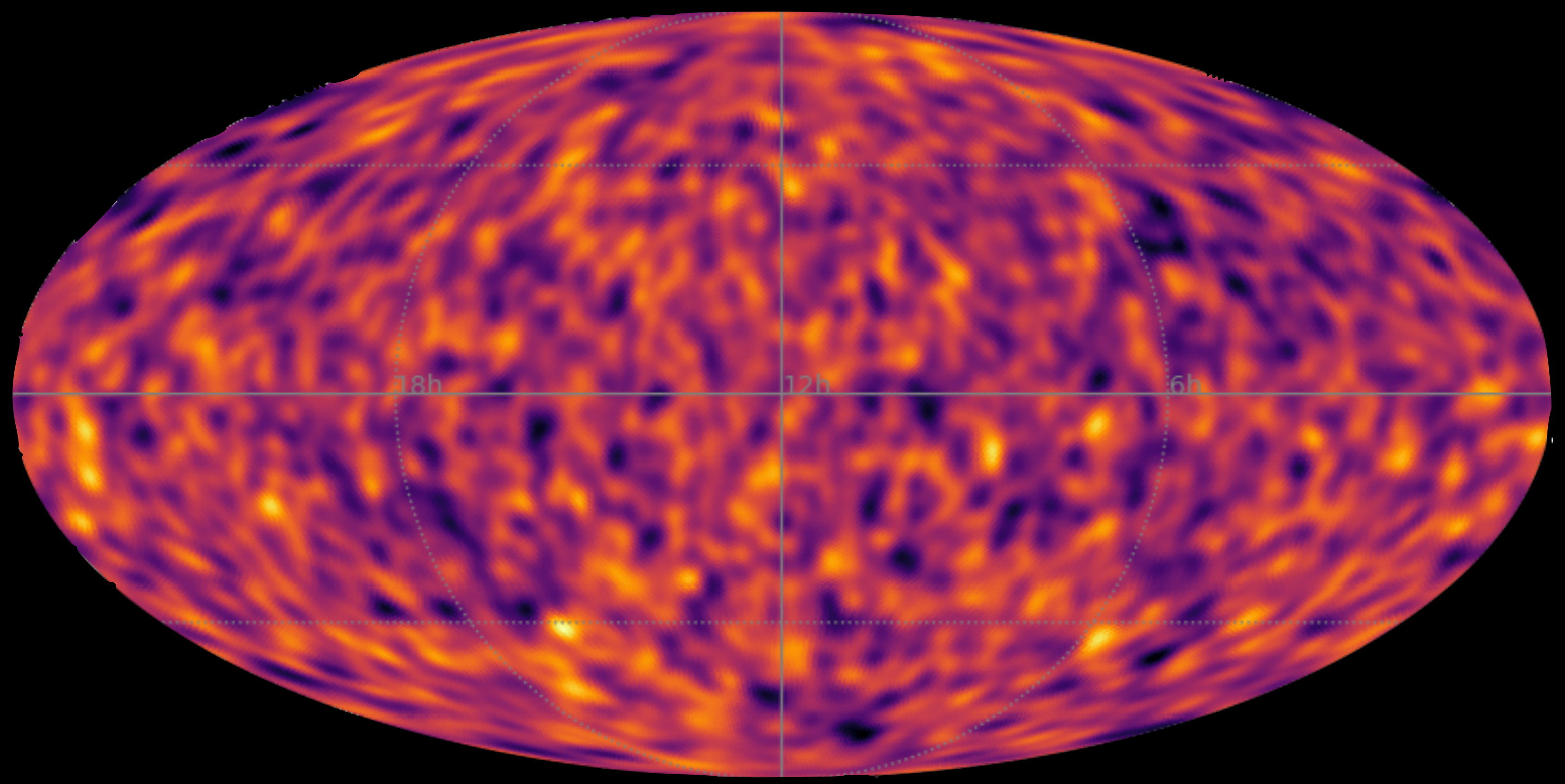
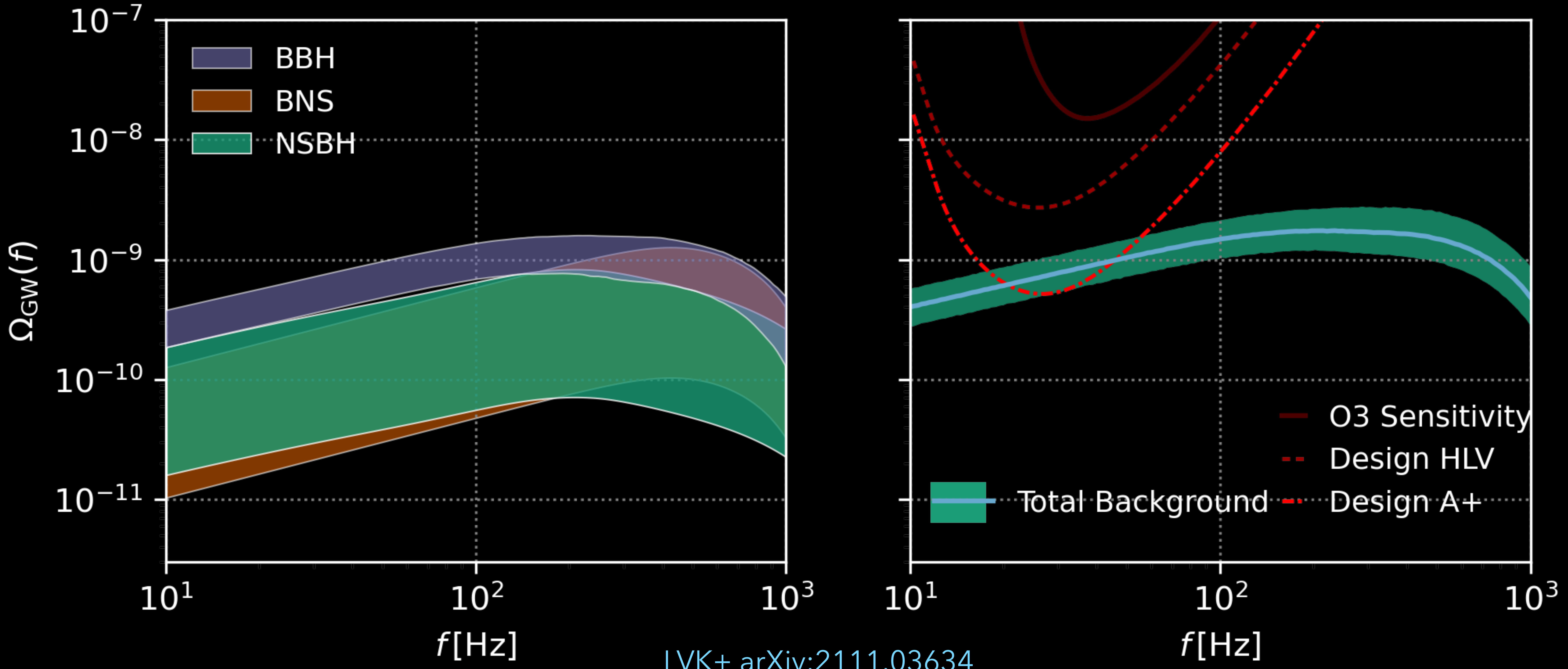


Searching for the Stochastic Gravitational-Wave Background

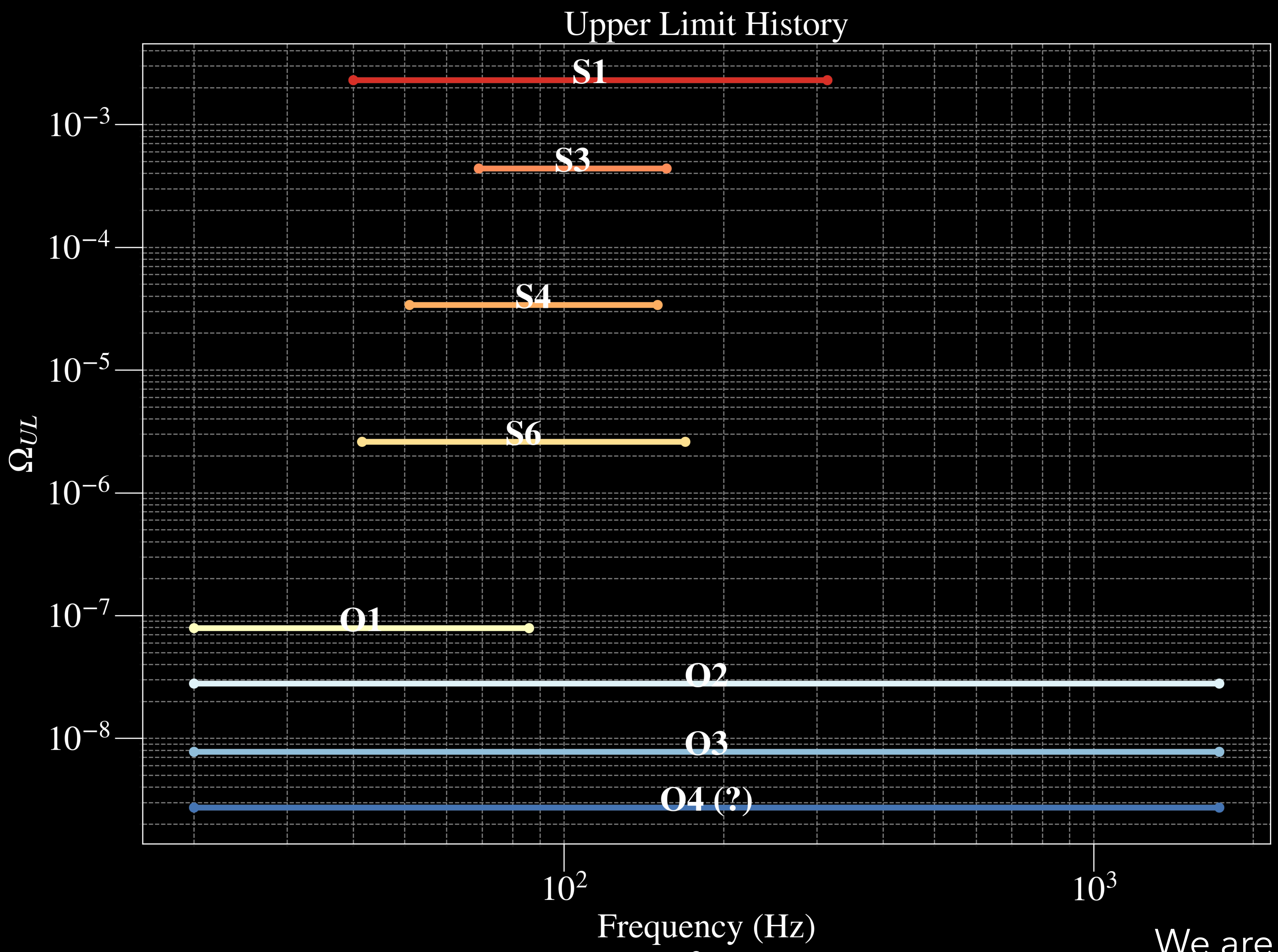
Jishnu Suresh
Henri Poincaré Fellow,
ARTEMIS, Observatoire de la Côte d'Azur

A stochastic gravitational-wave background (SGWB) is created by the superposition of individually undetectable signals.



LVK+ arXiv:2111.03634

The individual contributions expected from the collection of BNS, NSBH, and BBH mergers. While uncertainties on the energy density due to BNS and NSBH are due to Poisson uncertainties in their merger rates, our forecast for the SGWB due to BBHs includes systematic uncertainties associated with their imperfectly known mass distribution. (Right): Estimate of the total gravitational-wave background (green), as well as our current experimental sensitivity (red)



We are reaching there…

Component separation for stochastic gravitational-wave background

based on: arXiv:2310.05823, arXiv:2106.09593, arXiv:1904.05056

A stochastic gravitational-wave background (SGWB) is created by the superposition of individually undetectable signals.

SGWB energy density $$\Omega_{\mathrm{GW}}(f, \hat{\Omega}) = \frac{2\pi^2}{3H_0^2} f^3 \mathscr{P}(f, \hat{\Omega})$$

We usually assume $$\mathscr{P}(f, \hat{\Omega}) = H(f)\, \mathscr{P}(\hat{\Omega})$$

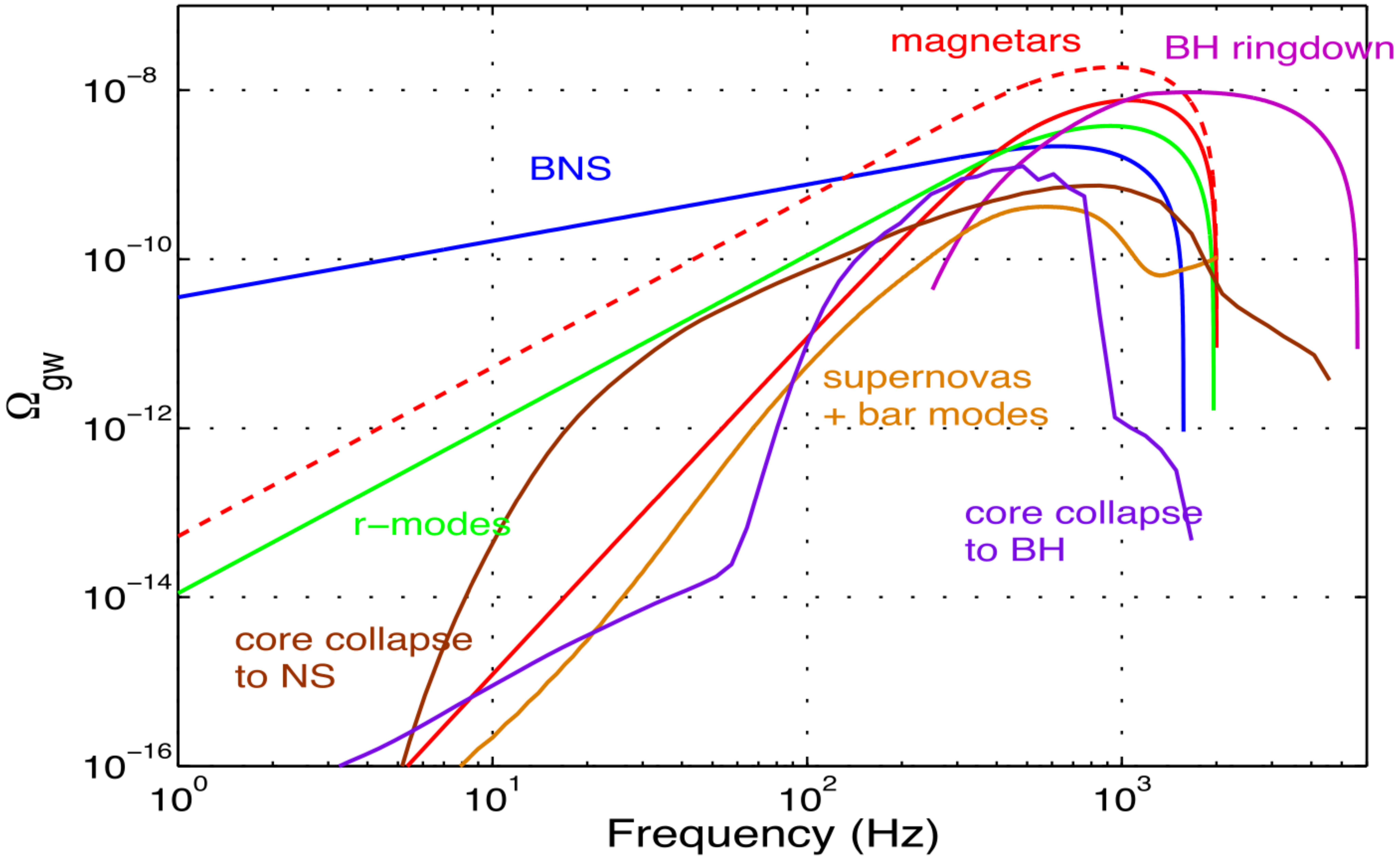
In our usual analysis, we assume a fiducial model for the spectral shape and perform the optimal filtering .

$$\tilde{Q}(f) \propto \frac{\gamma^{I*}_{ft,u} H(f)}{P_{\mathscr{I}_1}(t; f) P_{\mathscr{I}_2}(t; f)}$$

weight the frequencies which agree with the expected signal spectrum.

and de-weight the frequencies that correspond to large detector noise.

We often choose a power-law functional form for the SGWB template spectrum $$H(f) = \left(f/f_{\mathrm{ref}}\right)^{\alpha}$$



Adapted from T. Regimbau, Res. Astron. Astrophys. 11 369 2011 by F. DeLillo

- Separating the contribution of these sources to the total observed background.
- If we filter the data for each GWB component separately: we overestimate the amplitude of each GWB component and underestimate the error bars.
    - since other GWB component is also contributing to the correlated signal.
- We need to go beyond the single component analysis, to better extract the amplitudes of individuals GWB components.
    - Many methods have been proposed to disentangle these components.*
        - these are appropriate for estimating the parameters associated with different SGWB models.
    - Parida et. al [JCAP 04, 024 (2016)] proposed a method to jointly estimate the GWB components.
        - joint analysis takes into account the covariance between the spectral shapes.
    - We extended this to the astrophysical SGWB and anisotropic SGWB

* see G. Boileau+ PRD 103, 103529 (2021) for a nice summary of all the proposed methods

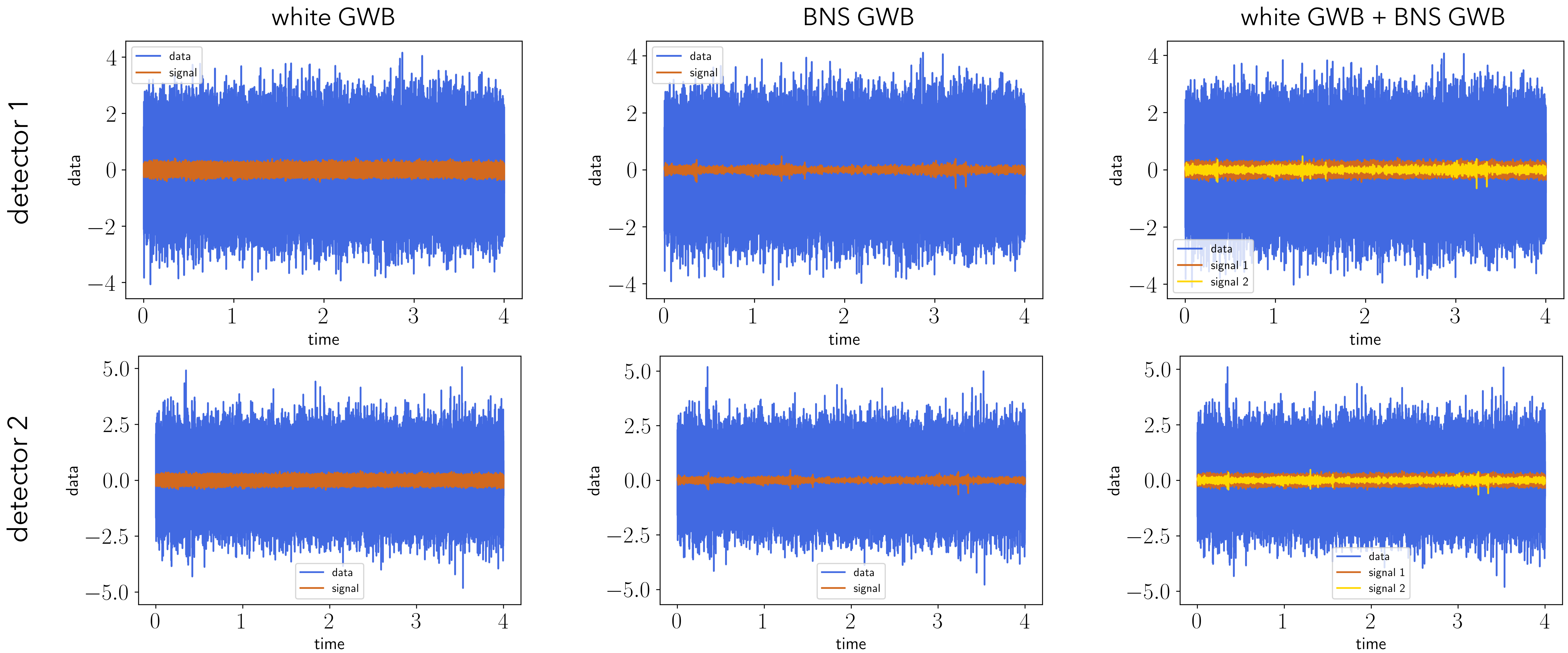
SGWB energy density $$\Omega_{\mathrm{GW}}(f,\hat{\Omega}) = \frac{2\,\pi^2}{3H_0^2}\, f^3\, \mathscr{P}(f,\hat{\Omega})$$

We usually assume $$\mathscr{P}(f,\hat{\Omega}) = H(f)\,\mathscr{P}(\hat{\Omega})$$

amplitude of the SGWB intensity $$\mathscr{P}(f,\hat{\Omega}) = \sum_{\alpha} H_\alpha(f)\,\mathscr{P}^{\alpha}(\hat{\Omega})$$

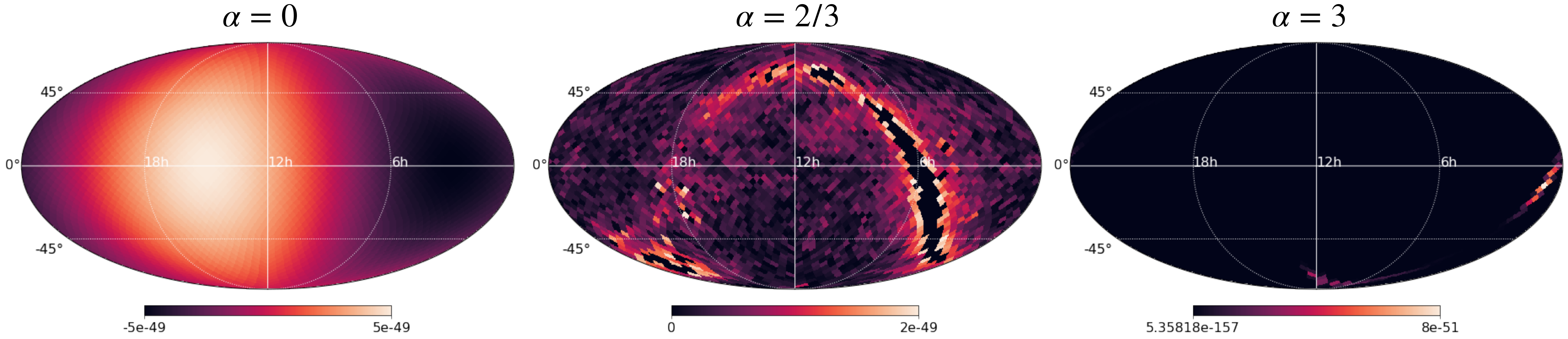
spectral shape of the background $H_\alpha(f)$ where α is the spectral index

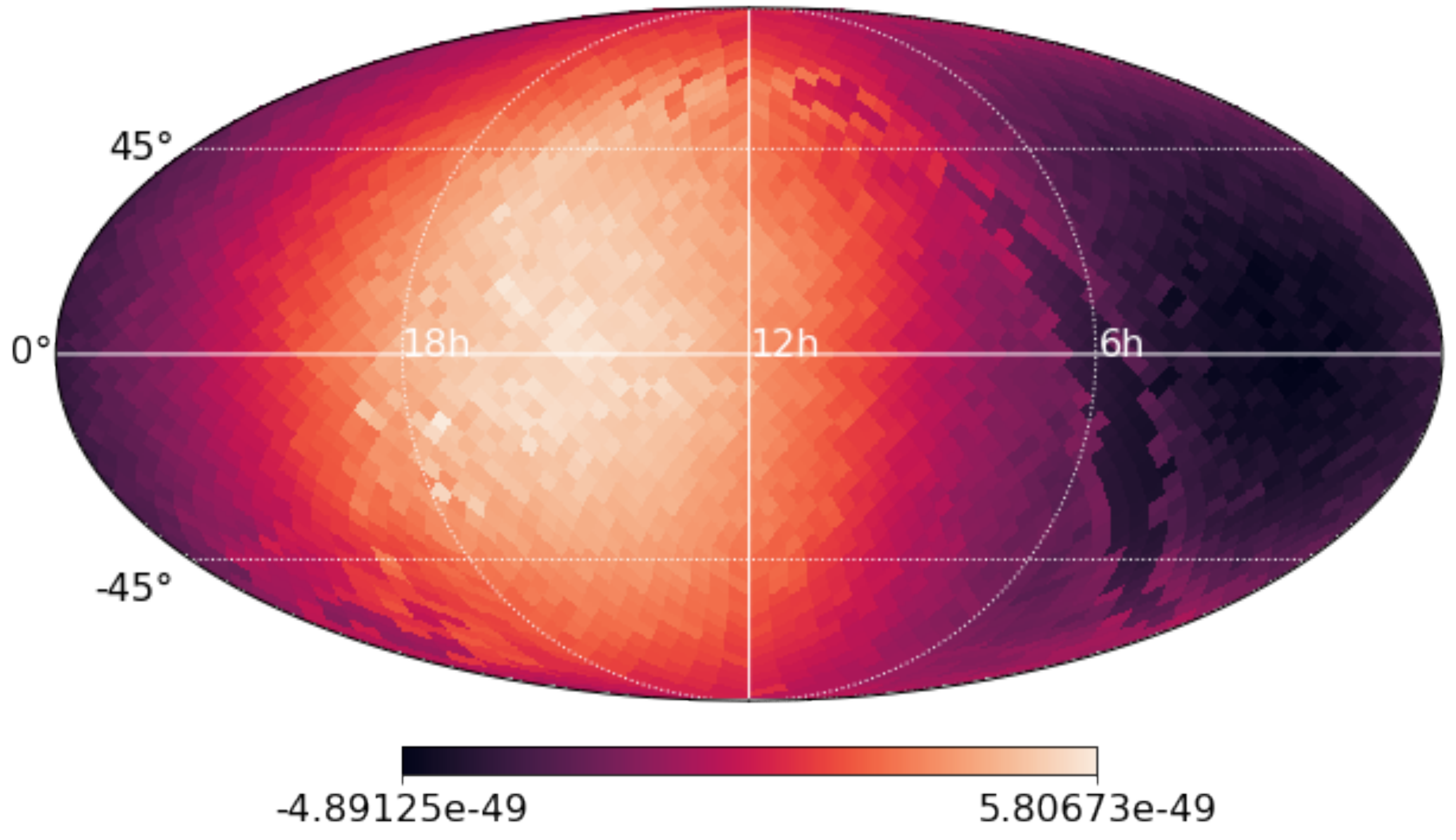
Injection Studies - Isotropic background



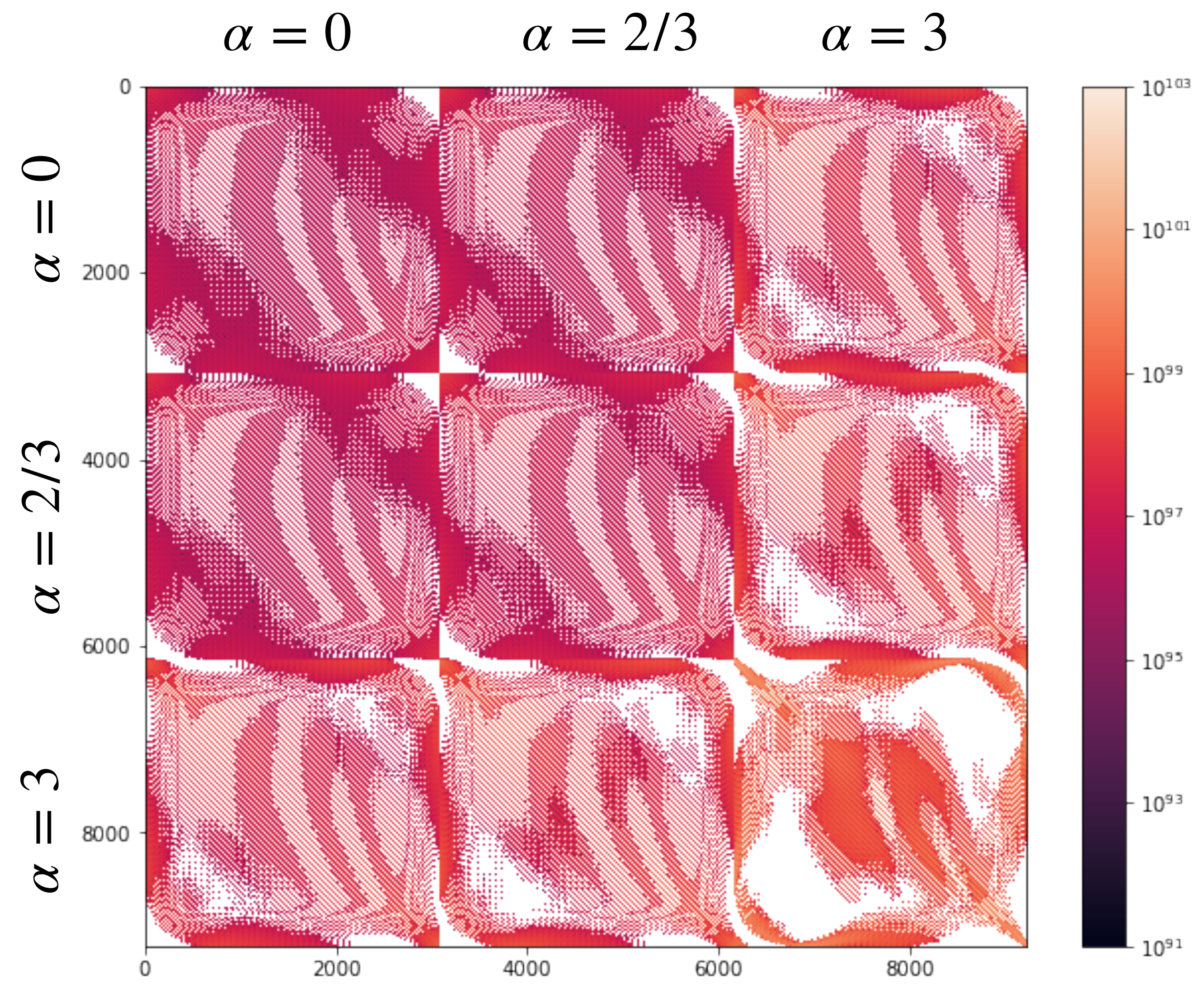
SGWB components	Expected amplitude	Using single-index component separation ± error bar	Using joint-index multi-component separation ± error bar
white GWB	8.89E-07	1.32e-06 ± 3.01e-07	8.24e-07 ± 6.08e-07
BNS GWB	7.21E-05	7.70e-05 ± 6.13e-06	7.13e-05 ± 1.24e-05

Injection Studies

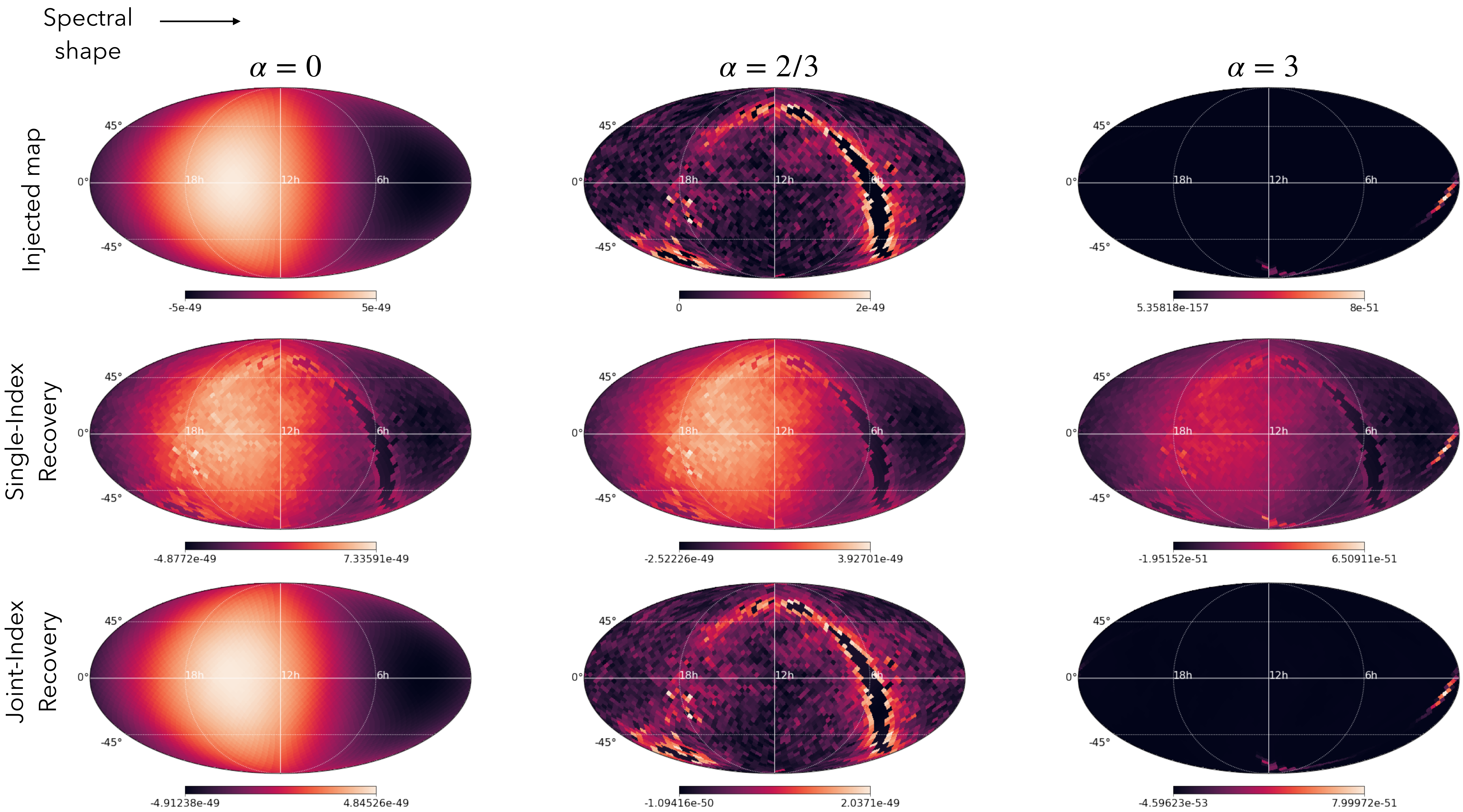




combined source



coupling matrix

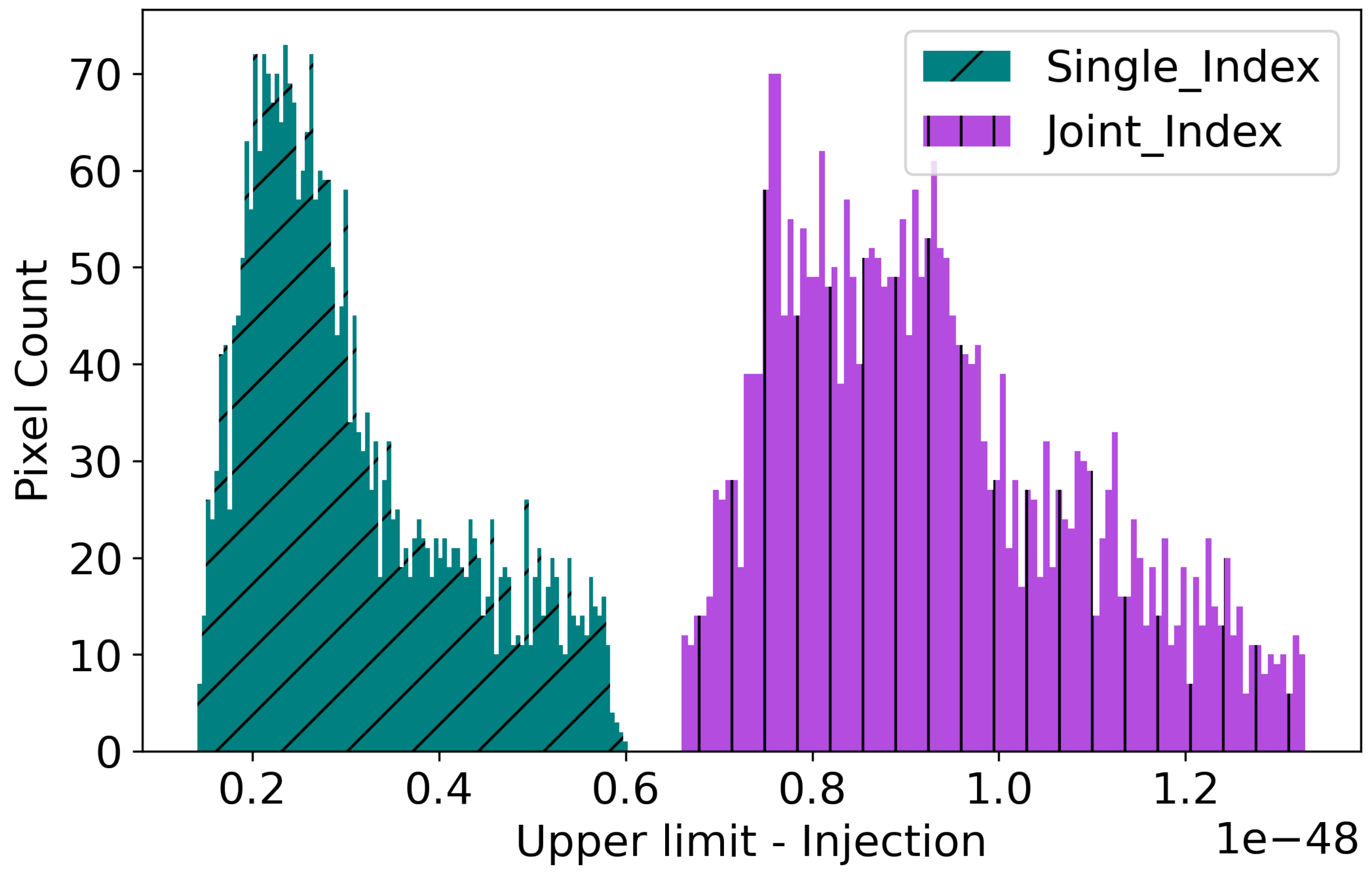

Spectral shape
$\alpha = 0$
$\alpha = 2/3$
$\alpha = 3$
Injected map
Single-Index Recovery
Joint-Index Recovery
45°
0°
-45°
18h
12h
6h
-5e-49
5e-49
0
2e-49
5.35818e-157
8e-51
-4.8772e-49
7.33591e-49
-2.52226e-49
3.92701e-49
-1.95152e-51
6.50911e-51
-4.91238e-49
4.84526e-49
-1.09416e-50
2.0371e-49
-4.59623e-53
7.99972e-51

Injection Studies

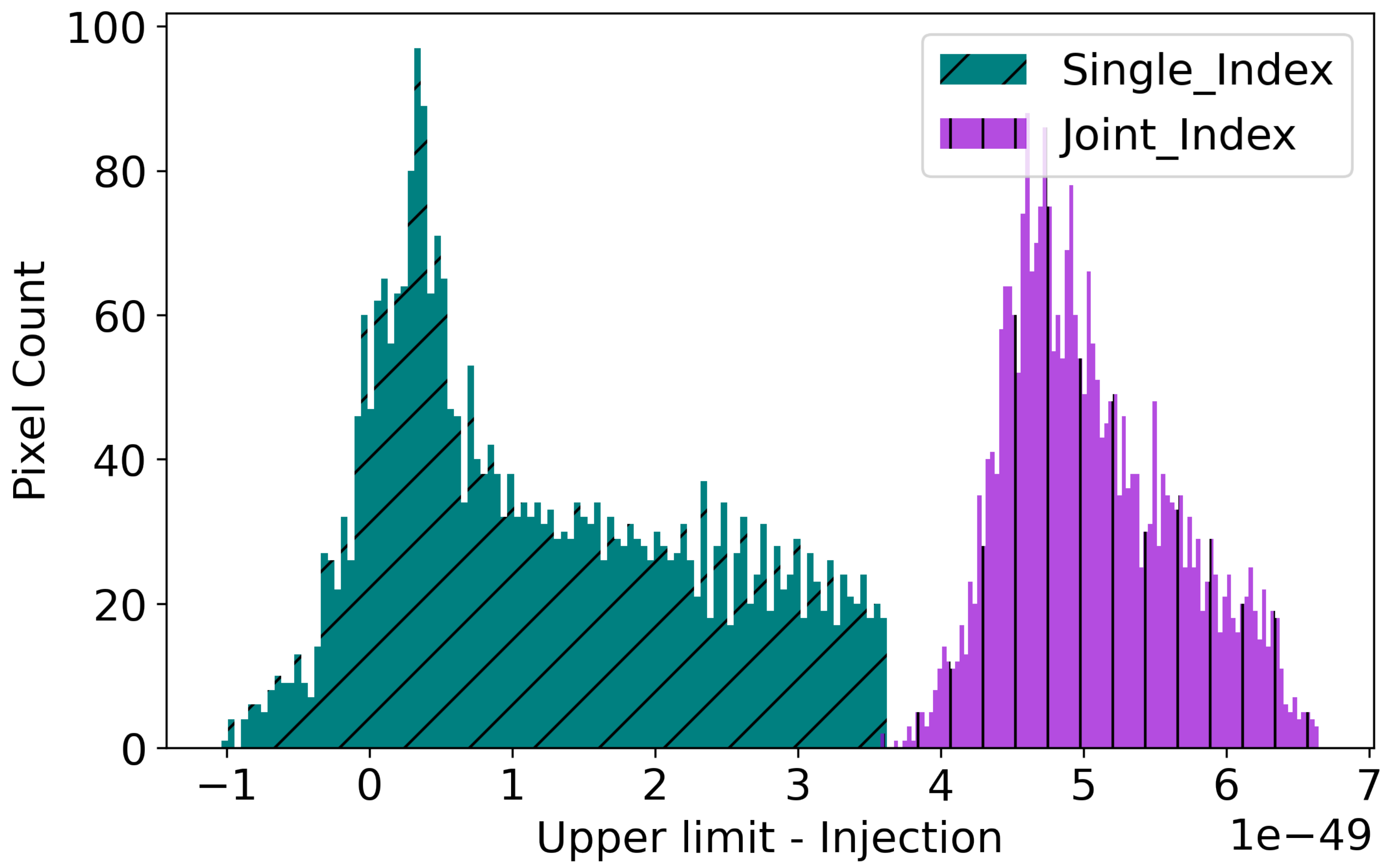
- Simulated 1000 noise realizations.
- Perform the injection study considering each noise realization.
- Recovered the source using single-index and joint-index multi-component separation methods.

95% confident UL - Injection

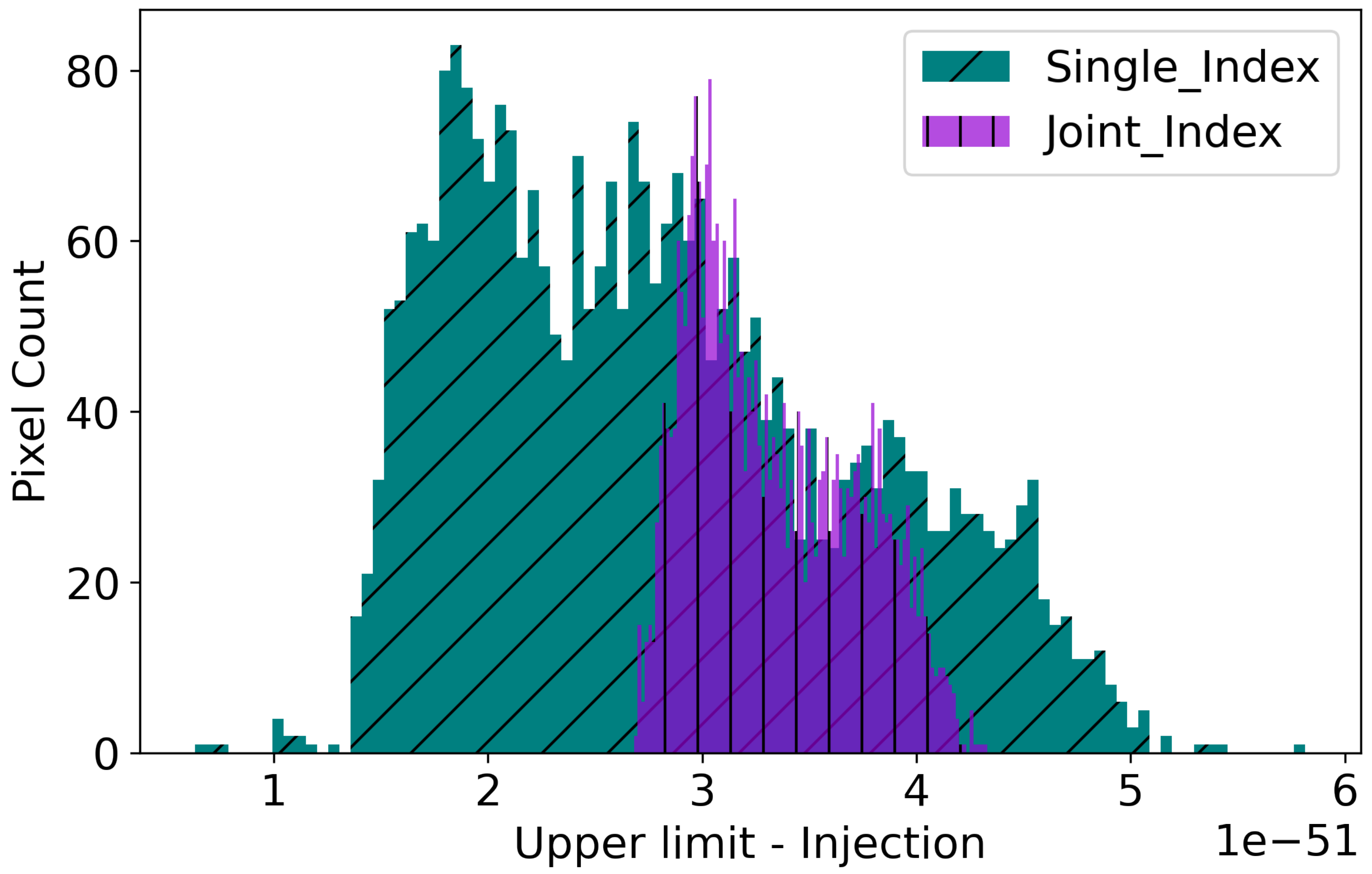
$\alpha = 0$



$\alpha = 2/3$



$\alpha = 3$

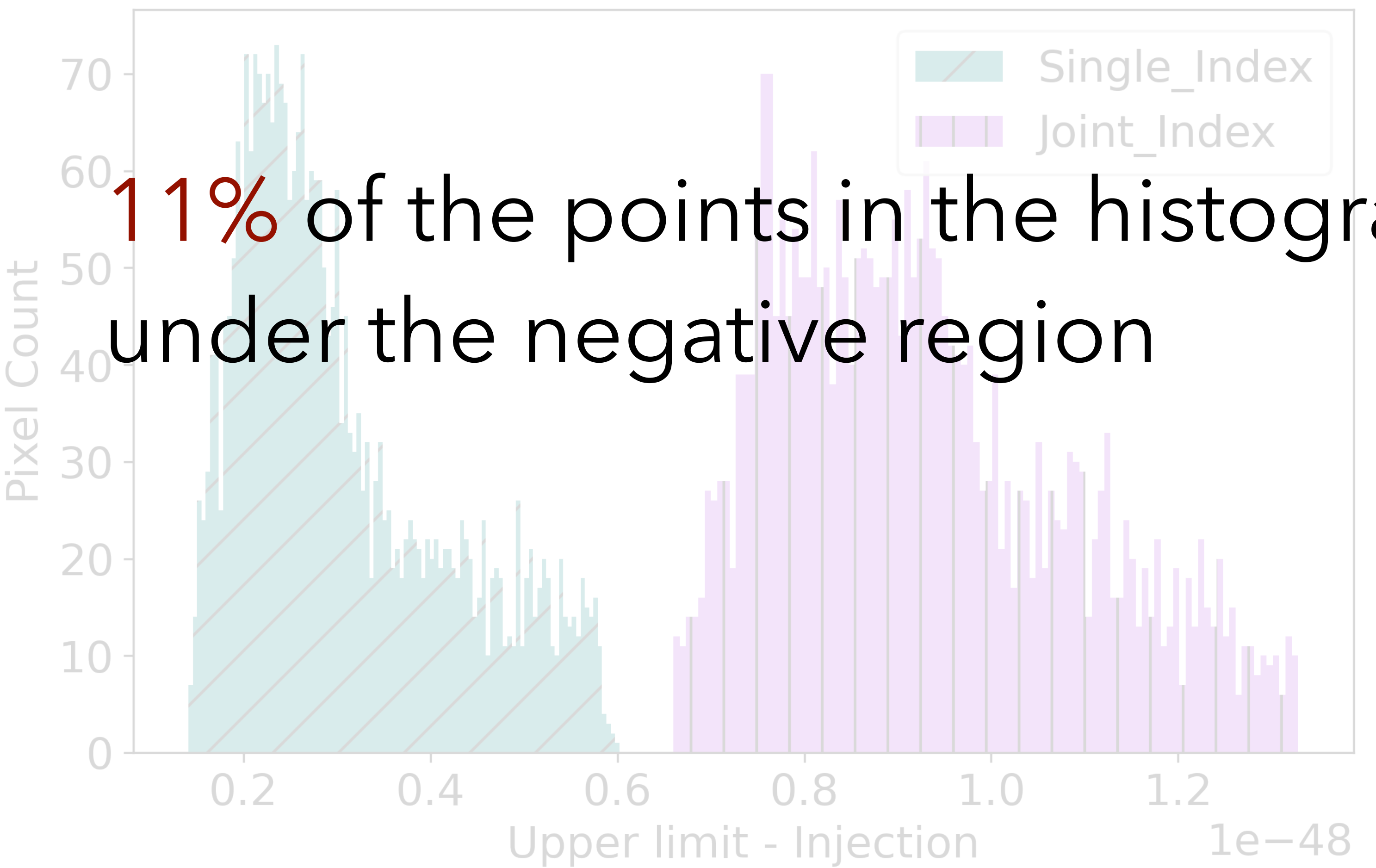


Upper limit from injection study

- Produced UL sky maps corresponding to the injection (injection strength is set to be close to the detectable limit).
- Histogram shows the difference between upper limit sky map and the injected sky map.

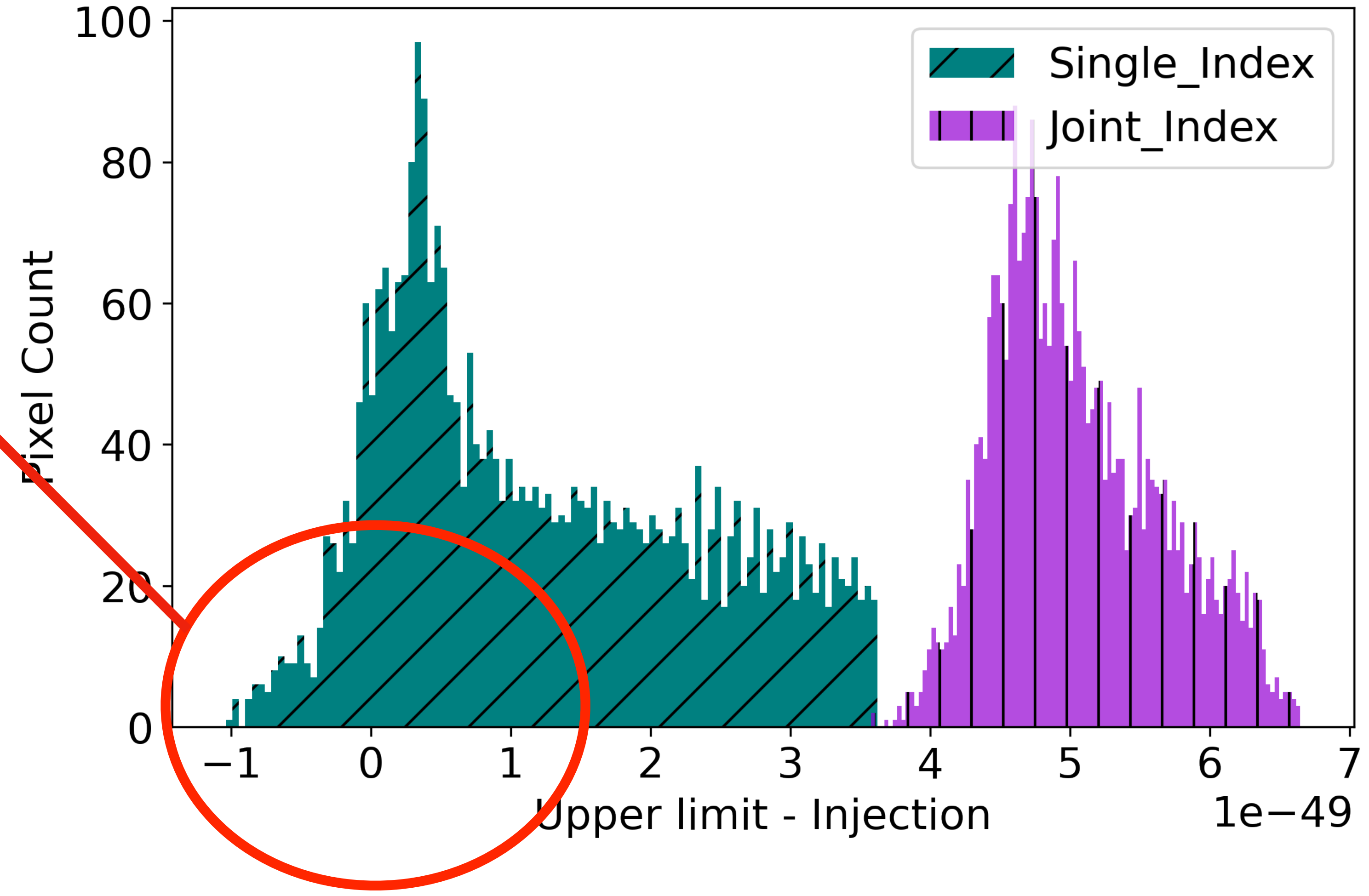
even if the detectors are not sensitive enough to detect SGWB, the joint-index multi-component estimator provides safer upper limits when one cannot ignore the existence of more than one component.

$\alpha = 0$

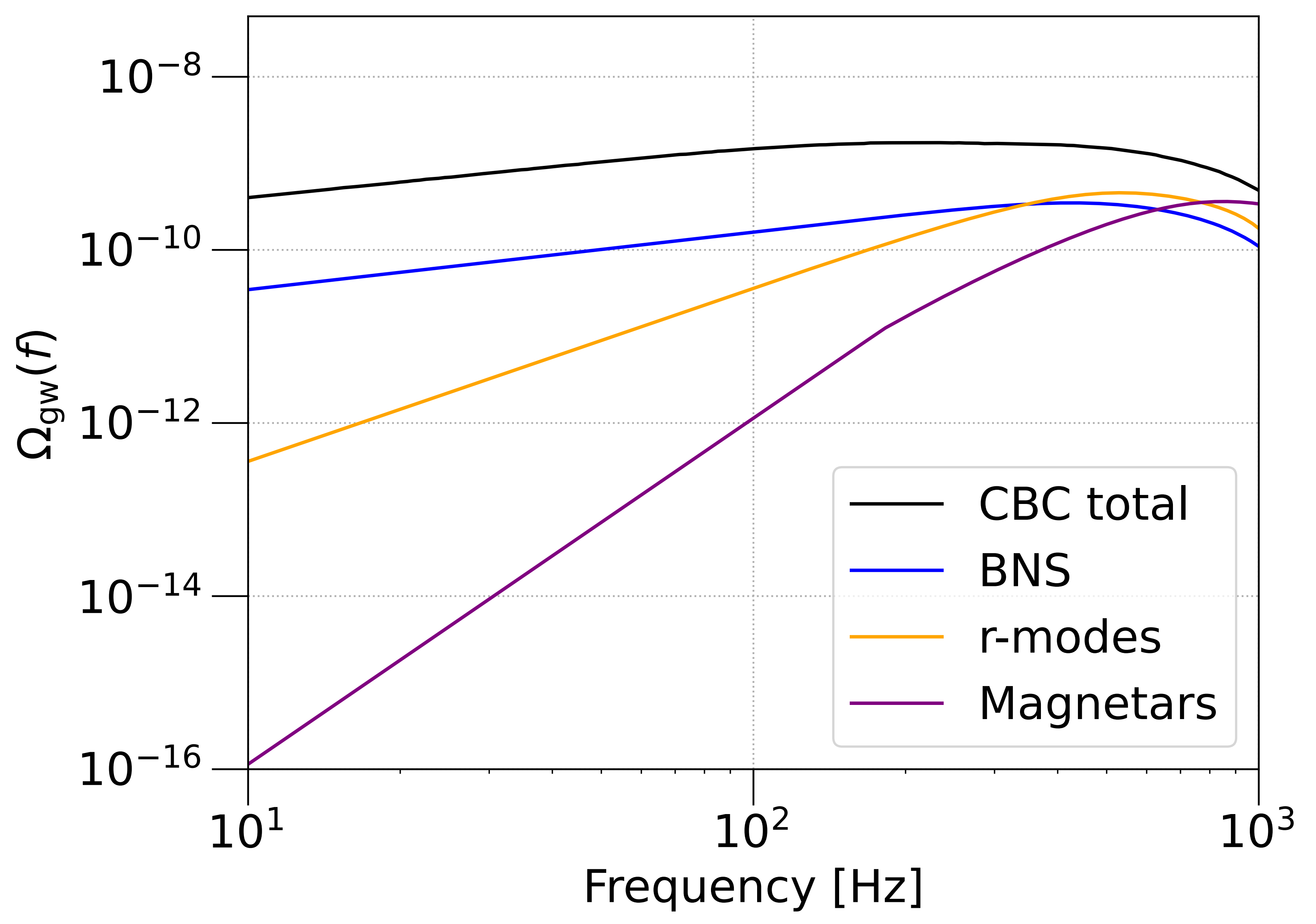


11% of the points in the histogram fall under the negative region

$\alpha = 2/3$



$\alpha = 3$



		CBC	r-mode		magnetars
	$\hat{\Omega}_0 = 1 \times 10^{-6}$	$\hat{\Omega}_{2/3} = 1 \times 10^{-6}$	$\hat{\Omega}_2 = 1 \times 10^{-6}$	$\hat{\Omega}_3 = 1 \times 10^{-6}$	$\hat{\Omega}_4 = 1 \times 10^{-6}$
$\alpha = \{0\}$	$(1.9420 \pm 0.0008) \times 10^{-5}$	-	-	-	-
$\alpha = \{2/3\}$	-	$(1.8066 \pm 0.0006) \times 10^{-5}$	-	-	-
$\alpha = \{2\}$	-	-	$(1.2519 \pm 0.0002) \times 10^{-5}$	-	-
$\alpha = \{3\}$	-	-	-	$(5.6650 \pm 0.0008) \times 10^{-6}$	-
$\alpha = \{4\}$	-	-	-	-	$(1.2262 \pm 0.0002) \times 10^{-6}$
$\alpha = \{0, 2/3, 2, 3\}$	$(-1.525 \pm 0.001) \times 10^{-4}$	$(2.045 \pm 0.002) \times 10^{-4}$	$(-7.061 \pm 0.004) \times 10^{-5}$	$(1.7369 \pm 0.0005) \times 10^{-5}$	-
$\alpha = \{0, 2/3, 2, 4\}$	$(1.18 \pm 0.01) \times 10^{-5}$	$(-1.30 \pm 0.01) \times 10^{-5}$	$(5.68 \pm 0.02) \times 10^{-6}$	-	$(1.054 \pm 0.0003) \times 10^{-6}$
$\alpha = \{0, 2/3, 3, 4\}$	$(-1.50 \pm 0.08) \times 10^{-6}$	$(4.18 \pm 0.07) \times 10^{-6}$	-	$(1.202 \pm 0.004) \times 10^{-6}$	$(9.898 \pm 0.005) \times 10^{-7}$
$\alpha = \{0, 2, 3, 4\}$	$(1.82 \pm 0.03) \times 10^{-6}$	-	$(1.29 \pm 0.02) \times 10^{-6}$	$(9.44 \pm 0.07) \times 10^{-7}$	$(1.0027 \pm 0.0006) \times 10^{-6}$
$\alpha = \{2/3, 2, 3, 4\}$	-	$(2.21 \pm 0.03) \times 10^{-6}$	$(6.6 \pm 0.3) \times 10^{-7}$	$(1.063 \pm 0.009) \times 10^{-6}$	$(9.970 \pm 0.007) \times 10^{-7}$
$\alpha = \{0, 2/3, 2, 3, 4\}$	$(1.0 \pm 0.2) \times 10^{-6}$	$(1.0 \pm 0.3) \times 10^{-6}$	$(1.00 \pm 0.07) \times 10^{-6}$	$(1.00 \pm 0.02) \times 10^{-6}$	$(1.0000 \pm 0.0009) \times 10^{-6}$

arXiv:2310.05823

Summary

- If we filter the data for each GWB component separately, we overestimate the amplitude of each GWB component and underestimate the error bars.
- We have shown that estimates and the upper limits can get severely biased in the single-index analysis when the actual signal is close to more than the detectable limit.
- Joint analysis accurately separates and estimates backgrounds with different spectral shapes and different sky distributions with no major bias.
- The upper limits set by the joint analysis are safer, though less strict than the individual analysis.
- While the results shown in this presentation are in the context of ground-based detectors, the methods are general and should be easily translated to LISA and PTA bands.

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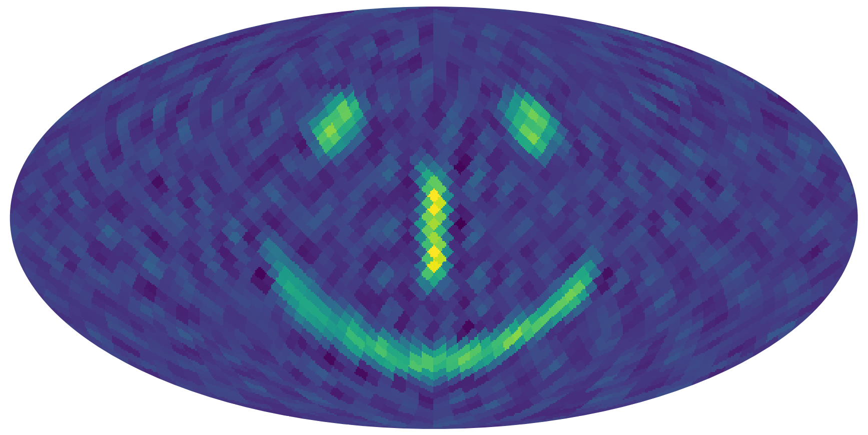

thank you!

Preliminaries

SGWB energy density $$\Omega_{\text{GW}}(f,\hat{\Omega}) = \frac{2\,\pi^2}{3H_0^2}\,f^3\,\mathscr{P}(f,\hat{\Omega}) \qquad (1)$$

amplitude of the SGWB intensity $$\mathscr{P}(f,\hat{\Omega}) = \sum_{\alpha} H_\alpha(f)\,\mathscr{P}^\alpha(\hat{\Omega}) \qquad (2)$$

spectral shape of the background $H_\alpha(f)$ where α is the spectral index

Usual Cross-Spectral Density (CSD) $$\langle \mathscr{C}^{\mathscr{I}} \rangle = \tau \sum_{\alpha} H_\alpha(f)\,\gamma^{I}_{ft,u}\,\mathscr{P}^{\alpha}_{u} \qquad (3)$$

likelihood function $$p(\mathscr{C}^{\mathscr{I}}\,|\,\mathscr{P}^{\alpha}_{u}) \propto \exp\Big[-\frac{1}{2}(\mathscr{C}^{\mathscr{I}} - \langle \mathscr{C}^{\mathscr{I}} \rangle)^{*}\mathscr{N}^{-1}(\mathscr{C}^{\mathscr{I}} - \langle \mathscr{C}^{\mathscr{I}} \rangle)\Big] \qquad (4)$$

clean map $$\hat{\mathscr{P}}^{\alpha}_{u} = \mathbf{\Gamma}^{-1} \cdot \mathbf{X} \qquad (5)$$

Preliminaries

dirty map

$$\mathbf{X} \equiv X_u^{\alpha} = \sum_{Ift} \gamma_{ft,u}^{I*} \frac{H_\alpha(f)}{P_{\mathcal{I}_1}(t;f) P_{\mathcal{I}_2}(t;f)} C^I(t;f) \qquad (1)$$

Fisher information matrix

$$\boldsymbol{\Gamma} \equiv \Gamma_{uu'}^{\alpha\beta} = \sum_{Ift} \frac{H_\alpha(f) H_\beta(f)}{P_{\mathcal{I}_1}(t;f)\, P_{\mathcal{I}_2}(t;f)} \gamma_{ft,u}^{I*} \gamma_{ft,u'}^{I} \qquad (2)$$

For a three spectral index case, we can write the convolution equation as

$$\begin{bmatrix} X_u^{\alpha 1} \\ X_u^{\alpha 2} \\ X_u^{\alpha 3} \end{bmatrix} = \begin{bmatrix} \Gamma_{uu'}^{\alpha 1 \alpha 1} & \Gamma_{uu'}^{\alpha 1 \alpha 2} & \Gamma_{uu'}^{\alpha 1 \alpha 3} \\ \Gamma_{uu'}^{\alpha 2 \alpha 1} & \Gamma_{uu'}^{\alpha 2 \alpha 2} & \Gamma_{uu'}^{\alpha 2 \alpha 3} \\ \Gamma_{uu'}^{\alpha 3 \alpha 1} & \Gamma_{uu'}^{\alpha 3 \alpha 2} & \Gamma_{uu'}^{\alpha 3 \alpha 3} \end{bmatrix} \begin{bmatrix} \hat{\mathscr{P}}_{u'}^{\alpha 1} \\ \hat{\mathscr{P}}_{u'}^{\alpha 2} \\ \hat{\mathscr{P}}_{u'}^{\alpha 3} \end{bmatrix} \qquad (3)$$

Coupling Matrix $C_{uu'}^{\alpha\beta}$

ML solution of the convolution equation

$$\hat{\mathscr{P}}_u^{\alpha} = \left[C_{uu'}^{\alpha\beta} \right]^{-1} \cdot X_{u'}^{\beta} \qquad (4)$$

	CBC		r-mode		magnetars
	$\hat{\Omega}_0 = 1 \times 10^{-6}$	$\hat{\Omega}_{2/3} = 1 \times 10^{-6}$	$\hat{\Omega}_2 = 1 \times 10^{-6}$	$\hat{\Omega}_3 = 1 \times 10^{-6}$	$\hat{\Omega}_4 = 1 \times 10^{-6}$
$\alpha = \{0\}$	$(1.9420 \pm 0.0008) \times 10^{-5}$	-	-	-	-
$\alpha = \{2/3\}$	-	$(1.8066 \pm 0.0006) \times 10^{-5}$	-	-	-
$\alpha = \{2\}$	-	-	$(1.2519 \pm 0.0002) \times 10^{-5}$	-	-
$\alpha = \{3\}$	-	-	-	$(5.6650 \pm 0.0008) \times 10^{-6}$	-
$\alpha = \{4\}$	-	-	-	-	$(1.2262 \pm 0.0002) \times 10^{-6}$
$\alpha = \{0, 2/3\}$	$(-1.5621 \pm 0.0004) \times 10^{-4}$	$(1.3276 \pm 0.0003) \times 10^{-4}$	-	-	-
$\alpha = \{0, 2\}$	$(-3.595 \pm 0.001) \times 10^{-5}$	-	$(2.1765 \pm 0.0004) \times 10^{-5}$	-	-
$\alpha = \{0, 3\}$	$(-9.516 \pm 0.009) \times 10^{-6}$	-	-	$(6.205 \pm 0.001) \times 10^{-6}$	-
$\alpha = \{0, 4\}$	$(7.665 \pm 0.008) \times 10^{-6}$	-	-	-	$(1.1886 \pm 0.0002) \times 10^{-6}$
$\alpha = \{2/3, 2\}$	-	$(-3.874 \pm 0.001) \times 10^{-5}$	$(2.7240 \pm 0.0005) \times 10^{-5}$	-	-
$\alpha = \{2/3, 3\}$	-	$(-9.227 \pm 0.007) \times 10^{-6}$	-	$(6.513 \pm 0.001) \times 10^{-6}$	-
$\alpha = \{2/3, 4\}$	-	$(6.305 \pm 0.006) \times 10^{-6}$	-	-	$(1.1696 \pm 0.0002) \times 10^{-6}$
$\alpha = \{2, 3\}$	-	-	$(-9.976 \pm 0.005) \times 10^{-6}$	$(8.751 \pm 0.002) \times 10^{-6}$	-
$\alpha = \{2, 4\}$	-	-	$(3.521 \pm 0.003) \times 10^{-6}$	-	$(1.0827 \pm 0.0002) \times 10^{-6}$
$\alpha = \{3, 4\}$	-	-	-	$(1.819 \pm 0.002) \times 10^{-6}$	$(9.141 \pm 0.003) \times 10^{-7}$
$\alpha = \{0, 2/3, 2\}$	$(2.253 \pm 0.001) \times 10^{-4}$	$(-2.5345 \pm 0.0009) \times 10^{-4}$	$(5.086 \pm 0.001) \times 10^{-5}$	-	-
$\alpha = \{0, 2/3, 3\}$	$(9.418 \pm 0.007) \times 10^{-5}$	$(-8.525 \pm 0.005) \times 10^{-5}$	-	$(8.154 \pm 0.002) \times 10^{-6}$	-
$\alpha = \{0, 2/3, 4\}$	$(-2.188 \pm 0.005) \times 10^{-5}$	$(2.278 \pm 0.004) \times 10^{-5}$	-	-	$(1.1288 \pm 0.0002) \times 10^{-6}$
$\alpha = \{0, 2, 3\}$	$(2.918 \pm 0.002) \times 10^{-5}$	-	$(-2.584 \pm 0.001) \times 10^{-5}$	$(1.2001 \pm 0.0003) \times 10^{-5}$	-
$\alpha = \{0, 2, 4\}$	$(-9.7 \pm 0.2) \times 10^{-7}$	-	$(3.839 \pm 0.006) \times 10^{-6}$	-	$(1.0745 \pm 0.0002) \times 10^{-6}$
$\alpha = \{0, 3, 4\}$	$(3.35 \pm 0.01) \times 10^{-6}$	-	-	$(1.385 \pm 0.002) \times 10^{-6}$	$(9.722 \pm 0.004) \times 10^{-7}$
$\alpha = \{2/3, 2, 3\}$	-	$(3.603 \pm 0.002) \times 10^{-5}$	$(-3.527 \pm 0.002) \times 10^{-5}$	$(1.3262 \pm 0.0003) \times 10^{-5}$	-
$\alpha = \{2/3, 2, 4\}$	-	$(-1.20 \pm 0.02) \times 10^{-6}$	$(4.076 \pm 0.008) \times 10^{-6}$	-	$(1.0708 \pm 0.0003) \times 10^{-6}$
$\alpha = \{2/3, 3, 4\}$	-	$(2.904 \pm 0.009) \times 10^{-6}$	-	$(1.255 \pm 0.002) \times 10^{-6}$	$(9.848 \pm 0.004) \times 10^{-7}$
$\alpha = \{2, 3, 4\}$	-	-	$(2.599 \pm 0.008) \times 10^{-6}$	$(5.32 \pm 0.04) \times 10^{-7}$	$(1.0289 \pm 0.0005) \times 10^{-6}$
$\alpha = \{0, 2/3, 2, 3\}$	$(-1.525 \pm 0.001) \times 10^{-4}$	$(2.045 \pm 0.002) \times 10^{-4}$	$(-7.061 \pm 0.004) \times 10^{-5}$	$(1.7369 \pm 0.0005) \times 10^{-5}$	-
$\alpha = \{0, 2/3, 2, 4\}$	$(1.18 \pm 0.01) \times 10^{-5}$	$(-1.30 \pm 0.01) \times 10^{-5}$	$(5.68 \pm 0.02) \times 10^{-6}$	-	$(1.054 \pm 0.0003) \times 10^{-6}$
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$\alpha = \{2/3, 2, 3, 4\}$	-	$(2.21 \pm 0.03) \times 10^{-6}$	$(6.6 \pm 0.3) \times 10^{-7}$	$(1.063 \pm 0.009) \times 10^{-6}$	$(9.970 \pm 0.007) \times 10^{-7}$
$\alpha = \{0, 2/3, 2, 3, 4\}$	$(1.0 \pm 0.2) \times 10^{-6}$	$(1.0 \pm 0.3) \times 10^{-6}$	$(1.00 \pm 0.07) \times 10^{-6}$	$(1.00 \pm 0.02) \times 10^{-6}$	$(1.0000 \pm 0.0009) \times 10^{-6}$