Gravitational Waves inspiral tests of General Relativity in the framework of the Einstein Telescope

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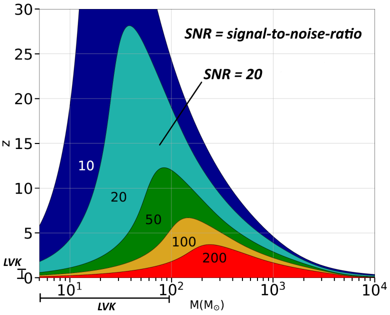
Prospects for the Einstein Telescope

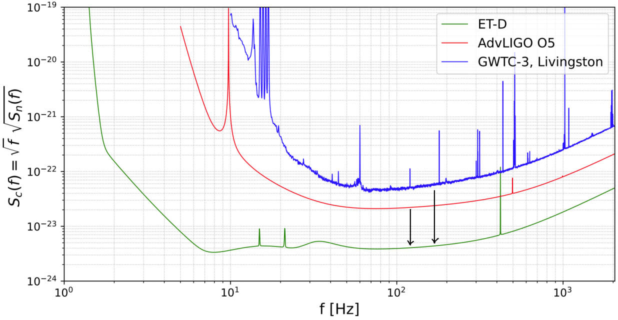
Theoretical and astrophysical implications

⇒ **Astrophysics:** Black Holes (BHs) properties and evolution

⇒ **Fundamental physics:** **Tests of General Relativity (GR)**



M. Maggiore +, JCAP, 2020



To better investigate its capabilities we need a **Parameter Estimation** (PE) study

Parameter estimation for coalescing binary systems

Bayesian inference

⇒ Let's assume our **time-domain Gravitational Wave (GW) signal** $s(t)$ as:

$$s(t) = h(t) + n(t)$$

⇒ Once $s(t)$ is detected, we want to *infer* **statistical information** about the physical parameters $\vec{\theta}$ (e.g. $M = m_1 + m_2, \chi_i$) related to the GW sources, such as Binary Black Hole (BBH):

$$\boxed{p(\vec{\theta}|s) \propto \mathcal{L}(s|\vec{\theta})\pi(\vec{\theta})}$$

The **posterior** $p(\vec{\theta}|s)$ must be stochastically sampled (e.g. MCMC methods)

The only computationally feasible way of performing PE on large populations ($\sim 10^5$ events$/yr$) is based on the **Fisher Information Matrix** (FIM) approach

M. Branchesi +, JCAP, 2023, *M. Maggiore +, JCAP, 2020*

Fisher information matrix

Approximations in the context of Bayesian analysis

$\Rightarrow$ By writing down the likelihood $\mathcal{L}(s|\vec{\theta})$, we implicitly assume a **noise model**:

$$\log \mathcal{L}(s|\vec{\theta}) \propto -\frac{1}{2}\langle n|n\rangle = -\frac{1}{2}\langle s - h(\vec{\theta})|s - h(\vec{\theta})\rangle$$

at fixed parameters $\vec{\theta}$, and compute the FIM in the following way:

$$\boxed{\Gamma_{ij} = \langle h_i|h_j\rangle} \quad , \quad h_i = \partial_i h|_{\vec{\theta}=\vec{\theta}_0}$$

Under the **large SNR limit**, a **stationary Gaussian noise** and a **flat prior**, the *inverse of the FIM* ($\boldsymbol{\Gamma}^{-1}$) *can be approximated to the covariance matrix of the Bayesian posterior*:

$$\boxed{p(\vec{\theta}|s) \propto \mathcal{L}(s|\vec{\theta})\pi(\vec{\theta}) \propto exp\left[-\tfrac{1}{2}\,\boldsymbol{\Gamma_{ij}}\,\delta\theta^i\delta\theta^j\right]} \quad , \quad \delta\theta^i = \theta^i - \theta^i_0$$

M. Vallisneri, Phys. Rev. D, 2008

Tests of GR: model-agnostic deviations

to the inspiral part of the GW phase predicted by GR

$\Rightarrow$ The **frequency-domain waveform models**, during the *inspiral phase*, can be given as:

$$\tilde{h}(f) = A(f)e^{i\Phi_{ins}(f)}$$

in which $\Phi_{ins}(f)$ is treated within the *Post-Newtonian (PN)* framework up to a certain nPN order:

$$\mathcal{O}\left[\left(\frac{v}{c}\right)^{2n}\right] = \mathcal{O}\left[\left(\frac{\pi GMf}{c^3}\right)^{2n/3}\right]$$

Our main aim is to introduce and constrain a **model-agnostic deviation** $\delta\Phi(f)$ in the *inspiral part of the phase* $\Phi_{ins}(f)$:

$$\Phi_{ins}(f) \rightarrow \Phi_{ins}(f) + \boldsymbol{\delta\Phi(f)}$$

N.Yunes +, Phys. Rev. D, 2016

Method: Adopt GWFISH to estimate errors on GW signals

simulated onto different detectors

⇒ Under the FIM formalism, we can easily **forecast the PE capabilities** (e.g. constraining model-agnostic $\delta\Phi$) of future generation detectors

Input: Binary parameters

- m_i, χ_i, d_L, ... $(i = 1, 2)$
- GW **models** based on **GR**
- **Detector type** (e.g. ET, LIGO)
- New injection of a **phase deviation** through $\delta\Phi$

→ GWFISH →

Dupletsa +, 2022

Output: Statistical errors σ by detector noise

- on BBH parameters
- on **GR deviations**, $\boldsymbol{\sigma_{\delta\Phi} \neq 0}$ represents our statistical constrain

Model-agnostic tests of the multipolar structure

arising with quadratic and cubic spin terms in the GW modified inspiral phase

We introduce **model-agnostic deviations** in the dominant spin-induced *quadrupole moment scalar* $\boldsymbol{M_2 = -\kappa\, m^3 \chi^2}$ through:

$$\kappa_i^{GR} = 1 \rightarrow \kappa_i = 1 + \delta\kappa_i$$

for each component $i = 1, 2$ of the coalescing binary

$\Rightarrow$ We want to constrain the symmetric combination $\boldsymbol{\delta\kappa_s = \frac{\delta\kappa_1 + \delta\kappa_2}{2}}$ entering the inspiral part of the GW modified phase:

$$\boxed{\delta\Phi = \delta\Phi(\boldsymbol{\delta\kappa_s})}$$

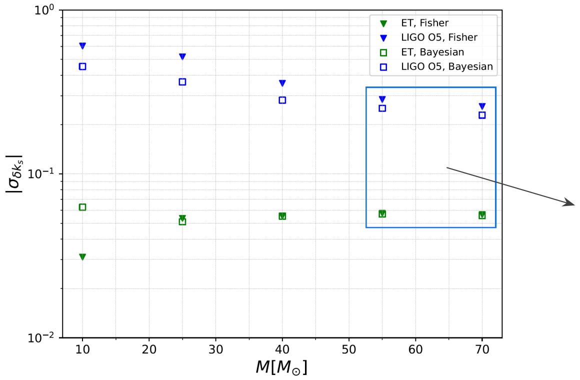
and test the Kerr BHs multipolar structure predicted by *'no-hair theorems'*

N.V. Krishnendu +, 2019

Results: statistical errors on different simulated events

Constraining quadrupolar deviations entering the GW modified inspiral phase

$\Rightarrow$ **GR deviations** are detectable in $\sigma_{\delta\kappa_s}$ and are **agnostic** to modified theories



- **5 different simulated events** with the same spins ($\chi_1 = 0.9$, $\chi_2 = 0.8$) mass ratio ($m_1/m_2 = 3$) and luminosity distance ($d_L = 400\ Mpc$)
- Test the **regime of validity** for **the FIM method**
- ET is expected to **reduce correlations** between $\delta\kappa_s$ and the other GW parameters

ET constraints can **improve by more than an order of magnitude** compared to future LVK

Model-agnostic tests of tidal effects

altering the inspiral part of the GW phase starting at 5PN

We introduce an inspiral phase deviation produced by possible **tidal effects**:

$$\delta\Phi(f) = -\frac{117}{256\eta}\mathbf{\Lambda}\left(\frac{\pi G M f}{c^3}\right)^{5/3} \quad , \quad \eta = \frac{m_1 m_2}{M^2}$$

where Λ is the *weighted tidal deformability* of the compact binary

$\Rightarrow$ We want to constrain Λ and possible deviations (*e.g exotic compact objects*) from its GR value:

$$\Lambda^{GR} = 0$$

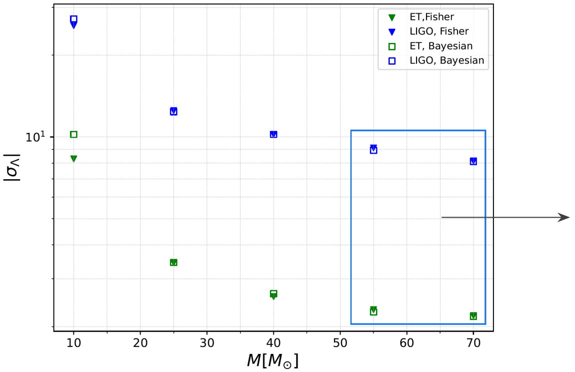
associated with a Kerr BH binary

M. Maggiore +, Oxford University Press, 2018

Results: statistical errors on different simulated events

Constraining possible tidal deviations entering the GW modified inspiral phase

$\Rightarrow$ **GR deviations** are detectable in σ_Λ and are **agnostic** to modified theories



- **5 different simulated events** with the same spins ($\chi_1 = \chi_2 = 0$) mass ratio ($m_1/m_2 = 3$) and luminosity distance ($d_L = 400\ Mpc$)
- Test the **regime of validity** for **the FIM method**
- **Less correlations** between Λ and the other GW parameters

ET constraints can **improve by more than an order of magnitude** compared to future LVK

Conclusions

and possible outcomes

Conclusions

- We **generalized GWFISH** to predict statistical errors on $\delta\kappa_s$ and Λ
- We have shown that **ET will improve statistical constraints** on $\delta\kappa_s$ and Λ by more than an order of magnitude compared to predicted future LIGO O5
- We have tested the validity of the FIM approach making a **fully Bayesian comparison**

Possible outcomes

- The investigation on possible causes of **false GR violations** (*Gupta +, 2024*)
- The implementation and test of **generic priors** in GWFISH (*Dupletsa +, 2024*)
- The development of an entire **CBC population analysis** (*Mancarella +, 2022*)

Thanks for the attention!

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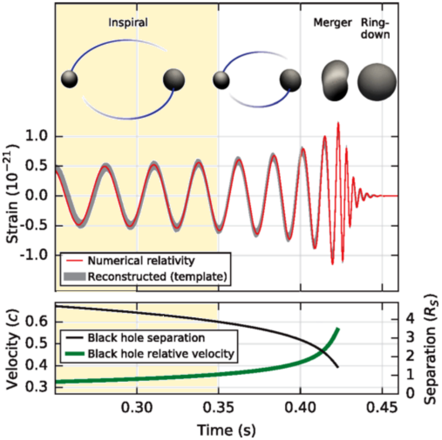
Gravitational Waves: a brief introduction

Emission by Compact Binary Coalescing (CBC) systems during the inspiral phase



B.P. Abbott +, PRL 116, 2016

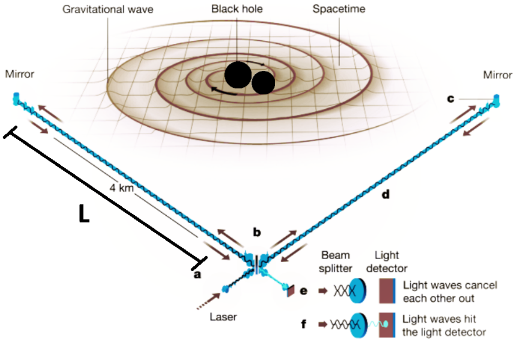
Inspiral phase main approximations

- **Slow-motion** condition
- **Weak-field** sources

Analytical treatment of the emitted Gravitational Wave (GW) **frequency** and **amplitude** of the two wave *polarizations* $h_{+,\times}$

Gravitational Waves: observations

Ground-based laser interferometers



M. Coleman +, Nature, 2019

We are able to measure the time-domain GW **strain** as:

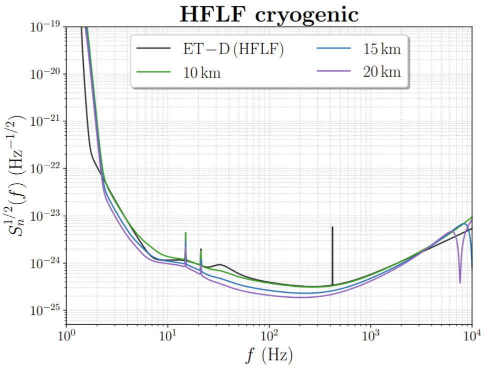
$$h = h_+ F_+ + h_\times F_\times \simeq \frac{\Delta L}{L}$$

But the signal is buried in the **detector noise** affecting the *sensitivity* and *frequency bandwidth*

Significant improvements are expected by future **third-generation interferometers** such as **the Einstein Telescope**

→ Each detector is composed by two interferometers, **”xylophone” configuration**:

Predicted noise Amplitude Spectral Density



M. Branchesi et al. JCAP07(2023)068

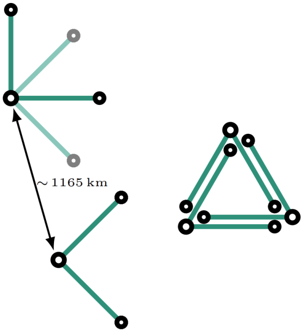
Improved sensitivity

- **High-frequency** (HF)
- **Low-frequency** (LF) **cryogenic**
- Up to **one order** of magnitude **better** respect to predicted Advanced LIGO O5 (*C. Cahillane et al. Galaxies, 2022*)

Noise ASD of a **single nested detector**, but we have more than one...

→ Two possible underground detector networks, **triangular** *vs* **separated 2L-shaped**:

Scheme of the two geometries



M. Branchesi et al. JCAP07(2023)068

Triangular configuration

- **"Null-stream"**
- $SNR \propto 1.5\ L$

2L-shaped configuration

- **Parallel** vs **misalligned**
- $SNR \propto 1.4\ L$

Fisher Information Matrix

Formalism

→ Let's assume our **time-domain GW signal** $s(t)$ as:

$$\boxed{s(t) = h_0(t) + n_0(t)}$$

where $h_0(t) = h(\vec{\theta}_0, t)$ is the signal with expected parameters $\vec{\theta}_0$ and $n_0(t)$ is a stationary Gaussian noise

→ Let's define a **weighted-noise inner product** $\langle \cdot | \cdot \rangle$ between two time-domain signals as:

$$\langle a|b \rangle = 4\Re \int_0^\infty \frac{\tilde{a}^*(f)\tilde{b}(f)}{S_n(f)} df$$

where the tilde denotes the Fourier transform and $S_n(f)$ is the one-sided noise PSD

The **Fisher Information Matrix (FIM)** is defined as:

$$\boxed{\Gamma_{ij} = -\langle \partial_i \partial_j \log P(s|\vec{\theta}) \rangle_n \Big|_{\vec{\theta}=\vec{\theta}_0}} \quad \text{where } \partial_i = \frac{\partial}{\partial \theta_i}$$

and $\langle \cdot \ \cdot \rangle_n$ is a noise average with fixed $\vec{\theta}$ and $P(s|\vec{\theta})$ is the likelihood for a data realization $s(t)$ conditioned on $\vec{\theta}$

Fisher Information Matrix

SNR, variance & sky localization area

→ For each k^{th} **single detector** of the network we can compute the FIM and the SNR:

- $\Gamma_{ij}^{k} = (h_i^k | h_j^k)$
- $SNR_k = \sqrt{(h^k | h^k)}$

→ For the **entire network** of n detector we can compute:

- $\Gamma_{ij} = \sum_{k=1}^{n} \Gamma_{ij}^{k}$
- $SNR = \sqrt{\sum_{k=1}^{n} SNR_k^2}$

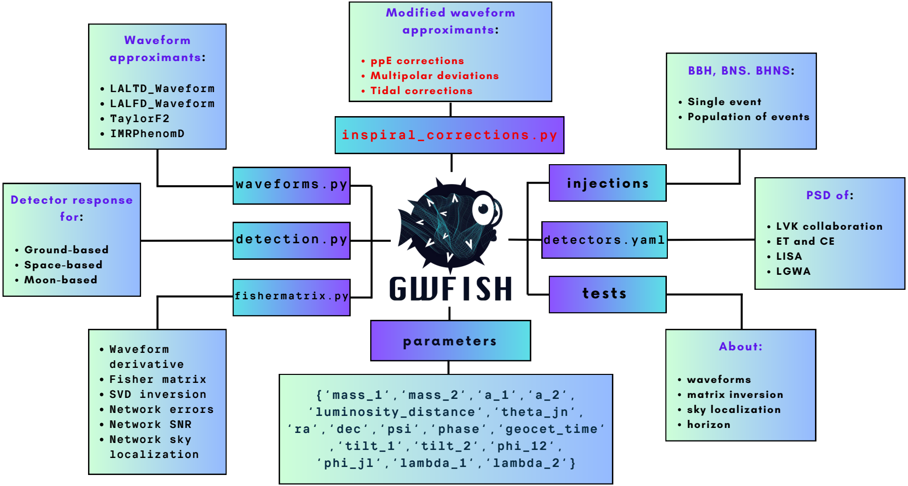
The **variance** σ_i^2 of the i^{th} parameter & the **sky localization** area $\Delta\Omega_{90\%}$ are:

$$\sigma_i^2 = \Gamma_{ii}^{-1}$$

$$\Delta\Omega_{90\%} = -2\pi \ln\left(\frac{1}{10}\right) |\sin\theta| \sqrt{\left(\Gamma_{\theta\theta}^{-1}\right)\left(\Gamma_{\phi\phi}^{-1}\right) - \left(\Gamma_{\phi\theta}^{-1}\right)^2}$$

GWFISH: an overview

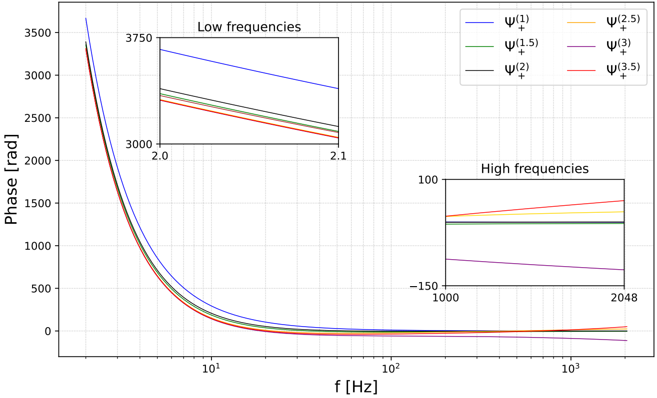
Flowchart of the functioning of modules and directories



TaylorF2

Early inspiral phase expansion

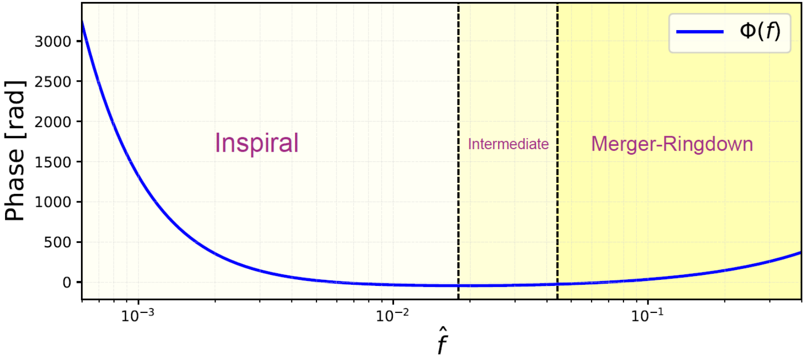
$$\Phi_{ins}(x) = \phi_c - \frac{3}{128\eta} \sum_{n=0}^{7} \left[\varphi_n + \varphi_n^{(l)} \ln\left(x^{3/2}\right) \right] x^{(n-5)/2} \quad , \quad x = \left(\frac{\pi G M f}{c^3} \right)^{2/3}$$



IMRPhenomD

Total phase expression

$$\Phi(f) = \Phi_{ins}\,\theta(f - f_{ins}) + \theta(f - f_{ins})\,\Phi_{int}\,\theta(f - f_{int}) + \Phi_{MR}\,\theta(f - f_{MR})$$



Multipolar structure of a Kerr BH

Spin-induced multipolar moments

$\Rightarrow$ In a specific class of coordinate systems called "Asymptotically Cartesian and Mass Centered" (ACMC), the metric is given by:

$$ds^2 = -\left(1 - c_{00}\right) dt^2 + \left(1 + c_{00}\right) dx_i^2 + c_{0i} dt dx_i$$

where c_{00} admits a spherical harmonic decomposition in terms of the *mass multipole moments* $M_{\ell m}$, while c_{0i} is decomposed in terms of the *current multipole moments* $S_{\ell m}$.

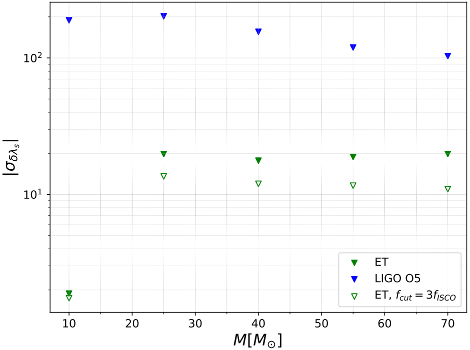
$\Rightarrow$ All the multipole moments of a Kerr BH can be uniquely determined by two parameters, its mass m and spin χ:

$$M_\ell^{\mathrm{BH}} + i S_\ell^{\mathrm{BH}} = m^{\ell+1} \left(i\chi\right)^\ell$$

Results: errors on multiple simulated events

Implications for possible GR deviations entering the GW modified phase

$\Rightarrow$ **GR deviations** are detectable in $\boldsymbol{\sigma_{\delta\lambda_s}}$ and are **agnostic** to modified theories



- **5 different simulated events** with the same spins ($\chi_1 = 0,5$, $\chi_2 = 0,4$) mass ratio ($m_1/m_2 = 3$) and luminosity distance ($d_L = 400\ Mpc$)
- ET property of **reducing the correlations** among parameters

ET constraints can **improve by more than an order of magnitude** compared to future LVK

Tidal properties of a Kerr BH

Weighted tidal deformability

⇒ The linear response of vacuum Kerr BHs to external static perturbations is predicted to be:

$$\tilde{\lambda}_\ell^{BH} = 0 \quad , \quad M_{ij} = -\tilde{\lambda}\mathcal{E}_{ij}$$

where M_{ij} is the *tidal-induced quadrupole moment* and $\mathcal{E}_{ij}$ is the *quadrupole tidal field*. Considering higher multipole contributions, the procedure can be extended to generic $\ell \neq 2$.

⇒ We can define the dimensionless tidal Love numbers $k_\ell \propto \tilde{\lambda}_\ell / m^{2\ell+1}$. For instance, in the Neutron Star case, $k_2 = \frac{3}{2}\frac{G\tilde{\lambda}_2}{R^5}$ from which we can define $\Lambda = \frac{2}{3}k_2\left(\frac{Gm}{Rc^2}\right)^{-5}$. In general, it is useful to introduce the **weighted tidal deformability** as:

$$\Lambda = \frac{16}{13}\frac{(m_1 + 12m_2)m_1^4\Lambda_1 + (m_2 + 12m_1)m_2^4\Lambda_2}{(m_1 + m_2)^5}$$