Gravitational wave signal of protoneutron star convection : a probe into PNS dynamo and magnetar formation

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Université Paris Cité
DE LA RECHERCHE À L'INDUSTRIE cea
VNIVERSITAT DE VALÈNCIA

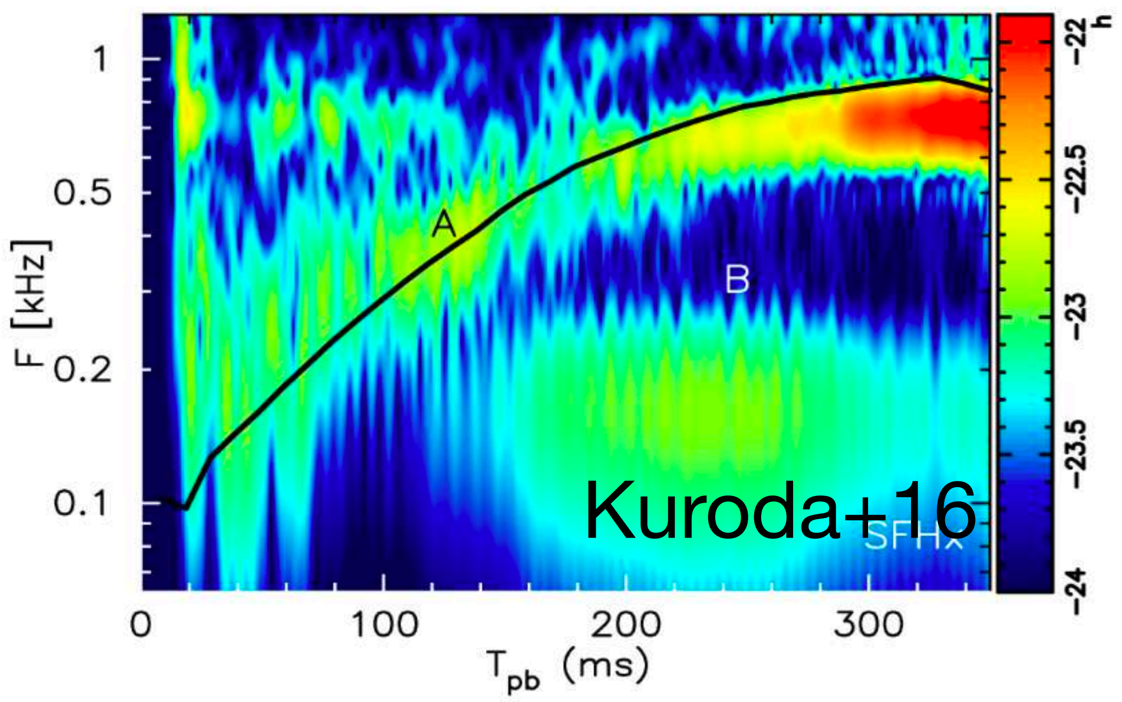
A complementary approach: CCSN models and PNS models

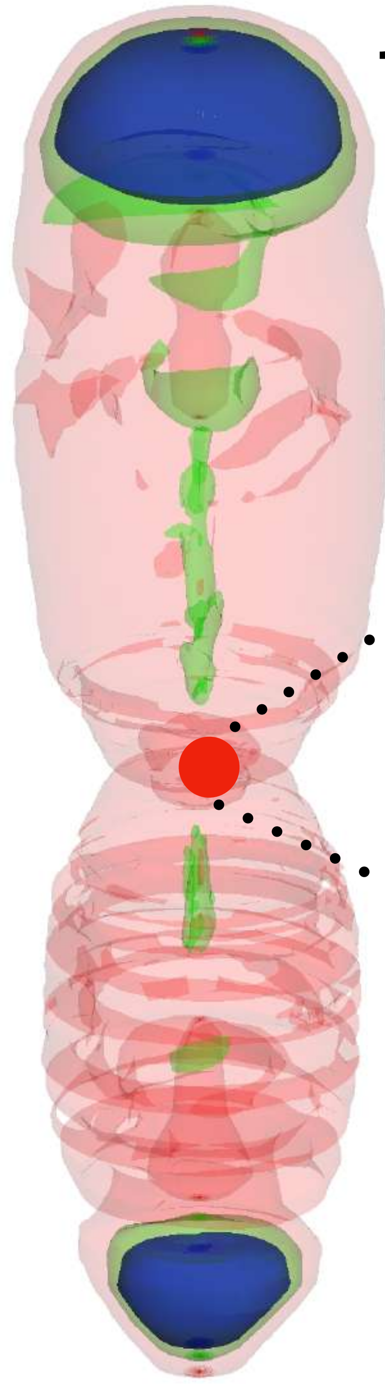
CCSN simulations

- Magnetorotational explosions & long GRBs

Next talk by Matteo Bugli

- Nucleosynthesis
- Multi-messenger observables

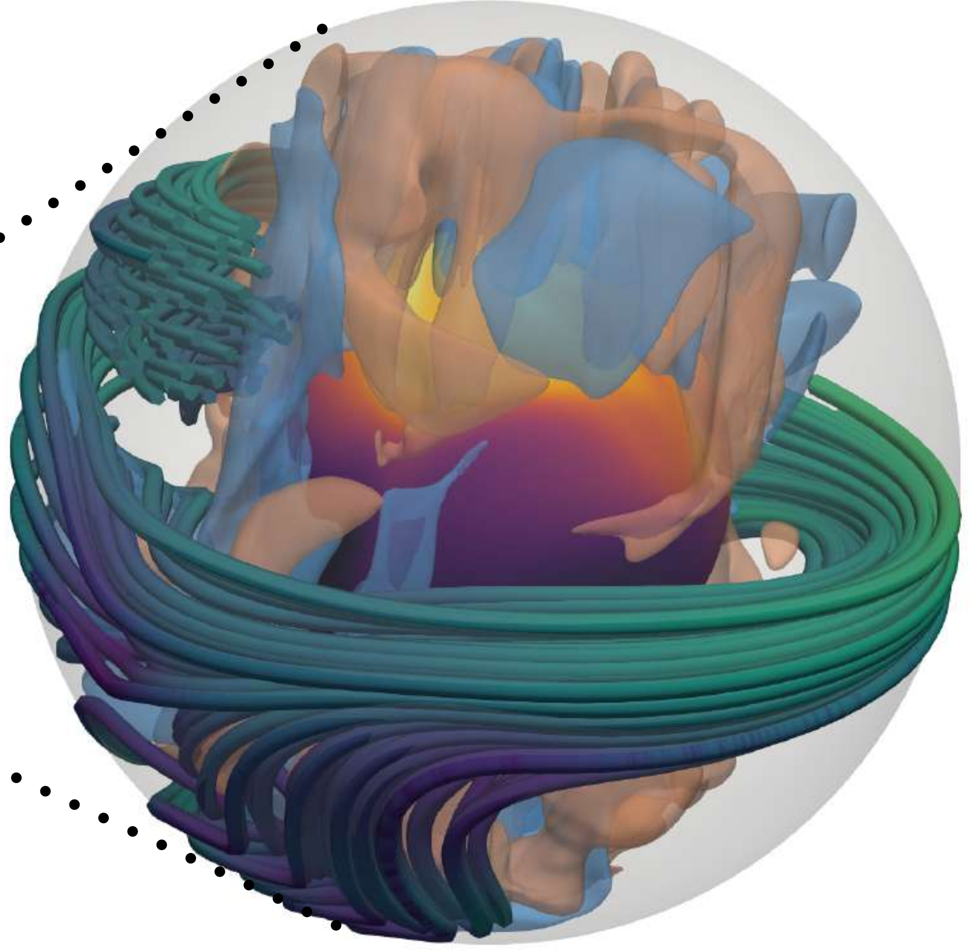




10^4 km

Bugli+21

10 km



3D-MHD PNS models

Study magnetar formation

- Fine characterisation of dynamo processes and large scale field generation
- Extensive parameter studies
- Derivation of physics informed scaling laws

0 **1 s** **10 s**

Bounce

g-modes, SASI

GW PNS convection signal ?

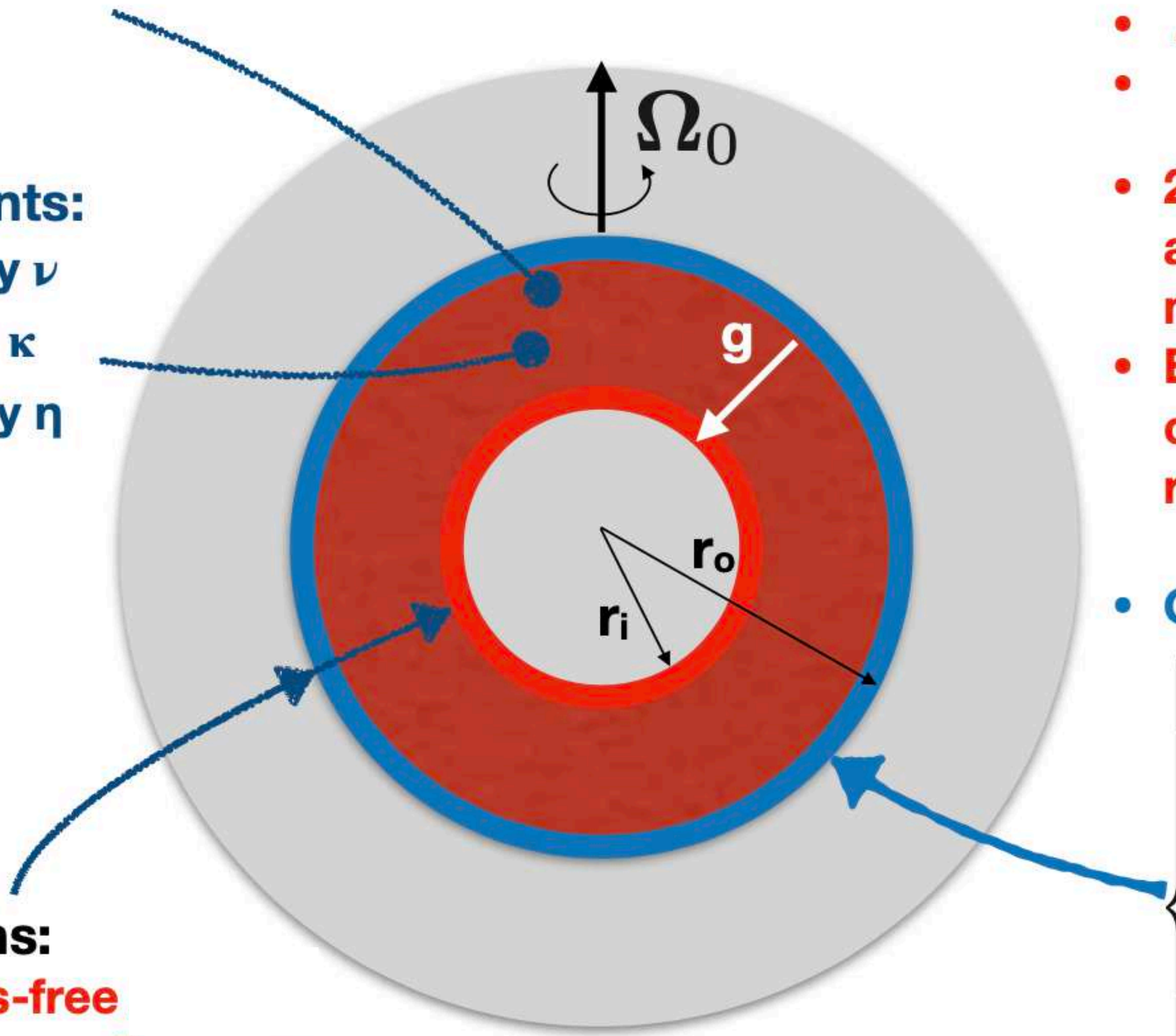
3D modelling with the MagIC code

Taken from 1D CCSN

Input:
- Temperature profile
- Density profile

Transport coefficients:
- Kinematic viscosity ν
- Thermal diffusivity κ
- Magnetic diffusivity η

github.com/magic-sph/magic



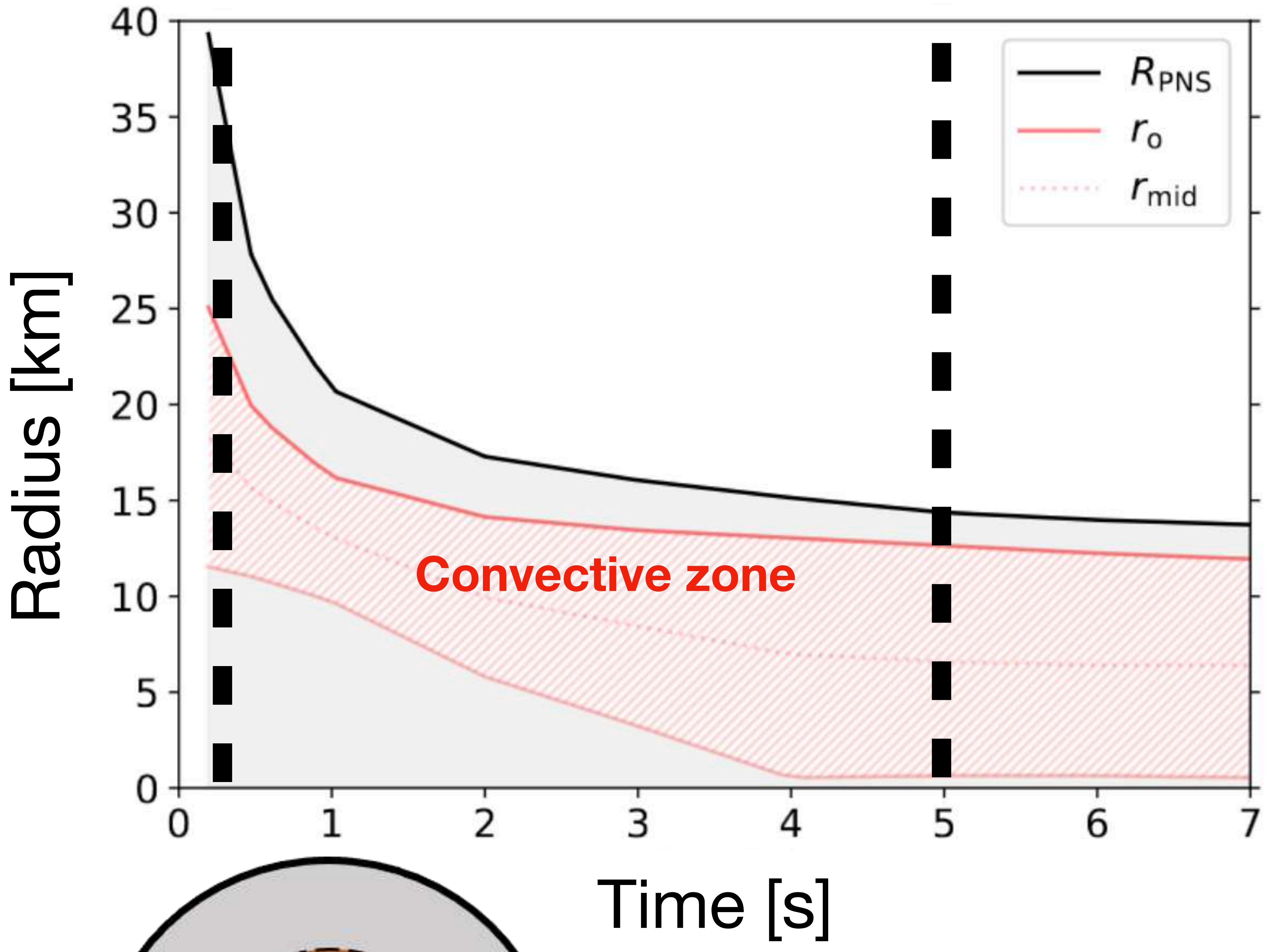
Boundary conditions:
- **Mechanical:** stress-free
- **Thermal:** fixed entropy flux
- **Magnetic:** perfect conductor ($B_{//}$)

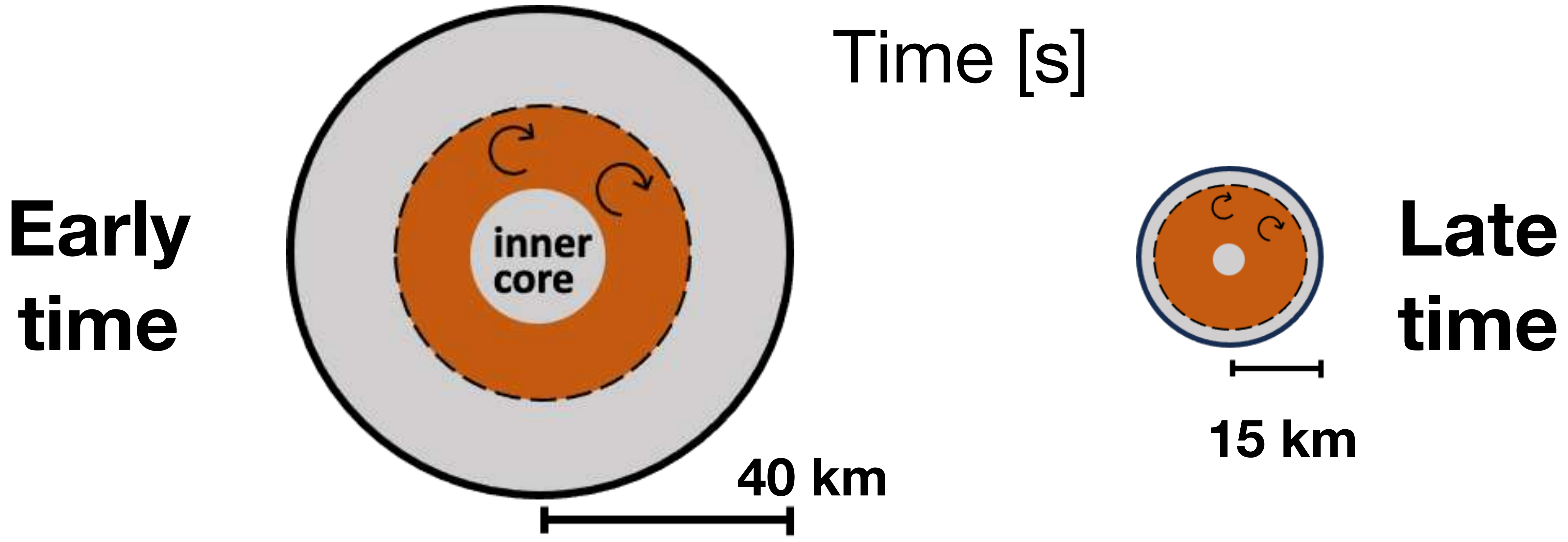
Hypothesis:
- Spherical geometry
- Adiabatic stratification
- Low Mach convection
- 2nd order diffusion approximation for the neutrino transport
- Electrical conductivity of degenerate, relativistic electrons

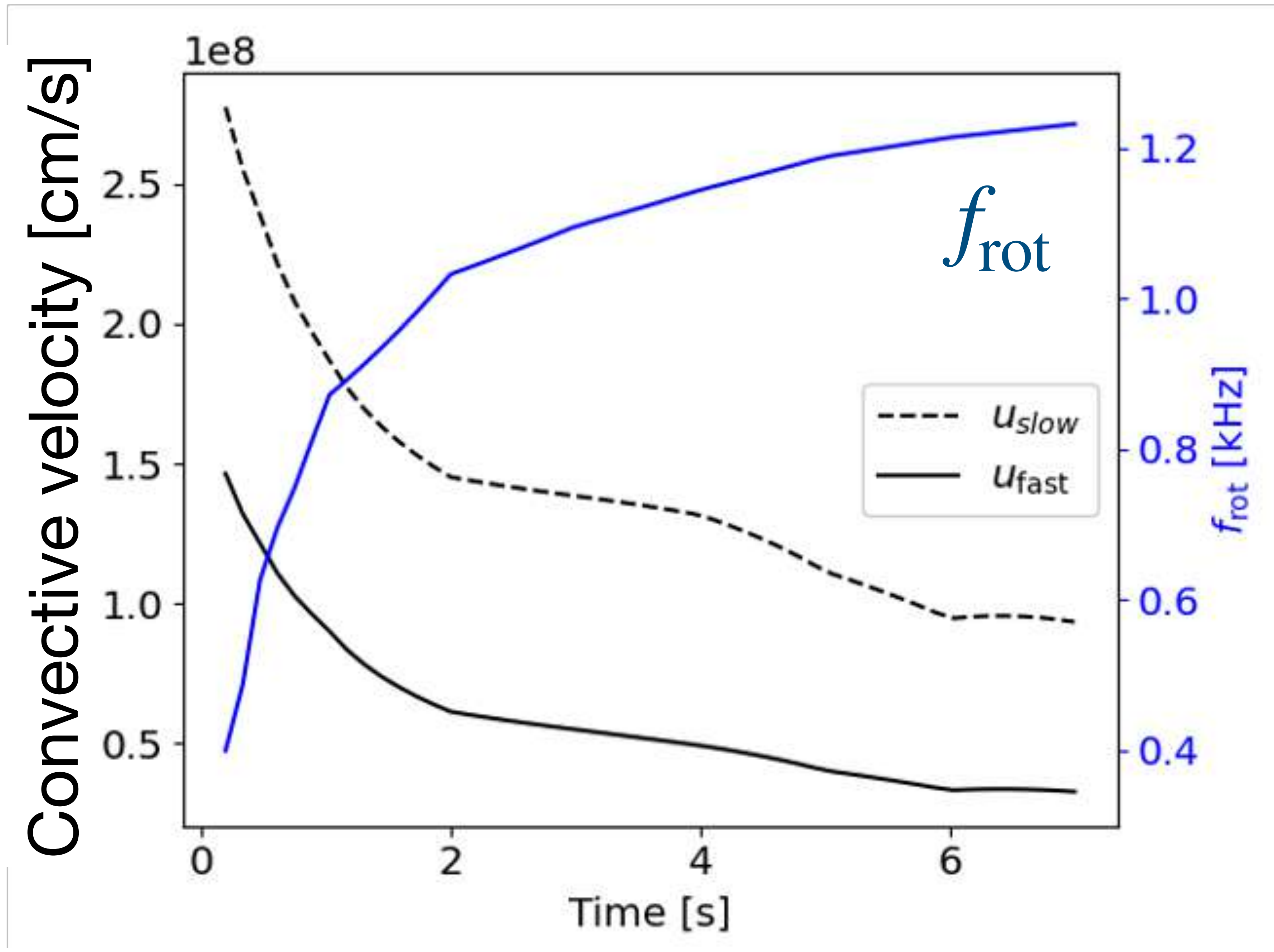
- Orders of magnitude

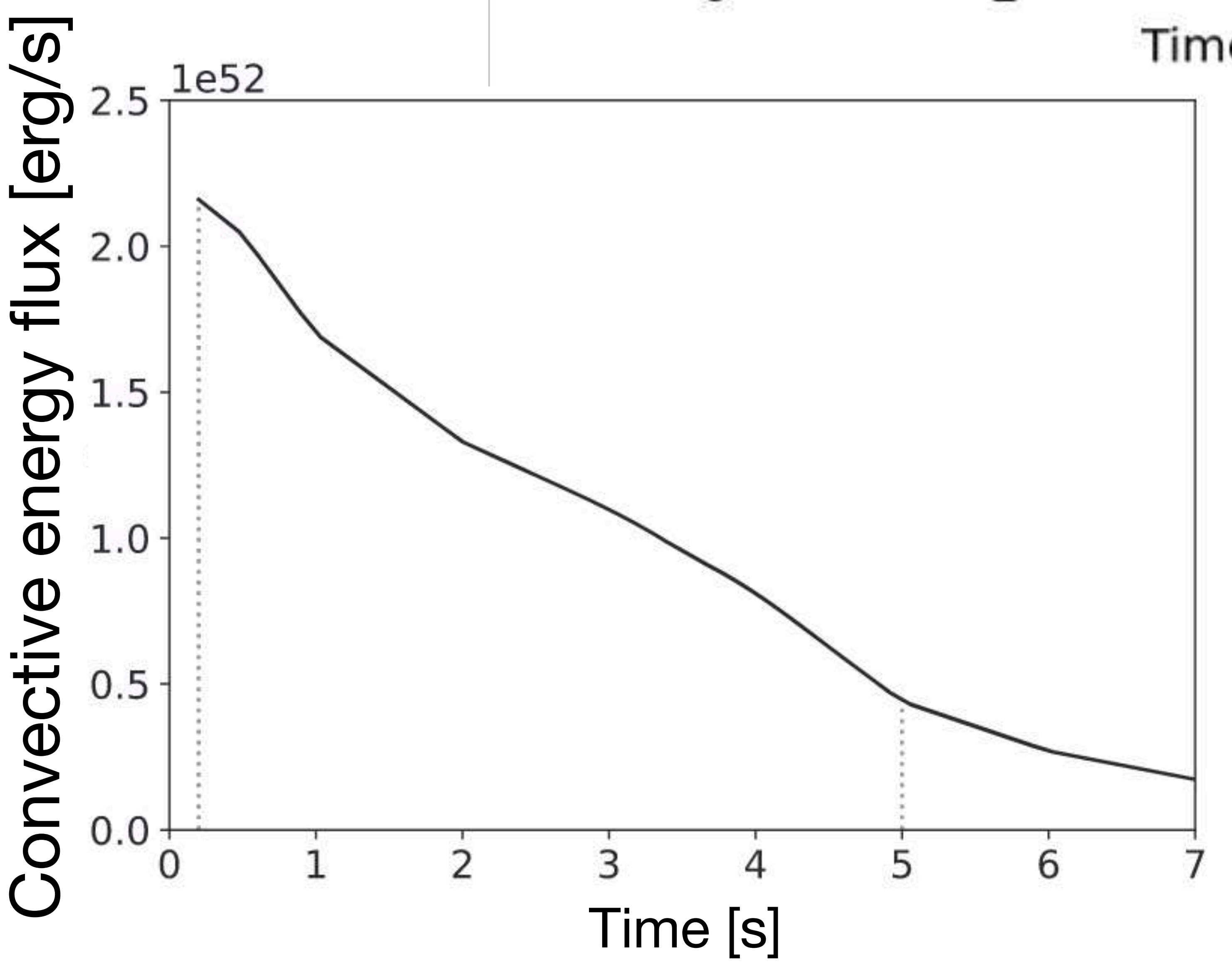
$$
\begin{cases}
\Phi_o \sim 10^{52}\,\mathrm{erg/s} \\
r_o \sim 25\,\mathrm{km} \\
T_o \sim 10^{11}\,\mathrm{K} \\
\varrho_o \sim 10^{13}\,\mathrm{g/cm^3} \\
\nu_o \sim 10^{10}\,\mathrm{cm^2/s} \\
\kappa_o \sim 10^{12}\,\mathrm{cm^2/s} \\
\eta_o \sim 10^{-3}\,\mathrm{cm^2/s}
\end{cases}
$$

Protoneutron star structure

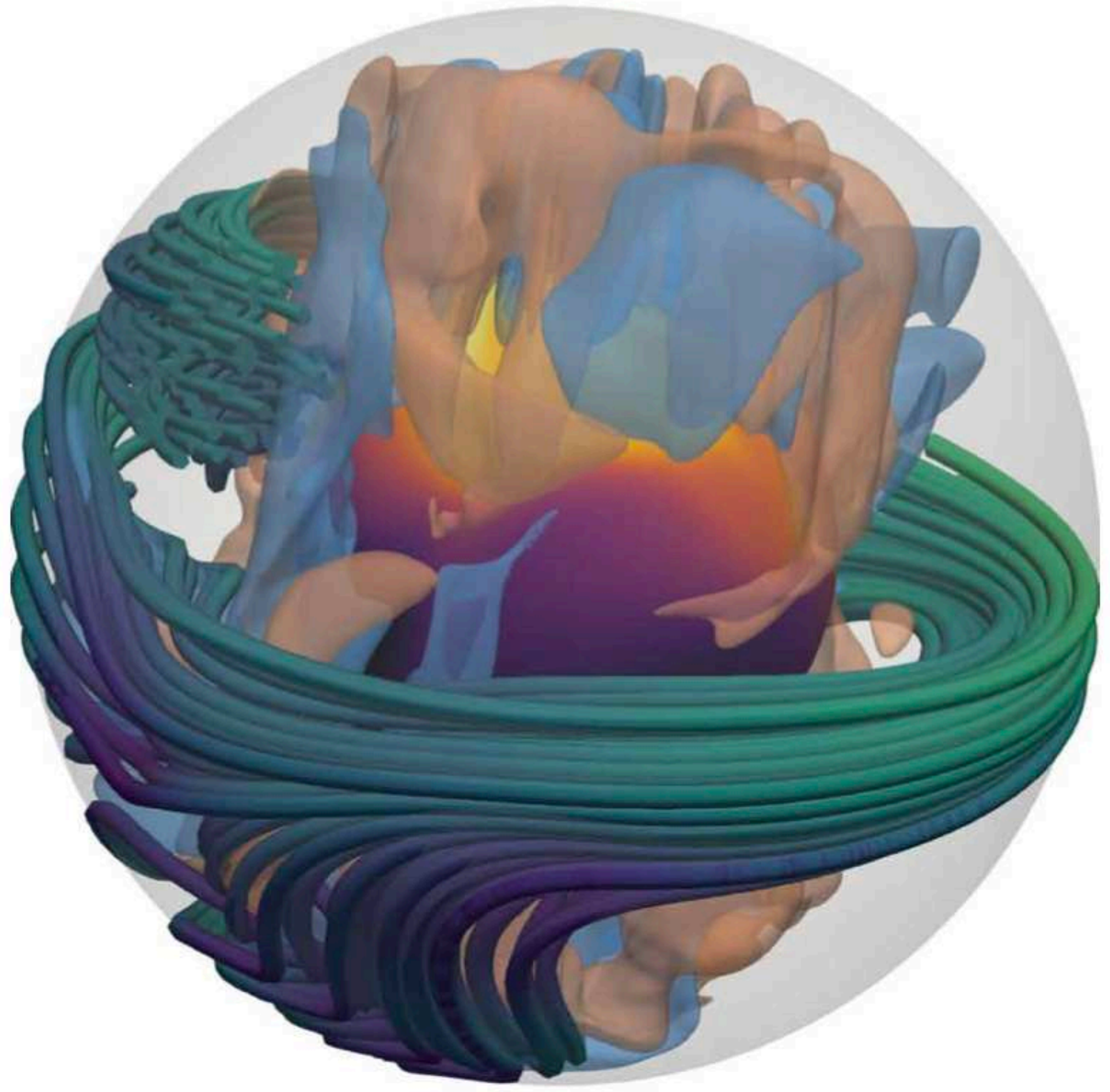






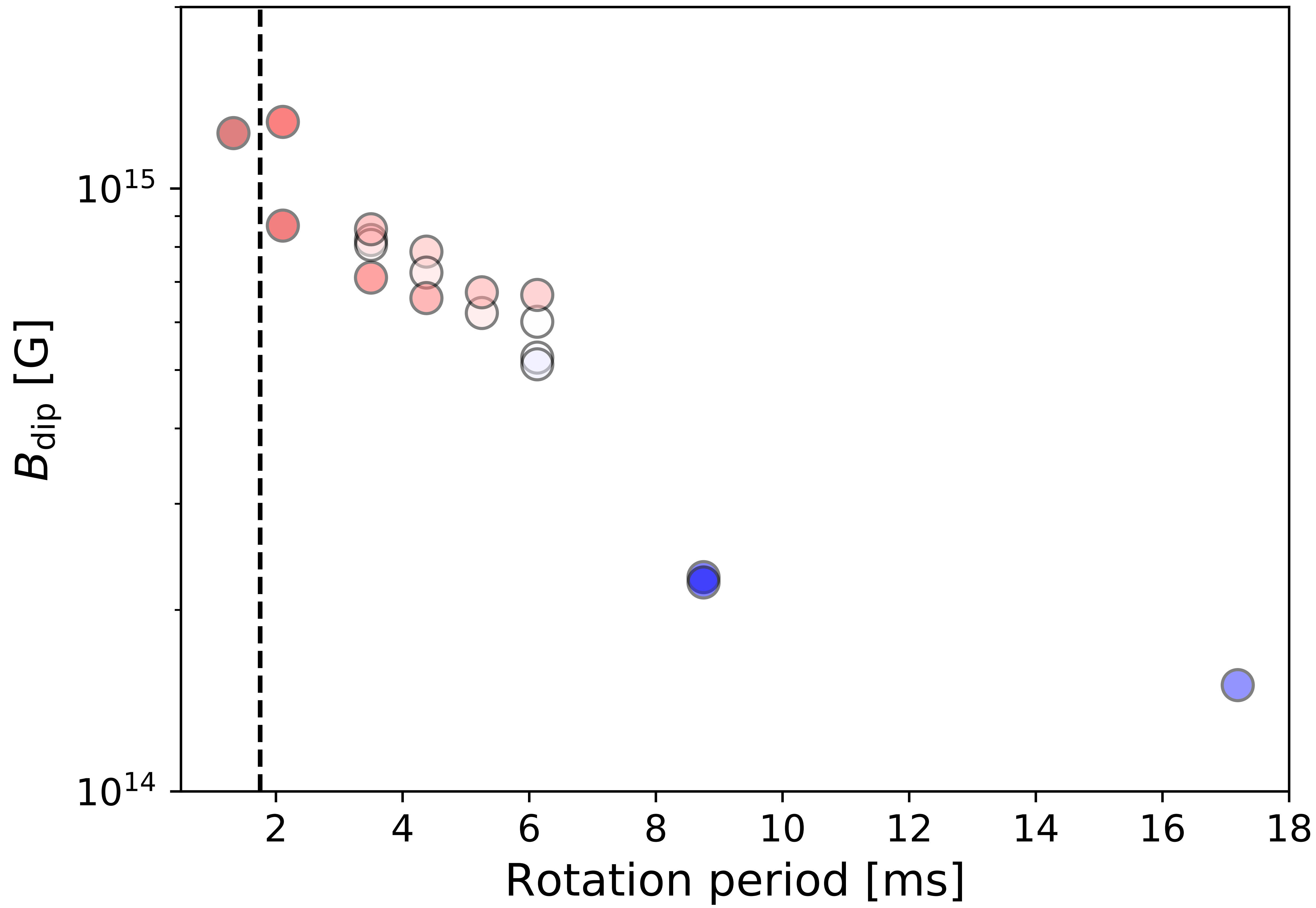


PNS convective dynamos



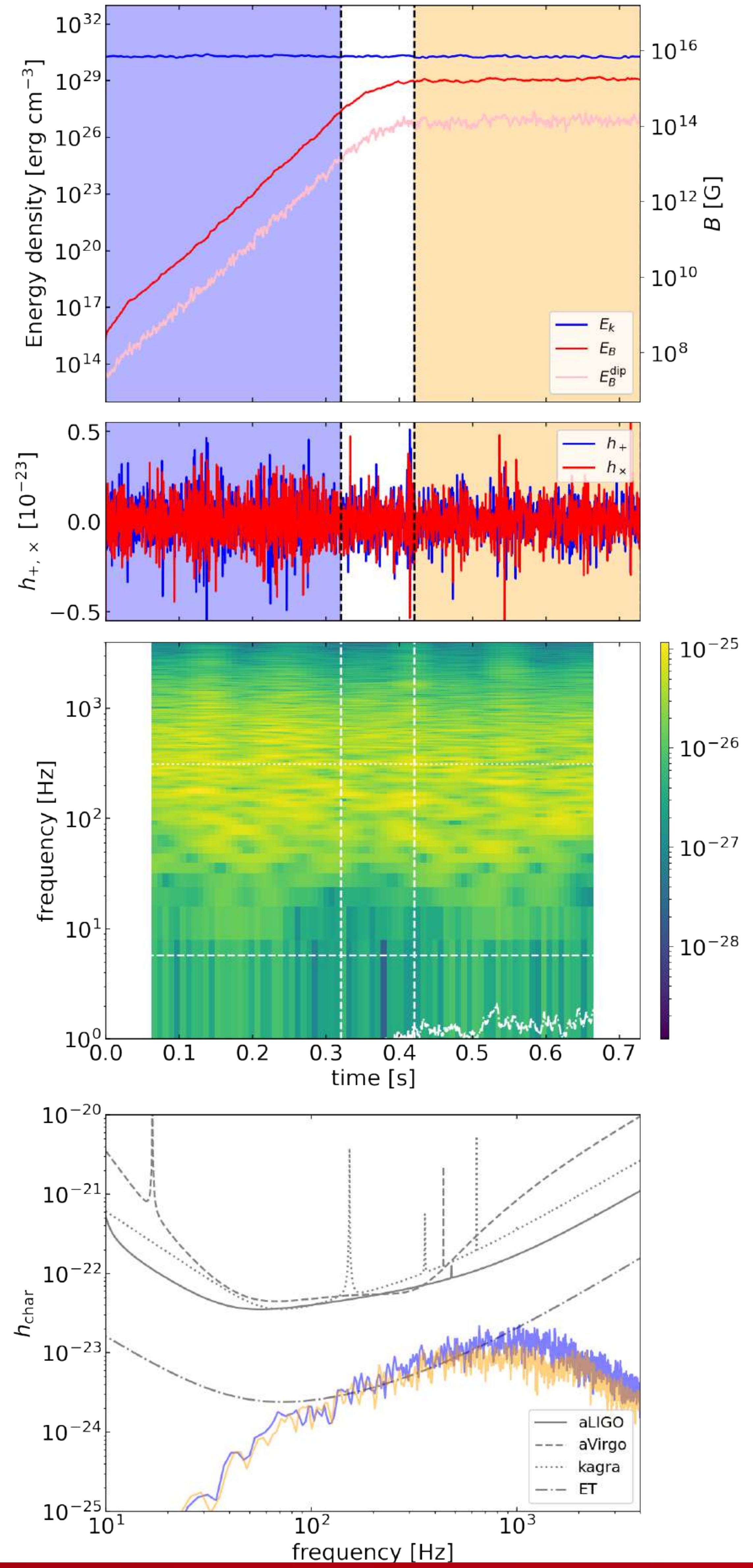
Strong field dynamo

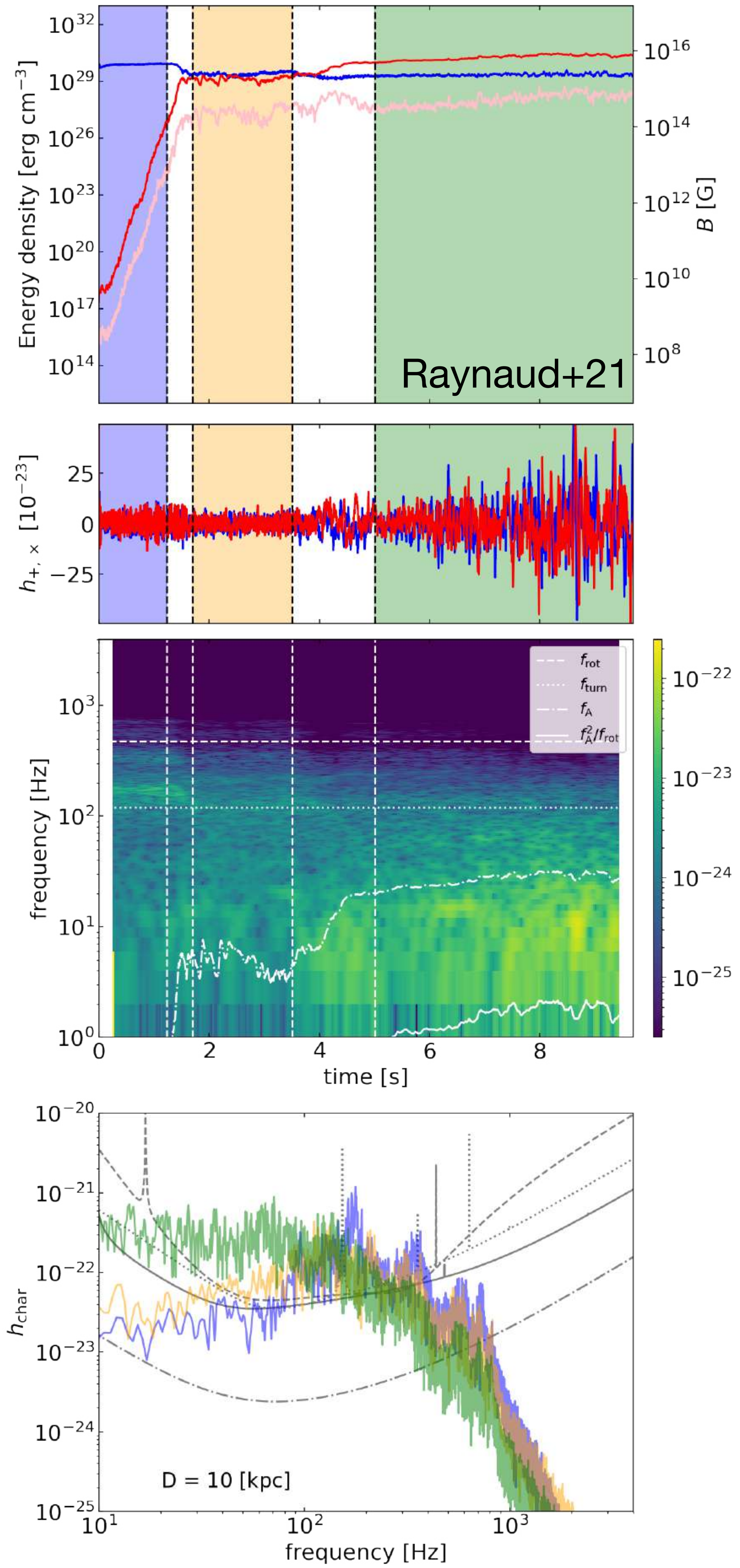
Dipole field strength



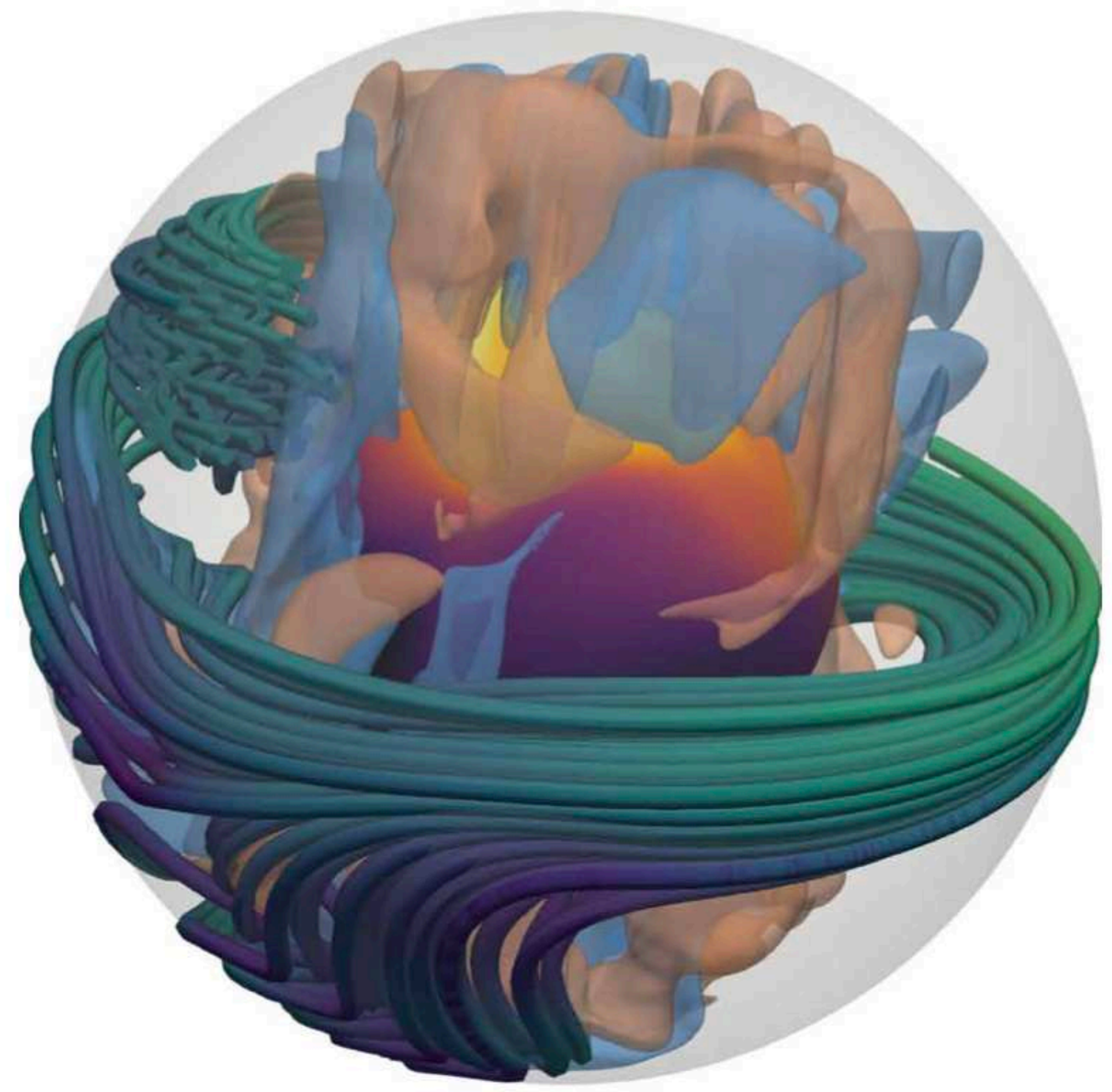
GW counterpart of PNS convective dynamos

$P = 175$ ms





$P = 2.1$ ms



~ "$\alpha\Omega$" dynamo

$$\frac{E_B}{E_{kin}} \lesssim 1$$

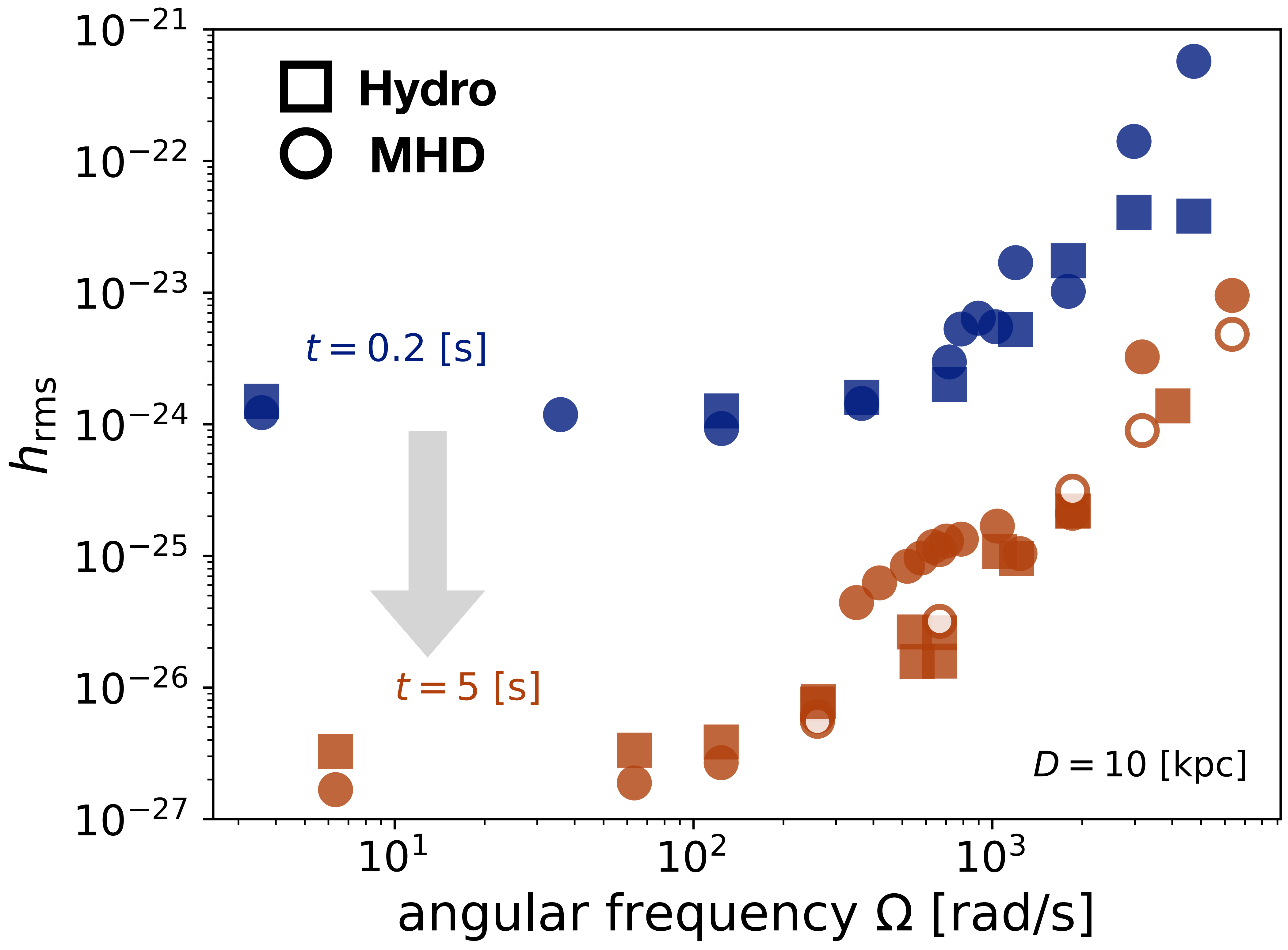
Strong field dynamo

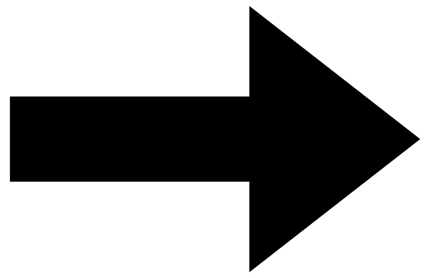
$$\frac{E_B}{E_{kin}} \propto \left(\frac{U}{\Omega d}\right)^{-1} \equiv Ro^{-1} \gg 1$$

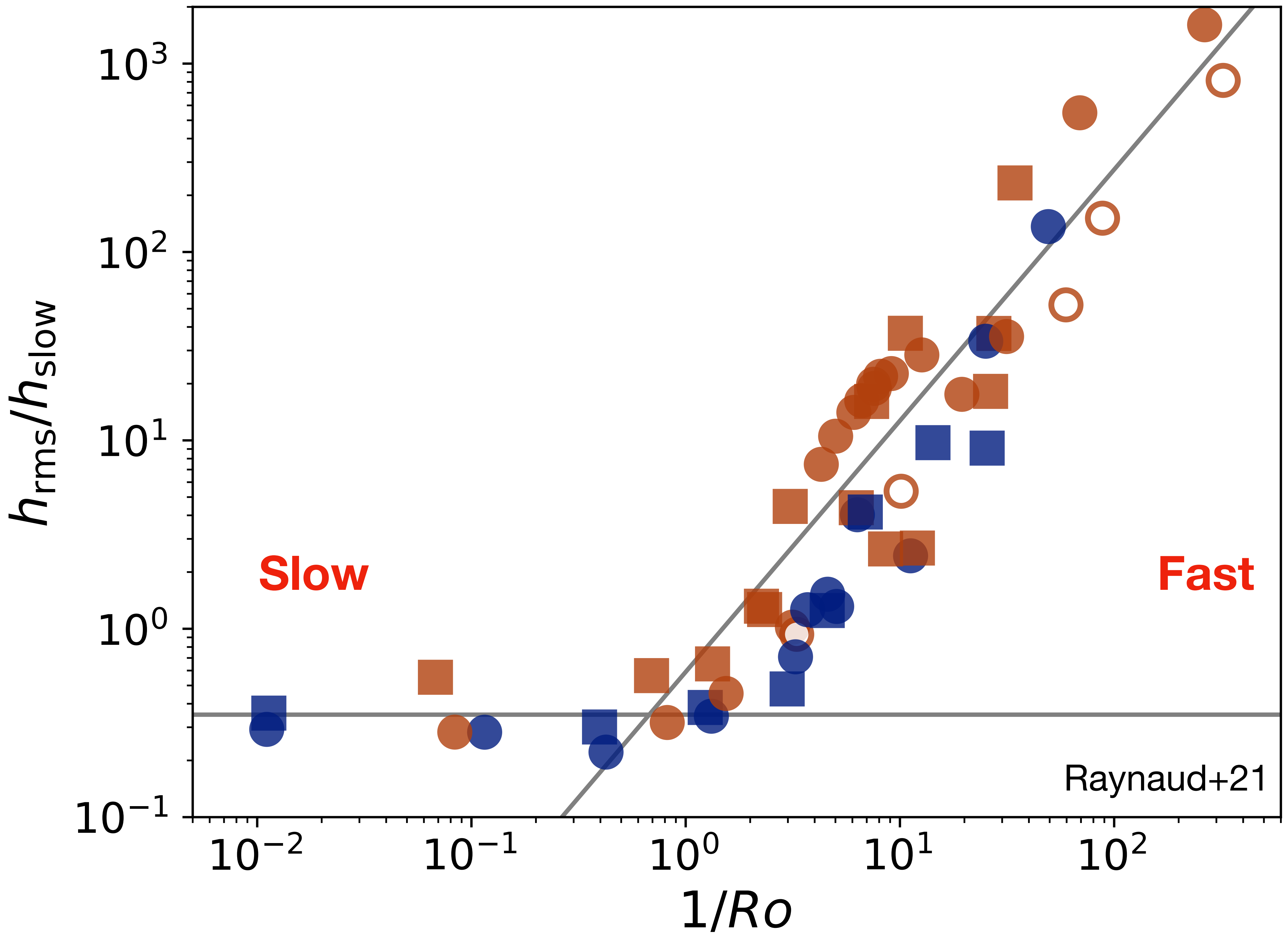
$$B_{dip} \sim 10^{15}\ \mathrm{G}$$
$$B_{tor} \sim 10^{16}\ \mathrm{G}$$

Raynaud+20

Amplitude scaling





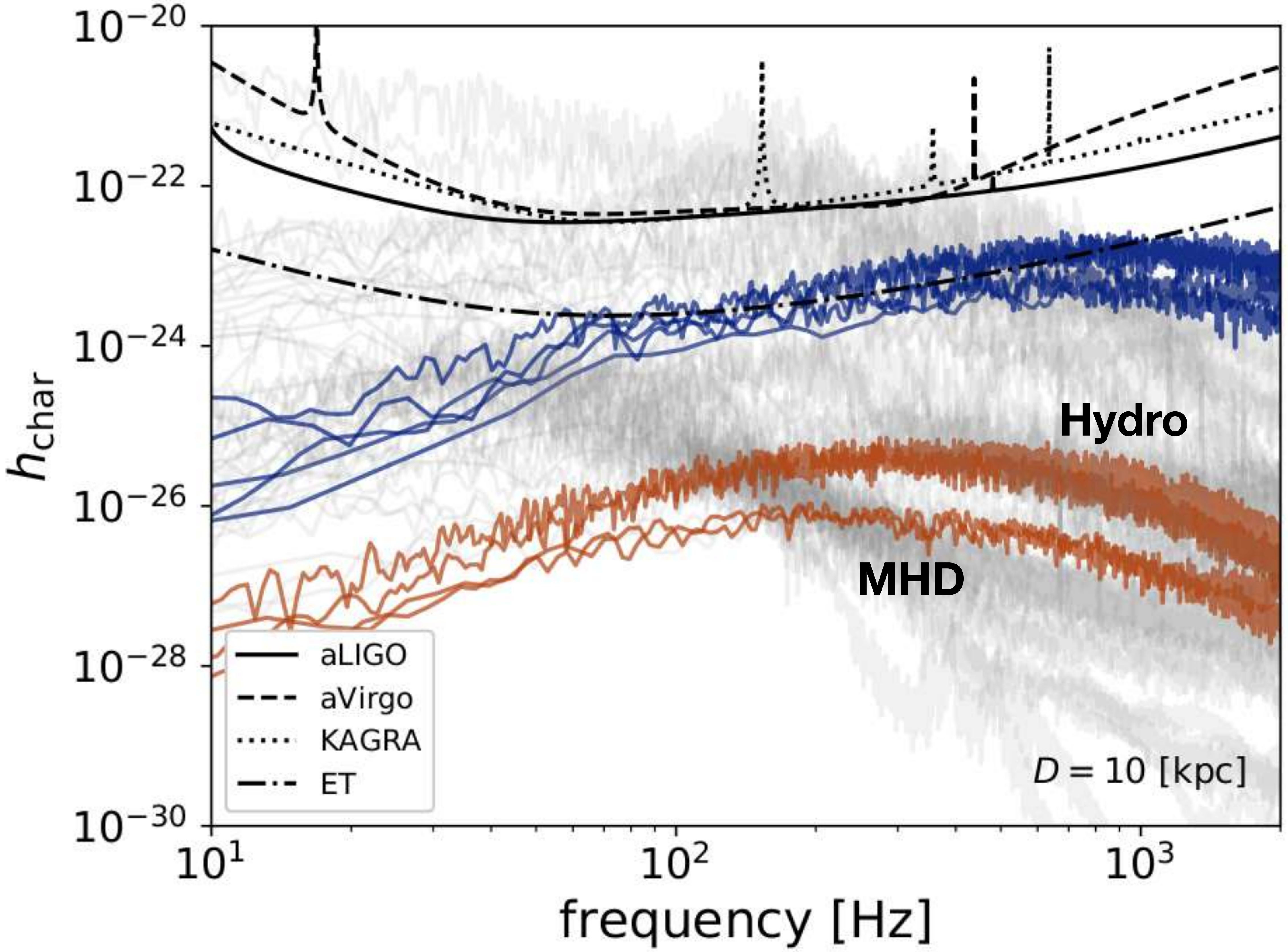


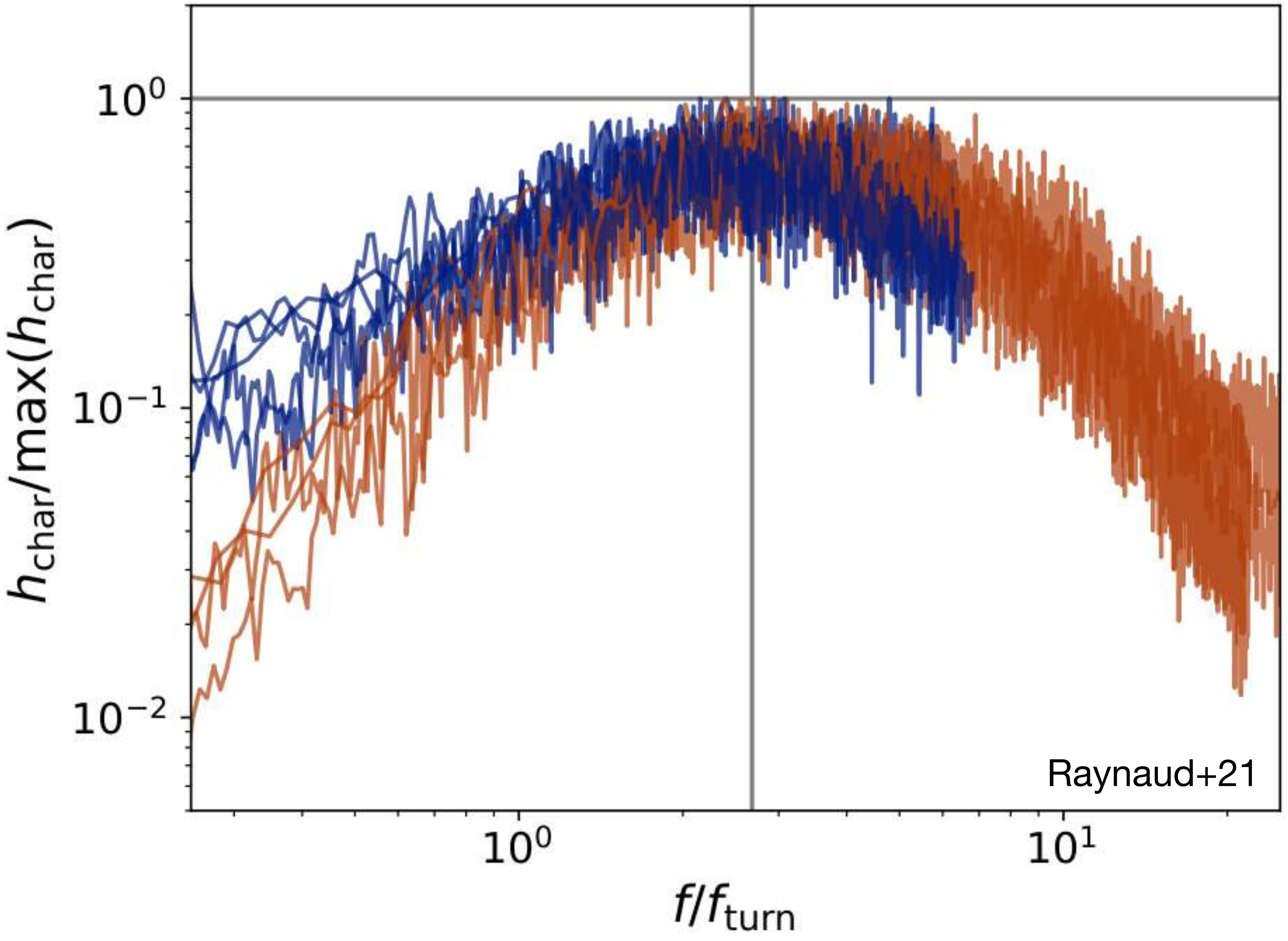
State-of-the-art rotating convection scalings (Aurnou+20)

- **Slow rotation:** $f_{turn} \gg f_{rot} \iff Ro \gg 1$
- **Fast rotation:** $f_{turn} \ll f_{rot} \iff Ro \ll 1$

with the Rossby number $Ro \equiv \frac{U}{\Omega d} \equiv \frac{f_{turn}}{f_{rot}}$

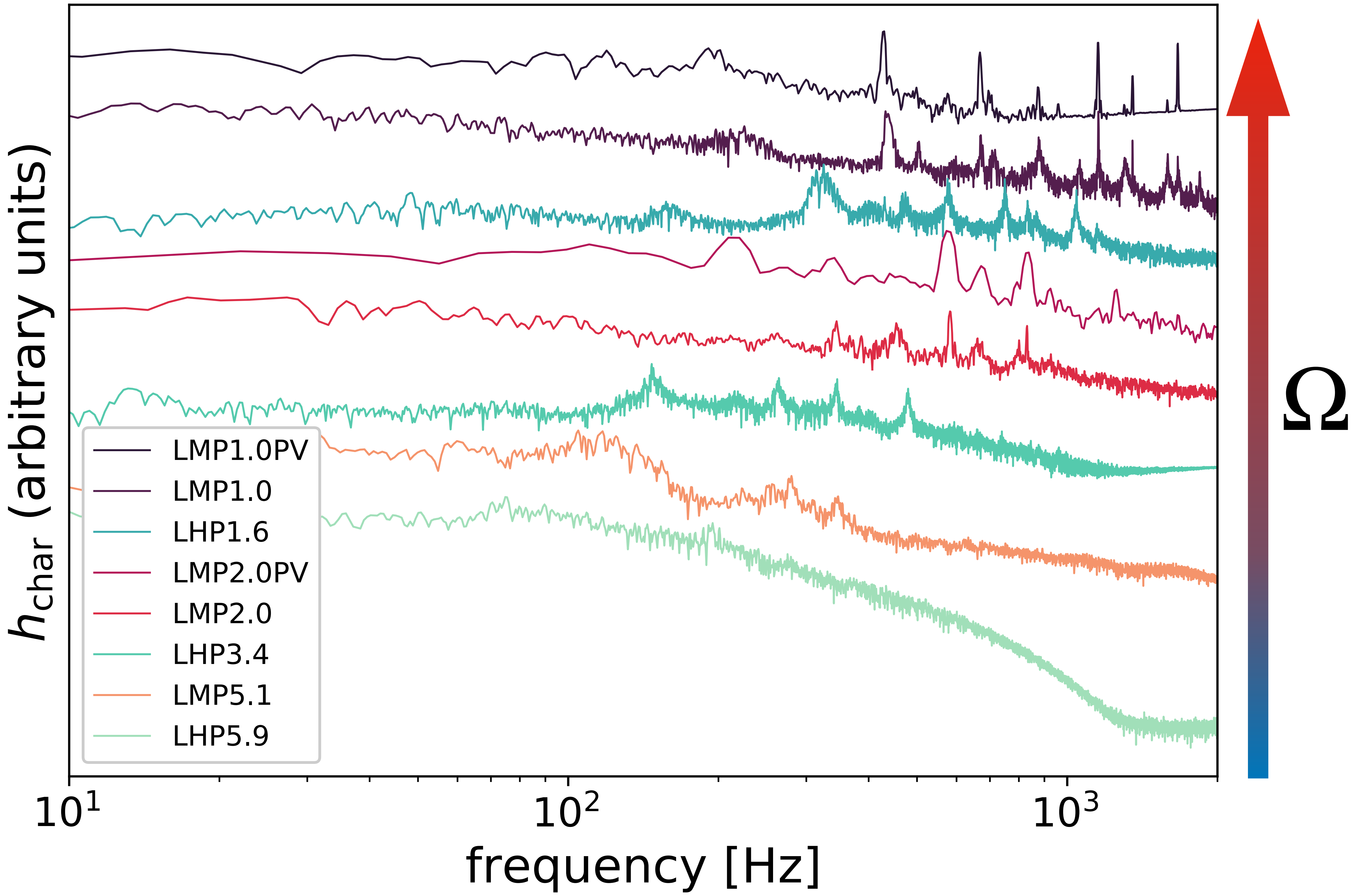
Frequency scaling: slow rotation

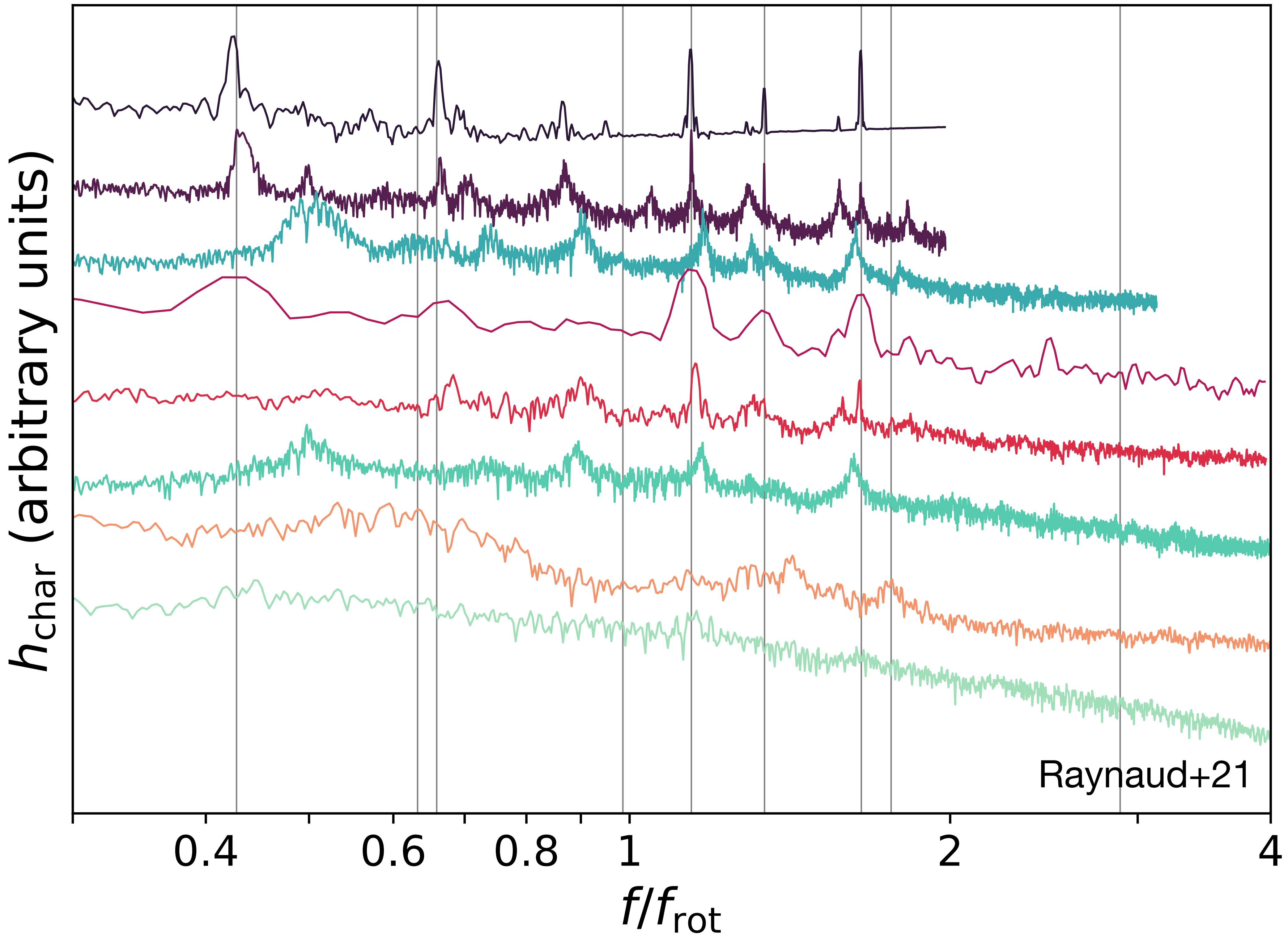




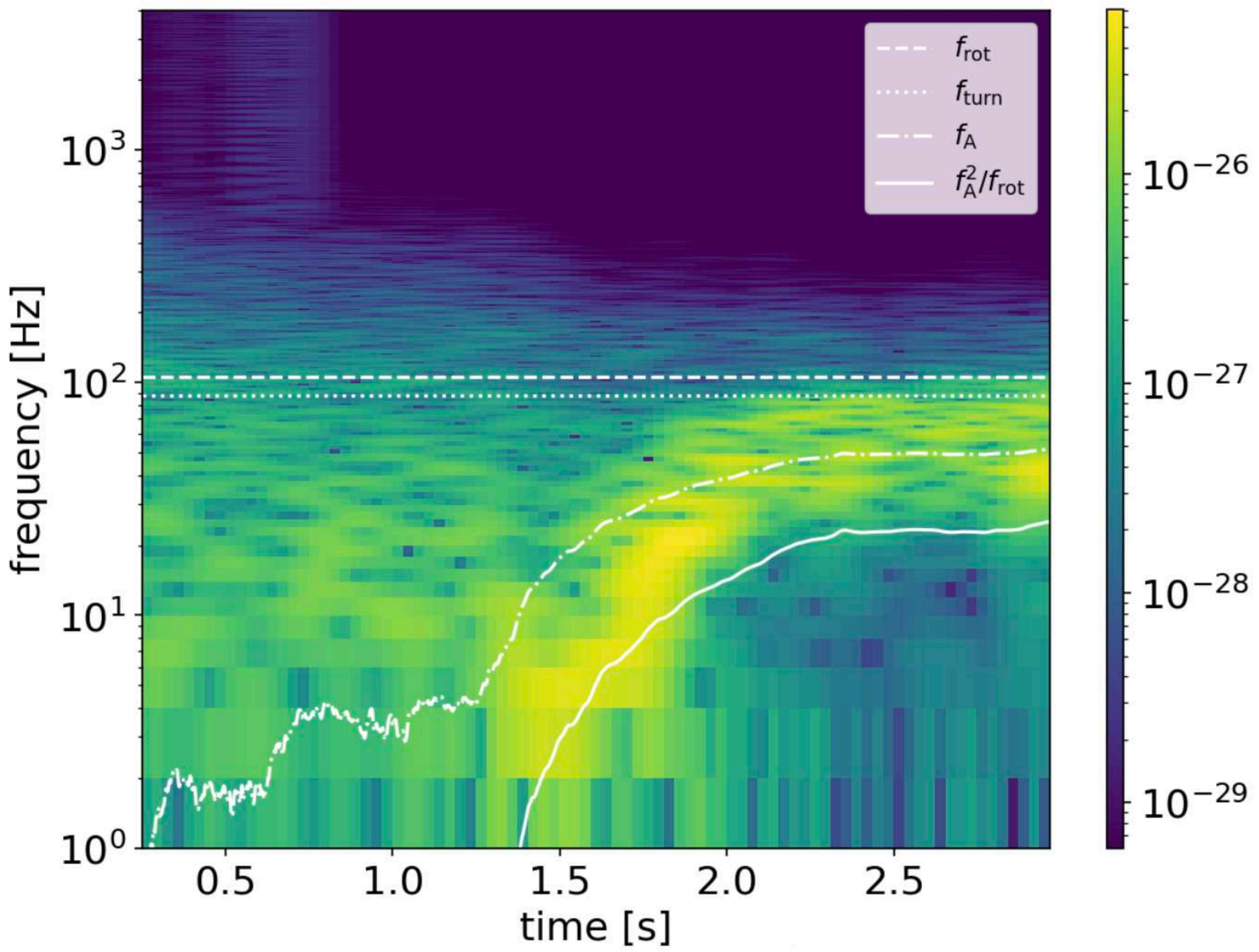
$$f_{\max} \propto f_{\mathrm{turn}} \equiv u_{\mathrm{rms}}/d$$

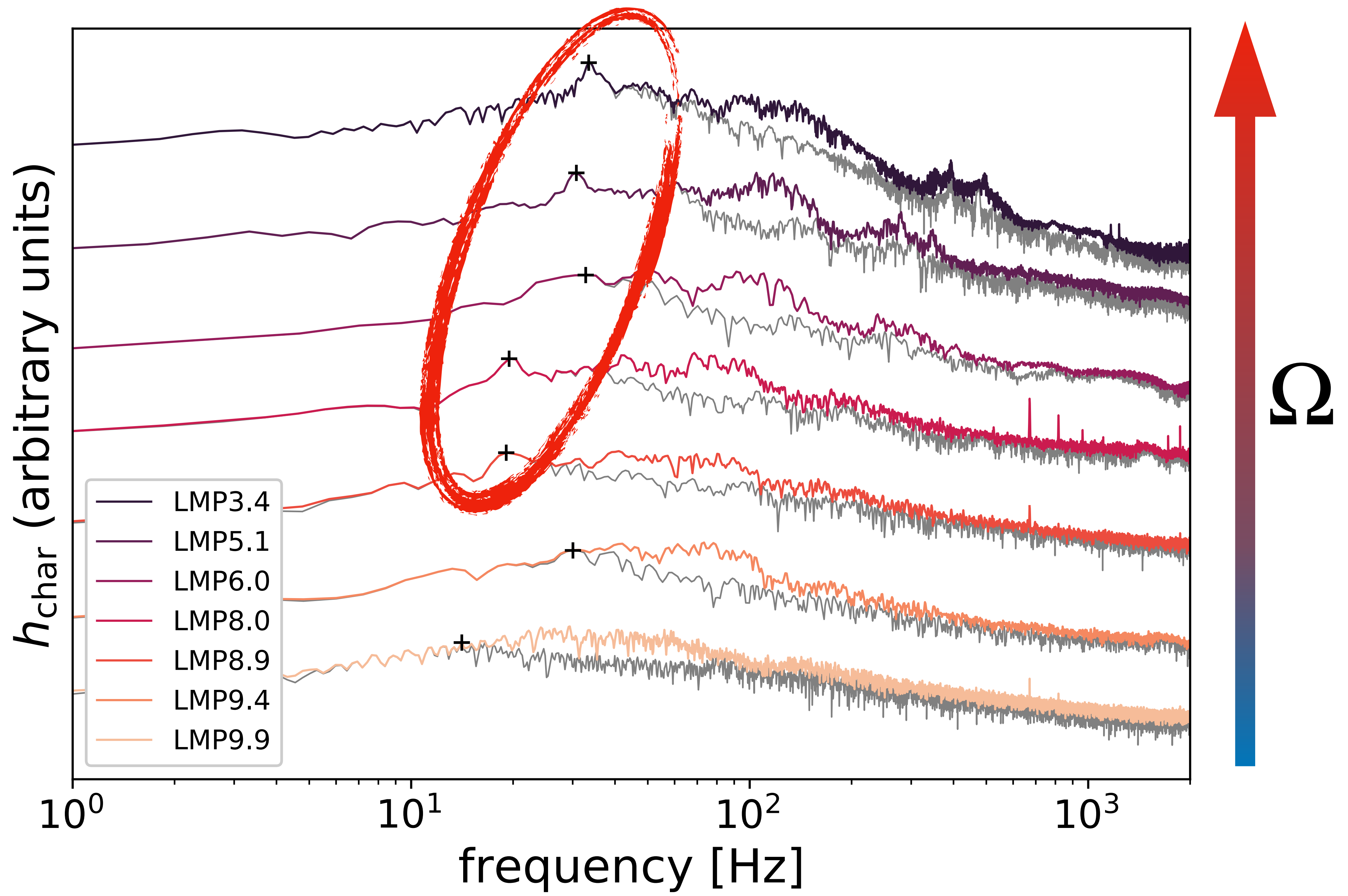
Frequency scaling: fast rotation





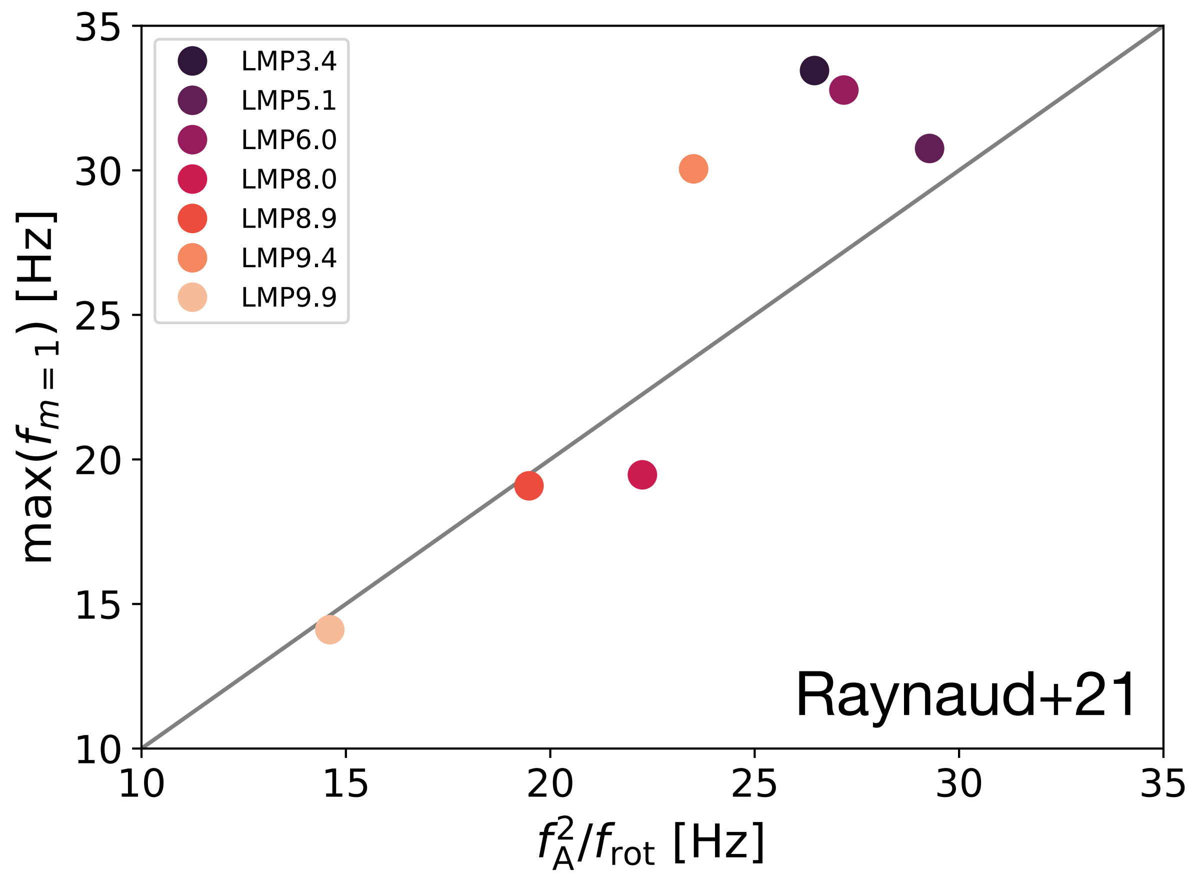
Inertial modes $f_{peaks} \propto f_{rot}$





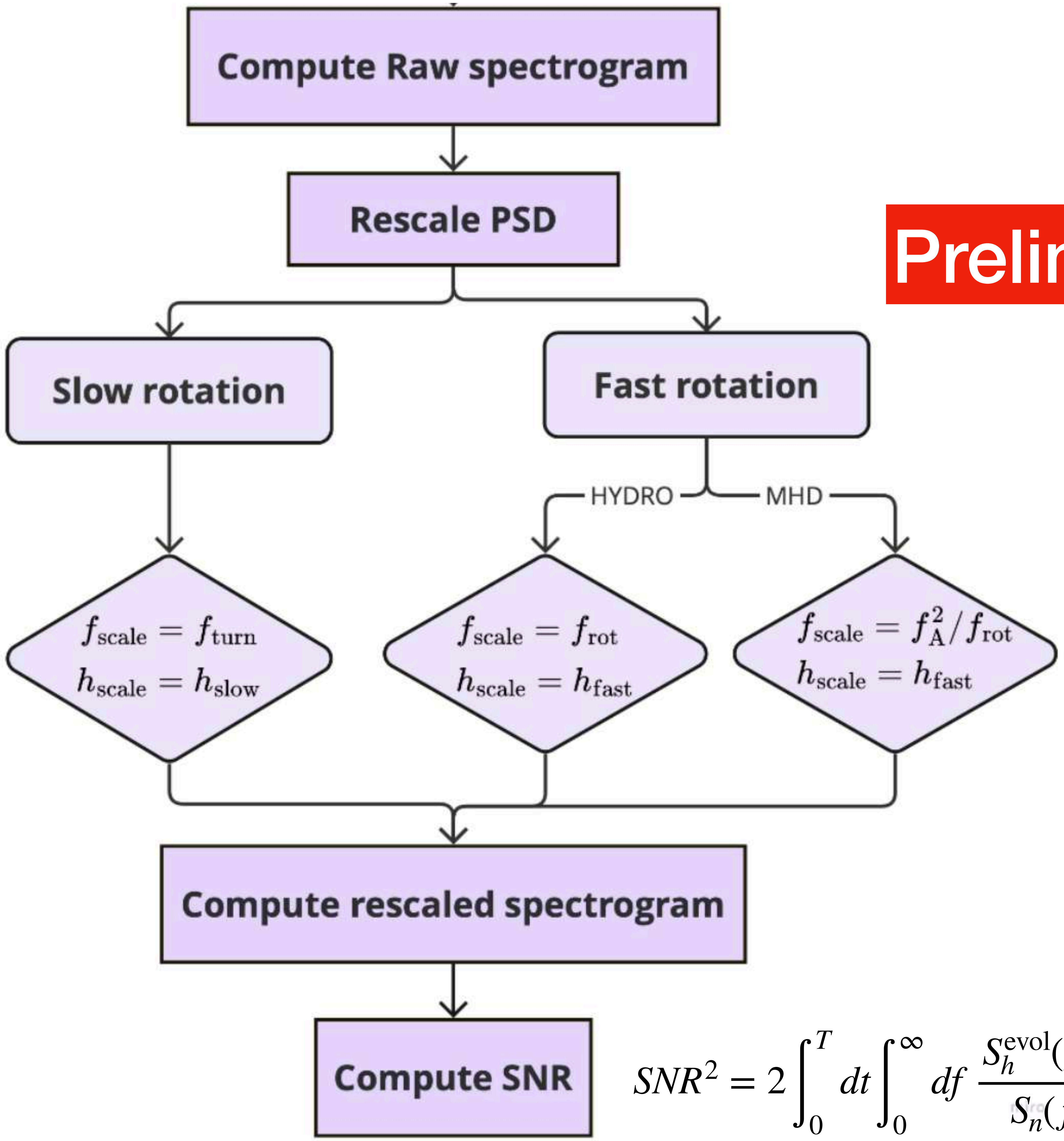
Rossby $m = 1$ mode modified by magnetic effects

$$f \propto f_A^2/f_{rot}$$



Hypotheses

- From the 3D models
  - Self-similarity of the PSD
  - Frequency & amplitude scaling relations
- From the 1D model
  - PNS evolution from 0.2 s to 7 s
- Angular momentum conservation $\Longrightarrow \quad \Omega(t)$
- Asymptotic regimes :
  - Slow rotation ($Ro \gg 1$)
  - Fast rotation ($Ro \ll 1$)

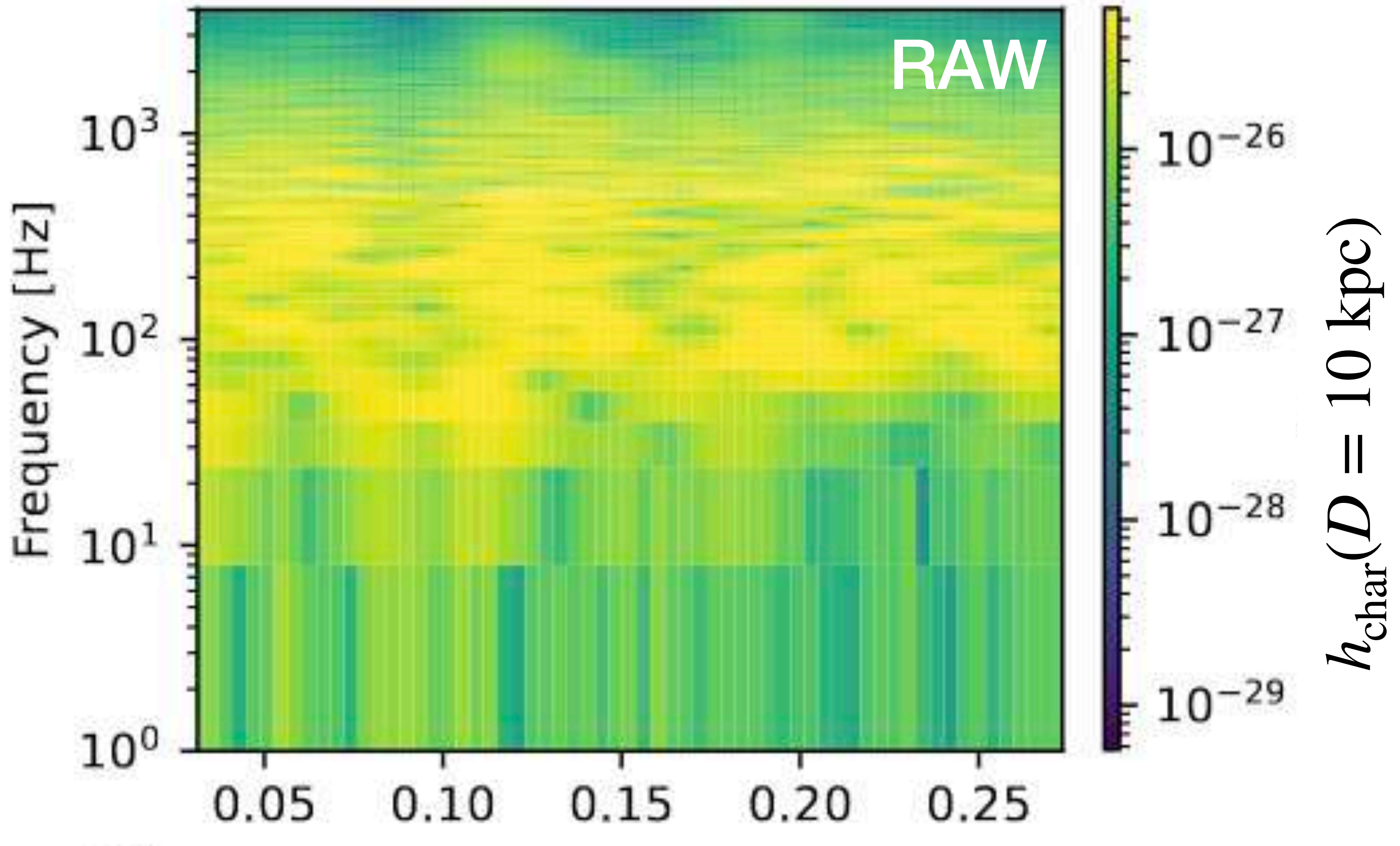


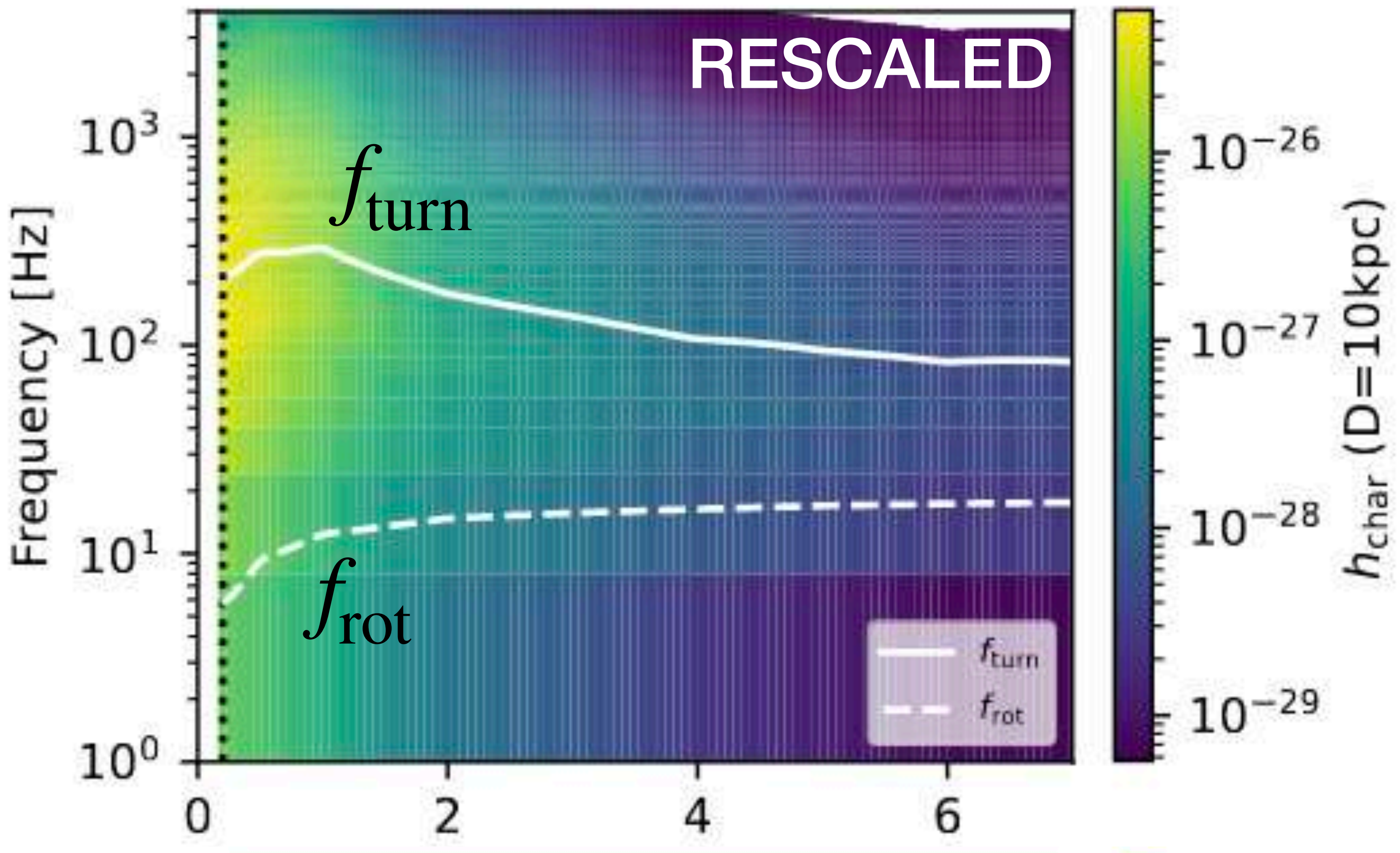
Preliminary !

$$SNR^2 = 2\int_0^T dt \int_0^\infty df \, \frac{S_h^{\rm evol}(f,t)}{S_n(f)} = \int_0^T \sigma(t) dt$$

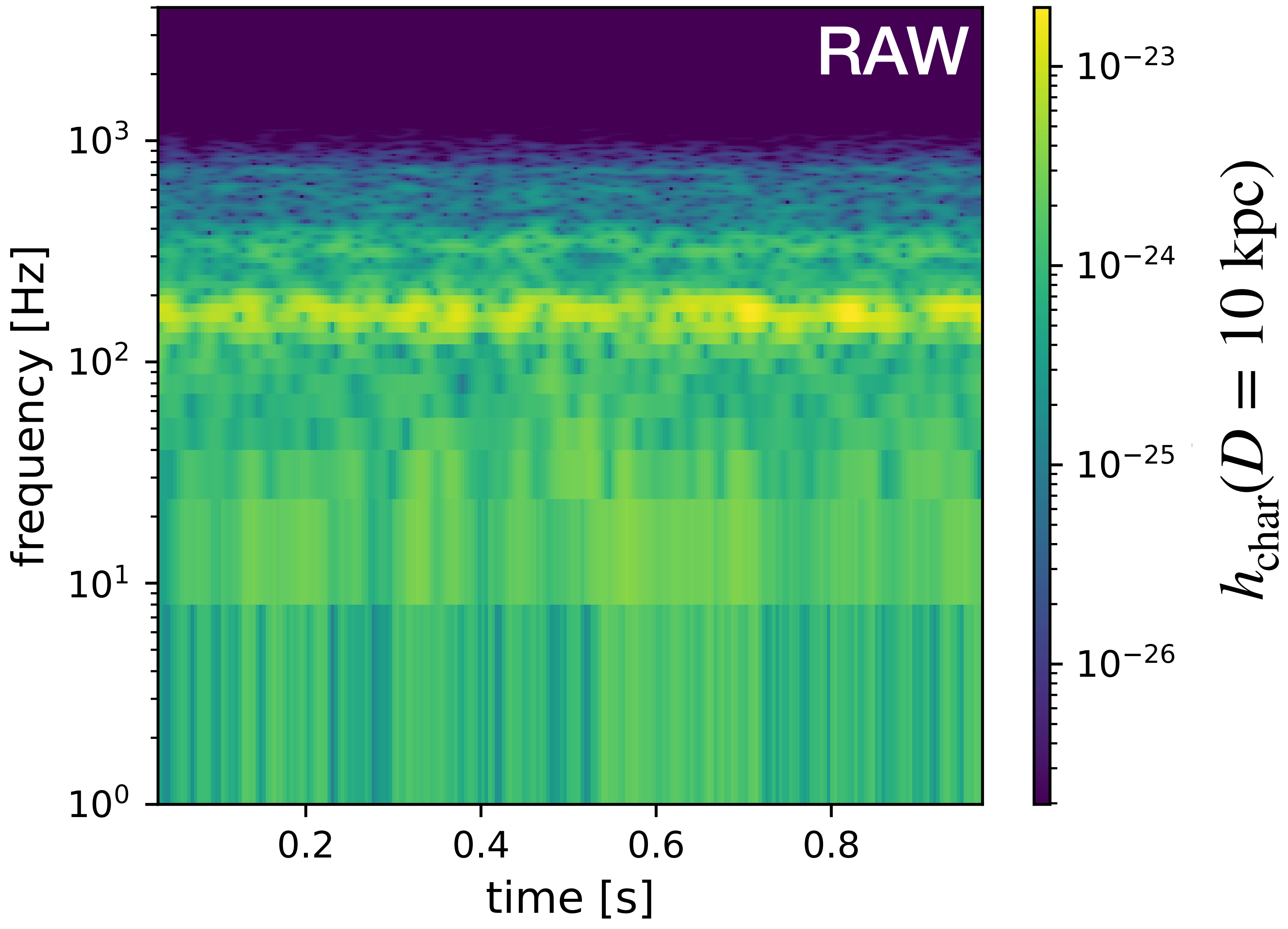
Preliminary proof of concept

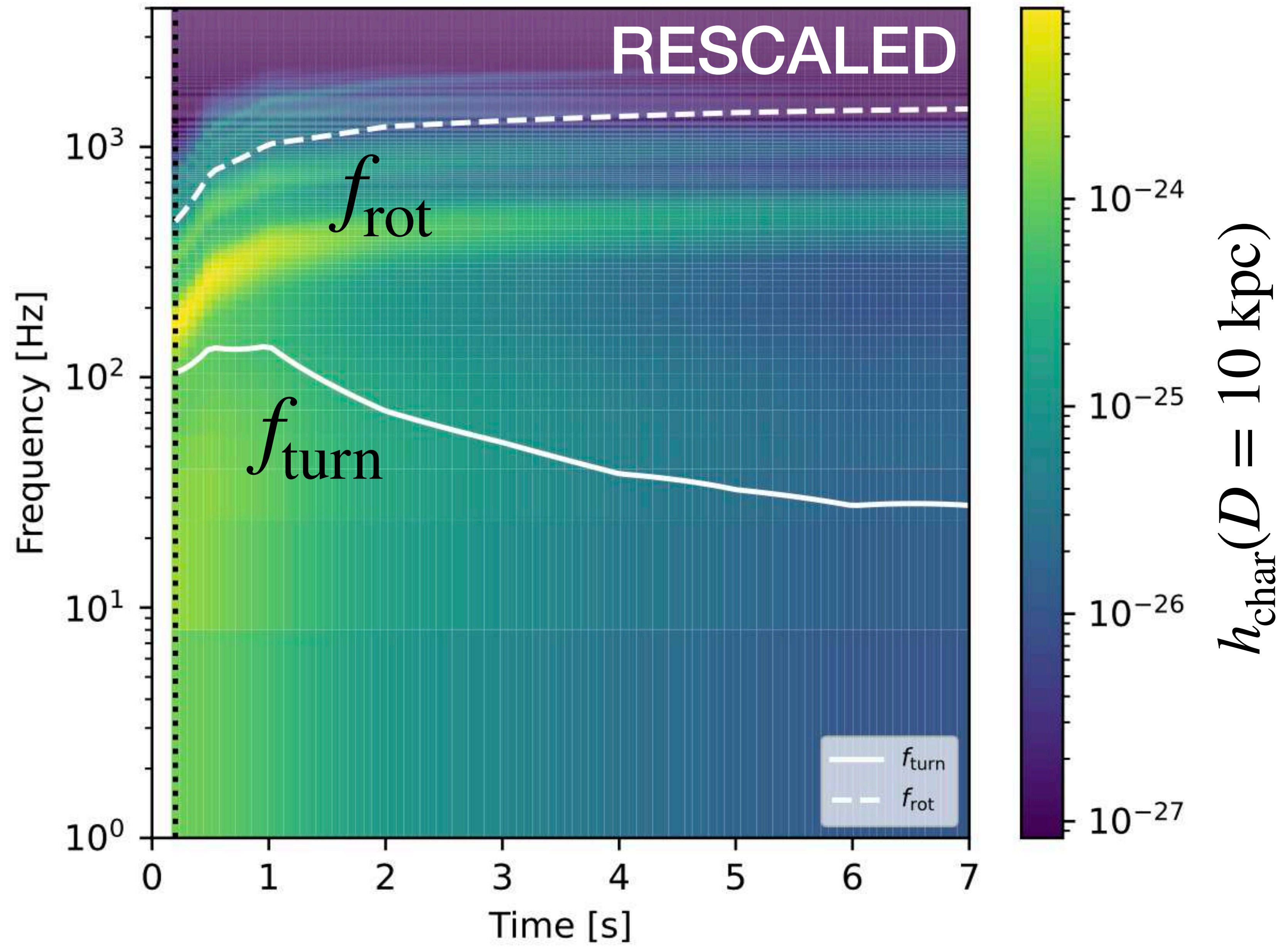
Slow rotation





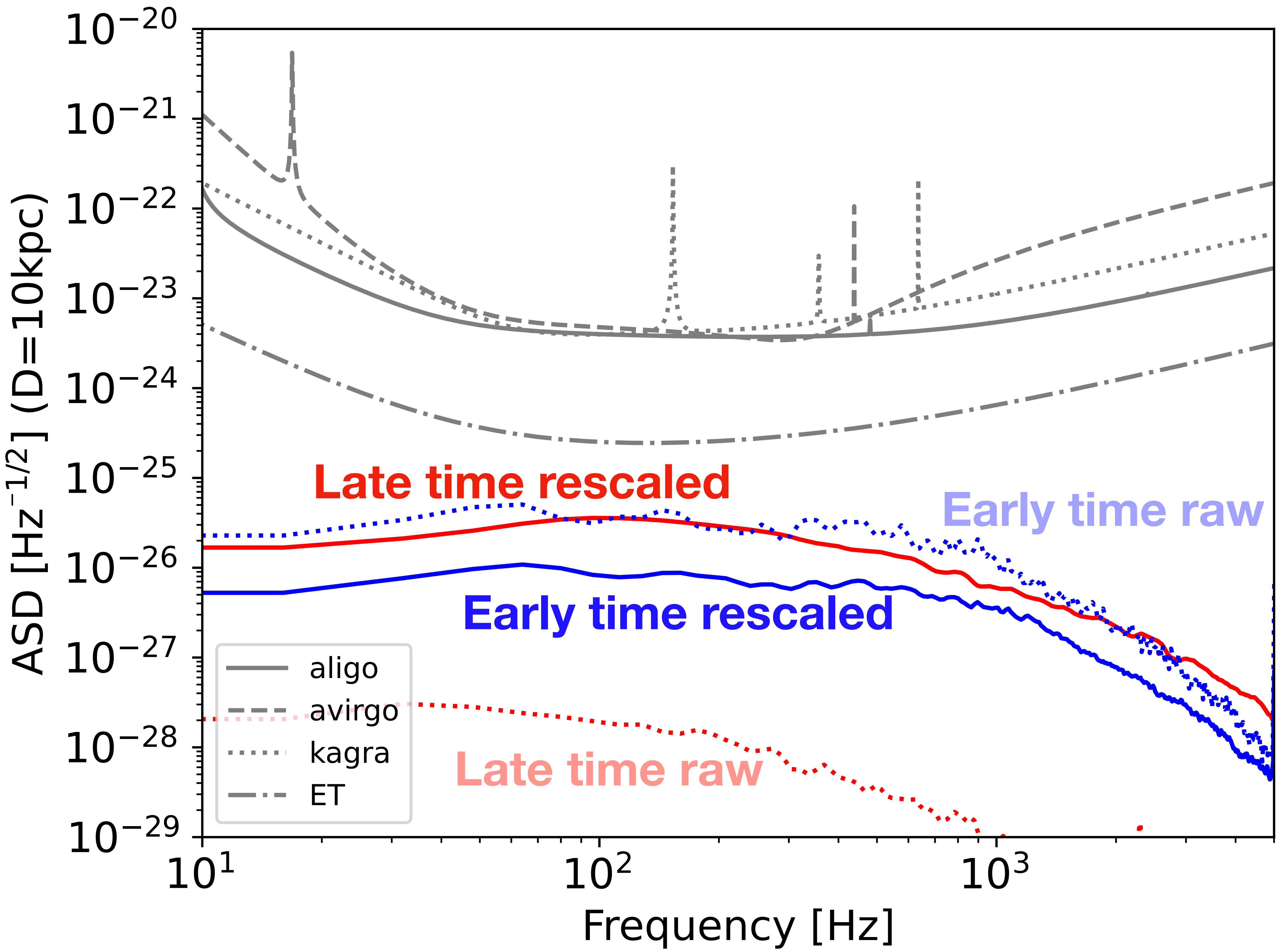
Fast rotation



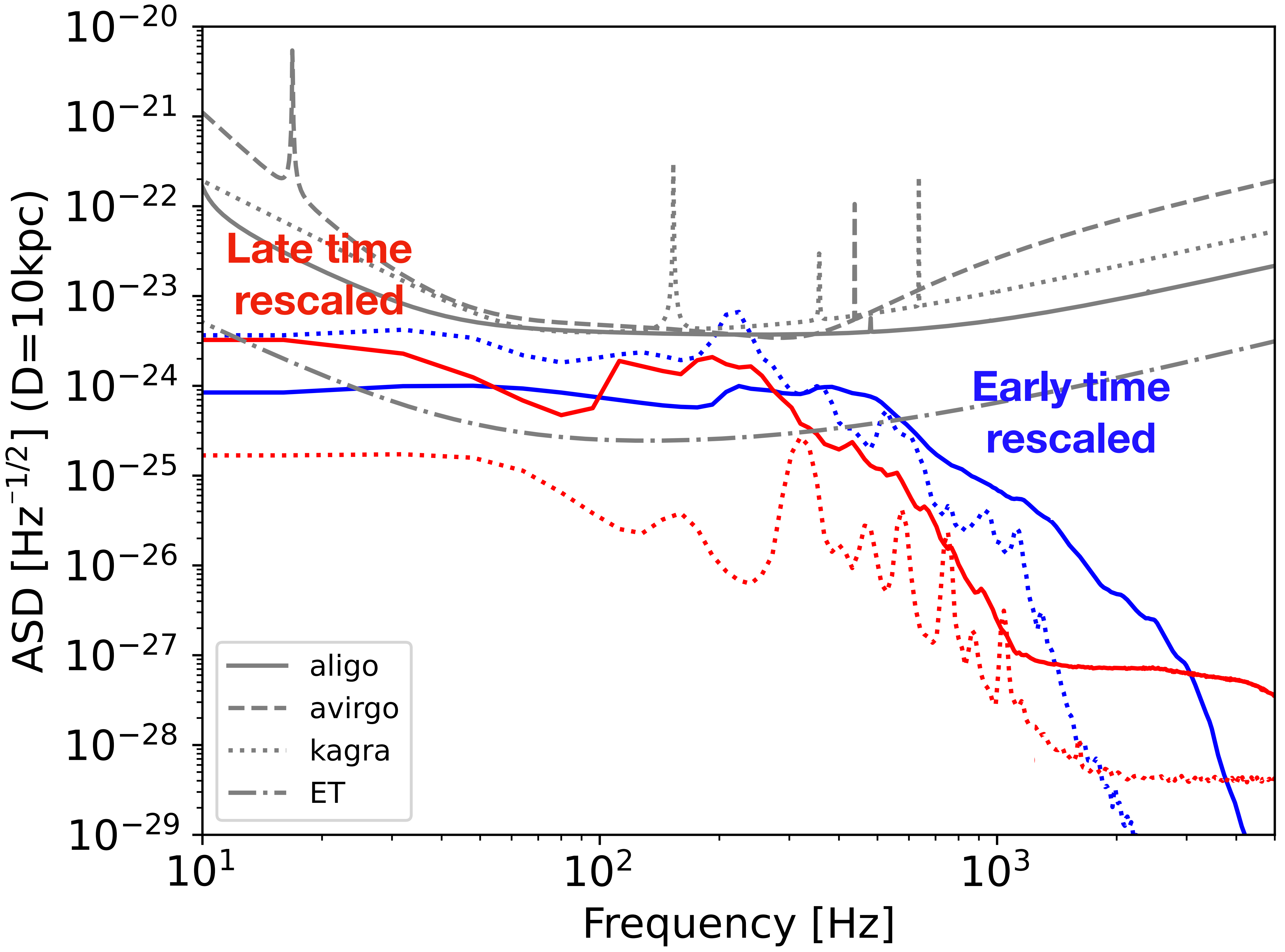


Rescaled spectra

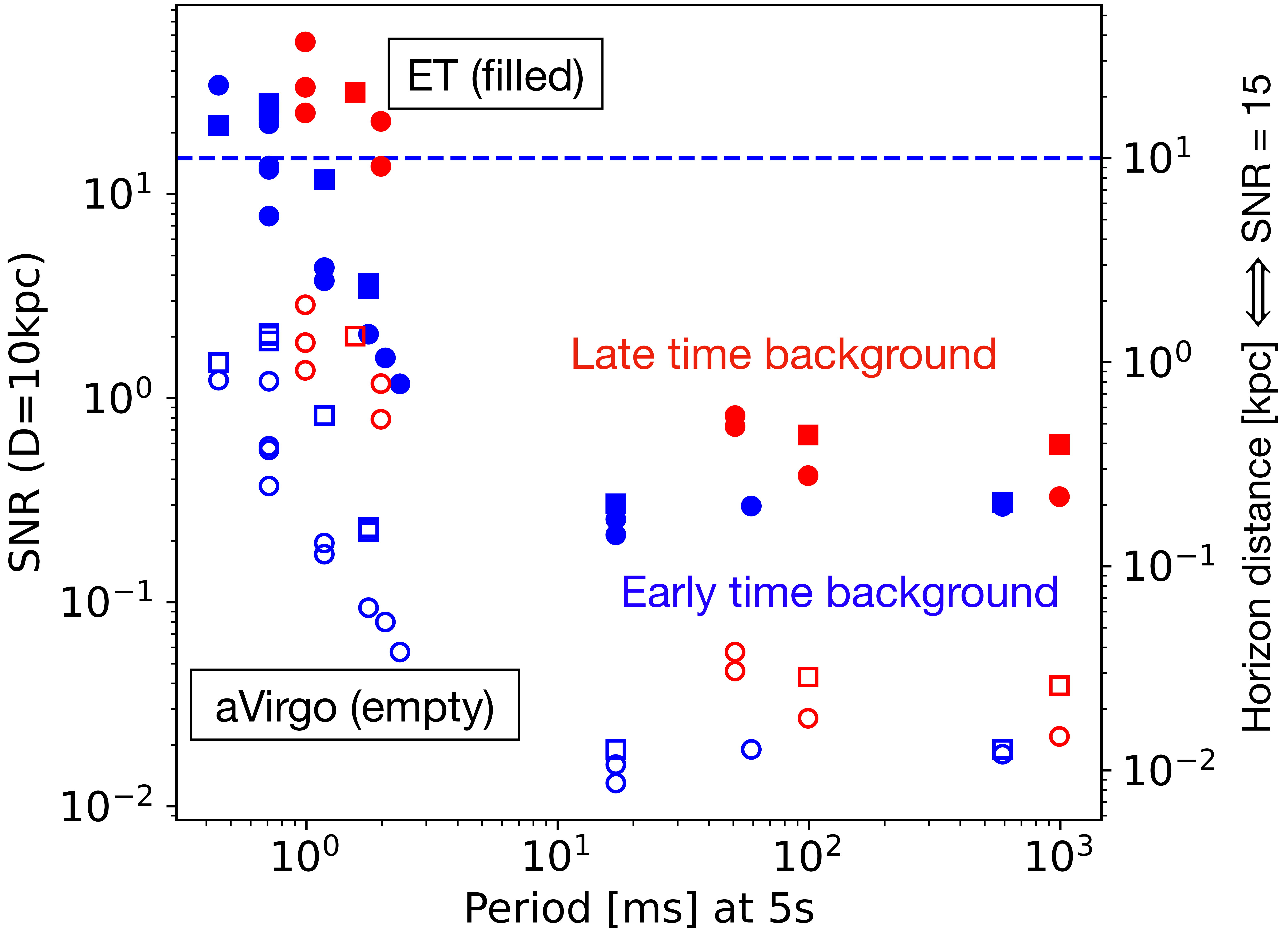
Slow rotation

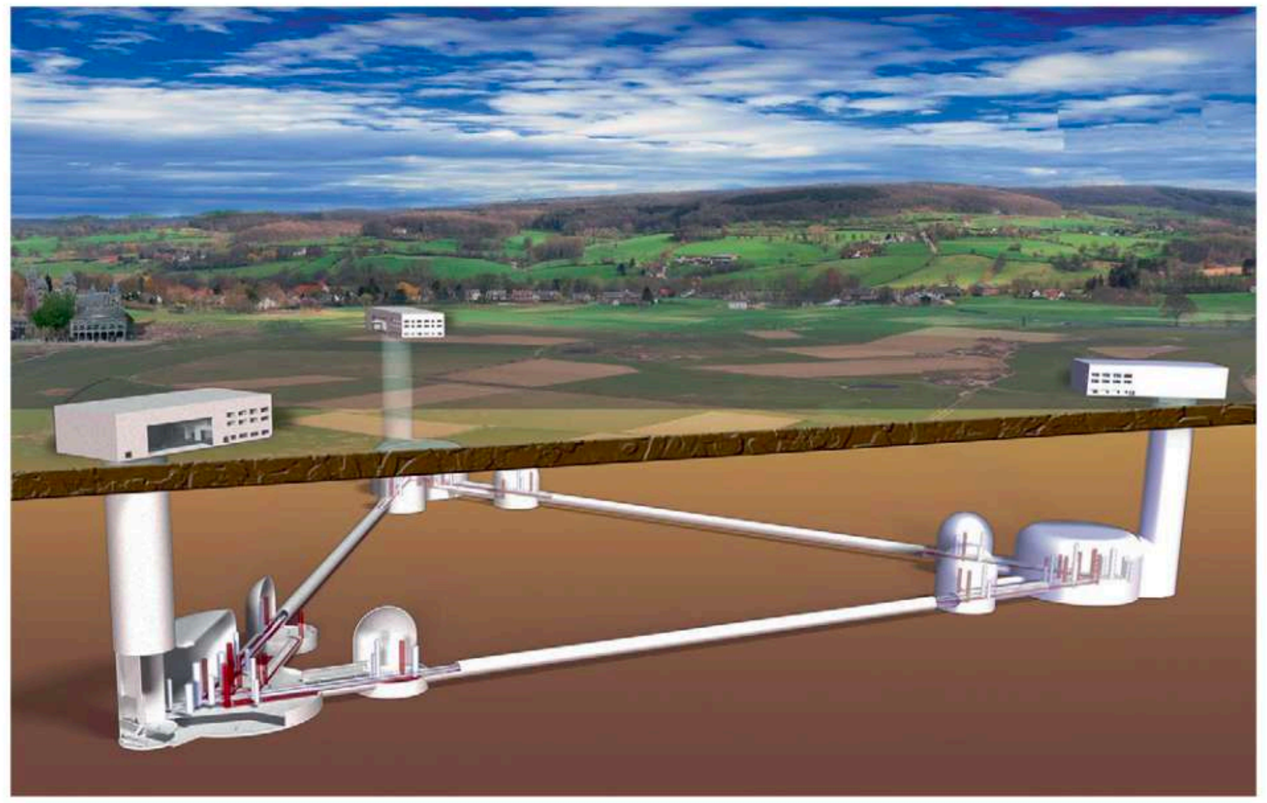


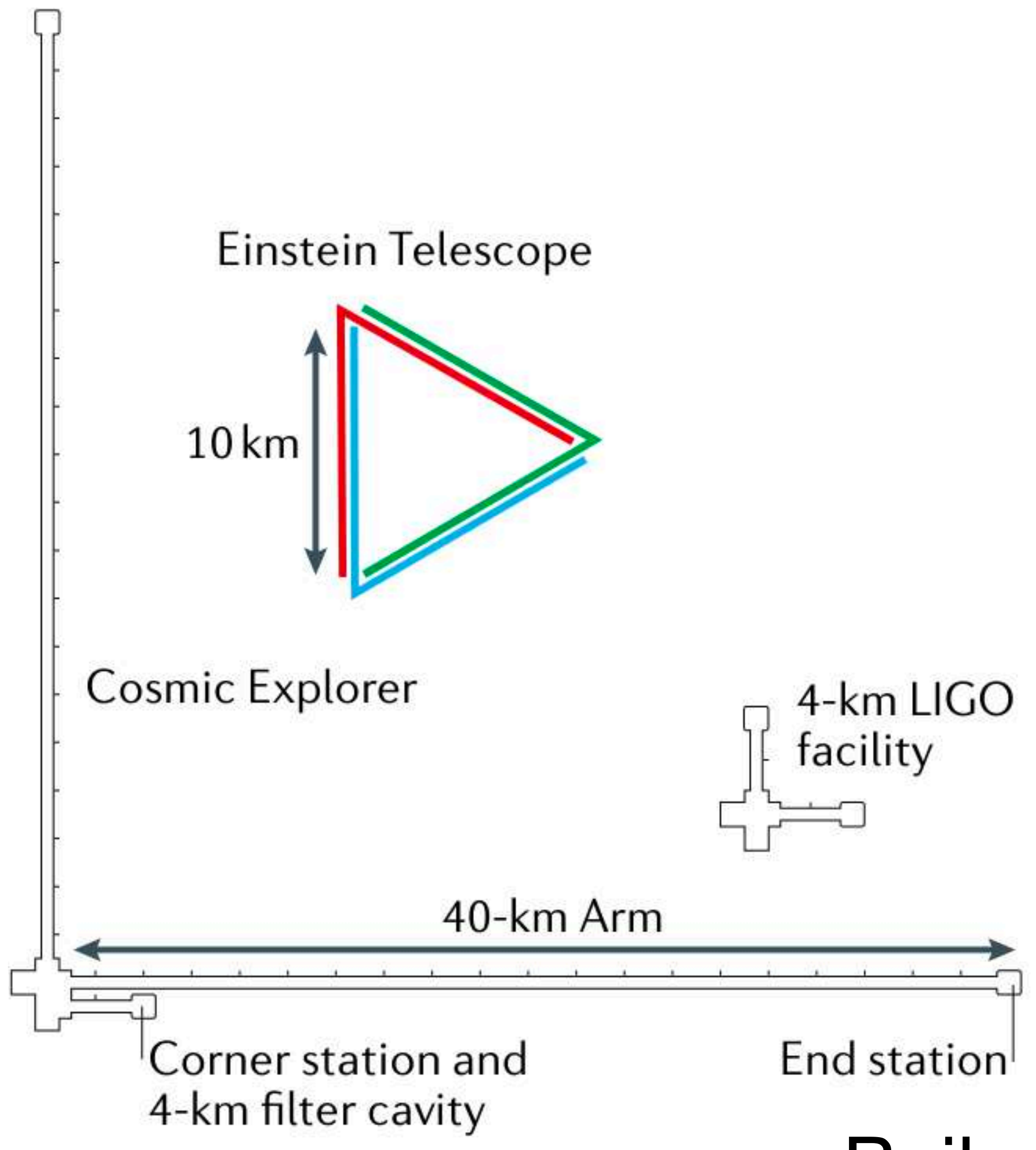
Fast rotation



SNR estimates with current and 3rd gen. Detectors







Bailes+21

Conclusion

Slow rotation $Ro \gg 1$

- Broad spectrum
- $f_{\rm max} \propto f_{\rm turn}$
- Weak impact of magnetic field
- SNR ~ O(0.1) @ 10 kpc with ET

Fast rotation $Ro \ll 1$

- $h_{\rm rms}$ strongly increases
- Complex spectra with inertial modes
- Possibly low frequency, strong field dynamo signature
- SNR ~ O(10) @ 10 kpc with ET

Perspectives

- Coupling with a stable zone to study the excitation of g-modes by turbulent convection
- Characterization of the different PNS dynamo scenarios

References
Raynaud+20,21

Dynamo scenarios:
Barrère+22,23,24 (submitted)
Reboul-Salze+21,22

Appendix

$$[d] = r_o - r_i, \quad [t] = d^2/\nu_o, \quad [S] = d\, \partial S/\partial r|_{r_o}, \quad [p] = \Omega \varrho_o \nu_o, \quad [B] = \sqrt{\Omega \varrho_o \mu_0 \eta_o}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\frac{\partial \mathbf{B}}{\partial t} = \underbrace{\nabla \times (\mathbf{u} \times \mathbf{B})}_{\text{Induction}} - \underbrace{\frac{1}{Pm} \nabla \times (\eta \nabla \times \mathbf{B})}_{\text{Dissipation}}$$

$$0 = \nabla \cdot (\tilde{\varrho} \mathbf{u})$$

$$\frac{D\mathbf{u}}{Dt} = -\overbrace{\boldsymbol{\nabla}\left(\frac{p}{E\tilde{\varrho}}\right)}^{\text{Pressure}} - \overbrace{\frac{2}{E}\mathbf{e}_z \times \mathbf{u}}^{\text{Coriolis}} - \overbrace{\frac{Ra}{Pr}\frac{d\tilde{T}}{dr} S \mathbf{e}_r}^{\text{Buoyancy}} + \overbrace{\mathbf{F}_\nu}^{\text{Viscosity}} + \overbrace{\frac{1}{EPm}\frac{1}{\tilde{\varrho}}(\boldsymbol{\nabla} \times \mathbf{B}) \times \mathbf{B}}^{\text{Lorentz}}$$

$$\frac{DS}{Dt} = \frac{1}{Pr\tilde{\varrho}\tilde{T}} \underbrace{\nabla \cdot \left(\kappa \tilde{\varrho} \tilde{T} \boldsymbol{\nabla} S\right)}_{\text{Heat flux}} + \frac{Pr}{Ra\tilde{\varrho}\tilde{T}} \left(\underbrace{\frac{\eta}{Pm^2 E} (\nabla \times \mathbf{B})^2}_{\text{Ohmic heating}} + \underbrace{Q_\nu}_{\text{Viscous heating}} \right)$$

Braginsky+95
Lantz+99