Vanishing of Quadratic Love Numbers of Schwarzschild Black Holes

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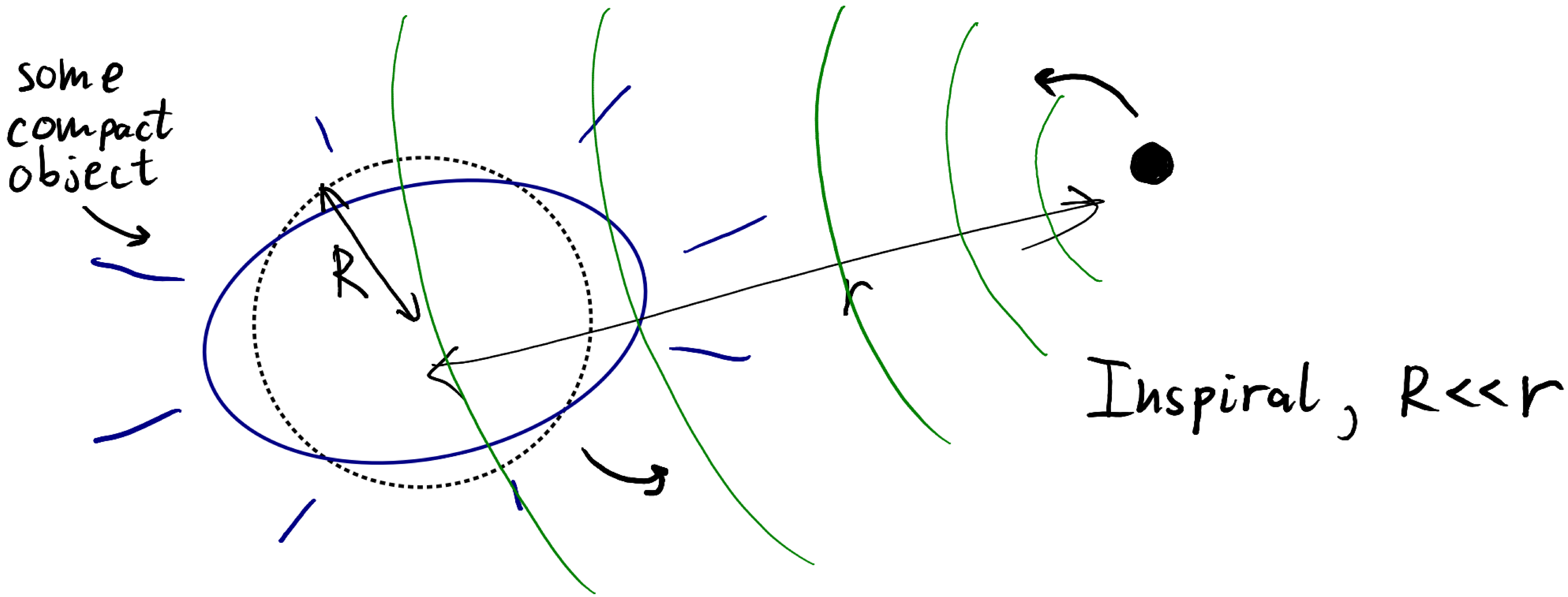
[based on 2312.05065 and 2410.03542 with M.M.Riva, L.Santoni, F.Vernizzi and S.Iteanu]

Reminder: Tidal effects and Love Numbers



[Fang & Lovelace '05] [...]

- $R \ll r$ => „small" tidal forces =>
  => Tidal effects encoded in Love numbers λ

Linear response: (Induced response) $= \lambda^{\text{lin}} \times$ (Background Tidal field)

i.e. deformations/ induced multipoles

Why do we care about Love Numbers?

[Abbott et al '18] [Annala et al '17]

- Constrained for GW170817 → constraint on EoS of neutron star

[Fang & Lovelace, Damour & Nagar, Le Tiec, Casals, ...]

- Black holes have $\lambda^{\mathrm{Lin}}_{\mathrm{BH}} = 0$! (pure GR, D=4, asymp. flat)

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Study subleading tidal effects

- for BHs „leading = 0"
- to put leading order prediction on firm ground

$$\boxed{\begin{pmatrix}\text{Tidal}\\ \text{deformations}\end{pmatrix} \sim \underbrace{\lambda^{\text{lin}}}_{\substack{=0\\ \text{BHs}}}\begin{pmatrix}\text{Tidal}\\ \text{field}\end{pmatrix} + \lambda^{\text{quad}}\begin{pmatrix}\text{Tidal}\\ \text{field}\end{pmatrix}^2 + \dots}$$

Quadratic Love numbers

- 0^{th} order in frequency (static)
- See [Gürlebeck'15] and [Poisson'20] on beyond linear tides

Love numbers

Worldline Effective Field Theory WEFT

* Clean and robust framework for general tidal effects
  (systematic account of nonlinearities and free of gauge ambiguity)

[Goldberger & Rothstein '04] [...]

$$S[g_{\mu\nu}, x^\mu] \overset{R \ll r}{=} \underbrace{\frac{1}{16\pi G}\int d^4x\sqrt{-g}\,R}_{\text{Einstein-Hilbert}} - \underbrace{M\int d\tau}_{\text{point particle}} + \int d\tau \sum_{\ell \geq 2}\left[E_{\mu_L} Q_E^{\mu_L} + B_{\mu_L} Q_B^{\mu_L}\right]$$

← Tidal effects

induced multipoles

- $E_{\mu_L} \equiv \nabla_{\langle\mu_1}\cdots\nabla_{\mu_{\ell-2}} C_{\mu_{\ell-1}\alpha\mu_\ell\beta}\, u^\alpha u^\beta \rangle_{STF}$

  derivatives of electric part of the Weyl tensor
  (even)

- $B_{\mu_L} \equiv \frac{1}{2}\nabla_{\mu_1}\cdots\nabla_{\mu_{\ell-2}}\, \varepsilon_{\gamma\mu_{\ell-1}}{}^{\alpha\beta} C_{\mu_\ell\delta\alpha\beta}\, u^\gamma u^\delta$

  magnetic part
  (odd)

Love numbers

Worldline Effective Field Theory WEFT

$$S[g_{\mu\nu}, x^\mu] = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,R - M\int d\tau + \int d\tau \sum_{\ell\geq 2}\left[E_{\mu_L} Q_E^{\mu_L} + B_{\mu_L} Q_B^{\mu_L}\right]$$

multipoles

Tidal effects

Non linear response

$$Q^E_{\mu_L} = \lambda^{lin} E_{\mu_L} + \lambda^{E^3}_{L L_1 L_2}\left(E_{\mu_{L_1}} E_{\mu_{L_2}}\right)_{\mu_L} + \lambda^{EB^2}_{L L_1 L_2}(BB)_{\mu_L} + O(E^3, EB^2)$$

(Linear) Love numbers

Quadratic Love Numbers

[Bern et al '20]

[Riva, Santoni, NS, Vernizzi '23]

[Iteanu, Riva, Santoni, NS, Vernizzi '24]

How to find Quadratic Love numbers?

- match BH perturbations on WEFT calculation

WEFT

Second order BH perturbations

$$g_{\mu\nu}^{\mathrm{WEFT}}(\lambda) \overset{!}{\underset{r \gg r_s}{=}} g_{\mu\nu}^{\mathrm{BHPT}}$$

$$\Downarrow$$

$$\lambda$$

Second order BH perturbation theory

$$g_{\mu\nu} = \bar{g}^{\,Sch}_{\mu\nu} + \delta g_{\mu\nu}$$

- Solving Einsteins eqs for $\delta g_{\mu\nu}$ perturbatively:

$$\delta g_{\mu\nu} = {}^{(1)}\delta g_{\mu\nu} + {}^{(2)}\delta g_{\mu\nu}$$

1st order (linear): $\hat{\mathcal{D}}^{(1)}\delta g_{\mu\nu} = 0 \xrightarrow{\text{solve}} {}^{(1)}\delta g_{\mu\nu}$

2nd order: $\boxed{\hat{\mathcal{D}}^{(2)}\delta g_{\mu\nu} = \mathcal{J}_{\mu\nu}\left[{}^{(1)}\delta g \circ {}^{(1)}\delta g\right]}$ ← plug into source

- Parametrisation and Regge-Wheeler gauge [Regge & Wheeler '57]

$$\delta g_{\mu\nu} = \delta g^{(even)}_{\mu\nu} + \delta g^{(odd)}_{\mu\nu}$$

master variable ↓

– focus on even: $\boxed{\delta g^{(even)}_{\mu\nu} \sim \delta g_{tt} \sim H_0(r,\theta,\varphi)} = \sum_{\ell m} H_0^{\ell m}(r)\, Y^{\ell}_{m}(\theta,\varphi)$

Second order BH perturbation theory

Linear solutions: ${}^{(1)}H_0^{\ell m} = \mathcal{E}_+^{\ell m} P_\ell^{(2)}\left(2\frac{r}{r_s}-1\right)$

$$\boxed{\left(\partial_r^2 + \frac{2r-r_s}{f(r)\,r^2}\partial_r - \frac{\lambda r^2 f(r) + r_s^2}{r^4 f^2(r)}\right) {}^{(2)}H_0^{\ell m}(r) = \sum_{\substack{\ell_1 m_1 \\ \ell_2 m_2}} S_{H_0\,\ell\ell_1\ell_2}^{m m_1 m_2}(r)}$$

sources, known functions of linear solutions

- Impose regularity of solution at the horizon

- Due to specific form of the source we find that all regular solutions at 2^{nd} order are finite polynomials in (r/r_s)!

[Iteanu, Riva, Santoni, NS, Vernizzi '24]

Matching to WEFT

[Iteanu, Riva, Santoni, NS, Vernizzi '24]

- BHPT gives ${}^{(2)}H_0^{\ell m} = \mathcal{E}_+^2 \left[A^{(BH)} r^n + \# r_s r^{n-1} + \dots \# r_s^{n-1} r + \# r_s^n \right]$

- We calculate ${}^{(2)}H_0$ and ${}^{(2)}h_0$ to leading order in r_s in WEFT

$$\text{WEFT:} \quad {}^{(2)}H_0^{\ell m} = A^{(EFT)} \mathcal{E}_+^2 r^n + \lambda_{\mathcal{E}^3} \frac{\#}{r^{\ell+1}}$$

Matching to WEFT

- BHPT gives ${}^{(2)}H_0^{\ell m} = \boxed{\varepsilon_+^2 \Big[A^{(BH)} r^n} + \# r_s r^{n-1} + \dots \# r_s^{n-1} r + \# r_s^n \Big]$

- We calculate ${}^{(2)}H_0$ and ${}^{(2)}h_0$ to leading order in r_s in WEFT

WEFT: $\quad {}^{(2)}H_0^{\ell m} = \boxed{A^{(EFT)} \varepsilon_+^2 r^n} + \lambda^{E^3} \dfrac{\#}{r^{\ell+1}}$

$$A^{(BH)} = A^{(EFT)}$$

→ we matced correctly

for BHs there are no $\dfrac{1}{r^{\ell+1}}$

just positive powers

⇓

$$\boxed{\begin{array}{ll} \lambda^{E^3}_{123} = 0 & \text{for all} \\ \lambda^{EB^2}_{123} = 0 & \text{multipoles!} \end{array}}$$

[Iteanu, Riva, Santoni, NS, Vernizzi '24]

Conclusions

* Quadratic static tidal response of Schwarzschild black holes vanishes for all multipoles

* Hint of hidden symmetries beyond linear perturbations?
  (see [Hui et al '21, '22] [Charalambous et al '21, '22] for linear perturbations)

* Simplicity of 2nd order BHPT $\longrightarrow$ smarter way of organising calculation?

* Extend to all orders, Kerr, RN ; consider $D>4$, AdS ...