Echo State Network for Dynamic Aperture prediction

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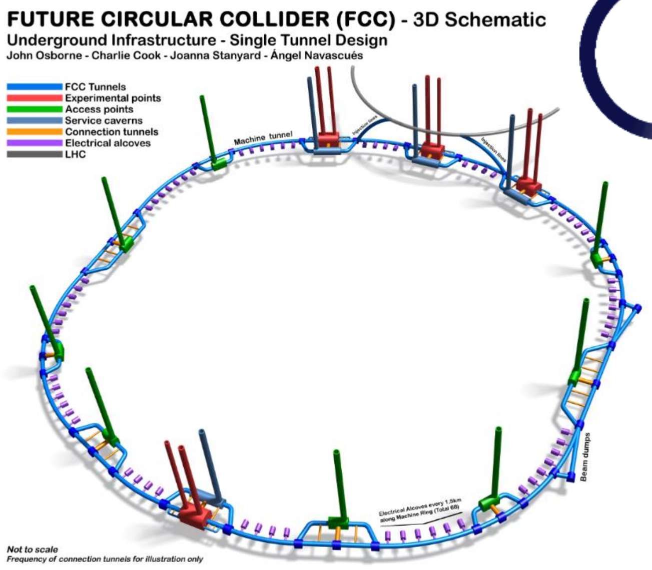
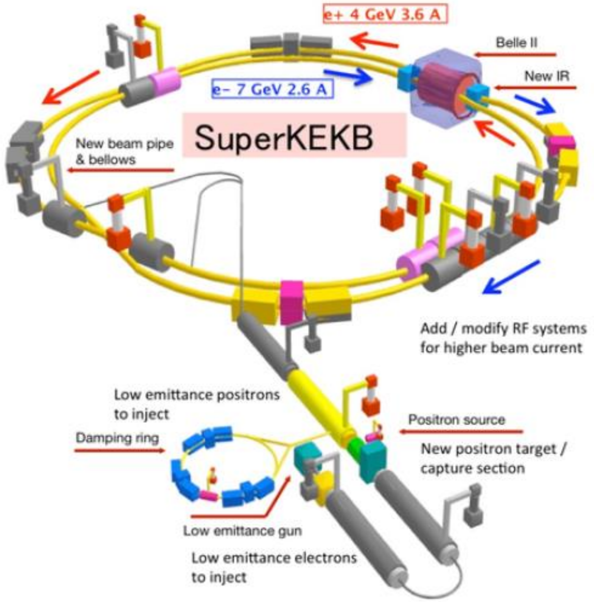
Outline

- Dynamic aperture
- ESN for DA prediction
- Results

FCC and SuperKEKB



FUTURE CIRCULAR COLLIDER



KEK

International FCC project (CERN as host lab)

Continue to study experimental high energy particle physics
Accelerator options: FCC-hh, FCC-ee, FCC-he

CDRs publié dans **European Physical Journal C (Vol 1) and ST (Vol 2-4)**

http://fcc-cdr.web.cern.ch/

Existing e+e- collider:
small size FCC-ee
proofs of principle of several design choices

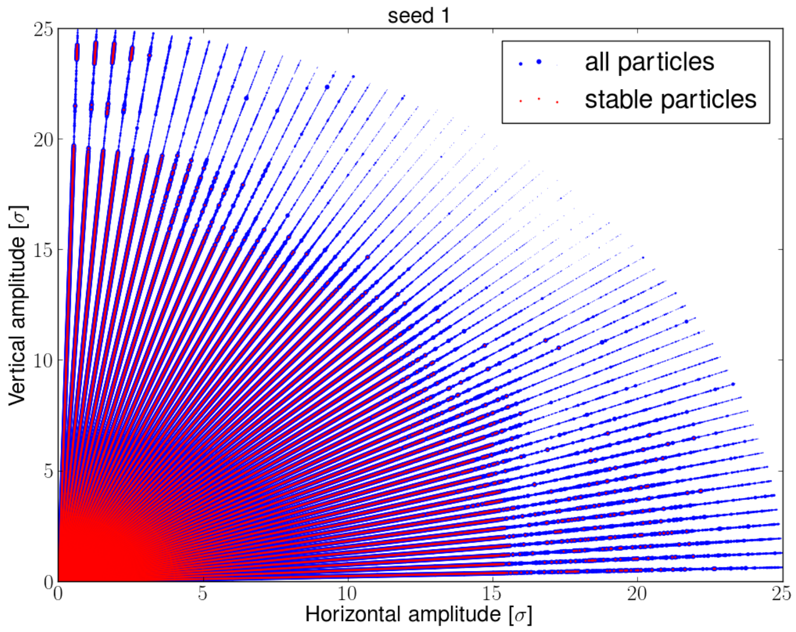
https://doi.org/10.1093/ptep/pts083

1. Dynamic Aperture (DA) prediction

Dynamic Aperture

Long-term Dynamic Aperture (DA):

- defines the region of stable motion of a particle in an accelerator
- is used to define tolerances on magnetic field quality
- is used to optimize the performance of the circular accelerators (beam losses, beam lifetime)
- corrects linear and non linear imperfections



Computed as the initial amplitude corresponding to particle lost after the 10^N revolutions in the accelerator (typically N=5,6)

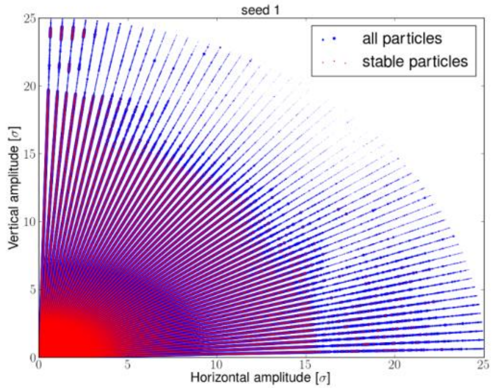
Particle tracking simulations:
From 1 day to one week or more…

Dynamic Aperture

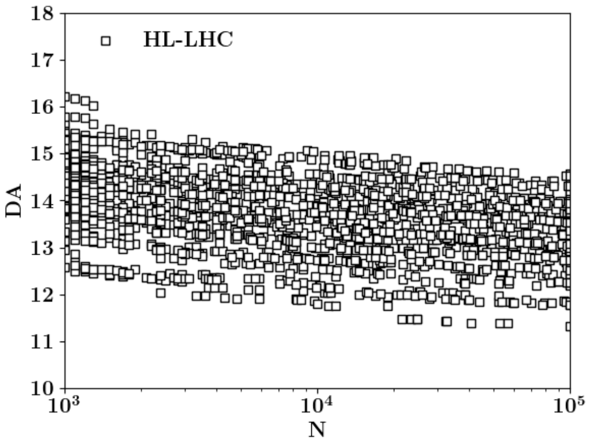
Bonded movement phase-space region

$$\mathcal{A}(N) = \int\int\int\int \chi(x_1, x_2, p_{x_1}, p_{x_2}) dx_1 dx_2 dp_{x_1} dp_{x_2}$$

$$\chi(x_1, x_2, p_{x_1}, p_{x_2}) = \begin{cases} 1 \text{ if the motion starting at } (x_1, x_2) \\ \text{with momentum } (p_{x_1}, p_{x_2}) \text{ is bounded after } N \text{ turns} \\ 0 \text{ else} \end{cases}$$



$$DA = \left(\frac{2\mathcal{A}(N)}{\pi^2}\right)^{\frac{1}{4}}$$



- Tracking simulations: initial particle amplitude of particle lost at 10^N revolutions for 60 different machine error configurations (seeds)
- Average over particle angles in x-y space: define a radius of the stable sphere (with r_s last stable radius at given angle and number of turns)

Analytical Scaling Laws and DA extrapolation

- Based on Nekhoroshev theorem

It proves the stability of a quasi integrable Hamiltonian system for finite but exponentially long time (N turns) on an open set of initial conditions (amplitudes)

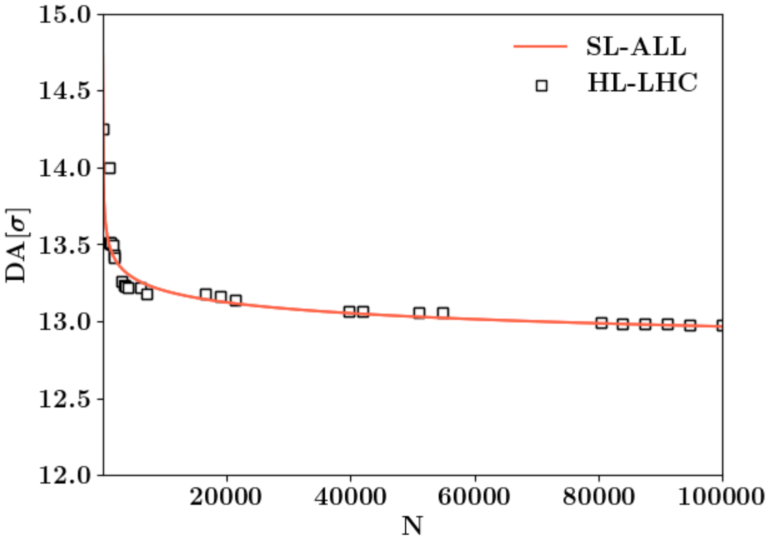
- Time evolution of DA (The Scaling Law):

$$DA^{SL} = \rho_* \left(\frac{\kappa}{2e}\right) \frac{1}{\ln(N)^\kappa}$$

[Phys. Rev. Acc. Beams 22 104003]

Two fitting parameters κ and ρ_*

It is used to fit (with Least Square Method) the DA data



- In order to improve the quality of the fit, **Gaussian Processes** are used to increase artificially the DA data

[https://www.mdpi.com/2078-2489/12/2/53]

AI applications

Goal

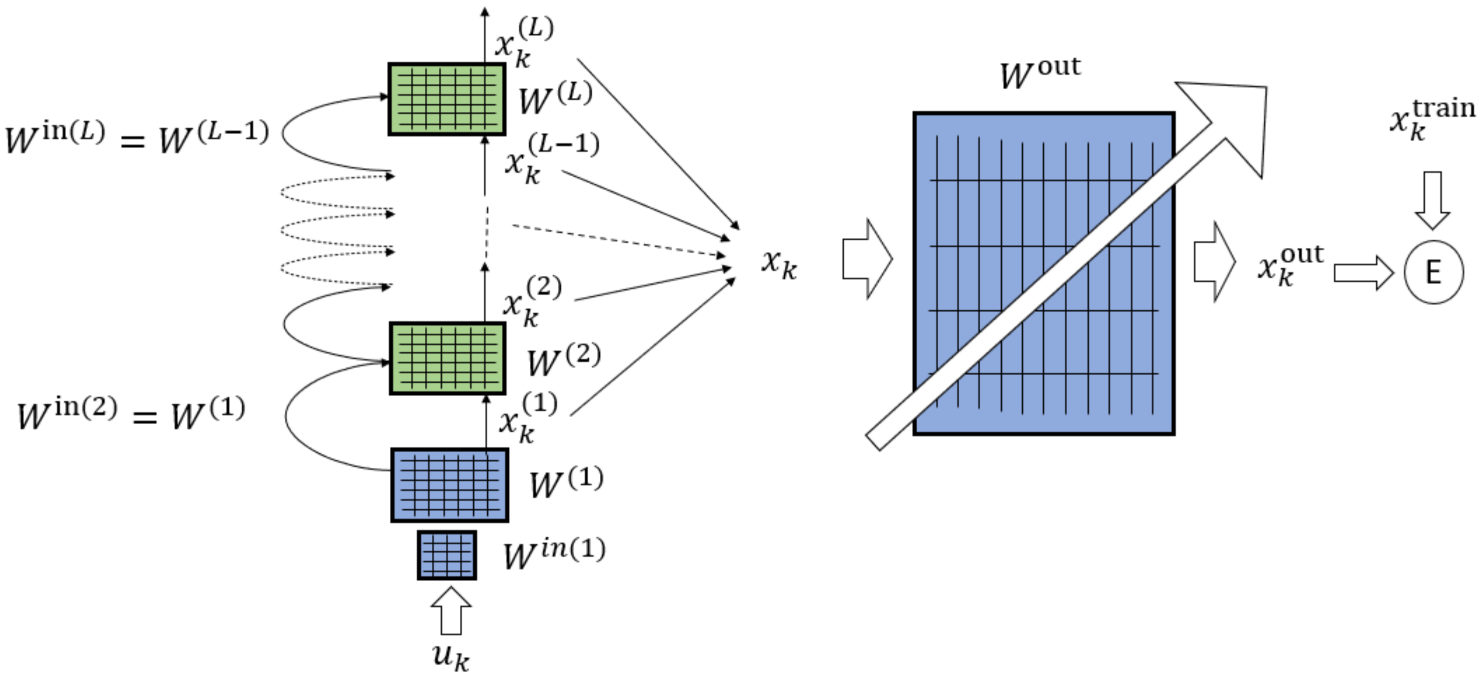
Predict the time evolution of Dynamic Aperture using tracking simulations for a limited number of turns and recurrent neural networks to generate (extrapolate) the numerical values at higher number of turns

- ✓ Fast surrogates models to speed up simulations of performance of the future Colliders
- ✓ Anomaly detection techniques and noise reduction techniques to improve Turn-by-Turn BPMs measurements

- ➢ Optimization of accelerator design and settings with Bayesian methods, reinforcement learning, etc?

2. ESN for DA prediction

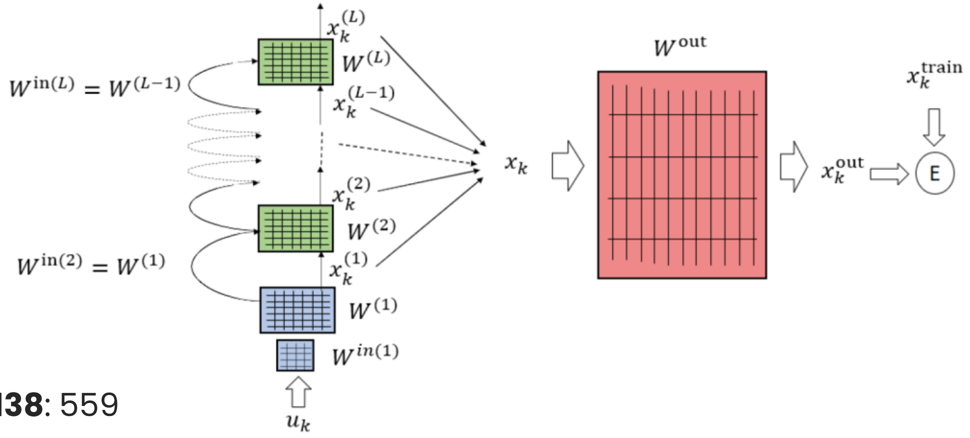
ECHO STATE NETWORKS (DeepESN)



- A class of recurrent neural network using the Reservoir Computing approach **[H. Jaeger 2001]**
- They have been proved to be an universal approximant of dynamical systems **[L. Grigoryeva and J.P. Ortega 2018]**
- The network hidden layer (the reservoir) is initialized randomly and not trained (no back propagation is required)
- The output layer only is trained (usually with linear regression) to compute the weight (W^{out}) that project the reservoir state into the predicted output

Echo State Networks (Deep ESN)

Reservoir Computing networks models and replicates the time evolution of Dynamic Aperture, allowing to speed−up tracking simulations for high energy hadron storage rings.



Deep Echo State Networks

M. Casanova, B.D. et al. , Eur. Phys. J. Plus (2023) **138**: 559

$$x_k^{(l)} = \left(1 - a\frac{\Delta t}{c}\right) x_{k-1}^{(l)} + \frac{\Delta t}{c} f\left(W^{(l-1)} x_{k-1}^{(l-1)} + W^{(l)} x_{k-1}^{(l)}\right) \quad l > 1$$

$$x_k^{out} = g(W^{out}[x_k, u_k])$$

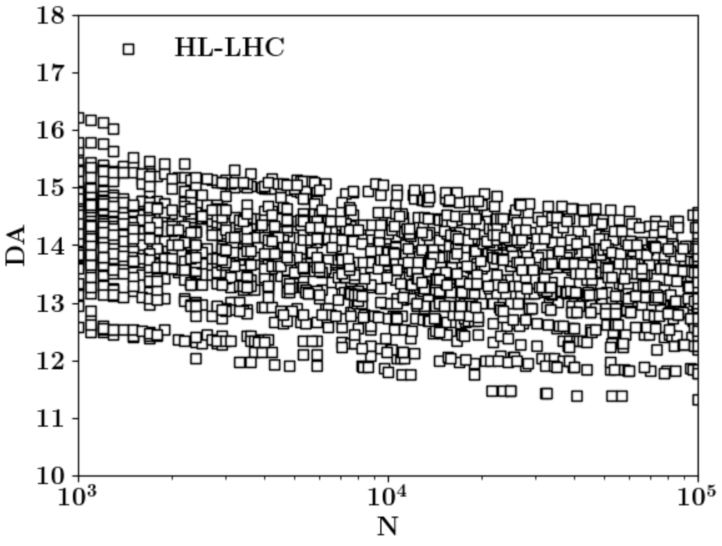
$$W^{out} = X^{target} X^T (X^T X + \beta I)^{-1}$$

where a denotes the leaking rate, c a global time constant, f a sigmoid function, g the output activation function, W^{in} the input weight matrix, W the reservoir matrix, W^{out} the output weight matrix, u the ESN input , x_k is the concatenation of all $x^{(l)}_k$ and x^{out} the ESN output

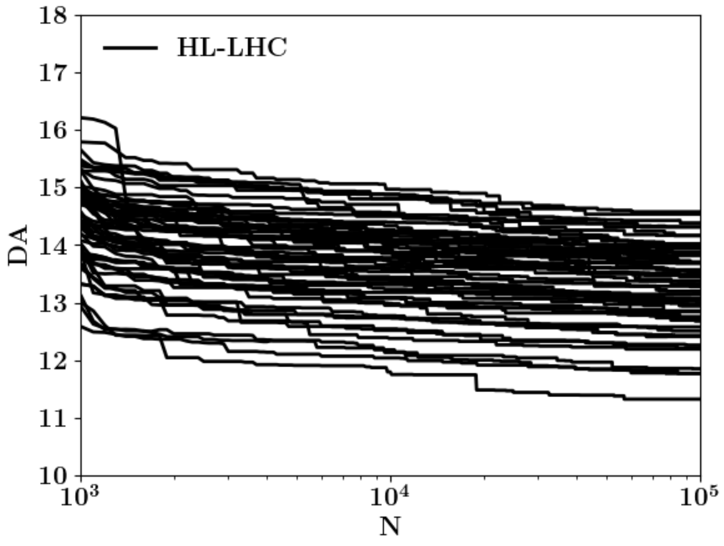
W^{in} and $W^{(l)}$ are randomly initialized

We impose the spectral radius (ρ) of the matrix $\frac{\Delta t}{c}|W| + \left(1 - a\frac{\Delta t}{c}\right) I < 1$ $\Rightarrow$ it guarantees the sufficient condition of the ESP

DATA preparation and splitting

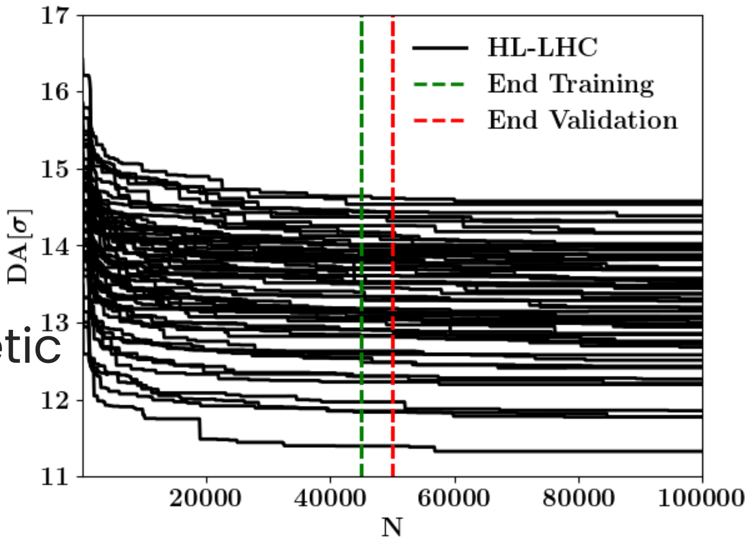


Stepwise constant function



Data splitting:

DA(N) [10, 10^4] train
DA(N)]10^4, 5 10^4] validation
DA(N)]5 10^4, 10^5] test



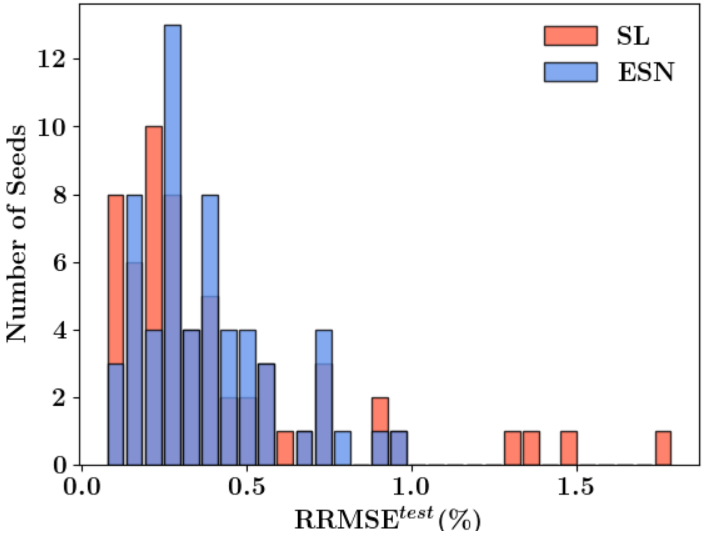
- High-Luminosity LHC, simulated
- 60 configurations → 60 distributions in magnetic lattice
- Nominal parameters

3. Results

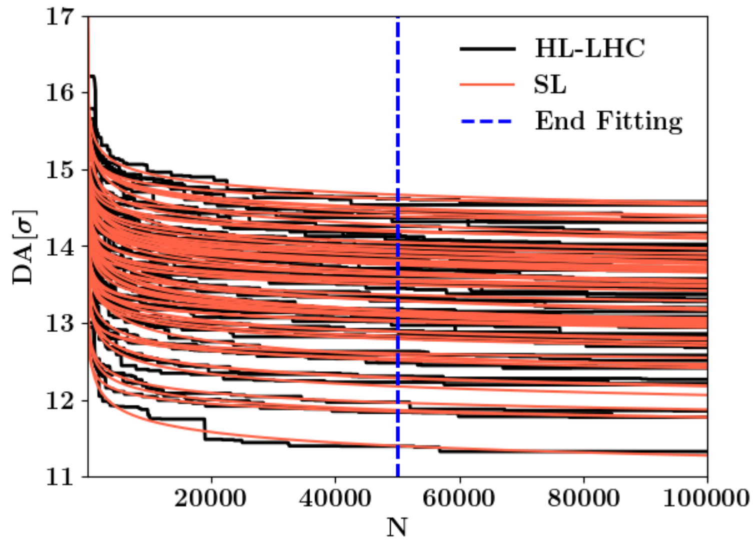
DA ESN prediction and comparison with Scaling Laws (SL)

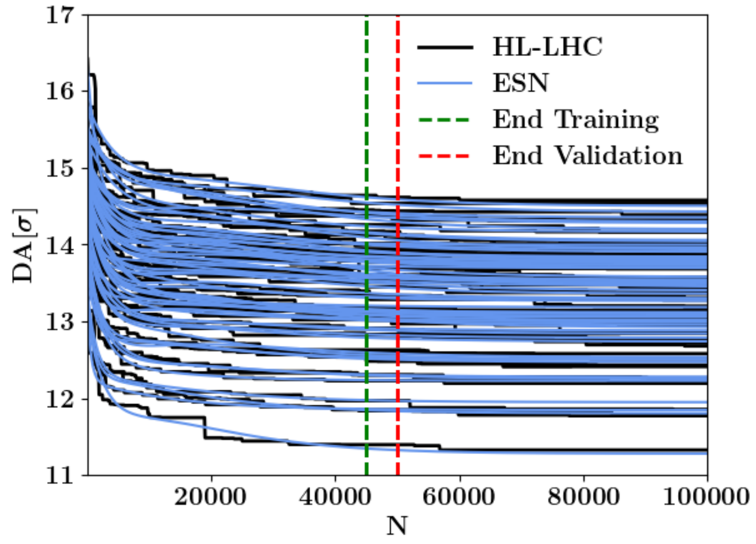
- **ESN** are able to give **physical predictions** for all seeds
- **ESN** predictions are better for the **outlier** seeds than **SL** fit (max MSE ~50% lower)
- **ESN** predictions are on average comparable to **SL** fit

$$\text{RRMSE}^{\text{test}} = 100\sqrt{\frac{\sum_{k=1}^{k_{\text{test}}}(x_{\text{mean},k}^{\text{out}} - x_k^{\text{test}})^2}{\sum_{k=1}^{k_{\text{test}}}(x_k^{\text{test}})^2}}$$



	Mean	Max	Min	Std
ESN	0.37	0.94	0.06	0.20
SL	0.42	1.78	0.07	0.35



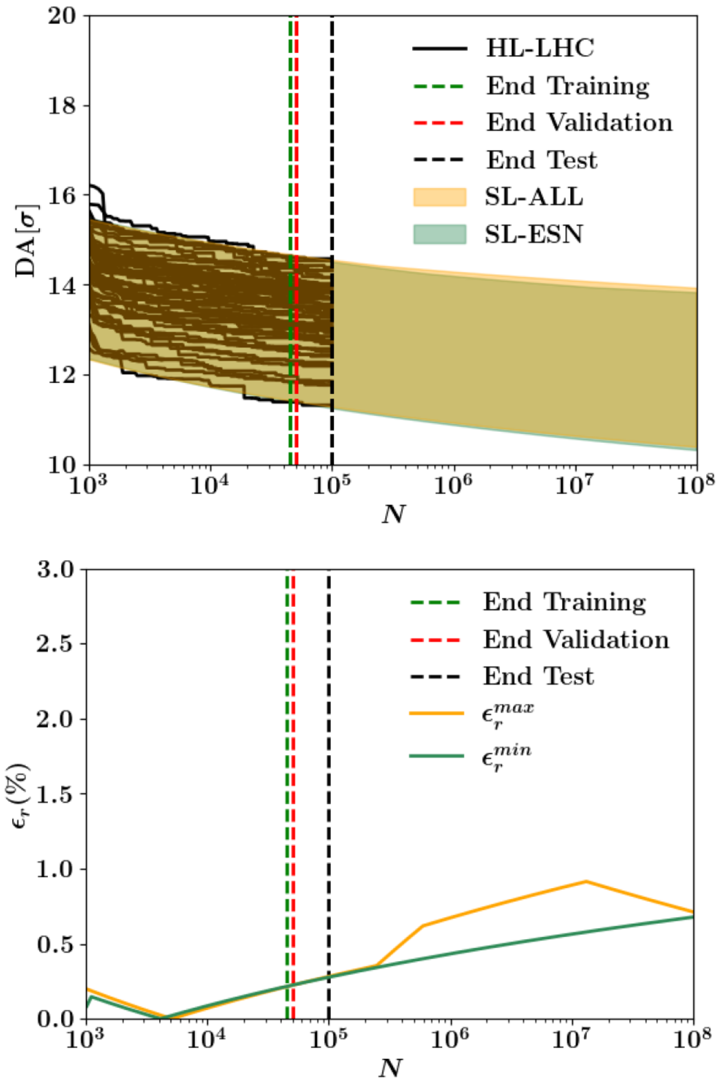


Forecast beyond the data set (HL-LHC)

We fit SL on ESN prediction up to 10^5 turns and we forecast until 10^8 turns (**SL-ESN**).
We compare the results with SL fit on tracking data and extrapolation up to 10^8 turns (**SL-ALL**).

The envelope defined by the error on the minimum and on the maximum DA value between the **SL-ALL** and **SL-ESN** is below 1%.

$$\varepsilon_r^{min,max} = \left(\frac{DA_{SL-ALL}^{min,max} - DA_{SL-ESN}^{min,max}}{DA_{SL-ALL}^{min,max}} \right)$$



ESN prediction robustness

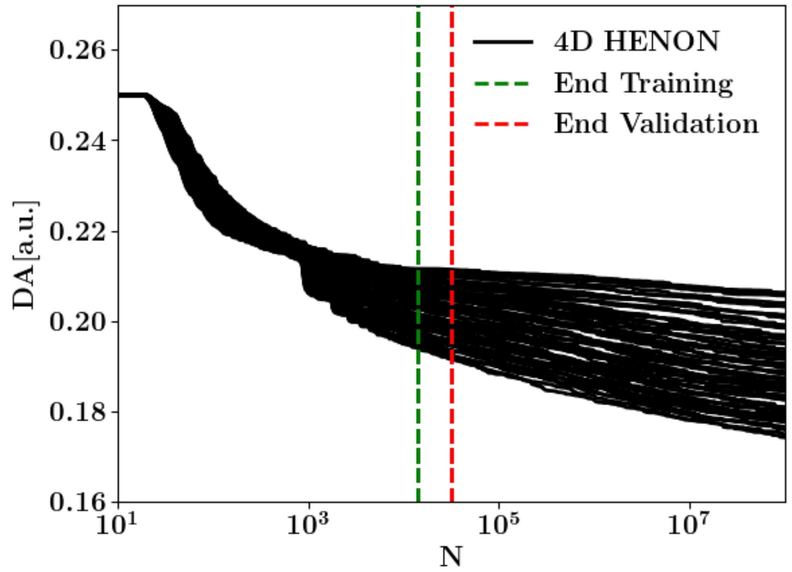
4D Henon map

$$\begin{pmatrix} x_1^{(n+1)} \\ p_{x1}^{(n+1)} \\ x_2^{(n+1)} \\ p_{x2}^{(n+1)} \end{pmatrix} = \mathrm{L} \begin{pmatrix} x_1^{(n)} \\ p_{x1}^{(n)} + \left(x_1^{(n)}\right)^2 - \left(x_2^{(n)}\right)^2 + \mu\left(\left(x_1^{(n)}\right)^3 - 3\left(x_2^{(n)}\right)^2 x_1^{(n)}\right) \\ x_2^{(n)} \\ p_{x2}^{(n+1)} - 2x_1^{(n)}x_2^{(n)} + \mu\left(\left(x_2^{(n)}\right)^3 - 3\left(x_1^{(n)}\right)^2 x_2^{(n)}\right) \end{pmatrix}$$

With $\mathrm{L} = \begin{pmatrix} R(w_{x1}^{(n)}) & 0 \\ 0 & R(w_{x2}^{(n)}) \end{pmatrix}$ w linear frequencies

$$w_{xi}^{(n)} = w_{xi0}\left(1 + \varepsilon \sum_{k=1}^{m} \varepsilon_k \cos(\Omega_k n)\right), \mathrm{i} = 1,2$$

ε allow for tune modulation



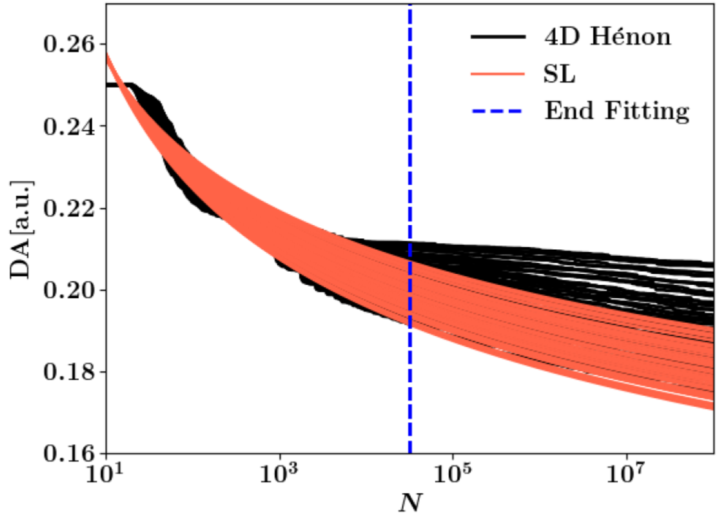
Henon map data splitting

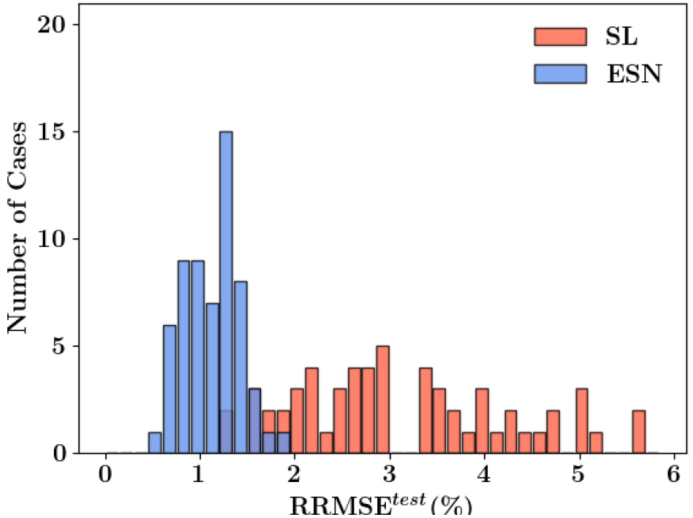
Henon Map results

- Tuning of the ESN hyper-parameters on Henon Map data

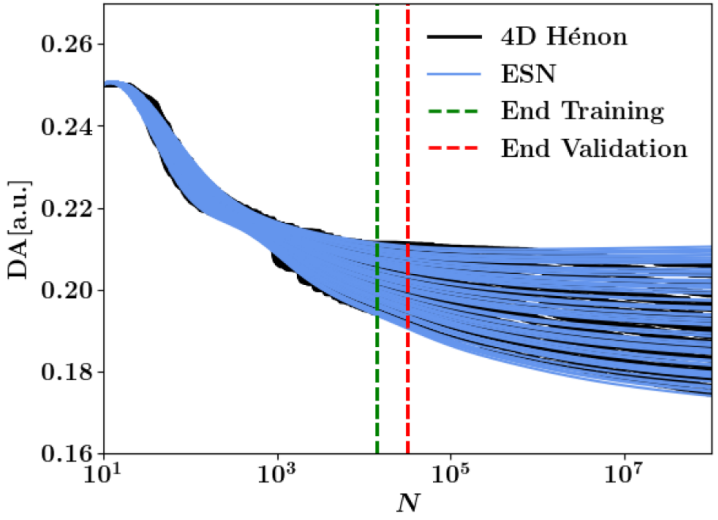
N_r	β	ρ	a	BI	L	Δt	f	s
20	9.10^{-6}	0.99	1	0	1	0.004	tanh	0

- ESN model performs better than SL on Henon Map data





	Mean	Max	Min	Std
ESN	1.13	1.89	0.59	0.28
SL	3.17	5.85	1.25	1.18



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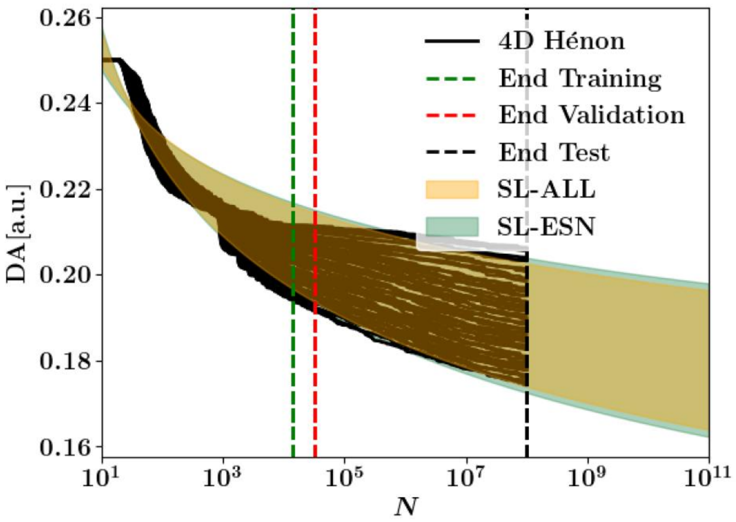
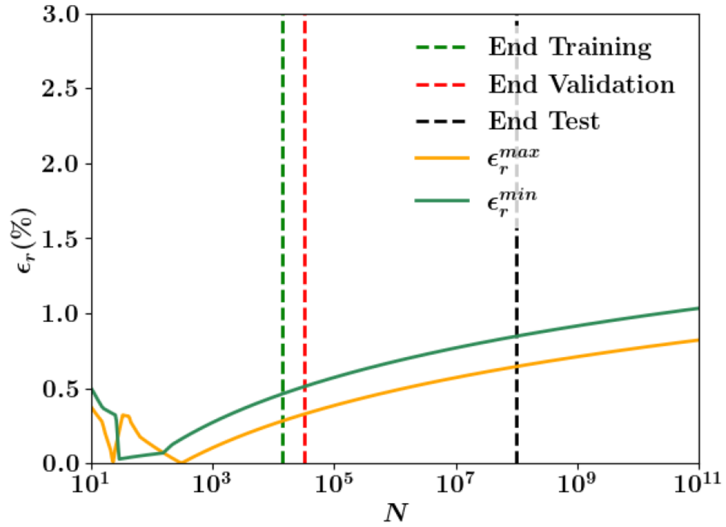




$$\varepsilon_r^{min,max} = \left(\frac{DA_{SL-ALL}^{min,max} - DA_{SL-ESN}^{min,max}}{DA_{SL-ALL}^{min,max}} \right)$$

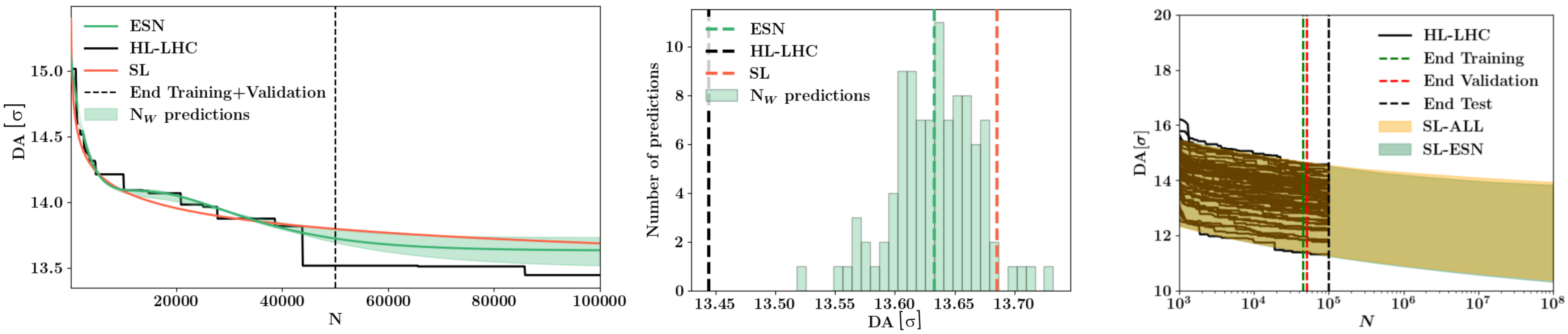
Ensemble approach

Training and validation:

- Generate randomly N_w =100 different pairs of input and reservoir matrices (W_{in}, W)
- Consider a set H of *hyper-parameters* (Nr, L, BI ρ, β, Δt)
- Train and validate the MSE on the validation data set for H and (W_{in}, W) pairs
- For each H and each seed, compute the **mean RRMSE over the 100** (W_{in}, W) pairs
- Choose the H set value which minimize the mean RRMSE in the validation data, on average over the seeds

Prediction:

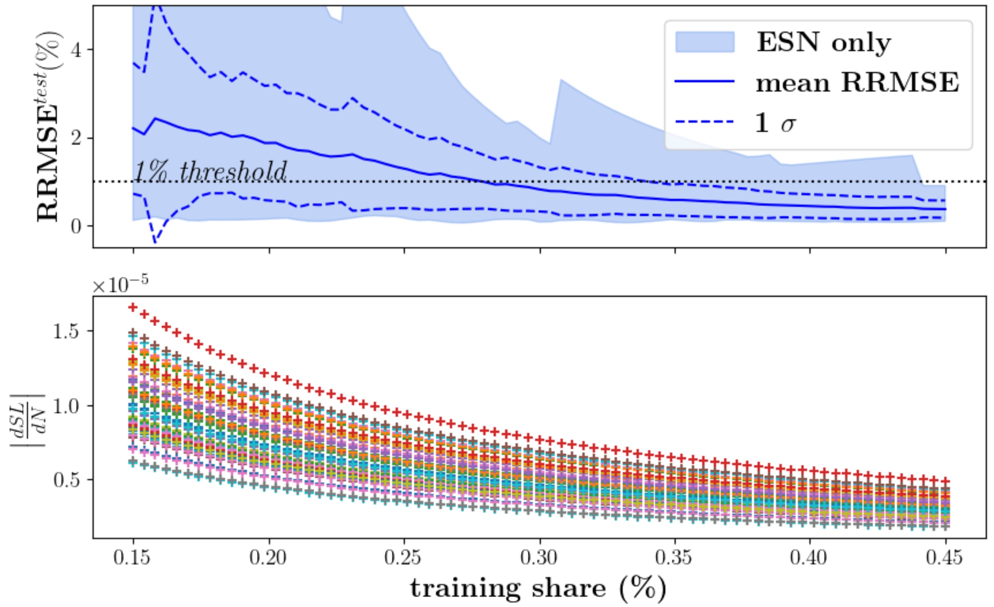
- For the selected H, compute the ESN output for the test data, for the 100 (W_{in}, W) pairs
- Compute the **mean of the ESN output over the 100** (W_{in}, W) pairs for each seed



Data splitting and hyper parameters search

Can we automatize and speed up as much as possible the determination of the data splitting and of the hyper-parameters ?

Data splitting



$$\frac{dDA}{dN}(N) = -\frac{bk}{N}\mathcal{B}^{-k}\ln\left(\frac{N}{N_0}\right)^{-k-1}$$

Bayesian search of the optimum hyper-parameters of the Deep ESN.

Python hyperopt library results:

Set #	L	N_r	BI	dt	β	ρ	Time(s)	$RRMSE^{test}_{max}$	$RRMSE^{test}_{mean}$
1	2	20	2	0.171	1.387	1.293	14801.7	2.261	0.925
2	2	15	2	0.138	0.379	0.537	18039.4	1.518	0.499
3	1	30	2	0.111	0.618	0.298	7557.7	1.901	0.552
4	1	20	2	0.025	1.877	1.259	6249.9	1.336	0.449
5	1	10	2	0.432	0.561	0.181	10438.5	2.674	1.112

Table: Set of optimized hyperparameters *via* hyperopt

L	N_r	BI	dt	β	ρ	$RRMSE^{test}_{max}$	$RRMSE^{test}_{mean}$
1	20	0	0.009	0.0224	0.19	0.892	0.365

Table: "by-hand" optimized set of parameters

Conclusion and Outlook

Long term DA can be predicted by Echo State Network (ESN)

- Combination of the scaling law and the ESN is the best approach.
- Tracking performed after 5e4 turns can be avoided by replacing it with the predictions of the ESN, gaining a factor 20 in CPU time.
- The partition of available data into training, validation, and test data sets could be obtained using an appropriate algorithm?
- Investigate the possibility of using ESN to improve the modelling of beam lifetime and luminosity evolution.
- The predictive power of ESN could be applied to other indicators of chaos ?

Perspectives

Fully automatize the optimum hyper parameters and the data splitting (training/validation/test) determination:

- Exploiting derivatives of the scaling law
- Bayesian search of hyper parameters

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THANK YOU