Non-linear gravitational waves in Horndeski gravity

Hugo Roussille, Jibril Ben Achour, Mohammad Ali Gorji

October 14^{th}, 2024

2401.05099
2402.01487

Huitième Assemblée Générale du GdR Ondes Gravitationnelles

Lab PHYS ENS de LYON

ENS DE LYON

Introduction

- Non-linear gravitational wave solutions known in GR (pp-waves, Kundt, Robinson-Trautman) [Robinson, Trautman '60; Ehlers, Kundt '62]
- Not directly useful for comparison to observations but crucial to explore non-linear radiative regime of a theory (ex: non-linear memory effects [Christodoulou '91])
- This work: investigate phenomenology of scalar-tensor mixing on non-linear gravitational waves in modified gravity
- Use an exact solution of a Horndeski theory as a toy model
- Take advantage of disformal transformations as solution-generating techniques

Scalar-tensor theories

Motivation: add a scalar degree of freedom ϕ to GR to effectively describe deviations at a given energy scale

- Simplest example: Einstein-scalar system (GR + minimally coupled scalar)
- Brans-Dicke: gravitational constant upgraded to a scalar field [Brans, Dicke '61]
- Horndeski: escape unicity theorems by introducing higher derivatives [Horndeski '74; Charmousis, Copeland+ '12]

Horndeski $=$ GR $\times$ Coupling $+$ Orders 0 and 1 in $\nabla\nabla\phi$ $+$ $(\nabla\nabla\phi)^2$

Disformal transformations of scalar-tensor theories

Disformal transformations

Generalization of conformal transformations [Bekenstein '93]

$$\tilde{g}_{\mu\nu} = A(\phi, X) g_{\mu\nu} + B(\phi, X)\phi_\mu \phi_\nu \qquad (X = \phi_\mu \phi^\mu)$$

- In vacuum: field redefinition $S[g_{\mu\nu}, \phi] \iff \tilde{S}[\tilde{g}_{\mu\nu}, \phi]$
- If one adds matter following geodesics:

$$S[g_{\mu\nu}, \phi] + \int \sqrt{-g_{\mu\nu}\, \mathrm{d}x^\mu\, \mathrm{d}x^\nu} \;\not\Longleftrightarrow\; \tilde{S}[\tilde{g}_{\mu\nu}, \phi] + \int \sqrt{-\tilde{g}_{\mu\nu}\, \mathrm{d}x^\mu\, \mathrm{d}x^\nu}$$

$\longrightarrow$ disformal transformations describe new physics when one probes spacetime through matter observables (ex: strain in a GW detector)

- Horndeski theories: stable under $A(\phi)$, $B(\phi)$

Solution-generating technique

Canonical term for scalar

$$S = \int \mathrm{d}^4x\,\sqrt{-g}\,(R - \frac{1}{2}X)$$

Exact solution $g_{\mu\nu}$

→ Disformal transformation →

Higher-order scalar-tensor theory (Horndeski, DHOST)

Exact solution $\tilde{g}_{\mu\nu}$

$\longrightarrow$ use disformal transformations as a solution-generating technique [Anson, Babichev+ ’21; Ben Achour, Liu+ ’20; Ben Achour, Liu+ ’20; Faraoni, Leblanc ’21]

- Simplest transformation: $\tilde{g}_{\mu\nu} = g_{\mu\nu} + B_0\phi_\mu\phi_\nu$
- Obtain a solution of Horndeski with $\tilde{F}_2(\tilde{X}) = \dfrac{1}{\sqrt{1 - B_0\tilde{X}}}$

Non-linear GW in GR

Usual linearised setup

- Add a perturbation to a background: $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$
- Obtain a wave propagation equation $\bar{\Box} h_{\mu\nu} = 0$

$\rightarrow$ sufficient for most cases but missing complex dynamics of the theory: non-linear memory effects [Christodoulou '91], colliding waves [Szekeres '70], solitons...

Simplest non-linear example: pp-wave

[Brinkmann '25; Bondi '57; Robinson, Trautman '60; Penrose '65]

$$\mathrm{d}s^2 = -H_{ab} x^a x^b \,\mathrm{d}u^2 + 2\,\mathrm{d}u\,\mathrm{d}v + \delta_{ab}\,\mathrm{d}x^a\,\mathrm{d}x^b$$

- describes propagation of a plane wave in vacuum along ∂_v
- property close to linearised waves: profiles H_{ab} can be added
- can define polarisations through components of H_{ab}

Metric element

Solution of Einstein-scalar theory [Tahamtan, Svitek '15; Tahamtan, Svitek '16]

$$S = \int \mathrm{d}^4x \sqrt{-g} \left(R - \frac{1}{2} \nabla_\mu \phi \nabla^\mu \phi \right)$$

$$\mathrm{d}s^2 = -K(x,y)\,\mathrm{d}w^2 - 2\,\mathrm{d}w\,\mathrm{d}\rho + \frac{\rho^2 - \chi(w)^2}{P(x,y)^2}(\mathrm{d}x^2 + \mathrm{d}y^2)$$

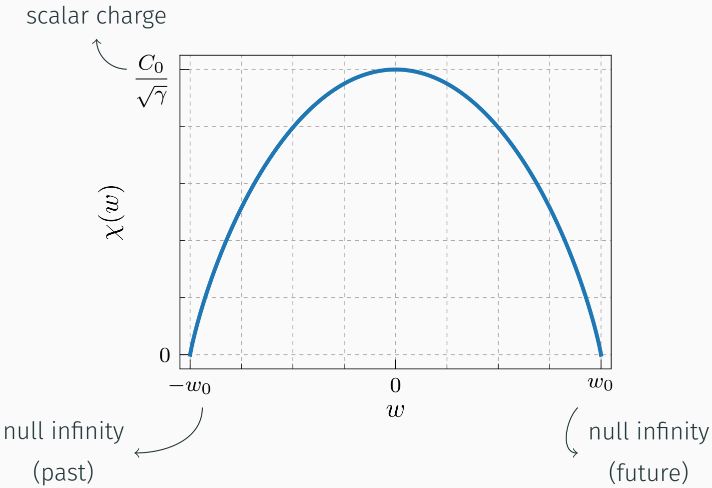
wave pulse

lightlike coordinate

$$\phi = \frac{1}{\sqrt{2}} \log \left(\frac{\rho - \chi(w)}{\rho + \chi(w)} \right)$$

- Wave propagation towards outgoing ρ
- Not spherically symmetric
- Presence of an apparent horizon
- Fully non-linear solution
- Petrov type II

Representing the wave pulse



- Curvature of 2D space (∂_x, ∂_y):

$$\mathcal{K} = \frac{\chi^2(w)}{C_0^2} K(x, y)$$

- Scalar pulse χ goes from 0 to max and back to 0
- **Longitudinal wave** generated by scalar field monopole
- Empty spacetime at remote past and future

Description of the scalar-tensor solution

$$\mathrm{d}s^2 = (-K(x,y) + B_0\phi_w^2)\,\mathrm{d}w^2 - 2(1 - B_0\phi_w\phi_\rho)\,\mathrm{d}w\,\mathrm{d}\rho + B_0\phi_\rho^2\,\mathrm{d}\rho^2 + \frac{\rho^2 - \chi(w)^2}{P(x,y)^2}(\mathrm{d}x^2 + \mathrm{d}y^2)$$

- Wave pulse χ unchanged: scalar monopole
- Apparent horizon and singularities unchanged qualitatively
- Remote past and future ($w \to \pm w_0$): empty non-spherical spacetime
- Petrov classification: type I while seed was type II $\to$ loss of algebraic speciality

GW content of spacetime

How can one read the polarisation content of a GW?

Linearised GW

Read off from the components of $h_{\mu\nu}$

$\longrightarrow$

Non-linear GW

No extraction of wave profile!

- **Main idea**: tidal effects experienced by a photon around its worldline $\bar{\gamma}$ [Penrose '76], with parallel transported null tetrad E^{μ}_{A} and parameter W
- In this setup, always recover pp-wave geometry:

$$\mathrm{d}s^2 = 2\,\mathrm{d}W\,\mathrm{d}V + \delta_{AB}\,\mathrm{d}X^A\,\mathrm{d}X^b - H_{AB}X^A X^B\,\mathrm{d}W^2$$

- Read off polarisation from components of $H_{AB} = \bar{R}_{\mu\nu\rho\sigma}E^{\mu}_{W}E^{\nu}_{A}E^{\rho}_{W}E^{\sigma}_{B}$

  evaluated on $\bar{\gamma}$

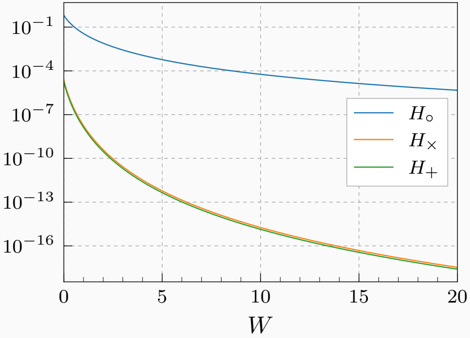
Waves in the transformed solution

$$H_{AB} = \begin{pmatrix} a_0 + B_0 a_1 + B_0^2 a_2 & 0 \\ 0 & a_0 + B_0 a_1 + B_0^2 a_2 \end{pmatrix} + B_0^2 \begin{pmatrix} +b_2 & c_2 \\ c_2 & -b_2 \end{pmatrix} + \mathcal{O}(B_0^3)$$

$a_0 + B_0 a_1 + B_0^2 a_2$ → scalar waves $H_\circ$

c_2 → $H_\times$

b_2 → H_+



- the disformal transformation sources *tensorial* gravitational waves
- keep the complete non-perturbative character of the metric

Conclusion

- First **exact radiative non-linear solution in Horndeski** beyond plane waves
- Contains non-linear superposition of shear and breathing modes generated by a scalar monopole
- Probe new effects in strong regime scalar-tensor gravity: **additional contribution to GWs**
- Open question: consequence for GWs in the case of scalar-tensor cosmology?

Thank you for your attention!