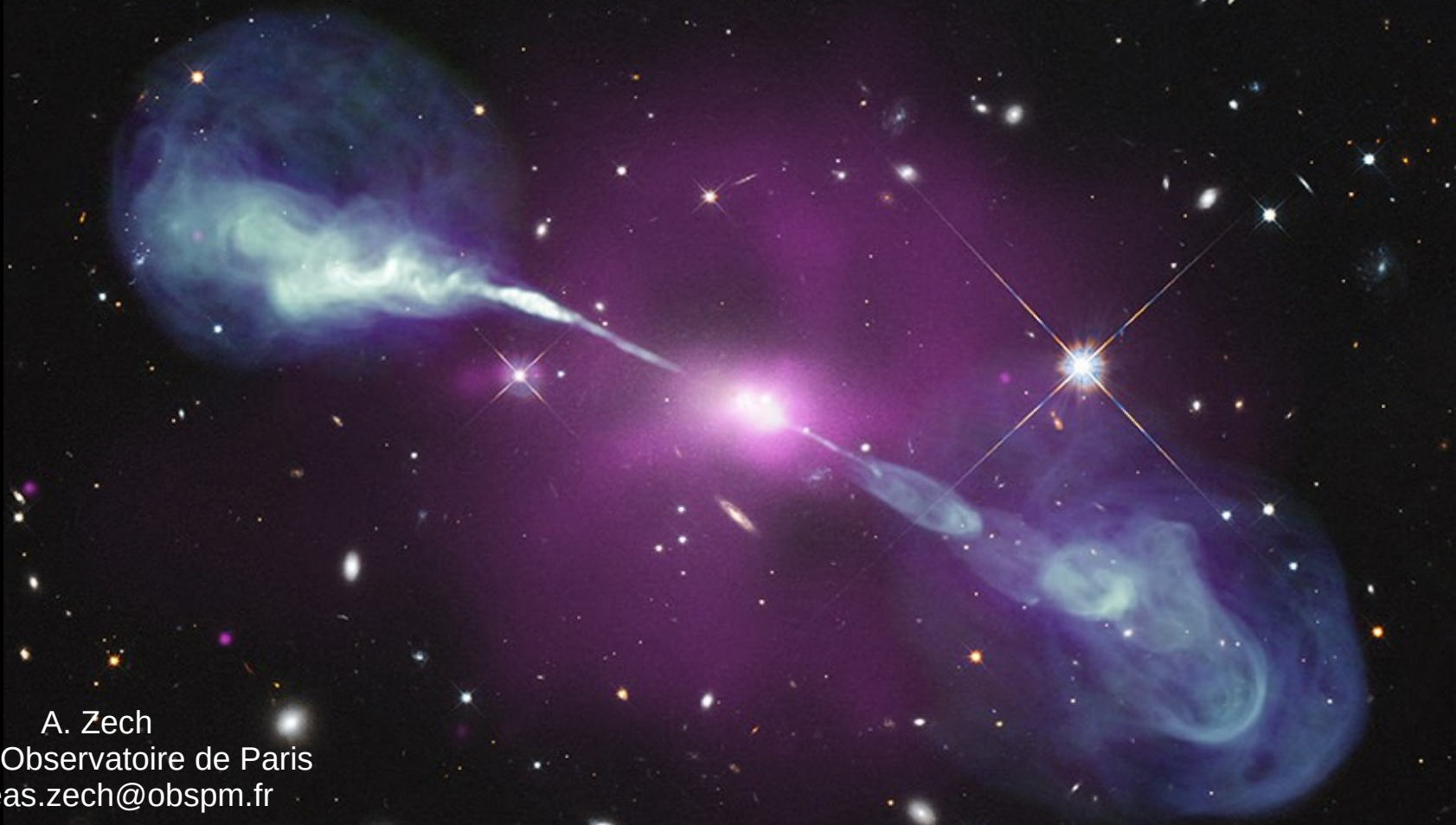


# Non-thermal emission from Active Galactic Nuclei



*Hercules A  
(NASA)*

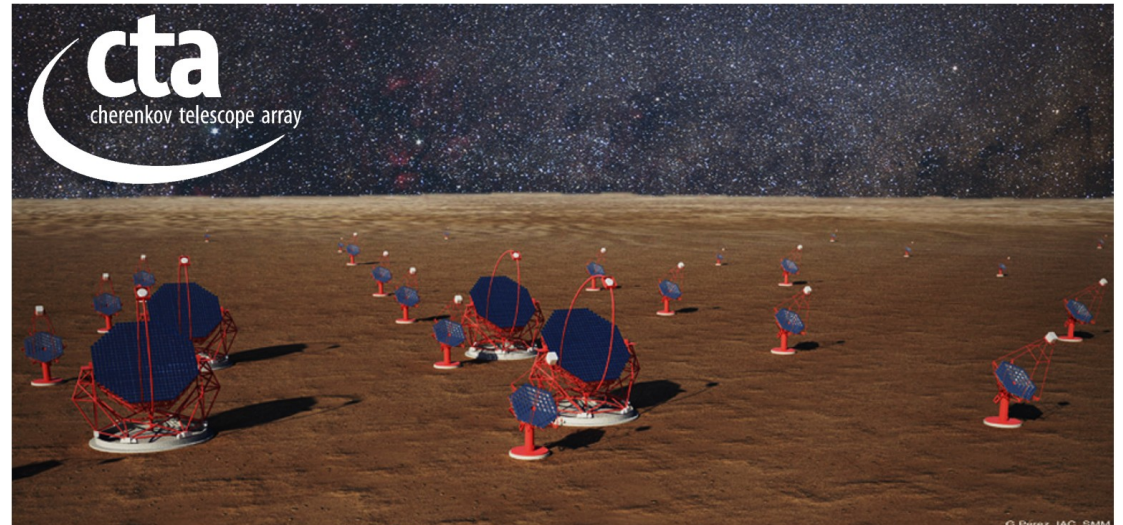
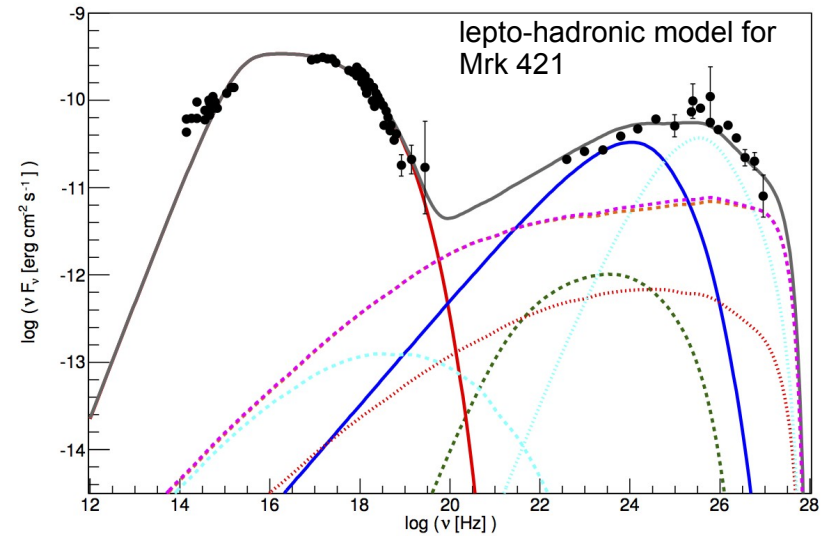
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Septembre 2024

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- very-high-energy gamma-ray astronomy
- > observations with the H.E.S.S. array in Namibia
- > preparation of the CTA (Cherenkov Telescope Array)
- development of emission models for the multi-wavelength emission from blazars



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# I ) non-thermal radiative processes

1) a short introduction to radiative processes

# thermal vs. non-thermal processes

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If the characteristics of the emitted radiation do not depend on the temperature of the source, the radiation is known as '**non-thermal**'.

- no underlying thermal equilibrium
- the emission process involves accelerated charges that interact with e.m. fields or particle collisions
- non-thermal emission is thus directly linked to a process of **particle acceleration** and **interaction**
- dominant emission processes at high energies (and the only process at Very High Energies), but also very important in radio astronomy.

# emission and absorption

---

Any emission process is associated with a corresponding absorption process :

- **emission coefficient**  $j = dE/(dV d\Omega dt)$  [erg s<sup>-1</sup> cm<sup>-3</sup> sr<sup>-1</sup>]  
 $j_\nu = dE/(dV d\Omega dt d\nu)$  [erg s<sup>-1</sup> cm<sup>-3</sup> sr<sup>-1</sup> Hz<sup>-1</sup>]

For an isotropic emitter  $P_\nu = 4\pi j_\nu$  is the radiated power per unit volume and frequency.

- **absorption coefficient**  $\alpha_\nu$  [cm<sup>-1</sup>]

defined by loss of intensity of a beam over a distance ds :  $dI_\nu = -\alpha_\nu I_\nu ds$

mass absorption coefficient  $\kappa_\nu = \alpha_\nu / \rho$  [cm<sup>2</sup> g<sup>-1</sup>]

optical depth  $\tau_\nu$  with  $d\tau_\nu = \alpha_\nu ds$

mean free path  $l_\nu = 1/\alpha_\nu$  [cm]

The absorption coefficient can account for “true absorption” and stimulated emission.

The specific intensity (=brightness)  $I_\nu = dE/(dA d\Omega dt d\nu)$  [erg s<sup>-1</sup> cm<sup>-2</sup> sr<sup>-1</sup> Hz<sup>-1</sup>]  
is the energy carried by all rays passing through dA within solid angle dΩ in time dt and frequency range dν .



# radiative transfer

---

The transfer equation describes the variation of intensity along a ray in an emitting and absorbing medium :

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

or equivalently

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu \quad \text{with} \quad S_\nu \equiv \frac{j_\nu}{\alpha_\nu} \quad \text{the "source function"}$$

with solution

$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + \int_0^{\tau_\nu} e^{-(\tau_\nu - \tau'_\nu)} S_\nu(\tau'_\nu) d\tau'_\nu \quad (\text{note: for scattering, } S_\nu \text{ depends on } I_\nu \dots)$$

For **thermal radiation** (= radiation emitted by matter in thermal equilibrium ):

$$S_\nu = B_\nu(T) \quad (= \text{Planck function})$$

$$j_\nu = a_\nu B_\nu(T) \quad (\text{Kirchhoff's law})$$

and for **blackbody radiation** (= thermal radiation in optically thick media; radiation itself in thermal equilibrium ) :

$$I_\nu = B_\nu(T)$$



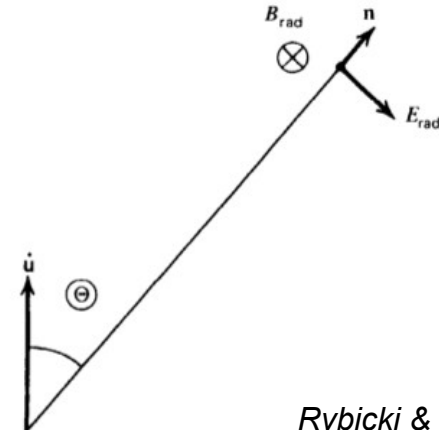
# accelerated charges: the Larmor formula

Charged particles with a varying velocity (vector) emit e.m. radiation.

The radiation is polarised. The radiation is 100% polarized in the plane of the acceleration vector and  $\bar{n}$ , the unit vector in the direction of the observer.

For a relativistically moving charge, the formula takes the following form

$$P = \frac{2q^2}{3c^3} \gamma^4 (a_{\perp}^2 + \gamma^2 a_{\parallel}^2)$$

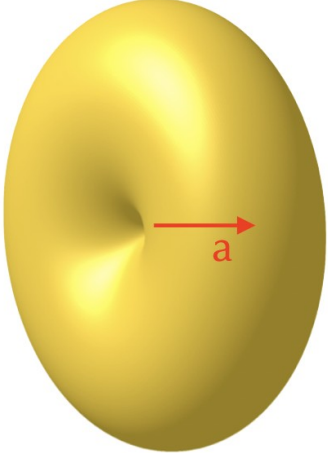
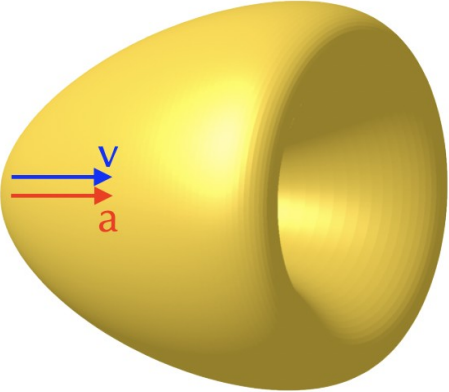



*Rybicki & Lightman*

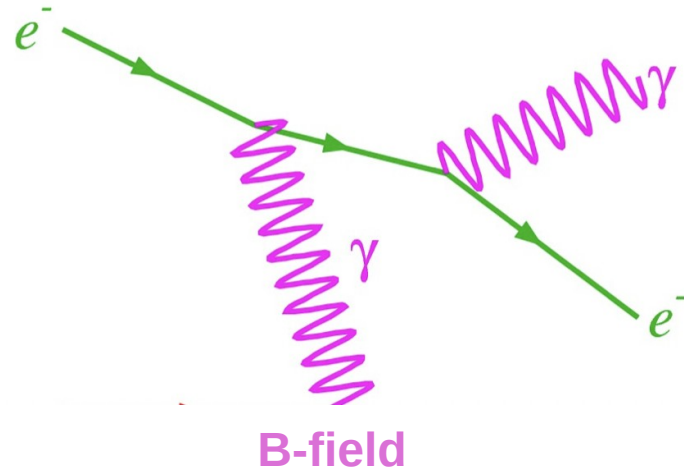
(depends on acceleration orthogonal ( $a_{\perp}$ ) and parallel ( $a_{\parallel}$ ) to the velocity of the particle; the radiation loss rate is given in the instantaneous rest frame of the particle)

The Larmor formula is the basis to describe radiation of charged particles in the fields of nuclei (**bremsstrahlung**) or in magnetic fields (**gyroradiation, cyclotron- and synchrotron-radiation**).

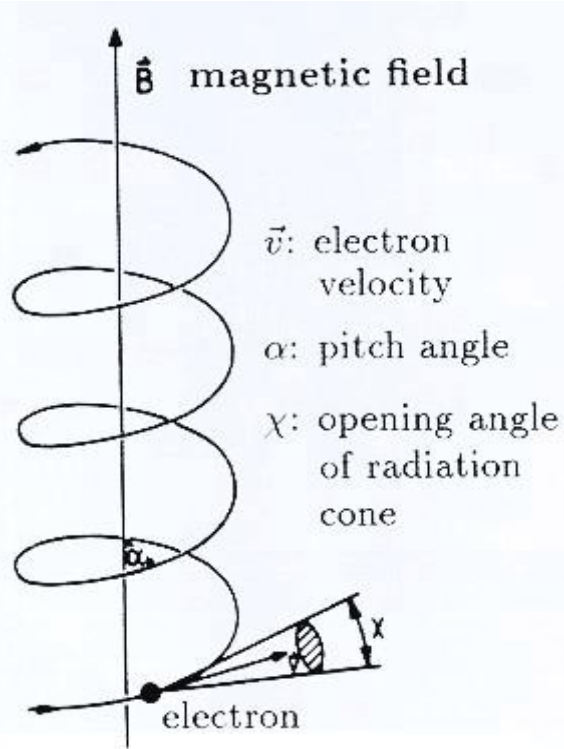
# emission from accelerated charges

	Non Relativistic	Any Regime	
		Parallel v,a	Perpendicular v,a
Total Power	$P = \frac{2q^2 a^2}{3c^3}$	$P = \frac{2q^2}{3c^3} \left( a_{\parallel}^2 \gamma^6 + a_{\perp}^2 \gamma^4 \right)$	
	$\frac{dP}{d\Omega} = \frac{q^2 a^2}{4\pi c^3} \sin^2 \theta$	$\frac{dP}{d\Omega}(\theta, \phi)$	
	Dipolar emission	Beamed emission	
Angular distribution			
Spectrum	$\frac{dP_{\nu}}{d\Omega} \propto \left  \text{FT}[\vec{E}] \right ^2$		

## 2) synchrotron radiation



# basic characteristics



- Synchrotron radiation arises from the interaction of a relativistic charged particle with a magnetic field.
- Very important continuous emission mechanism in astrophysics for non-thermal sources. (e.g. blazars, supernova remnants, pulsars...)
- Due to the highly relativistic energy of the electron, synchrotron emission is strongly forward beamed, i.e. emitted into a cone in the direction of the particle's motion.
- At lower (non-relativistic) energies, the emission is called gyroradiation or cyclotron radiation and has different characteristics.

# single-particle emission

---

Emission is defined by the relativistic Larmor formula:

$$P = \frac{2}{3} \frac{\gamma^4 q^2}{c^3} (\dot{v}_\perp^2 + \gamma^2 \dot{v}_\parallel^2)$$

The Lorentz force due to the magnetic field is given by:

$$\frac{d\vec{p}}{dt} = \frac{d}{dt}(\gamma m \vec{v}) = q(\vec{v} \times \vec{B}) \quad \rightarrow \quad \dot{v}_\perp = \frac{q v B \sin \alpha}{\gamma m} \quad ; \quad \dot{v}_\parallel = 0$$

$$P = - \left( \frac{dE}{dt} \right) = \frac{2}{3} \frac{q^4 B^2}{c m^2} \frac{v^2}{c^2} \gamma^2 \sin^2 \alpha$$

For an **isotropic distribution of velocities**:

$$\langle \sin^2 \alpha \rangle = \frac{1}{4\pi} \int \sin^2 \alpha d\Omega = \frac{2}{3} \quad \rightarrow \quad P = - \left( \frac{dE}{dt} \right) = \frac{4}{9} \frac{q^4 B^2}{c m^2} \frac{v^2}{c^2} \gamma^2$$

# single-particle emission

The total emitted power per frequency [erg s<sup>-1</sup> Hz<sup>-1</sup>] is:

$$P_{\omega}(x) = \frac{\sqrt{3} q^3 B \sin \alpha}{2 \pi m c^2} F(x)$$

$$F(x) = x \int_x^{\infty} K_{5/3}(\xi) d\xi$$

$K_{5/3}$  is the modified Bessel function of order 5/3

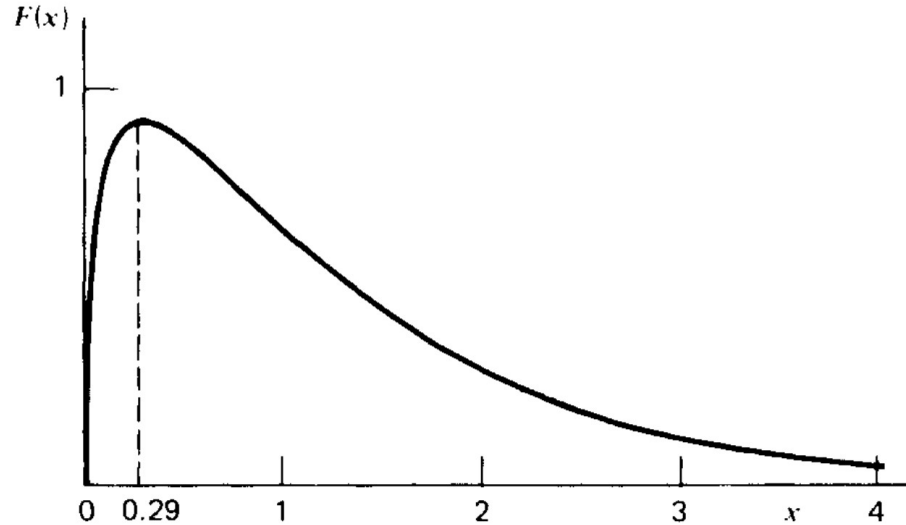
$$x = \frac{\omega}{\omega_c} = \frac{2 m c \omega}{3 \gamma^2 q B \sin \alpha}$$

The peak frequency of the emission is

$$\nu_{peak} = 0.29 \nu_c = 0.29 \frac{3 \gamma^2 e B \sin \alpha}{4 \pi m_e c}$$

Below  $\nu_{peak}$  :  $P_{\nu} \propto \nu^{1/3}$

Above  $\nu_{peak}$  : exponential decay



**Figure 6.6** Function describing the total power spectrum of synchrotron emission. Here  $x = \omega/\omega_c$ . (Taken from Ginzburg, V. and Syrovatskii, S. 1965, *Ann. Rev. Astron. Astrophys.*, 3, 297.)

Rybicki & Lightman

# emission from a particle distribution

For an electron distribution following a **power law with index  $p$**  :

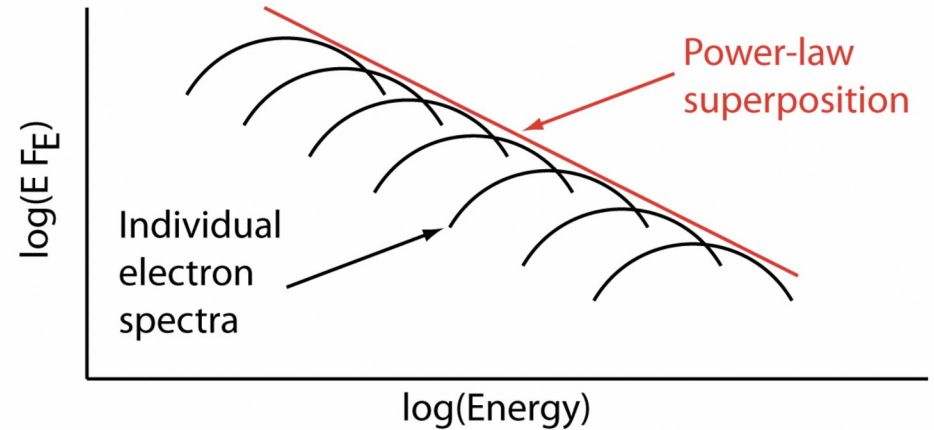
$$N(E) = C E^{-p}$$

the synchrotron spectrum is also given by a power law, with a **modified index  $s = (p-1) / 2$**

$$P_{\omega}(\omega) = \frac{\sqrt{3} q^3 C B \sin \alpha}{2 \pi m c^2 (p+1)} \Gamma\left(\frac{p}{4} + \frac{19}{12}\right) \Gamma\left(\frac{p}{4} - \frac{1}{12}\right) \left(\frac{m c \omega}{3 q B \sin \alpha}\right)^{-(p-1)/2}$$

The cutoff frequency is

$$\nu_{cut} \approx \frac{3}{4} \gamma_{max}^2 \frac{qB}{\pi m c}$$



(from Shu, Part II, p 178)



# synchrotron self-absorption

Synchrotron emission is accompanied by synchrotron absorption, in which a charged particle in a magnetic field absorbs a photon.

$$\alpha_\nu = -\frac{c^2}{8\pi\nu^2} \int dE P_\nu(E) E^2 \frac{\partial}{\partial E} \left[ \frac{N(E)}{E^2} \right]$$

For a power-law electron distribution with index  $p$  :

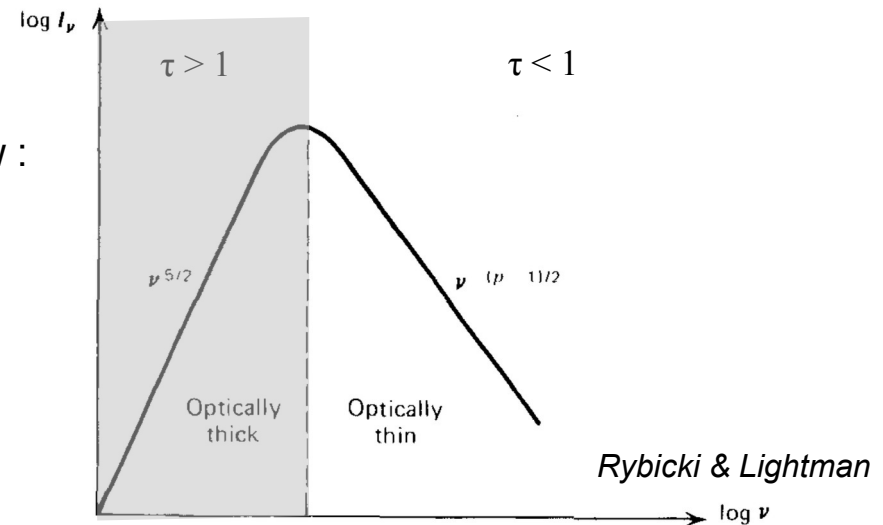
$$\alpha_\nu \propto \nu^{-(p+4)/2}$$

And the source function is independent from the initial power law :

$$S_\nu = \frac{j_\nu}{\alpha_\nu} = \frac{P_\nu}{4\pi\alpha_\nu} \propto \nu^{5/2}$$

In optically thick regions, synchrotron self-absorption can render the source opaque to long wavelengths.

- typically observable at radio frequencies
- low-frequency cutoff



**Figure 6.12** Synchrotron spectrum from a power-law distribution of electrons.

Note: One obtains the 5/2 slope when the synchrotron frequency of the emitting electron is inside the self-absorption range. One obtains a slope of 2 if there is self-absorption, but the radiation in that range is due to the low-energy tail of electrons radiating effectively at higher energies.

# synchrotron cooling

Synchrotron emission leads to energy loss

$$\frac{dE}{dt} = P_{iso} = \frac{4}{3} \sigma_T c \beta^2 \gamma^2 U_B, \quad U_B = \frac{B^2}{8\pi}$$

AGN jet

so the characteristic **cooling time scale** is

$$t_{syn} \equiv \frac{E}{P_{iso}} = \frac{3m_e c^2}{4\sigma_T c U_B \gamma \beta^2} \approx 16 \text{ yr} \left(\frac{1\text{G}}{B}\right)^2 \frac{1}{\gamma}$$

Location	Typical B (Gauss)	Cooling time
Interstellar medium	$10^{-6}$	$10^{10}$ yrs
Stellar atmosphere	1	5 days
Super massive Black Hole	$10^4$	$10^{-3}$ sec
White dwarf	$10^8$	$10^{-11}$ sec
Neutron star	$10^{12}$	$10^{-19}$ sec

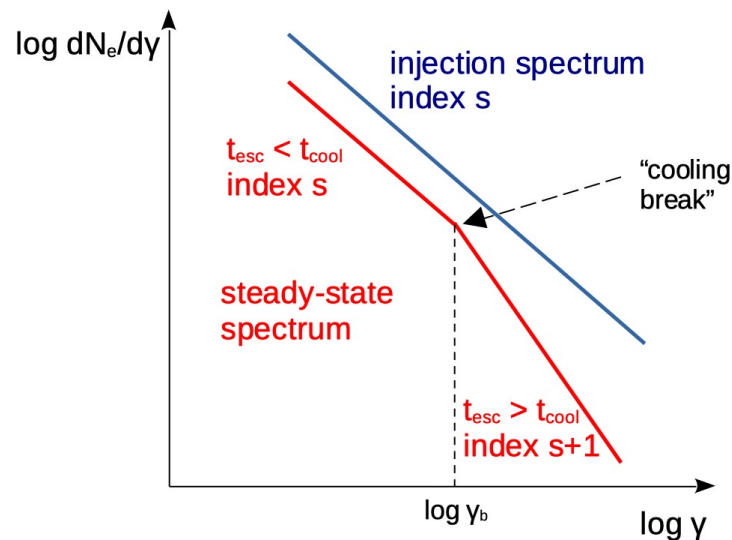
F.Perrotta  
( $\gamma \sim 1e3$ )

For continuous injection of a power-law electron distribution and particle escape, a **cooling break** develops :

$$\frac{dN_\gamma}{dt} = \frac{d}{d\gamma} (-\dot{\gamma} N_\gamma) + Q_{inj} - \frac{N_\gamma}{t_{esc}}$$

For  $Q_{inj} \propto \gamma^{-s}$ , steady-state electron spectrum is  $N_\gamma \propto \gamma^{-(s+1)}$

above the cooling break  $\gamma_c$ , where  $t_{syn}(\gamma) < t_{esc}(\gamma)$



# synchrotron cooling in GRBs

For sufficiently fast synchrotron cooling, the complete electron spectrum above the minimum injected energy  $\gamma_{min}$  is cooled (e.g. in Gamma Ray Bursts)

## fast cooling regime $\gamma_c < \gamma_{min}$

- all electrons cool rapidly on dynamic time-scale
- integrated spectrum of a flare above self-absorption

$$F_\nu \propto \nu^{1/3} \quad \text{below critical cooling frequency}$$

$$F_\nu \propto \nu^{-1/2} \quad \text{between critical cooling frequency and frequency of e- at } \gamma_{min}$$

$$F_\nu \propto \nu^{-p/2} \quad \text{above frequency of electrons at } \gamma_{min}$$

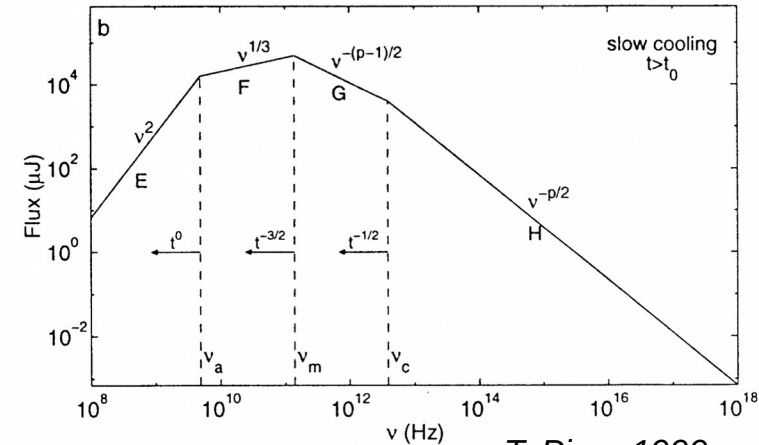
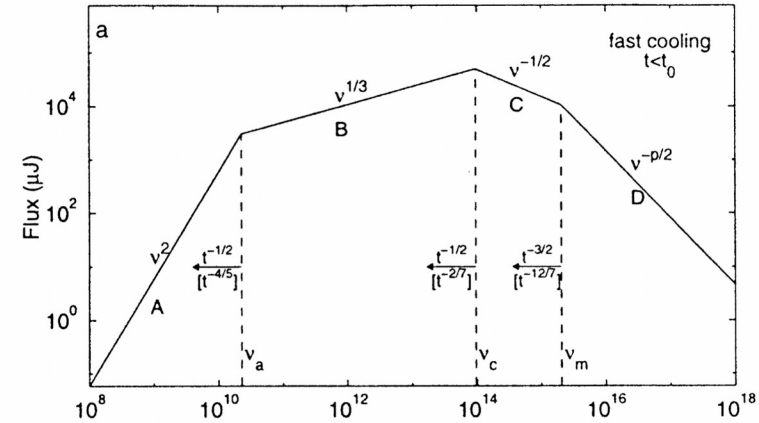
## slow cooling regime $\gamma_c > \gamma_{min}$

- only the most energetic electrons cool rapidly on dynamic time-scale
- integrated spectrum of a flare above self-absorption

$$F_\nu \propto \nu^{1/3} \quad \text{below frequency of electrons at } \gamma_{min}$$

$$F_\nu \propto \nu^{-(p-1)/2} \quad \text{between frequency of e- at } \gamma_{min} \text{ and critical cooling frequency}$$

$$F_\nu \propto \nu^{-p/2} \quad \text{above critical cooling frequency}$$



T. Piran 1999

# maximum energy of synchrotron photons

Maximum achievable energy of synchrotron photons for

$$t_{acc}(\gamma_{max}) = t_{loss}(\gamma_{max})$$

e.g. following *P. Kumar et al. 2012* for electron acceleration on relativistic shocks :

- energy gain by a factor of  $\sim 2$  in one Larmor time  $t_L = \frac{m_e c \gamma_e}{e B}$

- radiative energy loss during this time is  $\delta E \sim t_L \sigma_T B^2 \gamma_e^2 c / (6 \pi) \sim \sigma_T B \gamma_e^3 m_e c^2 / (6 \pi e)$

→ maximum electron energy for  $\delta E \sim \gamma_e m_e c^2 / 2$  i.e.  $\gamma_{e,max} \sim \sqrt{9 m_e^2 c^4 / (8 B e^3)}$

→ maximum synchrotron photon energy

$$\epsilon(\gamma_{e,max}) \sim \frac{h e B \gamma_{e,max}^2}{2 \pi m_e c} \sim \frac{9 m_e c^3 h}{16 \pi e^2} \text{ i.e. } \sim 50 \text{ MeV} \quad (\text{but } > \sim 100 \text{ GeV for protons})$$

( Note that the authors show how specific B-field configurations in GRBs can surpass this limit and lead to energies above 10 GeV in the observer frame in the electron scenario. )

# polarisation

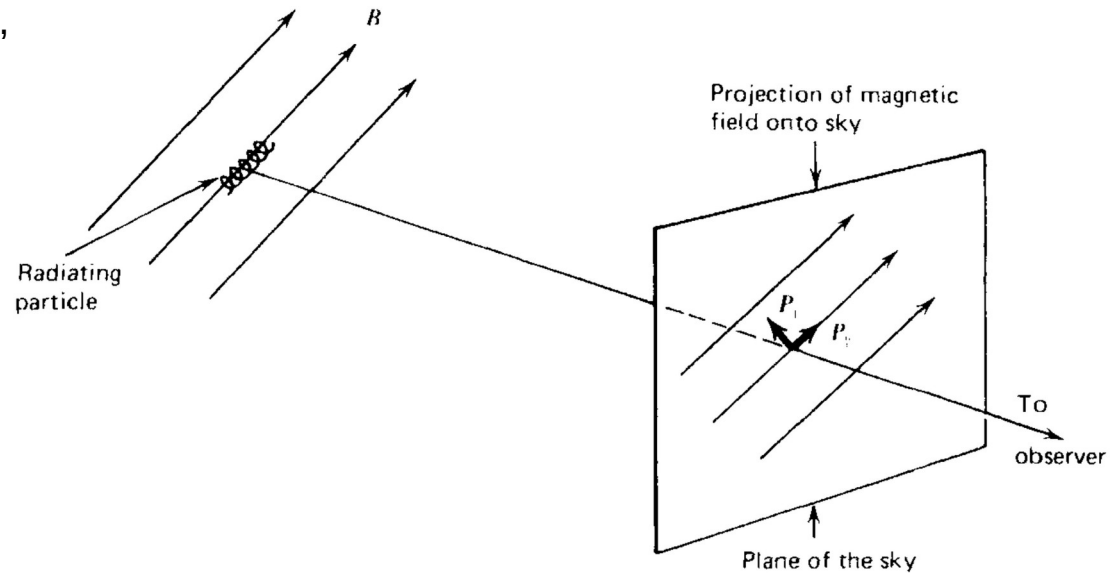
For an ordered magnetic field, synchrotron emission from a population of particles will be partially linearly polarised. The polarization fraction is derived from the power emitted in directions parallel and orthogonal to the projection of the magnetic field on the plane of the sky :

$$\Pi = \frac{P_{\perp} - P_{\parallel}}{P_{\perp} + P_{\parallel}} = 75\% \quad \text{monoenergetic particles, frequency integrated}$$

For particles with a power-law energy distribution with index  $p$  :

$$\Pi = \frac{p + 1}{p + \frac{7}{3}}$$

→ IXPE polarisation signatures in AGN jets



**Figure 6.7** *Decomposition of synchrotron polarization vectors on the plane of the sky.*

*Rybicki & Lightman*

# curvature radiation

= emission from charged particles following curved magnetic field lines, observed in strong B-fields (e.g. pulsar magnetosphere)

in the strong B-field, the relativistic particles quickly lose their velocity component orthogonal to the field line through synchrotron cooling :

$$\begin{aligned} v_{\perp} &\rightarrow 0 \\ v_{\parallel} &\rightarrow c \quad \text{if particle acceleration} \end{aligned}$$

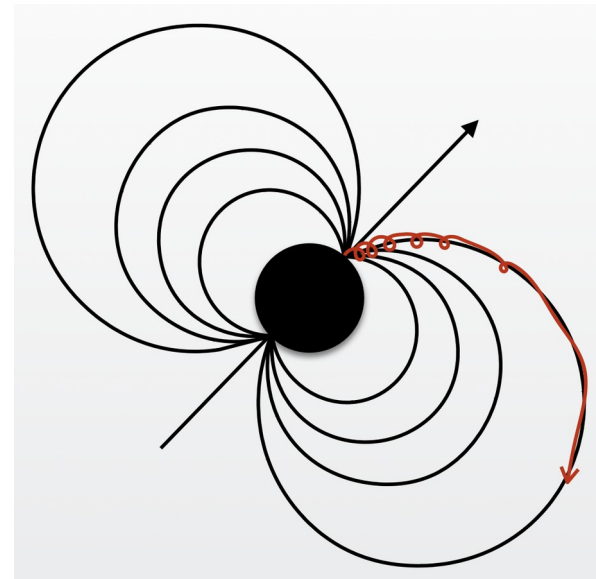
=> synchrotron emission is then weak, but the curved trajectory along the field line leads to curvature radiation

e.g. circular motion ( cf. Larmor formula)

$$P = \frac{2}{3} \frac{q c \gamma^4}{R^2}$$

$$v_c = \frac{3}{4\pi} \frac{c \gamma_{max}^3}{R} \approx 3 \left( \frac{\gamma_{max}}{10^7} \right)^3 \left( \frac{R}{10^6 \text{ cm}} \right)^{-1} \text{ GeV}$$

=> depending on the conditions, synch. and curv. rad. can coexist → synchro-curvature radiation



R.Belmont

# an example of synchrotron emission in astrophysics

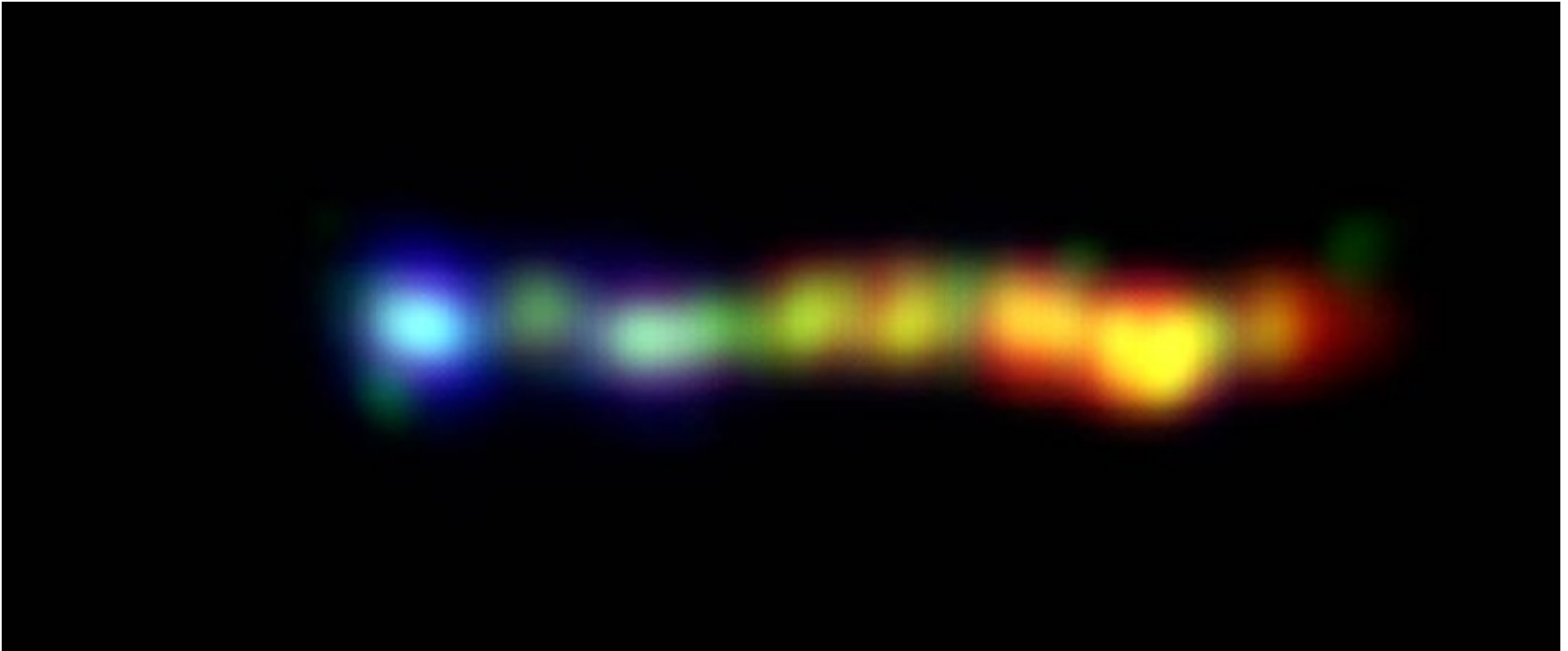
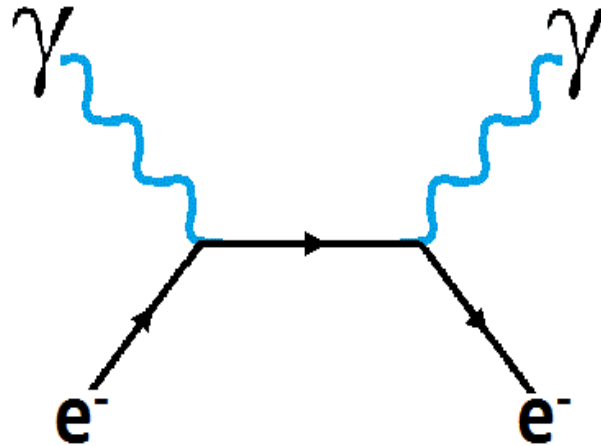


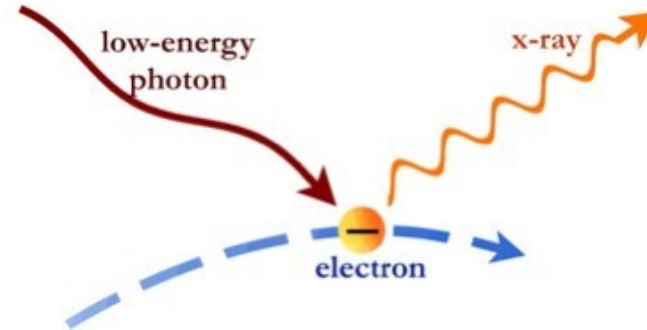
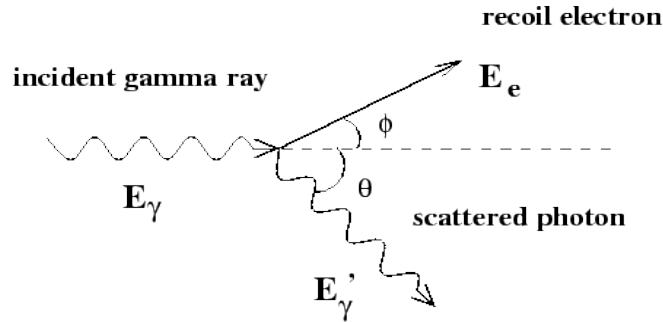
Image of the radio jet ( $> 100$  kpc) from the quasar 3C 273 from NASA's Hubble (optical, green), Chandra (X-ray, blue) and Spitzer (infrared, red) space telescopes.



### 3) Inverse Compton radiation



# basic characteristics



- Inverse Compton Scattering is the relativistic limit of Thomson scattering.
- Inverse Compton Scattering on high-energy electrons is an important source of X-ray and  $\gamma$ -ray radiation.
- Inverse Compton Scattering of Cosmic Microwave Background photons in regions of hot ionised gas leads to spectral distortions (Sunyaev-Zeldovich effect).
- (Inverse) Compton scattering is one of the main processes for energy exchange between matter and radiation (Comptonization).

# energy transfer

electron:  $p^\mu = [\gamma m_e c, \gamma m_e \vec{v}] \rightarrow p'^\mu = [\gamma' m_e c, \gamma' m_e \vec{v}']$

photon:  $k^\mu = \left[ \frac{\hbar \omega}{c}, \frac{\hbar \omega}{c} \vec{i}_k \right] \rightarrow k'^\mu = \left[ \frac{\hbar \omega'}{c}, \frac{\hbar \omega'}{c} \vec{i}_{k'} \right]$

energy-momentum conservation :

$$p^\mu + k^\mu = p'^\mu + k'^\mu$$

$$(p^\mu + k^\mu)(p_\mu + k_\mu) = (p'^\mu + k'^\mu)(p'_\mu + k'_\mu)$$

$$\rightarrow p^\mu p_\mu + 2p^\mu k_\mu + k^\mu k_\mu = p'^\mu p'_\mu + 2p'^\mu k'_\mu + k'^\mu k'_\mu$$

$$\rightarrow p^\mu k_\mu = p'^\mu k'_\mu$$

$$|^2$$

$$|\cdot k'_\mu$$

$$p^\mu k'_\mu + k^\mu k'_\mu = p'^\mu k'_\mu + k'^\mu k'_\mu$$

$$\rightarrow p^\mu k'_\mu + k^\mu k'_\mu = p'^\mu k'_\mu$$

$$p^\mu k'_\mu + k^\mu k'_\mu = p^\mu k_\mu$$

$$\rightarrow \frac{\omega'}{\omega} = \frac{1 - \left(\frac{v}{c}\right) \cos \theta}{\left[1 - \left(\frac{v}{c}\right) \cos \theta' + \left(\frac{\hbar \omega}{\gamma m_e c^2}\right) (1 - \cos \alpha)\right]}$$

$$\cos \alpha = \vec{i}_k \cdot \vec{i}_{k'} ; \cos \theta = \vec{i}_k \cdot \frac{\vec{v}}{v} ; \cos \theta' = \vec{i}_{k'} \cdot \frac{\vec{v}}{v}$$

For a **stationary electron** ( $v = 0, \gamma = 1$ ):  $\frac{\omega'}{\omega} = \frac{1}{\left[1 + \left(\frac{\hbar \omega}{m_e c^2}\right) (1 - \cos \alpha)\right]} \rightarrow \frac{\Delta \lambda}{\lambda} = \frac{\lambda' - \lambda}{\lambda} = \frac{\hbar \omega}{m_e c^2} (1 - \cos \alpha)$

# energy transfer

---

If the electron is more **energetic** than the photon ( $\gamma m_e c^2 \gg \hbar\omega$ ), we get the following:

$$\frac{\omega' - \omega}{\omega} \approx \frac{v}{c} \frac{\cos \theta' - \cos \theta}{1 - \frac{v}{c} \cos \theta'}$$

- energy transfer possible in both directions
- frequency change  $\sim v / c$
- for random scattering angles, no net increase in the photon energy to first order, but there is a net gain to second order (i.e.  $\sim v^2 / c^2$ ) -> **Inverse Compton Scattering**

Average energy gain for photons:

$$\hbar\bar{\omega} = \frac{4}{3} \gamma^2 \left(\frac{v}{c}\right)^2 \hbar\omega_0 \approx \frac{4}{3} \gamma^2 \hbar\omega_0$$

Maximum energy gain for photons:

$$\hbar\omega_{max} \approx 4\gamma^2 \hbar\omega_0$$

exact for  $\gamma\hbar\omega \ll m_e c^2$

Electrons with Lorentz factors of  $\sim 1000$  can thus convert radio- into UV-photons, IR-photons into X-rays and optical photons into soft  $\gamma$ -rays !

# cross-section

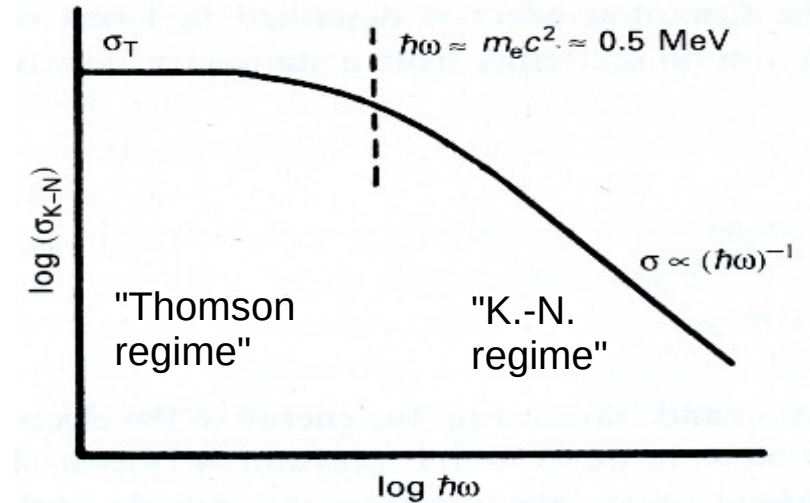
If  $\hbar\omega \lesssim m_e c^2$  in the rest frame of the electron (or  $\gamma\hbar\omega \lesssim m_e c^2$  in lab frame), the Thomson cross-section is still a good approximation. Otherwise the full quantum relativistic cross-section has to be used (Klein Nishina formula) :

$$\sigma_{KN} = \sigma_T \frac{3}{8} \frac{1}{\varepsilon} \left( \left[ 1 - \frac{2(\varepsilon+1)}{\varepsilon^2} \right] \ln(2\varepsilon+1) + \frac{1}{2} + \frac{4}{\varepsilon} - \frac{1}{2(2\varepsilon+1)^2} \right)$$

with:  $\varepsilon = \frac{\hbar\omega}{m_e c^2}$  ( in the rest frame of the electron )

$$\varepsilon < 1 \rightarrow \sigma_{KN} \approx \sigma_T$$

$$\varepsilon > 1 \rightarrow \sigma_{KN} \approx \pi r_e^2 \frac{1}{\varepsilon} \left( \ln 2\varepsilon + \frac{1}{2} \right)$$



*scheme of the K.N. cross-section (Longair)*

In the ultra-relativistic limit ("Klein-Nishina regime"), the cross-section decreases roughly as  $1 / \varepsilon$  and (inverse) Compton scattering becomes inefficient.

# emission spectrum

In the Thomson limit :

For an isotropic distribution of electrons of density  $N$  and Lorentz factor  $\gamma \gg 1$ , traveling through  $F_0$  photons of energy  $\epsilon_0$  per unit area / time / sr, the number of emitted photons (!) per unit volume / time / sr is :

$$j(E) = \frac{N \sigma_T F_0}{2 \gamma^2 \epsilon_0} (1-x) \quad \text{with} \quad x \equiv \frac{E}{4 \gamma^2 \epsilon_0} \quad \text{for} \quad 0 < x < 1$$

For electrons with a power-law energy distribution with index  $p$ , one finds an IC emission distribution following a power law with index

$$s = \frac{p-1}{2}$$

as for synchrotron emission.

Multiple up-scattering can occur in optically thick media, but is usually negligible in applications to AGNs. (cf. “Kompaneets Equation” in the non-relat. limit)

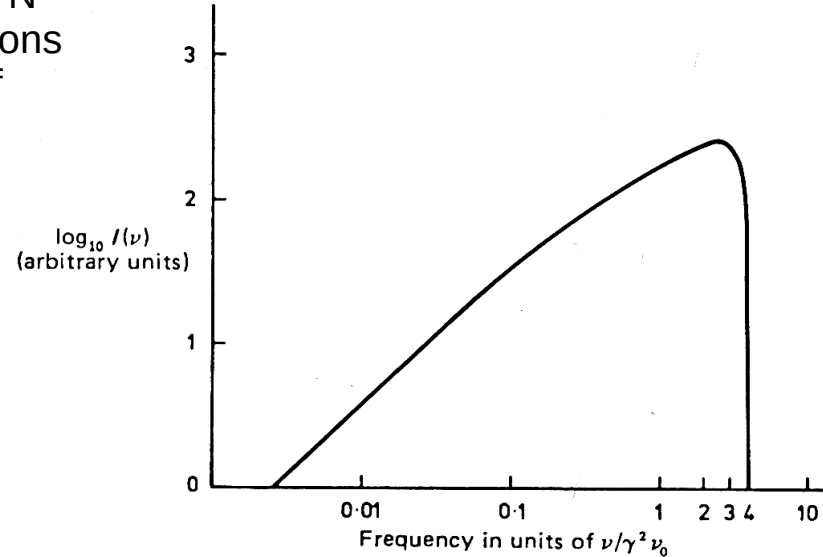


Figure 4.8. The emission spectrum of inverse Compton scattering;  $\nu_0$  is the frequency of the unscattered radiation. (From G. R. Blumenthal and R. J. Gould (1970). *Rev. Mod. Phys.*, **42**, 237.)

# emission spectrum Thomson vs. KN regime

e.g. IC scattering on black body emission in microquasar

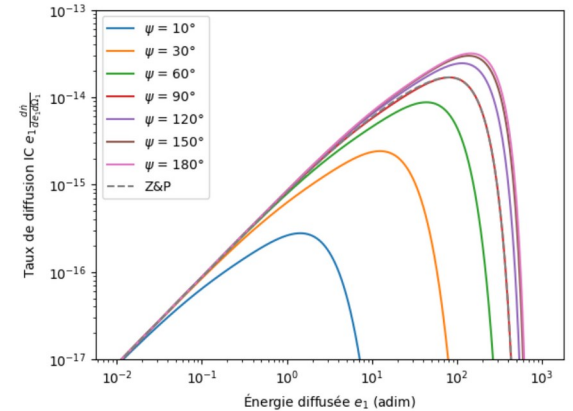
In the **Thomson regime**, IC emission on monoenergetic electrons conserves the spectral form of the target photons. Electrons transfer only a small part of their kinetic energy. The initial photon distribution is shifted to a higher energy with  $\epsilon_1 \approx \gamma^2 \epsilon_0$ .

In the **Klein-Nishina regime**, information on the target photon spectrum is lost and the upscattered photon energies peak around the electron energies. Electrons transfer a large fraction of their kinetic energy in a single interaction. Photon energies are shifted to  $\epsilon_1 \approx \gamma m_e c^2$  for  $\gamma \gg 1$

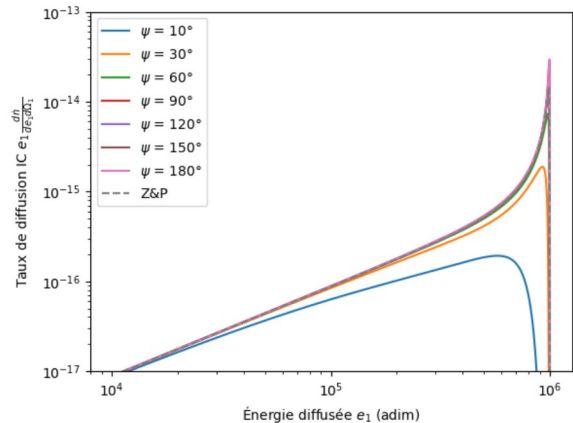
→ This sets an upper limit to the attainable energy !

Master thesis C. Kazantsev 2024:

*emission of a microquasar seen under different angles*



(a) Régime Thomson : électrons mono-énergétiques avec  $\gamma = 10^3$



(b) Régime Klein-Nishina : électrons mono-énergétiques avec  $\gamma = 10^6$



# polarisation

---

Inverse Compton scatterings reduce the initial polarisation degree of emission.

However, IC upscattering of polarised emission can still lead to a high degree of polarisation.  
e.g. synchrotron self-Compton emission, where the synchrotron emission is upscattered by its parent electron population.

The observed polarisation depends on the specific configuration of the emission scenario and is difficult to generalise.

for a detailed treatment of several scenarios, cf. *H. Krawczynski 2012*.

# Inverse Compton cooling

Energy loss follows similar expression as for synchrotron cooling:

$$\frac{dE}{dt} = \frac{4}{3} \sigma_T c \beta^2 \gamma^2 U_{ph} \quad (\text{in Thomson regime})$$

with  $U_{ph}$  the energy density of the target photons.

For an electron population in the presence of a magnetic field and photon field :

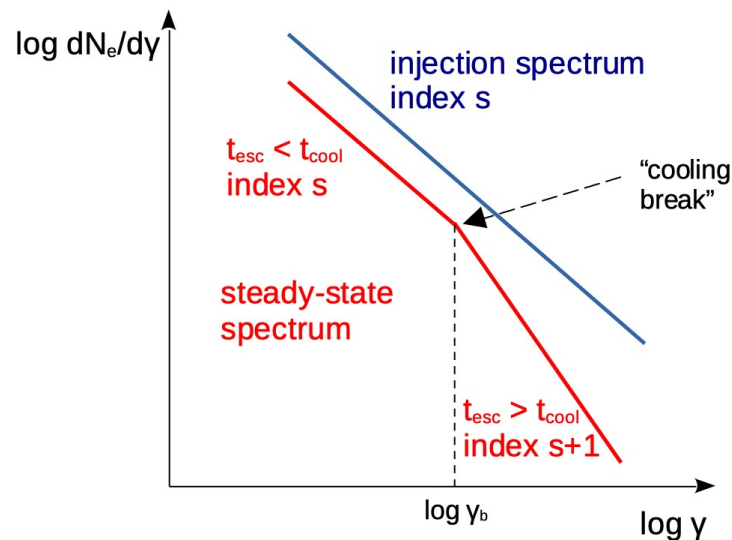
$$\frac{P_{IC}}{P_{synch}} = \frac{U_{ph}}{U_B} \quad (\text{in Thomson regime})$$

If this ratio  $> 1$ , “Inverse Compton catastrophe” is possible, i.e. all the energy of the electrons is lost at the highest energies.

Inverse Compton cooling time scale :

$$t_{IC} = \frac{3m_e c^2}{4\sigma_T c U_{ph} \gamma \beta^2}$$

For continuous injection of a power-law electron distribution and particle escape, IC cooling can again lead to a “cooling break” in the steady-state spectrum.



# Compton rocket & Compton drag

Interaction of plasma with anisotropic photon fields has an effect on the bulk motion of the electron population.

**cold matter** : radiation pressure from an anisotropic photon field

$$f = (\sigma_T/c)S$$

**hot plasma** : more efficient by about a factor of  $\gamma^2$

*“Owing to the Doppler effect, particles moving towards the main light source will scatter photons of higher energy and with a higher rate than those moving outwards. This will naturally lead to an anisotropic inverse Compton emission, most of it going back to the main photon source.”*

*(T. Vuillaume et al. 2015)*

*( Compton rocket effect of bulk acceleration )*

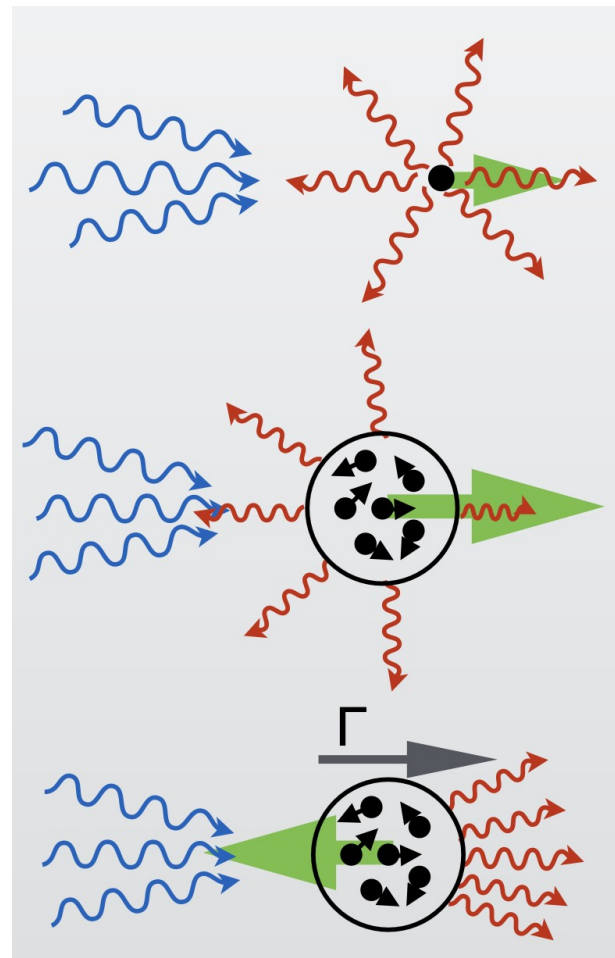
**hot plasma with relativistic bulk motion ( $\Gamma$ ) :**

emission beamed in forward direction leads to recoil force

*( Compton drag / inverse rocket effect of bulk deceleration )*

Important effect on a relativistic pair-plasma.

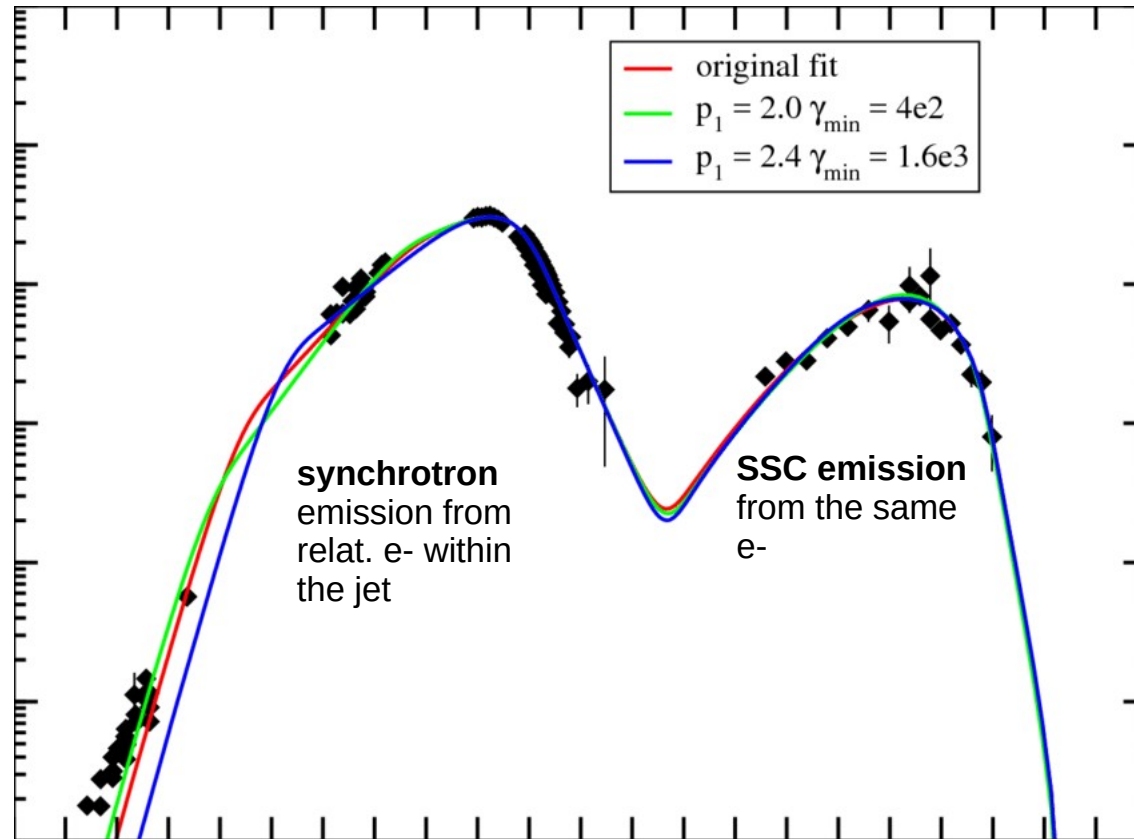
Much less important for an electron-proton plasma.



R.Belmont

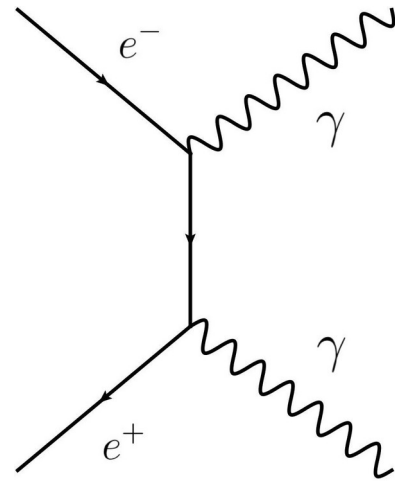
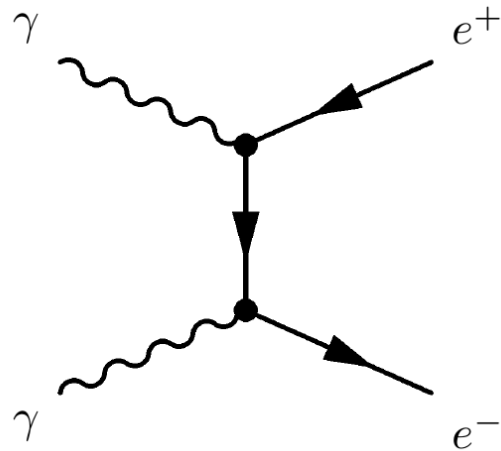
38

# an example in astrophysics



Synchrotron Self Compton (SSC) models for the emission from the blazar Mrk 421

## 4) pair production & annihilation



# pair annihilation characteristics

---

Electron-positron can proceed in two ways:

## 1. $e^+ e^- \rightarrow 2 \gamma$

When the electron and positron are almost at rest when interacting, both photons have energy 0.511 MeV. When they interact "in flight", there is a range of possible photon energies.

## 2. positronium

If the velocity of the positron and electron is small, positronium, i.e. a bound states of an electron and a positron can form.

25% of the p.a. form in the singlet  $^1S_0$  state ("para-positronium"), 75% in the triplet  $^3S_1$  state ("ortho-positronium").

The **singlet state decays into two  $\gamma$ -rays**, both with energy 0.511 MeV (lifetime:  $1.25 \times 10^{-10}$  s).

The **triplet state decays into three  $\gamma$ -rays**, with a range of energies, with maximum energy 0.511 MeV (lifetime:  $1.5 \times 10^{-7}$  s).

(More frequently, the triplet state annihilates with nearby electrons in a few ns though...)

# pair annihilation cross-section

The formation of positronium is only possible in regions that are not too dense and not too hot. The characteristic shape of the 0.511 MeV spectral line is thus a diagnostic tool in  $\gamma$ -ray astronomy.

Cross-section for  $e^+ e^-$  annihilation

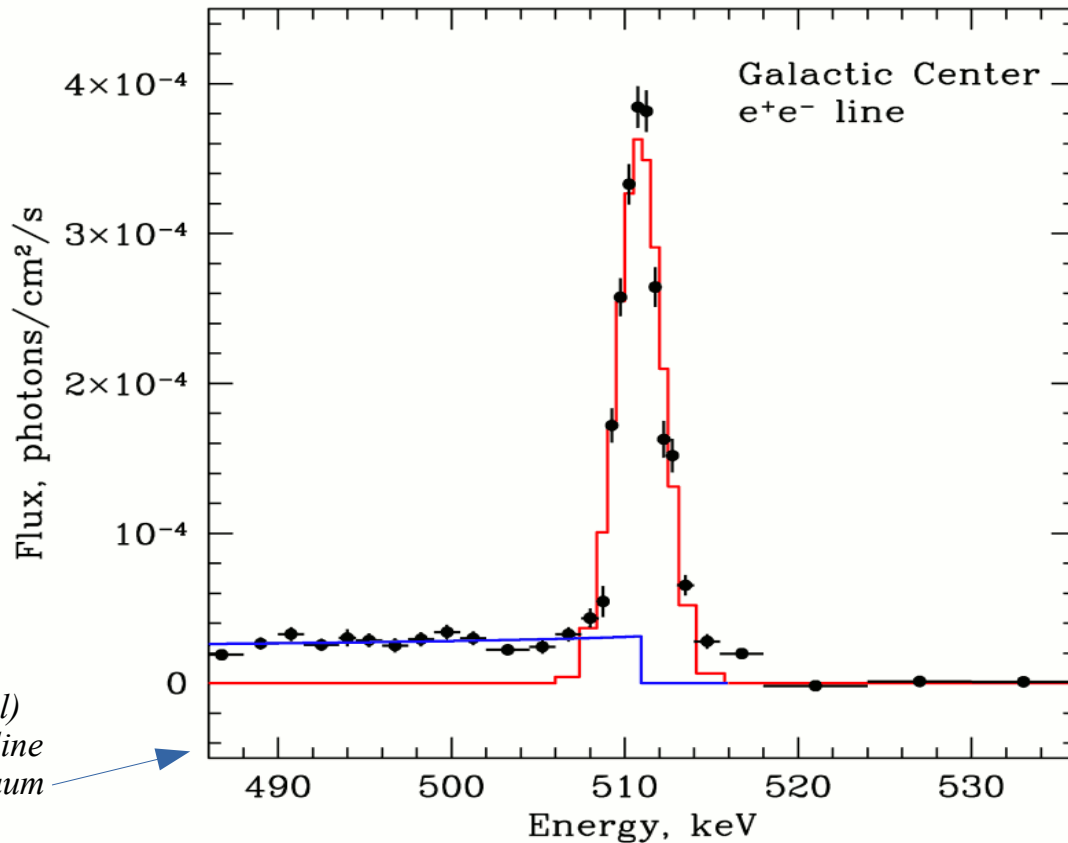
extreme relativistic limit:

$$\sigma = \frac{\pi r_e^2}{\gamma} [\ln 2\gamma - 1] \quad \text{with} \quad r_e = \frac{\alpha \hbar}{m_e c}$$

thermal  $e^-$  and  $e^+$  ( $\sim 25$  meV,  $E_{\text{kin}} \sim kT$ ):

$$\sigma \approx \frac{\pi r_e^2}{\beta}$$

*Spectrum of the  $e^+e^-$  annihilation radiation detected by SPI (Integral) towards the GC region after 3.5 million seconds exposure. The red line shows the positron annihilation line, the blue line shows the continuum spectrum associated with the three photon ortho-positronium decay.*





# pair production characteristics

---

Conversion of high-energy photons into particle / antiparticle pairs.  
In most environments, production of  $e^+e^-$  pairs is dominant over other particle species (low energy threshold, large cross-section).

The target photon can be a free photon (astrophysical / cosmological radiation fields) or a virtual photon (e.m. field of a nucleus, pulsar magnetosphere...).

P.p. by a single photon not possible in free space.

I. energy conservation:  $\hbar\omega = 2\gamma m_e c^2 \quad \rightarrow \quad p = 2\gamma m_e v = \frac{\hbar\omega}{c^2} v$

II. momentum conservation:  $\frac{\hbar\omega}{c} \neq p$

Important energy loss mechanism at the highest photon energies.

# pair production threshold in $\gamma - \gamma$ interactions

$$k_1^\mu = \left[ \frac{\varepsilon_1}{c}, \frac{\varepsilon_1}{c} \vec{i}_1 \right] ; k_2^\mu = \left[ \frac{\varepsilon_2}{c}, \frac{\varepsilon_2}{c} \vec{i}_2 \right] ; p_{1,CM}^\mu = [mc, \vec{0}] ; p_{2,CM}^\mu = [mc, \vec{0}]$$

$$k_1^\mu k_{1\mu} = k_2^\mu k_{2\mu} = 0 \\ p_1^\mu p_{1\mu} = p_2^\mu p_{2\mu} = p_{1,CM}^\mu p_{1,CM\mu} = p_{2,CM}^\mu p_{2,CM\mu} = m^2 c^2$$

Conservation of the four-momentum square (Lorentz scalar!) between lab frame and center of momentum frame :

$$k^\mu k_\mu = (p^\mu p_\mu)_{CM} \\ (k_1^\mu + k_2^\mu) \cdot (k_{1\mu} + k_{2\mu}) = (p_1^\mu + p_2^\mu) \cdot (p_{1\mu} + p_{2\mu}) \\ k_1^\mu k_{1\mu} + 2 k_1^\mu k_{2\mu} + k_2^\mu k_{2\mu} = p_1^\mu p_{1\mu} + 2 p_1^\mu p_{2\mu} + p_2^\mu p_{2\mu} \\ 2 k_1^\mu k_{2\mu} = 4(m^2 c^2) \\ 2 \left( \frac{\varepsilon_1 \varepsilon_2}{c^2} - \frac{\varepsilon_1 \varepsilon_2}{c^2} \cos \theta \right) = 4 m^2 c^2 \rightarrow \varepsilon_2 = \frac{2 m^2 c^4}{\varepsilon_1 (1 - \cos \theta)}$$

The **threshold** occurs when the angle between the two photons  $\theta = \pi$ , hence follows for  $e^+e^-$  pair production:

$$\varepsilon_2 \geq \frac{m_e^2 c^4}{\varepsilon_1} = \frac{0.26 \times 10^{12}}{\varepsilon_1 [eV]} eV$$

e.g. a photon of energy  $> \sim 100$  TeV will undergo pair production even with the low energy photons of the CMB.

# pair production cross-section in $\gamma$ - $\gamma$ interactions

## 1. cross-section in the classical regime

$$\sigma = \sigma_T \frac{3}{8} \left( 1 - \frac{m_e^2 c^4}{\omega^2} \right)^{\frac{1}{2}} ; \quad \omega = \sqrt{(E_1 E_2)}$$

valid for:  $\omega \approx m_e c^2$

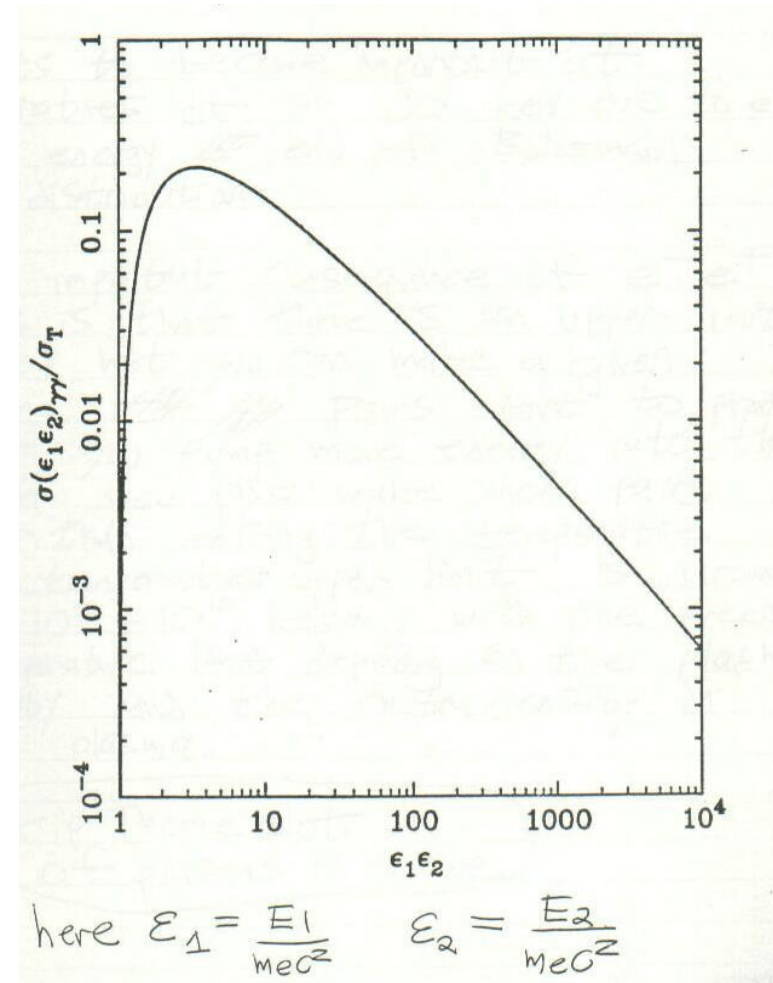
## 2. cross-section in the ultra-relativistic limit

$$\sigma = \sigma_T \frac{3}{8} \frac{m_e^2 c^4}{\omega^2} \left[ 2 \ln \left( \frac{2\omega}{m_e c^2} \right) - 1 \right]$$

valid for:  $\omega \gg m_e c^2$

peak :  $E_1 E_2 = 2(m_e c^2)^2$

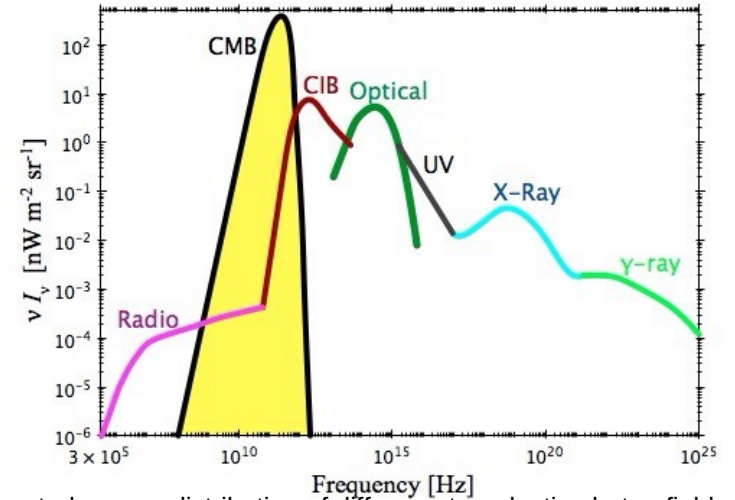
With the help of these cross-sections, one can determine the opacity of the interstellar and intergalactic medium for high energy photons and determine the flux of generated positrons.



# pair production in astrophysics

## diffuse photon backgrounds:

- EBL (extragalactic background light = CIB + optical bkg.) : direct/ dust-reprocessed starlight emitted during the history of the universe in the optical/infrared.
- CMB (cosmic microwave background) : "relic" radiation decoupling from the matter content of the universe after the epoch of recombination

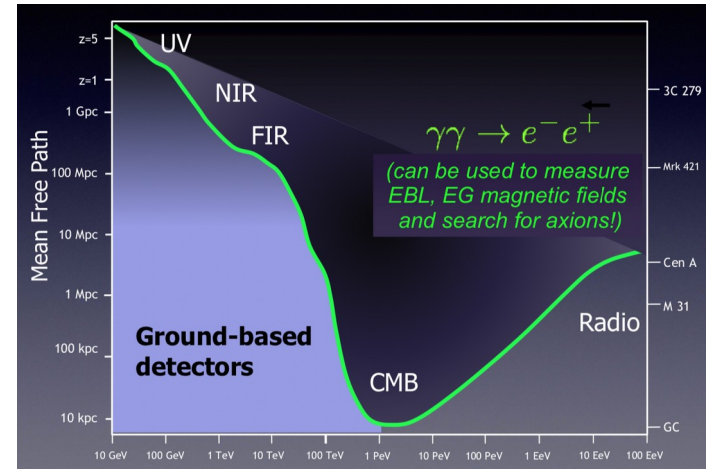


spectral energy distribution of diffuse extragalactic photon fields

## y-ray absorption on diffuse photon fields:

- PeV y-rays absorbed on  $10^{-4} - 10^{-3}$  eV photons (e.g. CMB)*
- TeV y-rays absorbed on optical / IR photons (e.g. EBL)*
- GeV y-rays absorbed on UV / soft X-rays (e.g. BLR)*

y-y opacity on ambient (stellar, dust,...) photon fields inside the source (e.g. AGN) can prevent transmission of TeV y-rays.



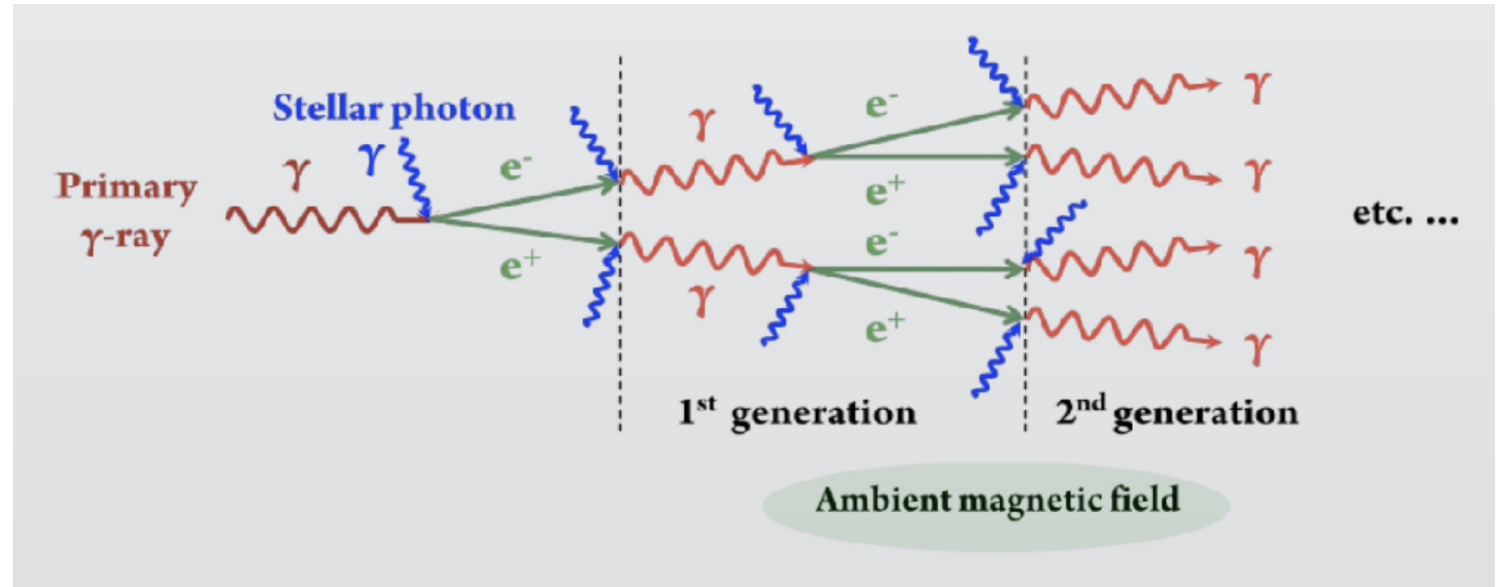
H. della Volpe

# pair cascades

In optically thick targets, the combination of pair production and a radiative process can lead to electromagnetic cascades that dissipate the energy of an initial high-energy particle.

examples :

- $\gamma$ -ray or  $e^-/e^+$  induced **synchrotron-pair cascade** in emission regions with strong B-fields and ambient photon fields

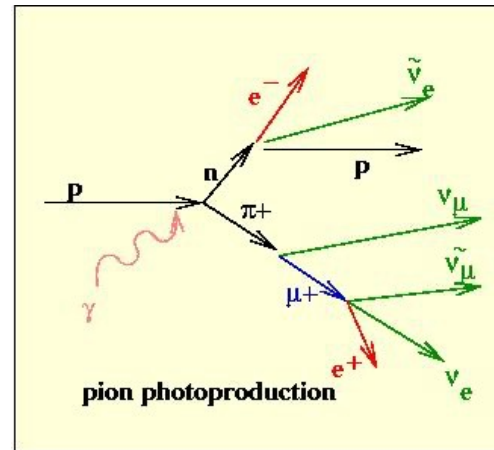
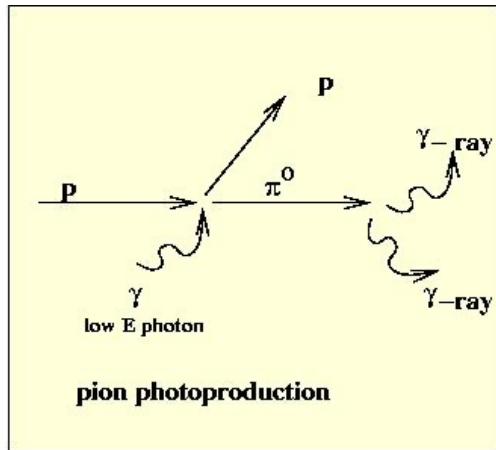


*R.Belmont*

- $\gamma$ -ray or  $e^-/e^+$  induced **IC – pair cascades** in the interstellar / intergalactic medium (upscattering of CMB / EBL / stellar photons and pair production)

- $\gamma$ -ray or  $e^-/e^+$  induced **electromagnetic air shower** (pair-production & bremsstrahlung in the atmosphere)

## 5) Pion production & decay



# pion production reactions & threshold

---

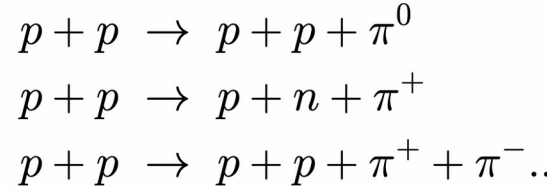
Hadronic interactions at high energies lead to the production of mesons, and in particular the lightest  $\pi$ -mesons ( $\pi^0$ ,  $\pi^+$ ,  $\pi^-$ ) with masses  $m_{\pi^\pm} \sim 139.6 \text{ MeV}/c^2$  and  $m_{\pi^0} \sim 134.97 \text{ MeV}/c^2$ .

- pion production through **nucleon-nucleon** interactions ( e.g.  $p + p \rightarrow p + p + \pi^0$  )
  - requires dense environment (e.g. protons accelerated in supernova remnants colliding with ISM or molecular clouds)
- photo-pion-production through **nucleon-photon** interactions ( e.g.  $p + \gamma \rightarrow p + \pi^0$  )
  - require dense target photon fields
- the same holds for interactions of **nuclei** ( e.g.  $A + p \rightarrow A + p + \pi^0$  ,  $A + \gamma \rightarrow A + \pi^0$  )

# pion production reactions & threshold

---

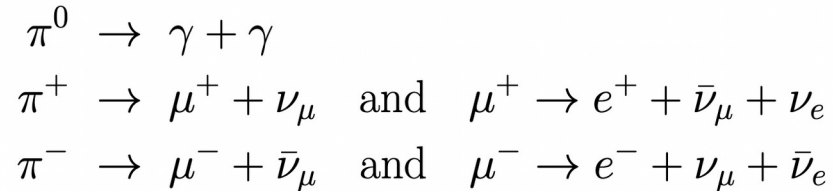
## 1. inelastic collisions of protons (or nuclei) with **ambient matter**



At high energies, the three types of pions are produced with similar probabilities.

The pions decay very quickly :  $\tau_{1/2 \pi^{+-}} \sim 2.5\text{e-}8 \text{ s}$  ,  $\tau_{1/2 \pi^0} \sim 8.4\text{e-}17 \text{ s}$

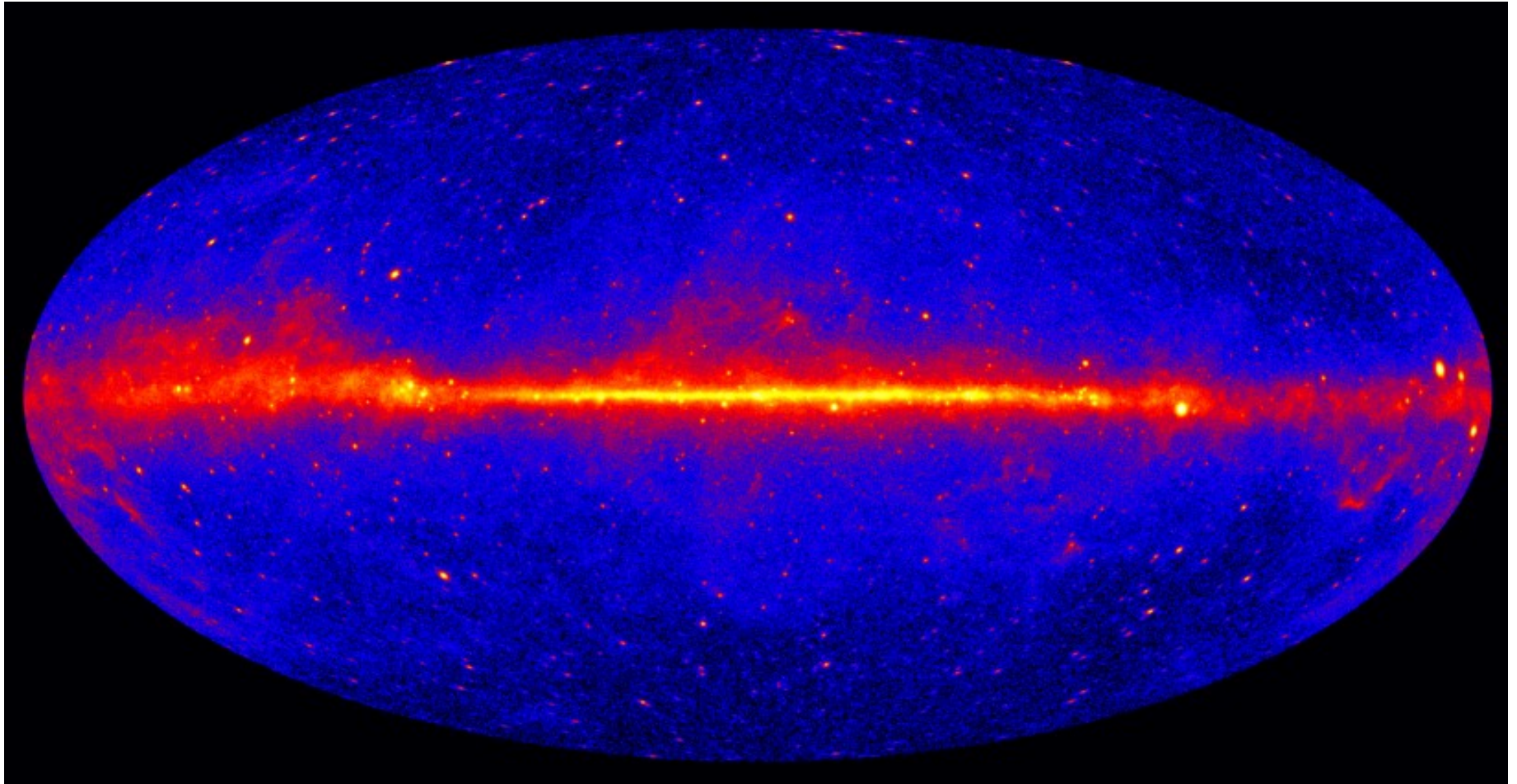
Main decay channels :



Note the production of neutrinos that is accompanying any charged pion decays !



# an example in astrophysics



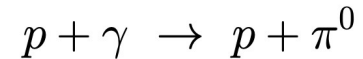
5 years of Fermi-LAT data  $> 1$  GeV, Gal. Coord.

# pion production reactions & threshold

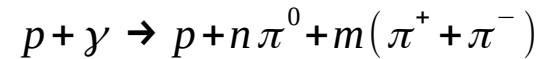
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## 2. inelastic collisions of protons (or nuclei) with **ambient photon fields**

dominant reactions :

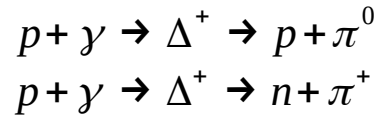


at high energies, multi-pion production dominates :



# py pion production cross section

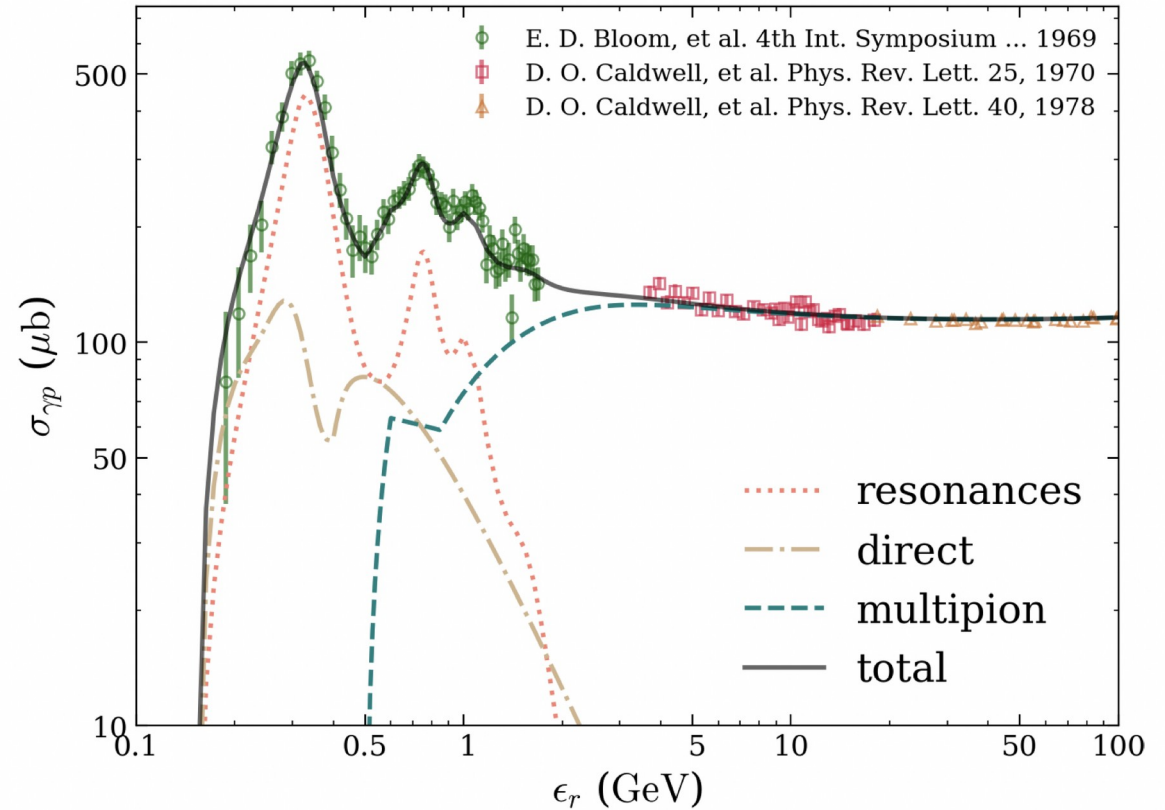
In  $p\gamma$  interactions, Delta resonances play an important role close to the threshold :



An example in astrophysics is the GZK effect, i.e. collision of ultra-high-energy cosmic rays with photons from the CMB.  
→ Suppression of UHECR proton flux above about  $\sim 6 \times 10^{19}$  eV.

Widely used MC code including detailed cross-sections : SOPHIA

<https://www.uibk.ac.at/projects/he-cosmic-sources/tools/sophia/index.html.en>



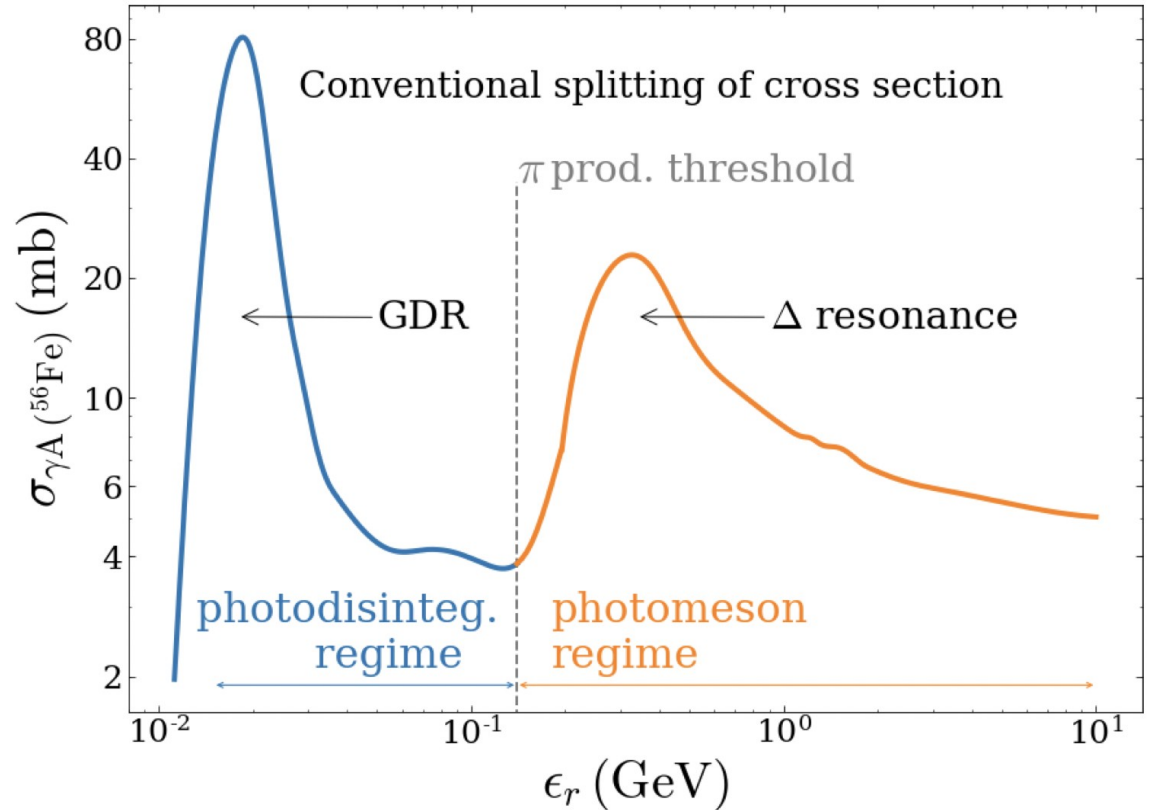
*L. Morejon et al JCAP11(2019)007*

# A<sub>γ</sub> pion production

“The total inelastic photonuclear cross section for <sup>56</sup>Fe as a function of photon energy in the nucleus’ rest frame illustrates the general shape for nuclei.

- The photodisintegration portion (in blue) refers to photon energies  $\epsilon_r$  below the photopion production threshold ( $\sim 140$  MeV)

- The photomeson portion (in orange) refers to photon energies above the photopion production threshold.”



GDR = giant dipole resonance

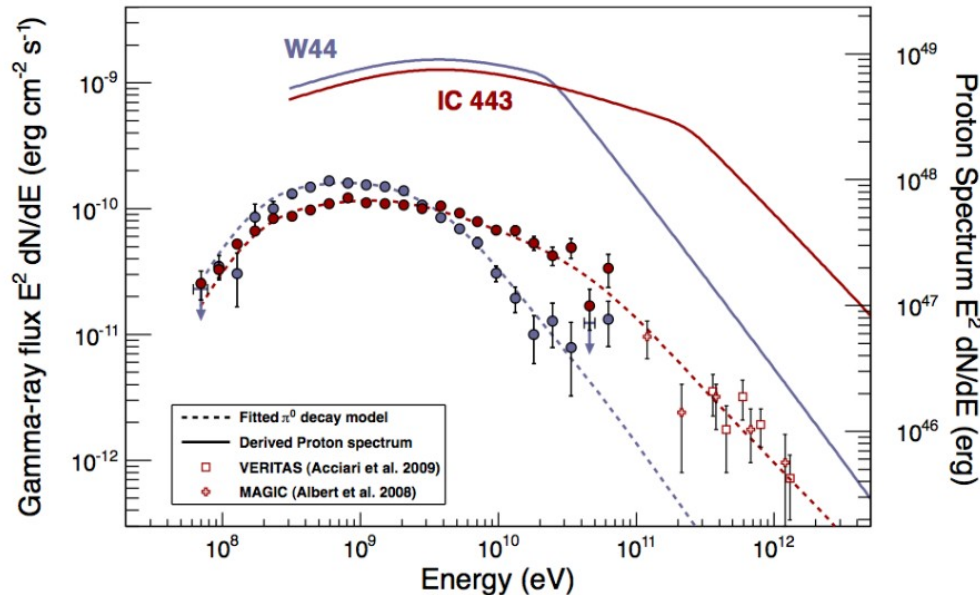
L. Morejon et al JCAP11(2019)007

# $\gamma$ -ray spectrum from pion decay

Resulting spectrum from neutral pion decay ( $\pi^0 \rightarrow 2\gamma$ ):

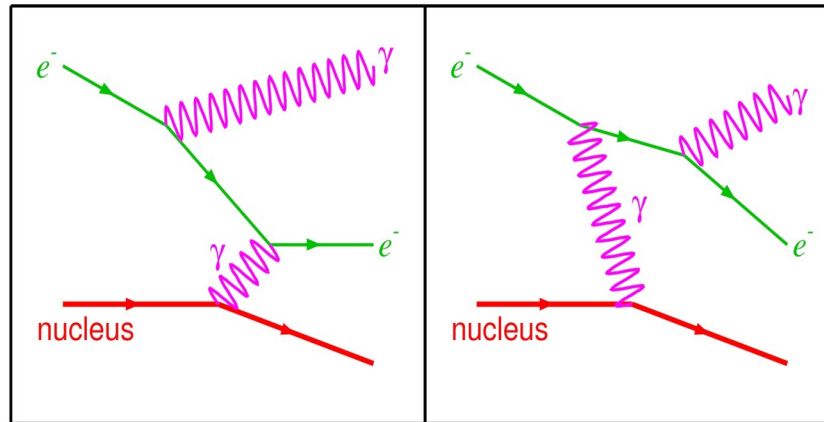
In the pion's center of momentum frame, the two photons have an energy of  $\frac{1}{2} m_{\pi^0} c^2 \sim 67.5$  MeV.

- Broad bump at 67.5 MeV in the differential  $\gamma$ -ray spectrum, independently of the shape of the pion distribution.
- Characteristic low-energy bump in the SED between 100 MeV and a few GeV.



GeV/TeV  $\gamma$ -ray spectra from SNR  
(Ackermann et al., *Science* 339 (2013) 807):  
detection of the pion-decay signature  
in the Fermi spectra of IC443 and W44

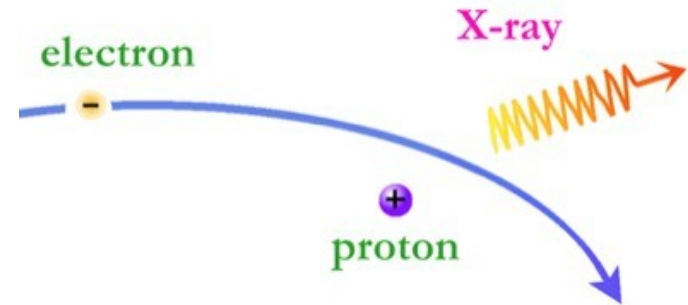
## 6) Bremsstrahlung



<http://www-zeus.physik.uni-bonn.de/~brock>

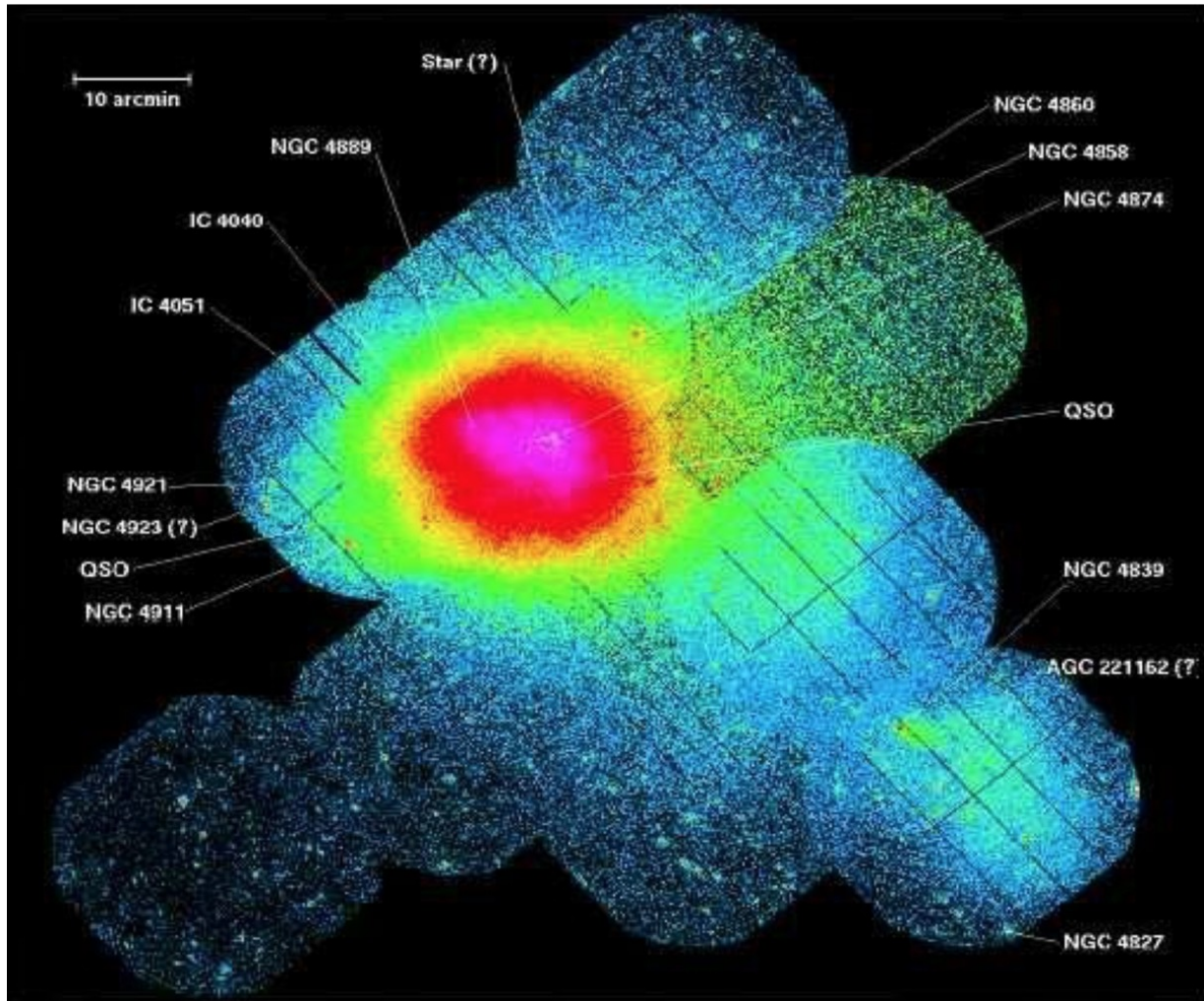
# bremsstrahlung

- "Bremsstrahlung" (braking radiation = "free-free emission") is the electromagnetic radiation of electrons that are accelerated (or decelerated) in the electrostatic field (Coulomb field) of a nucleus.
- Significance for High Energy Astrophysics: one of the principal sources of thermal radiation; hot ionised gas emits free-free radiation (radio emission from ionised hydrogen, X-ray emission from binaries and clusters of galaxies)





# e.g. bremsstrahlung emission from Coma cluster



The intra-cluster medium (ICM) in galaxy clusters emits thermal X-rays from bremsstrahlung.

Here the Coma cluster (size  $\sim 1$  Mpc,  $z = 0.0232$ ).



# bremsstrahlung - Bethe-Heitler formula

A full relativistic quantum treatment results in the bremsstrahlung formula derived by Bethe and Heitler:

$$-\left(\frac{dE}{dt}\right) = \frac{Z(Z+1.3)e^6 N}{16\pi^3 \epsilon_0^3 m_e^2 c^4 \hbar} \cdot E \cdot \left[ \ln\left(\frac{183}{Z^{1/3}}\right) + \frac{1}{8} \right]$$

- $(Z+1.3)$  accounts for interactions between the high energy electron and the bound electrons
- $Z^{-1/3}$  accounts for screening of nuclei by bound electrons (scales with the radius of the atom).
- the energy loss rate in the relativistic case is proportional to the kinetic energy  $E$  of the h.e. electron.

## Radiation Length

for highly relativistic electrons:  $v \approx c$

$$\begin{aligned} -\frac{dE}{dt} &= -\frac{dE}{dx} c \rightarrow -\frac{dE}{dx} \propto E \\ -\frac{dE}{dx} &\equiv \frac{E}{X_0} \rightarrow E(x) = E_0 \exp\left(-\frac{x}{X_0}\right) \end{aligned}$$

# bremsstrahlung - radiation length

The constant  $X_0$  is called «**radiation length**». When traversing one radiation length, the electron loses a fraction of  $(1 - 1/e)$  of its energy. It is often described as a "grammage"  $\xi_0 = \rho X_0$ .

$$-\frac{dE}{d\xi} = -\frac{dE}{dt} \frac{1}{\rho c} = \frac{E}{\rho X_0} = \frac{E}{\xi_0} \rightarrow E(\xi) = E_0 \exp\left(-\frac{\xi}{\xi_0}\right)$$

$\xi_0$  is also called the **stopping power** of a material. We can find a numerical expression by replacing  $\rho$  with  $N^* M_{\text{atom}} / N_A$  and by inserting the Bethe-Heitler result for  $-dE/dt$ :

$$\xi_0 = \frac{7160 M_{\text{atom}} / (\text{g mol}^{-1})}{Z(Z+1.3) \left[ \ln(183 Z^{-\frac{1}{3}}) + \frac{1}{8} \right]} \text{ kg m}^{-2}$$

$N$	atomes per volume
$M_{\text{atom}}$	molar mass
$N_A$	Avogadro's constant ( $\sim 6.022 \times 10^{23} \text{ mol}^{-1}$ )

radiation lengths for different materials:

<b>hydrogen</b>	$\xi_0 = 580 \text{ kg/m}^2$	$X_0 = 6.7 \text{ km}$
<b>air</b>	$\xi_0 = 365 \text{ kg/m}^2$	$X_0 = 280 \text{ m}$
<b>lead</b>	$\xi_0 = 58 \text{ kg/m}^2$	$X_0 = 5.6 \text{ mm}$

see also: <http://pdg.lbl.gov/2015/AtomicNuclearProperties/>

# thermal bremsstrahlung

Consider a gas in a thermal (**Maxwell Boltzmann**) distribution at temperature  $T$ :

$$N_{e,v}(v)dv = 4\pi N_e \left(\frac{m_e}{2\pi kT}\right)^{3/2} v^2 \exp\left(\frac{-m_e v^2}{2kT}\right) dv$$

To derive the bremsstrahlung emission from this gas, one integrates the non-relativistic emission formula over the velocity distribution of the electrons. One arrives at the **emissivity** ( in  $[\text{ergs s}^{-1} \text{cm}^{-3} \text{Hz}^{-1}]$ ):

$$\frac{dW}{dV dt df} \approx 6.8 \times 10^{-38} Z^2 N_e N_i T^{-1/2} \exp\left(\frac{-hf}{kT}\right) g(T, f)$$

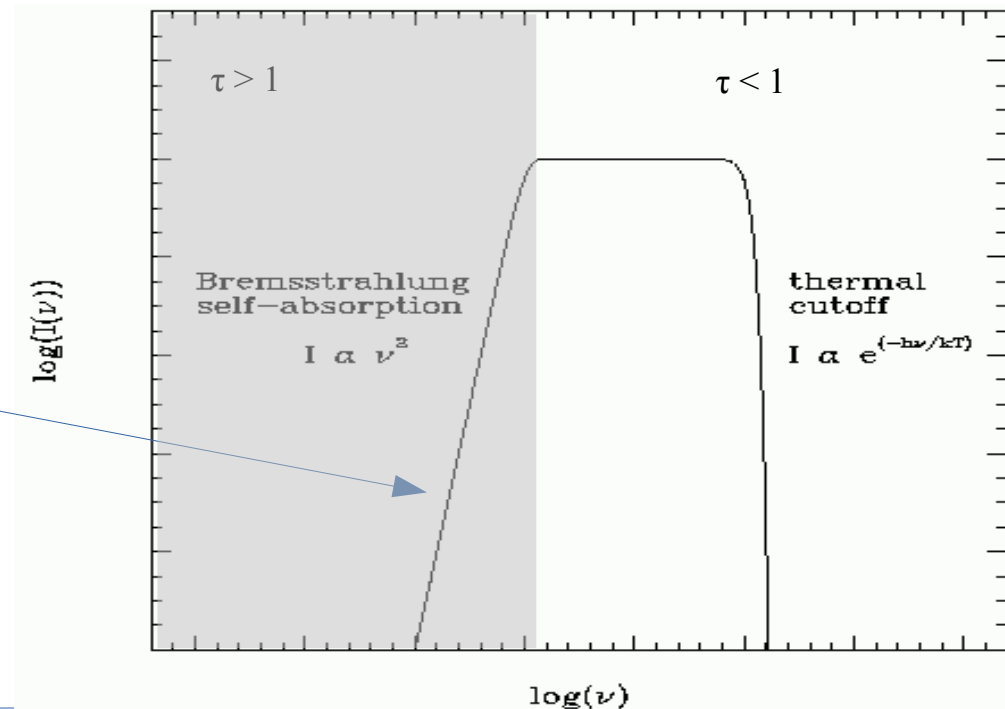
$g(T, f)$  is the Gaunt factor (includes quantum corrections)

- One can observe bremsstrahlung only from media that are sufficiently **optically thin**. In this case, the radiation can escape from the emission region without (or with little) interaction with the medium.
- In **optically thick** media, radiation is (completely) absorbed. The spectral shape is given by an equilibrium of absorption and emission processes, leading to blackbody radiation. Thermal radiation becomes blackbody radiation for optically thick media.

# bremsstrahlung self-absorption

In astrophysics, bremsstrahlung (self-)absorption is important in dense regions (e.g. compact HII regions which lie close to star formation regions).

At low frequencies, one observes the Rayleigh-Jeans tail of the black-body spectrum.



# non-thermal bremsstrahlung

---

Non-thermal bremsstrahlung arises from an **underlying non-thermal electron distribution**. To derive the bremsstrahlung spectrum, one has to integrate over the velocity distribution of the electrons (using the Bethe-Heitler formula for relativistic electrons).

It can be shown that a power-law distribution of relativistic electrons

$$N(E) \propto E^{-\Gamma}$$



leads to a distribution of photons that follows **the same power-law**:

$$N_{\gamma}(\epsilon) \propto \epsilon^{-\Gamma}$$

Bremsstrahlung from ultra-relativistic electrons interacting with cold interstellar matter is thought to contribute to the low-energy  $\gamma$ -ray flux detected from our galaxy (in the range of 30 - 100 MeV).

# bibliography (section I)

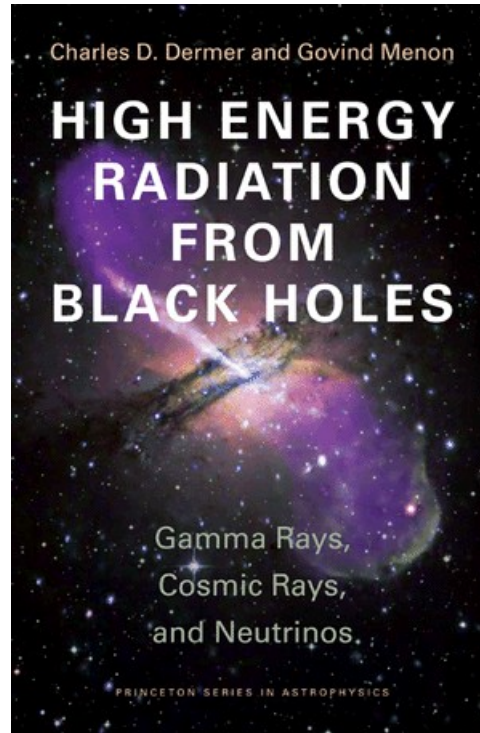
further reading for a deeper understanding ...

PHYSICS TEXTBOOK

George B. Rybicki  
Alan P. Lightman

WILEY-VCH

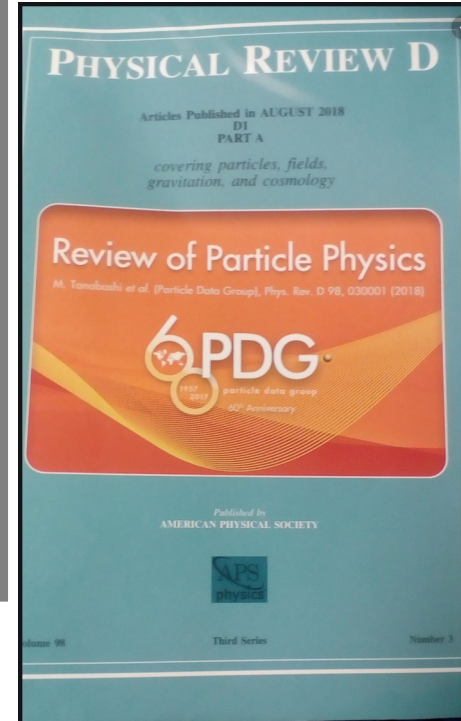
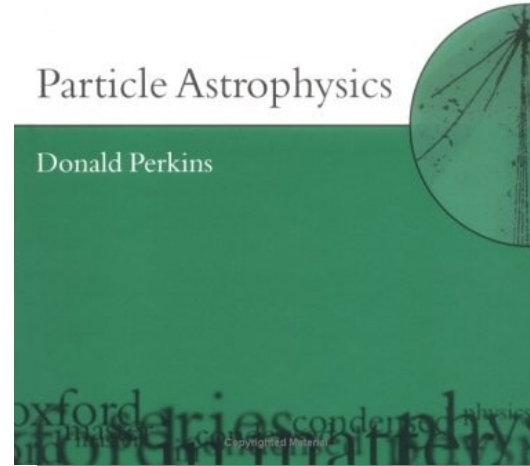
## Radiative Processes in Astrophysics



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