Probing General Relativity with the inspiral of Massive Black Hole Binaries

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L2IT

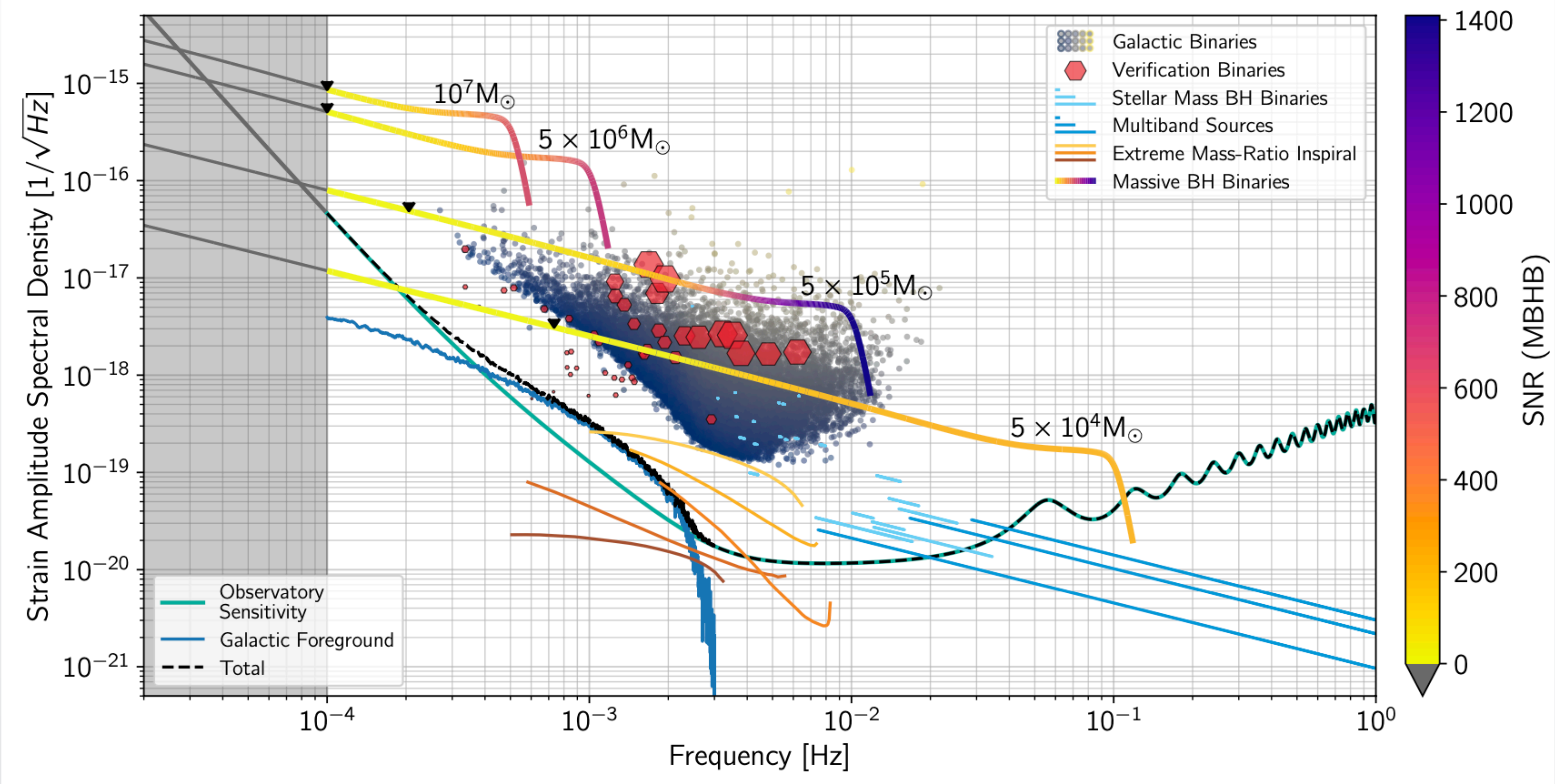
I. LISA: landscape

LISA sources:

- Massive BH Binaries (MBHBs),
- Extreme Mass-Ratio Inspirals (EMRIs),
- Galactic Binaries,
- Stochastic Backgrounds (SGWB),
- Stellar Origin BHs.

masses	$10^3 M_\odot$ to $10^7 M_\odot$
redshift	up to z ~ 15

The high signal-to-noise ratio (SNR) reached by LISA sources will allow us to probe General Relativity



Credits: LISA Definition Study Report [arxiv:2402.07571]

II. Probing General Relativity with GWs

Inspiral

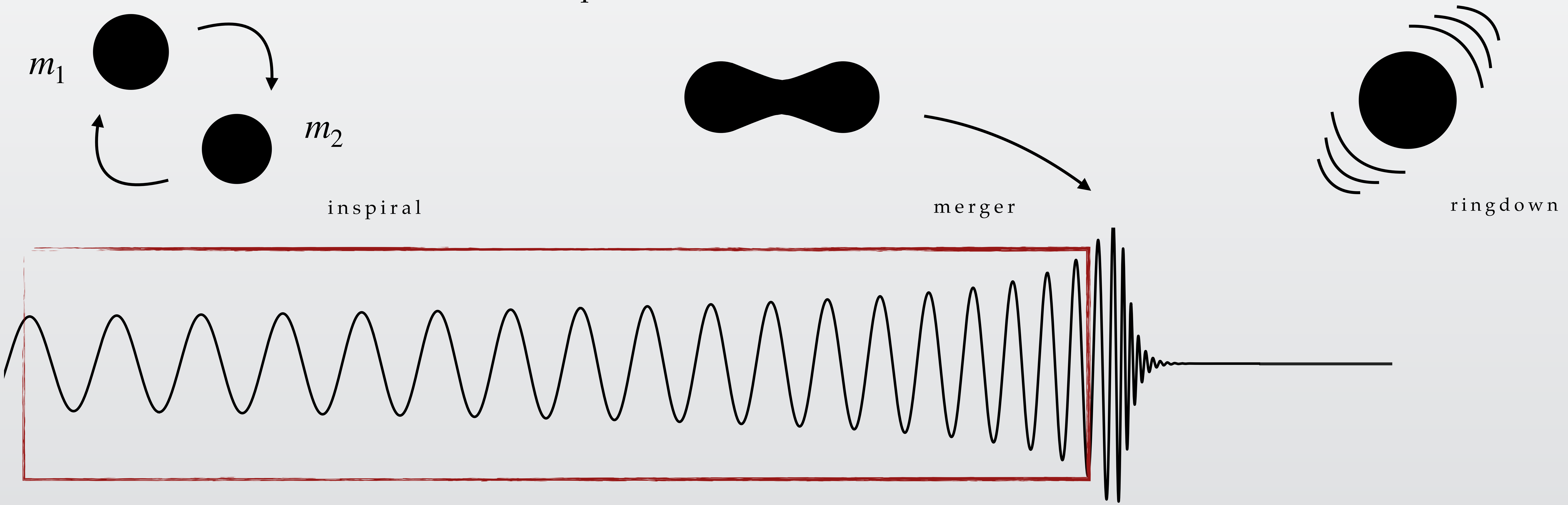
long inspiral allow for high-precision gravity tests (e.g. propagation effects, extra-polarizations …)

Merger

characterized by nonlinear effects of gravity becoming a challenging regime to model, requiring full non-linear evolutions of the field equation

Ringdown

During the ringdown, the GWs emitted by a perturbed BH settle to equilibrium with parameters that depend on the mass and spin of the remnant Kerr BH



II. Probing General Relativity with GWs

Estimate the accuracy of GR deviation constraints with GWs.
Could waveform systematics mimic false GR deviation?

- **Present**

LIGO - Virgo - KAGRA (**LVK**) collaboration
is already testing GR with current GWs observations of **Stellar BBHs**

Limits of ground-based detectors
- low SNR (order ~ 10-30)
- short signals

Test	Section	Quantity	Parameter	Improvement w.r.t. GWTC-2
RT	IV A	p-value	p-value	Not applicable
IMR	IV B	Fractional deviation in remnant mass and spin	$\left\{\frac{\Delta M_f}{\bar{M}_f}, \frac{\Delta \chi_f}{\bar{\chi}_f}\right\}$	1.1–1.8
PAR	V A	PN deformation parameter	$\delta\hat{\phi}_k$	1.2–3.1
SIM	V B	Deformation in spin-induced multipole parameter	$\delta\kappa_s$	1.1–1.2
MDR	VI	Magnitude of dispersion	$\lvert A_\alpha \rvert$	0.8–2.1
POL	VII	Bayes Factors between different polarization hypotheses	$\log_{10} \mathcal{B}^{X}_{T}$	New Test
RD	VIII A 1	Fractional deviations in frequency (pyRing)	$\delta\hat{f}_{221}$	1.1
	VIII A 2	Fractional deviations in frequency and damping time (pSEOB)	$\{\delta\hat{\tau}_{220}, \delta\hat{f}_{220}\}$	1.7–5.5
ECH	VIII B	Signal-to-noise Bayes Factor	$\log_{10} \mathcal{B}_{S/N}$	New Test

Tests of General Relativity with GWTC-3 (2021) [arxiv:2112.06861]

- **Future**

LISA will hopefully observe signals of **MBHBs** with 10^3 - $10^7 M_\odot$

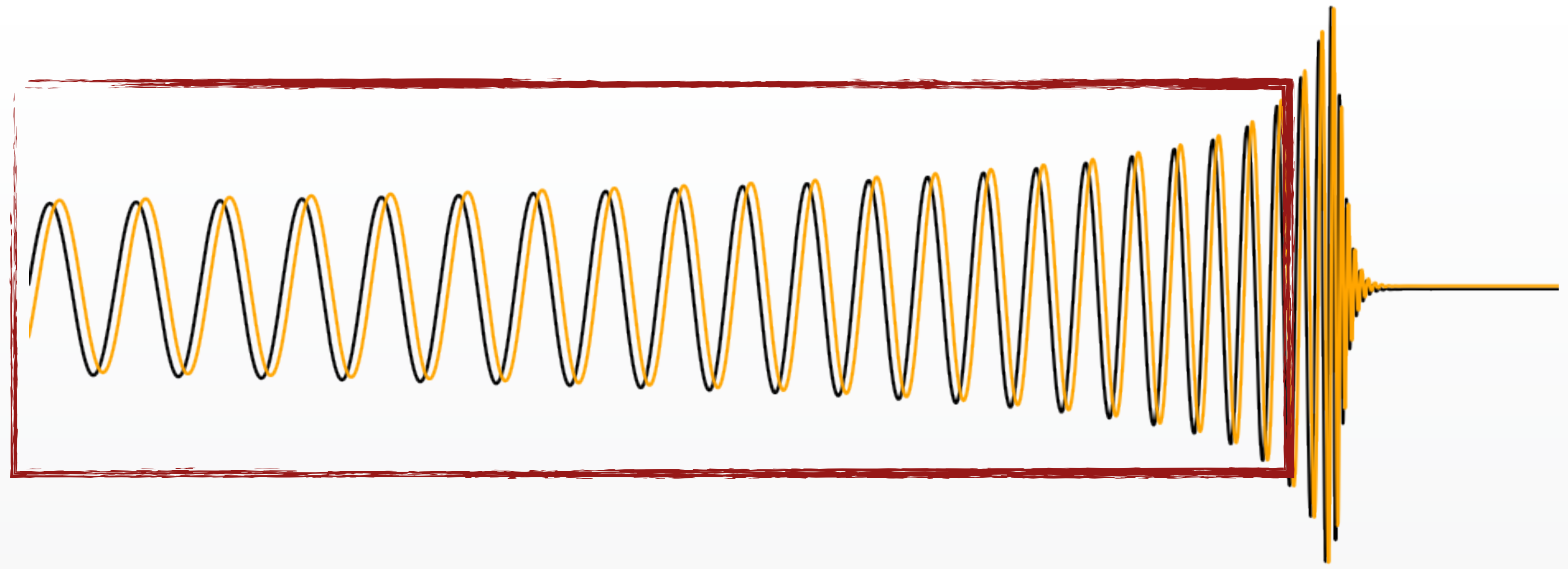
Why could LISA be better?
- high SNR (order ~ 100-1000)
- long signals (much higher number of cycles in band)

$\Downarrow$

- Test of GR (TGR) with LISA observations
- Impact of waveform systematics

Flexible Theory Independent (FTI) method

A. K. Mehta, et. al. Phys. Rev. D 107, 044020 (2023)



1. The polarizations $h_+, h_\times$ decomposed in spin-weighted spherical harmonics

$$h_+ - ih_\times = \sum_{l\geq 2} \sum_{m=-l}^{l} -2Y_{l,m} h_{lm}$$

During the inspiral,
in Stationary Phase Approximation (SPA)
each mode can be written as

$$\tilde{h}_{lm}(f) = A_{lm}(f) e^{-i\psi_{lm}^{GR_{SPA}}(f)}$$

for each mode holds $\psi_{lm}^{GR_{SPA}}(\frac{mf}{2}) \sim \frac{m}{2}\psi_{22}(f)$

2. GR phase in PN theory

$$\psi_{lm}^{GR_{SPA}}(f) \sim \frac{1}{v^5}\frac{m}{2}\left[\sum_{n=0}^{7} \phi_n^{PN} v^n + \sum_{n=5}^{6} \phi_{n(log)}^{PN} v^n \log v\right]$$

$$v = (GM\omega/c^3)^{1/3}, \quad \omega = 2\pi f/m$$

3. we add a generic deviation to the GR phase

PN deviation parameters to infer

$$\delta\psi_{lm}(f) \sim \frac{1}{v^5}\frac{m}{2}\left[\sum_{n=-2}^{7} \delta\phi_n^{PN} v^n + \delta k_s \phi_{4,ks}^{PN} v^4 + \delta k_s \phi_{6,ks}^{PN} v^6 + \sum_{n=5}^{6} \delta\phi_{n(log)}^{PN} v^n \log v\right]$$

signal with GR deviation $\tilde{h}_{lm}(f) = A_{lm}(f) e^{-i(\psi_{lm}^{GR_{SPA}} + \delta\psi_{lm})}$

Flexible Theory Independent (FTI) method

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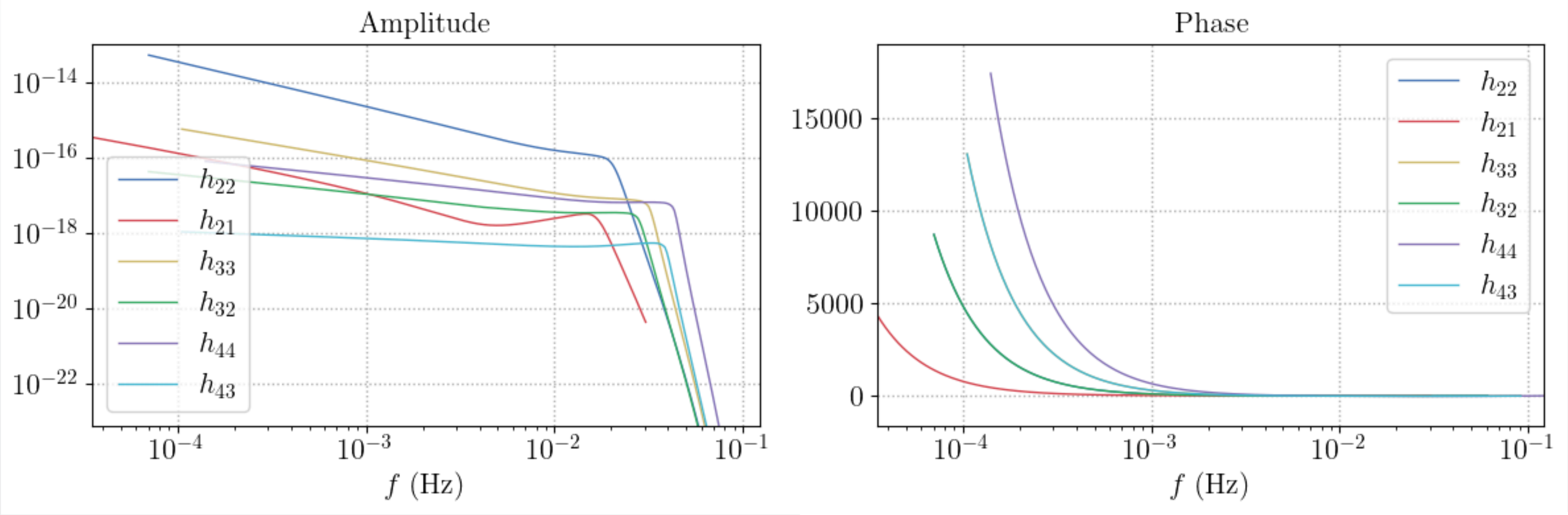
example with GR deviation, $\delta\phi_3 = 0.3$:
$m_1 = 10^6 M_\odot$, q = 1.4, $\chi_1 = 0.5$, $\chi_2 = 0.2$



We smoothly attach the non-GR waveform to the GR one, near merger with a window function W

GR merger

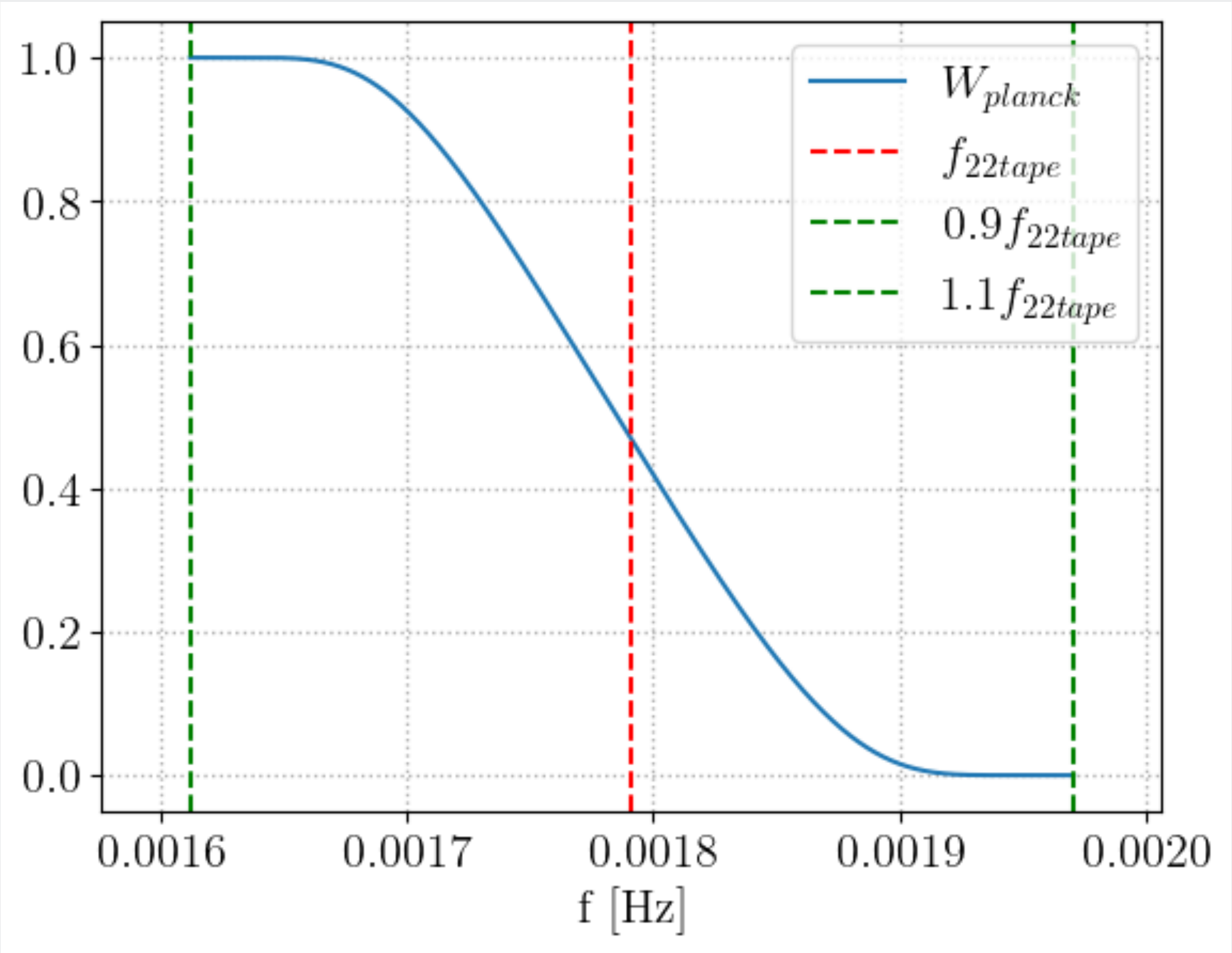
Why?

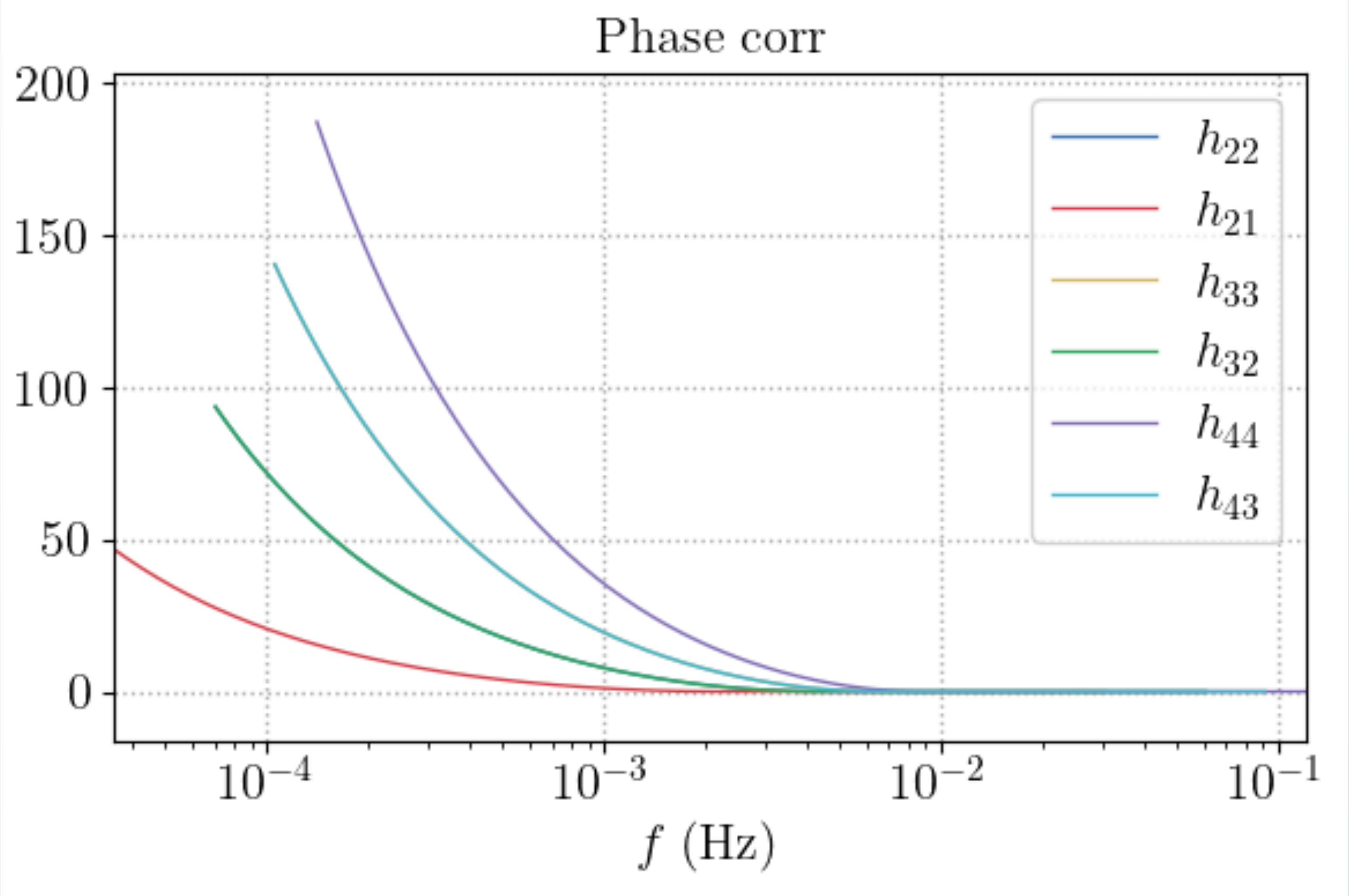
- no analytical solution for the merger
- PN framework holds during the inspiral

Where?

$$f_{22tape} = \alpha f_{22Peak}\ ,\ \alpha = 0.35$$

the choice of the tapering frequency and window's width is purely phenomenological



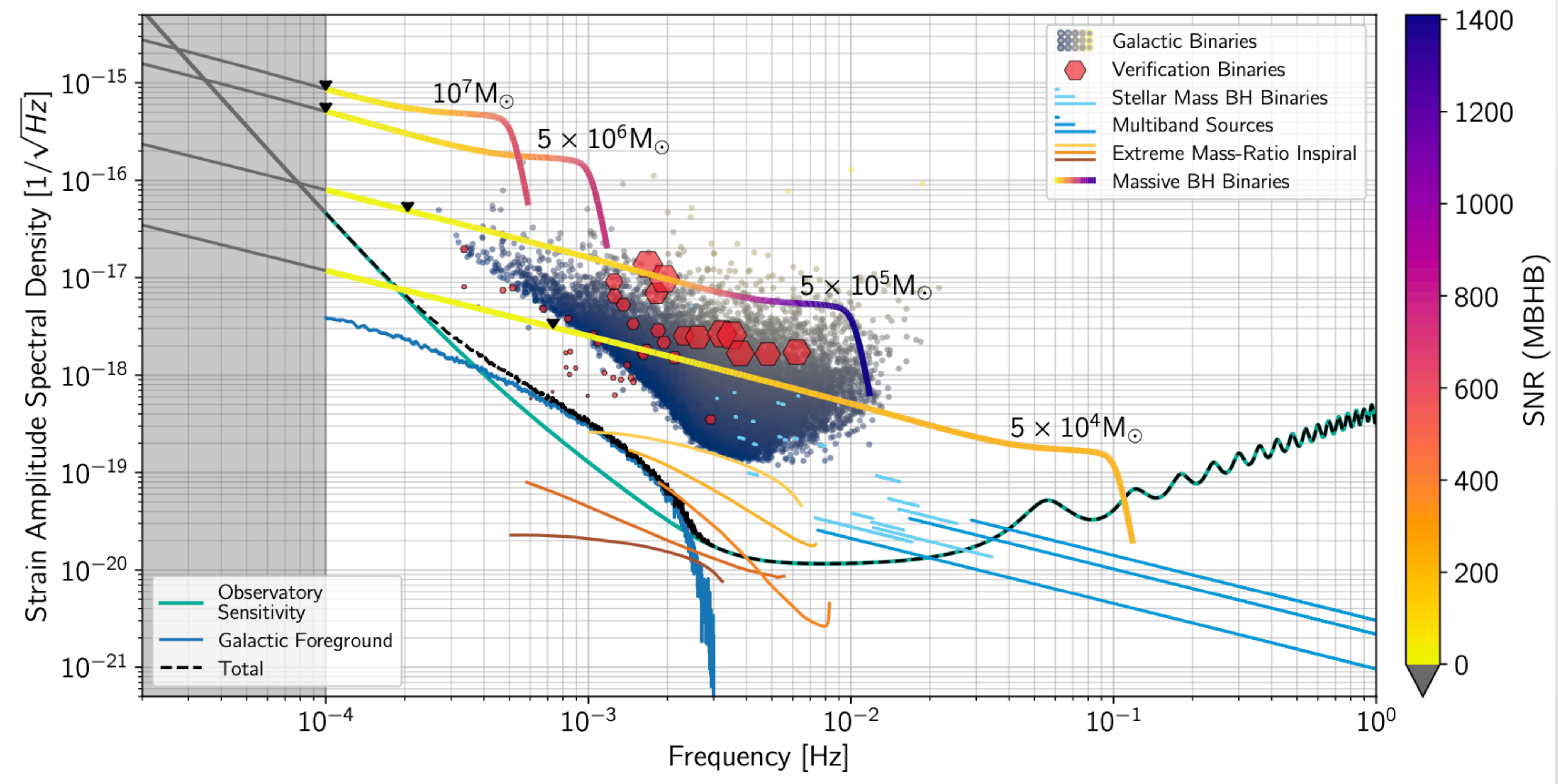


We estimate parameter uncertainties with Fisher information matrix. For measurements with additive Gaussian noise, the Fisher information matrix is written as

$$F_{ij} = (\partial_i h \,|\, \partial_j h)$$

derivative respect component i of the vector param. θ

The inverse of the Fisher matrix $\Sigma = F^{-1}$ is the Gaussian covariance matrix which gives an estimate of the true parameter uncertainties $\sigma(\delta\phi_i)$. (In the presented analysis we are injecting GR, $\delta\phi_i = 0$)



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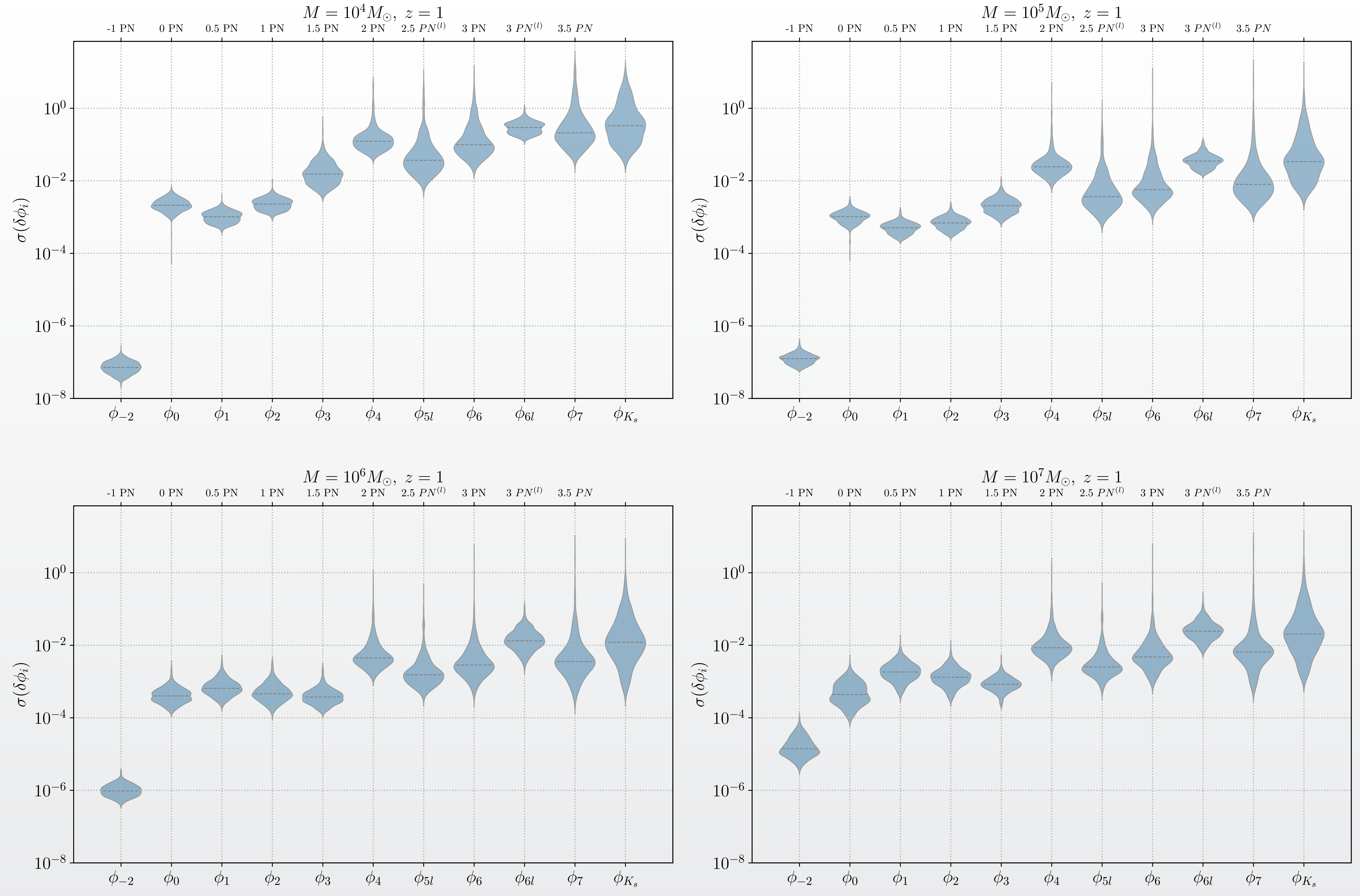
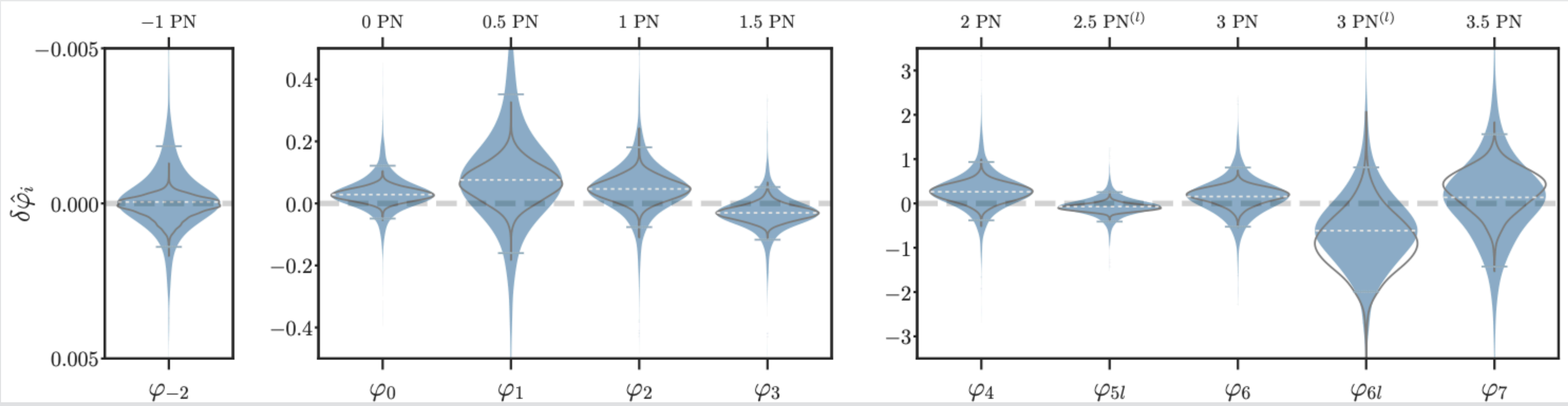
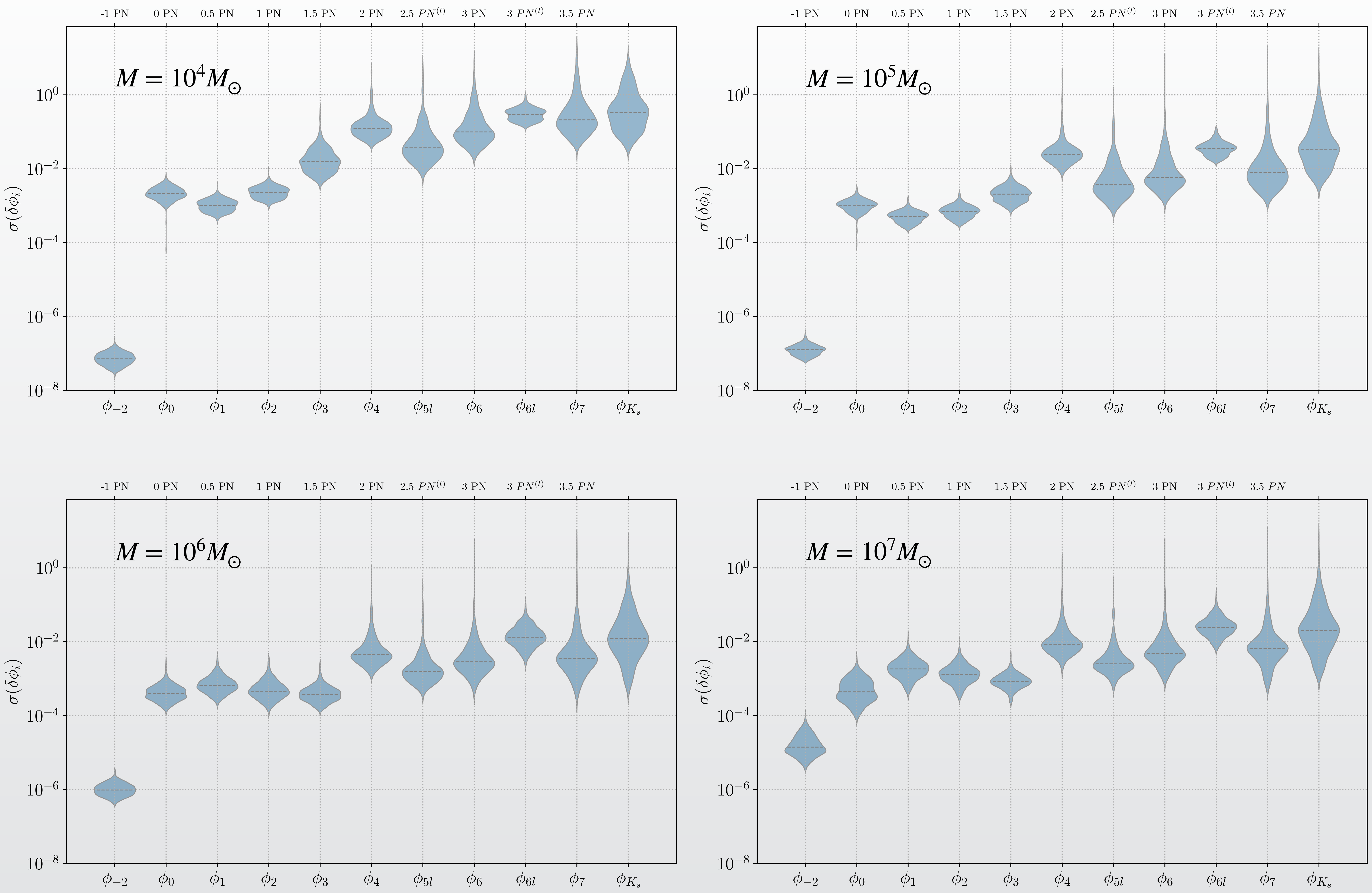


As expected, signals from MBHBs significantly improve the parameter uncertainties $\sigma(\delta\phi_i)$. Our projected constraints decrease by ~ 2 orders of magnitude compared to the last LVK analysis on GWTC-3.



Constraints on the $\delta\phi_i$ parameters in the LVK context. [arxiv:2112.06861]

$z = 1$



Those are constraints for 500 Fisher analysis with fixed primary Mass $[10^4 - 10^5 - 10^6 - 10^7]\ M_\odot$, redshift $z = 1$, and random:

$q \in [0,\ 8]$

$\chi_1, \chi_2 \in [-1,\ 1]$

angles

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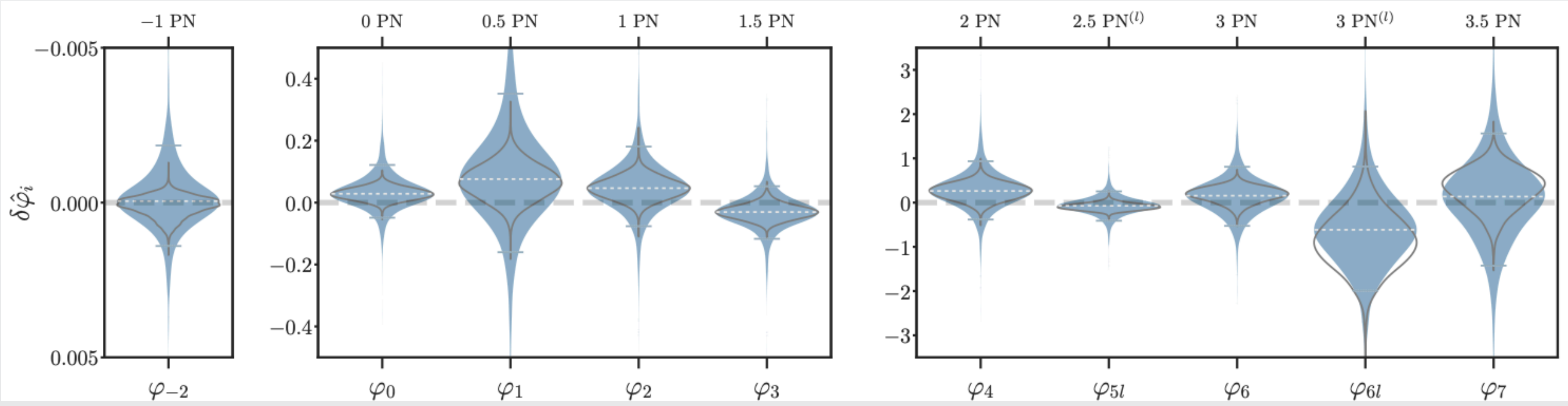

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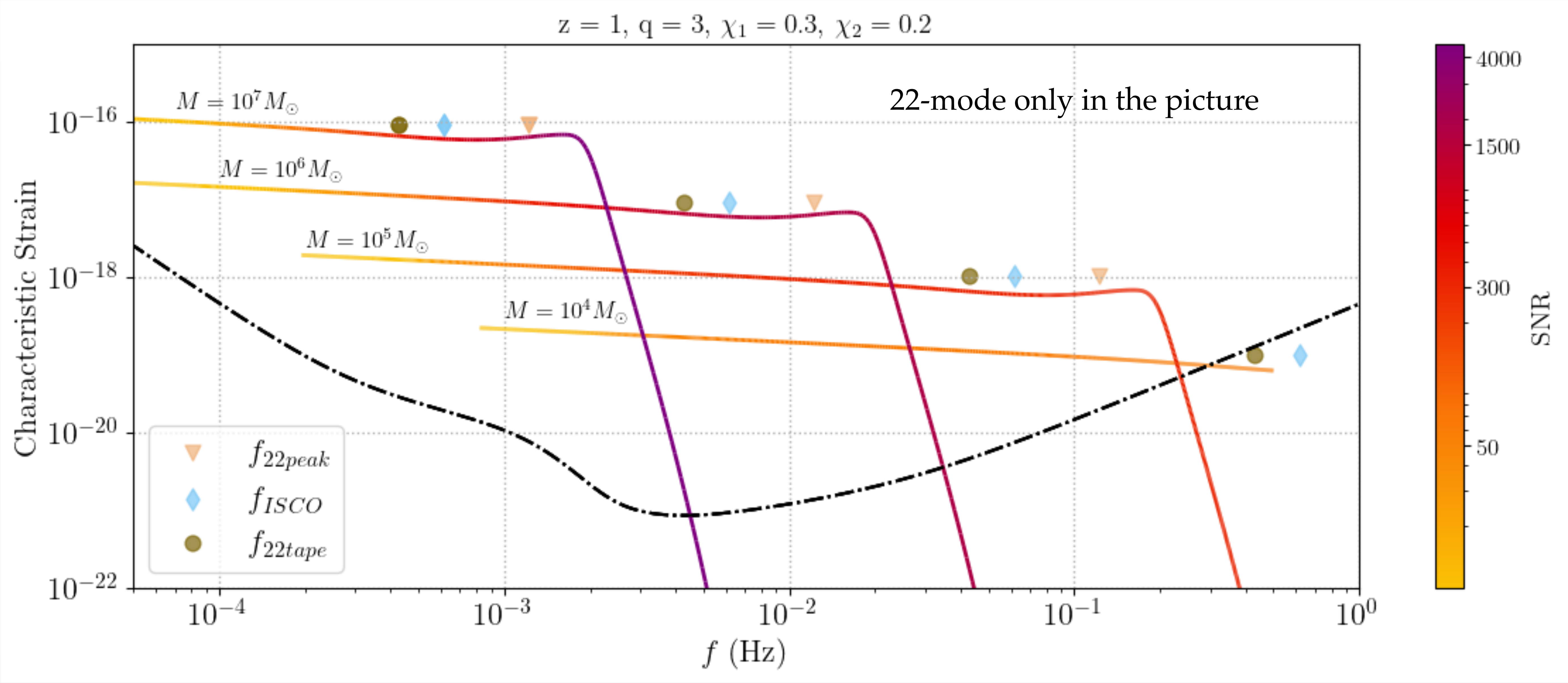
How do our constraints change using only the inspiral?

inspiral-only vs IMR with GR merger analysis



why?

- the SPA approximation and the PN framework do not hold anymore when approaching the merger.
- being an inspiral test, is it consistent to include information driven by the merger-ringdown?
- is it consistent to use a GR merger when searching for GR deviation?

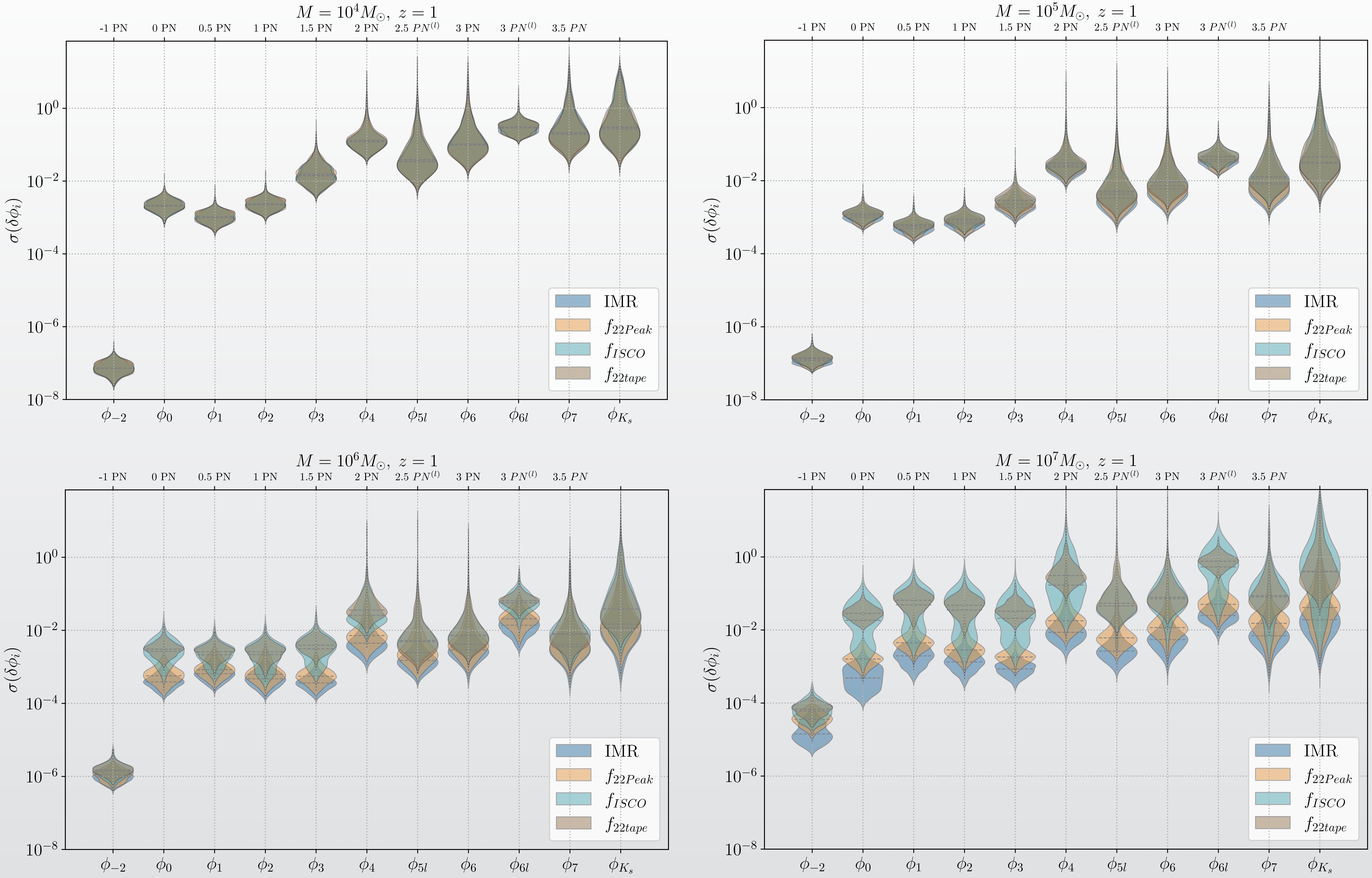
how?

- we cut the data at a certain frequency

this is well-defined compared to setting to zero the signal after a certain point

limitation: during the analysis, we used higher-modes in the signals, but $f_{lmPeak} = \frac{m}{2} f_{22Peak}$, for $m \neq 2$ the cut is not exactly at the mode-Peak

IMR vs cut at: f_{22Peak} - f_{ISCO} - f_{22tape} with $SNR > 10$



what these plots are telling us?

- for **inspiral-dominated** signals, $M \in [10^4,\ 10^5]M_\odot$ there are no appreciable differences between the different analysis
- moving to **merger-dominated** signals $M \in [10^6,\ 10^7]M_\odot$ the information given by the merger-ringdown becomes more and more important for constraining the PN deviation parameters

focusing on IMR vs f_{22Peak} : including also the *ringdown* part of the signal gives you better constraints

injection and Parameter Estimation

signal parameters

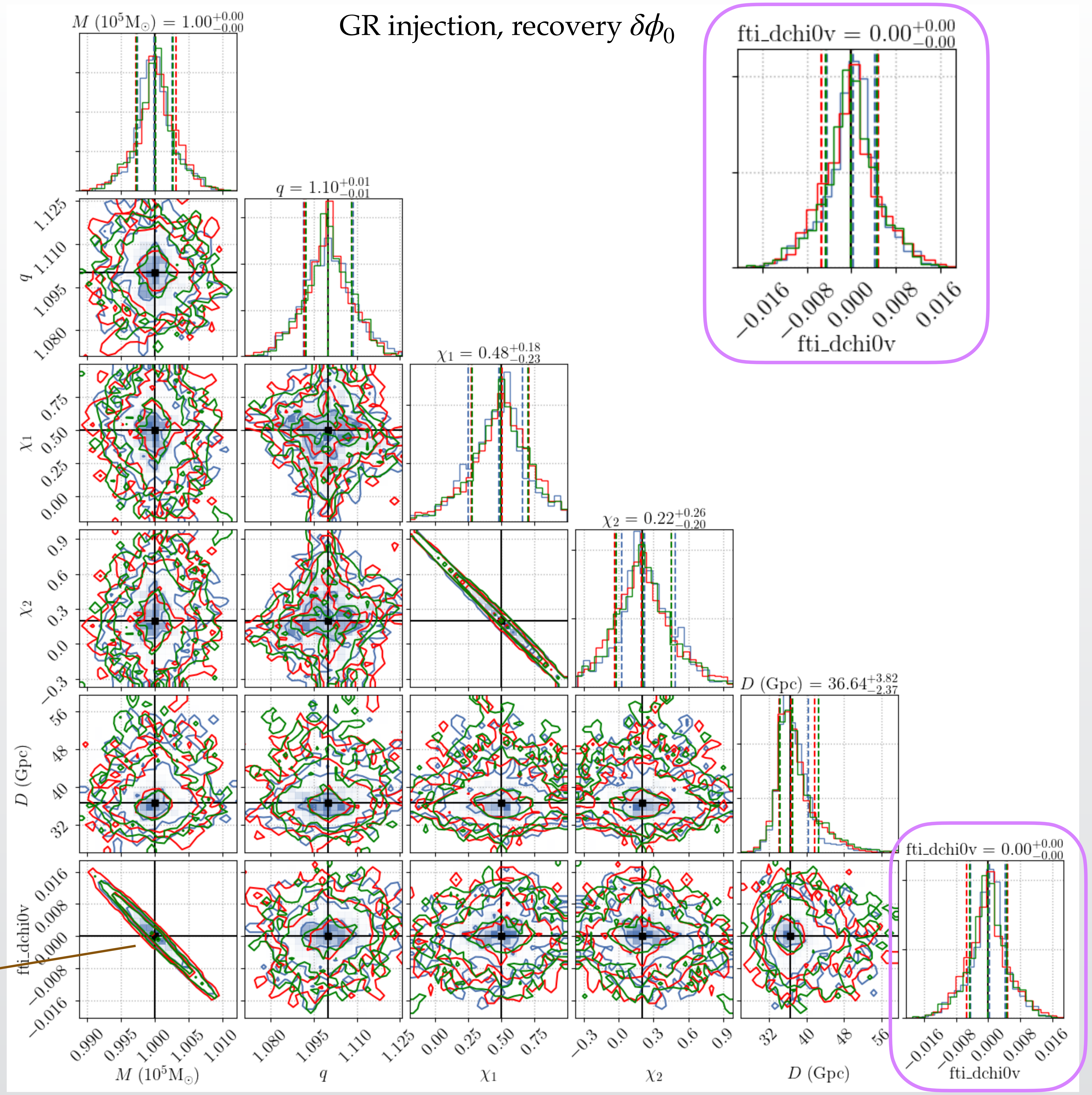
$M = 10^5$ or $10^7\, M_\odot$
$q = 1.1$
$\chi_1 = 0.5, \chi_2 = 0.2$
$z = 1$

SNR = 66 or 1306

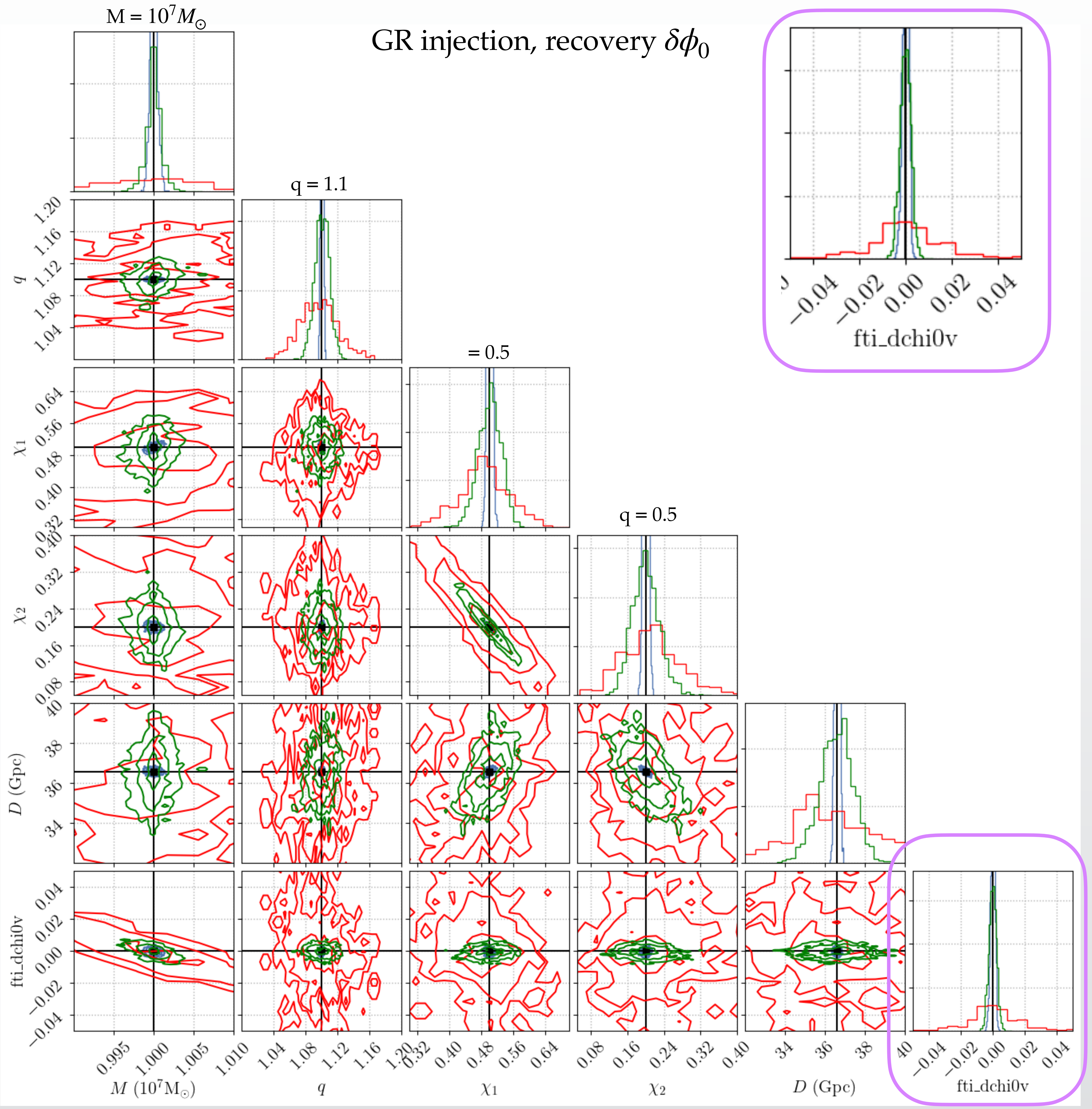
In the analysis, we allow for one deviation parameter $\delta\phi_i$ at the time

$$\psi_{lm}^{0PN}(f) \sim \frac{1+\delta\hat{\phi}_0}{M_c^{5/3}} f^{-5/3}$$

$M = 10^5 M_\odot$, SNR = 66



$M = 10^7 M_\odot$, SNR = 1306



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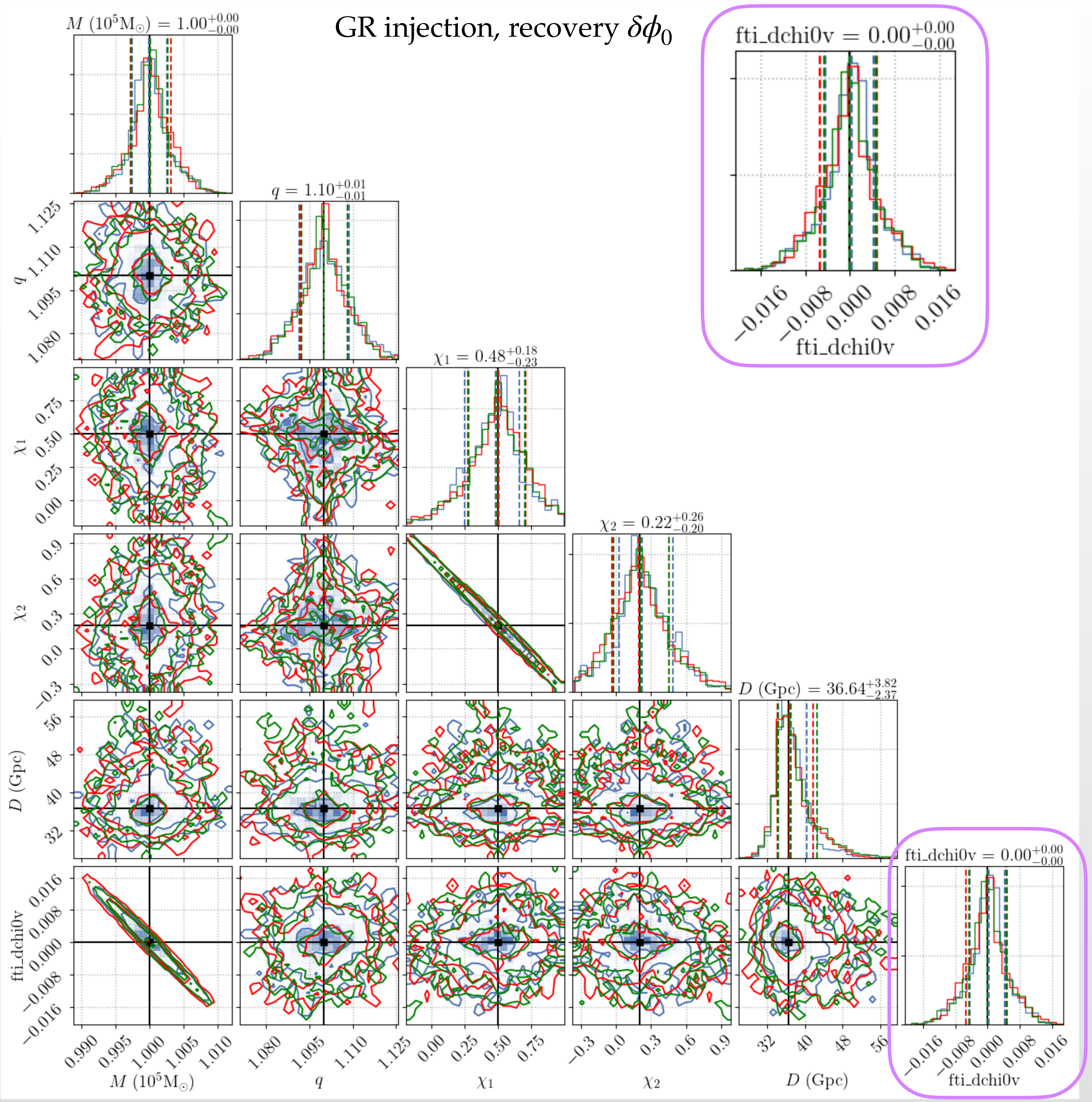

Next steps:

- How well can we constrain each deviation parameter, with Bayesian inference, exploring the parameter space?

- *multi-parameter tests* (more PN deviation parameters simultaneously tested)
  Such tests could be not very effective due to high correlations among themselves.
  Could Principal Component Analysis (PCA) help?

- What's the impact of waveform systematics on this test?
- Could systematics mimic false GR deviation, and how?

Thanks for the attention!

happy to take questions

Flexible Theory Independent (FTI) method

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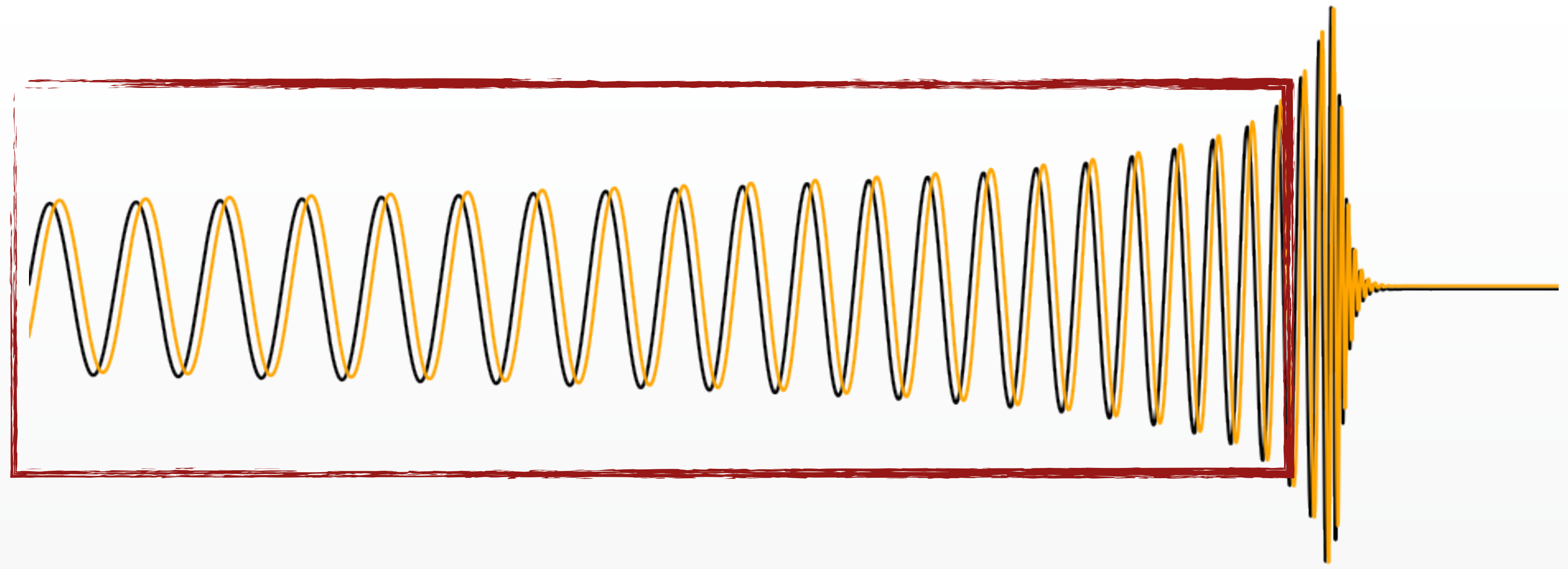

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During the inspiral,
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each mode can be written as

$$\tilde{h}_{lm}(f) = A_{lm}(f) e^{-i\psi_{lm}^{GR_{SPA}}(f)}$$

for each mode holds $\psi_{lm}^{GR_{SPA}}(\frac{mf}{2}) = \frac{m}{2}\psi_{22}(f) + const(m, \Delta\phi_{lm}, \Delta\phi_{22})$

2. GR phase in PN theory

$$\psi_{lm}^{GR_{SPA}}(f) = \frac{3}{128\eta v^5}\frac{m}{2}\left[\sum_{n=0}^{7}\phi_n^{PN} v^n + \sum_{n=5}^{6}\phi_{n(log)}^{PN} v^n \log v\right] + o(v^8)$$

$$v = (GM\omega/c^3)^{1/3}, \quad \omega = 2\pi f/m, \quad \eta = m_1 m_2/M^2$$

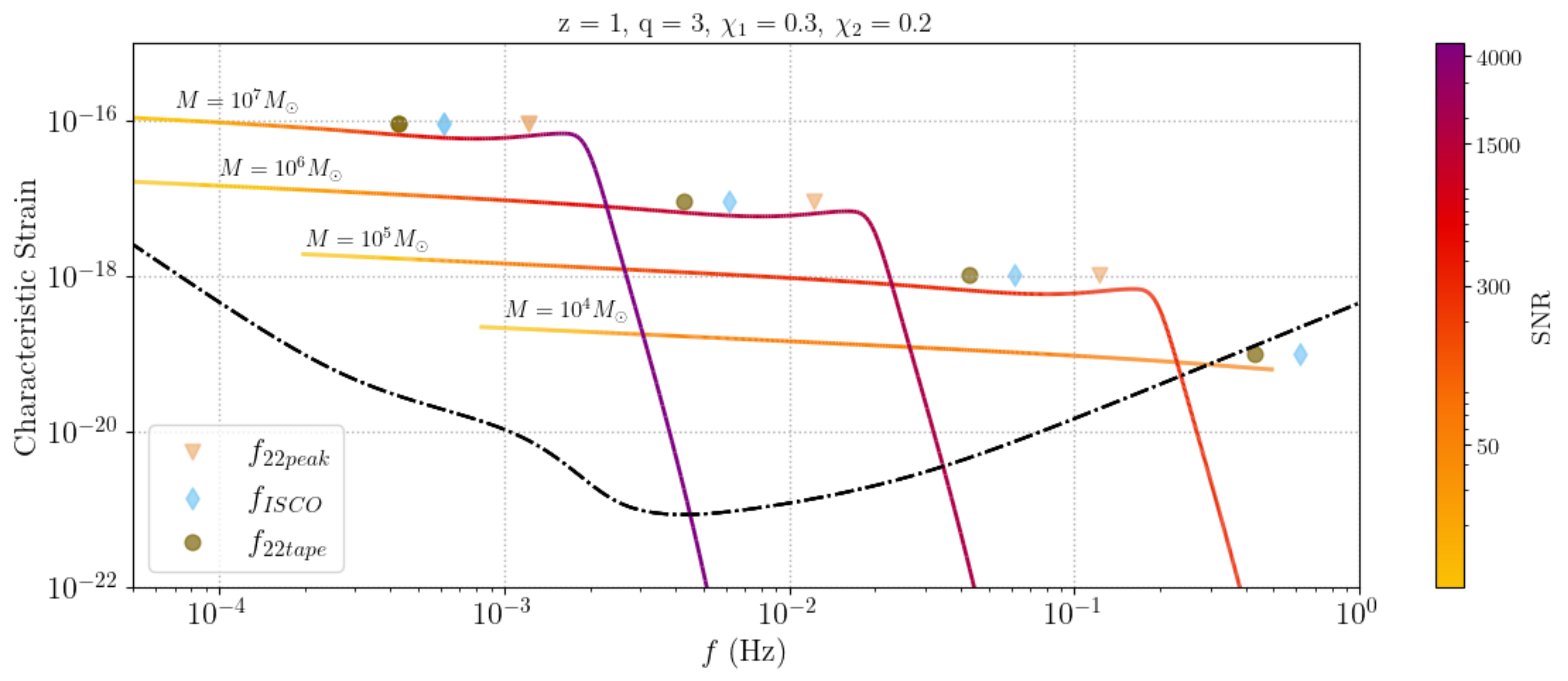
3. we add a generic deviation to the GR phase

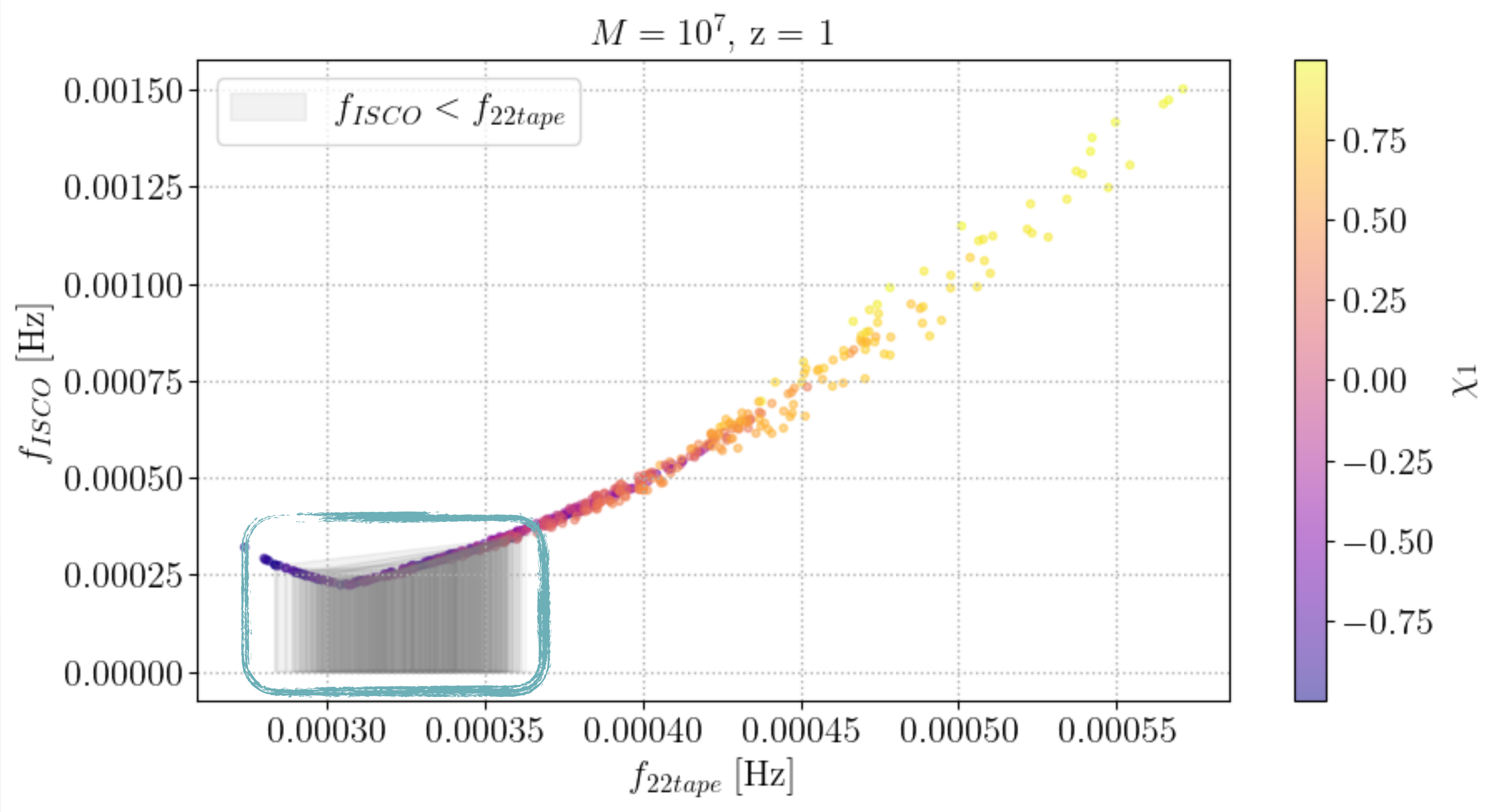
PN deviation parameters to infer

$$\delta\psi_{lm}(f) = \frac{3}{128\eta v^5}\frac{m}{2}\left[\sum_{n=-2}^{7}\delta\phi_n^{PN} v^n + \delta k_s \phi_{4,ks}^{PN} v^4 + \delta k_s \phi_{6,ks}^{PN} v^6 + \sum_{n=5}^{6}\delta\phi_{n(log)}^{PN} v^n \log v\right] + O(v^8)$$

signal with GR deviation $\tilde{h}_{lm}(f) = A_{lm}(f) e^{-i(\psi_{lm}^{GR_{SPA}} + \delta\psi_{lm})}$

Why for analyses with cut at f_{ISCO} larger distributions appear?



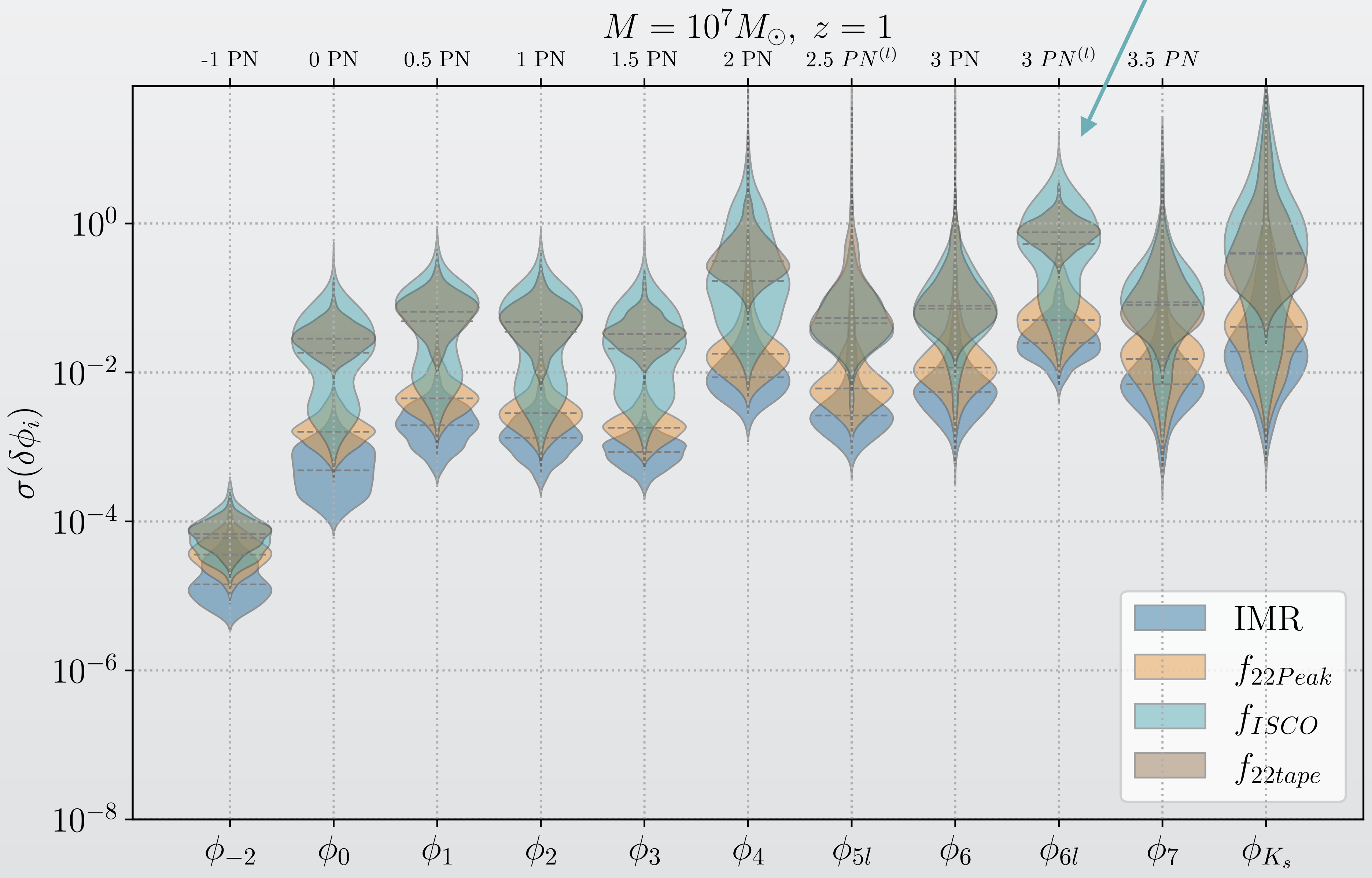


There is part of the parameter space where $f_{ISCO} < f_{22tape}$

$\Downarrow$

2 different behaviors in the constraints for merger-dominated signals cutting at ISCO

as expected signals with $f_{ISCO} < f_{22tape}$ gives worst constraints



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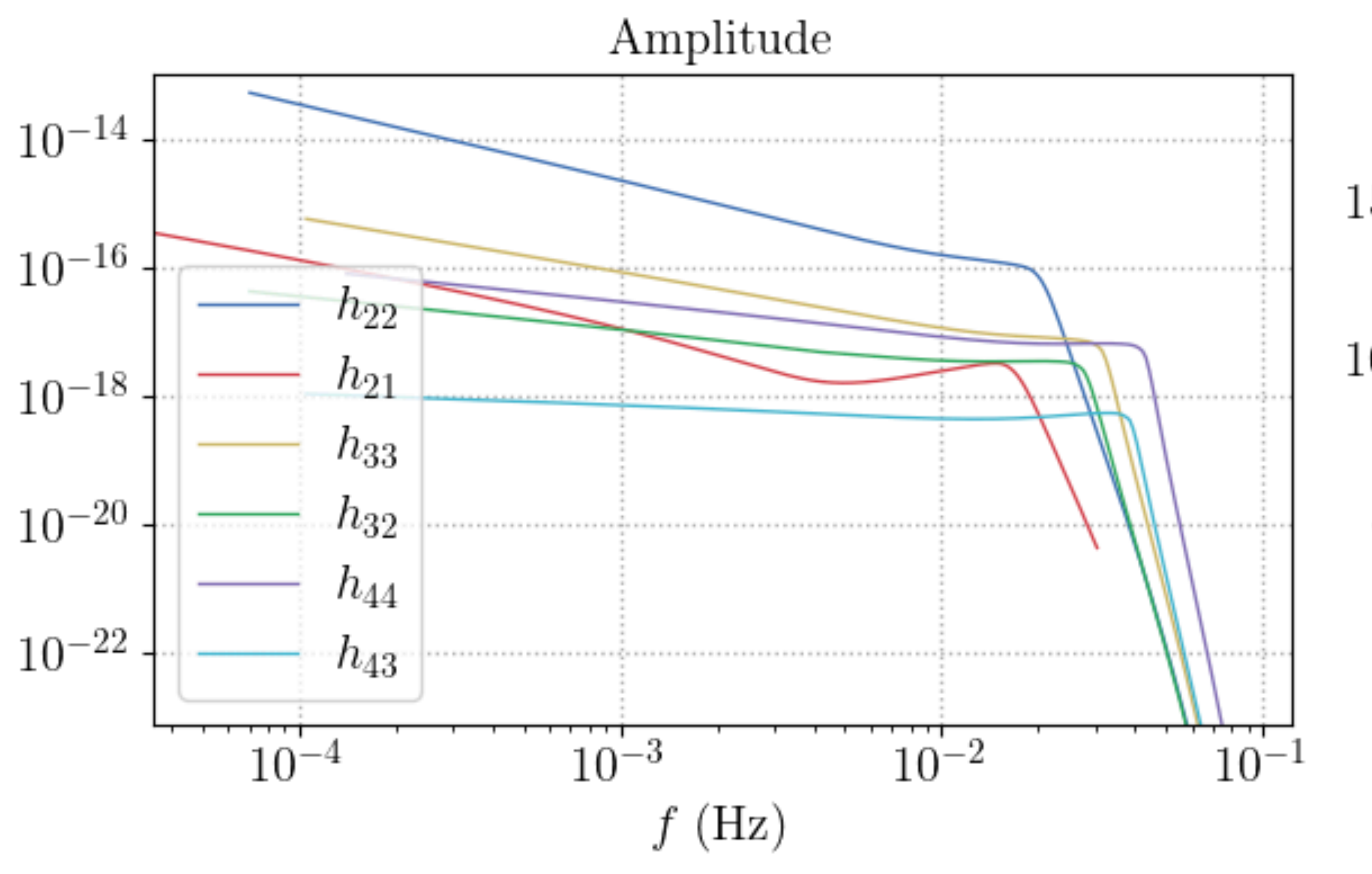
example with GR deviation, $\delta\phi_3 = 0.3$:

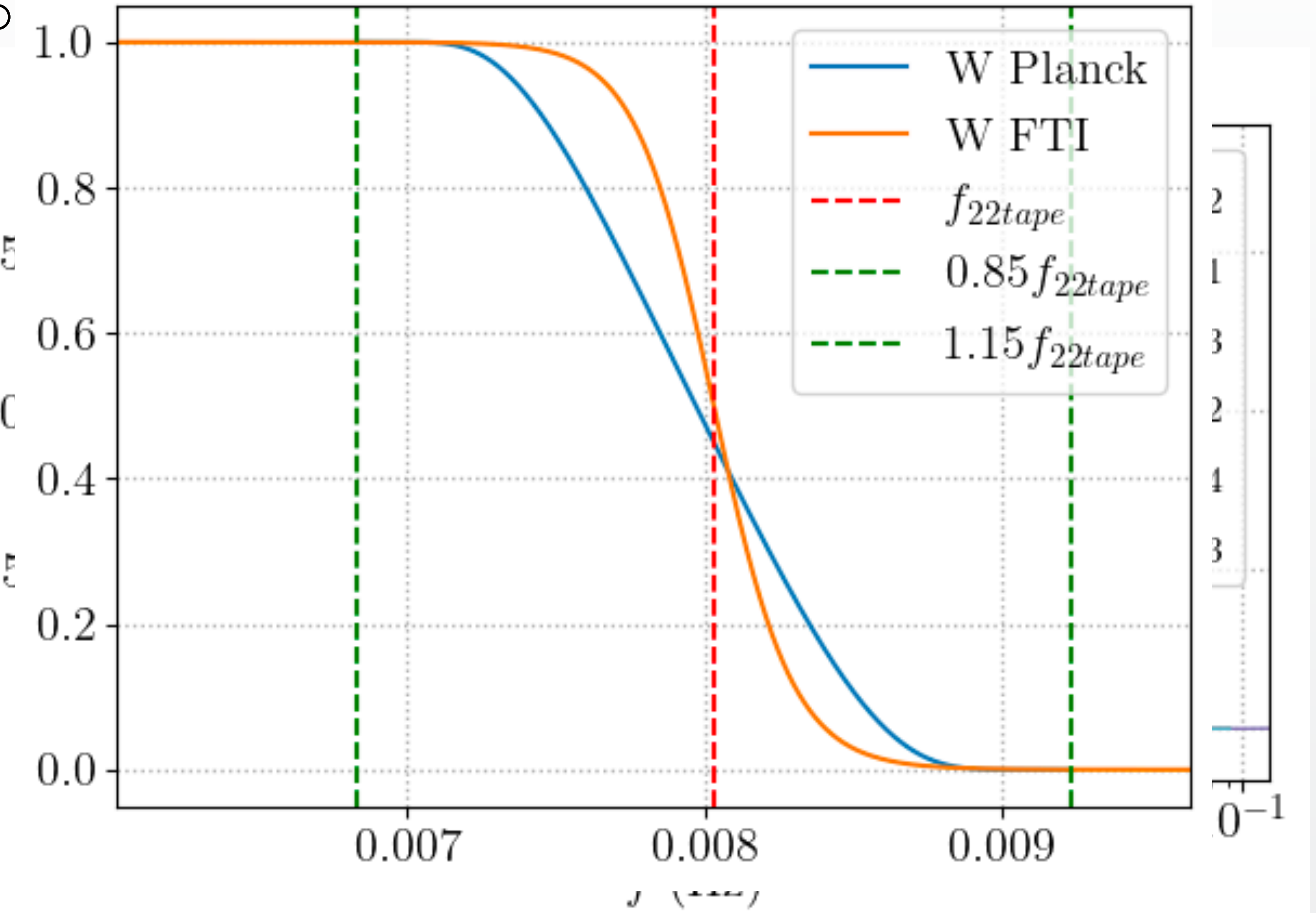
$m_1 = 10^6 M_\odot$

$$\delta\psi_{\ell m} = \int_{f^{ref}_{\ell m}}^{f} df' \int_{f^{peak}_{22}}^{f'} df'' \delta\psi''_{\ell m} \times W(f''\nu^{tape},$$

tapering function, to attach smoothly the
with deviations to the GR wf, before mer

GR merger



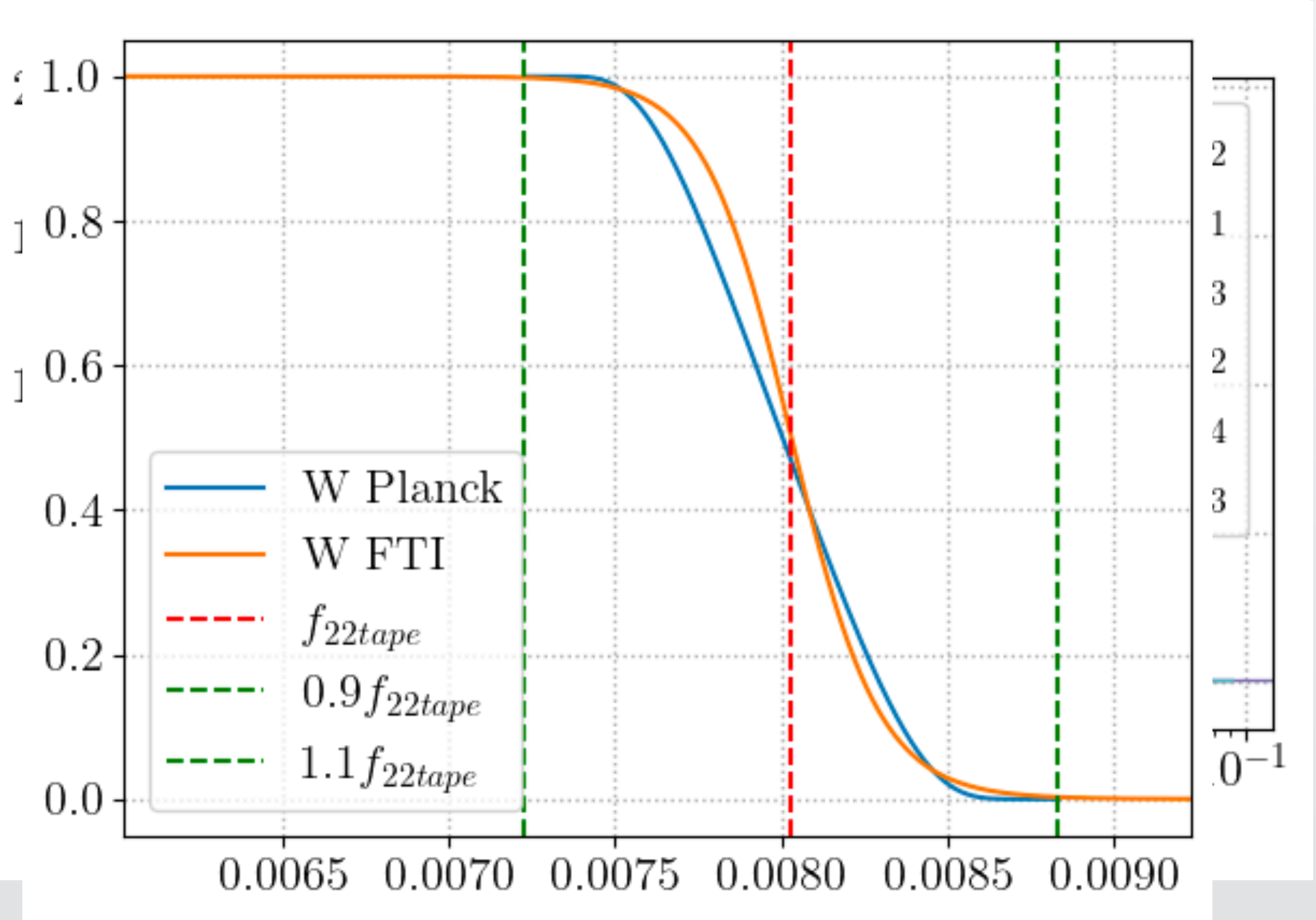


Due to the long inspiral of MBHBs, integrating twice on a fine frequency grid could be computationally expensive

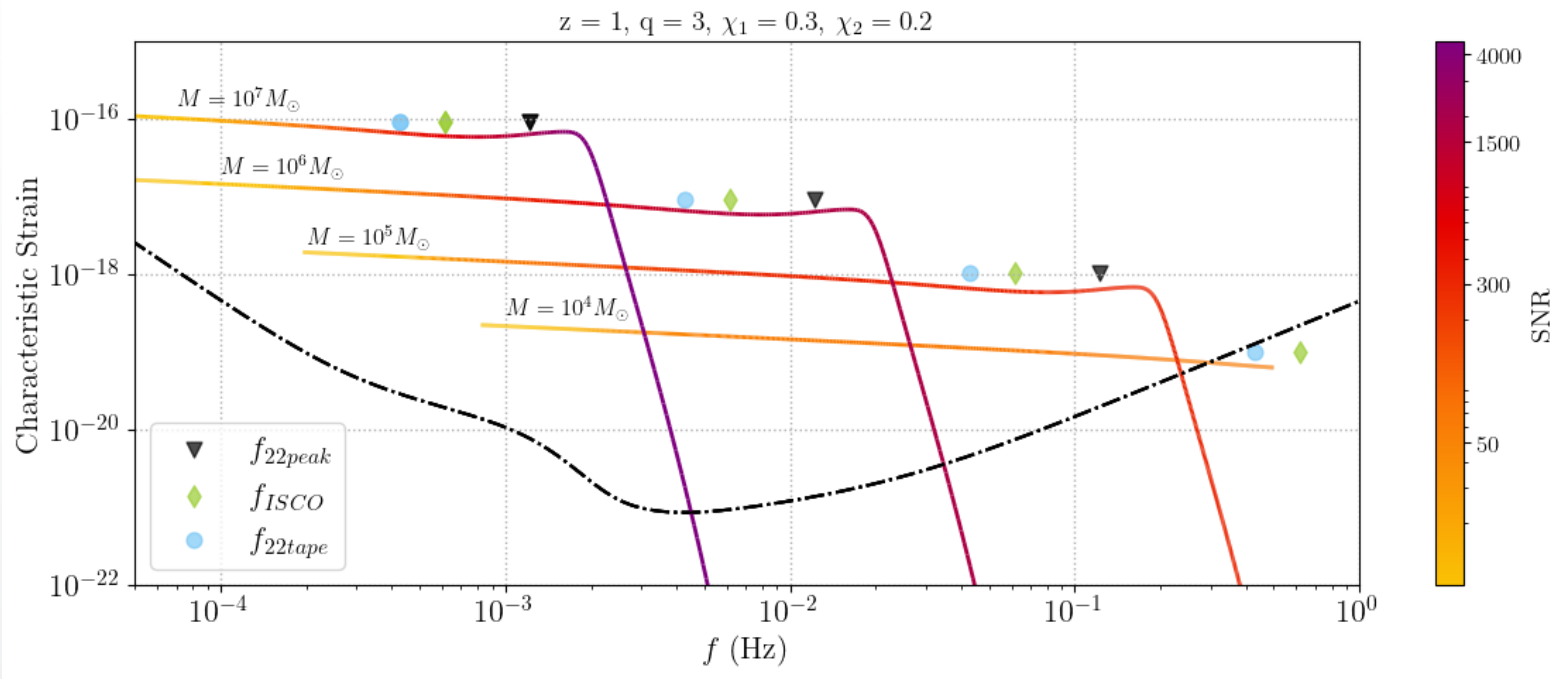
we split the frequency grid into 3 parts to void this problem:

- f < $(1-\beta)\, f_{tape}$: $\quad \delta\psi_{\ell m} = \delta\psi^{analytic}_{\ell m}$
- $(1-\beta)\, f_{tape} < f < (1+\beta)\, f_{tape}$: $\quad \delta\psi_{\ell m} = \delta\psi^{integr}_{\ell m}$
- $f > (1+\beta)\, f_{tape}$: $\quad \delta\psi_{\ell m} = 0$

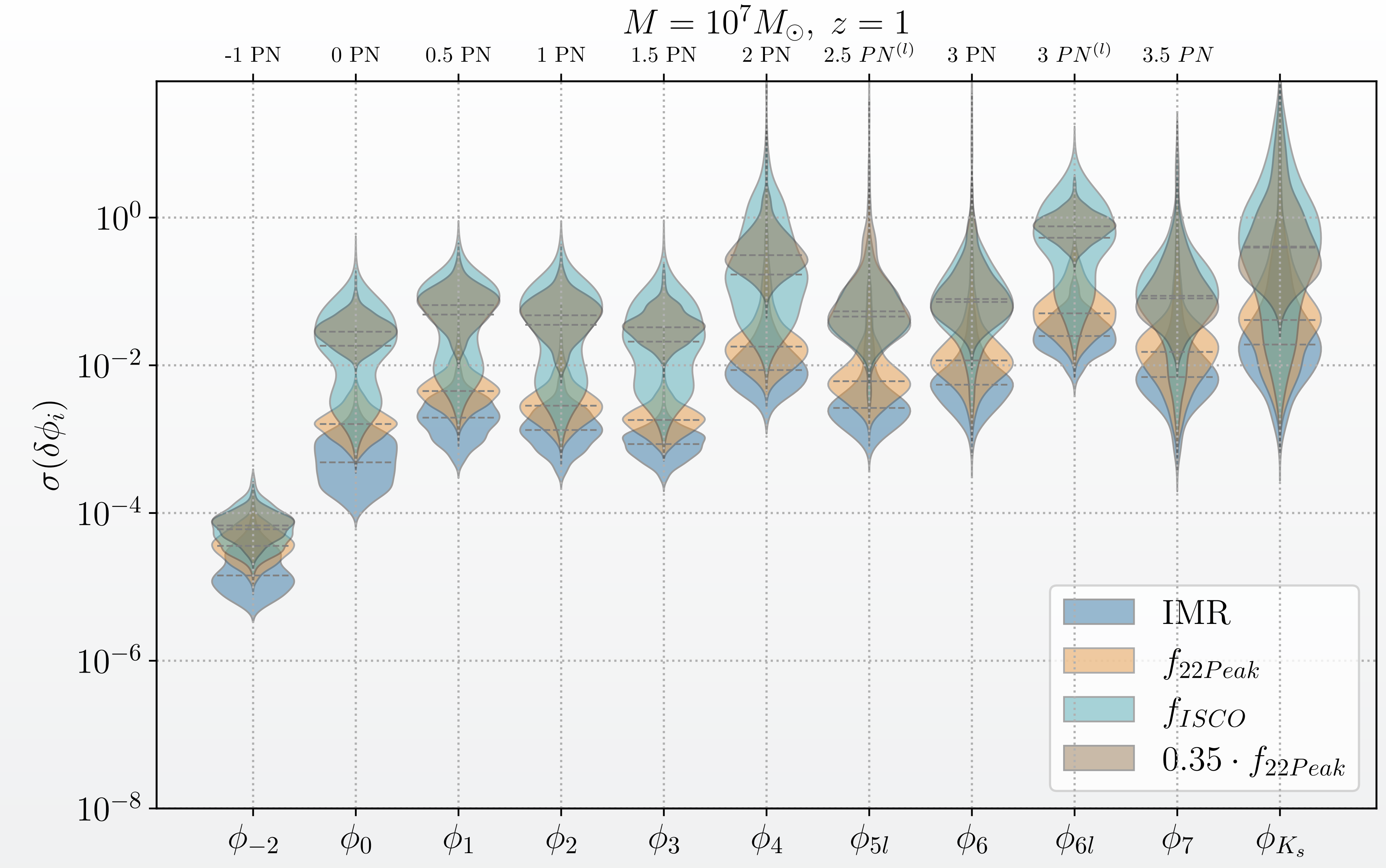
$f_{22tape} = \alpha f_{22Peak}$, $\alpha = 0.35$



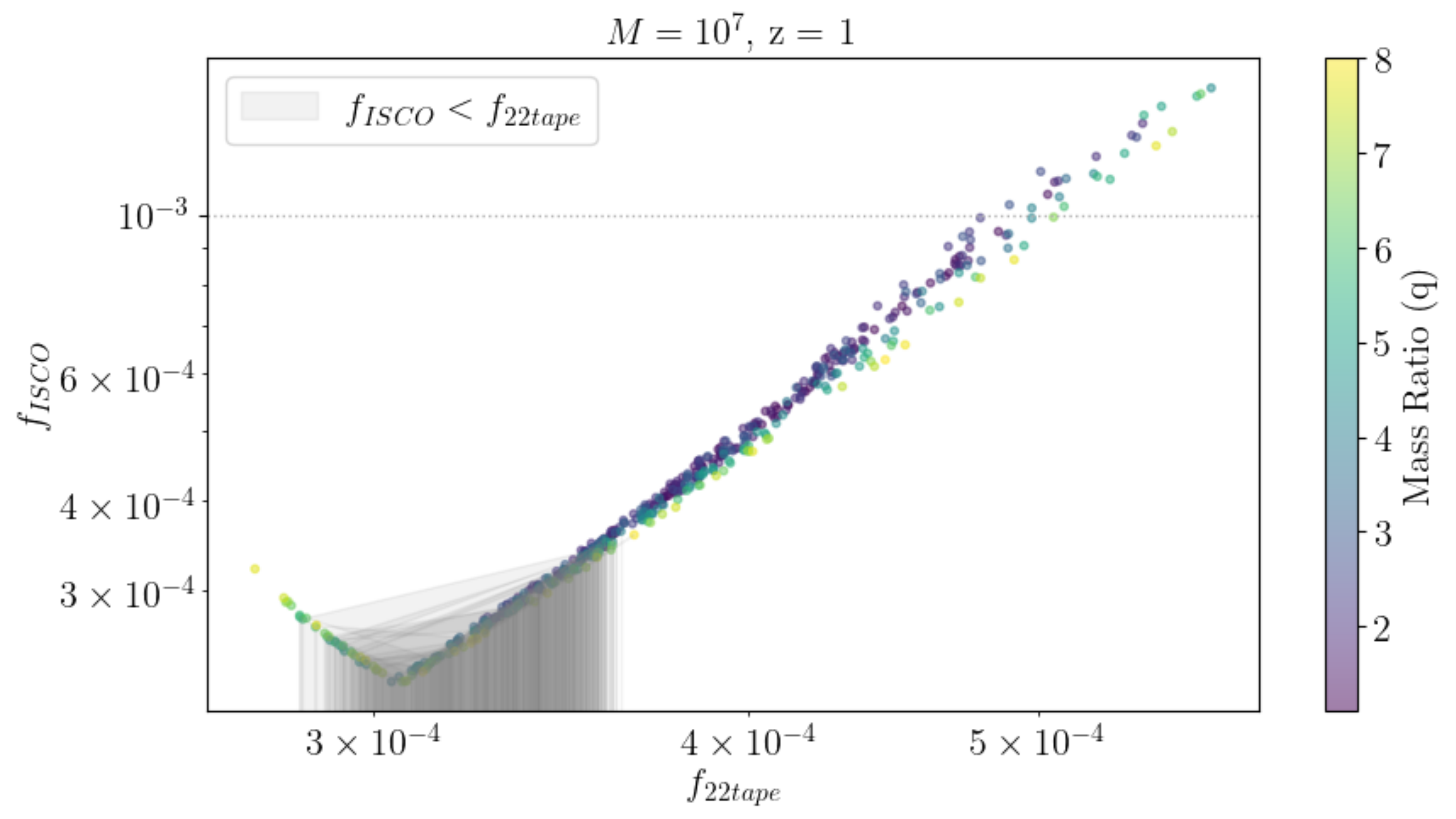
For now, the choice of the window's width is purely phenomenological



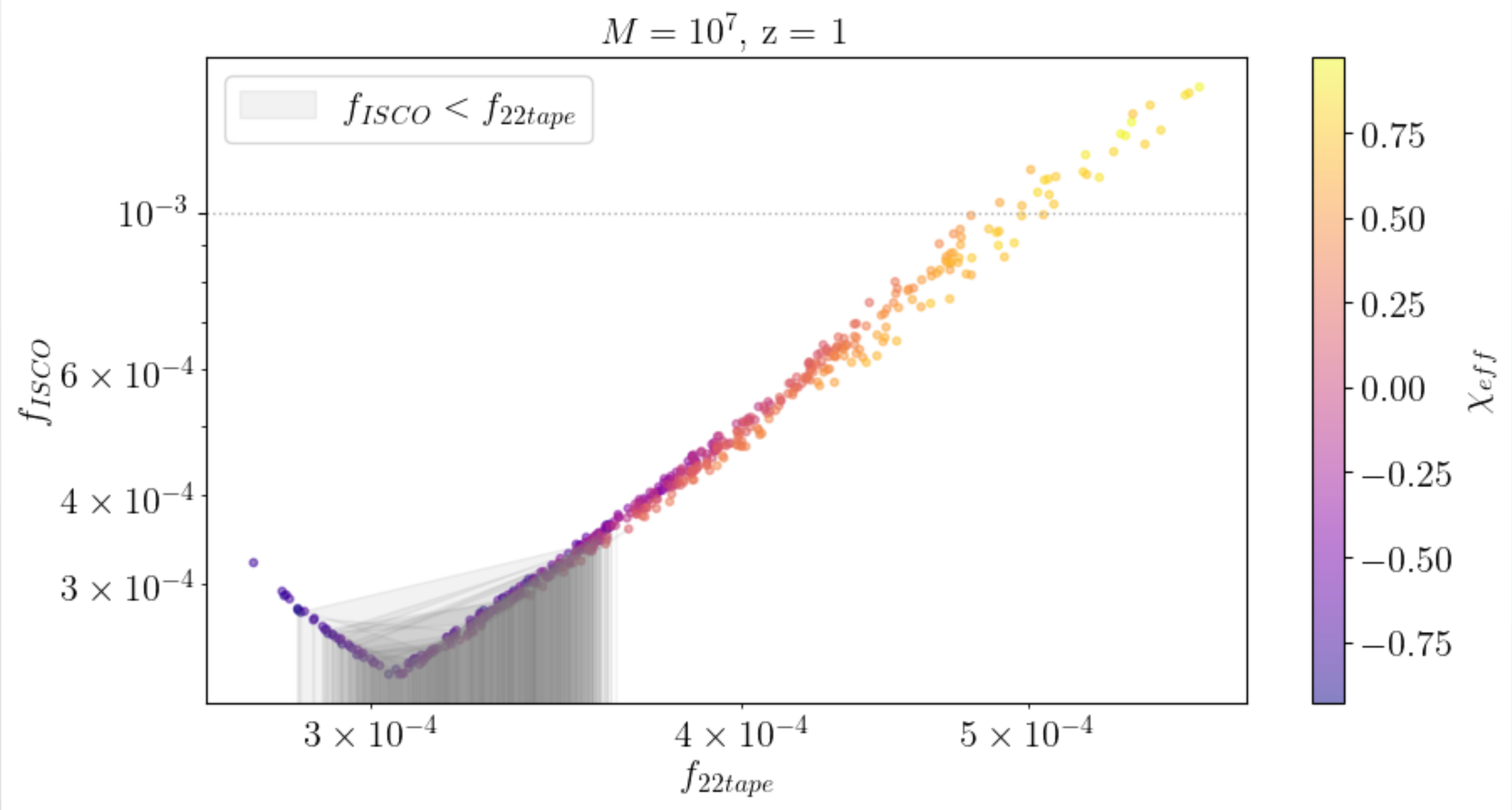
z = 1, q = 3, χ_1 = 0.3, χ_2 = 0.2
$M = 10^7 M_\odot$
$M = 10^6 M_\odot$
$M = 10^5 M_\odot$
$M = 10^4 M_\odot$
Characteristic Strain
f (Hz)
SNR
f_{22peak}
f_{ISCO}
f_{22tape}



$M = 10^7 M_\odot,\ z = 1$
-1 PN
0 PN
0.5 PN
1 PN
1.5 PN
2 PN
2.5 $PN^{(l)}$
3 PN
3 $PN^{(l)}$
3.5 PN
$\sigma(\delta\phi_i)$
IMR
f_{22Peak}
f_{ISCO}
$0.35 \cdot f_{22Peak}$
ϕ_{-2} ϕ_0 ϕ_1 ϕ_2 ϕ_3 ϕ_4 ϕ_{5l} ϕ_6 ϕ_{6l} ϕ_7 ϕ_{K_s}

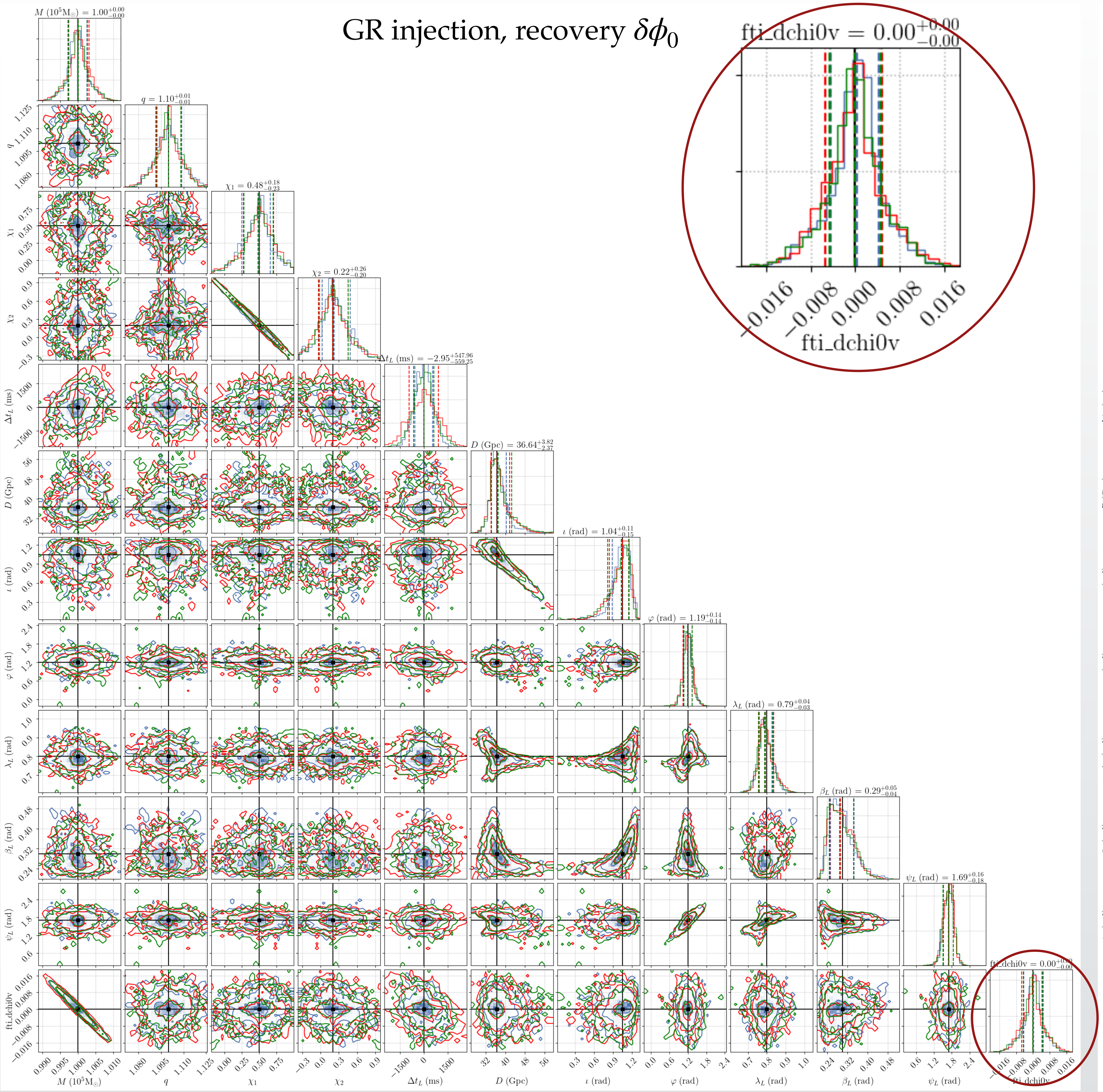


$M = 10^7$, z = 1
$f_{ISCO} < f_{22tape}$
f_{ISCO}
f_{22tape}
Mass Ratio (q)

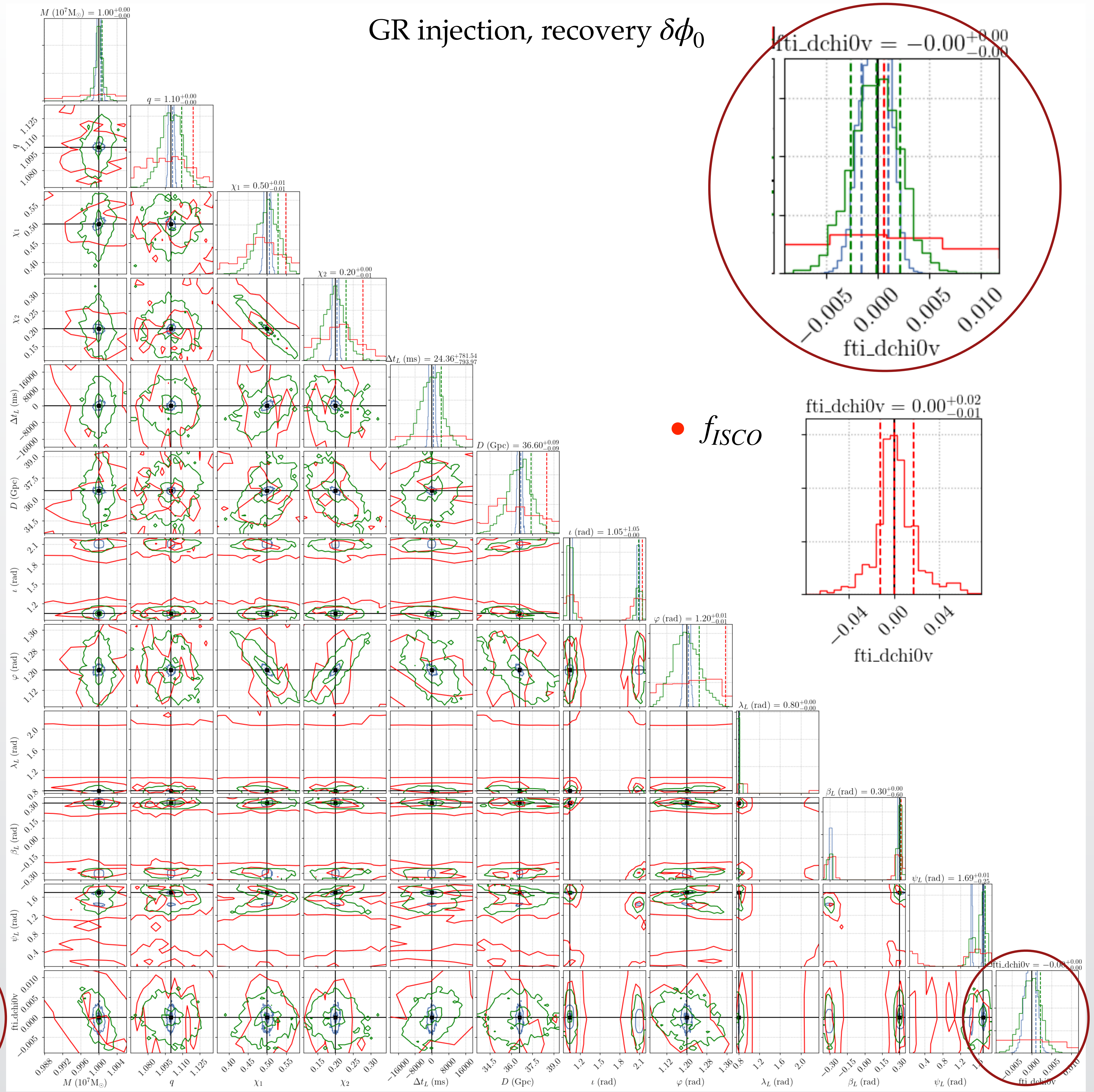


$M = 10^7$, z = 1
$f_{ISCO} < f_{22tape}$
f_{ISCO}
f_{22tape}
χ_{eff}

M = $10^5 M_\odot$, SNR = 66



M = $10^7 M_\odot$, SNR = 1306



SNR distribution