

Probing Cosmic Expansion with Gravitational Wave-Large Scale Structure Correlations

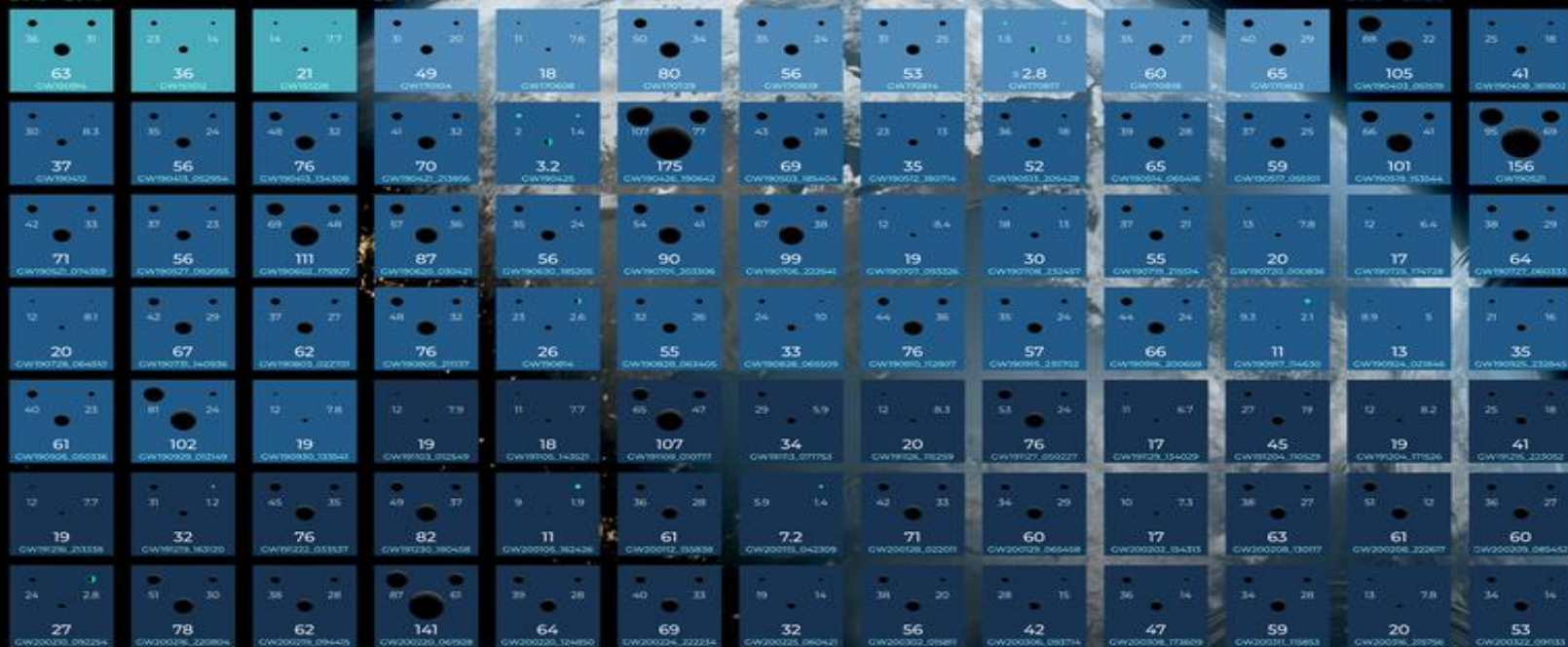
Syantani Bera

University of the Balearic Islands, Spain

OBSERVING
01
RUN
2015 - 2016

02
2016 - 2017

03a+b
2019 - 2020



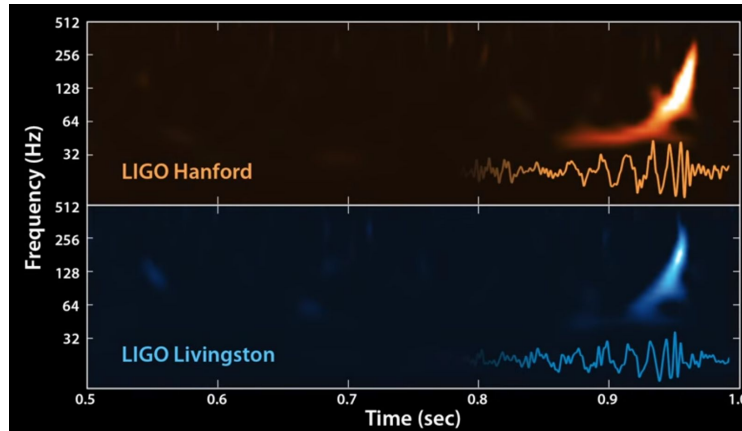
GRAVITATIONAL WAVE MERGER DETECTIONS

SINCE 2015



Credit: Carl Knox (OzGrav, Swinburne University of Technology)

Distance measurement from Gravitational Waves



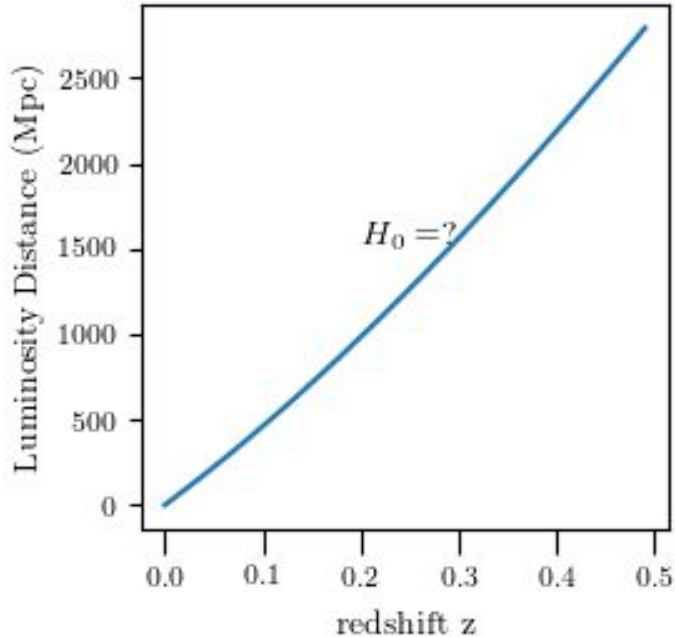
$$h(t) = \frac{M_z^{5/3} f(t)^{2/3}}{d_L} F(\iota, \theta) \cos(\Phi(t))$$

M_z : Redshifted chirp mass

ι : inclination angle

$\Phi(t)$: Accumulated phase

Measuring H_0 with “standard sirens”



- ❑ Luminosity distance - redshift curve depends on the value of the Hubble parameter H_0
 $d_L \sim cz/H_0$ low redshift
- ❑ Luminosity distance - GW observation
- ❑ Redshift - Electromagnetic counterpart

Thus an independent estimate of H_0 is possible

GW170817

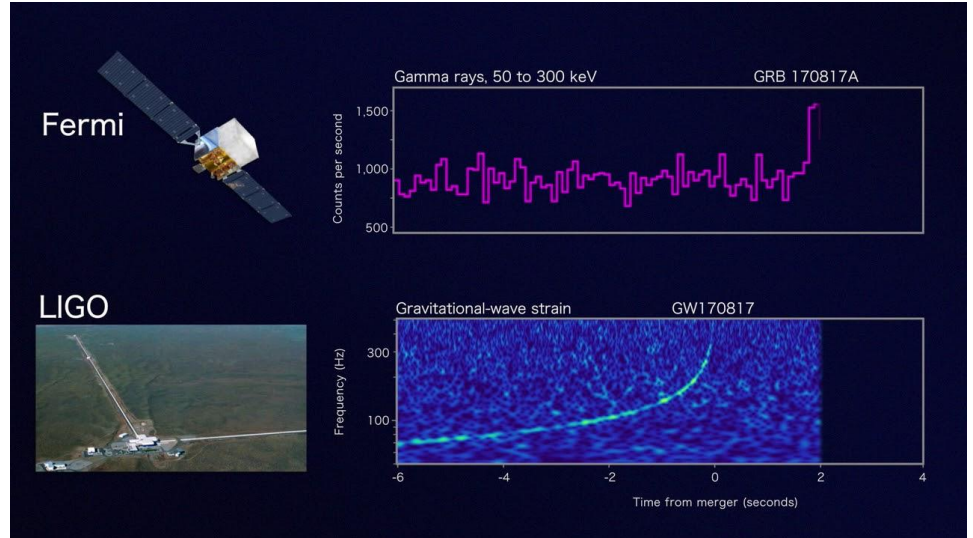
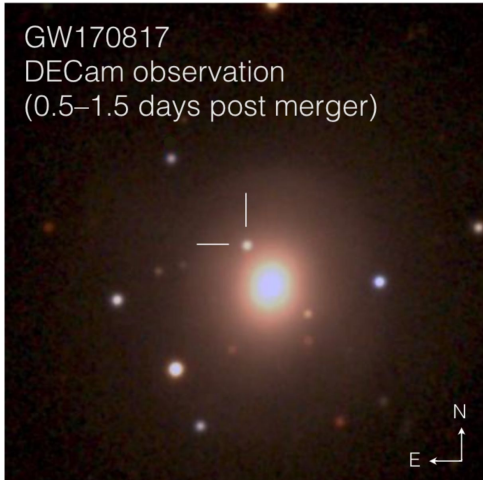
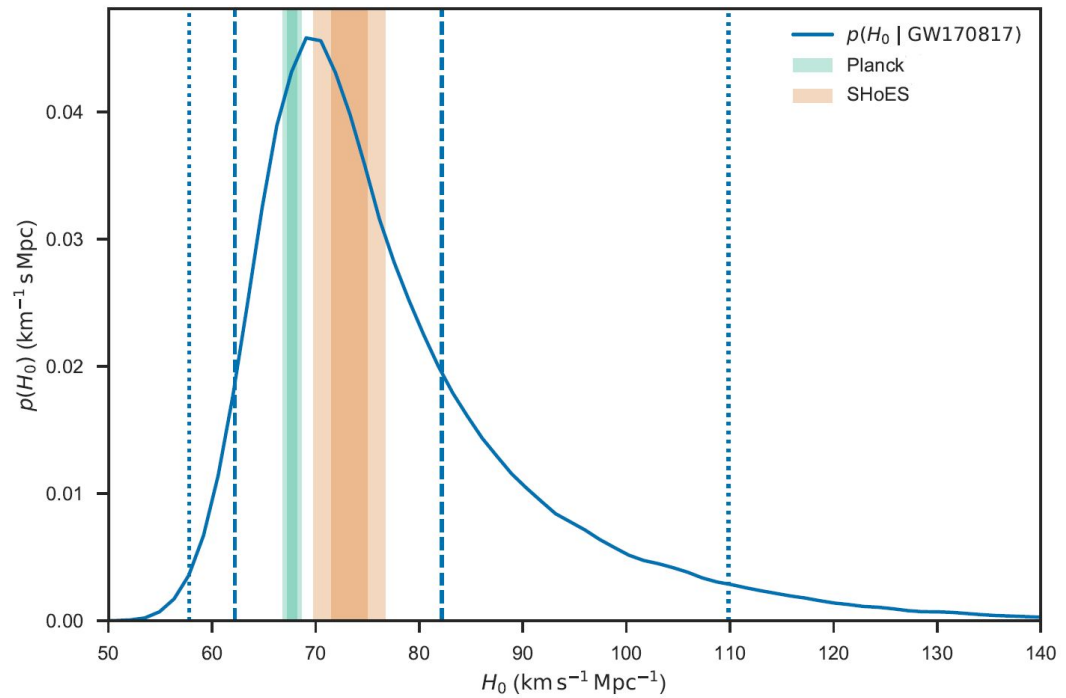


Image: <https://www.ligo.org>

- ❑ The only GW event detected along with a GRB: **GRB 170817A**
- ❑ Luminosity distance ~ **40 Mpc**
- ❑ Host identification : **NGC 4993**

For the majority of the detected events, host identification is not possible

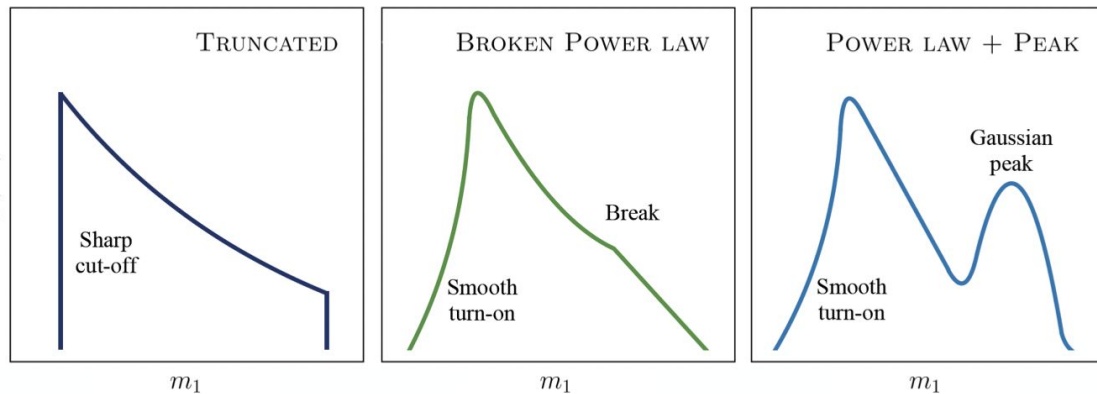
Constraints on Hubble constant from GW170817



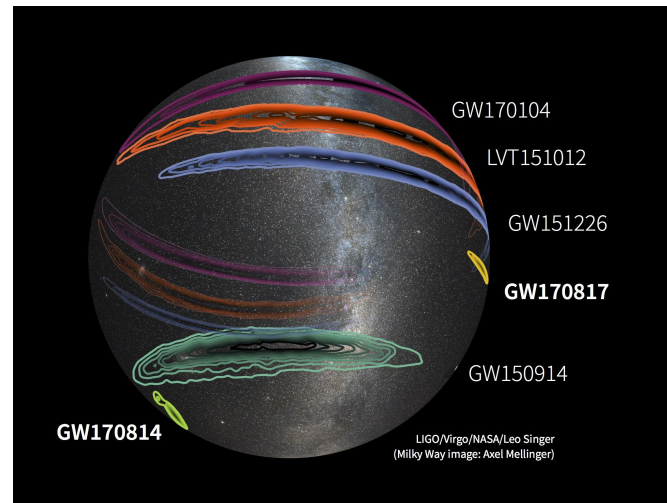
Abbott et. al. (2017)

Measurement of H_0 with $\sim 15\%$ accuracy at 68.3% confidence

Inferring H_0 using population statistics



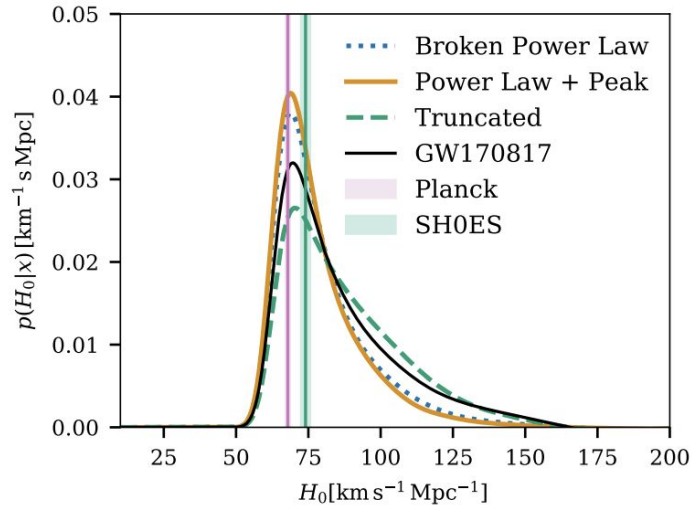
Abbott et al. (2021)



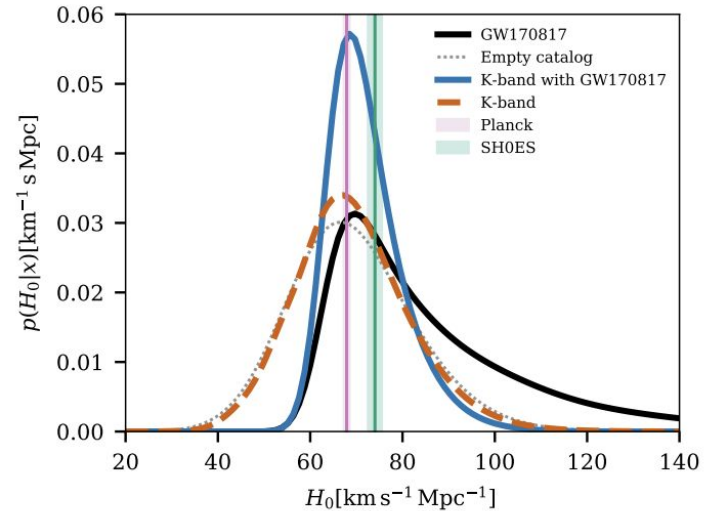
Credit : Leo Singer

- Map an astrophysically motivated source mass distribution to the detector frame - thus extract the redshift distribution (icarogw) Abbott et al. (2023)
- Consider galaxies (with known redshifts) in the localization region as potential hosts and compute H_0 distribution for each potential host (gwcosmo) Schutz(1986)

Constraints from GWTC-3



Method 1: Spectral Siren technique (icarogw)



Method 2: Galaxy Catalogue technique (gwcosmo)

An alternative approach: *The Large Scale Structures*

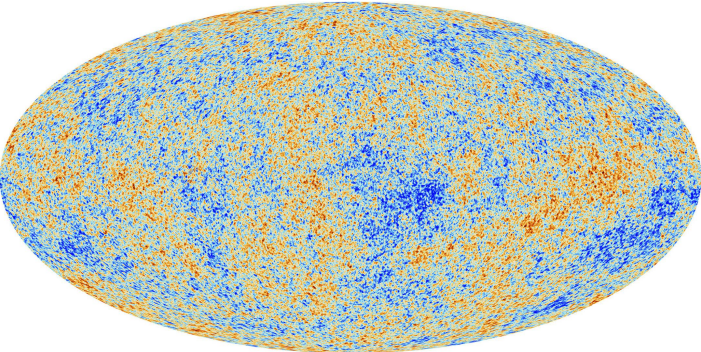


Image: ESA

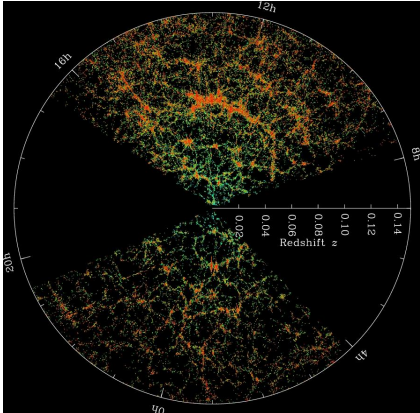


Image: SDSS

The Millennium simulation (z=0)

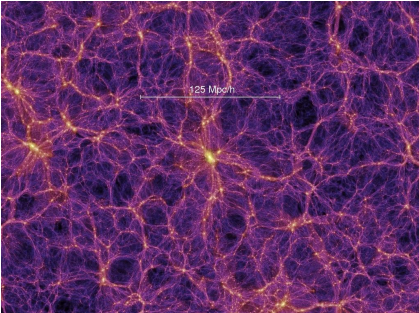


Image: <https://wwwmpa.mpa-garching.mpg.de>

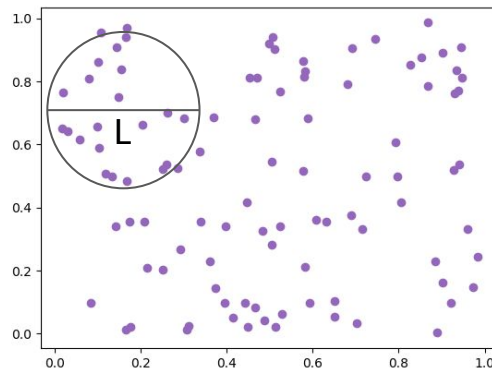
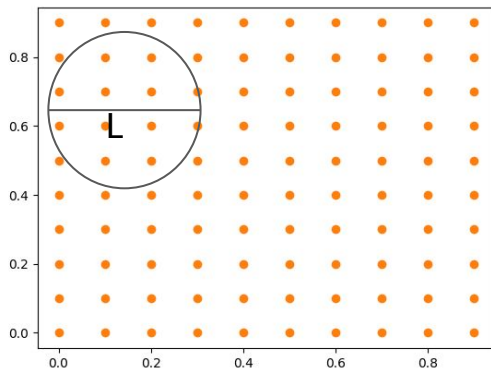
Measures of clustering: Density Contrast and cross-correlation

$$\delta(\mathbf{x}) \sim \frac{\rho(\mathbf{x})}{\bar{\rho}} - 1$$

$$\xi(\mathbf{x}, \mathbf{x}') \sim \langle \delta(\mathbf{x})\delta(\mathbf{x}') \rangle$$

Angular cross-correlation

$$w(\theta, \theta') \sim \langle \delta(\theta)\delta(\theta') \rangle$$



Clustering $\sim N/W - 1$
 L : Clustering length

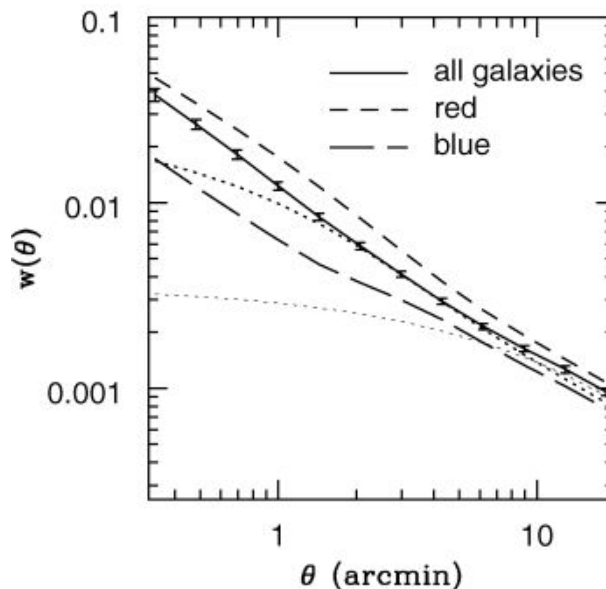
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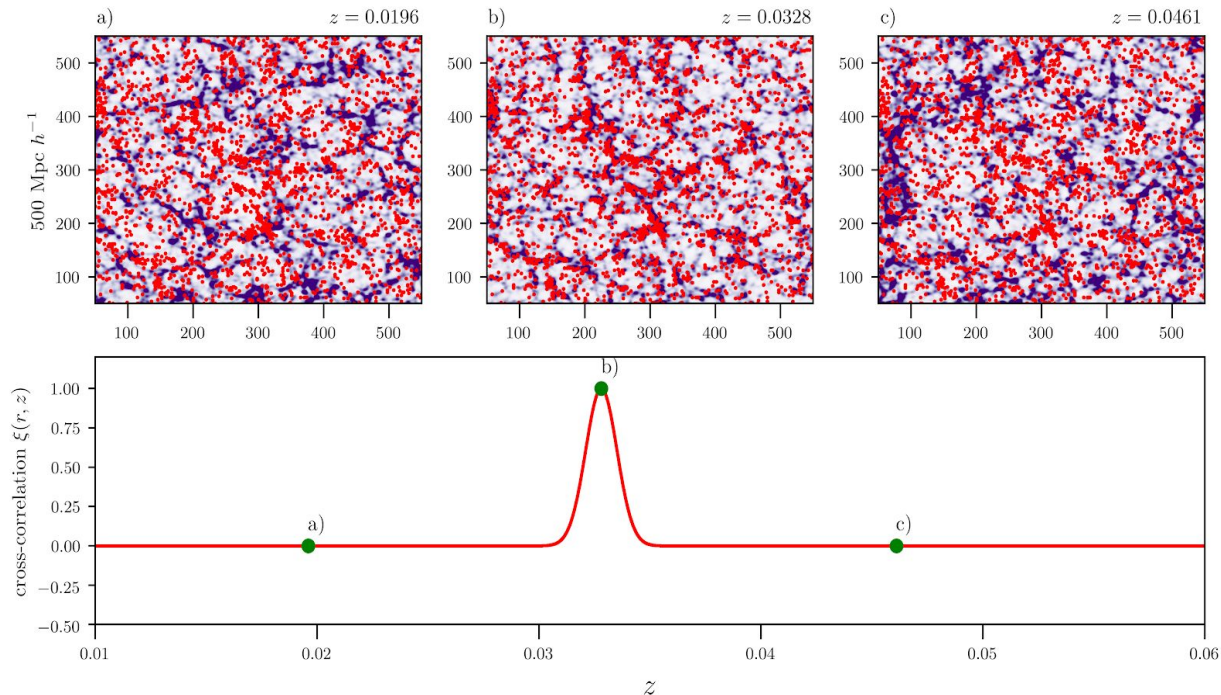
Inferring redshift from LSS distribution

Red : BBH sources at a fixed unknown redshift

Blue: Galaxy distribution at different redshift slices

The BBH distribution is a part of the same large scale structure as the galaxies.

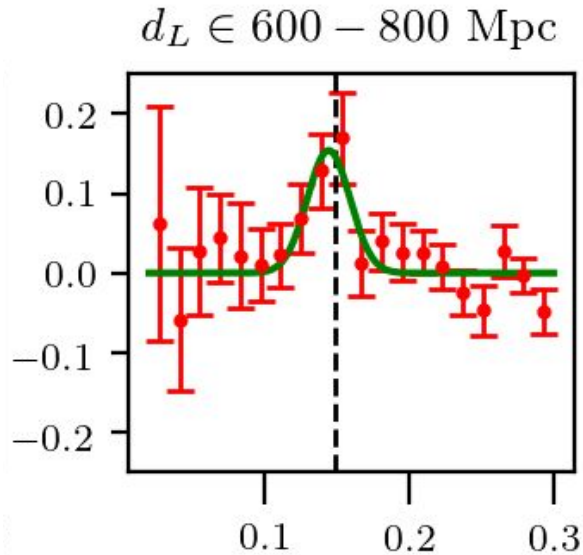
Cross-correlation of the two distributions provide a redshift estimate for the unknown BBH population



A realistic Simulation of the catalogs

- ❑ The true locations of the GW events are sampled from the dark matter distribution of a cosmological N-body simulation (Big-MultiDark Planck)
- ❑ Massive dark matter halos act as galaxy markers in our simulation.
- ❑ Realistic simulation of the GW events and parameter estimations run using BILBY: A free Bayesian Inference library for GW (Ashton et al. 2019)
- ❑ 3 detector network (Advanced Ligo L +H + Advanced Virgo): combined SNR threshold of 8

Modelling the cross-correlation

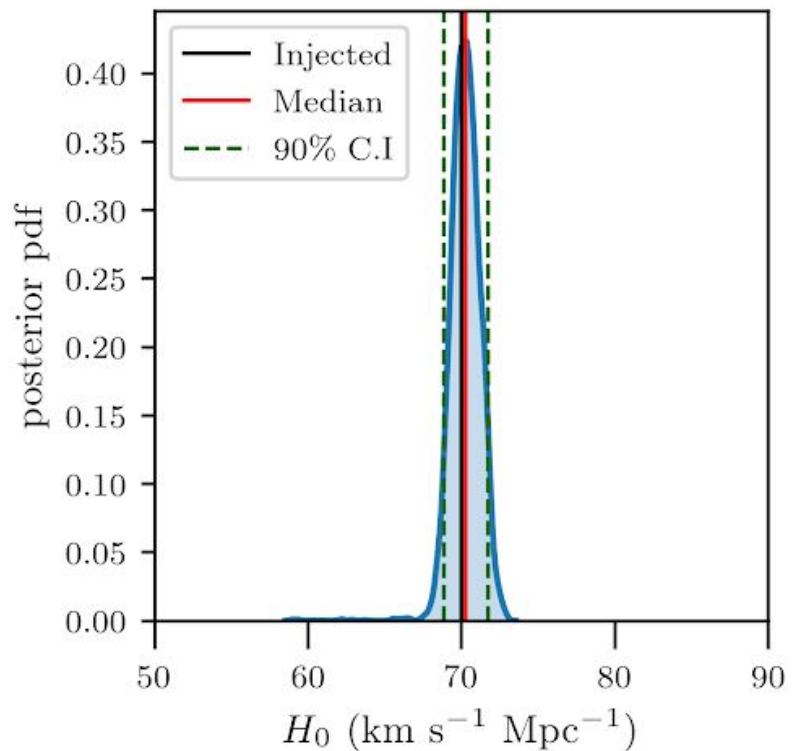
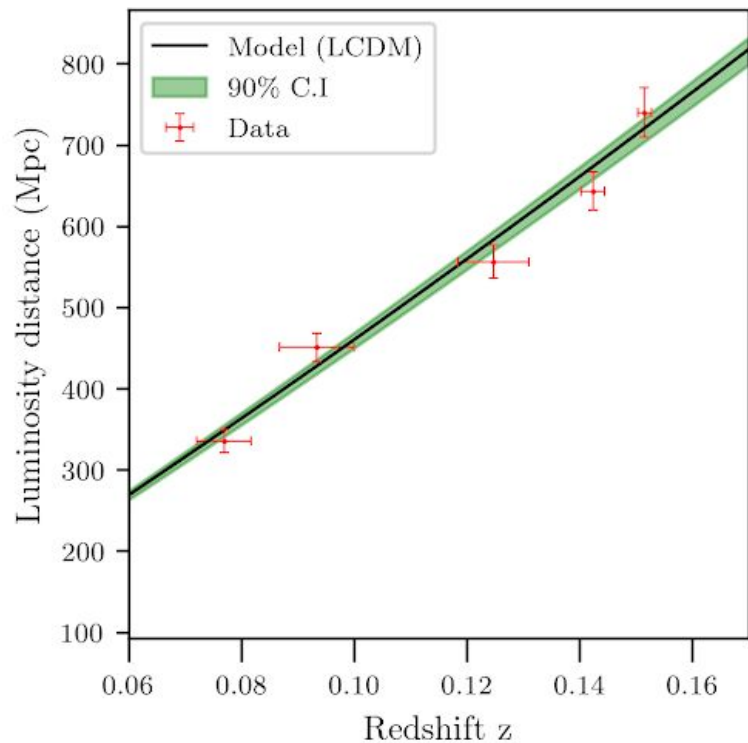


Assume power law three-dimensional cross-correlation function:

$$\xi_{\text{gw,g}}(r) = \left[\frac{r}{r_0} \right]^{-\gamma}$$

$$w(\leq \theta_{\text{max}}, z, z') \propto \exp \left[-\frac{(z - z')^2}{2\sigma_z^2} \right]$$

Hubble-Lemaitre diagram : 500 events



An event-by-event analysis

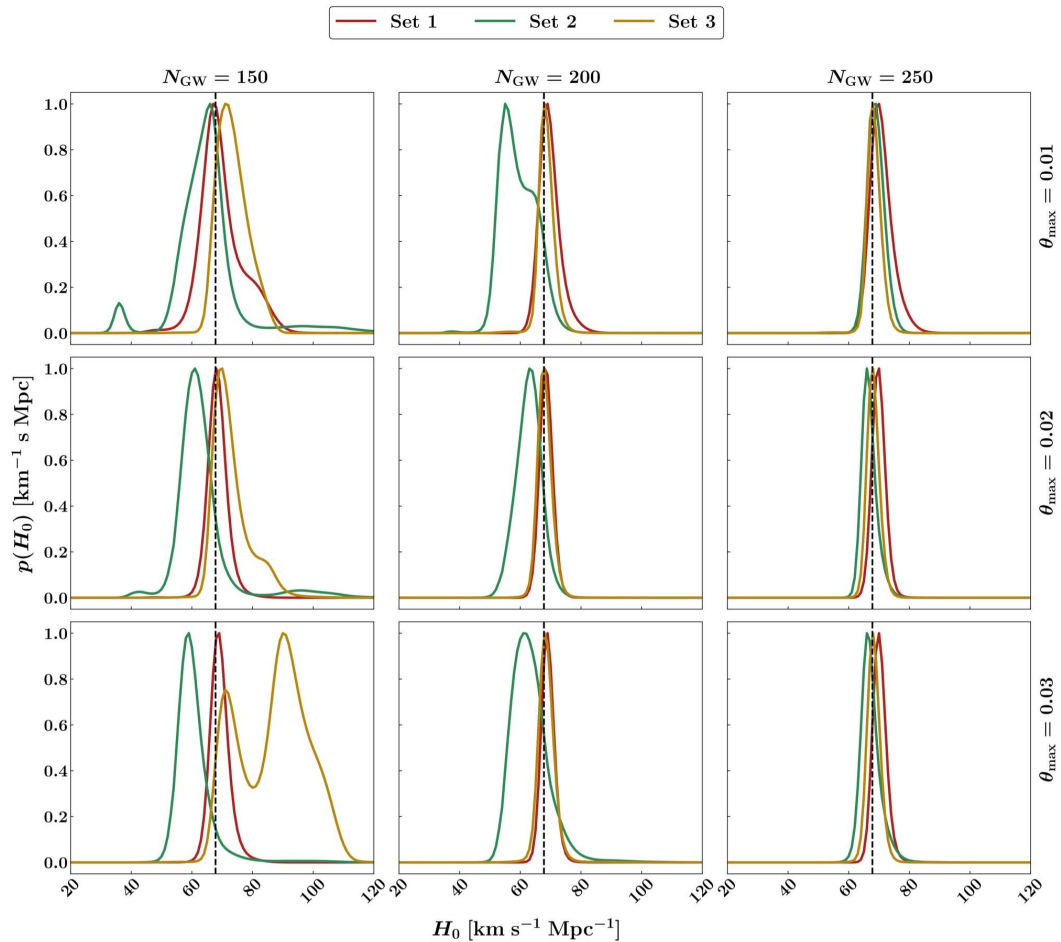
$$p(H_0 | \mathbf{d}_{\text{strain}}, \mathbf{d}_g^{\text{obs}}) = \int p(H_0, \mathbf{d}_{\text{gw}} | \mathbf{d}_{\text{strain}}, \mathbf{d}_g^{\text{obs}}) d\mathbf{d}_{\text{gw}}$$
$$\propto \int \mathcal{L}(\mathbf{d}_{\text{strain}}, \mathbf{d}_g^{\text{obs}} | H_0, \mathbf{d}_{\text{gw}}) P(H_0, \mathbf{d}_{\text{gw}}) d\mathbf{d}_{\text{gw}}$$



For each GW event, the posterior is obtained by marginalizing over localization uncertainties \mathbf{d}_{gw}

Assuming independent probability distributions, the single-event posteriors can be combined as :

$$P(H_0 | \{\mathbf{d}_{\text{strain}}\}, \{\mathbf{d}_g^{\text{obs}}\}) \propto P(H_0) \prod_i \mathcal{L}(\mathbf{d}_{\text{strain}_i}, \mathbf{d}_{g_i}^{\text{obs}} | H_0)$$
$$\propto P(H_0) \prod_i \int \mathcal{L}(\mathbf{d}_{\text{strain}_i}, \mathbf{d}_{g_i}^{\text{obs}} | H_0, \mathbf{d}_{\text{gw}}) P(\mathbf{d}_{\text{gw}}) d\mathbf{d}_{\text{gw}}$$



-Set 1,2,3 : Different realizations of randomly generated events upto 1000 Mpc, SNR>12

-Dependence on **sample size** and **correlation scale**

-Injected value of $H_0 = 67$ km/s/Mpc

Ghosh, More, **SB**, Bose (arXiv: 2312.16305)

Take-Home Message

Incorporating information from large-scale structure correlations is **crucial** to a more robust inference of the background cosmology

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Incorporating information from large-scale structure correlations is **crucial** to a more robust inference of the background cosmology

- With 3G gravitational wave detectors such as Einstein Telescope, Cosmic Explorer and space based LISA, the future looks promising.
- Euclid will map the large-scale structures in the universe very accurately up to high redshifts.

ACKNOWLEDGMENTS

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Extra Slides

Waveform simulation: inputs

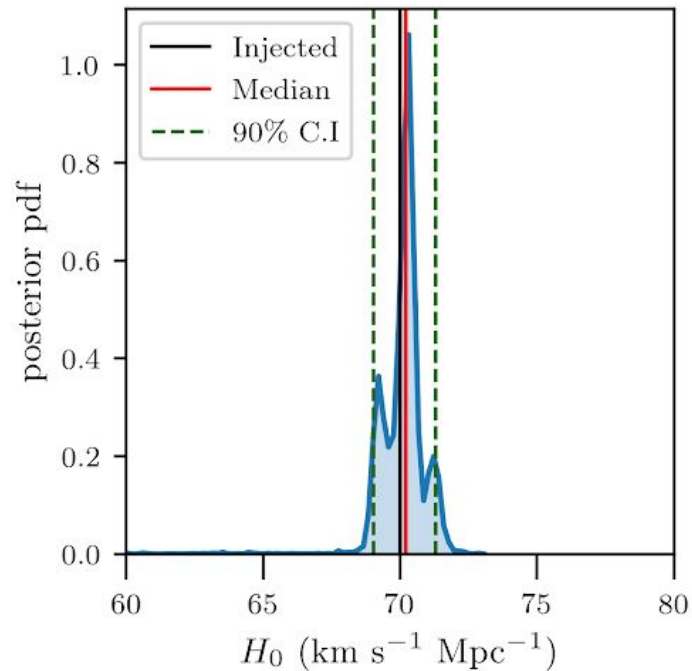
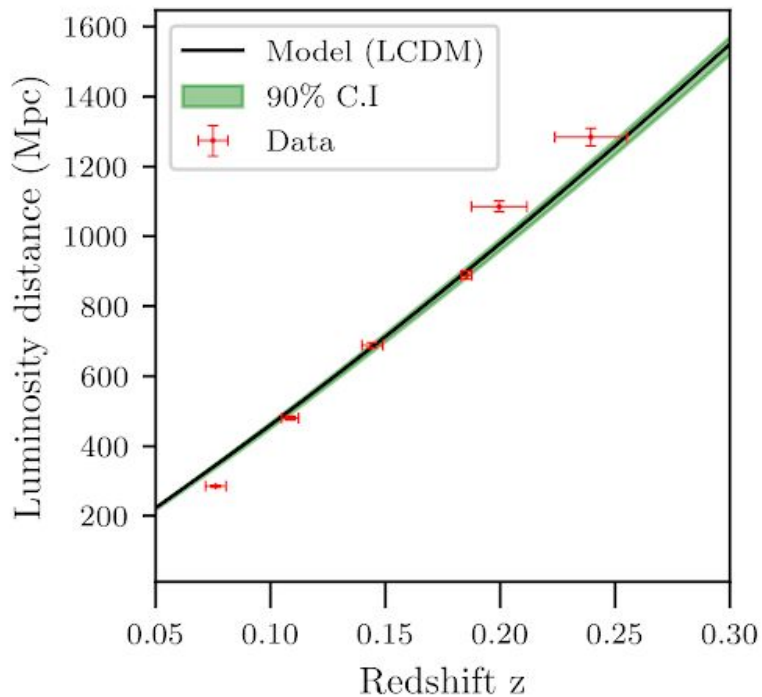
Detectors	Sensitivity
Livingston	Advanced LIGO
Hanford	Advanced LIGO
Virgo	Advanced Virgo

Injection Parameters		
Parameters	Distribution	Limits
$m_{1,2}$	uniform	$[10, 35] M_{\odot}$
$\chi_{1,2}$	uniform	$[0, 0.8]$
ϕ_{12}, ϕ_{jl}	uniform	$[0, 2\pi)$
$\cos \theta_{1,2}, \cos \iota$	uniform	$[-1, 1)$
ψ, ϕ_c	Fixed	0

Detection criteria: At least two of the detectors SNR above a threshold value of 5 each, the third an SNR greater than 2.5, and network SNR of greater than 8.

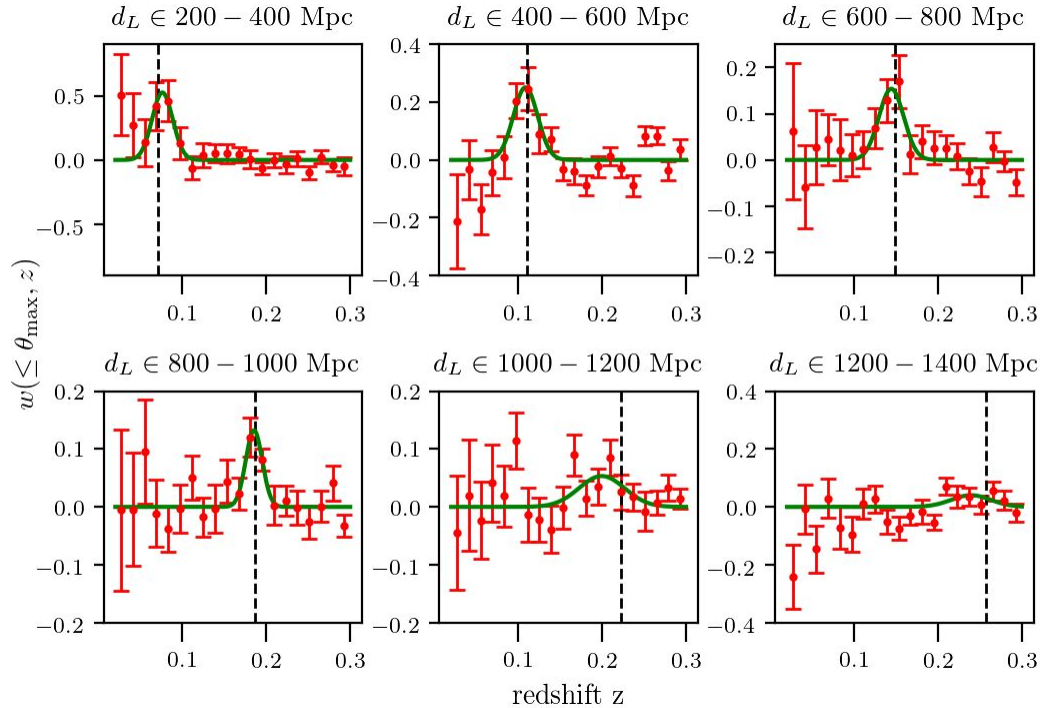
Hubble-Lemaître diagram : 5000 events

Red points: d_L inferred from BBH merger waveforms, redshift from cross-correlations



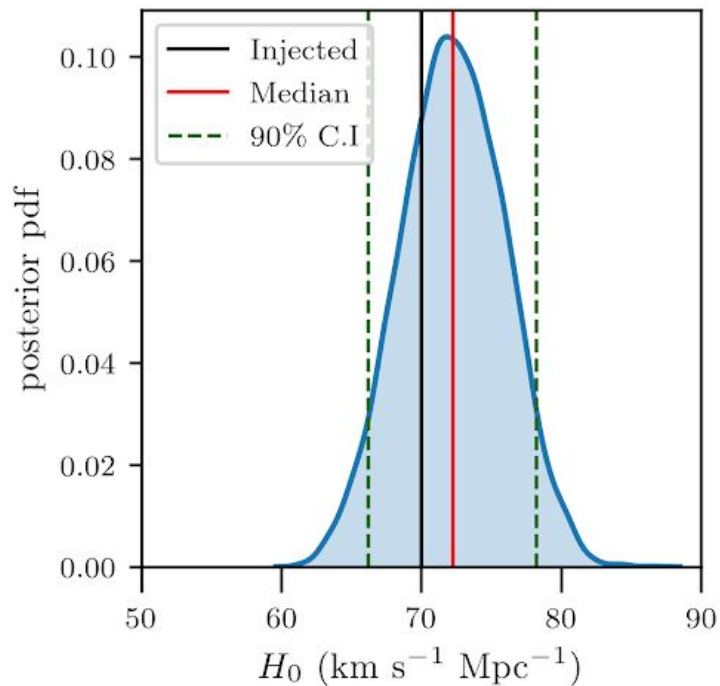
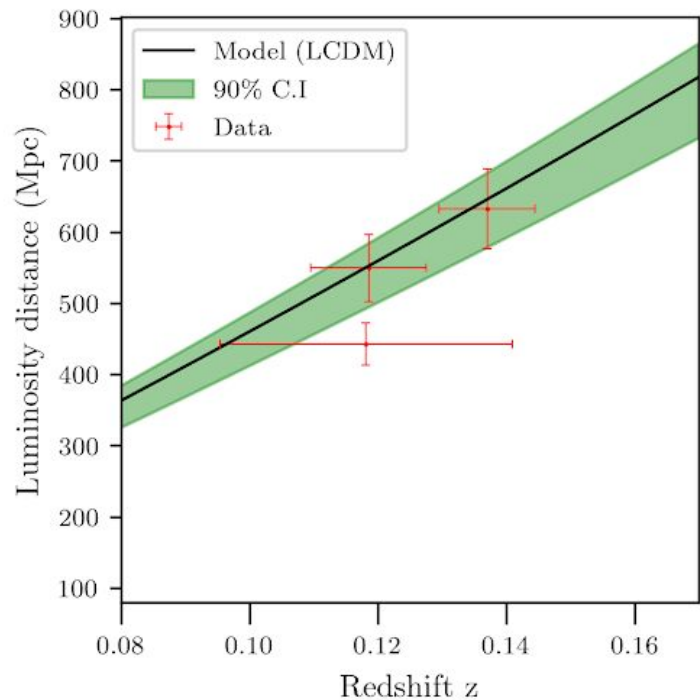
Black solid line: the true value of H_0 in the simulation
Dashed lines: 90 percent credible interval

Redshift from angular cross-correlation



- 5000 BBH mergers divided into 6 bins in the inferred luminosity distances
- The mock galaxies are divided into 20 redshift bins
- Red points are the measured cross-correlations with error bars, peaking at the correct redshift
- The injected value of $H_0 = 70$ km/s/Mpc gives an average redshift of the GW sources in each bin (black vertical line)

Hubble-Lemaitre diagram : 50 events

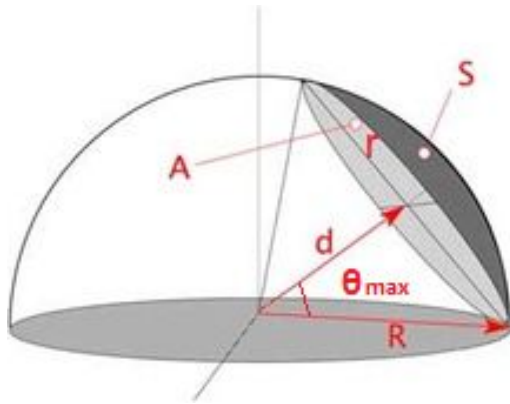


Constraints from the three samples

Constraints on H_0			
No. of GW events	Max d_L (Mpc)	Injected H_0 ($\text{km s}^{-1} \text{Mpc}^{-1}$)	Constraints on H_0 ($\text{km s}^{-1} \text{Mpc}^{-1}$)
5100	1400	70	$70.22^{+1.09}_{-1.18}$
500	900	70	$70.26^{+1.47}_{-1.40}$
50	900	70	$72.24^{+5.98}_{-6.05}$

The error bars signify 90% credible interval around the the median of H_0 posterior

Angular Cross-correlation Estimator



We count the number of galaxy-BBH pairs which have an angular separation θ_{\max} or less in the actual catalog and in a randomly distributed catalog.

Angular cross-correlation estimator

$$w(\leq \theta_{\max}) = \frac{n_{D_1 D_2}(\leq \theta_{\max})}{n_{R_1 R_2}(\leq \theta_{\max})} - 1$$

D_1, D_2 : Data catalogs

R_1, R_2 : Random catalogs