Revisiting 2PN mechanics of binary black holes

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La huitième assemblée générale du GdR Ondes Gravitationnelles
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In collaboration with Tom Colin, Manuel Alva Morales, Laura Bernard

Plan of the talk

- Introduction and theory

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- Results

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Introduction and theory

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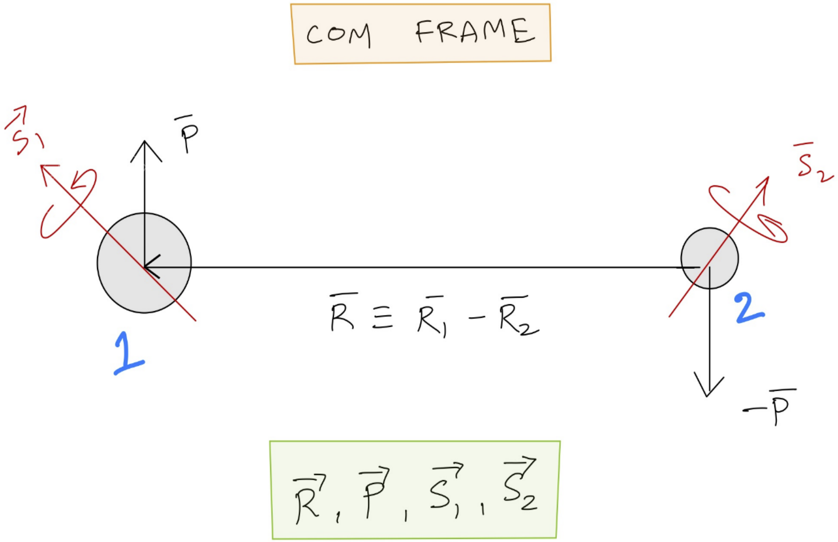
- In inspiral state, at 2$^{\text{nd}}$ post-Newtonian (2PN) order; $\frac{v^2}{c^2} \ll 1$.
- Arbitrary spins and eccentricity.

Phase space of spinning PN BBHs



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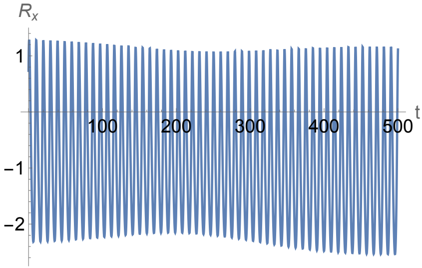

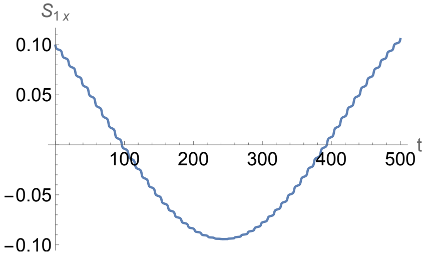

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- **fast variables:** $\vec{R}, \vec{P}$.

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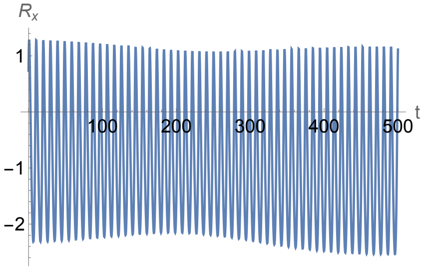

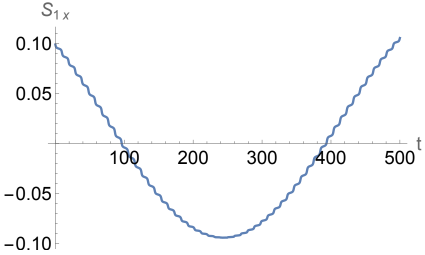

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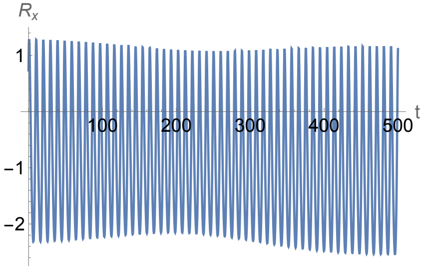

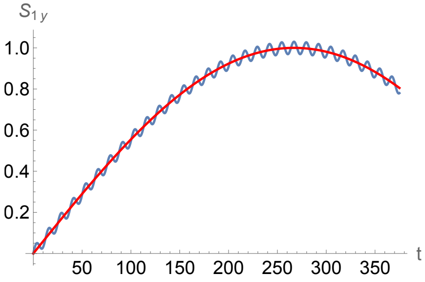

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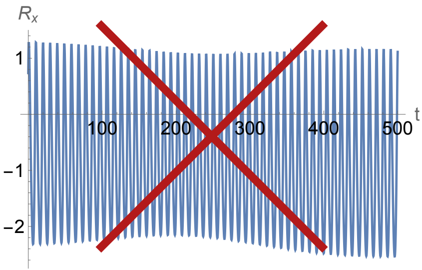

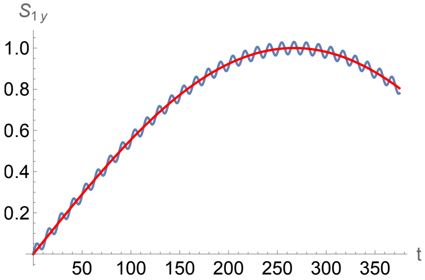

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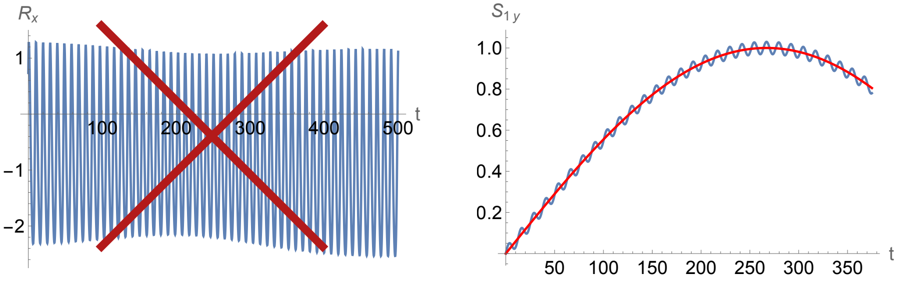

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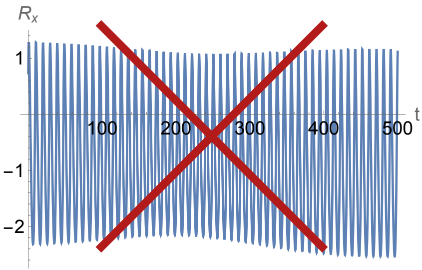

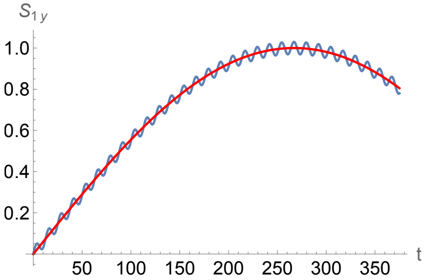

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- **Phase space variables:** $\vec{S}_1, \vec{S}_2, \vec{L}$. Total number $= 6$ since $\dot{S}_1 = \dot{S}_2 = \dot{L} = 0$.

2PN orbit-averaged system

The equations of motion with $M_1, M_2, M, \mu, d, \lambda$ being constants. [Racine (2008)]

$$\frac{d\boldsymbol{S}_1}{dt} = \frac{1}{2d^3}\left\{\left[4 + \frac{3M_2}{M_1} - \frac{3\mu}{M_1}\lambda\right]\boldsymbol{L} + \boldsymbol{S}_2\right\} \times \boldsymbol{S}_1$$

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$$\frac{d\boldsymbol{L}}{dt} = \frac{1}{2d^3}\left\{\boldsymbol{S} + 3\left[1 - \frac{\mu}{M}\lambda\right]\boldsymbol{S}_0\right\} \times \boldsymbol{L}$$

$$\boldsymbol{S}_0 = \left(1 + \frac{M_2}{M_1}\right)\boldsymbol{S}_1 + \left(1 + \frac{M_1}{M_2}\right)\boldsymbol{S}_2.$$

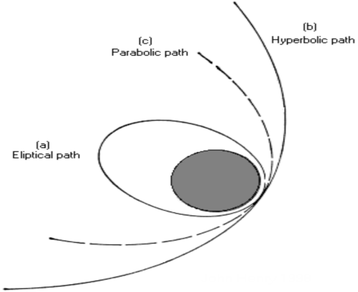
$$\boldsymbol{S} = \boldsymbol{S}_1 + \boldsymbol{S}_2$$

Separatrix

Informally, a boundary separating qualitatively, two very different kinds of trajectories.

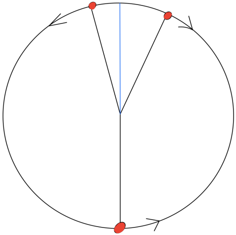
Sun-earth system

$H = 0.$



Pendulum

$H = 2mgl.$



[Image credit: pages.uoregon.edu]

Integrable Hamiltonian systems and action-angle variables

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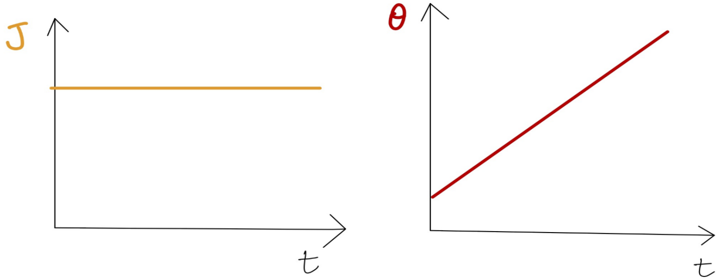

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It's nice to have integrable systems (they occur rarely), and extra nice to have action-angle variables.

Results

Results: Hamiltonian system

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This is a Hamiltonian system with the Hamiltonian

$$H = \frac{1}{4d^3 m_1 m_2 L^2}\left\{-3\left(m_2\vec{L}\cdot\vec{S}_1+(1\leftrightarrow 2)\right)^2 \right.$$
$$\left. +2L^2\left(m_2(4m_1+3m_2)\vec{L}\cdot\vec{S}_1+\frac{1}{2}m_1m_2\vec{S}_1\cdot\vec{S}_2+(1\leftrightarrow 2)\right)\right\} \qquad (1)$$

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- Matches with the numerical and the already-existing non-AA-based analytical solutions [Kesden & others - 2014]

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$$\mathcal{J}_3 = \frac{1}{4\mathcal{L}(\sigma_1-\sigma_2)\left(\mathcal{C}_1^2 - 4\mathcal{L}^2({\mathcal{S}_1}^2+{\mathcal{S}_2}^2)\right)}\Big[\mathcal{C}_1^3\mathcal{C}_2(\sigma_1+\sigma_2) + 4\mathcal{C}_1\mathcal{L}^2\Big\{{\mathcal{S}_1}^2(\mathcal{C}_2(\sigma_1-\sigma_2)+2\sigma_1)$$
$$+{S_2}^2(\mathcal{C}_2(\sigma_2-\sigma_1)+2\sigma_2)\Big\} - (S_{\rm eff}\cdot L)\left\{16L^2\left({S_1}^2+{S_2}^2\right) + 4\mathcal{C}_1^2\mathcal{C}_2\right\}\Big]$$

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Ongoing: Construct solutions for full (w/o orbit-averaging) 2PN spinning, eccentric systems.