ROXAS: a pseudospectral evolution code based on primitive variables dedicated to isolated neutron stars

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APC - work carried in LUTh

arxiv:2212.10853 + code paper soon on arXiv

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Content

Context

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- Valencia formulation [Banyuls et al., 1997]: conservative formulation of GR-hydro equations
- Development of high-resolution shock capturing (HSRC) methods

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- Valencia formulation [Banyuls et al., 1997]: conservative formulation of GR-hydro equations
- Development of high-resolution shock capturing (HSRC) methods
- Conservative formulation comes with recovery procedures which use iterative algorithm to recover primitive variables from conservative ones: $(D, S_i, \tau) = f(e, p, T, U_i)$.
  - Be source of code failure (non-convergence of iterative algorithms, non-analytical EoS)
  - Computationally consuming (each grid point, each time step)

Writing a full GR hydrodynamic code

Goal

Evolve an isolated super/hypermassive NS and extract GW signals + parametric study on properties of matter (varying the equation of state (EoS)).

Code written in C++ , based upon LORENE[1].

Steps:

- Newtonian simulations in spherical symmetry.
- Relativistic simulations in spherical symmetry.
- **Development of new numerical tools to use multidomain approach with deformed domains.**
- **Axisymmetric/non-axisymmetric simulations.**

[1] https://lorene.obspm.fr

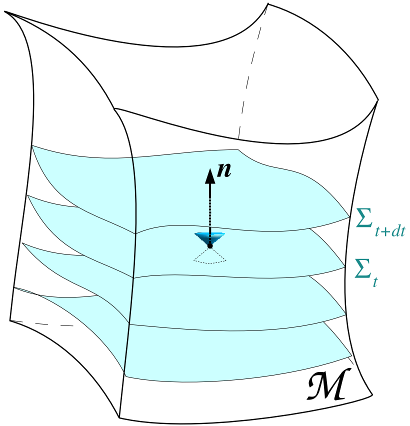

Figure: 3+1 foliation in spacelike hypersurfaces

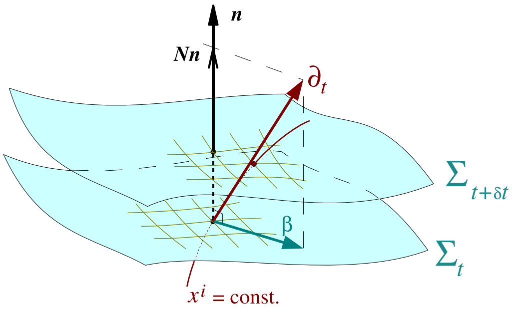

Figure: Illustration of the lapse and the shift

Physical variables are called "primitive" variables:

- e the energy density, p the pressure
- n_B the baryon number density
- m_B a baryon mass
- $H = \ln\left(\frac{e+p}{m_B n_B}\right)$ the log-enthalpy
- U_i the Eulerian velocity field
- v_i the coordinate velocity field
- c_s the sound speed

Content

Conservative variables

The variables $D = m_B n_B \Gamma^2$, $S_j = (e+p)\Gamma^2 U_j$ and $\tau = (e+p)\Gamma^2 - p$, with $\Gamma = (1 - U_i U^i)^{-1/2}$ the Lorentz factor, are conserved in the sense that $\mathbf{u} = (D, S_j, \tau)$ obeys an equation that looks like

$$\partial_t \mathbf{u} + \mathrm{div}(F(\mathbf{u})) = \mathrm{source}$$

but the knowledge of e, p, U_i is compulsory to solve Einstein equations to compute the metric. The reference code CoCoNuT [Dimmelmeier et al., 2005] is based upon the conservative formulation of hydrodynamics.

Recovery procedures are needed to recover the primitive variables in multiple steps of CoCoNuT and account for a significant part of the computation:

- Solving for the metric
- Solving for the Riemann problems in each grid cell at each timestep

Full GR equations, using only primitive variables

From the principles of stress-energy and baryon number conservation:

$$\nabla_\mu (n_B u^\mu) = 0, \ \nabla_\mu T^{\mu\nu} = 0,$$

the following holds for a barotropic, non-reactive perfect fluid:

$$\begin{aligned}
\partial_t U_i = & -v^j D_j U_i - D_i N - \frac{N}{\Gamma^2}\left(D_i H - \frac{\Gamma^2 (1 - c_s^2)}{\Gamma^2 - c_s^2 (\Gamma^2 - 1)} U_i U^j D_j H \right) \\
& + U_j D_i \beta^j + U_i U^j D_j N \\
& + \frac{N c_s^2}{\Gamma^2 - c_s^2(\Gamma^2 - 1)} U_i D_j U^j + \frac{N \Gamma^2 (c_s^2 - 1)}{\Gamma^2 - c_s^2(\Gamma^2 - 1)} U_i U^j U^l K_{jl} \\
\partial_t H = & -v^i D_i H - c_s^2 N \frac{\Gamma^2}{\Gamma^2 - c_s^2(\Gamma^2 - 1)} \left[U^i U^j K_{ij} - \frac{U^i}{\Gamma^2} D_i H + D_i U^i \right]
\end{aligned}$$

This new set of equations is **covariant** within the 3+1 formalism.
e, p, c_s are recovered in a single call to the EoS **(no iteration)**.

Content

ROXAS: Numerical ingredients [Very soon^{tm} on arXiv, currently under Virgo internal review]

ROXAS: **R**elativistic **O**scillations of non-a**X**isymmetric neutron st**A**r**S**

- Pseudospectral methods (space) + explicit finite-difference scheme (time).
- xCFC formulation of metric equations (elliptic partial differential equations).
- Rigid rotation.
- GW extracted with quadrupole formula (+ improvements to first PN order).

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ROXAS: **R**elativistic **O**scillations of non-a**X**isymmetric neutron st**A**r**S**

- Two distinct grids:
  1. Metric grid: spherical domains up to spatial infinity.
  2. Hydro grid: one nucleus + shells.
- Hydro grid adapted to the surface of the star: deformed domains (*).
- Multi-domain matching (*).
- Filters (*).
- Well-balanced formulation of the hydro equations (**).

(*) these numerical techniques were tested and validated on the scalar wave equation.
(**) tested also in the Newtonian framework

Hydro grid: deformed domains

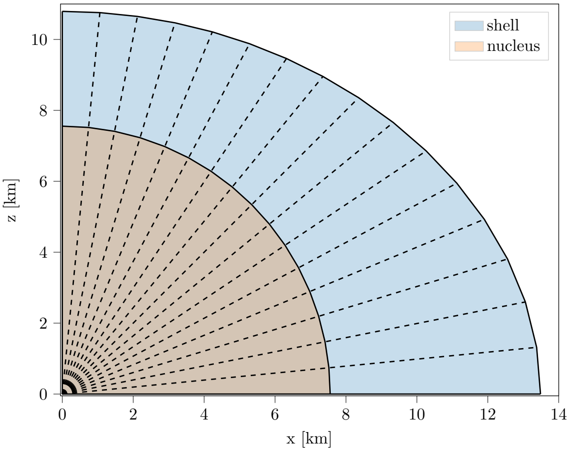

Figure: Deformed external domain (rotating star).

Numerical oscillations

BU4 model: rigidly rotating polytrope, $f_{\rm rot} = 673\,\mathrm{Hz}$.
GW extraction with quadrupole formula.

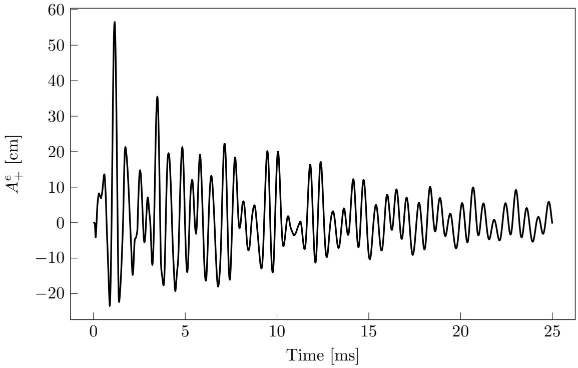

Figure: Waveform (BU4 model).

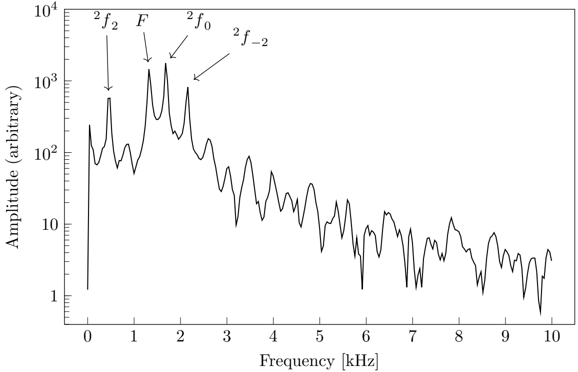

Figure: Spectrum (BU4 model).

Non-axisymmetric modes [Krüger et al., 2010; Krüger and Kokkotas, 2020]

Model	$f_{\rm rot}$ [Hz]	M [$M_\odot$]	r_e [km]	${}^2f_{-2}$ [kHz] Cowling	${}^2f_{-2}$ [kHz] CFC	Relative difference (%) Cowling	Relative difference (%) CFC/GR
BU0	0	1.400	11.99	1.881	1.565	0.00	0.83
BU1	347	1.431	12.30	2.259	1.953	0.09	0.56
BU2	487	1.465	12.64	2.364	2.052	0.09	0.10
BU3	590	1.502	13.03	2.443	2.123	0.61	0.24
BU4	673	1.542	13.49	2.473	2.153	0.57	0.09
BU5	740	1.585	14.03	2.480	2.158	0.44	1.44
BU6	793	1.627	14.70	2.477	2.143	0.40	1.82
BU7	831	1.665	15.55	2.396	2.105	1.88	3.94
SLy4	191	1.361	9.34	2.640	2.185	×	3.7

Content

Summary and outlook

Assets:

- Very light code (3D simulations on laptop/office computer).
- Short simulation time.
- No recovery procedure.
- Radius of the star followed by the grid.
- Very good frequency extraction (although CFC).
- Soon to be published/open source (paper soon on arXiv).
- GWs extracted with quadrupole formula.

Drawbacks:

- In practice: no shock treatment.
- Spectral methods are unforgiving.
- Only oscillations/post-merger phase.

Outlook:

- Getting rid of CFC (low priority).
- Implement differentially rotating profiles.
- Implement EoS with compositional and temperature dependence.

Thank you!

Figure: Roxas, the eponymous cat.

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