omega)$ $\tilde{E}_h(\omega)$

CASE II: LINEAR SIGNALS

$$\Delta\omega_d$$

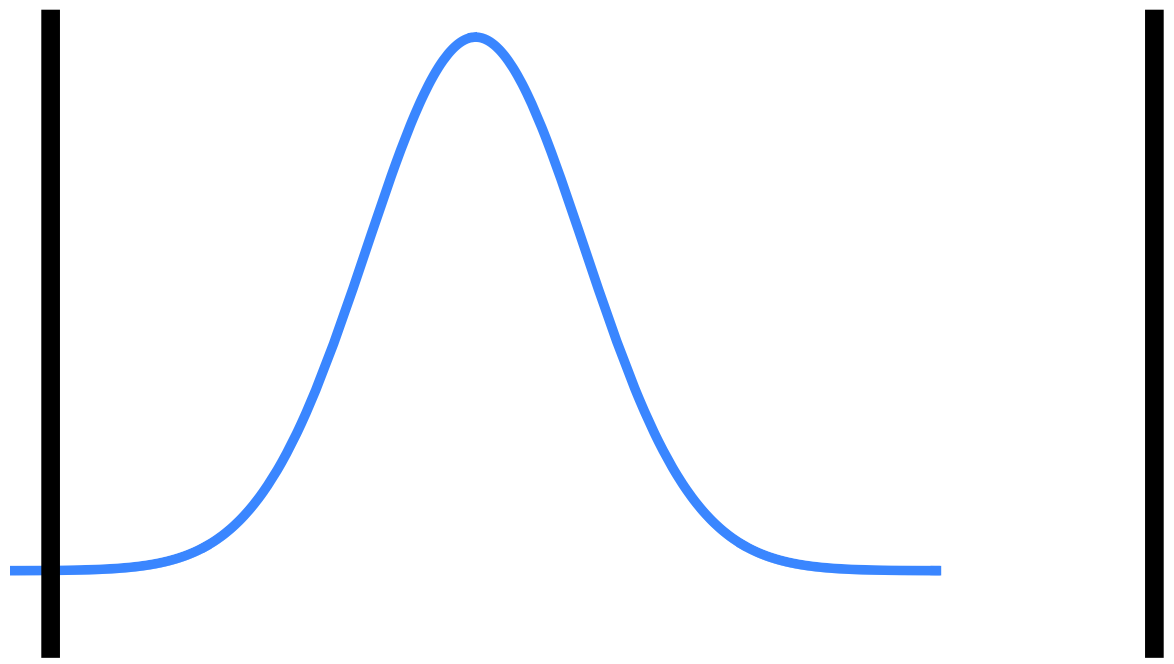
Detector Bandwidth



$$\tilde{E}_0(\omega) \qquad \tilde{E}_h(\omega)$$

CASE II: LINEAR SIGNALS

In absence of a signal,
the detector is not empty



$$P^{\min}_{\text{noise}} \simeq \frac{2\pi\omega}{t_{\text{int}}}\left(1+\sqrt{\frac{P_{\text{in}}t_{\text{int}}}{2\pi\omega}}\right)$$

NARROW LINEAR SIGNAL

$$\frac{P_{\rm sig}}{P_{\rm noise}} \simeq 1 \quad \rightarrow \quad h_{\rm min} \gtrsim \sqrt{\frac{2\pi}{U_{\rm in} t_{\rm int}} \frac{1}{\mathcal{T}}}$$

Same as quadratic!


STATISTICS INTERLUDE

STOCHASTIC BACKGROUND

Stationary Gaussian Process with Zero Mean

$$\langle h(t) \rangle = 0$$

$$\langle h(t) h(t') \rangle = H(t' - t)$$

SNR

Single Detector

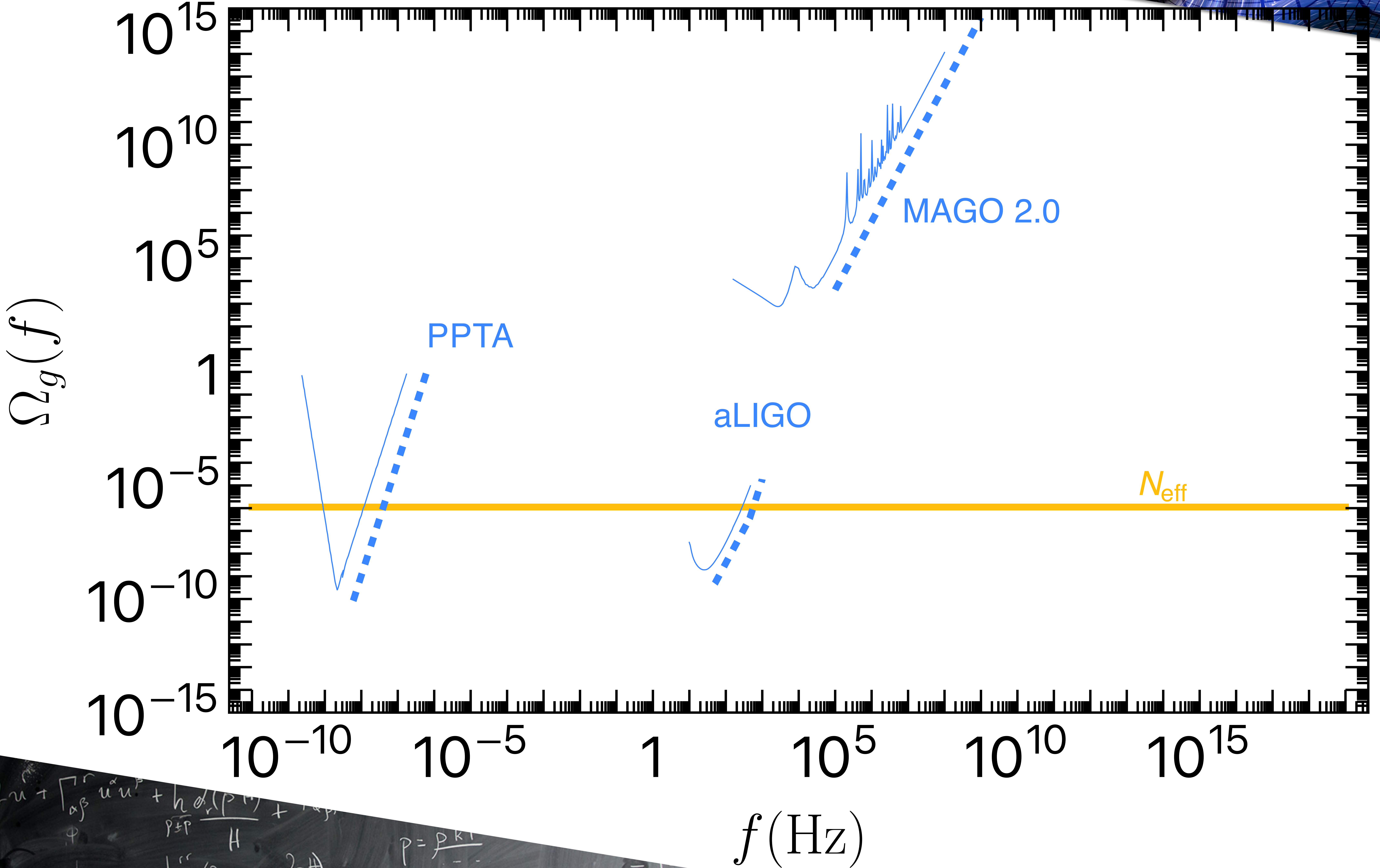
$$\mathrm{SNR} \simeq \left(\frac{S_h(\omega_s)}{S_n(\omega_s)} \right)^{1/2}$$

Two Detectors Optimal Filtering

$$\mathrm{SNR} = \left(t_{\mathrm{int}} \int d\omega \Gamma^2(\omega) \frac{S_h^2(\omega)}{S_n^2(\omega)} \right)^{1/4}$$

$$\Omega_g \to \Omega_g \times \frac{1}{\sqrt{\Delta\omega t_{\mathrm{int}}}}$$

If the total energy density is not fixed



10^{15}
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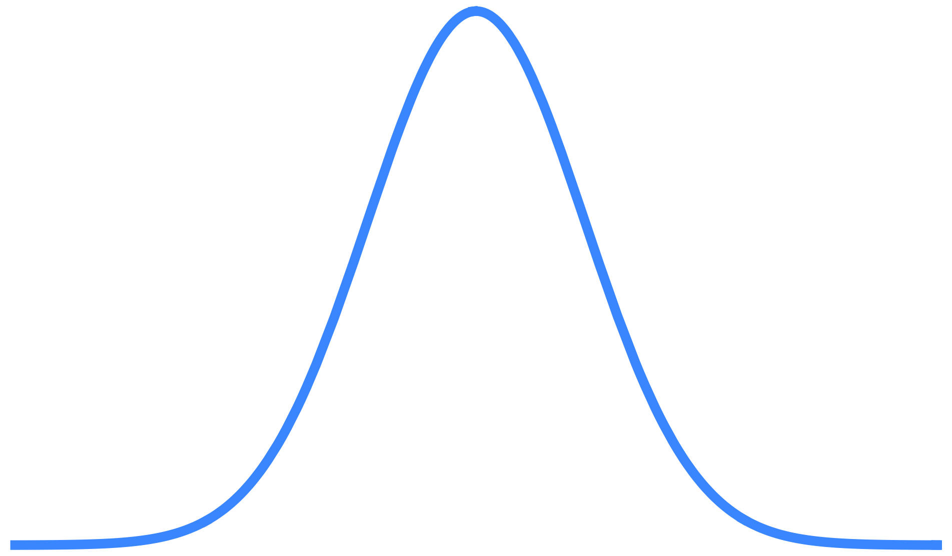
$\Omega_g(f)$
MAGO 2.0
PPTA
aLIGO
$N_{\rm eff}$
10^{-10}
10^{-5}
1
10^{5}
10^{10}
10^{15}
$f\,(\mathrm{Hz})$


BEYOND THE QUANTUM LIMIT

NO QUALITATIVE CHANGE FOR QUADRATIC SIGNALS

$$P_{\mathrm{SQL}} \simeq \frac{2\pi\omega}{t_{\mathrm{int}}}$$



$$P_{\mathrm{min}} \simeq \frac{2\pi\omega}{t_{\mathrm{int}}}$$

LINEAR

$$P_{\text{noise}}^{\text{min}} \simeq \frac{2\pi\omega}{t_{\text{int}}} \left(1 + \sqrt{\frac{P_{\text{in}} t_{\text{int}}}{2\pi\omega}} \right)$$

$$\simeq \frac{2\pi\omega}{t_{\text{int}}} \left(1 + \sqrt{N_\gamma} \right)$$

LINEAR

In principle

$$P_{\text{noise}}^{\text{min}} \rightarrow \frac{2\pi\omega}{t_{\text{int}}}$$

Heisenberg Limit

In practice

$$P_{\text{noise}}^{\text{min}} \to \frac{2\pi\omega}{t_{\text{int}}} \quad \longrightarrow \quad \sqrt{N_\gamma} \to 1$$

You need to control a huge number of photons (a billion for LIGO) at the single photon level


WHAT DID WE LEARN?

QUADRATIC vs LINEAR

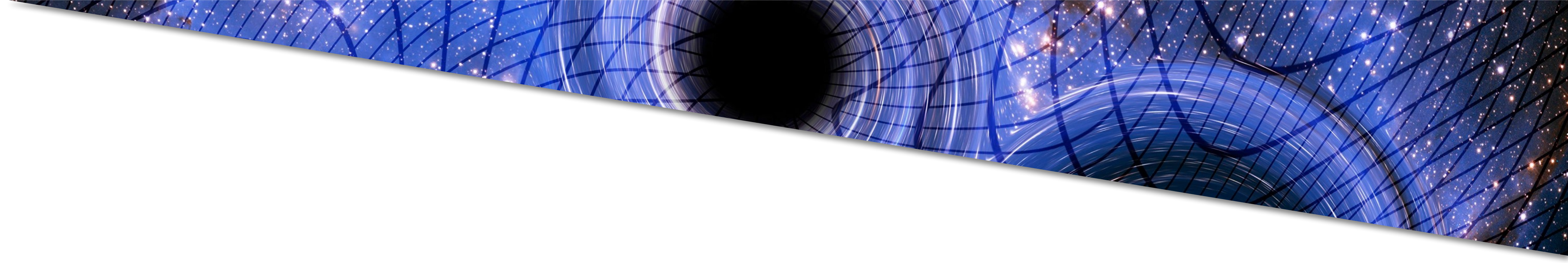
$$h_{\mathrm{min}} \gtrsim \sqrt{\frac{2\pi}{U_{\mathrm{in}} t_{\mathrm{int}}}} \frac{1}{\mathcal{T}}$$

ENERGY DENSITY

$$\Omega_g^{\min}(\omega) \simeq \frac{\omega^3 h_{\min}^2 t_{\rm int}}{3H_0^2}$$

$$U_{\rm in}^{\rm BBN} = 10^{12}\, J \left(\frac{\omega}{2\pi \times 80{\rm kHz}}\right)\left(\frac{10^{-6}}{\Omega_g}\right)\left(\frac{10^{12}}{\mathcal{T}^2 \frac{\omega}{\Delta\omega}}\right)$$

QUADRATIC vs LINEAR



QUADRATIC

$$P_{\text{noise}}^{\text{min}} \simeq \frac{2\pi\omega}{t_{\text{int}}}$$

LINEAR

$$P_{\text{noise}}^{\text{min}} \simeq \frac{2\pi\omega}{t_{\text{int}}}\left(1+\underline{\sqrt{\frac{P_{\text{in}}t_{\text{int}}}{2\pi\omega}}}\right)$$

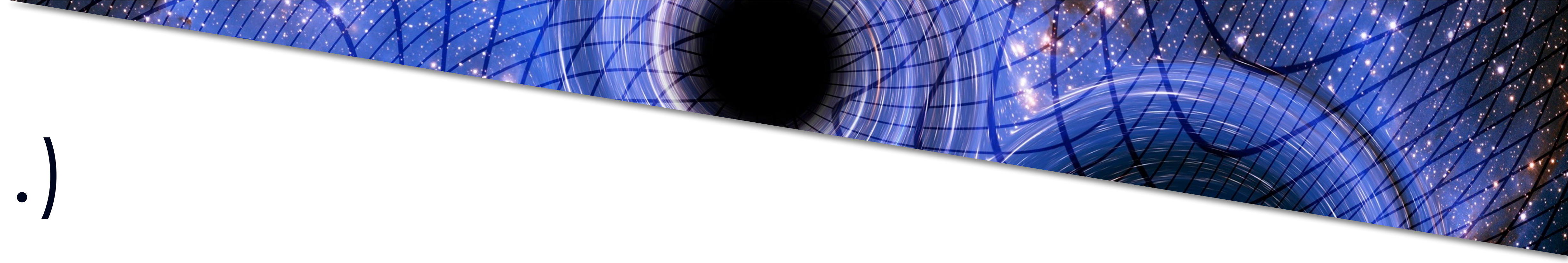
We can gain from quantum techniques

THE MISSING PIECE

$$h_{\rm min} \gtrsim \sqrt{\frac{2\pi}{U_{\rm in} t_{\rm int}} \boxed{\frac{1}{\tau}}}$$

???

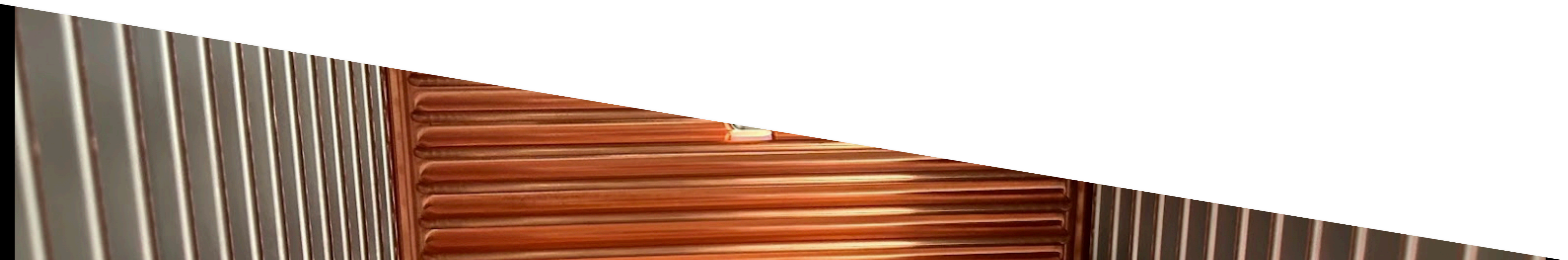

TRANSFER FUNCTIONS

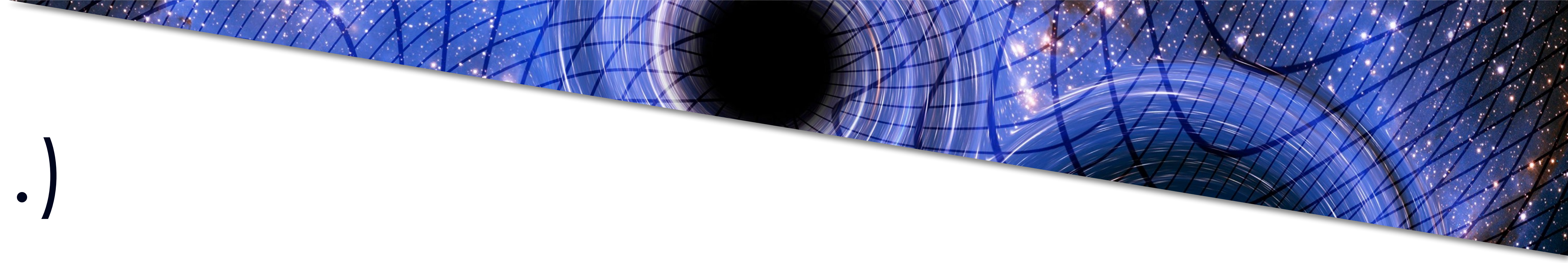


MOST DETECTORS (LIGO, OPTOMECHANICAL, WEBER BARS…)

$$\left(\omega_m^2 - \omega^2 + i\frac{\omega\omega_m}{Q_m}\right)\tilde{u}_m(\omega) \simeq -\frac{1}{2}\omega_g^2 V^{1/3}\tilde{h}(\omega,\omega_g)$$

The wave excites a mechanical mode



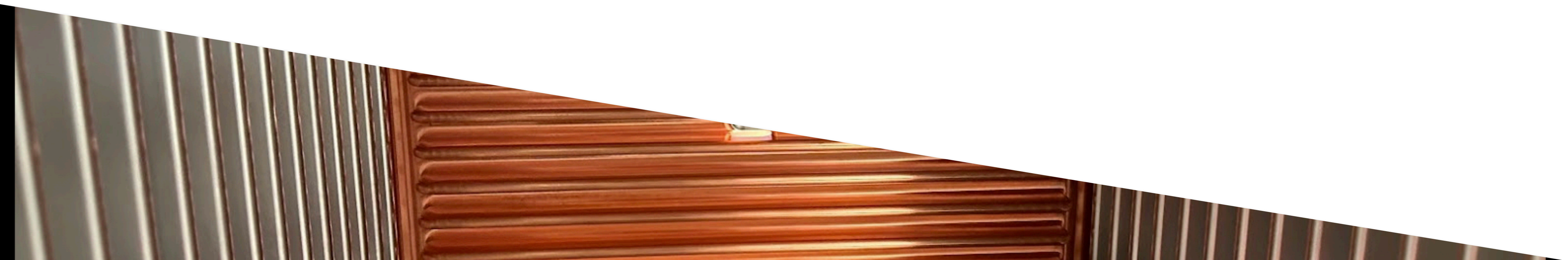


EVERYTHING ELSE
(LIGO, OPTOMECHANICAL, WEBER BARS…)

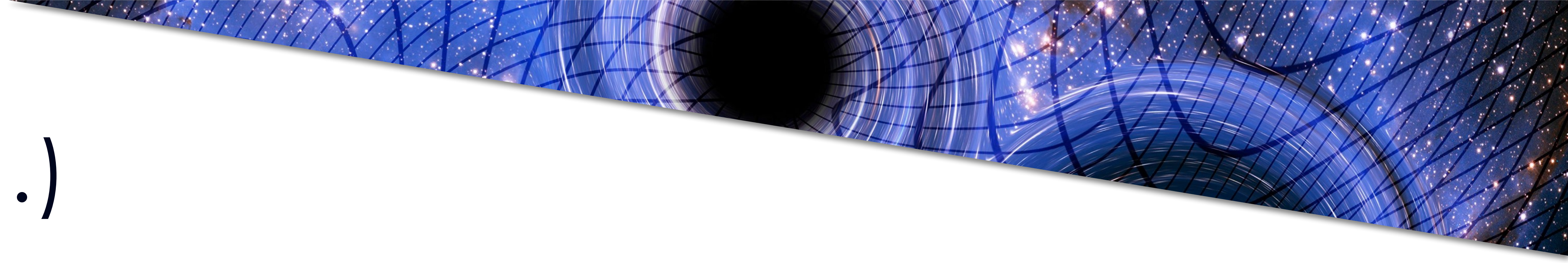
$$\left(\omega_m^2 - \omega^2 + i\frac{\omega\omega_m}{Q_m}\right)\tilde{u}_m(\omega) \simeq -\frac{1}{2}\omega_g^2 V^{1/3}\tilde{h}(\omega,\omega_g)$$

$$\left(\omega_1^2 - \omega^2 + i\frac{\omega\omega_1}{Q}\right)\tilde{E}_h(\omega) \simeq -2\omega_1^2 V^{-1/3}\int d\omega'\,\tilde{u}_m(\omega'-\omega)\tilde{E}_0(\omega')$$

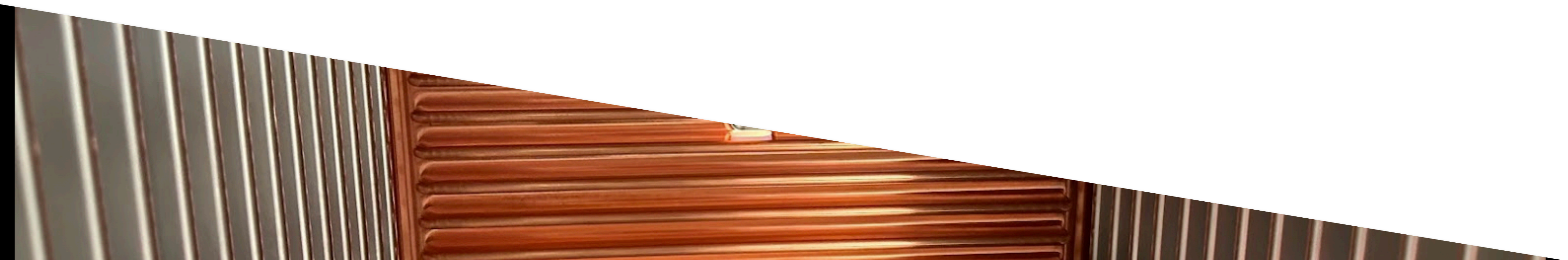
An EM resonator does the readout



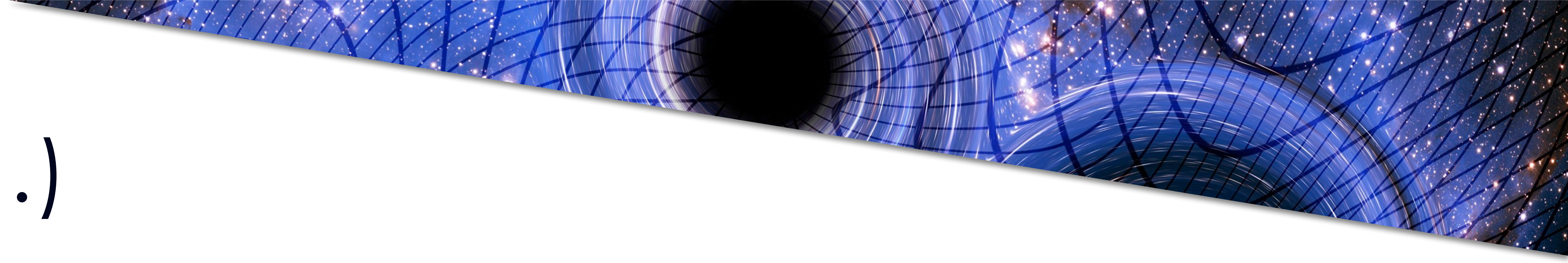
EVERYTHING ELSE
(LIGO, OPTOMECHANICAL, WEBER BARS…)



$$\mathcal{T}^2(\omega) = \frac{\omega_g^4 \omega_1^4}{\left((\omega_1^2 - \omega^2)^2 + \frac{\omega^2 \omega_1^2}{Q^2}\right)\left((\omega_m^2 - \omega_g^2)^2 + \frac{\omega_g^2 \omega_m^2}{Q_m^2}\right)}$$

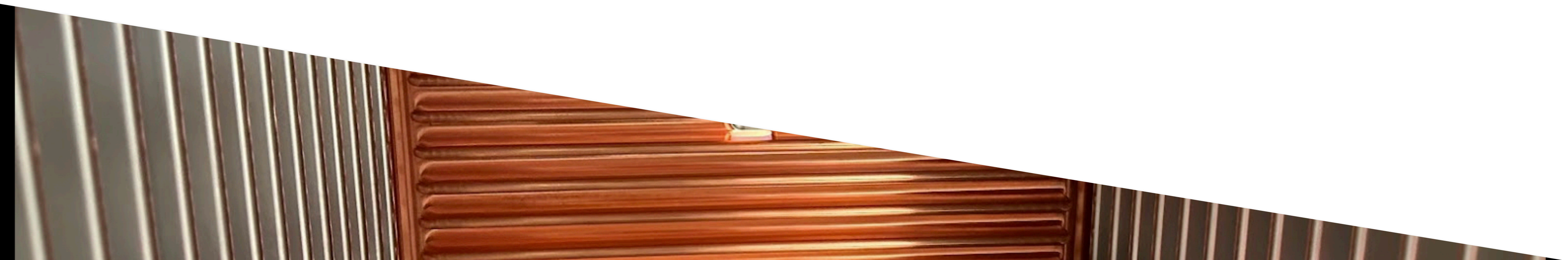


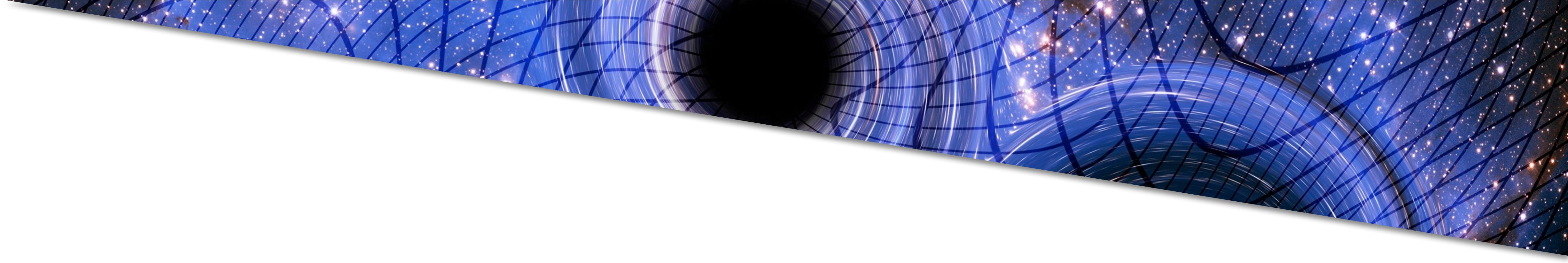
EVERYTHING ELSE
(LIGO, OPTOMECHANICAL, WEBER BARS…)



$$\mathcal{T}^2_{\mathrm{LIGO}} \simeq \frac{\omega_L^2}{\left(4\omega_g^2 + \frac{\omega_L^2}{Q^2}\right)} \simeq \frac{\omega_L^2 L_{\mathrm{eff}}^2}{(4\omega_g^2 L_{\mathrm{eff}}^2 + 1)}$$

$$\omega_0 \simeq \omega_1 \simeq \omega_L \gg \omega_g \gg \omega_m$$

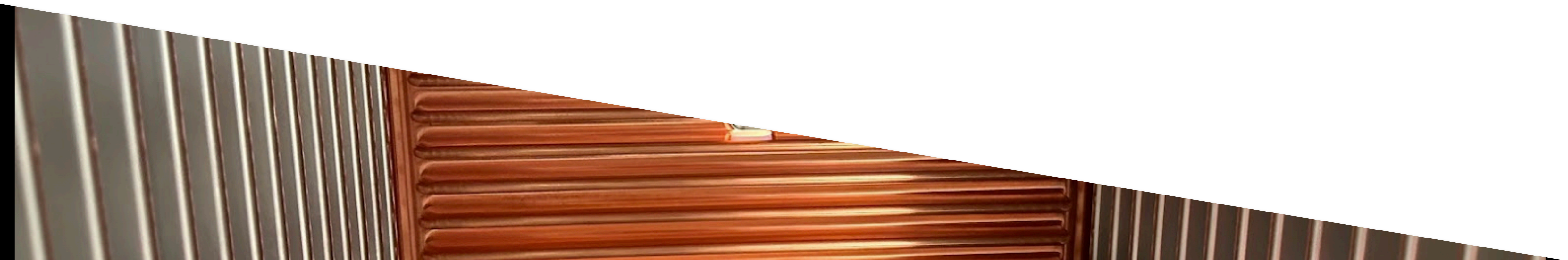


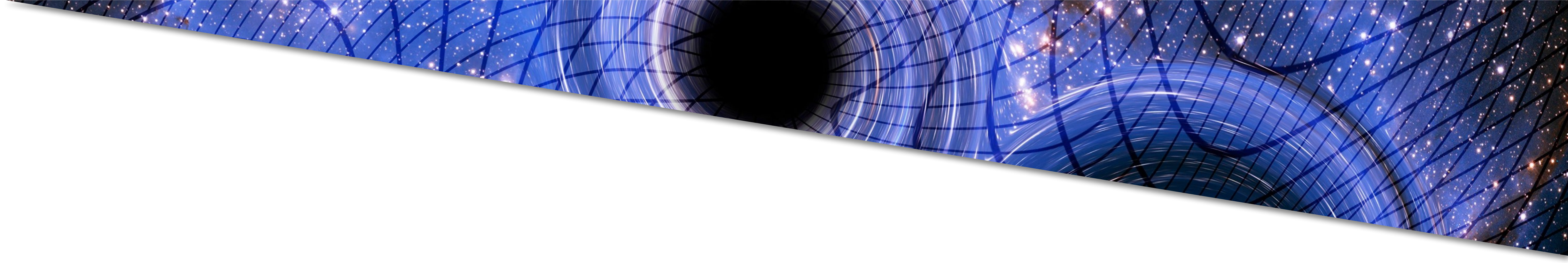


INTERFEROMETERS vs RESONATORS

$$\mathcal{T}_{\mathrm{LIGO}} \lesssim 10^{10}$$

$$\mathcal{T}_{\mathrm{res}} \simeq Q \lesssim 10^{12}$$

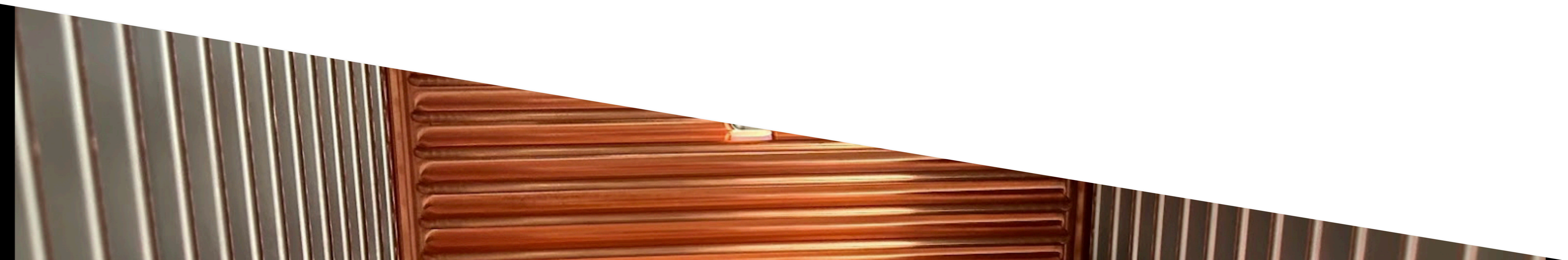




INTERFEROMETERS vs RESONATORS

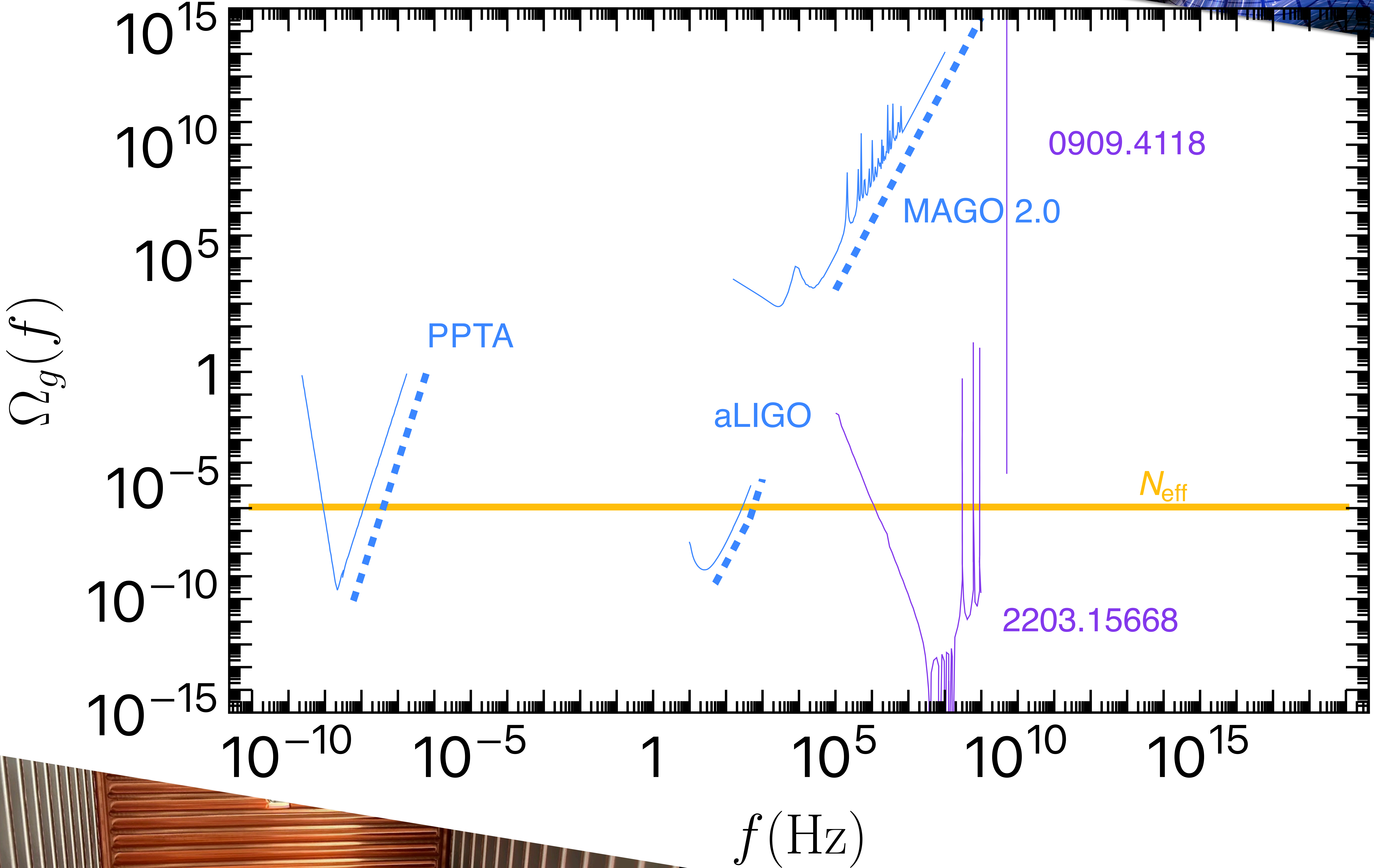
$$\Omega_g \sim \left(\mathcal{T}_{\text{int}}\right)^{-2}$$

$$\Omega_g \sim \left(\mathcal{T}_{\text{res}} \sqrt{\frac{\omega}{\Delta\omega}} \simeq \sqrt{Q}\right)^{-2}$$

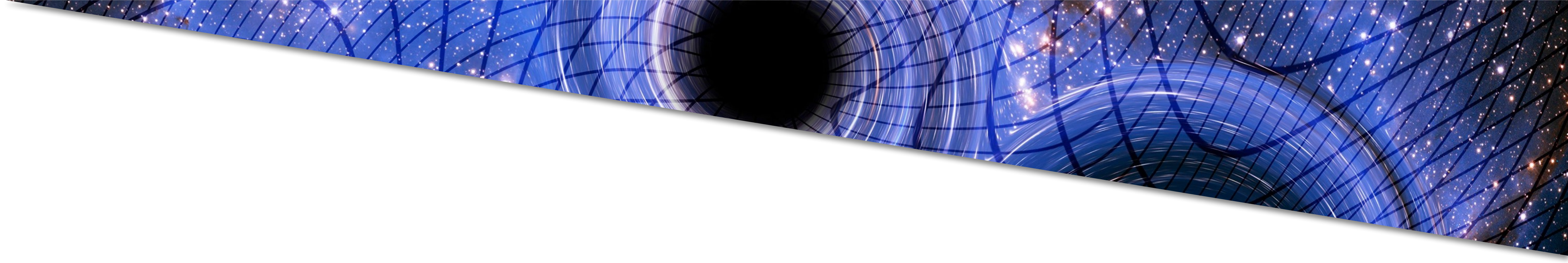



SUSPICIOUS SENSITIVITIES

SOME SUSPICIOUS RESULTS

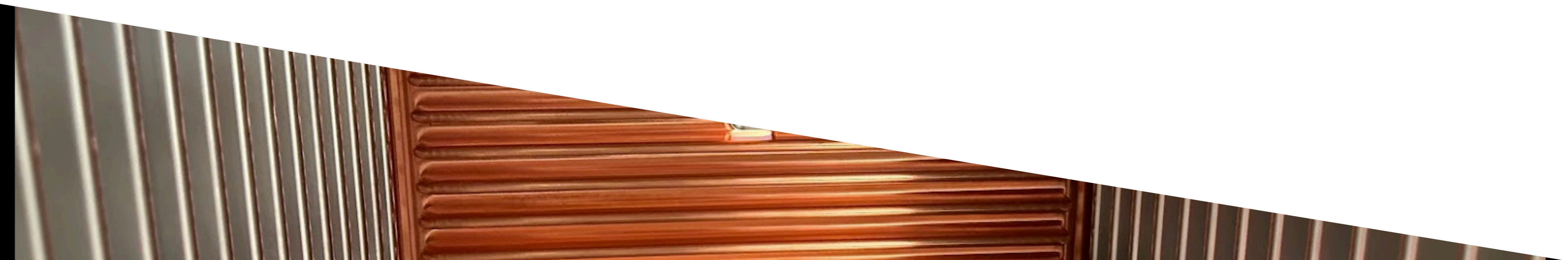


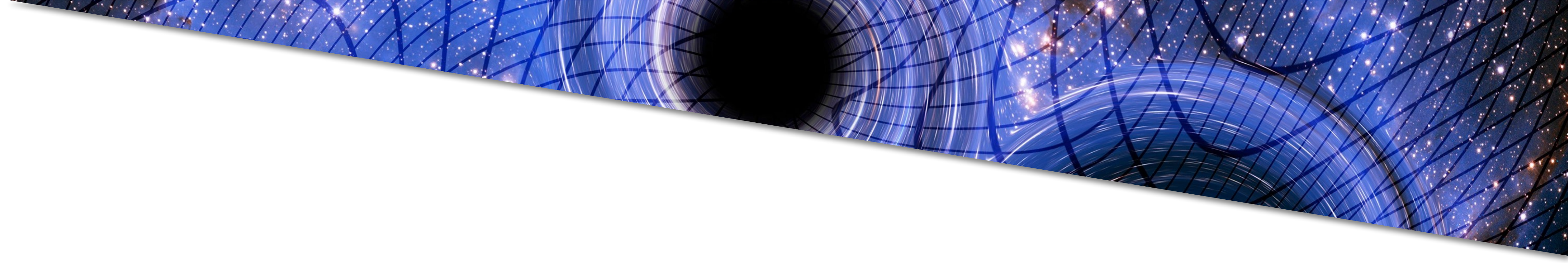
LI-BAKER



$$h_{\mathrm{min}} \gtrsim 10^{-24} \qquad h_{\mathrm{claim}} \gtrsim 10^{-30}$$

F. Li et al., Phys. Rev. D 80, 064013 (2009), 0909.4118



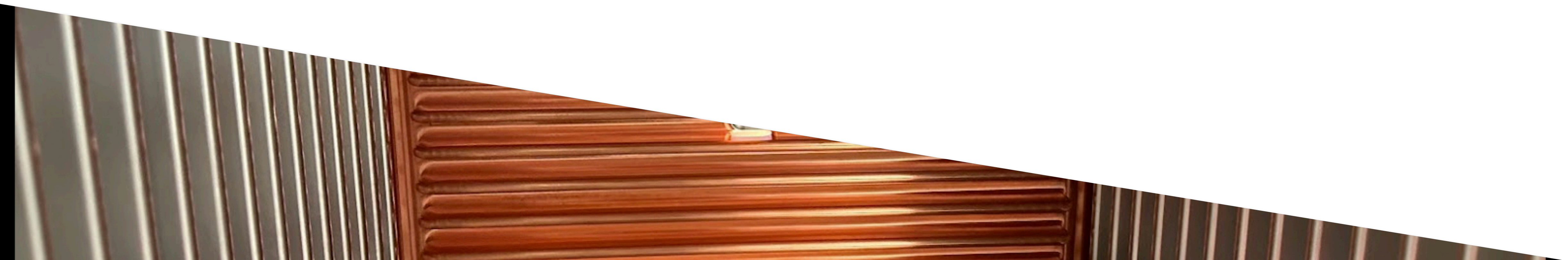


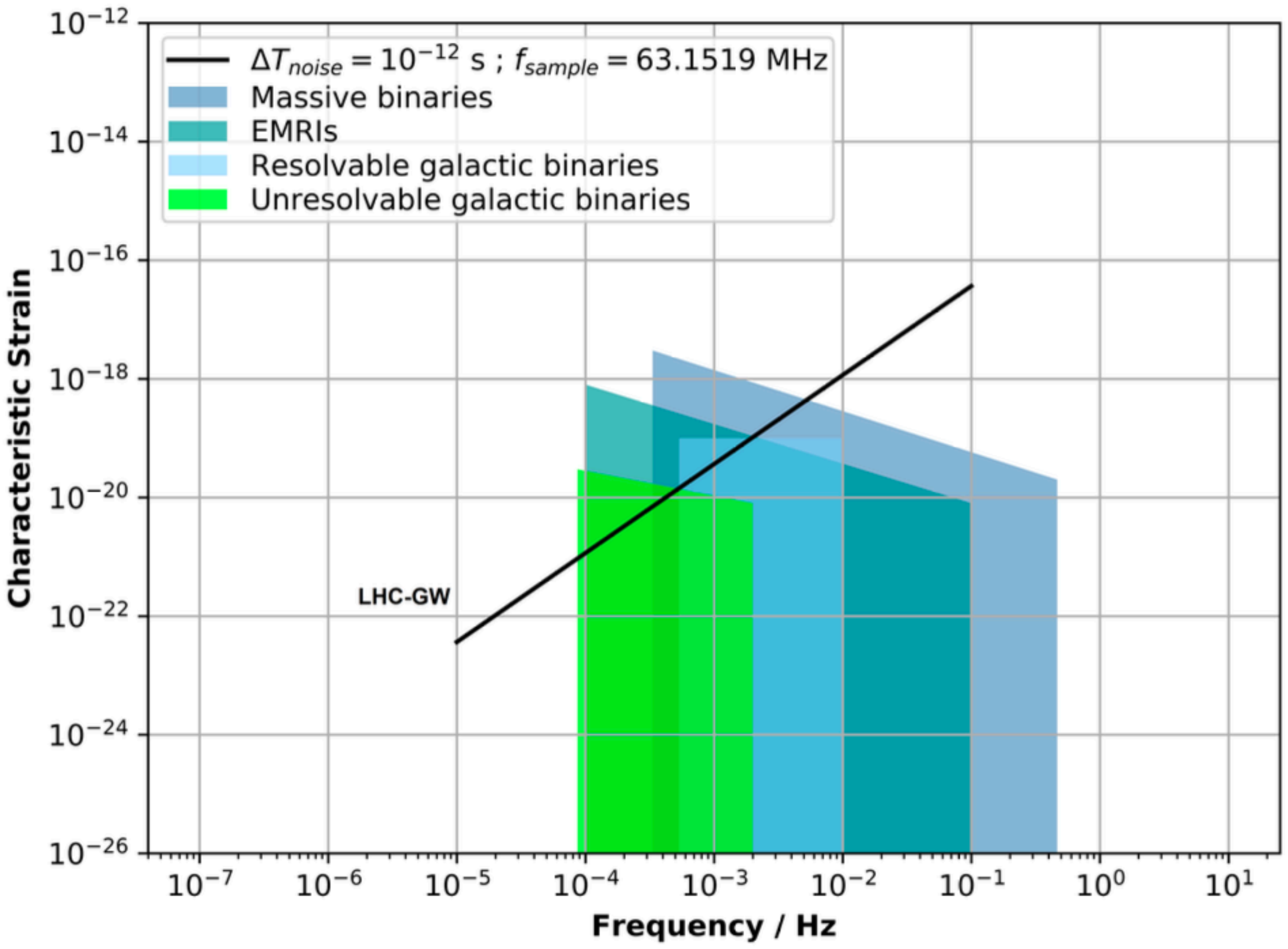
LINEAR CAVITIES

$$h_{\text{min}} \gtrsim 10^{-20}$$

$$h_{\text{claim}} \gtrsim 10^{-24} - 10^{-36}$$

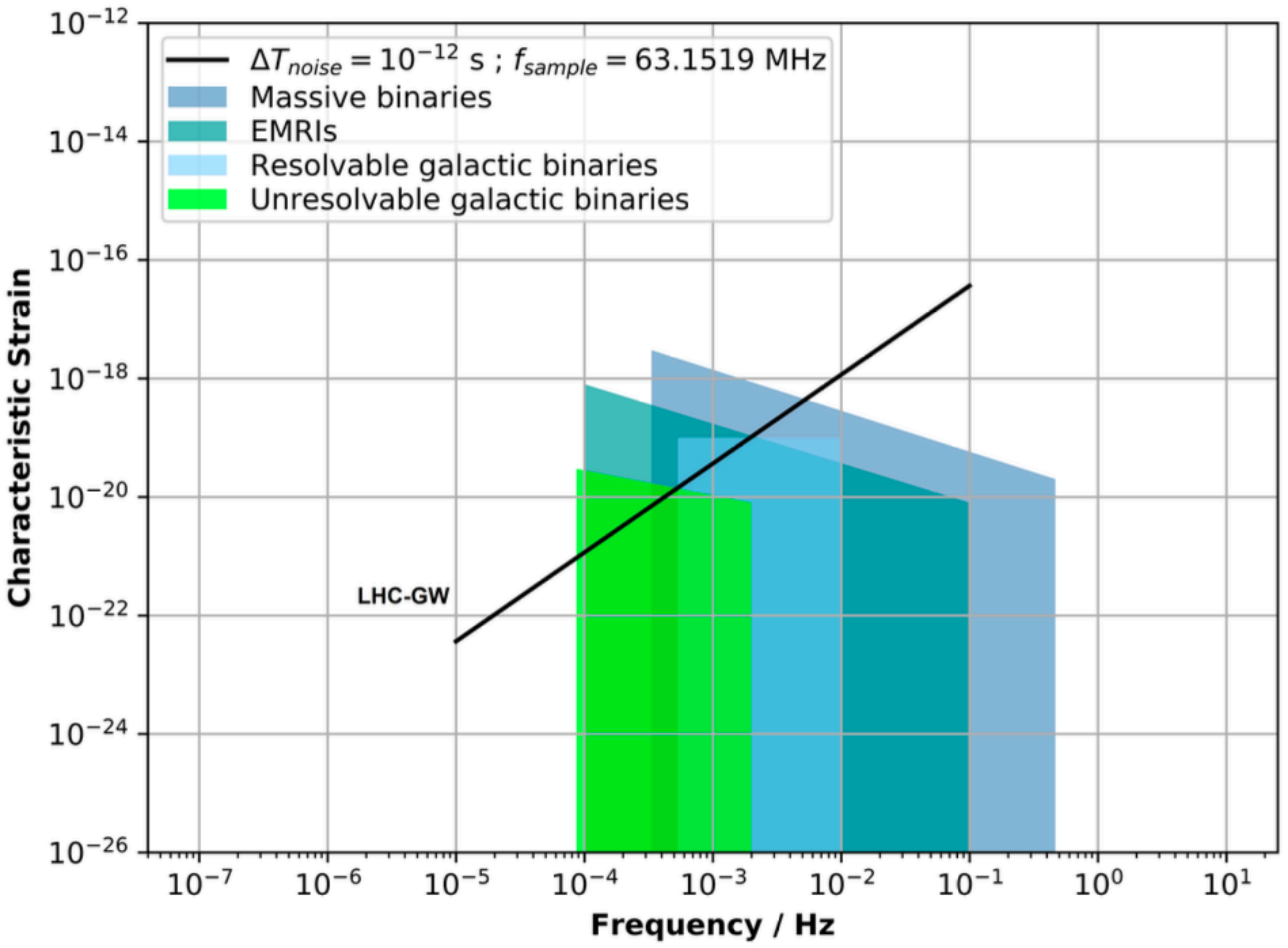
N. Herman, L. Lehoucq, and A. Fúzfa, Phys. Rev. D 108, 124009 (2023), 2203.15668





Rao, Bruggen, Lisle

Phys.Rev.D 102 (2020) 12, 122006, *Phys.Rev.D* 105 (2022) 6, 069903 (erratum)

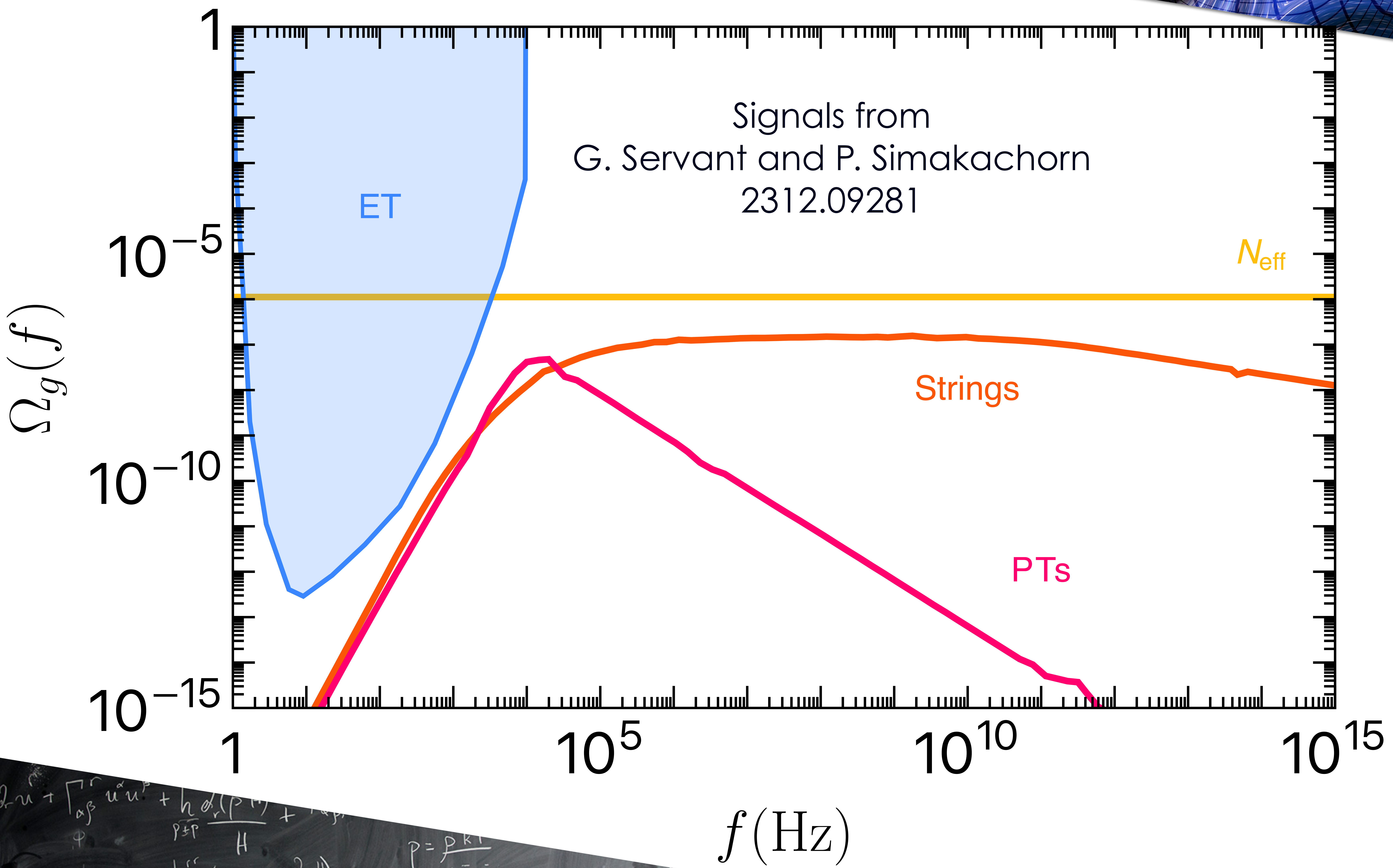


$$h \gtrsim 10^{-11} \qquad \mathcal{T}_{\rm LHC} \simeq \frac{\omega_g^2}{(10\ \rm Hz)^2}$$

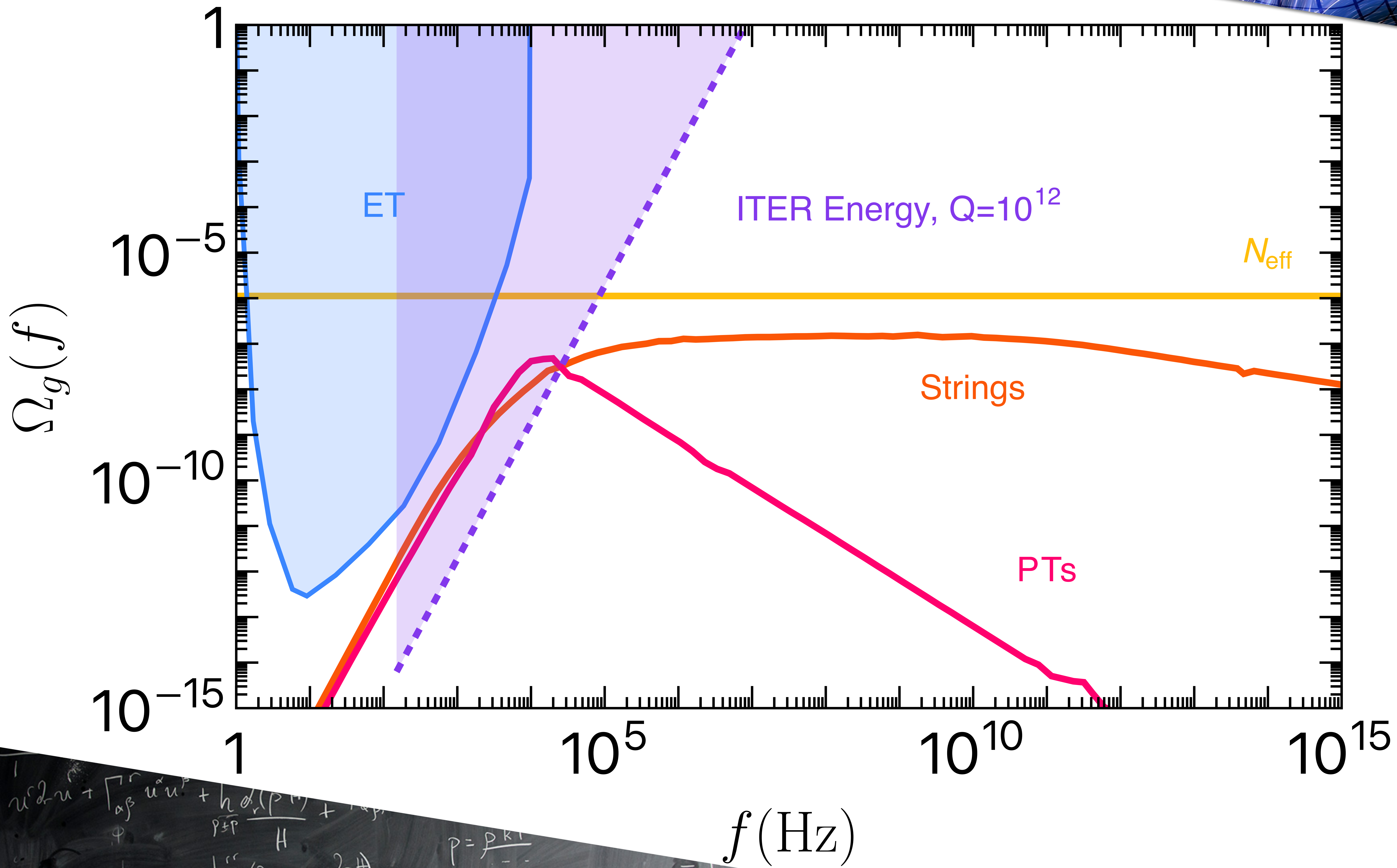
ACKNOWLEDGED IN
2301.08331

CONCLUSION

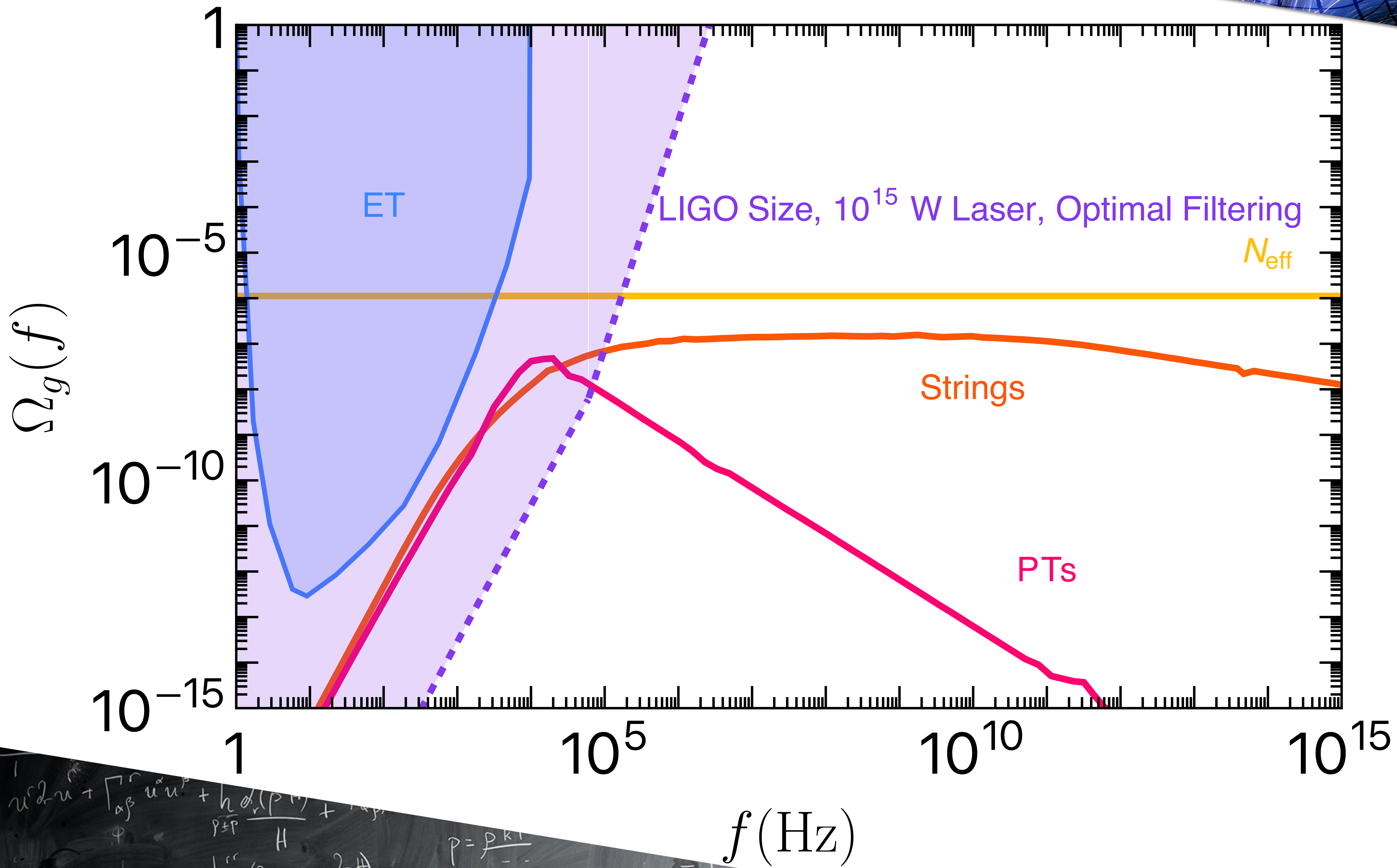
SOME LARGE SIGNALS



"CRAZY" RESONATOR

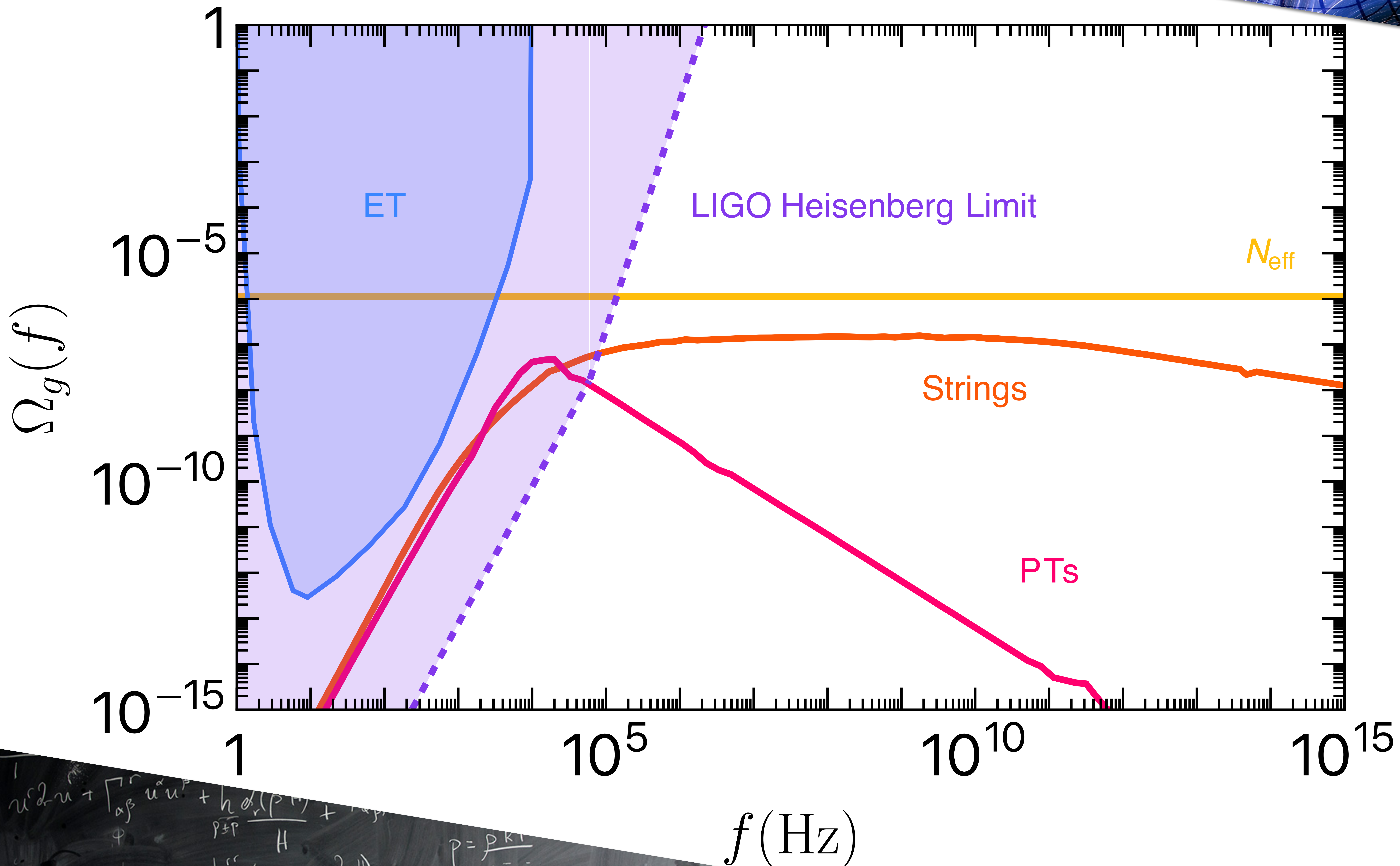


"CRAZY" LASER



ET
LIGO Size, 10^{15} W Laser, Optimal Filtering
N_{eff}
Strings
PTs
1
10^{-5}
10^{-10}
10^{-15}
$\Omega_g(f)$
1
10^5
10^{10}
10^{15}
$f\,(\mathrm{Hz})$

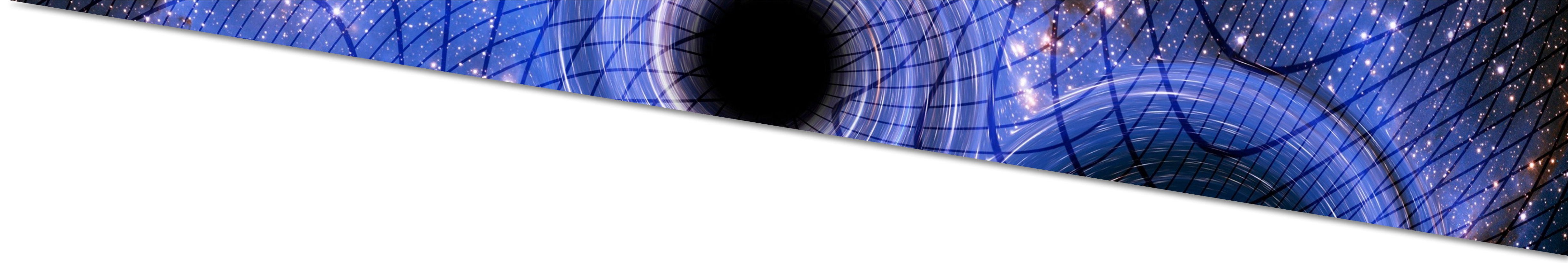
"CRAZY" QUANTUM LASER



CONCLUSION

$$\Omega_g(\omega_g) \sim \omega_g^3 h^2$$

BACKUP

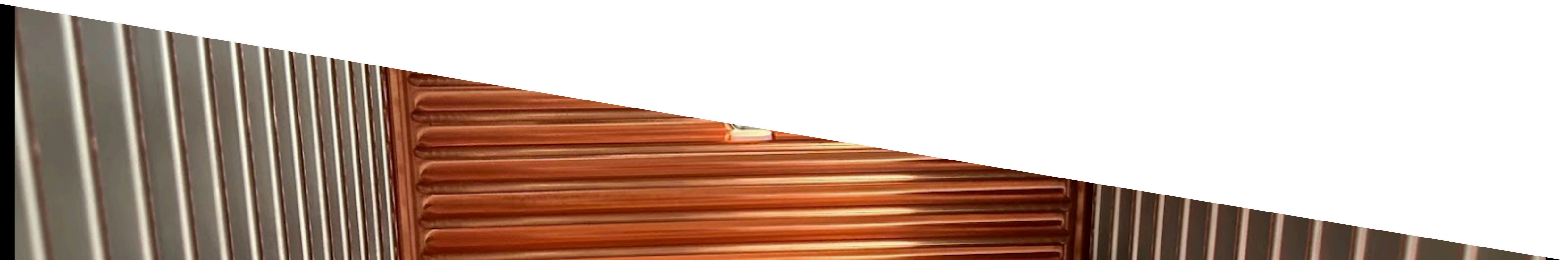


EM RESONATOR

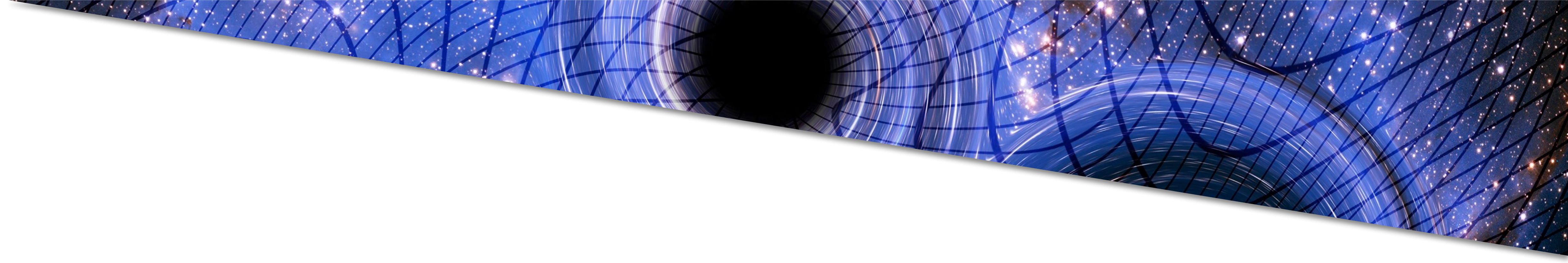
$$\left(\omega_1^2 - \omega^2 + i\frac{\omega\omega_1}{Q}\right)\tilde{E}_h(\omega) \simeq -C_g(\omega_0 \pm \omega_g)^2(\omega_g V^{1/3})^2 h E_0$$

$$\frac{\omega_s^2\omega_1^2}{Q^2}U_{\text{sig}} = h^2 U_{\text{in}}(\omega_g V^{1/3})^4(\omega_0 \pm \omega_g)^4$$

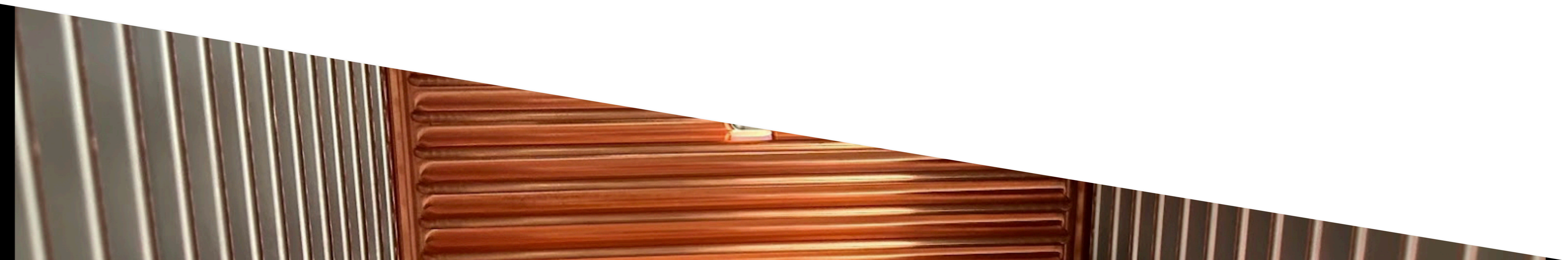
$$\mathcal{T}^2 = (\omega_g V^{1/3})^4 \frac{(\omega_0 \pm \omega_g)^4}{\omega_s^2\omega_1^2} Q^2$$



EM RESONATOR

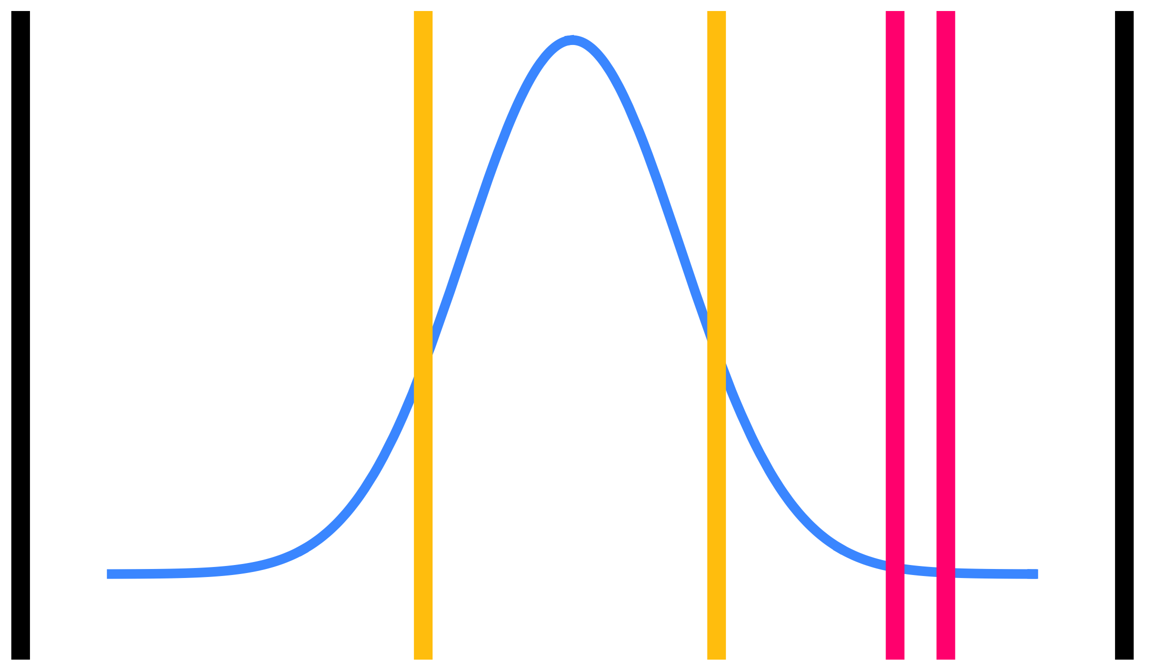


$$\mathcal{T}^2 \simeq \begin{cases} Q^2\,, & \mathrm{ADMX-like}: \omega_s = \omega_g = \omega_1,\ \omega_0 = 0\,, \\ Q^2\,\dfrac{\omega_g^4}{\omega_0^4}\,, & \mathrm{MAGO-like}: \omega_s = \omega_0 = \omega_1 \gg \omega_g\,. \end{cases}$$



BROAD SIGNAL

Detector Bandwidth $\Delta\omega_d$



t_{int}^{-1}

Resolution

Signal Width $\Delta\omega_s$

$$\Delta\omega \equiv \max[\min[\Delta\omega_d, \Delta\omega_s], t_{\text{int}}^{-1}]$$

BROAD SIGNAL

$$h_{\min}(\Delta f)\mathcal{T} \simeq \begin{cases} \sqrt{\dfrac{2\pi}{U_{\rm in}}}\left(\dfrac{\Delta\omega}{2\pi t_{\rm int}}\right)^{1/4} & \mathcal{O}(h^2) \\ \sqrt{\dfrac{2\pi}{U_{\rm in} t_{\rm int}}} & \mathcal{O}(h) \end{cases}$$

$$\gtrsim \sqrt{\frac{2\pi}{U_{\rm in} t_{\rm int}}}$$

Same as before

SIGNALS (strings)

$$V = \frac{\lambda}{2}\left(|\phi|^2 - v^2\right)$$

$$\mu \simeq v^2 \begin{cases} 1 \\ \log \frac{m_\phi}{H} \end{cases}$$

$$G_N \mu = 10^{-5}$$

SIGNALS

PT: $\frac{\beta}{H} = 7 \quad \alpha = 10 \quad T \simeq 10^{10}\ \text{GeV}$

InfKin: $H_I \simeq 10^{16}\ \text{GeV}$ + late time kination from QCD axion DM

SAME BUT QUANTUM (EM RESONATOR)

Perfect Resonator + GW

$$H_0 = \sum_n \omega_n a_n^\dagger(t) a_n(t) - h(\omega_g L)^2 C \sum_{m,n} \omega_m(\omega_m \pm \omega_g) a_n^\dagger a_m + \ldots + h(\omega_g L)^2 \omega_g B_0 (C_1 a_{n^*} + C_2 a_{n^*}^\dagger)$$

Measurement Port + Intrinsic Losses

$$H_R = \int d\omega \left\{ \omega b^\dagger(\omega) b(\omega) + g(\omega) [b(\omega) a^\dagger(t) - b^\dagger(\omega) a(t)] \right\}$$

SAME BUT QUANTUM (EM RESONATOR)

Intrinsic Losses

$$\dot{a}(t) = i[H_0, a(t)] - \frac{\kappa}{2} a(t) + \sqrt{\kappa_m} a_{\text{in}}^{m}(t) + \sqrt{\kappa_\ell} a_{\text{in}}^{\ell}(t)$$

$$\dot{a}^\dagger(t) = i[H_0, a^\dagger(t)] - \frac{\kappa}{2} a^\dagger(t) + \sqrt{\kappa_m} a_{\text{in}}^{m,\dagger}(t) + \sqrt{\kappa_\ell} a_{\text{in}}^{\ell,\dagger}(t)$$

Measurement Port

SQUEEZING

$$X = \frac{a + a^\dagger}{\sqrt{2}} \qquad Y = i\frac{a - a^\dagger}{\sqrt{2}}$$

SQUEEZING

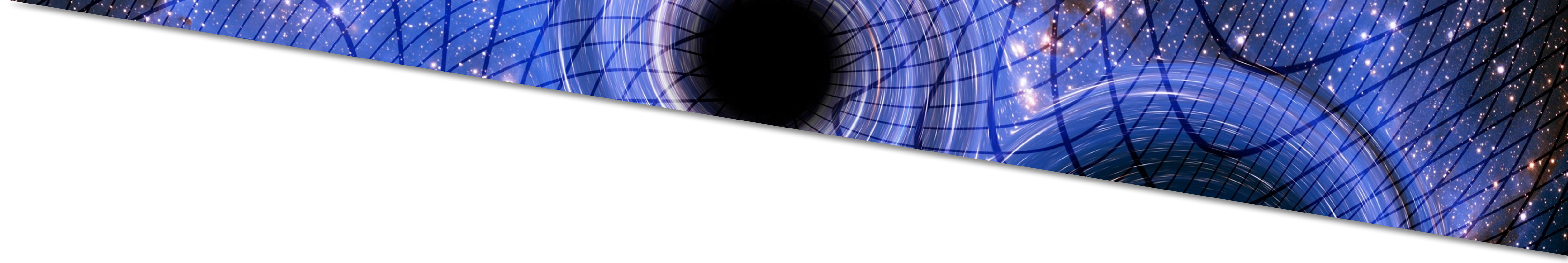
$$S^{\text{out}}_{Y_m Y_m} = \frac{1}{\kappa^2 + 4\Omega^2}\left(\left[(\kappa_m - \kappa_l)^2 + 4\Omega^2\right] S^{\text{in}}_{Y_m Y_m} + 4\kappa_l \kappa_m S^{\text{in}}_{Y_l Y_l}\right)$$

Measurement Port

Intrinsic Losses

$$\frac{\kappa}{2} = \frac{\omega_n}{Q_n}$$

SQUEEZING



$$S^{\text{out}}_{Y_m Y_m} = \frac{1}{\kappa^2 + 4\Omega^2}\left(\left[(\kappa_m - \kappa_l)^2 + 4\Omega^2\right] S^{\text{in}}_{Y_m Y_m} + 4\kappa_l \kappa_m S^{\text{in}}_{Y_l Y_l}\right)$$

SQL: $S^{\text{in}}_{Y_m Y_m} = S^{\text{in}}_{Y_l Y_l} = 1/2$

SQUEEZING

$$S^{\text{out}}_{Y_m Y_m} = \frac{1}{\kappa^2 + 4\Omega^2}\left(\left[(\kappa_m - \kappa_l)^2 + 4\Omega^2\right] S^{\text{in}}_{Y_m Y_m} + 4\kappa_l \kappa_m S^{\text{in}}_{Y_l Y_l}\right)$$

SV: $$S^{\text{in}}_{Y_m Y_m} = \frac{e^{-2r}}{2} \ll S^{\text{in}}_{Y_l Y_l} = 1/2$$

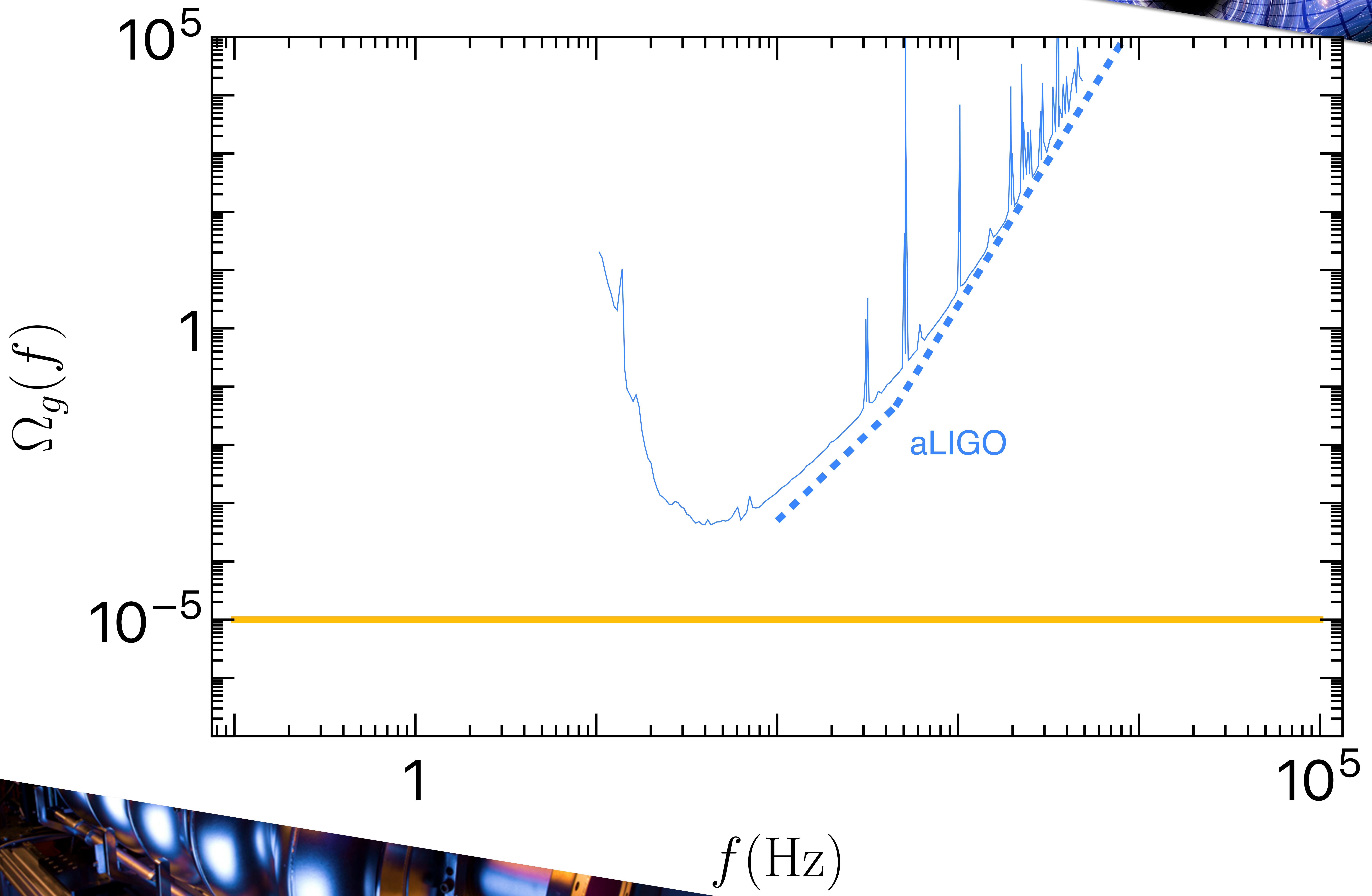
MANY OTHER IDEAS

1. Quantum non-demolition measurements: for instance "speedometers" for interferometers, strongly suppress back action from the laser

$$S_{hh} \simeq \frac{\kappa^2 + 4\Omega^2}{4U_{\rm in}\omega_L^2} + \frac{16U_{\rm in}\omega_L^2}{L^4 M_{\rm mirror}\Omega^4(\kappa^2+4\Omega^2)}$$

2. Entanglement of input photons to get an advantage that scales like N when combining N interferometers

....



10^5
1
10^{-5}
$\Omega_g(f)$
aLIGO
1
10^5
$f\,(\mathrm{Hz})$

BANDWIDTH SQUEEZING

$$Q_{\rm cpl} = Q_{\rm int}/(T/\omega_s)$$

$$\omega_s \to \omega_s e^{-2r}$$

$$h \to h e^{-r}$$