Current status of the cosmological analysis pipelines in LVK

B. Revenu, Subatech (CNRS, Nantes)

GDR, Marseille, oct 2024

1. GW and cosmology

Cosmology with GWs

GW from mergers give access to d_L:

signal amplitude: $$< h > \propto \ M_c^{5/3} \ f^{2/3} \ r^{-1}$$

time to coalescence: $$\tau \propto \ M_c^{-5/3} \ f^{-8/3}$$

so that: $$r \propto \ < h >^{-1} \ \tau^{-1} \ f^{-2}$$ (independent of time)

measuring <h>, τ (in seconds) and f gives the distance r to the CBC:

$$r = 780 \text{ Mpc} \frac{1}{10^{23} \ < h > \ \tau} \left(\frac{100 \text{ Hz}}{f} \right)^2$$

in an expanding universe, r is d_L and masses are multiplied by (1+z)

Cosmology with GWs

In a flat LCDM model, d_L is related to cosmology:

$$d_L = \frac{(1+z)\,c}{H_0}\int_0^z \frac{\mathrm{d}z'}{\sqrt{\Omega_m(1+z')^3+\Omega_\Lambda}} \overset{z\ll 1}{\approx} \frac{c\,z}{H_0}$$

(need large-z events to infer Ω_m and Ω_Λ)

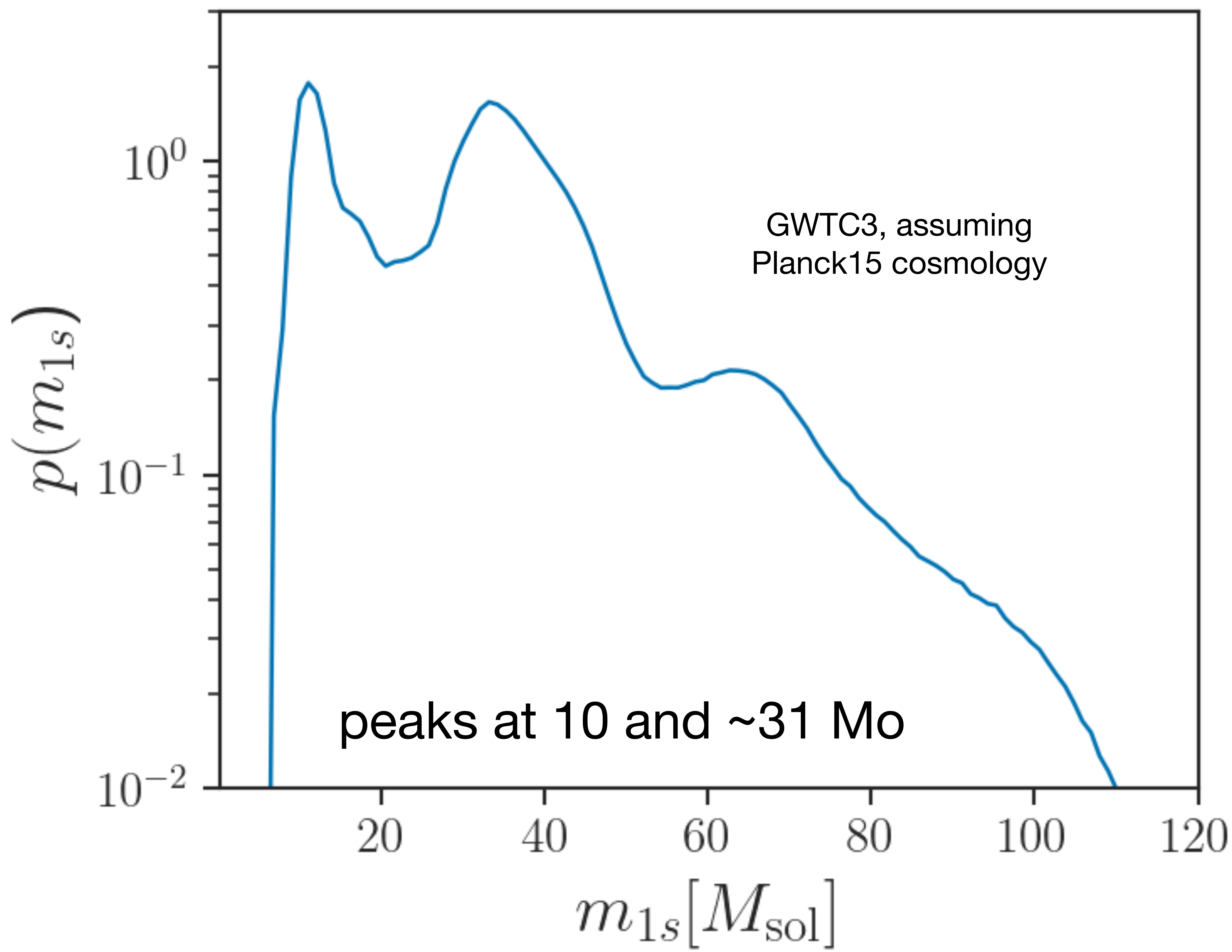
$$m_{\rm det} = (1+z)\,m_{\rm s} \xrightarrow{\text{if } m_s \text{ is known!}} z = \frac{m_{\rm det}}{m_{\rm s}} - 1 \xrightarrow{\text{if } m_s \text{ is known!}} H_0 = \frac{c}{d_L}\frac{m_{\rm det}-m_{\rm s}}{m_{\rm s}}$$

redshift-mass degeneracy:

any feature in the mass spectrum helps breaking the degeneracy: this is the **spectral analysis** (GW data only)

Cosmology with GWs

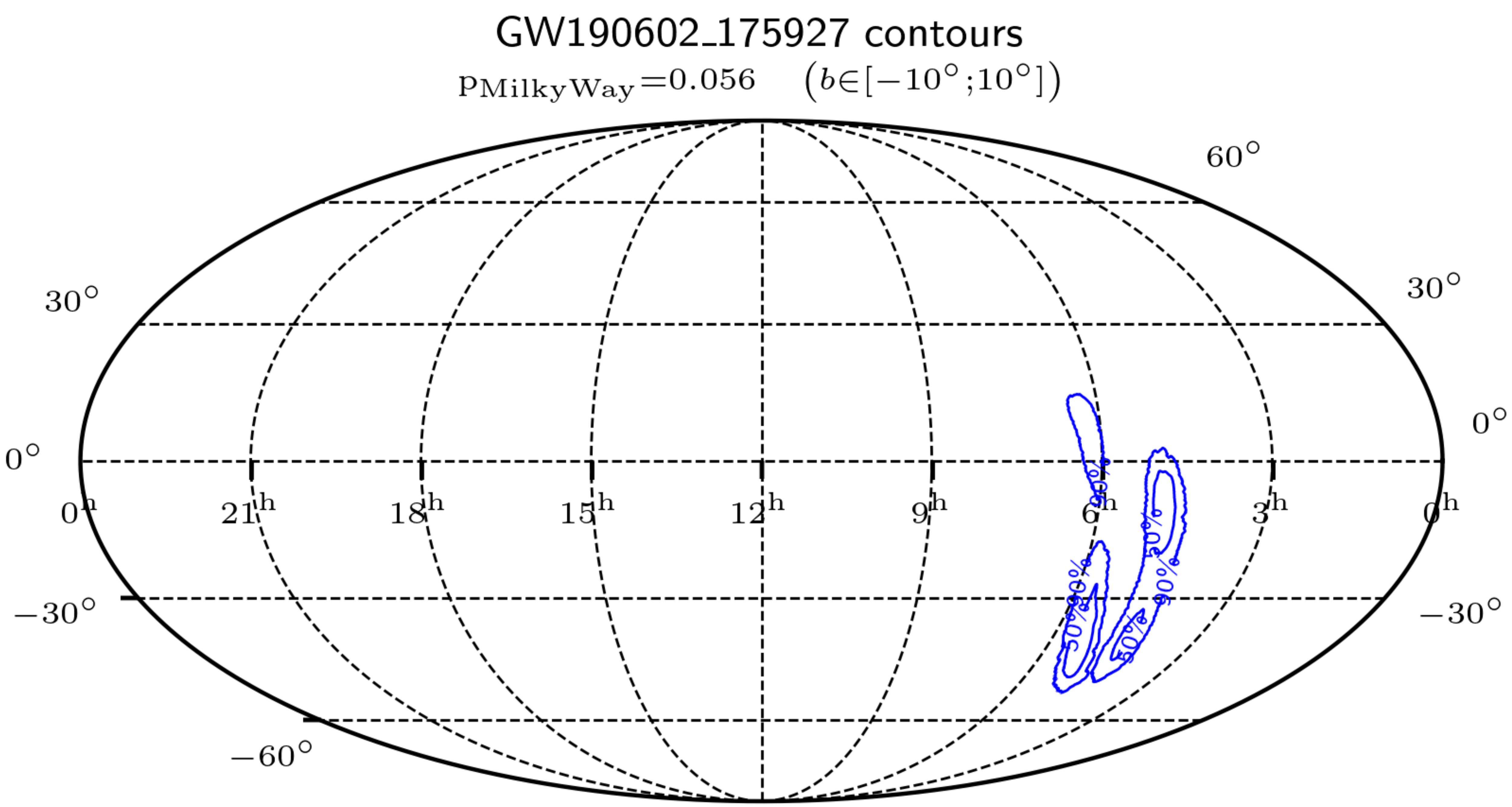
there are features in the mass spectrum:

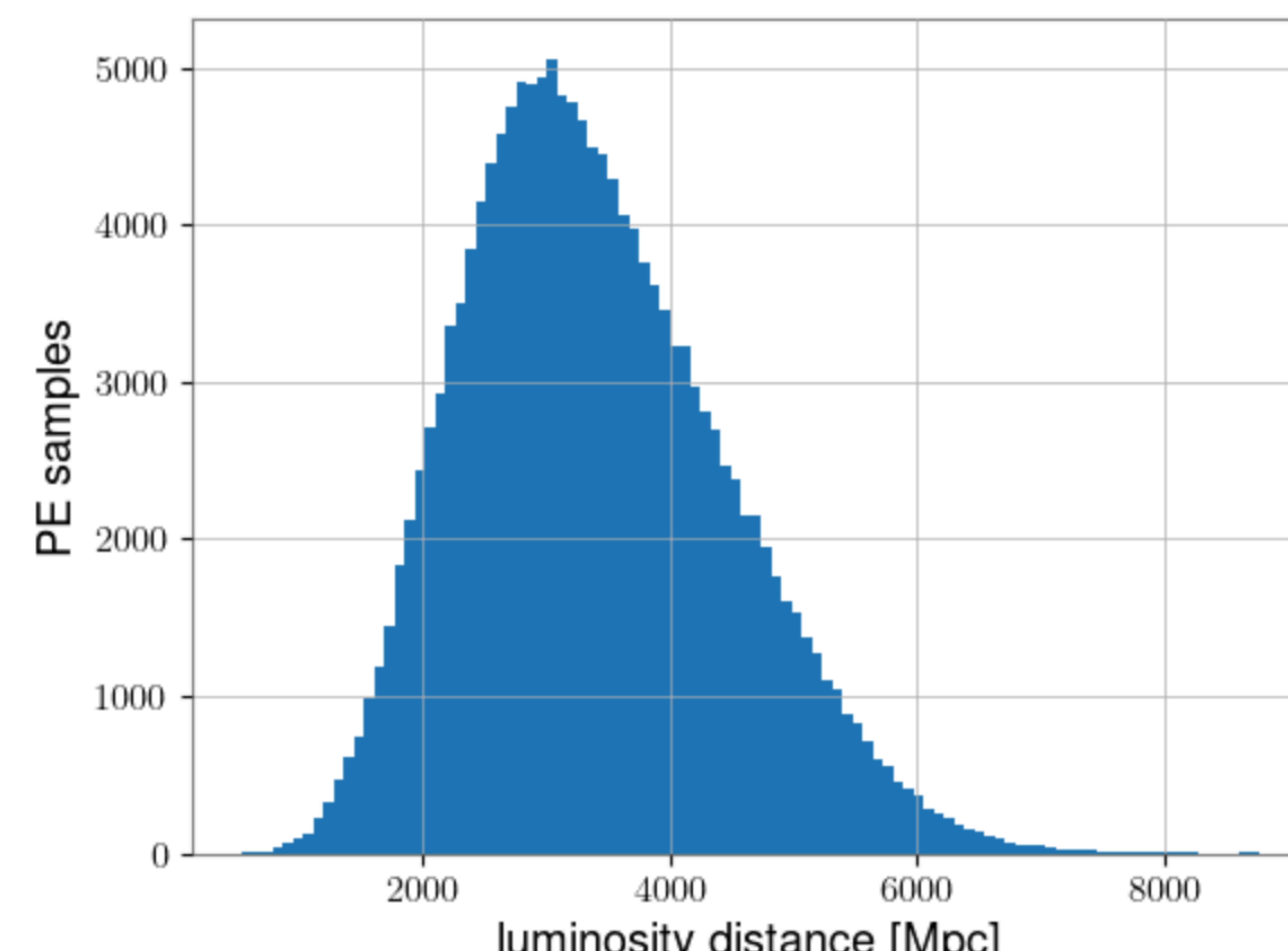


Cosmology with GWs

We add another source of information
under the assumption: CBCs are hosted in galaxies

the measured GW signal depends on the sky position:
we compute a sky map of the possible source locations (3D: ra, dec, d_L),
with correlations





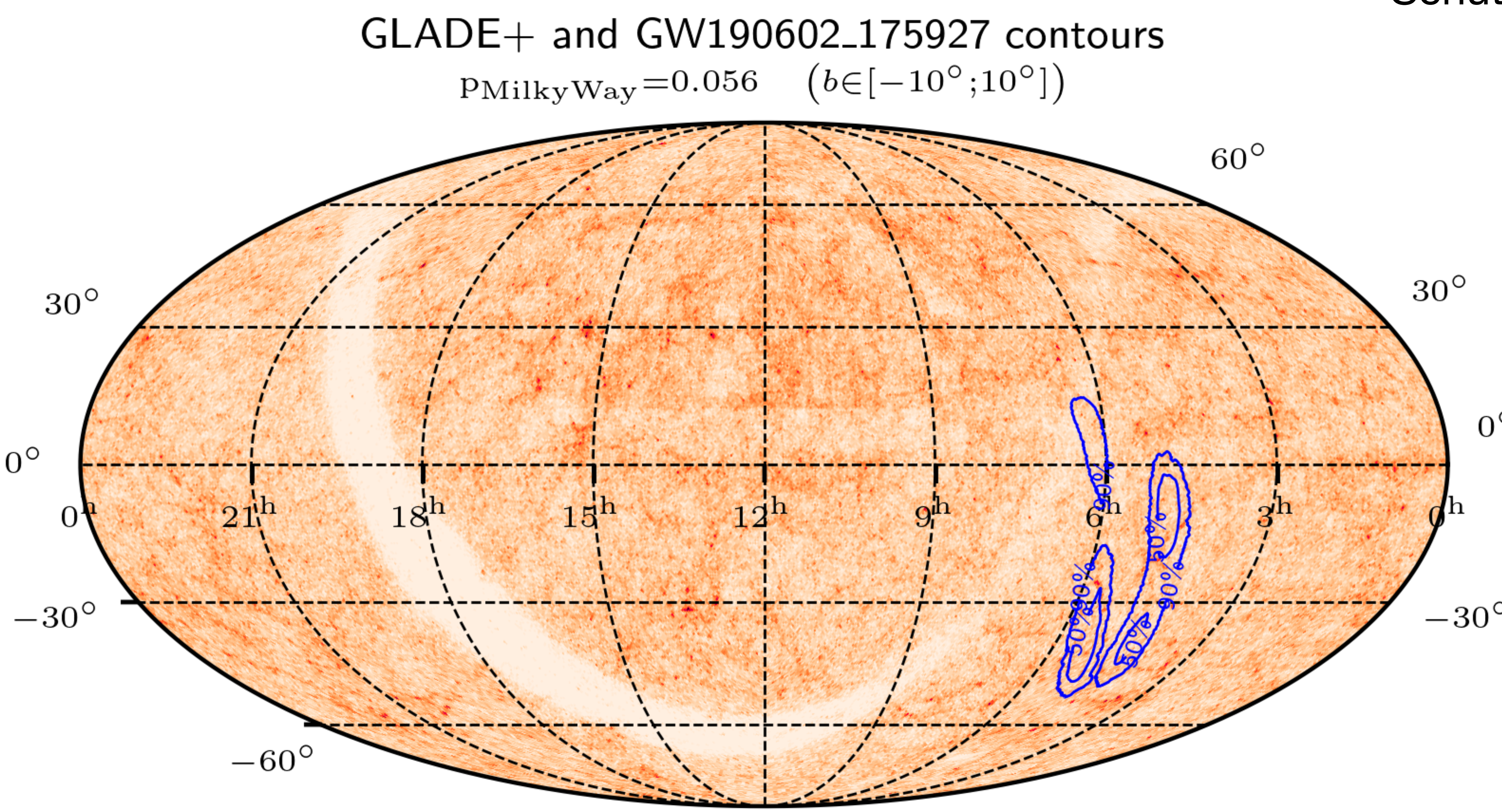
Cosmology with GWs

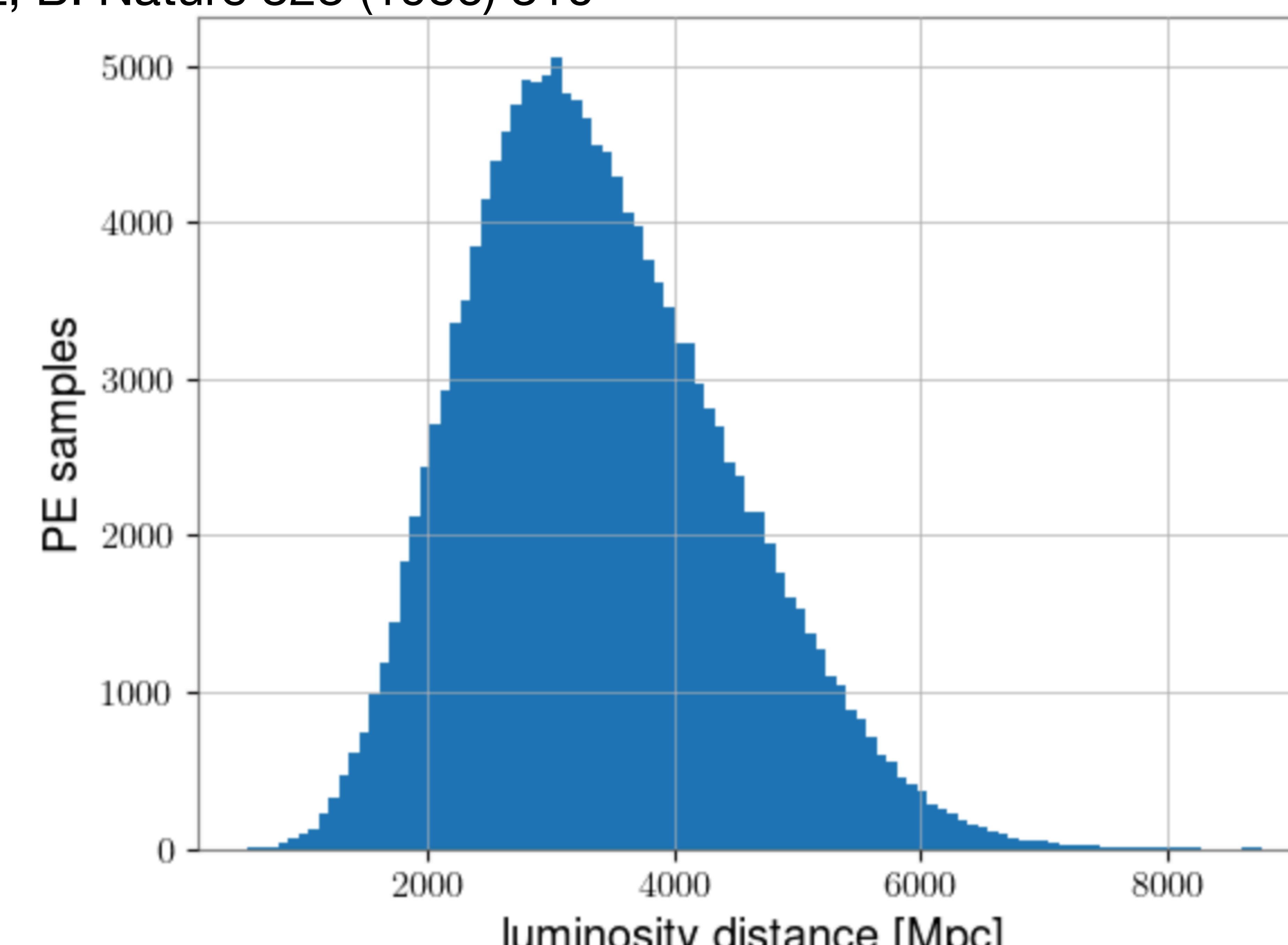
We add another source of information
under the assumption: CBCs are hosted in galaxies

the measured GW signal depends on the sky position:
we compute a sky map of the possible source locations (3D: ra, dec, d_L),
with correlations

add a galaxy catalogue: **galaxy-catalogue analysis**

Schutz, B. Nature 323 (1986) 310





GLADE+: Dalya et al, MNRAS 514, 1403–1411 (2022)

2. The model

Cosmology with GWs

Goal of the analysis:

describe the rates and distributions of mass, distance, spins

For a single event:

$$\theta = (m_{1d}, m_{2d}, d_L, \iota, \mathrm{ra}, \mathrm{dec}, \chi ...)$$

For a set of events:

$p(m_{1d}, m_{2d}, d_L, \chi, ...)$ detector frame

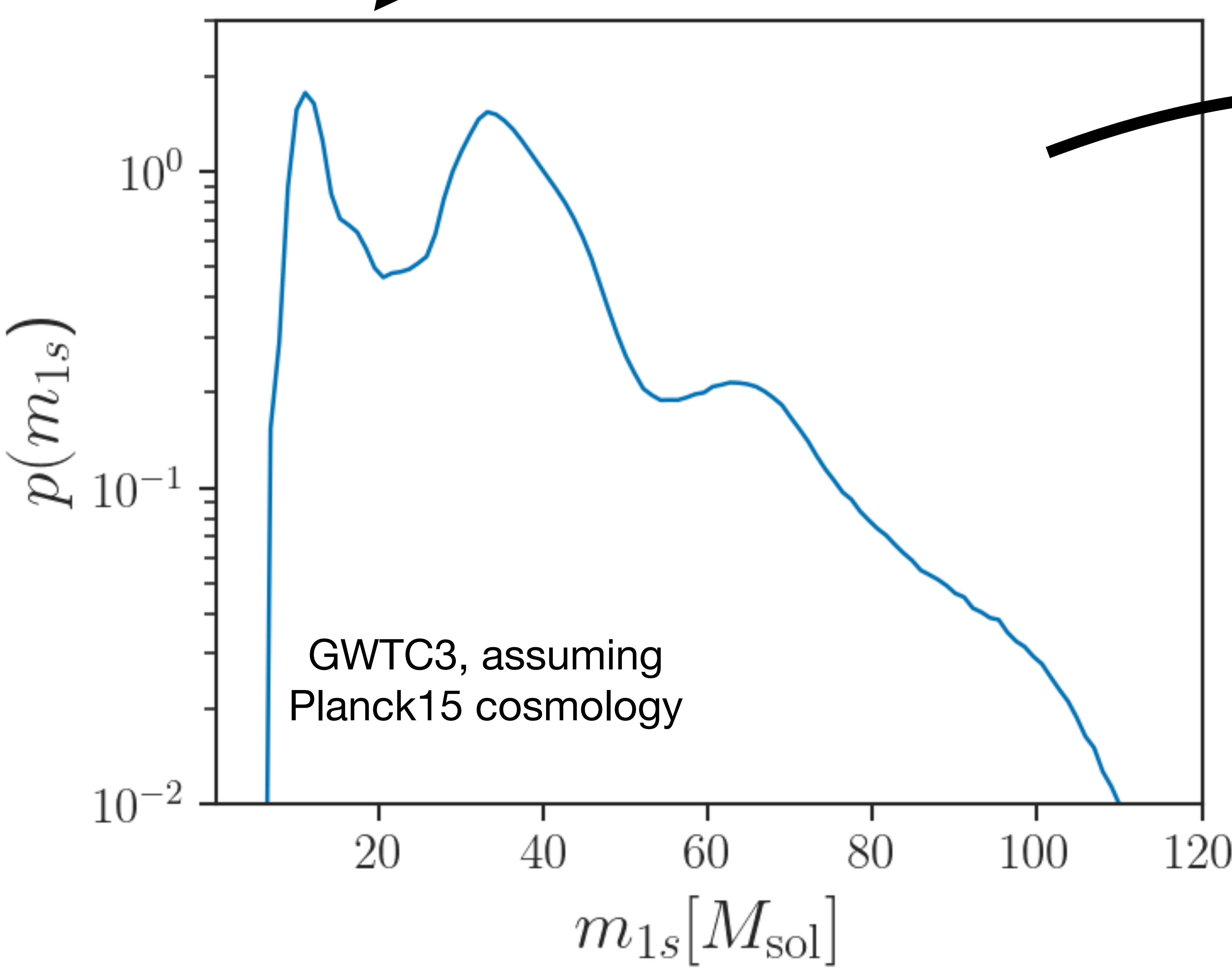
$p(m_{1s}, m_{2s}, z, \chi, ...)$ source frame (astrophysics)

$p(m_{1s})p(m_{2s}|m_{1s})p_{\mathrm{CBC}}(z)p(z)p(\chi)...$ simplification: separing variables

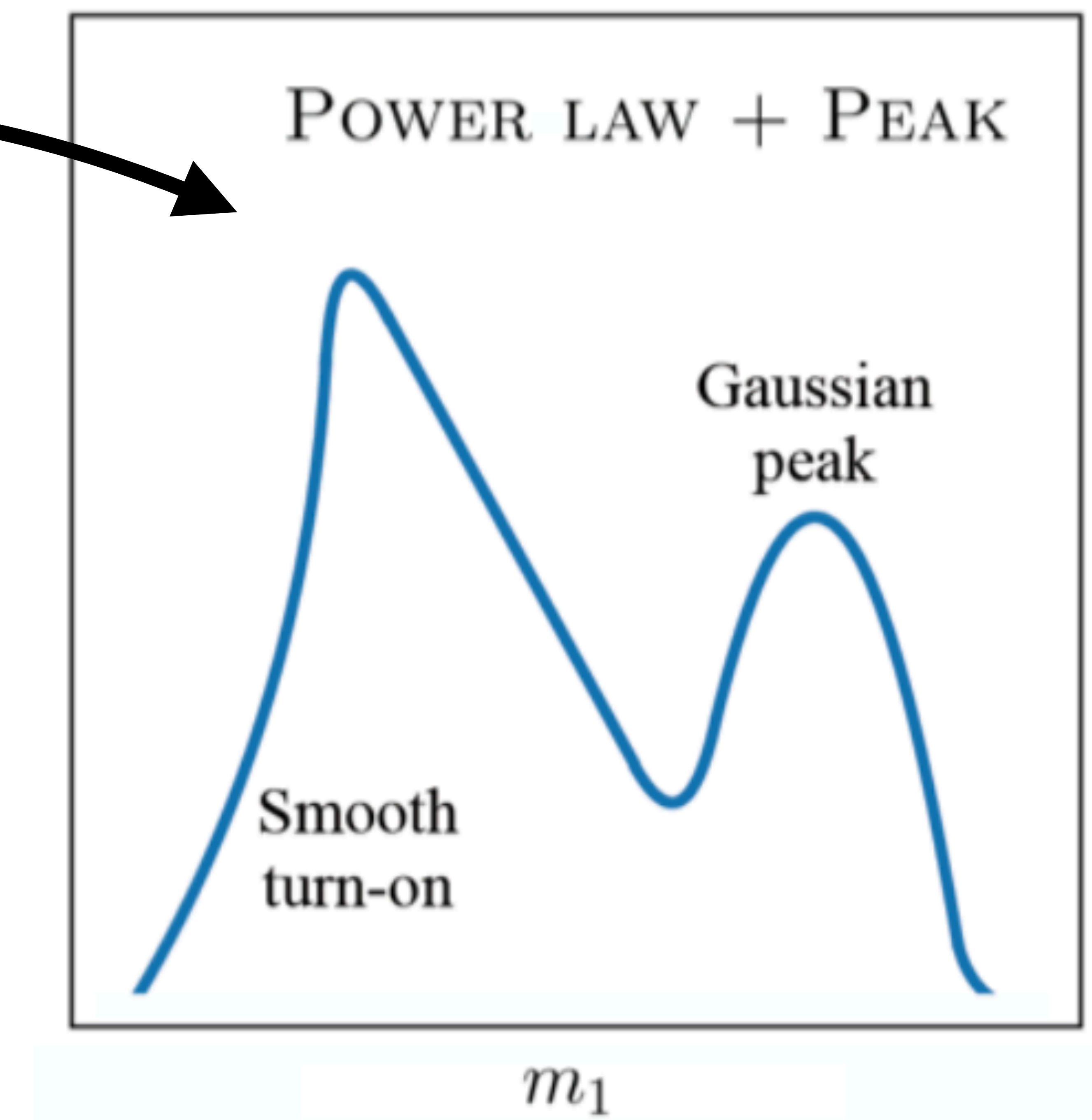
Cosmology with GWs

$$p(m_{1s})p(m_{2s}|m_{1s})p_{\mathrm{CBC}}(z)p(z)p(\chi)...$$

$$\propto m_{2s}^{\beta},\ \ m_{2s} \in [M_{\min}; m_{1s}]$$



LVK, *Astrophys.J.* 949 (2023) 2, 76



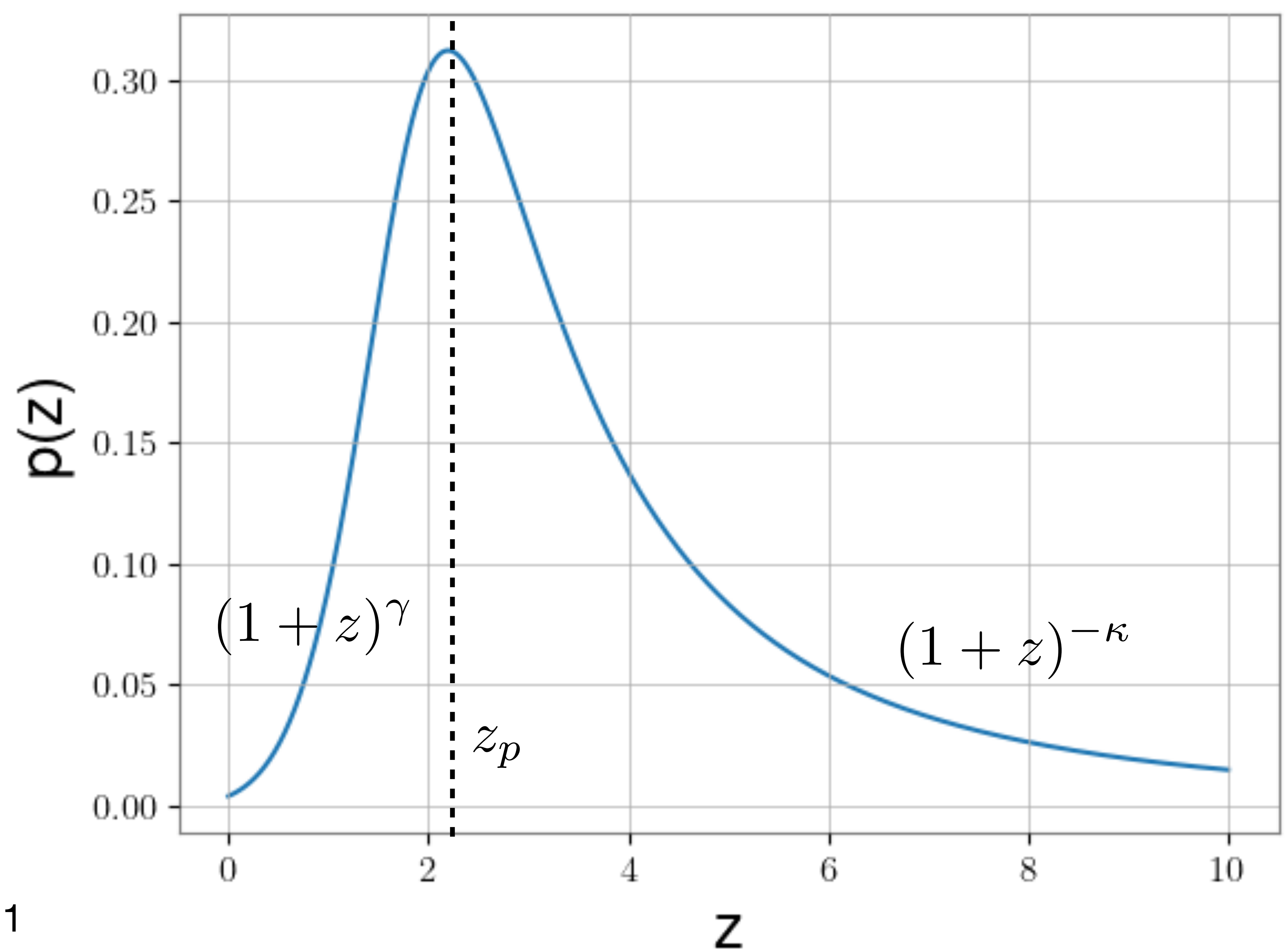
Cosmology with GWs

$$p(m_{1s})p(m_{2s}|m_{1s})p_{\rm CBC}(z)p(z)p(\chi)...$$

$$R_0 f(z) = R_0 \frac{\left(1+\frac{1}{(1+z_p)^{\gamma+\kappa}}\right)(1+z)^{\gamma}}{1+\left(\frac{1+z}{1+z_p}\right)^{\gamma+\kappa}} \ \mathrm{Gpc}^{-3}\ \mathrm{yr}^{-1}$$

Model for redshift distribution of CBCs: the binary black holes distribution follows the star formation rate

Madau-Dickinson



Cosmology with GWs

$$p(m_{1s})p(m_{2s}|m_{1s})p_{\rm CBC}(z)p(z)p(\chi)...$$

$$R_0 f(z) = R_0 \frac{\left(1+\frac{1}{(1+z_p)^{\gamma+\kappa}}\right)(1+z)^{\gamma}}{1+\left(\frac{1+z}{1+z_p}\right)^{\gamma+\kappa}} \ \mathrm{Gpc}^{-3}\ \mathrm{yr}^{-1}$$

Model for redshift distribution of CBCs:
the binary black holes distribution
follows the star formation rate

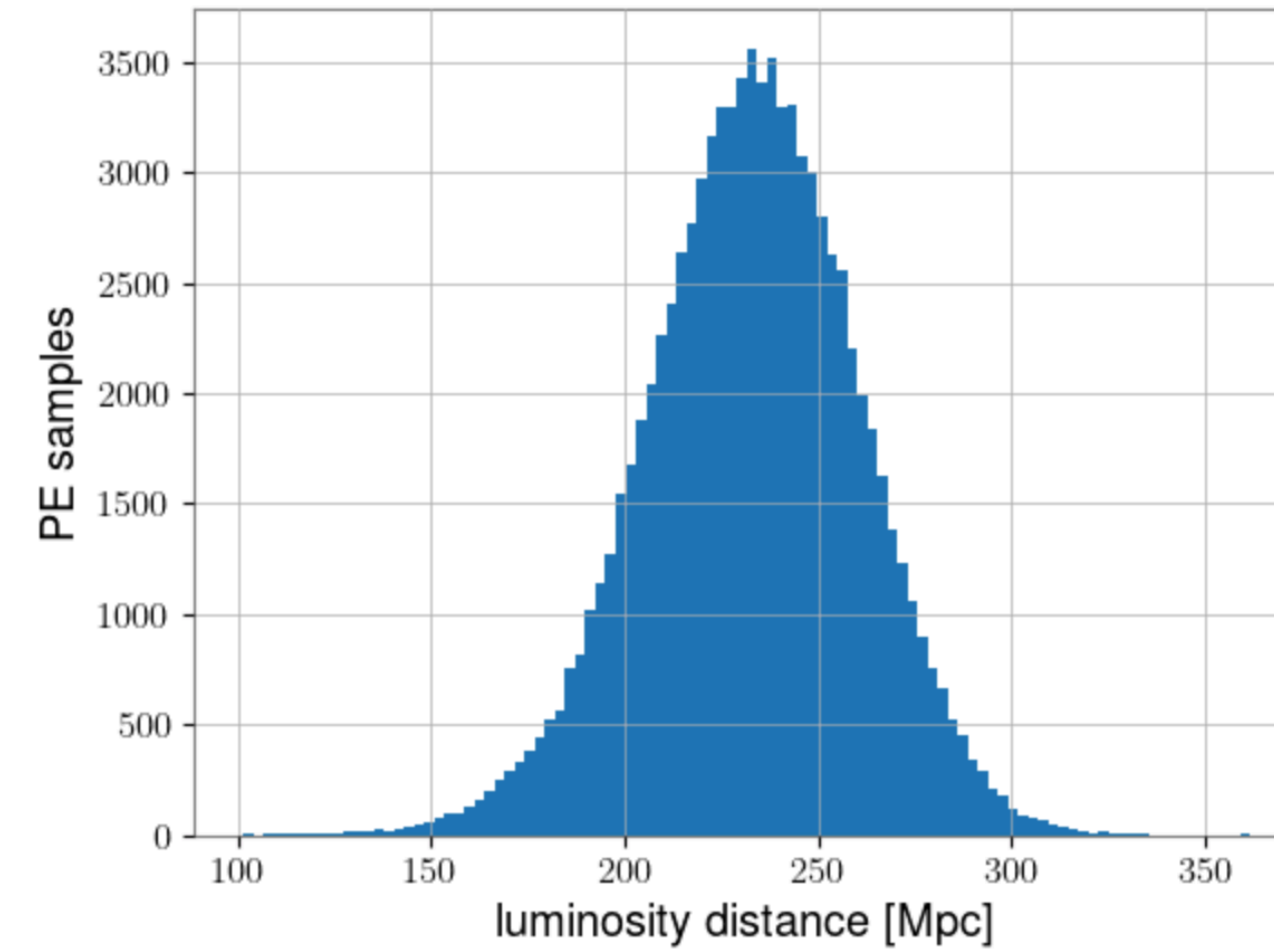
either uninformative prior:
uniform in comoving volume (spectral analysis)

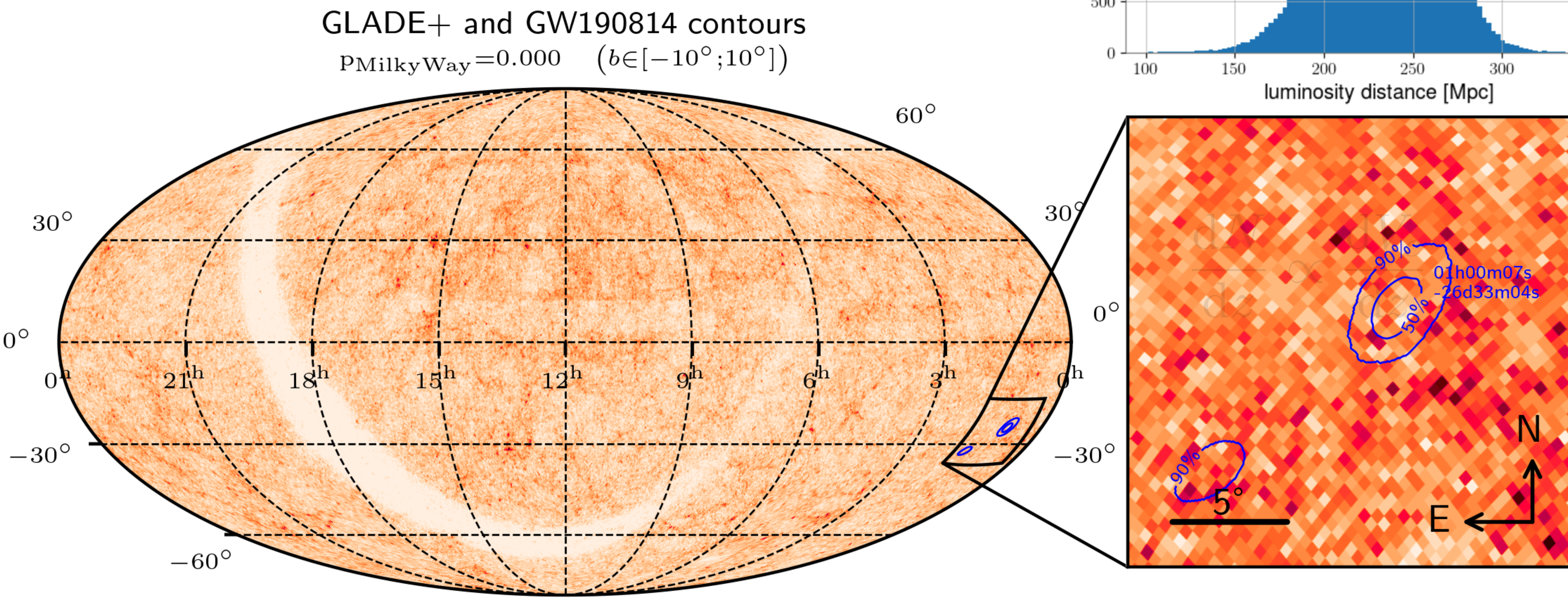
$$\mathrm{d}N \propto \mathrm{d}V_c : \quad \frac{\mathrm{d}N}{\mathrm{d}z} \propto \frac{\mathrm{d}V_c}{\mathrm{d}z}$$

or use a galaxy catalogue as an
informative prior for the host galaxies:

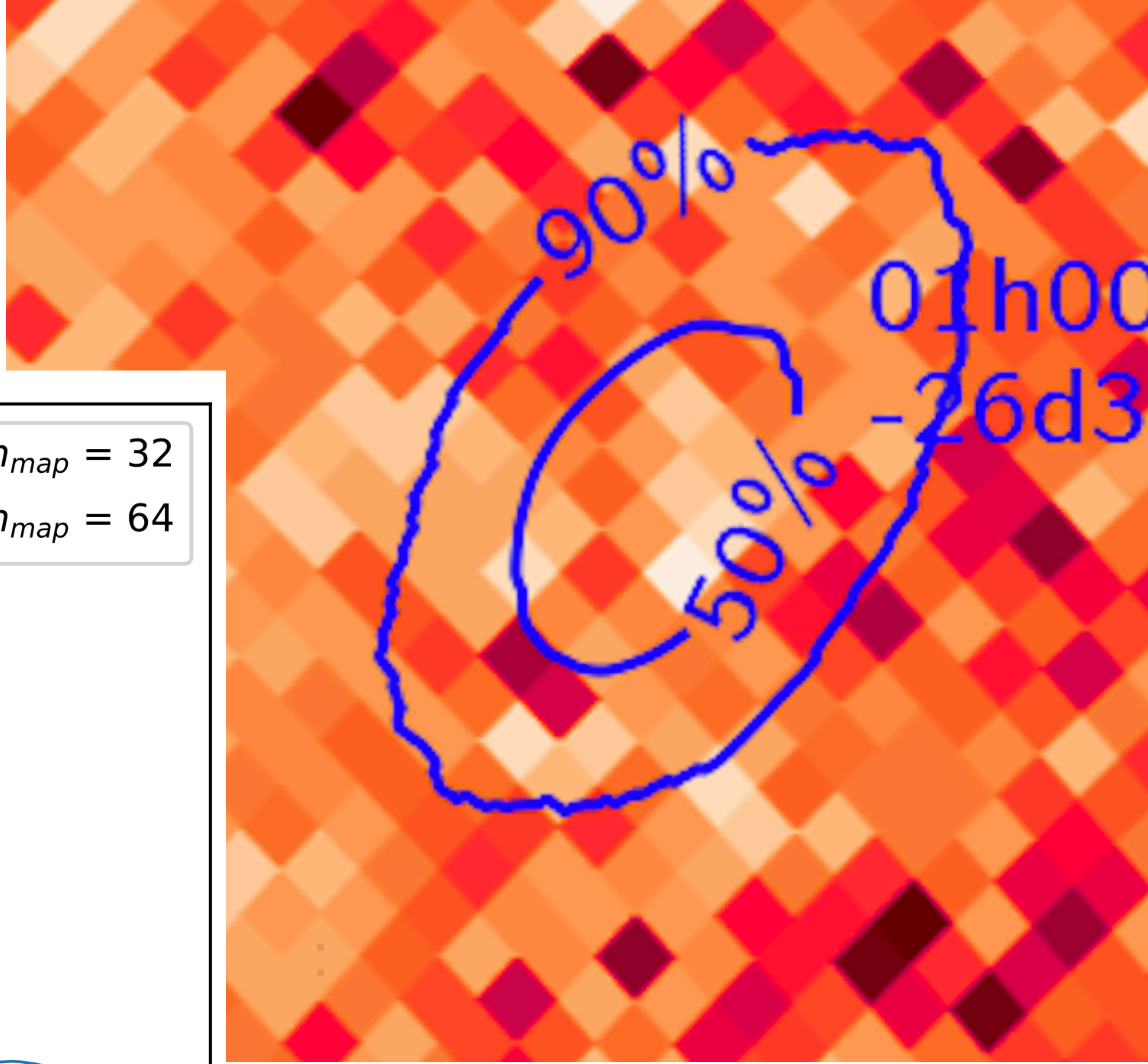
$$p_{\rm cat}(z|\{\hat{z}_g\}) = \frac{1}{N_{\rm gal}} \sum_i^{N_{\rm gal}} \delta(z - \hat{z}_g^i),$$

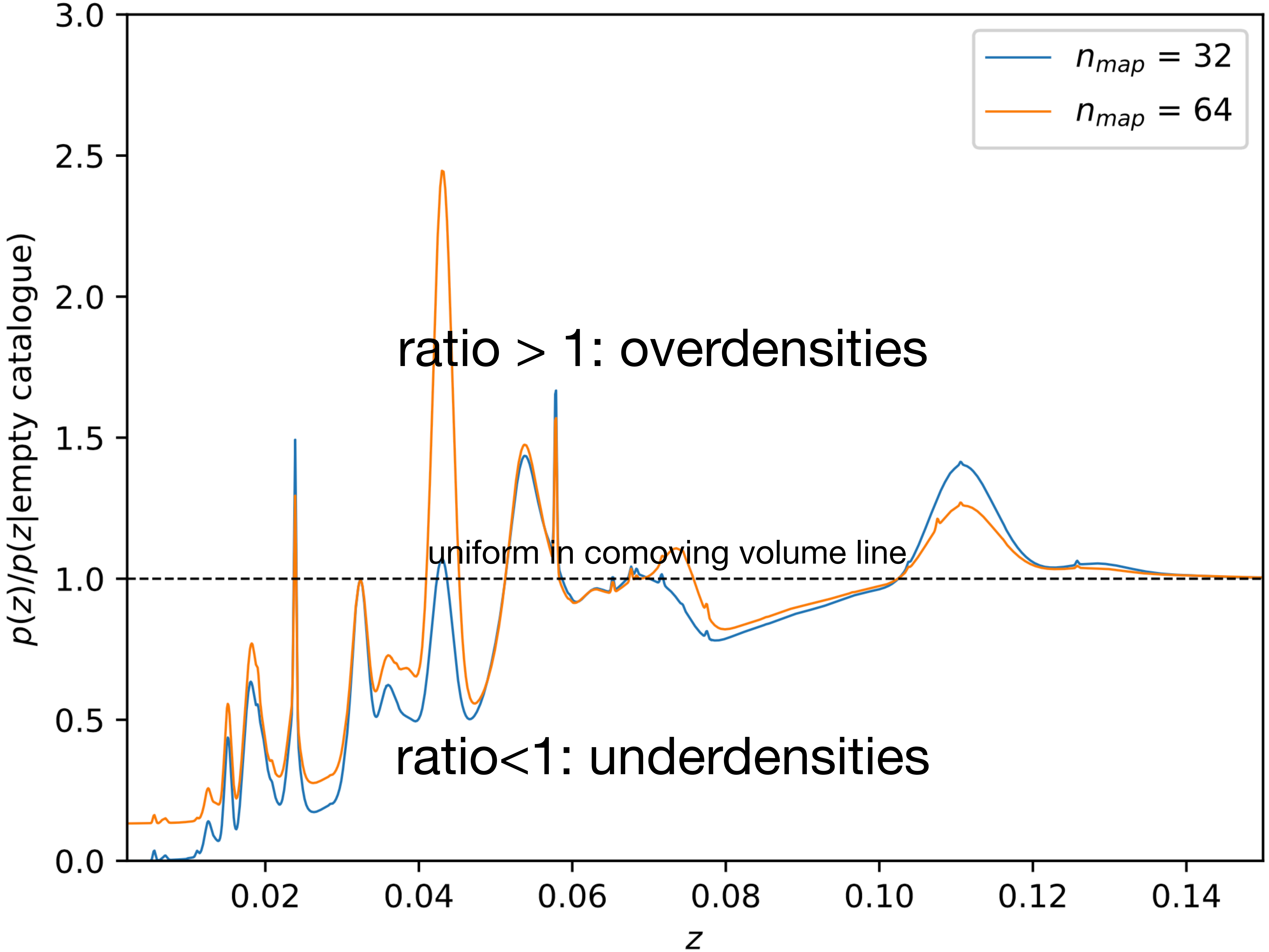
Cosmology with GWs





Cosmology with GWs





spectral analysis corresponds to an "empty" catalogue, i.e. with unobserved galaxies following a uniform-in-comoving volume distribution

Cosmology with GWs

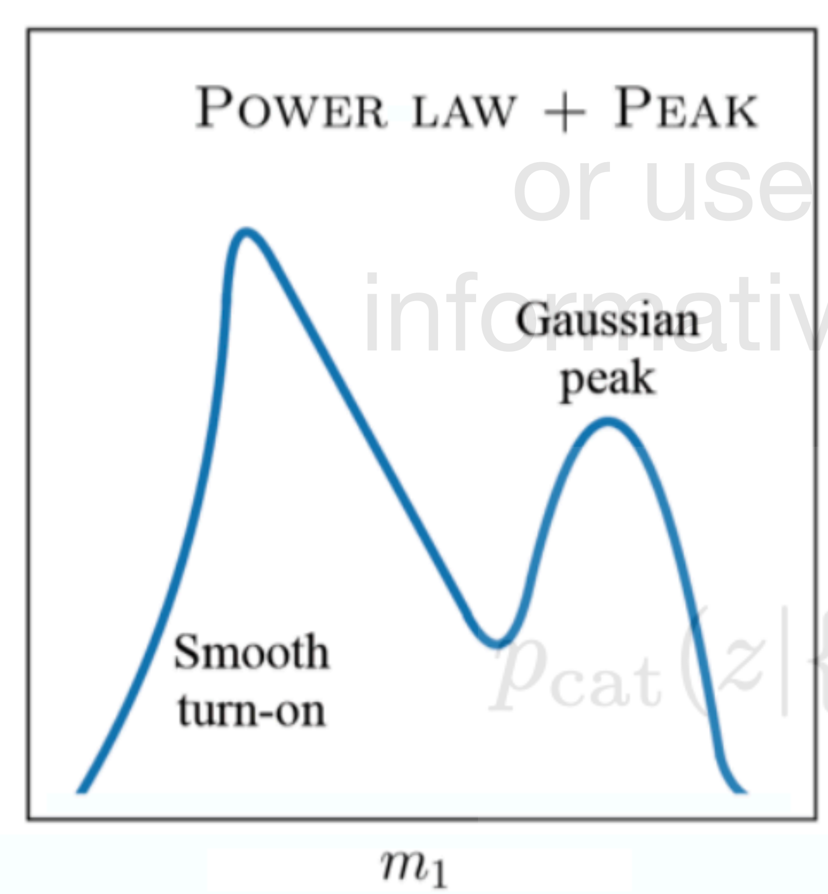
$$p(m_{1s})p(m_{2s}|m_{1s})p_{\rm CBC}(z)p(z)p(\chi)...$$

all hyper-parameters
(common to all GW events)
are in a random vector:

$$\Lambda = (H_0, \alpha, \beta, \mu_g, \sigma_g, \lambda_g, M_{\rm min}, M_{\rm max}, z_p, \gamma, \kappa ...)$$

cosmology — mass distribution — rates Madau-Dickinson

Model for the binary black holes distribution follows the star formation rate



or use a galaxy catalogue as an informative prior for the host galaxies:

$$p_{\rm cat}(z|\{\hat{z}_g\}) = \frac{1}{N_{\rm gal}} \sum_i^{N_{\rm gal}} \delta(z - \hat{z}_g^i).$$

Cosmological Analysis Pipelines in LVK

$$\Lambda = (H_0, \alpha, \beta, \mu_g, \sigma_g, \lambda_g, M_{\rm min}, M_{\rm max}, z_p, \gamma, \kappa...)$$

Two (public) independent codes are available for cosmology:

1) **icarogw**: S.Mastrogiovanni et al (Astron.Astrophys. 682 (2024) A16, Phys. Rev. D, 104, 062009 (2021), Phys. Rev. D, 108, 042002 (2023)

2) **gwcosmo**: R. Gray et al (JCAP 12 (2023) 023, MNRAS 512 (2022) 1127, Phys. Rev. D 101 (2020) 122001)

Historically:
icarogw = spectral analysis
gwcosmo = galaxy catalogue analysis

since ~ 1 year, both codes can infer jointly all hyper-parameters
with/without a galaxy catalogue

3. Bayesian analysis

Mandel et al. 1809.02063,
Thrane and Talbot 1809.02293
Vitale et al. 2007.05579

Statistical framework

single event GW likelihood

astrophysical distributions

hyper-parameters

$$\mathcal{L}(x|\Lambda, N) = e^{-N_{\rm exp}(\Lambda)} {N_{\rm exp}}^{N_{\rm obs}} \prod_{i=1}^{N_{\rm obs}} \frac{\int \mathcal{L}(x_i|\theta,\Lambda)\pi(\theta|\Lambda)\,{\rm d}\theta}{\frac{N_{\rm exp}}{N}(\Lambda)}$$

gives the events rate

selection bias correction
e.g. SNR>11

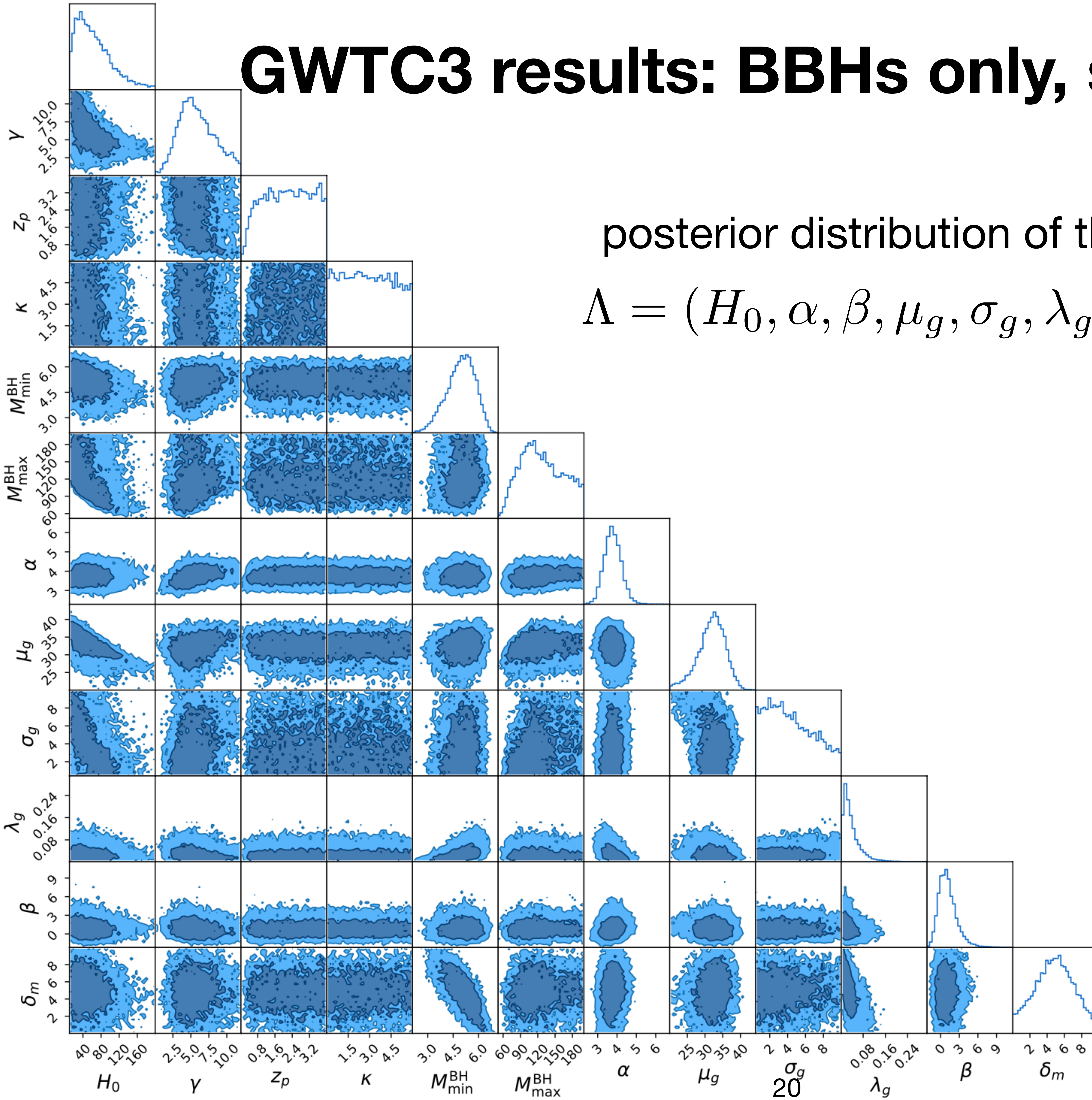
θ: individual event parameters $\theta = (m_{1s}, m_{2s}, d_L, \iota, {\rm ra}, {\rm dec}, \chi ...)$

$\pi(\theta|\Lambda)$: proba for a CBC to have individual parameters θ given Λ

single event likelihood: $\mathcal{L}(x_i|\theta,\Lambda) = \frac{\mathcal{L}(\theta|x_i,\Lambda)p(x_i|\Lambda)}{p_{\rm PE}(\theta|\Lambda)} \propto \frac{\mathcal{L}(\theta|x_i,\Lambda)}{p_{\rm PE}(\theta|\Lambda)}$

4. Results

GWTC3 results: BBHs only, spectral



posterior distribution of the 12 hyper-parameters

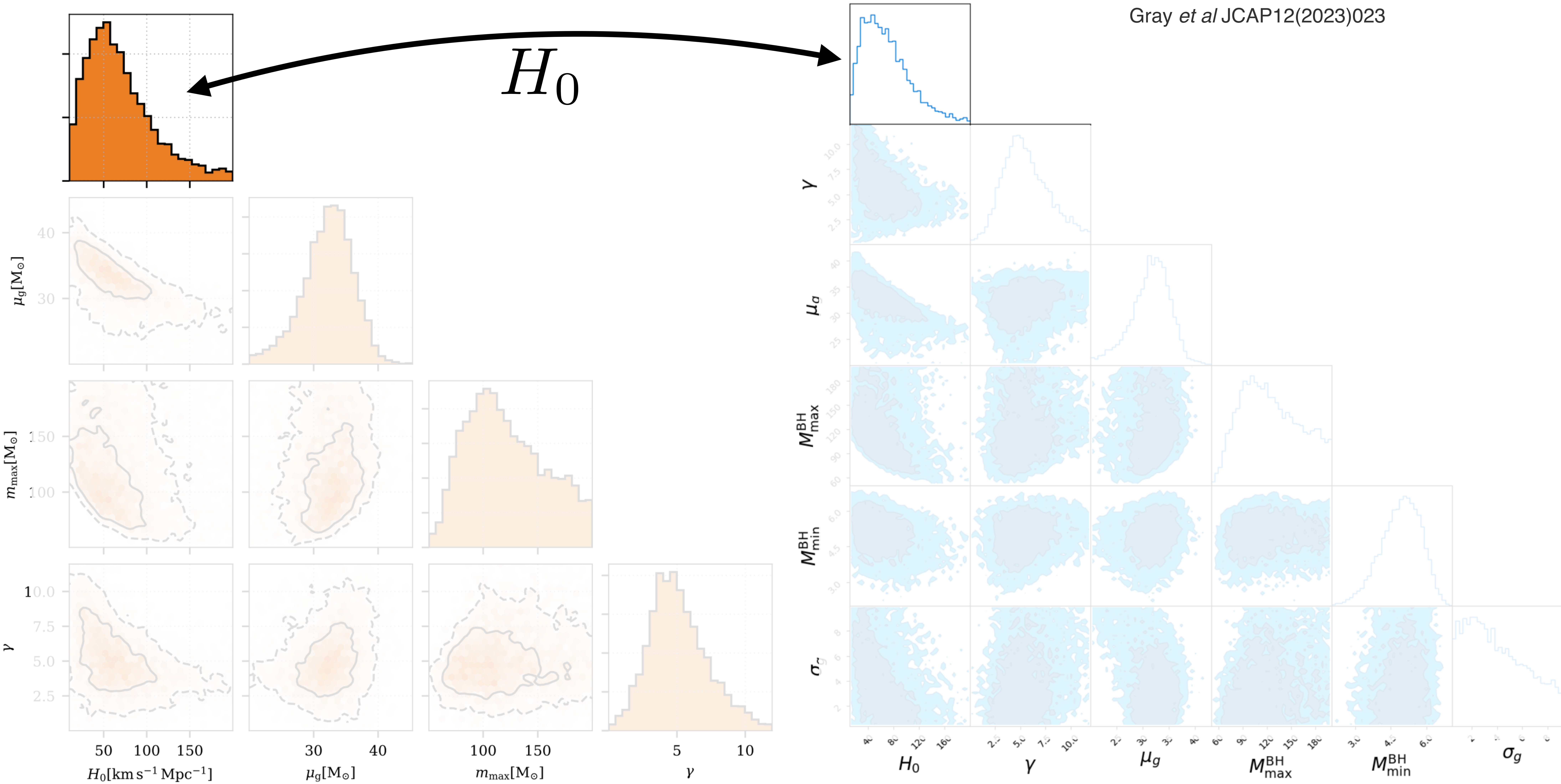
$$\Lambda = (H_0, \alpha, \beta, \mu_g, \sigma_g, \lambda_g, M_{\rm min}, M_{\rm max}, z_p, \gamma, \kappa...)$$

Gray *et al* JCAP12(2023)023

GWTC3 results: BBHs only, spectral

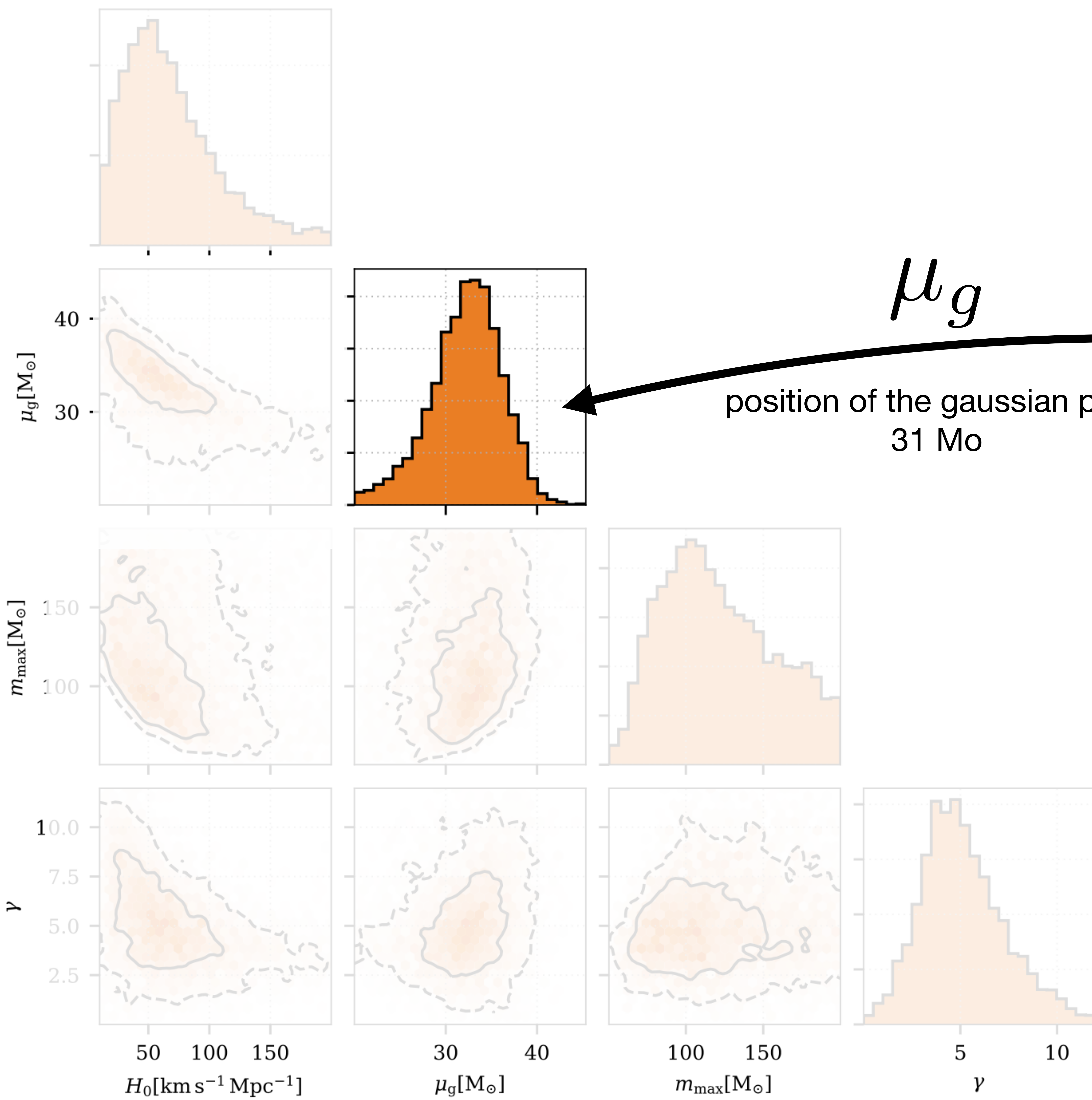
LVK, *Astrophys.J.* 949 (2023) 2, 76
but results in 2021

Gray *et al* JCAP12(2023)023



GWTC3 results: BBHs only, spectral

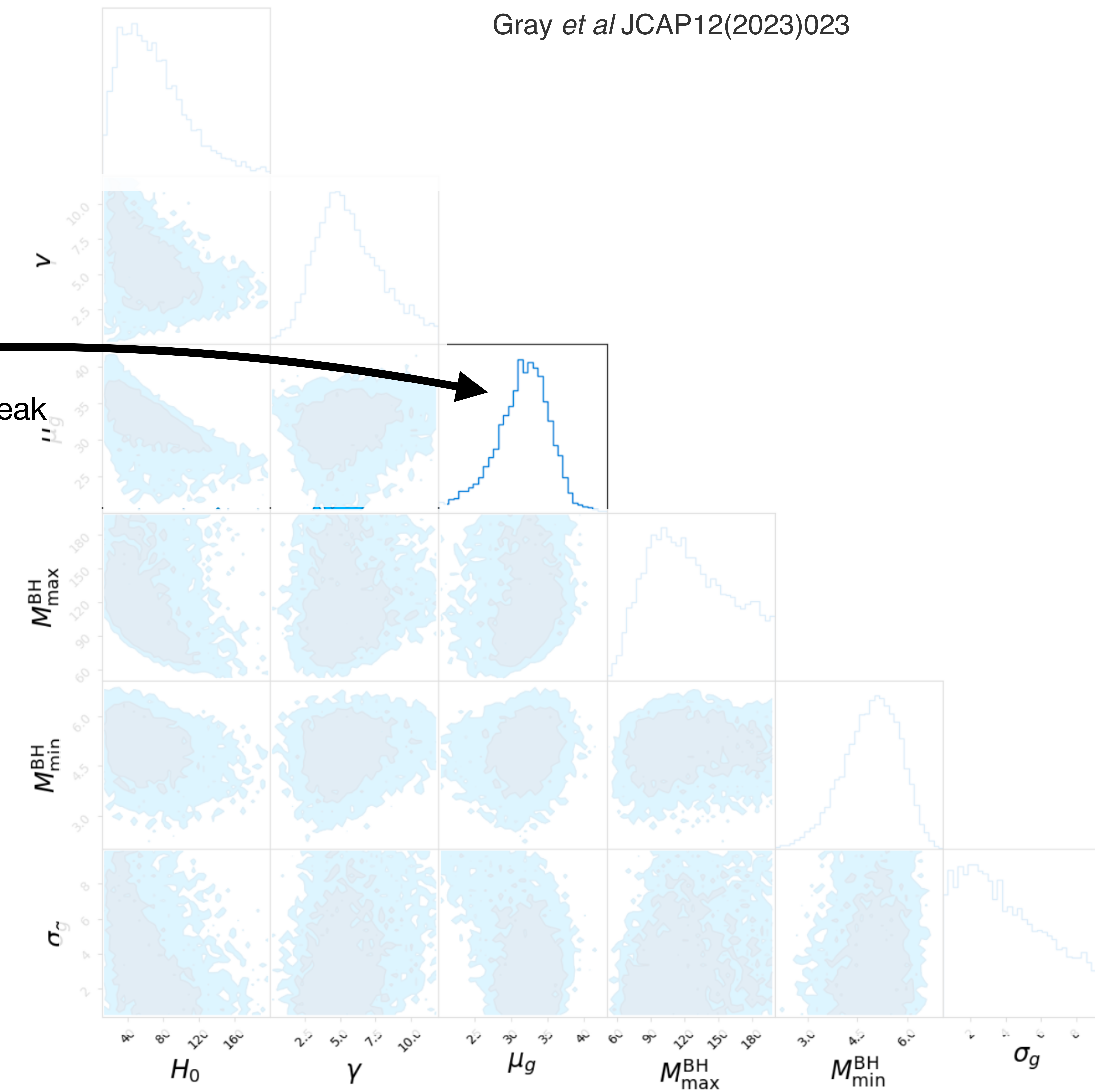
LVK, *Astrophys.J.* 949 (2023) 2, 76
but results in 2021



μ_g

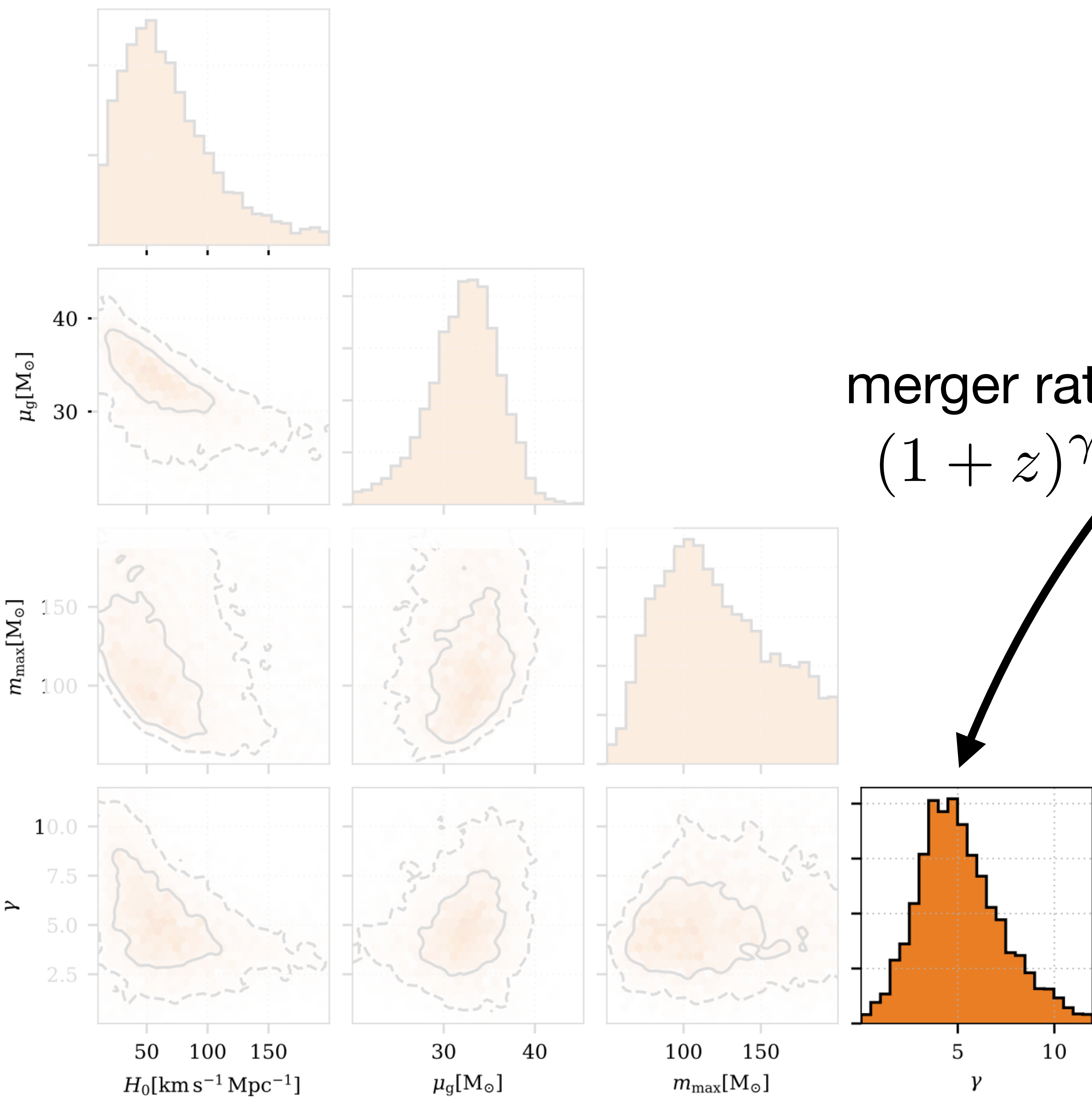
position of the gaussian peak
31 Mo

Gray *et al* JCAP12(2023)023



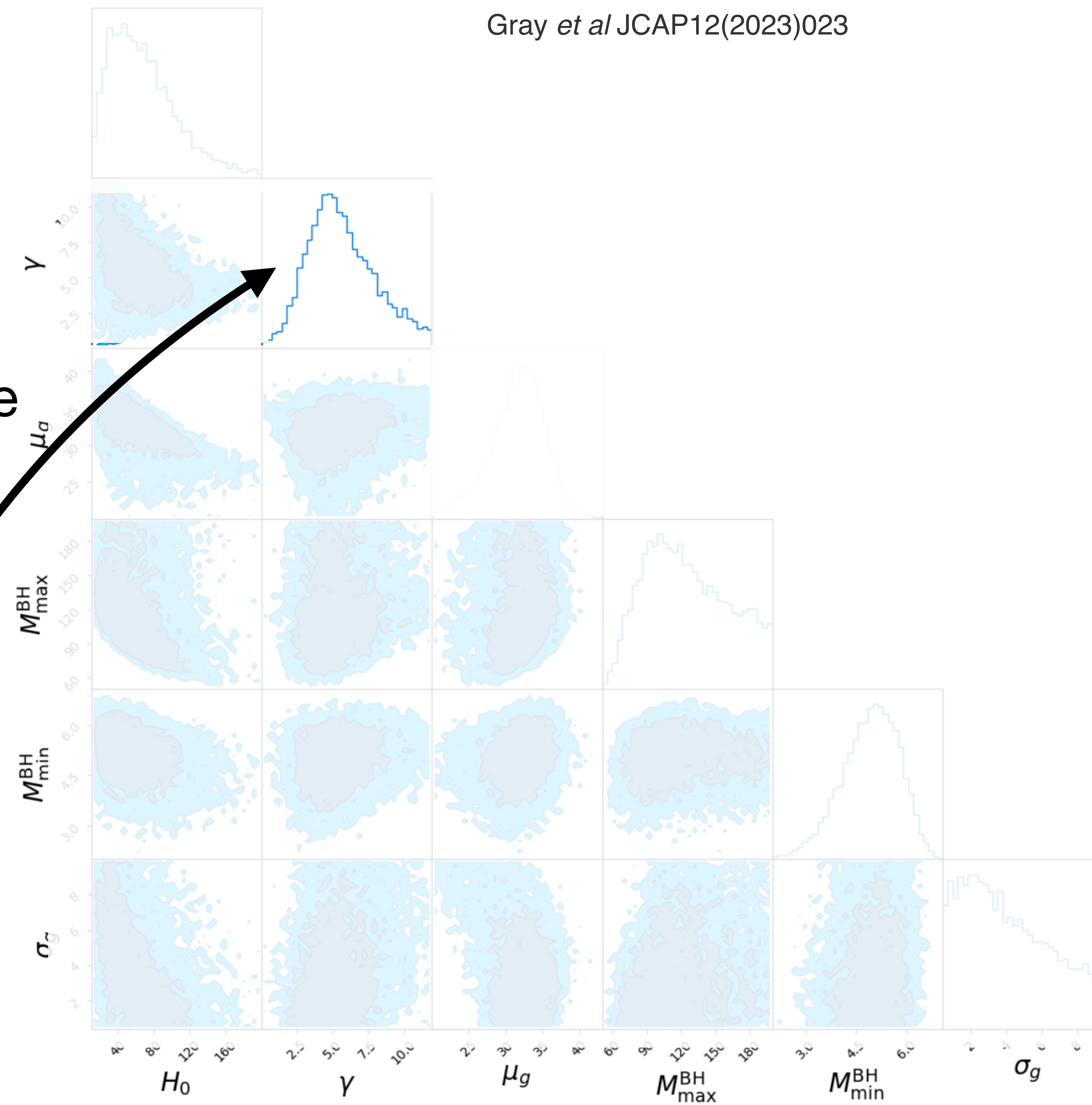
GWTC3 results: BBHs only, spectral

LVK, *Astrophys.J.* 949 (2023) 2, 76
but results in 2021



merger rate
$$(1+z)^\gamma$$

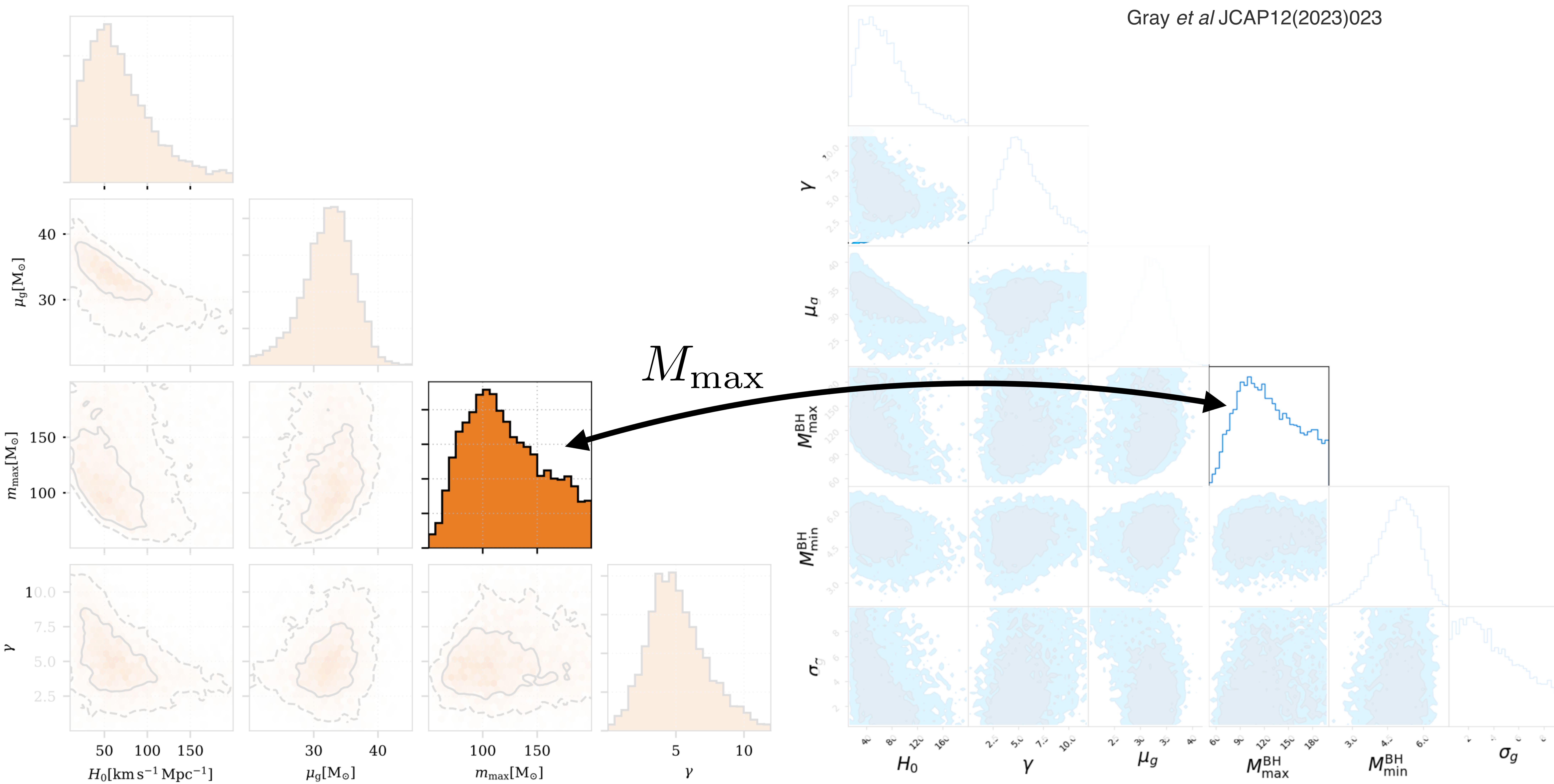
Gray *et al* JCAP12(2023)023



GWTC3 results: BBHs only, spectral

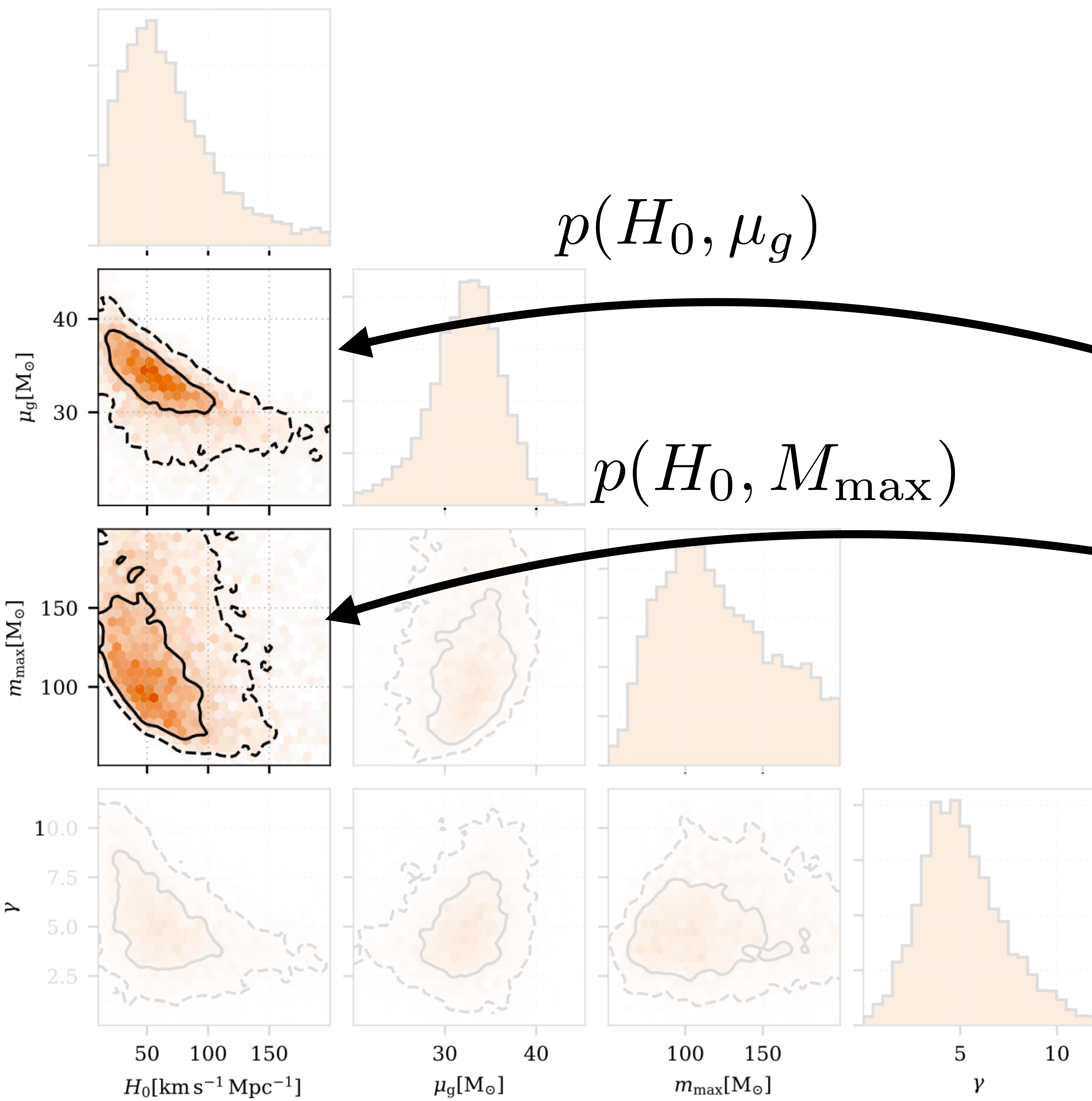
LVK, *Astrophys.J.* 949 (2023) 2, 76
but results in 2021

Gray *et al* JCAP12(2023)023



GWTC3 results: BBHs only, spectral

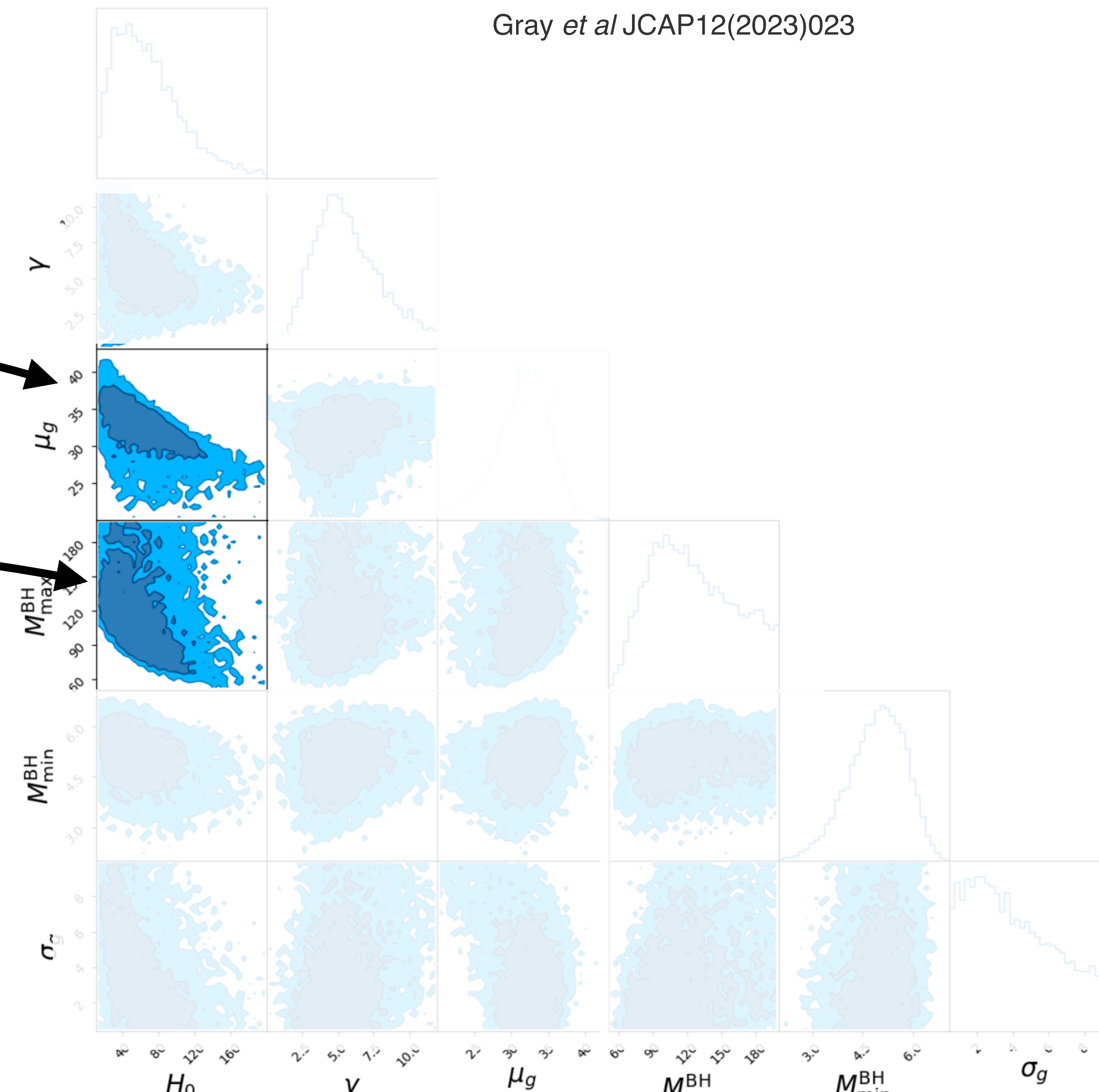
LVK, *Astrophys.J.* 949 (2023) 2, 76
but results in 2021



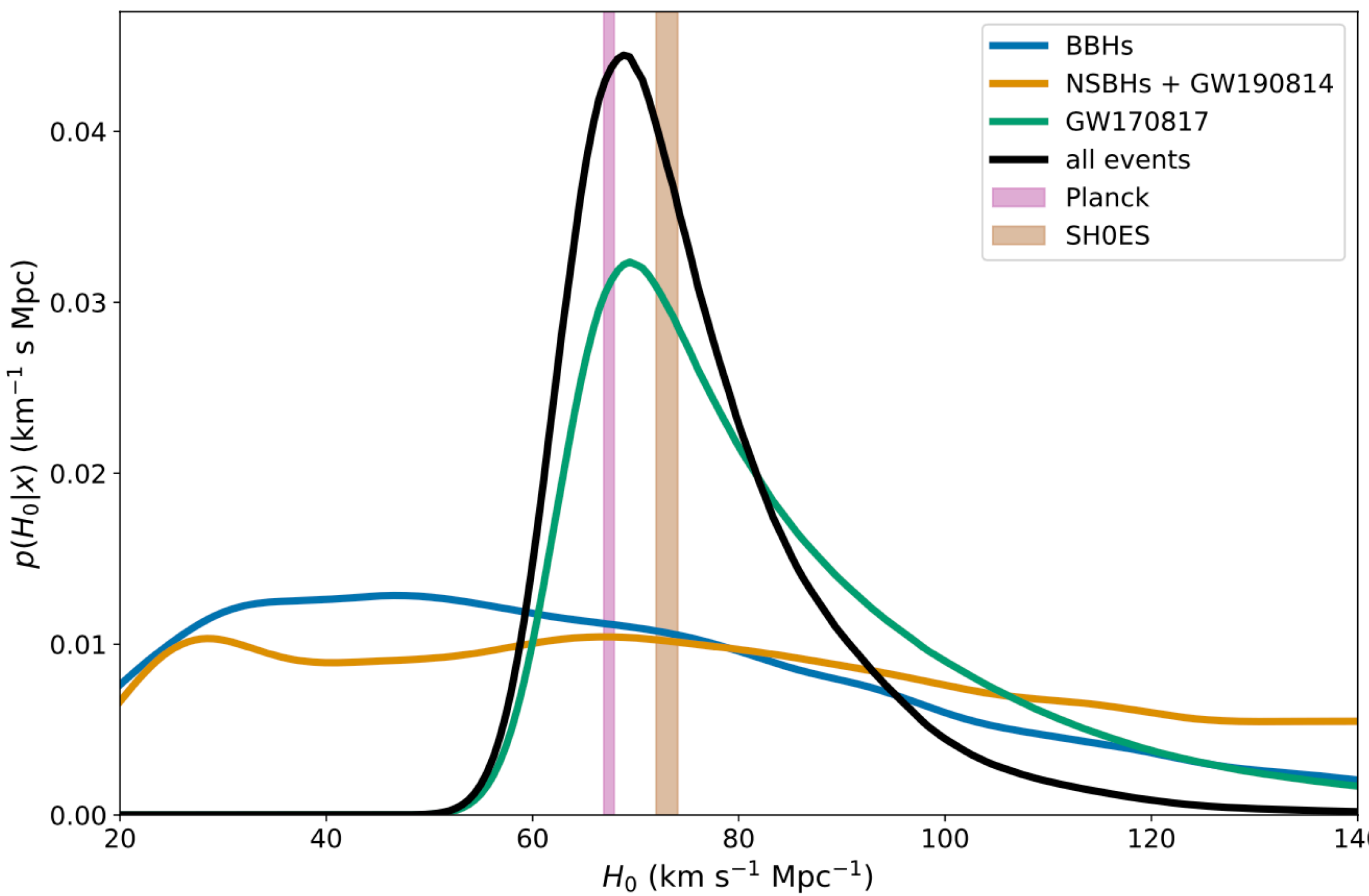
$$p(H_0, \mu_g)$$

$$p(H_0, M_{\max})$$

Gray *et al* JCAP12(2023)023



GWTC3 - BBHs, NSBHs, BNS - H_0



current result: (Gray *et al* JCAP12(2023)023)

$$H_0 = 69^{+12}_{-7}\ \mathrm{km\ s^{-1}\ Mpc^{-1}}$$

previous result: (LVK, *Astrophys.J.* 949 (2023) 2, 76)

$$H_0 = 68^{+8}_{-6}\ \mathrm{km\ s^{-1}\ Mpc^{-1}}$$

Conclusion

- cosmology results with the O4a events are currently being produced
- other mass models are available + use a single mass model for all CBCs (done in icarogw)
- probably the most important: use a **deeper galaxy catalogue**
    - currently GLADE+ (Dalya et al, MNRAS 514, 1403–1411 (2022)), 22 million of galaxies
    - upcoming: UpGLADE, 2 billion of galaxies, much more complete
- allow mass models evolving with redshift
- Mock Data Challenge: study of systematics, for instance using a Mock catalogue with mass-z dependence