# Multiple QCD axions.

or *The "QCD axion sum rule"*

**The Axion Quest 2024 conference - 20th Rencontres du Vietnam**

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Based on *"The QCD axion sum rule"* **JHEP 04 (2024) 056** 2305.154

in collaboration with B. Gavela and M. Ramos

# The QCD axion

- ➔ Solves the Strong CP problem
- ➔ Excellent Dark Matter candidate

[Peccei+Quinn 77]
[Weinberg, 78]
[Wilczek, 78]

[Abbot+Sikivie, 83]
[Dine and W. Fischler, 83]
[Preskil et al, 91]

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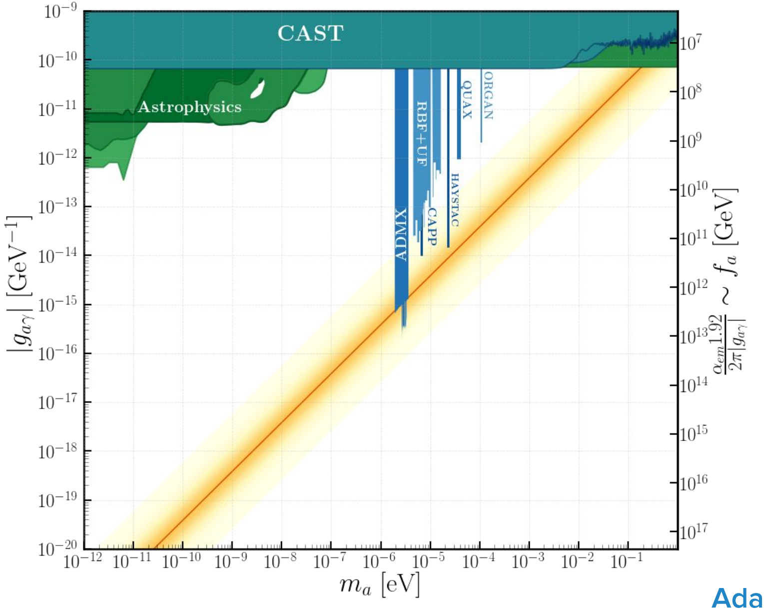


Adapted from AxionLimits
[Ciaran O'hare, 20]

Based on **2305.15465** - Pablo Quílez

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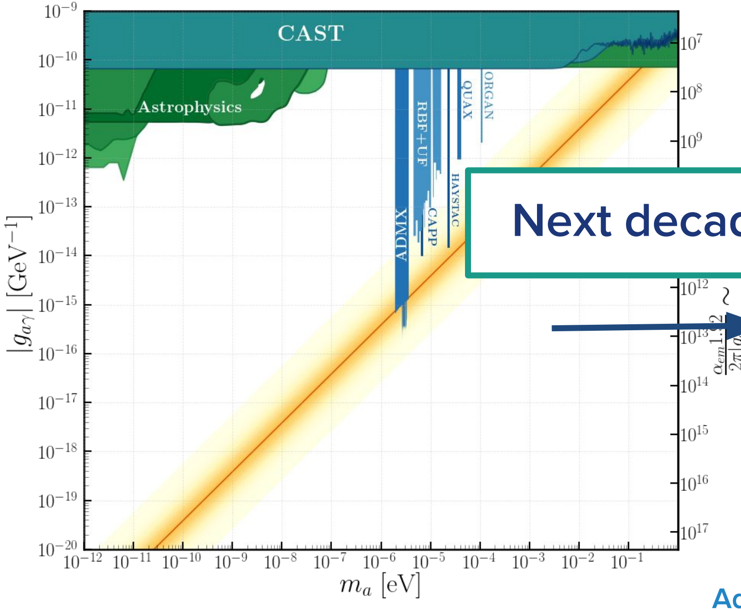
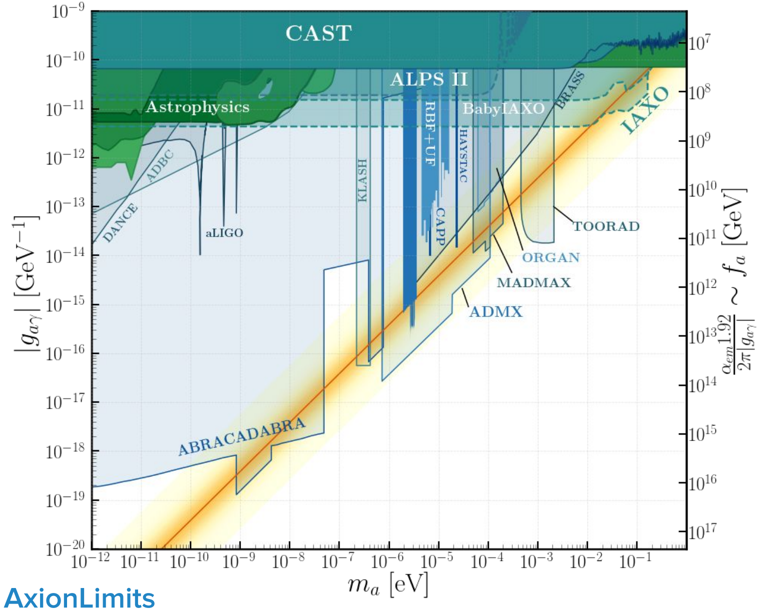



**Next decades**

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# The axion solution

[Peccei+Quinn 77]

➔ Strong CP problem $\mathcal{L} \supset \bar{\theta}_{\text{QCD}} \frac{\alpha_s}{8\pi} G\tilde{G}$

Neutron EDM
(Electric Dipole Moment)

**Why is it so small?** $\bar{\theta} \lesssim 10^{-10}$

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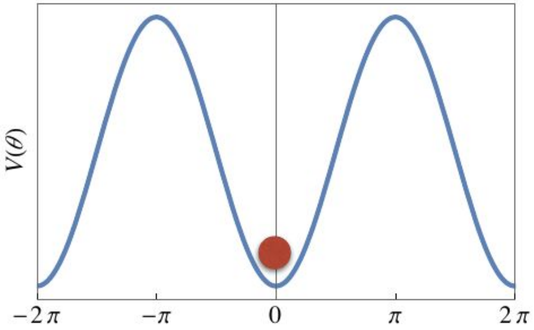
(Electric Dipole Moment)

**Why is it so small?** $\bar{\theta} \lesssim 10^{-10}$

- ➔ If $\bar{\theta}$ were a scalar field, its vev would be zero [Vafa+Witten, 84]

$$\bar{\theta} \frac{\alpha_s}{8\pi} G\tilde{G} \quad \longrightarrow \quad \left(\bar{\theta} - \frac{a}{f_a}\right) \frac{\alpha_s}{8\pi} G\tilde{G}$$




$\bar{\theta} - \frac{a}{f_a}$

$$\bar{\theta}_{\rm eff} = \langle \bar{\theta} - \frac{a}{f_a} \rangle = 0$$

# The pGoldstone boson of the PQ sym.

- ➔ Introduce a $U(1)_{PQ}$ symmetry (classically exact): [Peccei+Quinn 77]
  - Spontaneously broken $\longrightarrow$ pGoldstone Boson: AXION [Weinberg, 78] [Wilczek, 78]
  - Anomalous: explicitly broken by QCD instantons $\longrightarrow$ massive

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$$\mathcal{L} \supset \frac{a}{f_a}\frac{\alpha_s}{8\pi}G\tilde{G} \quad \longrightarrow \quad m_a^2 f_a^2 \simeq m_\pi^2 f_\pi^2 \frac{m_u m_d}{(m_u+m_d)^2}$$

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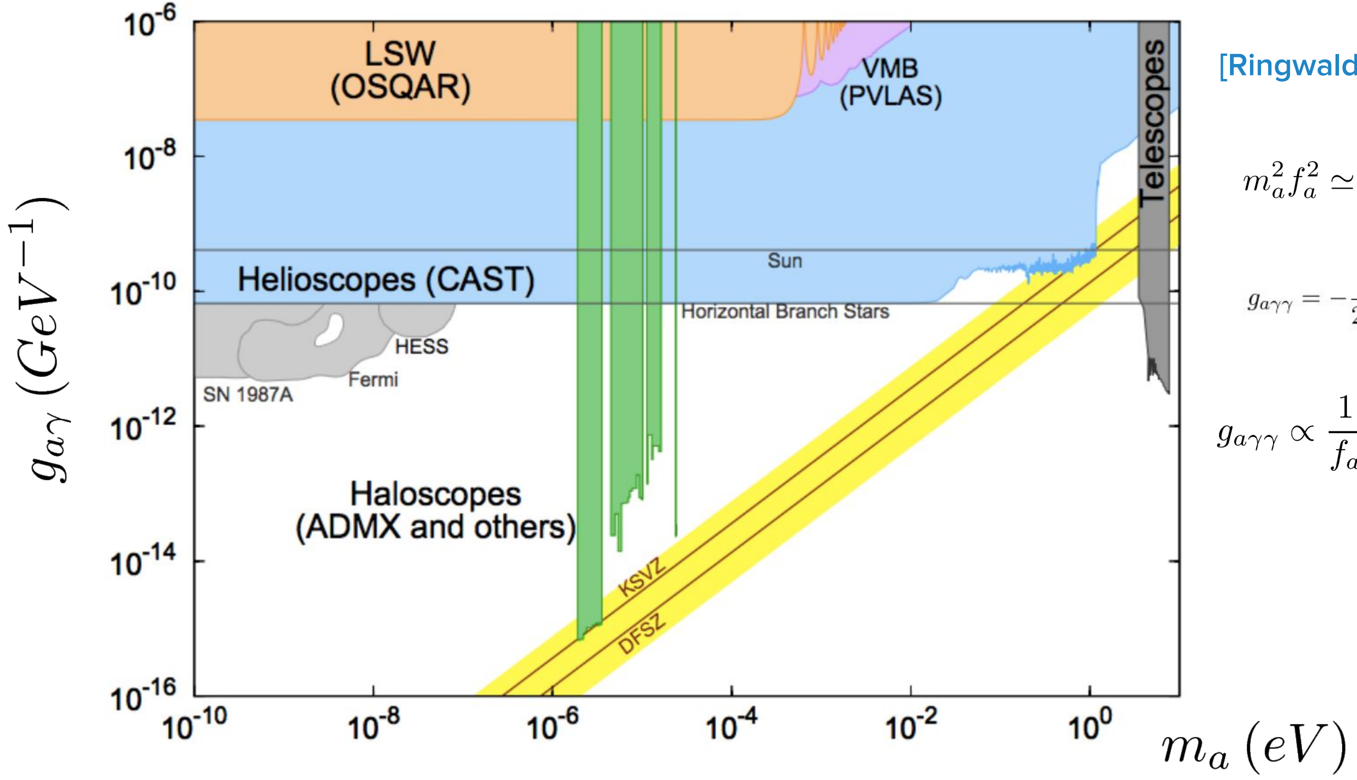
➔ Couplings: $\mathcal{L} \supset \frac{1}{4} g_{a\gamma\gamma} a F\tilde{F}$

$$g_{a\gamma\gamma} \propto \frac{1}{f_a} \implies g_{a\gamma\gamma} \propto m_a \qquad g_{a\gamma\gamma} = -\frac{1}{2\pi f_a}\alpha_{\rm em}\left(\frac{E}{N} - 1.92(4)\right)$$

# Invisible axion parameter space




[Ringwald, PDG 17]

$$m_a^2 f_a^2 \simeq m_\pi^2 f_\pi^2 \frac{m_u\, m_d}{(m_u + m_d)^2}$$

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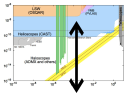
# Beyond the canonical band

Summary slide from Patras 2021 Review talk: “True axions beyond the canonical band” P. Quilez

$g_{a\gamma}$



A) Photophilic/photophobic axions

1. Single scalar: Playing with fermionic representations

“Preferred axion window”
[Di Luzio, Mescia, Nardi, 16]
[Di Luzio, Mescia, Nardi, 18]

“Axion from monopoles”
[Sokolov, Ringwald, 21]

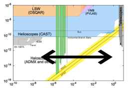
2. Multiple scalars: Alignment in field space

“Clockwork axion”
[Farina et al, 17]
[Coy, Frigerio, 17]
[Kim et al, 04]
[Choi et al, 14 and 16]
[Kaplan et al 16]
[Giudice et al 16]

“KNP alignment”
[Agrawal et al 17]
[Kim et al, 04]
+ Refs in FIPs report [2102.12143]

“Multi-higgs models”
[Di Luzio, Mescia, Nardi, 17]
[Di Luzio, Giannotti, Nardi, Visinelli, 16]
[Darmé, Di Luzio, Giannotti, Nardi, 20]

$m_a$



B) Heavy/even lighter axions

1. Heavy axions: extra instantons

[Rubakov, 97]
[Berezhiani et al ,01]
[Fukuda et al, 01]
[Hsu et al, 04]
[Gianotti, 05]
[Hook et al, 14]
[Chiang et al, 16]
[Khobadize et al,]
[Dimopoulos et al, 16]
[Gherghetta et al, 16]
[Agrawal et al, 17]
[Gaillard, Gavela, Houtz, Rey PQ, 18]
[Fuentes-Martin et al, 19]
[Csaki et al, 19]
[Gherghetta et al, 20]

2. Even lighter QCD axion

[Hook, 18]
[Luzio, Gavela, PQ, Ringwald, 21]
[Luzio, Gavela, PQ, Ringwald, 21]

**More PQ breaking**

$$\partial_\mu j^\mu_{\rm PQ} = G\tilde{G} + \ldots$$

# The single QCD axion line

$$\mathcal{L} \supset \frac{a}{f_a}\frac{\alpha_s}{8\pi}G\tilde{G}$$

$$\delta\mathcal{L} \equiv -\frac{i}{2}\frac{0.011\,e}{m_n}\frac{a}{f_a}\bar{n}\sigma_{\mu\nu}\gamma_5 n F^{\mu\nu}$$

$$\equiv g_{a\gamma n}$$

$$m_a^2 f_a^2 \simeq m_\pi^2 f_\pi^2 \frac{m_u\, m_d}{(m_u + m_d)^2}$$

**Coupling to the nEDM**

**Axion mass**

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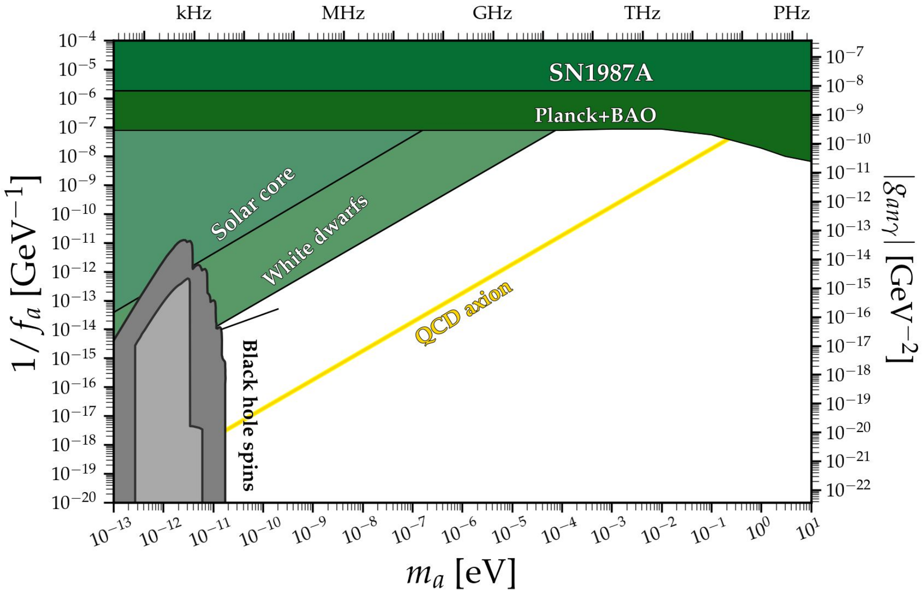


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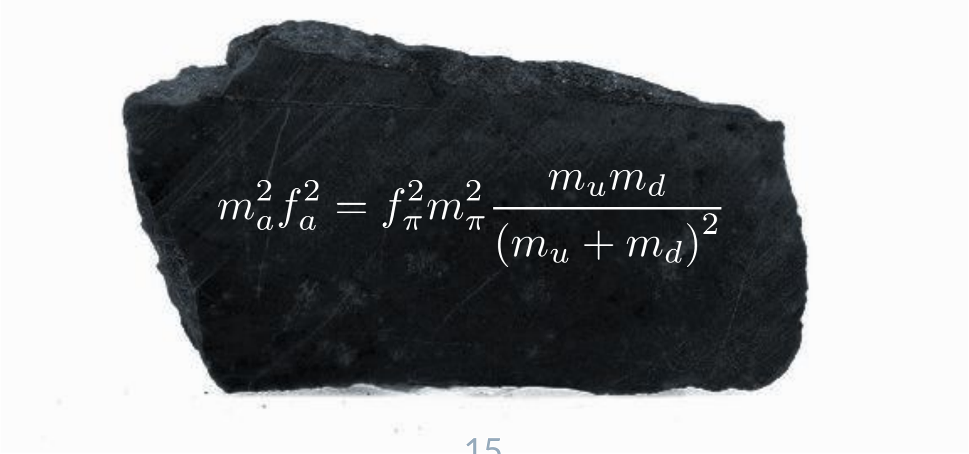
“

***Can the QCD axion deviate from the standard $m_a$-$f_a$ relation being QCD the only source of PQ breaking?***

“

***Or is it written in stone?***



“

***How many QCD axions can there be?***

“

***If there are other scalar singlets in Nature, other ALPs,***

***What are the consequences of a general mixing with the QCD axion?***

# Multiple QCD axions

# Multiple QCD axions

$$\mathcal{L} = \frac{\alpha_s}{8\pi}\left(\sum_{k=1}^{N}\frac{\hat{a}_k}{\hat{f}_k} - \bar{\theta}\right) G\widetilde{G} - V_B(\hat{a}_1,\, \hat{a}_2, \ldots, \hat{a}_N)\,,$$

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$$\exists\ \textit{true}\ U(1)_{\rm PQ} \Longleftrightarrow \text{classically exact and only broken by QCD}$$

# Multiple QCD axions

$$\mathcal{L} = -\frac{1}{2}\chi_{\rm QCD}\left(\sum_{k=1}^{N}\frac{\hat{a}_k}{\hat{f}_k}\right)^2 - V_B(\hat{a}_1,\, \hat{a}_2, \ldots, \hat{a}_N)\,,$$

$$\mathcal{L} \supset -\frac{1}{2}\hat{a}_k \hat{\mathbf{M}}^2_{kl}\hat{a}_l \quad \text{with} \quad \hat{\mathbf{M}}^2 = \hat{\mathbf{M}}^2_A + \hat{\mathbf{M}}^2_B\,,$$

Diagonalize mass matrix

$$\mathcal{L} \supset -\frac{1}{2}m_i^2 a_i^2$$

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Compute coupling to gluons of the N mass eigenstates

$$\mathcal{L} \supset \frac{\alpha_s}{8\pi}\frac{a_i}{f_i}G\widetilde{G}$$

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# Multiple QCD axions

$$\mathcal{L} = -\frac{1}{2}\chi_{\rm QCD}\underbrace{\left(\sum_{k=1}^{N}\frac{\hat{a}_k}{\hat{f}_k}\right)}_{\hat{a}_{G\widetilde{G}}/F}^{2} - V_B(\hat{a}_1,\, \hat{a}_2, \ldots, \hat{a}_N)\,,$$

Diagonalize mass matrix

$$\mathcal{L} \supset -\frac{1}{2}m_i^2 a_i^2$$

Compute coupling to gluons of the N mass eigenstates

$$\frac{1}{f_i} \equiv \frac{\langle a_i \mid \hat{a}_{G\widetilde{G}}\rangle}{F}$$

$$\mathcal{L} \supset \frac{\alpha_s}{8\pi}\frac{a_i}{f_i}G\widetilde{G}$$

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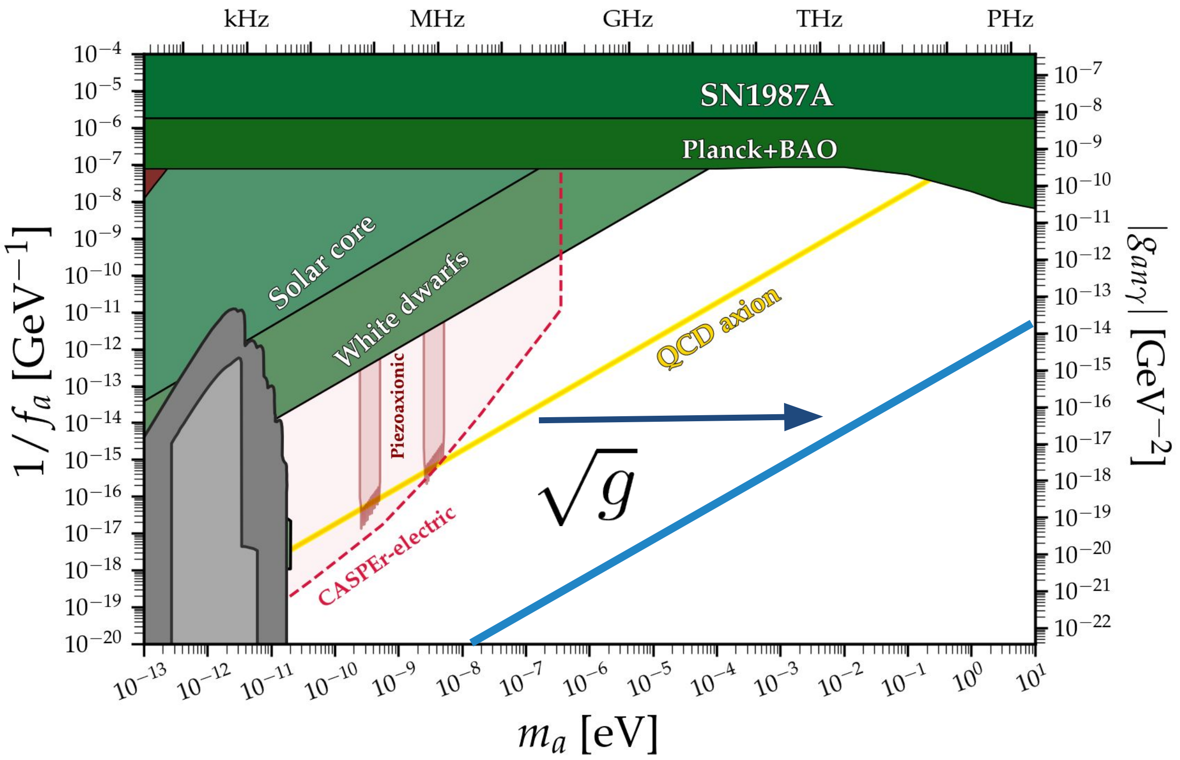
# Deviation from the QCD line: $g_i$-factor

$$m_i^2 f_i^2 \equiv f_\pi^2 m_\pi^2 \frac{m_u m_d}{(m_u+m_d)^2} \times g_i$$

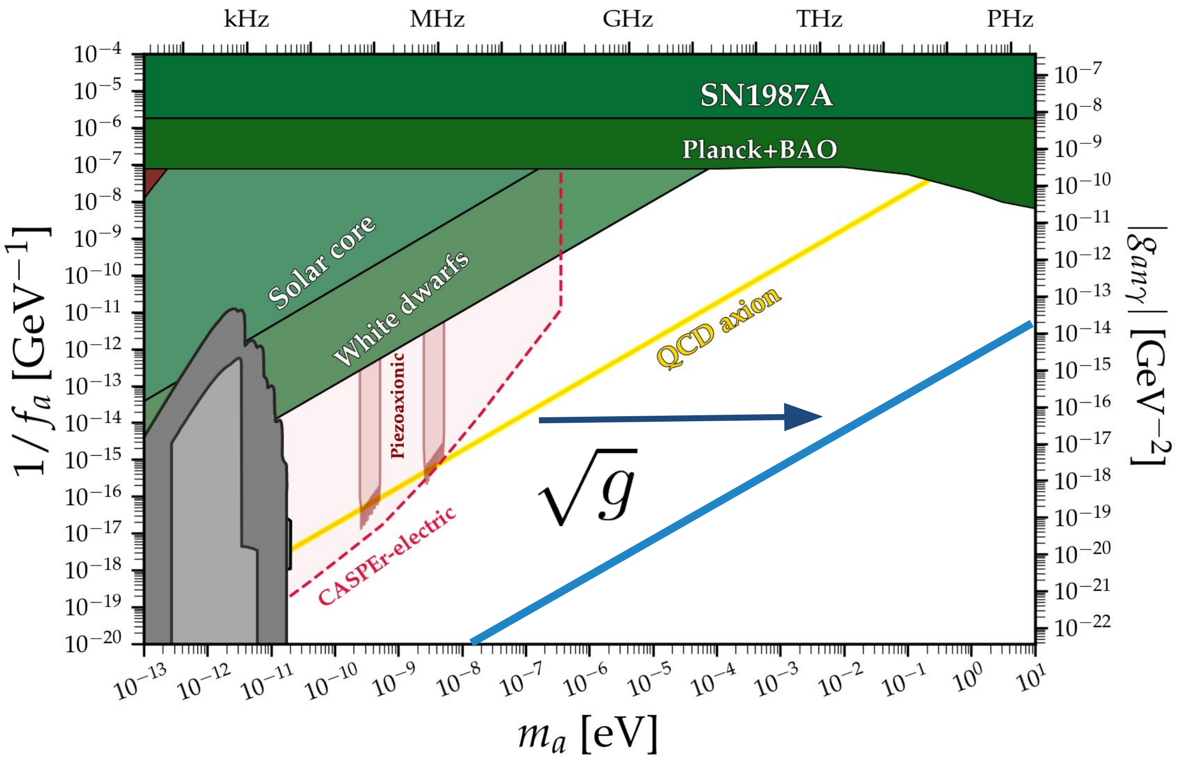


# Deviation from the QCD line: $g_i$-factor

$$\equiv \chi_{\rm QCD}$$

$$m_i^2 f_i^2 \equiv f_\pi^2 m_\pi^2 \frac{m_u m_d}{(m_u + m_d)^2} \times g_i$$

$$g_i \equiv \frac{m_i^2 f_i^2}{\chi_{\rm QCD}} = \frac{m_i^2 F^2}{\left|\langle a_i \mid \hat{a}_{G\tilde{G}}\rangle\right|^2 \chi_{\rm QCD}}$$



# Toy example: N=2

$$m_i^2 f_i^2 \equiv f_\pi^2 m_\pi^2 \frac{m_u m_d}{(m_u + m_d)^2} \times g_i$$

$$\mathcal{L}_{N=2} = \frac{\alpha_s}{8\pi}\left(\frac{\hat{a}_1}{\hat{f}} + \frac{\hat{a}_2}{\hat{f}} + \bar{\theta}\right) G\widetilde{G} - \frac{1}{2}\hat{m}_2^2\,\hat{a}_2^2\,. \quad \longrightarrow V_{N=2} = \frac{1}{2}\chi_{\rm QCD}\left[\left(\frac{\hat{a}_1}{\hat{f}} + \frac{\hat{a}_2}{\hat{f}}\right)^2 + r\,\hat{a}_2^2\right]$$

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- ➔ Note that there are 2 relevant axion linear combinations:
  - The one that couples to $G\widetilde{G}$

$$\hat{a}_{G\widetilde{G}} = \frac{1}{\sqrt{2}}(\hat{a}_1 + \hat{a}_2)$$

  - The one implementing the PQ symmetry:

$$a_{\rm PQ} = \hat{a}_1\,,$$

$$PQ:\ \hat{a}_1 \longrightarrow \hat{a}_1 - \bar{\theta}\hat{f} \text{ allows to fully reabsorb } \bar{\theta}$$

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$$a_{\rm PQ} = \hat{a}_1\,, \qquad \hat{a}_{G\widetilde{G}} = \frac{1}{\sqrt{2}}(\hat{a}_1 + \hat{a}_2)$$

**Limit r** $\rightarrow \infty$: The mass eigenstates read,

$$a_1 \simeq \hat{a}_1\,, \text{with} \qquad g_1 \to 1 \qquad \Longrightarrow a_1 = a_{\text{QCD-like}}$$
$$a_2 \simeq \hat{a}_2\,, \text{with} \qquad g_2 \to \infty \qquad \Longrightarrow a_2 = a_{\text{decoupled}}$$

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**Limit r → 0:** The mass eigenstates read,

$$a_1 \simeq \frac{1}{\sqrt{2}}(\hat{a}_1 - \hat{a}_2)\,, \text{with} \qquad g_1 \to \infty \qquad \Longrightarrow \; a_1 = a_{\text{decoupled}}$$

$$a_2 \simeq \frac{1}{\sqrt{2}}(\hat{a}_1 + \hat{a}_2)\,, \text{with} \qquad g_2 \to 1 \qquad \Longrightarrow \; a_2 = a_{\text{QCD-like}}$$

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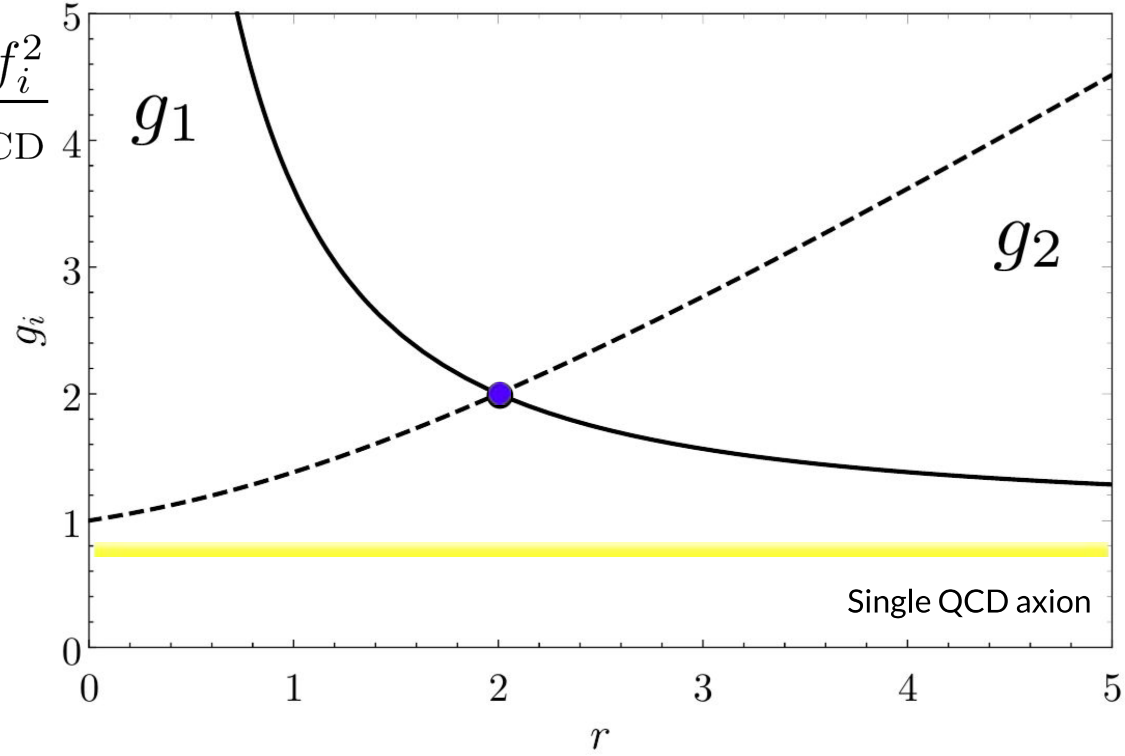
# Toy example: N=2

$$m_i^2 f_i^2 \equiv f_\pi^2 m_\pi^2 \frac{m_u m_d}{(m_u+m_d)^2} \times g_i$$

$$g_i = \frac{m_i^2 f_i^2}{\chi_{\rm QCD}}$$




$$g_{1(2)} = \frac{2\sqrt{4+r^2}}{\sqrt{4+r^2} \pm (r-2)},$$

$$g = 1$$

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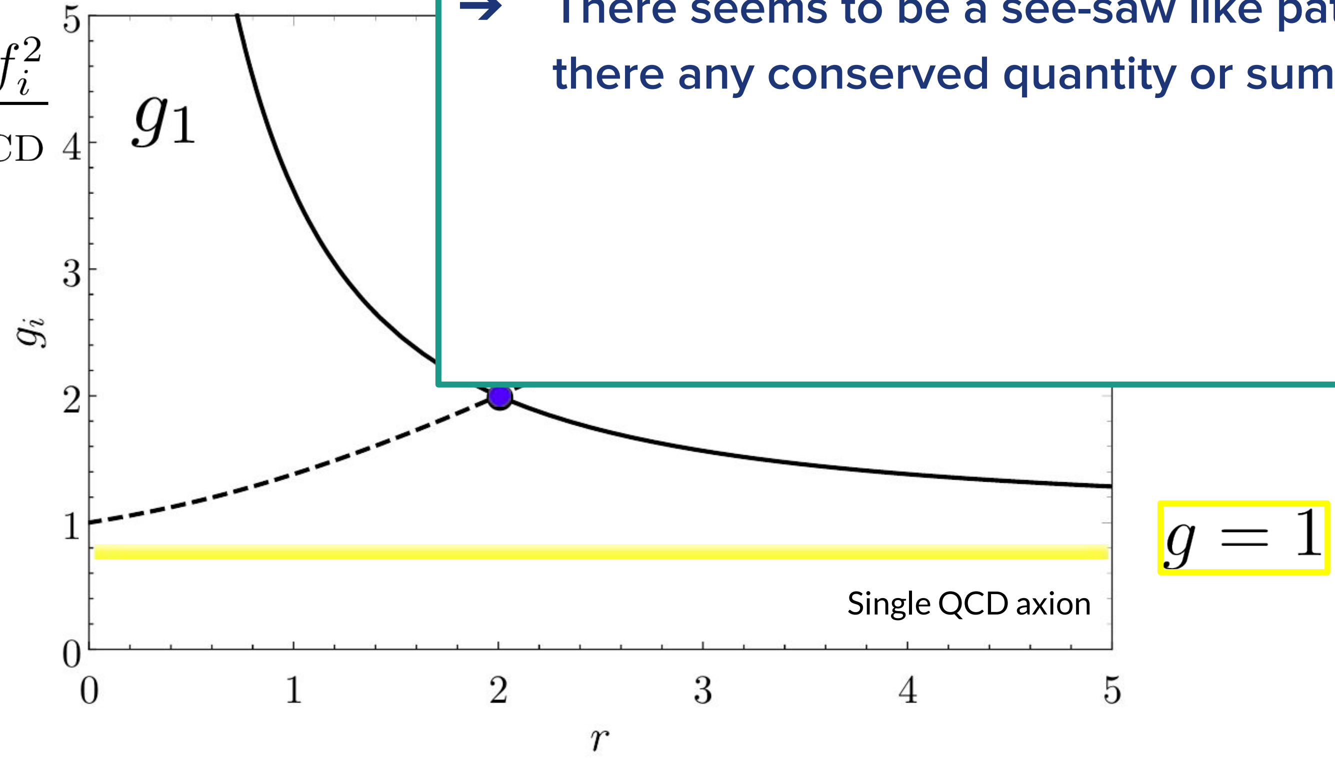


➔ There seems to be a see-saw like pattern, is there any conserved quantity or sum rule?

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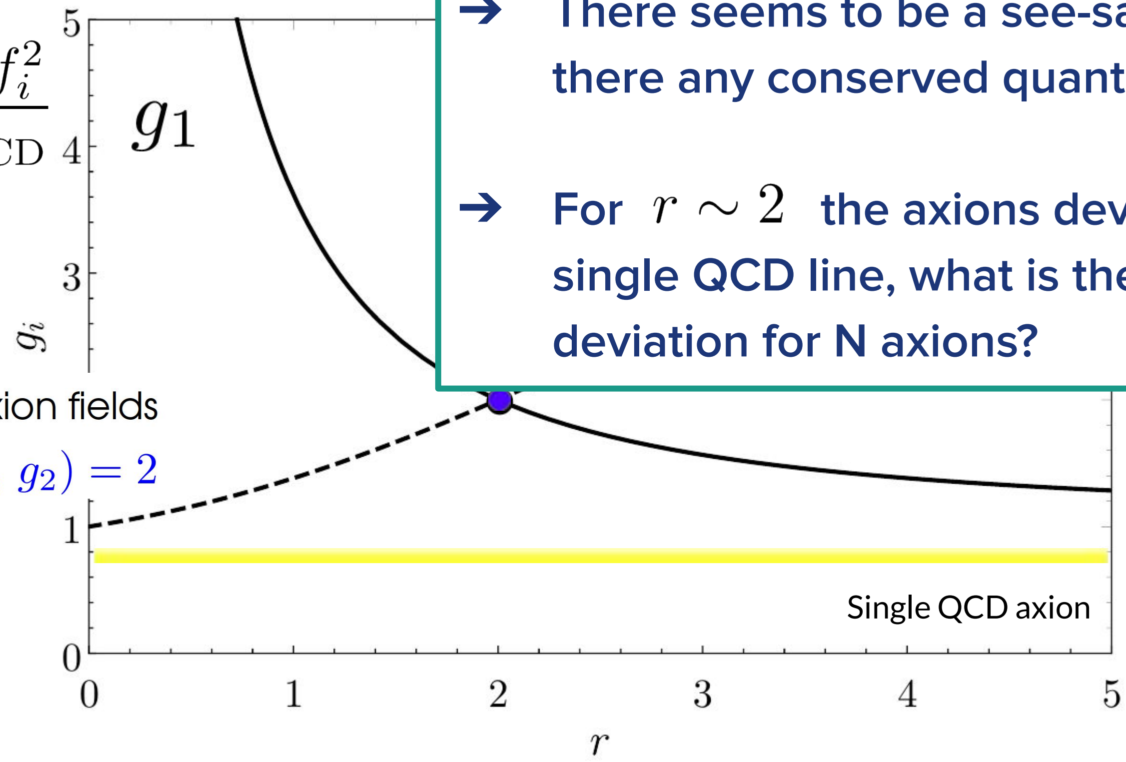


$g = 1$

- ➔ There seems to be a see-saw like pattern, is there any conserved quantity or sum rule?
- ➔ For $r \sim 2$ the axions deviate from the single QCD line, what is the maximum deviation for N axions?

- 2 QCD maxion fields

$\max\min(g_1,\, g_2) = 2$

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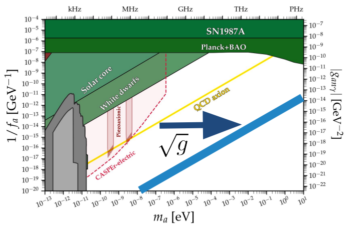
# QCD-axionness

$$\frac{1}{g_i} = f_\pi^2 m_\pi^2 \frac{m_u m_d}{(m_u+m_d)^2} \frac{1}{m_i^2 f_i^2} = \frac{m_a^2 f_a^2}{m_i^2 f_i^2}\Big|_{\text{single QCD axion}}$$

$$\frac{1}{g_i} = \frac{\chi_{\text{QCD}}}{m_i^2 f_i^2}$$

- Inverse of the distance to QCD line
- Fraction of its mass stemming from QCD



# QCD-axionness: a sum rule from true PQ

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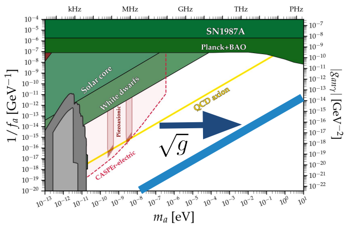


$$\exists\, U(1)_{PQ} \implies \sum_{i=1}^{N} \frac{1}{g_i} = 1\,,$$

# Toy example: N=2

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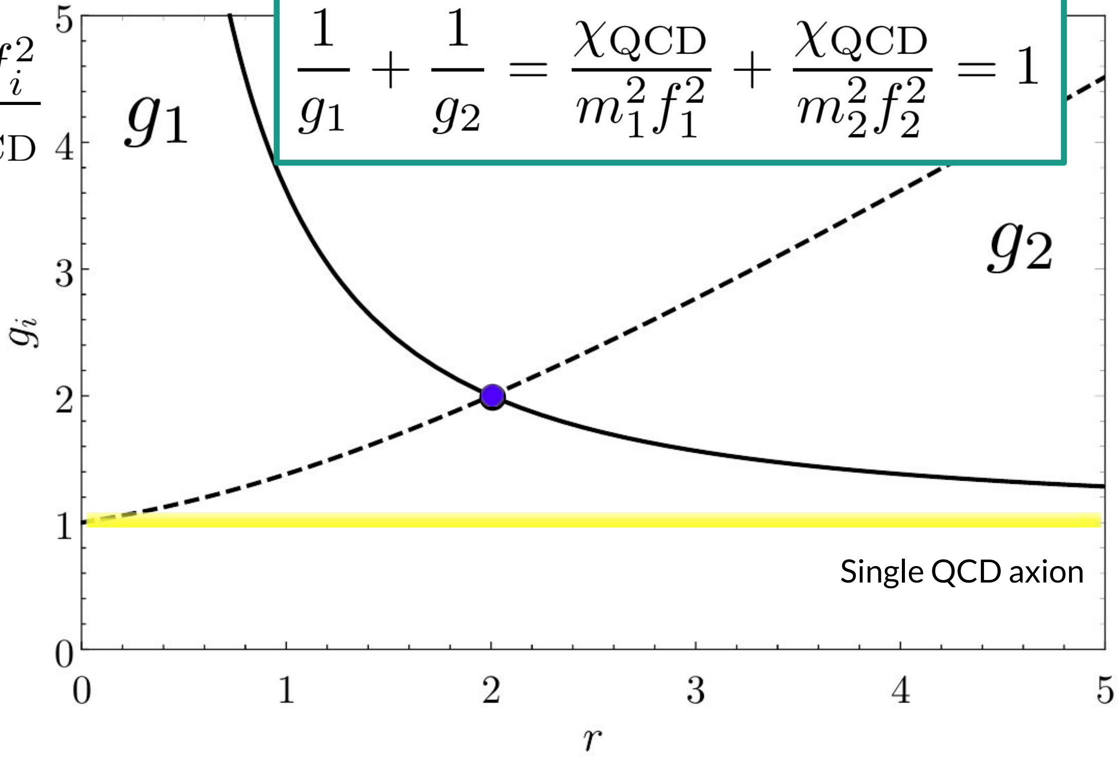
$$g_i = \frac{m_i^2 f_i^2}{\chi_{\rm QCD}}$$

$$\frac{1}{g_1} + \frac{1}{g_2} = \frac{\chi_{\rm QCD}}{m_1^2 f_1^2} + \frac{\chi_{\rm QCD}}{m_2^2 f_2^2} = 1$$

$$g_{1(2)} = \frac{2\sqrt{4+r^2}}{\sqrt{4+r^2} \pm (r-2)},$$

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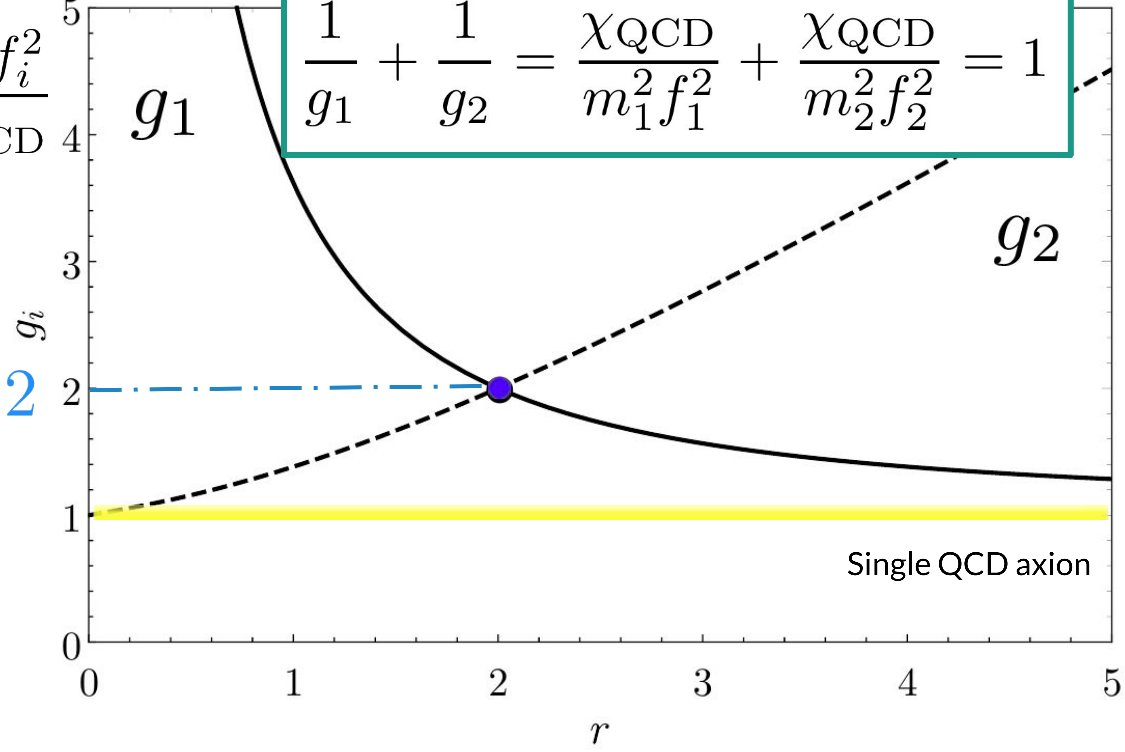
$$g_i = \frac{m_i^2 f_i^2}{\chi_{QCD}}$$

$$\frac{1}{g_1} + \frac{1}{g_2} = \frac{\chi_{QCD}}{m_1^2 f_1^2} + \frac{\chi_{QCD}}{m_2^2 f_2^2} = 1$$

$$g_{1(2)} = \frac{2\sqrt{4+r^2}}{\sqrt{4+r^2} \pm (r-2)},$$

$g = 2$




$g = 1$

# QCD-axionness: a sum rule from true PQ

$$\frac{1}{g_i} = \frac{\chi_{\rm QCD}}{m_i^2 f_i^2}$$

$$\exists\, U(1)_{PQ} \implies \sum_{i=1}^{N} \frac{1}{g_i} = 1\,,$$

Mass Eigenstates

$$\frac{1}{g_i} = \frac{\langle a_{\rm PQ} \mid a_i \rangle \langle a_i \mid a_{G\tilde{G}} \rangle}{\langle a_{\rm PQ} \mid a_{G\tilde{G}} \rangle}$$

Based on **2305.15465** - Pablo Quílez

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Combination that couples to gluons

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Combination whose shift sym. is only broken by QCD

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Gluon interaction basis $\neq$ Mass basis $\neq$ PQ basis

Multiple QCD axions will deviate from the standard relation

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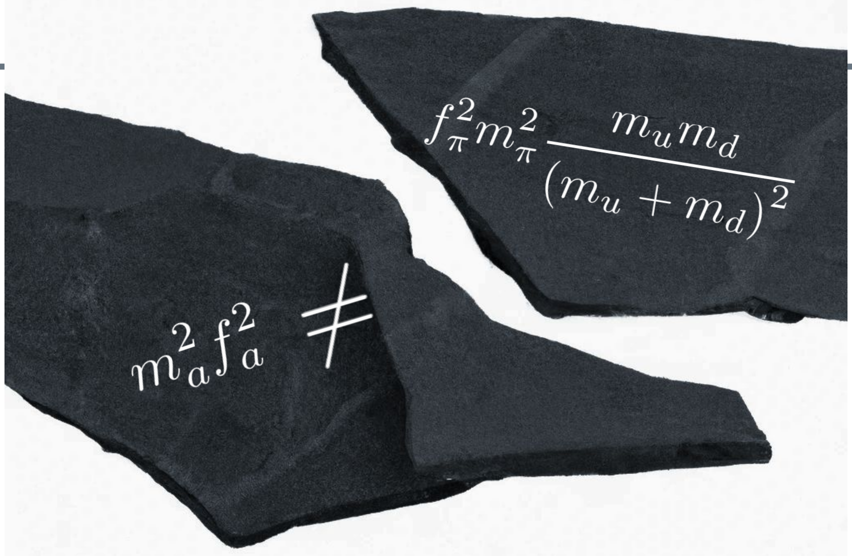
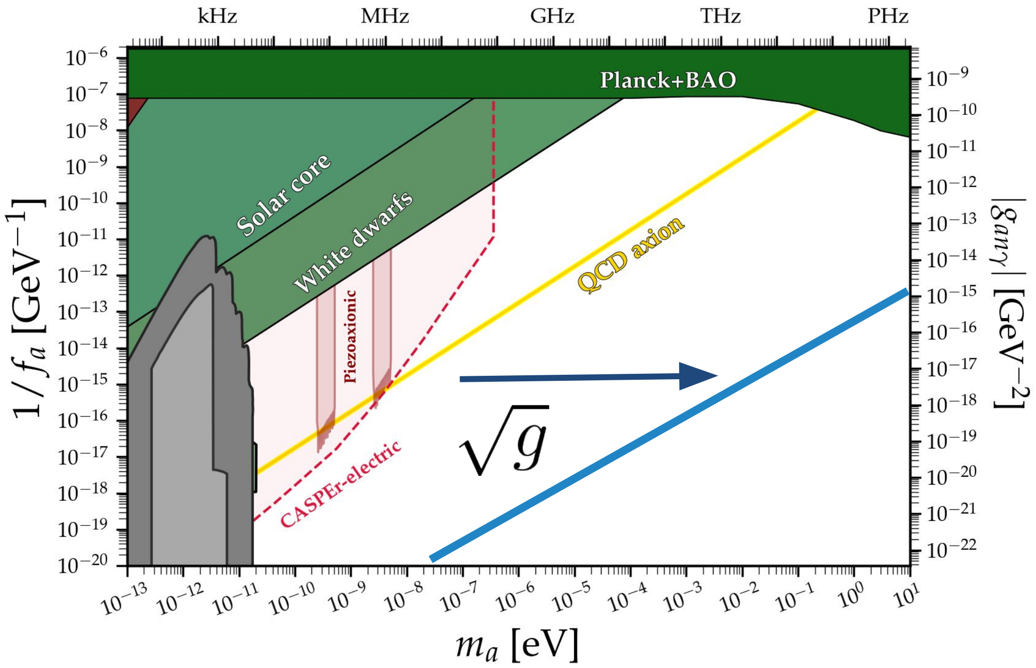


# Experimental consequences

$$\frac{1}{g_i} = \frac{\chi_{\rm QCD}}{m_i^2 f_i^2} \qquad \sum_{i=1}^{N} \frac{1}{g_i} = 1$$

1) $$g_i \geq 1 \quad = \left(1 + \frac{F^2}{\chi_{\rm QCD}} \frac{\langle a_i|\mathbf{M}_B^2|a_i\rangle}{\left|\langle a_i|\hat{a}_{G\widetilde{G}}\rangle\right|^2}\right)$$



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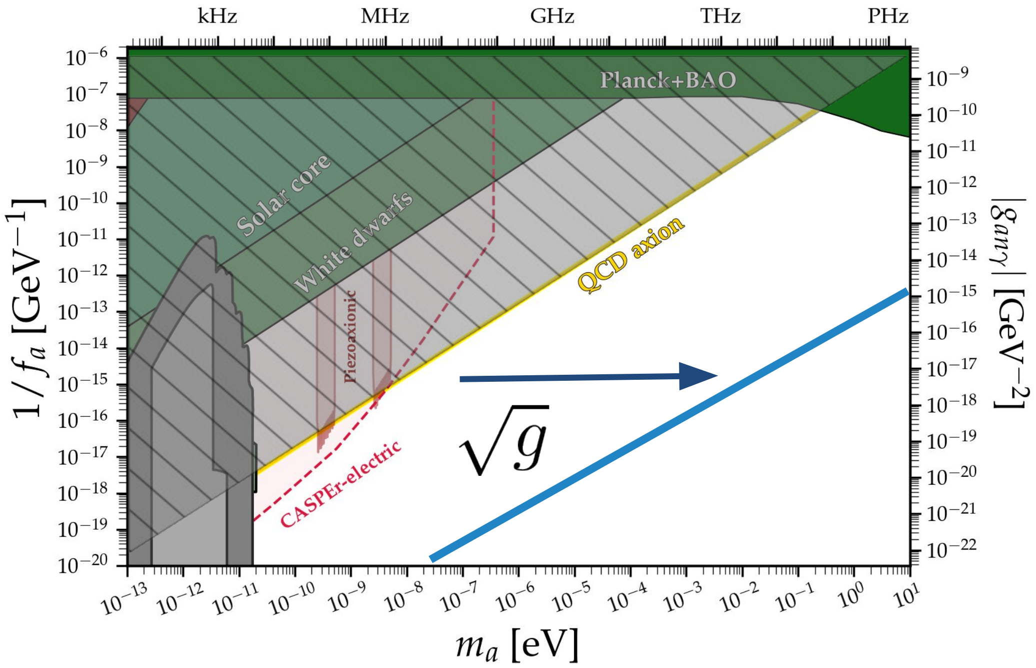
**1)** $g_i \geq 1 \quad = \left(1 + \frac{F^2}{\chi_{QCD}} \frac{\langle a_i|\mathbf{M}_B^2|a_i\rangle}{\left|\langle a_i|\hat{a}_{G\tilde{G}}\rangle\right|^2}\right)$



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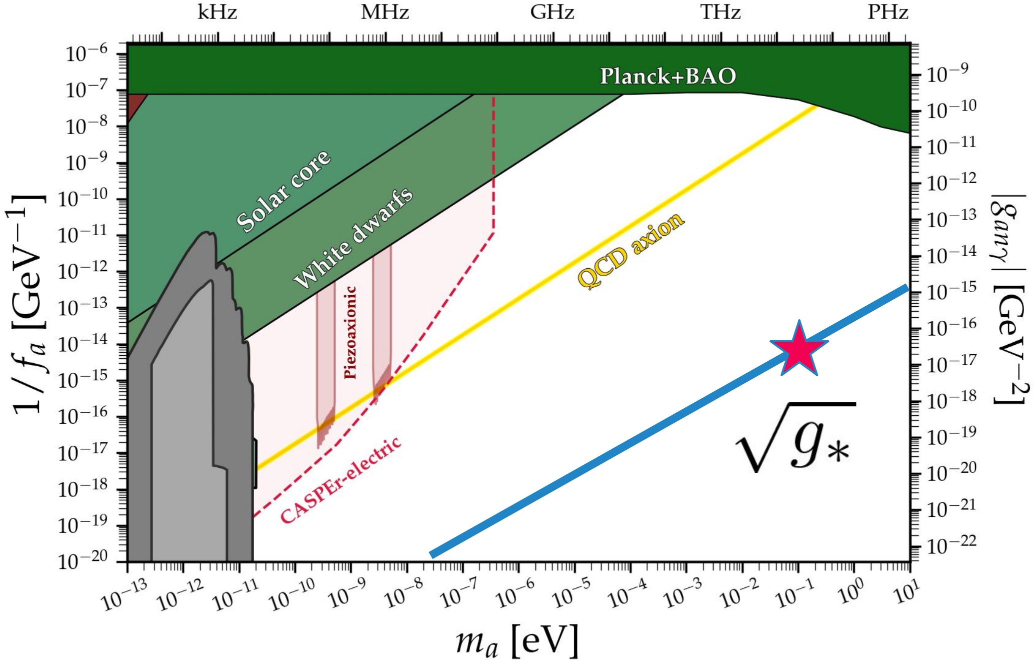

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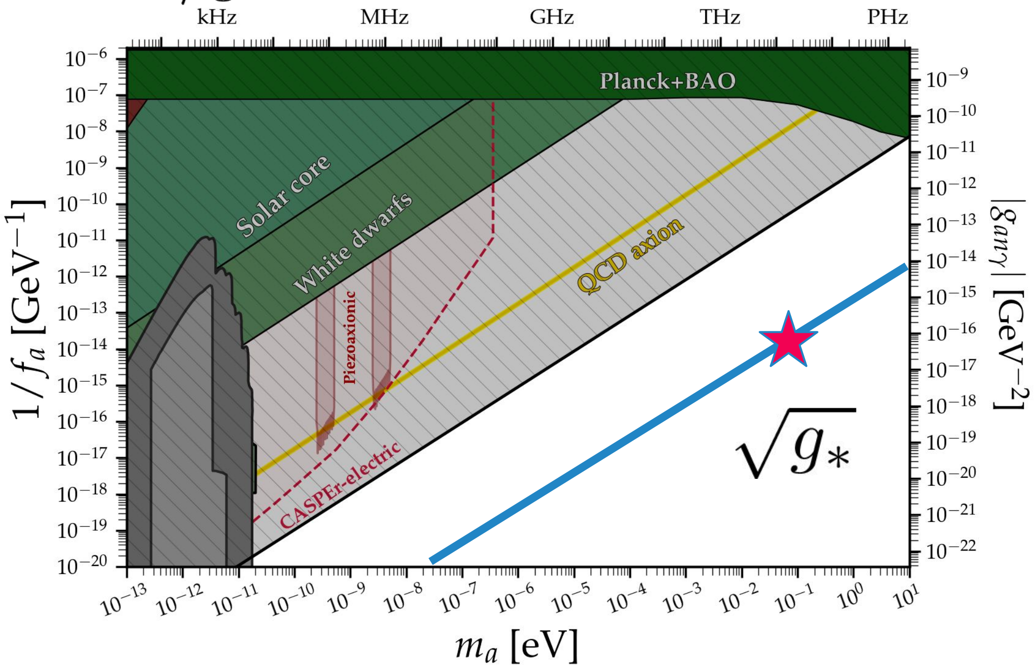


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# Experimental consequences $\frac{1}{g_i} = \frac{\chi_{\text{QCD}}}{m_i^2 f_i^2}$ $\sum_{i=1}^{N} \frac{1}{g_i} = 1$

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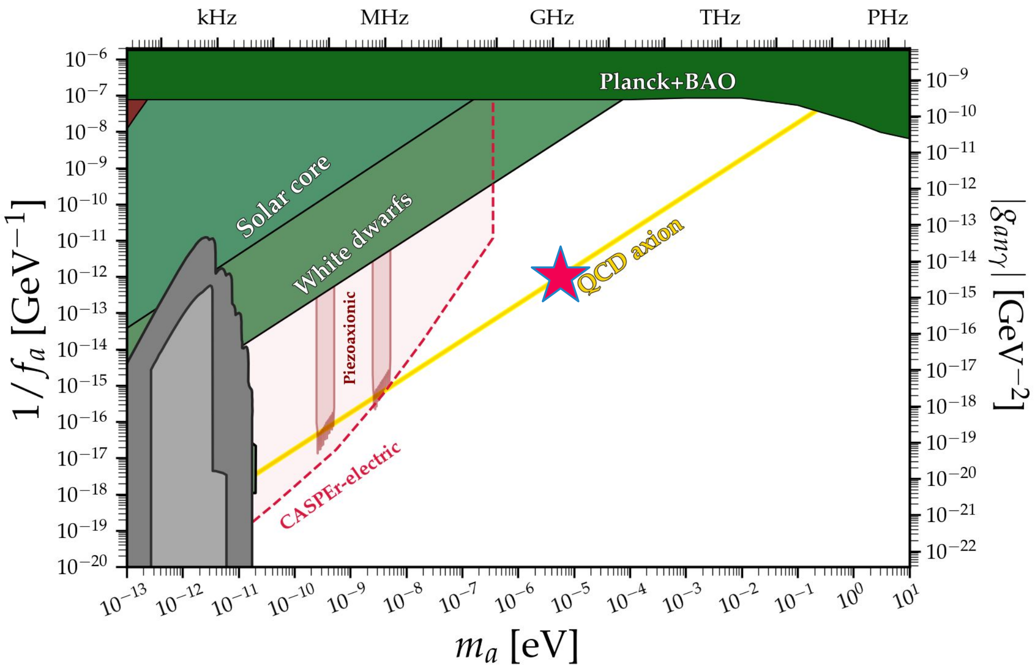


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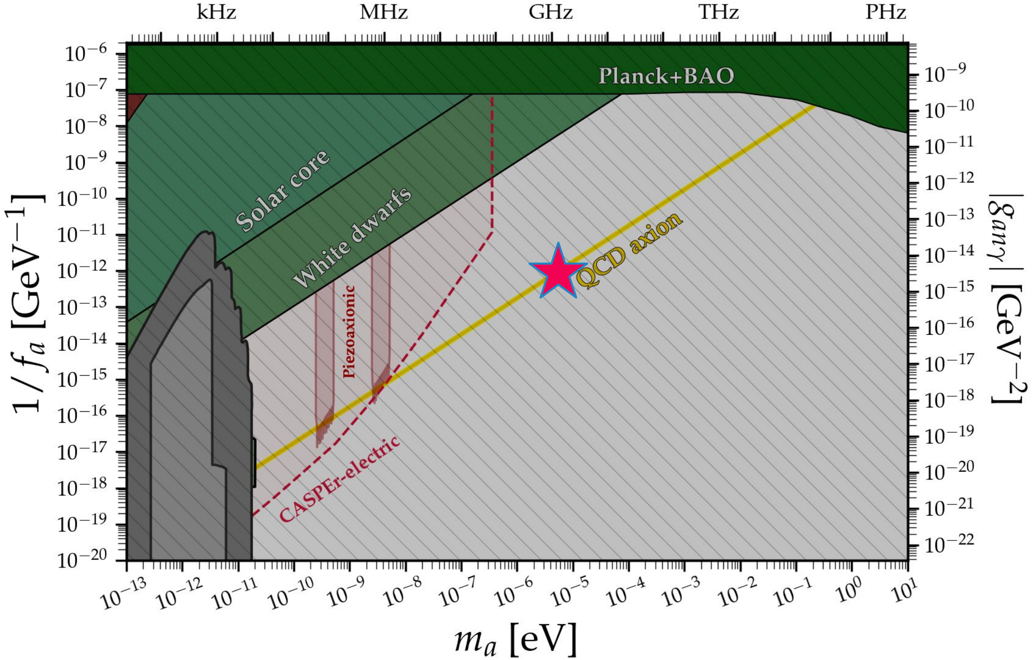


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# Experimental consequences $\frac{1}{g_i} = \frac{\chi_{\rm QCD}}{m_i^2 f_i^2}$ $\sum_{i=1}^{N} \frac{1}{g_i} = 1$

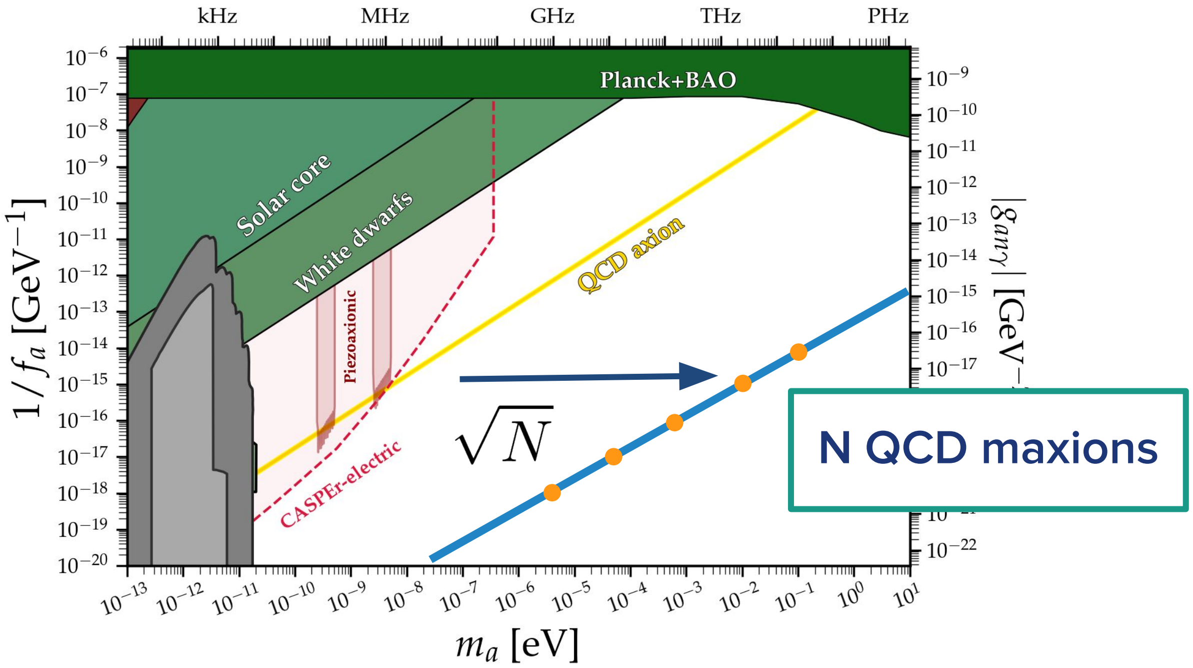
**4)** $$\max_{\mathbf{M}^2} \left\{ \min_i \{g_i\} \right\} = N \quad \Longrightarrow \quad g_i = N, \ \forall \, i \, .$$

# QCD Maxions

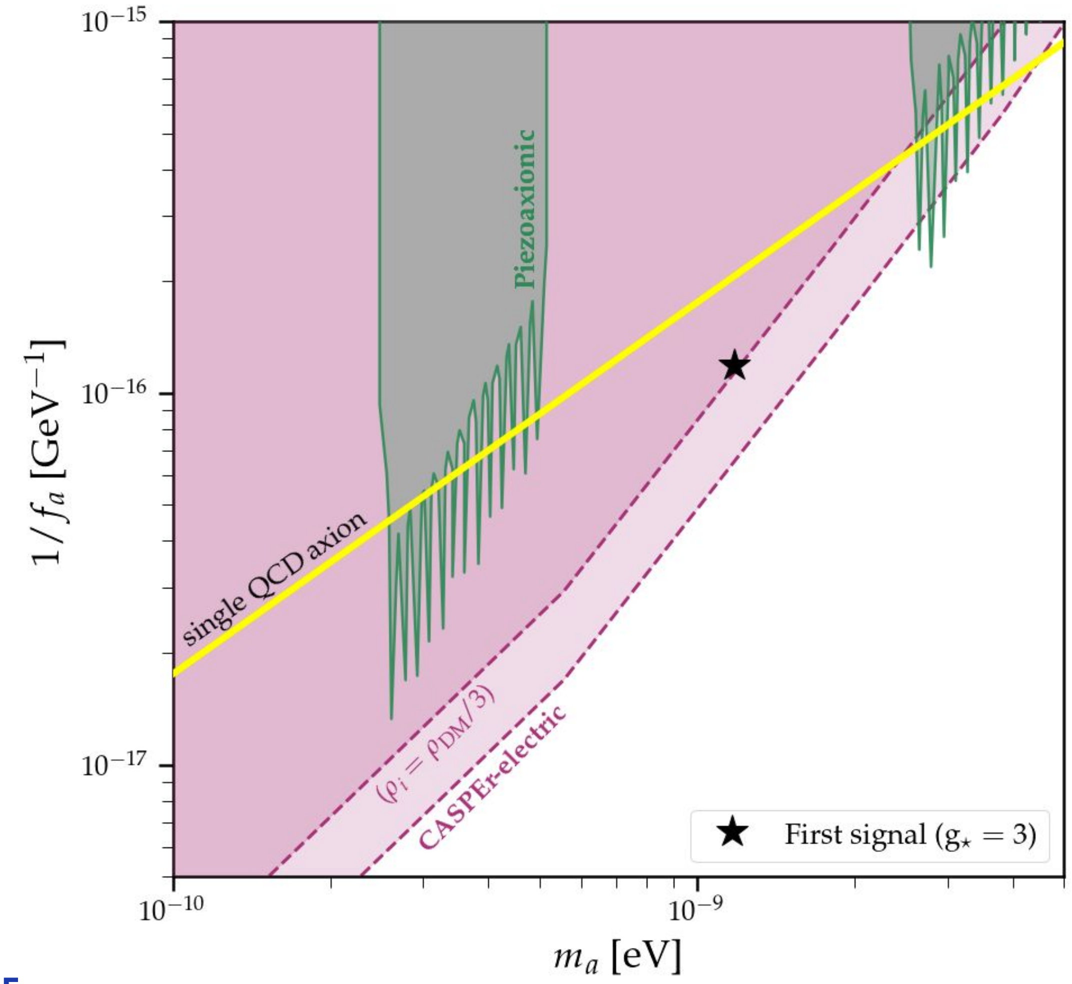
- = Maximally deviated QCD axions
- = Multiple QCD axions

$$\sum_{i=1}^{N} \frac{1}{g_i} = 1$$

4) $$\max_{\mathbf{M}^2}\left\{\min_i\{g_i\}\right\} = N \quad \Longrightarrow \quad g_i = N,\ \forall\, i\,.$$



$$\sum_{i=1}^{N} \frac{1}{g_i} = 1$$

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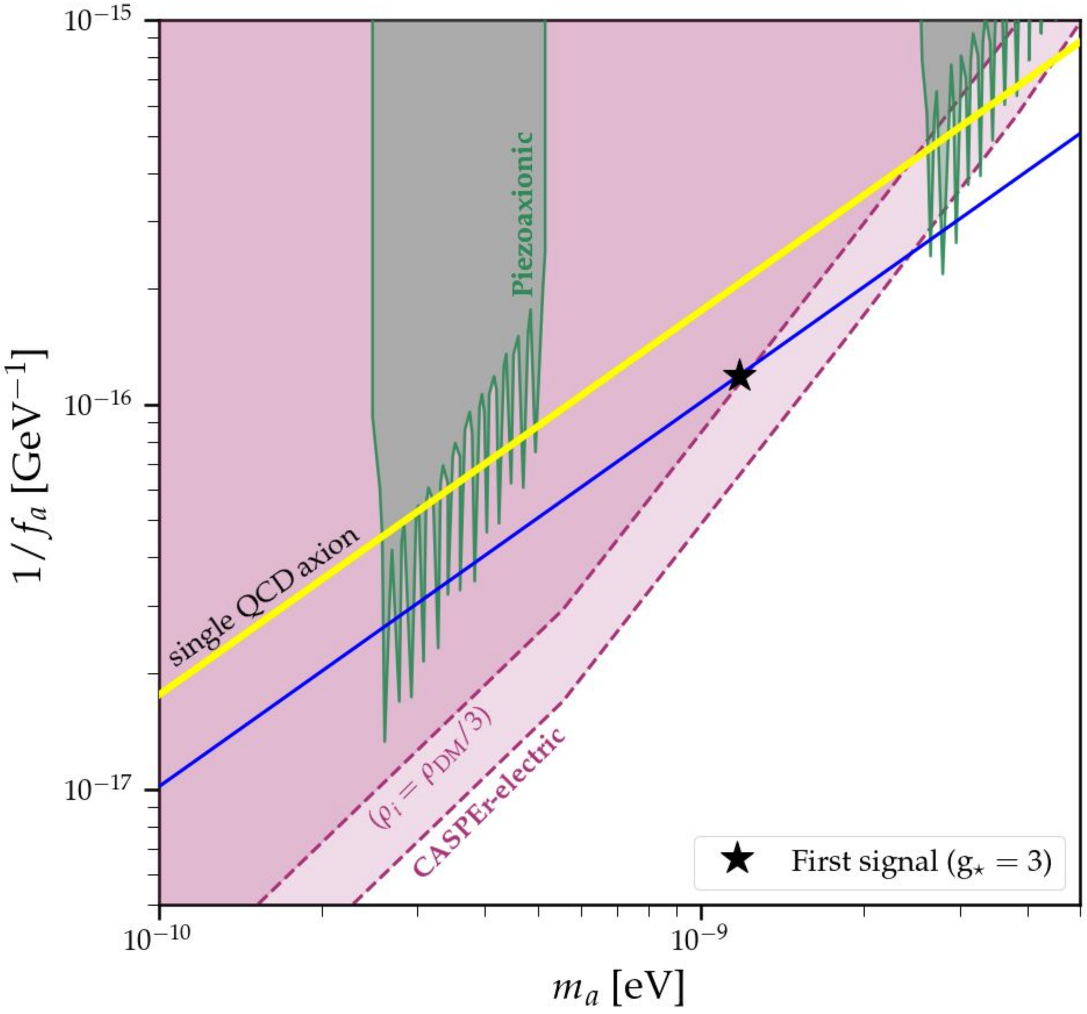
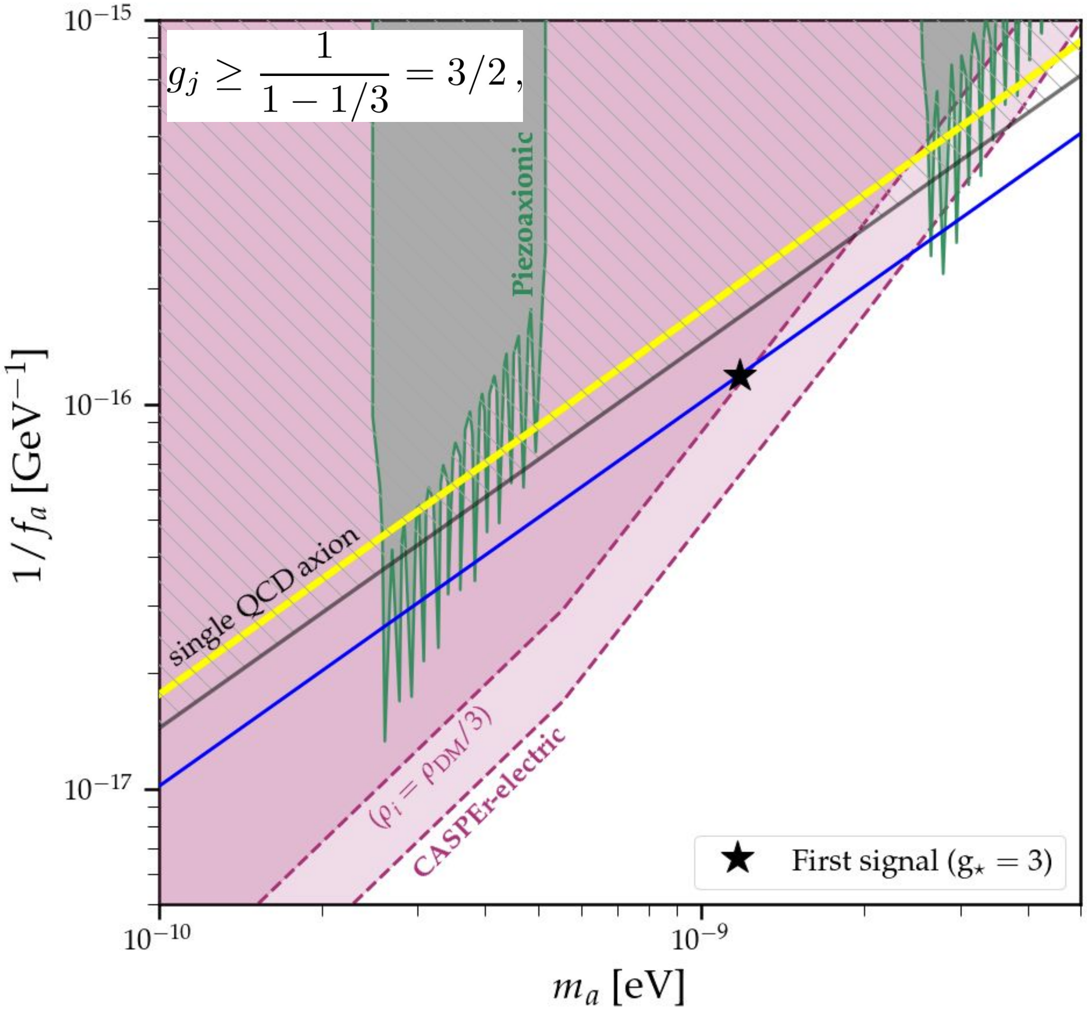



$$\sum_{i=1}^{N} \frac{1}{g_i} = 1$$

Based on **23C**

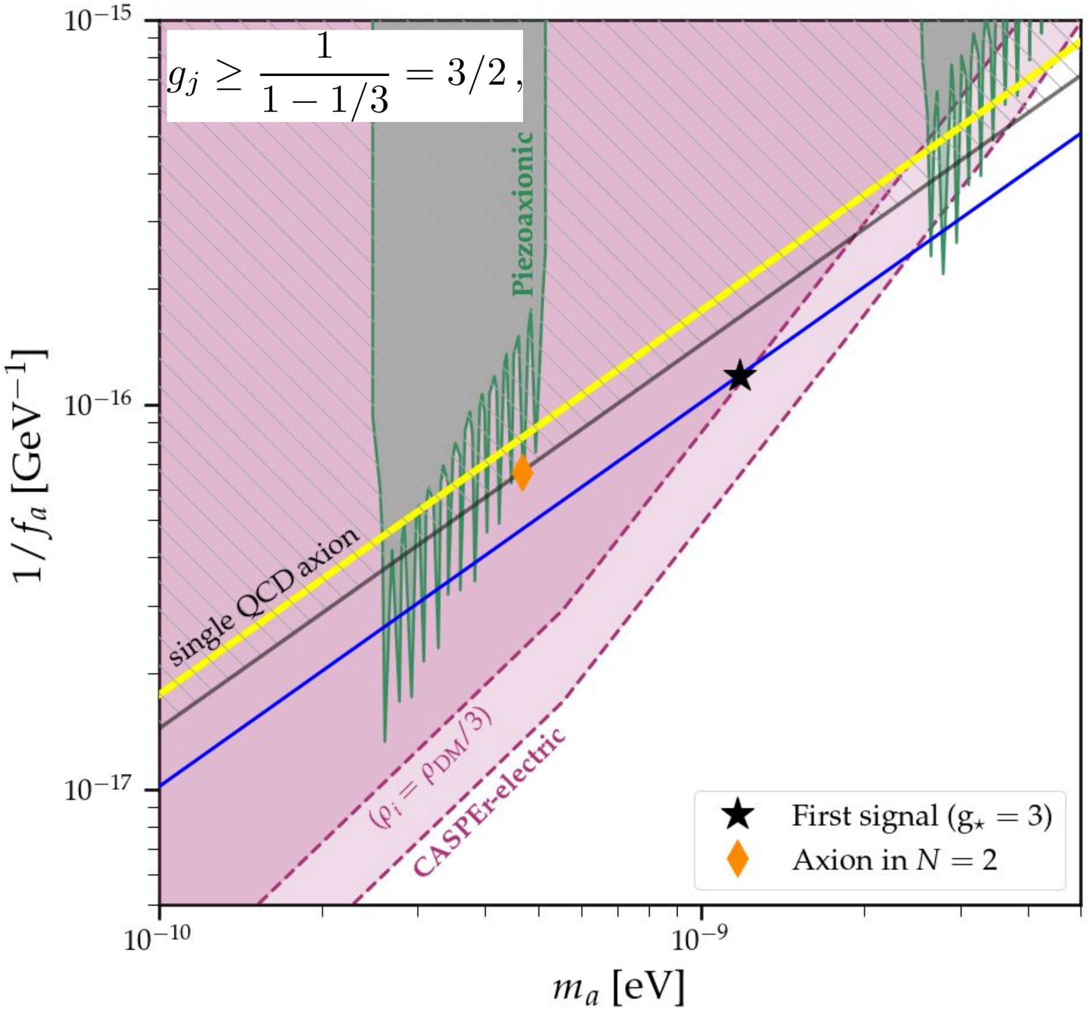



$$\sum_{i=1}^{N} \frac{1}{g_i} = 1$$

Based on **230**

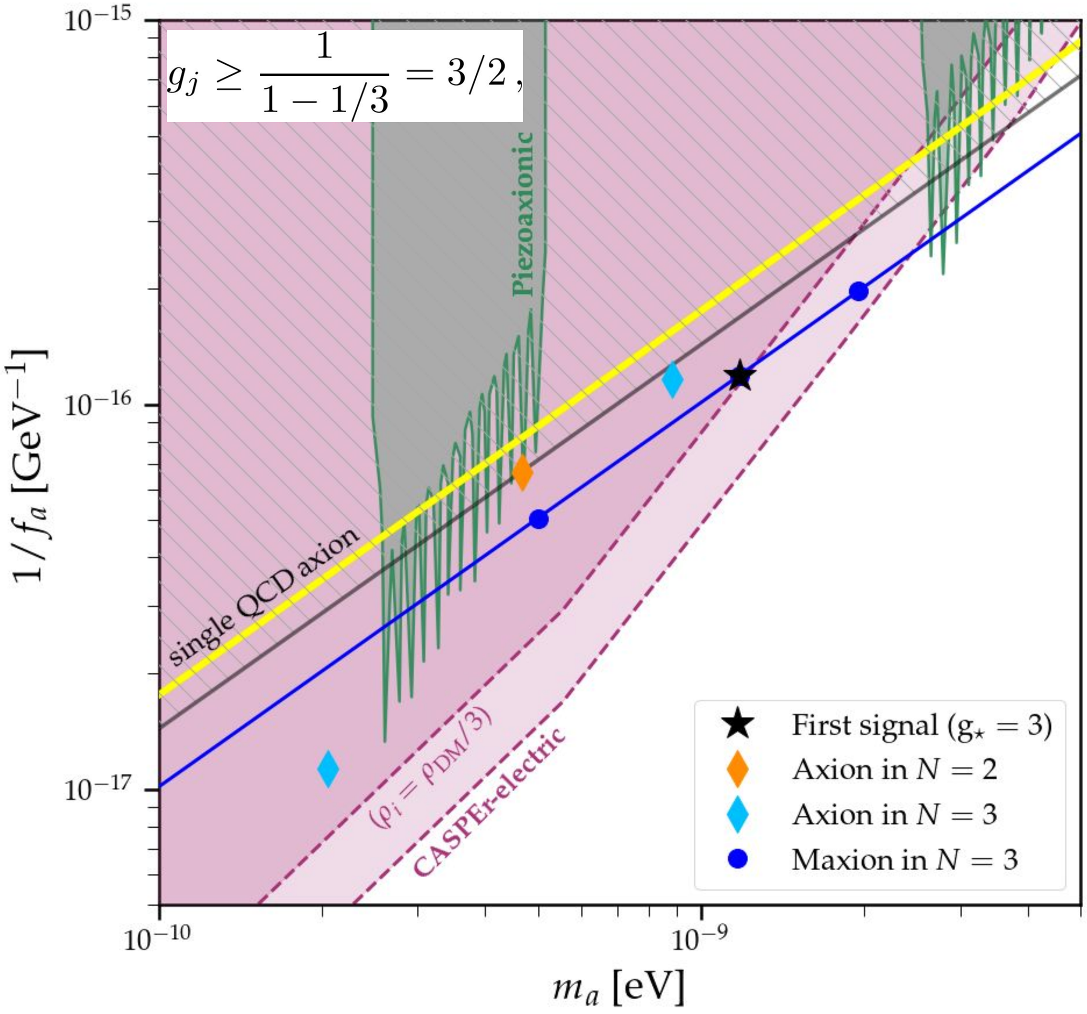
$$g_j \geq \frac{1}{1 - 1/3} = 3/2\,,$$

$$\sum_{i=1}^{N} \frac{1}{g_i} = 1$$




Based on **230**

$$\sum_{i=1}^{N} \frac{1}{g_i} = 1$$




Based on **230**

MATHEMATICAL PHYSICS

# Neutrinos Lead to Unexpected Discovery in Basic Math



*Three physicists wanted to calculate how neutrinos change. They ended up discovering an unexpected relationship between some of the most ubiquitous objects in math.*

PETER B. DENTON, STEPHEN J. PARKE, TERENCE TAO, AND XINING ZHANG

ABSTRACT. If $A$ is an $n \times n$ Hermitian matrix with eigenvalues $\lambda_1(A), \dots, \lambda_n(A)$ and $i, j = 1, \dots, n$, then the $j^{\text{th}}$ component $v_{i,j}$ of a unit eigenvector $v_i$ associated to the eigenvalue $\lambda_i(A)$ is related to the eigenvalues $\lambda_1(M_j), \dots, \lambda_{n-1}(M_j)$ of the minor $M_j$ of $A$ formed by removing the $j^{\text{th}}$ row and column by the formula

$$|v_{i,j}|^2 \prod_{k=1; k \neq i}^{n} (\lambda_i(A) - \lambda_k(A)) = \prod_{k=1}^{n-1} (\lambda_i(A) - \lambda_k(M_j)) .$$

We refer to this identity as the *eigenvector-eigenvalue identity* and show how

# QCD Maxion conditions

$$\sum_{i=1}^{N} \frac{1}{g_i} = 1$$

4) $$\max_{\mathbf{M}^2} \left\{ \min_i \{g_i\} \right\} = N \qquad \Longrightarrow \quad g_i = N, \ \forall\, i\,.$$

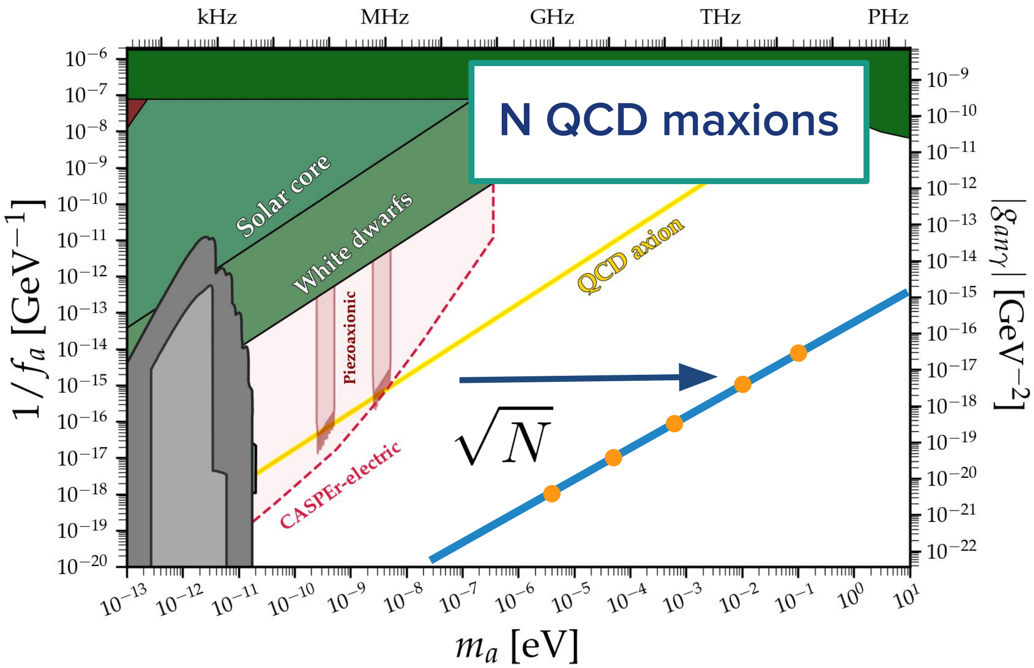
$$\mathcal{B}^{\mathbf{M}^2}_{N-k} = N\, \frac{\chi_{\mathrm{QCD}}}{F^2}\, \mathcal{B}^{\mathbf{M}^2_1}_{N-k-1}\,.$$

$$p_{\mathbf{M}^2}(\lambda) = \sum_{k=0}^{N} \frac{(-1)^{N-k}}{(n-k)!}\, \mathcal{B}^{\mathbf{M}^2}_{n-k}\, \lambda^k$$

$$\mathrm{tr}\,\mathbf{M}^2 = \sum_{i=1}^{N} m_i^2 = N \frac{\chi_{\mathrm{QCD}}}{F^2}\,.$$

$$m = N(N+1)/2.$$

m-parameter family of Maxion matrices,

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# QCD Maxion conditions

$$\sum_{i=1}^{N} \frac{1}{g_i} = 1$$

4) $\max_{\mathbf{N}} \{ \min\{g_i\} \} = N \implies g_i = N, \; \forall i \, .$

$\mathcal{B}^{\mathbf{M}^2}_{N-k} =$

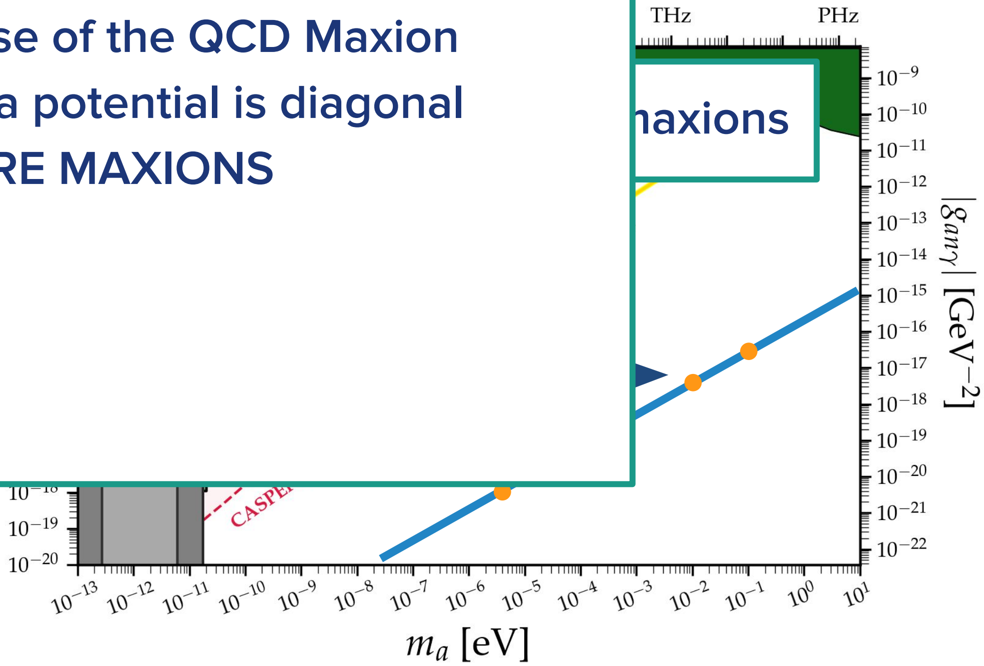
$p_{\mathbf{M}^2}(\lambda)$

$\mathrm{tr}\, \mathbf{M}^2$

As a particular case of the QCD Maxion

If the starting extra potential is diagonal

**LAGUERRE MAXIONS**




$$m = N(N+1)/2.$$

m-parameter family of Maxion matrices,

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# QCD Maxion conditions

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$p_{\mathbf{M}^2}(\lambda)$

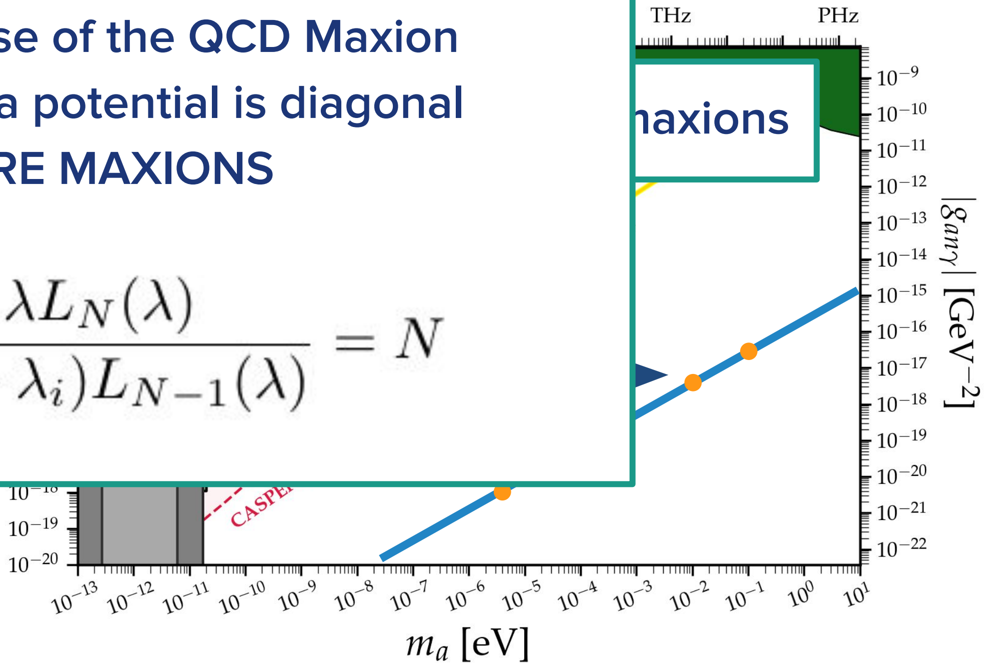
$\mathrm{tr}\, \mathbf{M}^2 =$

**As a particular case of the QCD Maxion**

**If the starting extra potential is diagonal**

**LAGUERRE MAXIONS**

$$g_i \equiv \lim_{\lambda \to \lambda_i} \frac{\lambda L_N(\lambda)}{(\lambda - \lambda_i) L_{N-1}(\lambda)} = N$$




m-parameter family of Maxion matrices,

# Coupling to photons

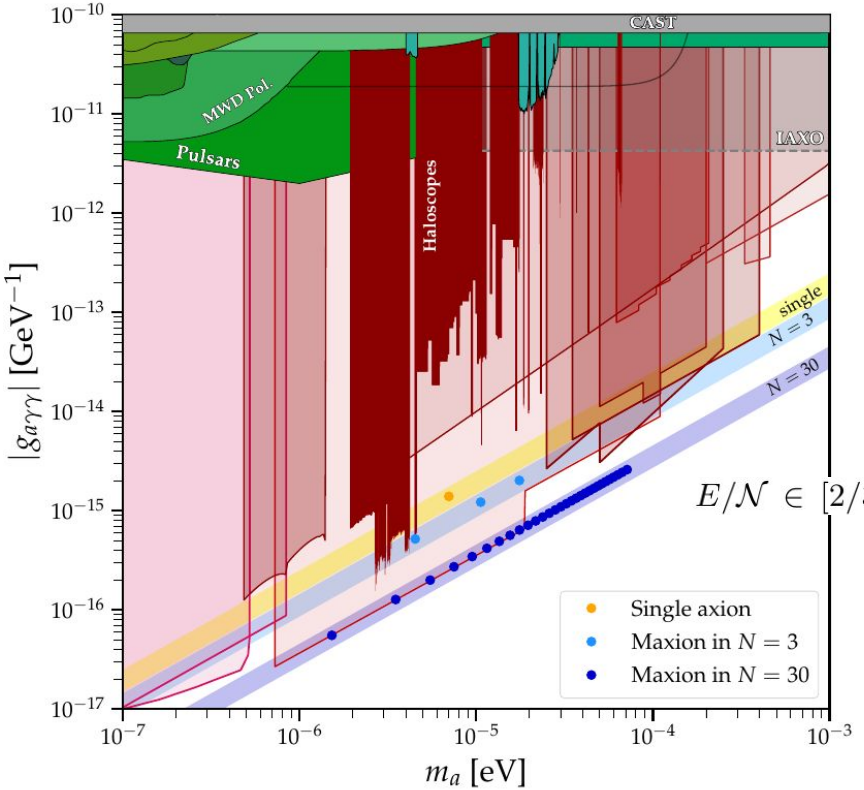
Same E/N

$$\delta\mathcal{L} = \frac{1}{4}\sum_{k=1}^{N} g^0_{\hat{a}_k\gamma\gamma}\,\hat{a}_k F\widetilde{F} \equiv \frac{\alpha_{em}}{8\pi}\sum_{k=1}^{N}\frac{E}{\mathcal{N}}\frac{\hat{a}_k}{\hat{f}_k}F\widetilde{F}\,,$$

All the results apply to photons if all $a_k$ have the same E/N

$$\frac{m_i^2}{g^2_{a_i\gamma\gamma}} = \left.\frac{m_a^2}{g^2_{a\gamma\gamma}}\right|_{\text{single QCD axion}} \times g_i\,.$$

$$\frac{(2\pi)^2\chi_{\rm QCD}}{\alpha^2_{em}}\left[\frac{E}{\mathcal{N}} - 1.92\right]^{-2}\sum_{i=1}^{N}\frac{g_{a_i\gamma\gamma}}{m_i^2} = 1\,.$$




$E/\mathcal{N} \in [2/3, 8/3]$.

# Caveats

- ➔ For sizable effects, extra masses need to be of the order of the QCD contribution
- ➔ Difficult to measure precisely gluon coupling, theoretical uncertainty,

$$g_{an\gamma} = e\frac{C_{\mathrm{EDM}}}{f_i} = (3.7 \pm 1.5) \times 10^{-3} \left(\frac{1}{f_i}\right) \frac{1}{\mathrm{GeV}}$$

(But typically very precise in masses/frequencies, detectable multiple signals)

- ➔ Coupling to photons more precise, but has model dependencies.
- ➔ Most experiments rely on DM

$$\sqrt{\rho_{\mathrm{DM},local}} \times g_{aXX},$$

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# Conclusions

- ➔ Multiple QCD axions can solve the strong CP problem!
- ➔ A sum rule from the PQ symmetry links the possible values $\{m_i, f_i\}$.
- ➔ Finding one axion gives us a lot of information on the posible others
- ➔ The maximum deviation for N axions is $\sqrt{N}$
- ➔ Outlook:
  - ◆ Modified experimental bounds for multiple axions?
  - ◆ DM production for multiple axions?
  - ◆ Topological defects, etc.
  - ◆ Connection to PQ quality problem
  - ◆ Motivated UV scenarios/patterns: string axiverse? Extra dim?

Francesca's talk

Kiwoon Choi's talk
Ryosuke's talk

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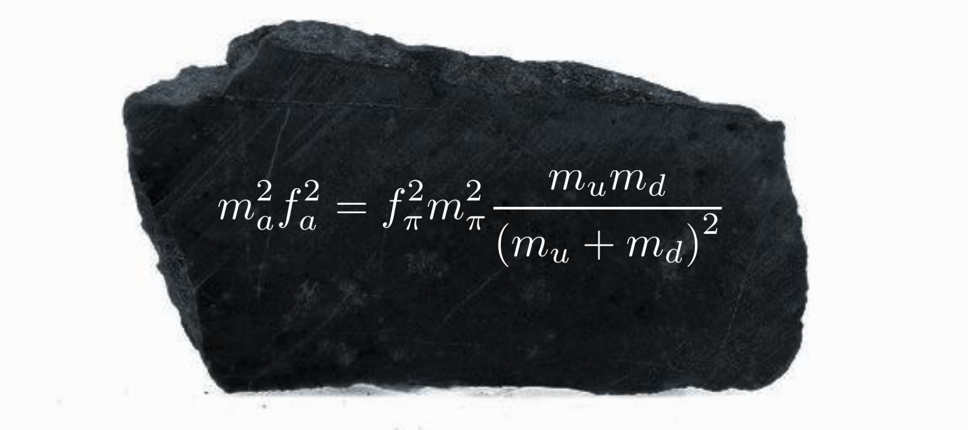
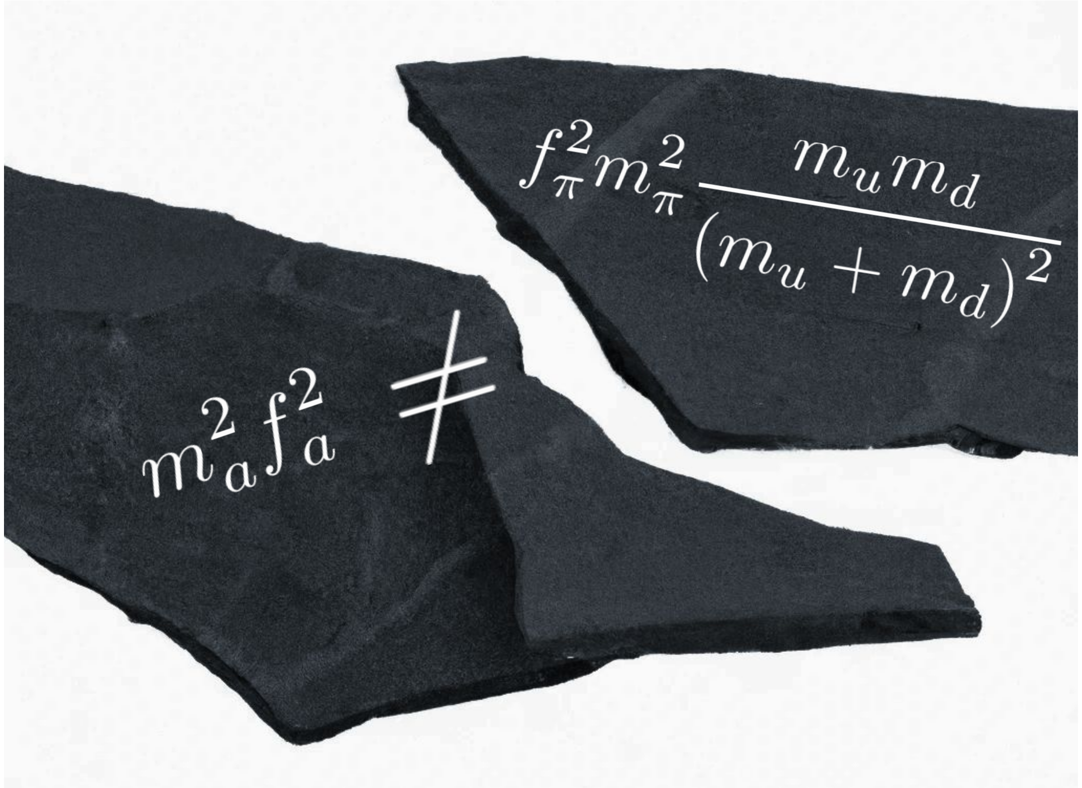
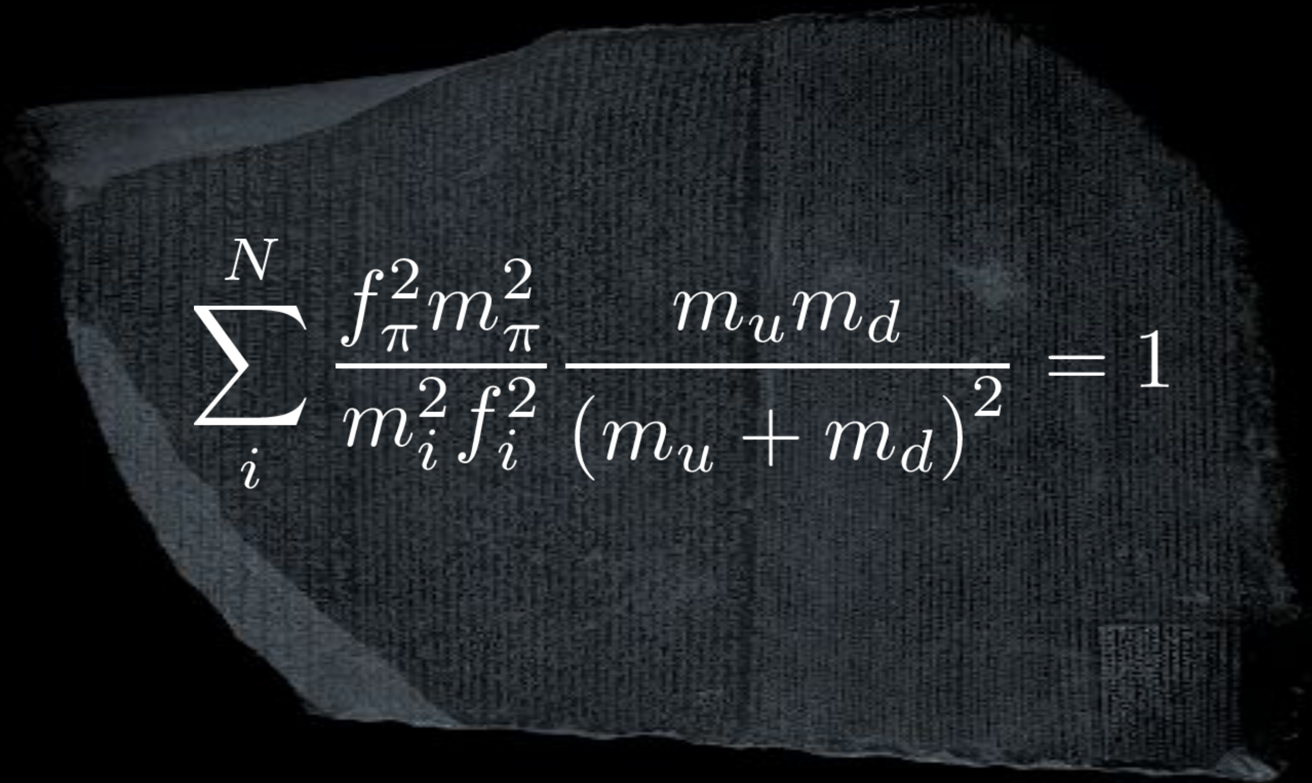
“

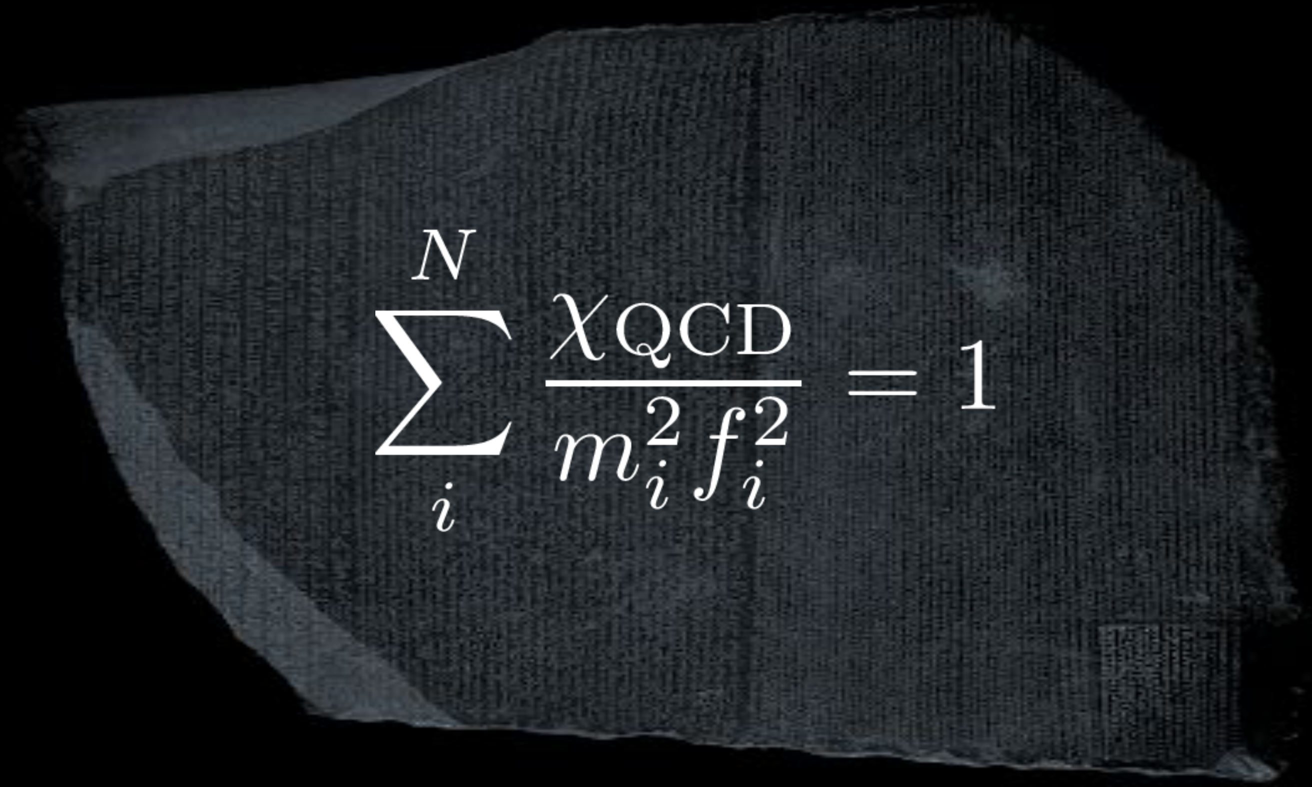
*Or is it written in stone?*




$m_a^2 f_a^2 \neq f_\pi^2 m_\pi^2 \frac{m_u m_d}{(m_u + m_d)^2}$



$$\sum_{i}^{N} \frac{f_\pi^2 m_\pi^2}{m_i^2 f_i^2} \frac{m_u m_d}{(m_u + m_d)^2} = 1$$



$$\sum_{i}^{N} \frac{\chi_{\mathrm{QCD}}}{m_i^2 f_i^2} = 1$$

# Thank you

# Back up slides

# Exact diagonalization: N=2

$$m_i^2 f_i^2 \equiv f_\pi^2 m_\pi^2 \frac{m_u m_d}{(m_u+m_d)^2} \times g_i$$

$$\equiv \chi_{\rm QCD}$$

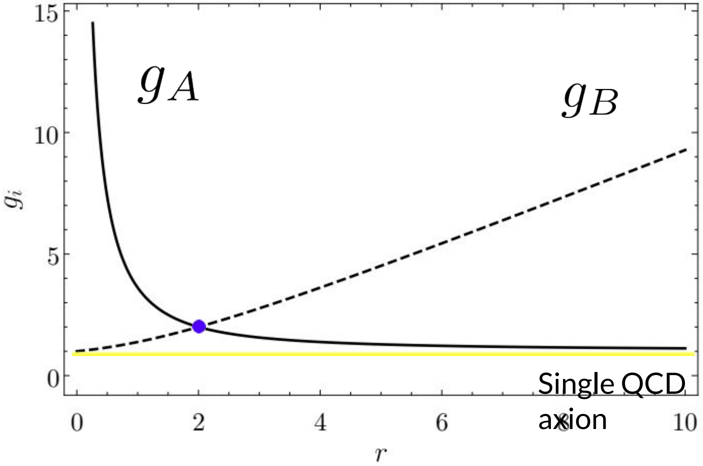
$$\mathcal{L}_{N=2} = \frac{\alpha_s}{8\pi}\left(\frac{\hat{a}_1}{\hat{f}} + \frac{\hat{a}_2}{\hat{f}} + \bar{\theta}\right) G\widetilde{G} - \frac{1}{2}\hat{m}_2^2\,\hat{a}_2^2\,. \qquad \longrightarrow V_{N=2} = \frac{1}{2}\chi_{\rm QCD}\left[\left(\frac{\hat{a}_1}{\hat{f}} + \frac{\hat{a}_2}{\hat{f}}\right)^2 + r\,\hat{a}_2^2\right]$$

$$m_{1,2}^2 = \frac{\chi_{\rm QCD}}{2\hat{f}^2}\left(2 + r \mp \sqrt{4+r^2}\right)$$

$$g_{1(2)} = \frac{2\sqrt{4+r^2}}{\sqrt{4+r^2} \pm (r-2)}\,,$$

$$a_1 = \frac{2\hat{a}_1 + \hat{a}_2\left(r - \sqrt{4+r^2}\right)}{\sqrt{2}\sqrt{4+r^2-r\sqrt{4+r^2}}}\,,$$

$$a_2 = \frac{2\hat{a}_2 + \hat{a}_1\left(-r + \sqrt{4+r^2}\right)}{\sqrt{2}\sqrt{4+r^2-r\sqrt{4+r^2}}}\,.$$



# Multiple QCD axions

$$\mathcal{L} = \frac{\alpha_s}{8\pi}\left(\sum_{k=1}^{N}\frac{\hat{a}_k}{\hat{f}_k} - \bar{\theta}\right) G\widetilde{G} - V_B(\hat{a}_1, \hat{a}_2, \ldots, \hat{a}_N) \to \frac{\alpha_s}{8\pi}\left(\frac{\hat{a}_{G\widetilde{G}}}{F} - \bar{\theta}\right) G\widetilde{G} - V_B^{\mathrm{R}}(\hat{a}_{G\widetilde{G}}, \ldots)$$

$$\frac{1}{F^2} = \sum_{k=1}^{N}\frac{1}{\hat{f}_k^2}$$

In the rotated basis,

$$\boxed{\mathbf{M}^2 \equiv \mathbf{R}\,\hat{\mathbf{M}}^2\mathbf{R}^T}$$

$$\mathbf{M}^2 = \mathbf{M}_A^2 + \mathbf{M}_B^2 = \begin{pmatrix} b_{11} & \mathbf{X}^\dagger \\ \mathbf{X} & \mathbf{M}_1^2 \end{pmatrix} = \frac{\chi_{\mathrm{QCD}}}{F^2}\begin{pmatrix} 1 & 0 \\ 0 & \mathbf{0} \end{pmatrix} + \begin{pmatrix} b_{11} - \frac{\chi_{\mathrm{QCD}}}{F^2} & \mathbf{X}^\dagger \\ \mathbf{X} & \mathbf{M}_1^2 \end{pmatrix},$$

$$\boxed{\exists U(1)_{PQ} \implies \lim_{\chi_{\mathrm{QCD}}\to 0} \det \mathbf{M}^2 = 0 \implies \det \mathbf{M}_B^2 = 0} \quad \langle \hat{a}_0 | a_{G\widetilde{G}} \rangle \neq 0$$

Applying Schur's formula for invertible $\mathbf{M_1}$,

$$\det \mathbf{M}_1^2 \left(b_{11} - \frac{\chi_{\mathrm{QCD}}}{F^2} - \mathbf{X}^\dagger\mathbf{M}_1^{-2}\mathbf{X}\right) = 0$$

$$\Rightarrow \boxed{\frac{\det \mathbf{M}^2}{\det \mathbf{M}_1^2} = \left(b_{11} - \mathbf{X}^\dagger\mathbf{M}_1^{-2}\mathbf{X}\right) = \frac{\chi_{\mathrm{QCD}}}{F^2}}$$

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# Multiple QCD axions

$$\mathcal{L} = \frac{\alpha_s}{8\pi} \underbrace{\left( \sum_{k=1}^{N} \frac{\hat{a}_k}{\hat{f}_k} - \bar{\theta} \right)}_{\hat{a}_{G\widetilde{G}}/F} G\widetilde{G} - V_B(\hat{a}_1,\, \hat{a}_2, \ldots, \hat{a}_N)\,,$$

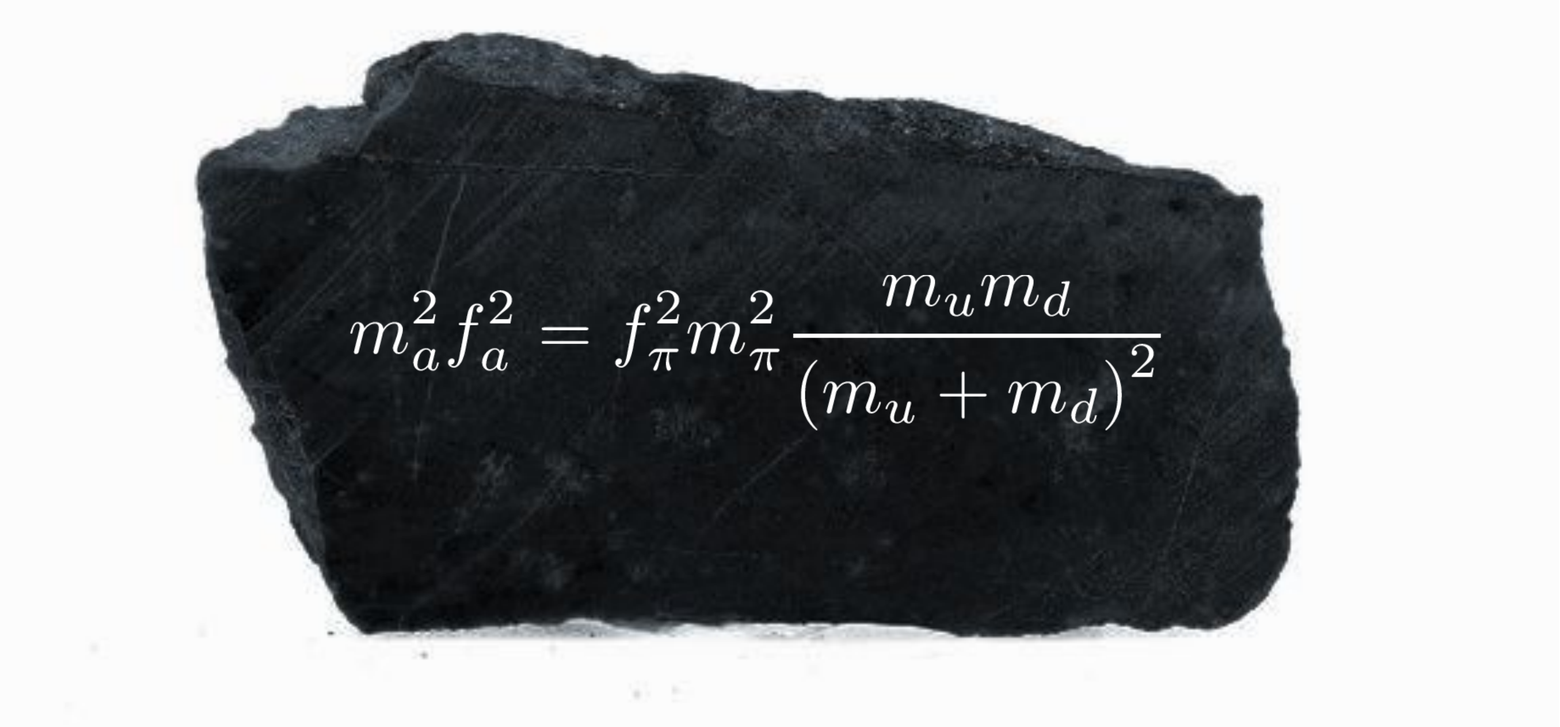
# Multiple QCD axions

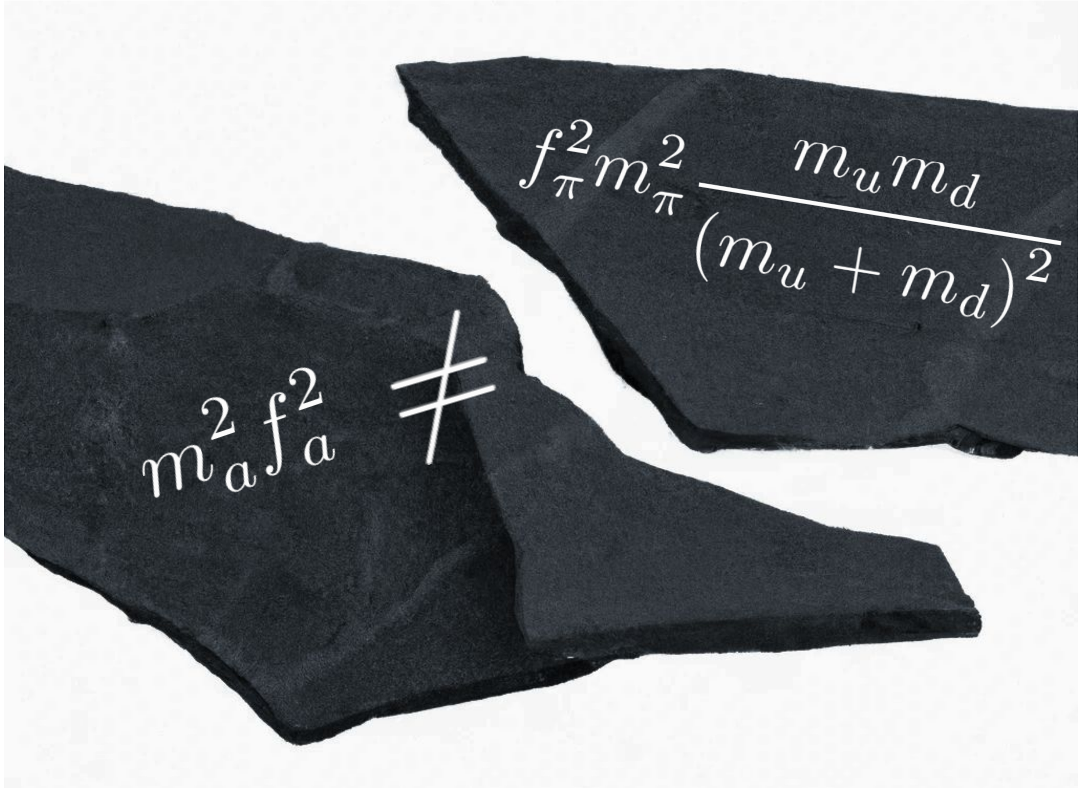
$$\mathcal{L} = \frac{1}{2}\chi_{\text{QCD}} \Big( \underbrace{\sum_{k=1}^{N} \frac{\hat{a}_k}{\hat{f}_k}}_{\hat{a}_{G\tilde{G}}/F} \Big)^2 + V_B\left(\tilde{a}_1, \ldots, \tilde{a}_{N-1}\right).$$

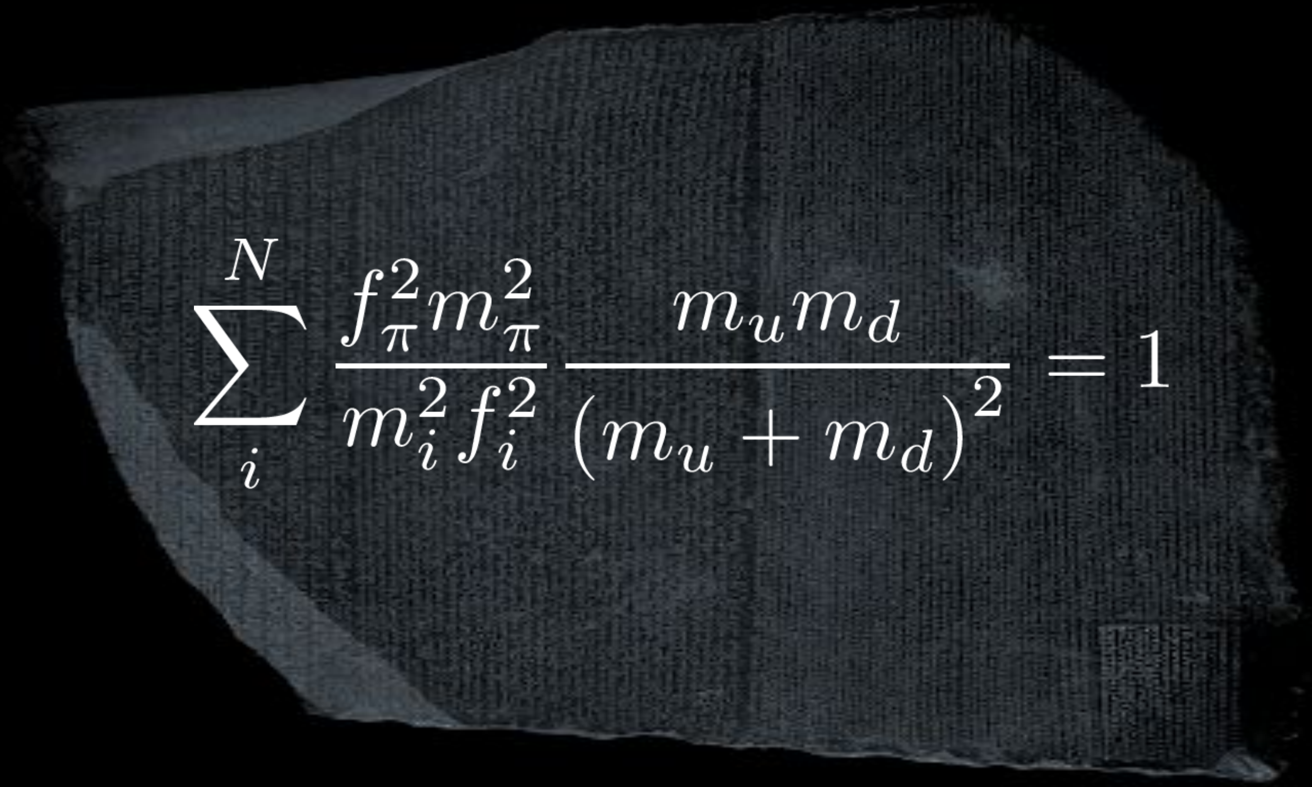
$$\mathcal{L} \supset -\frac{1}{2}\hat{a}_k \hat{\mathbf{M}}^2_{kl}\hat{a}_l \quad \text{with} \quad \hat{\mathbf{M}}^2 = \hat{\mathbf{M}}^2_A + \hat{\mathbf{M}}^2_B\,,$$

$$\exists\, U(1)_{PQ} \implies \lim_{\chi_{\text{QCD}}\to 0} \det \hat{\mathbf{M}}^2 = 0 \implies \det \hat{\mathbf{M}}_B = 0\,.$$

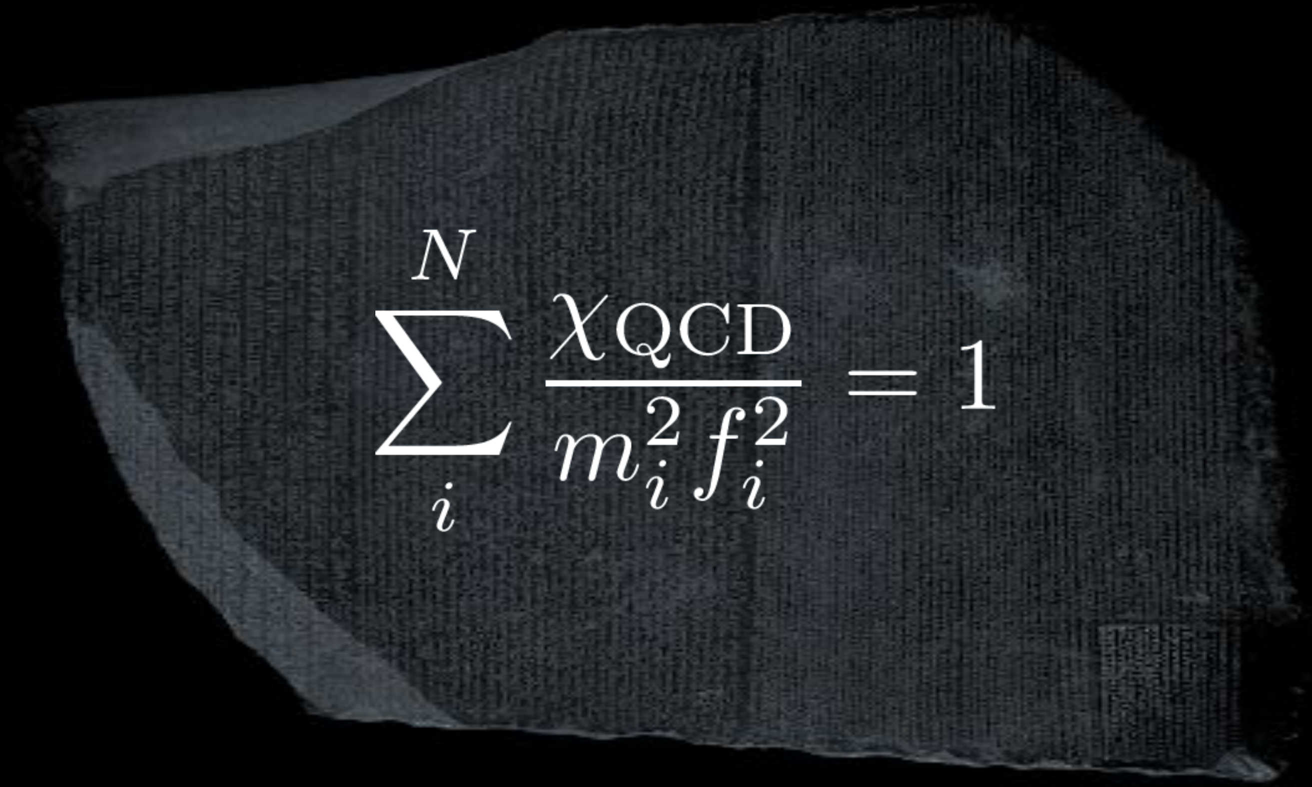
$$\text{Rank}\left[\hat{\mathbf{M}}^2_A\right] = 1$$


$$m_a^2 f_a^2 = f_\pi^2 m_\pi^2 \frac{m_u m_d}{(m_u + m_d)^2}$$


$m_a^2 f_a^2 \neq$
$f_\pi^2 m_\pi^2 \frac{m_u m_d}{(m_u + m_d)^2}$



$$\sum_{i}^{N} \frac{f_\pi^2 m_\pi^2}{m_i^2 f_i^2} \frac{m_u m_d}{(m_u + m_d)^2} = 1$$

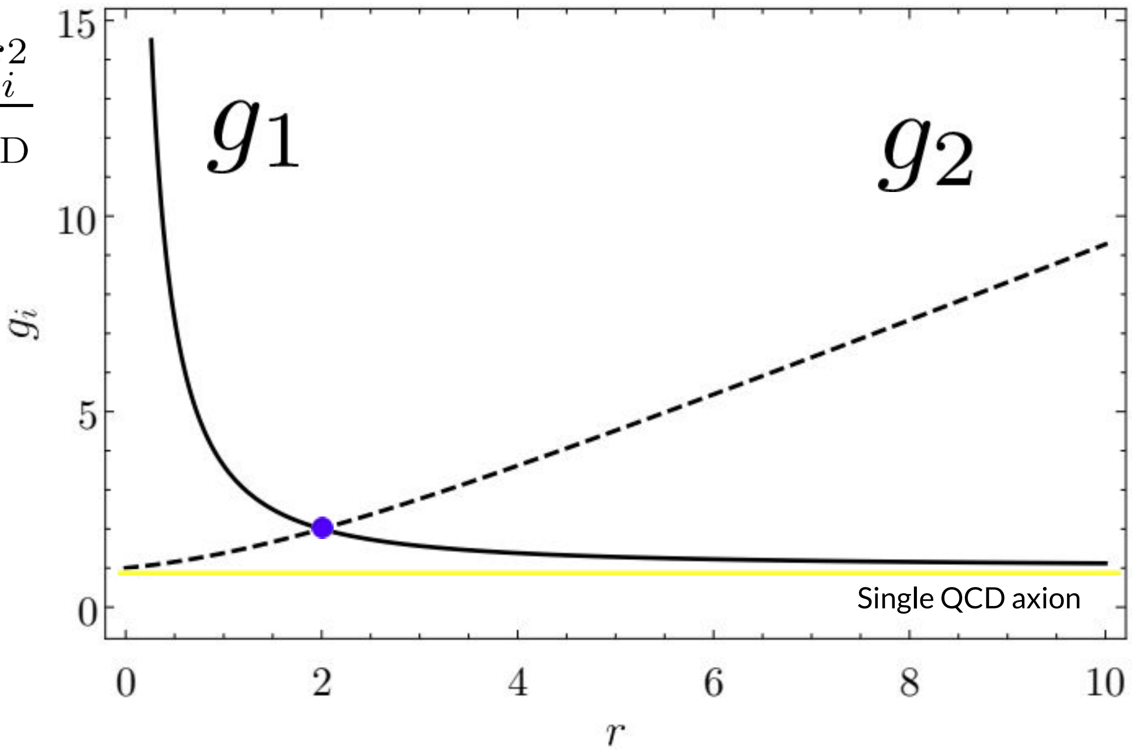


$$\sum_{i}^{N} \frac{\chi_{\mathrm{QCD}}}{m_i^2 f_i^2} = 1$$

# Toy example: N=2

$$m_i^2 f_i^2 \equiv f_\pi^2 m_\pi^2 \frac{m_u m_d}{(m_u + m_d)^2} \times g_i$$

$$g_i = \frac{m_i^2 f_i^2}{\chi_{\rm QCD}}$$




$$g_{1(2)} = \frac{2\sqrt{4+r^2}}{\sqrt{4+r^2} \pm (r-2)}\,,$$

$$g = 1$$

Based on **2305.15465** - Pablo Quílez

# Toy example: N=2

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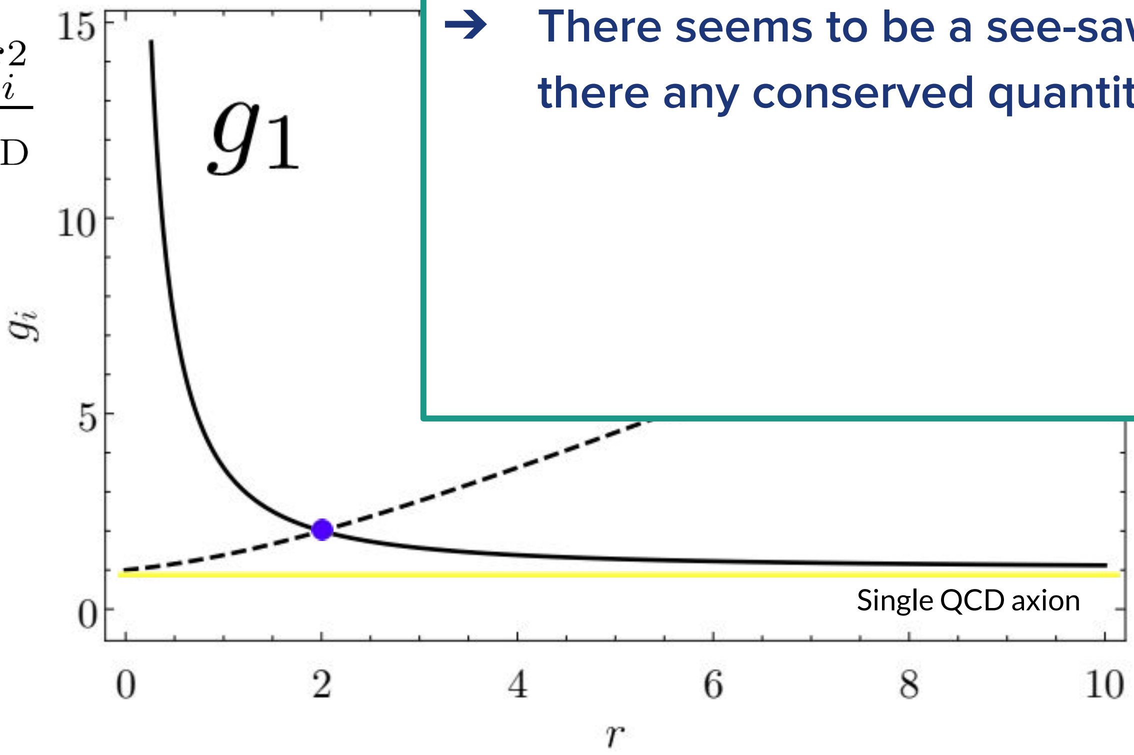


- ➔ There seems to be a see-saw like pattern, is there any conserved quantity or sum rule?

Based on **2305.15465** - Pablo Quílez

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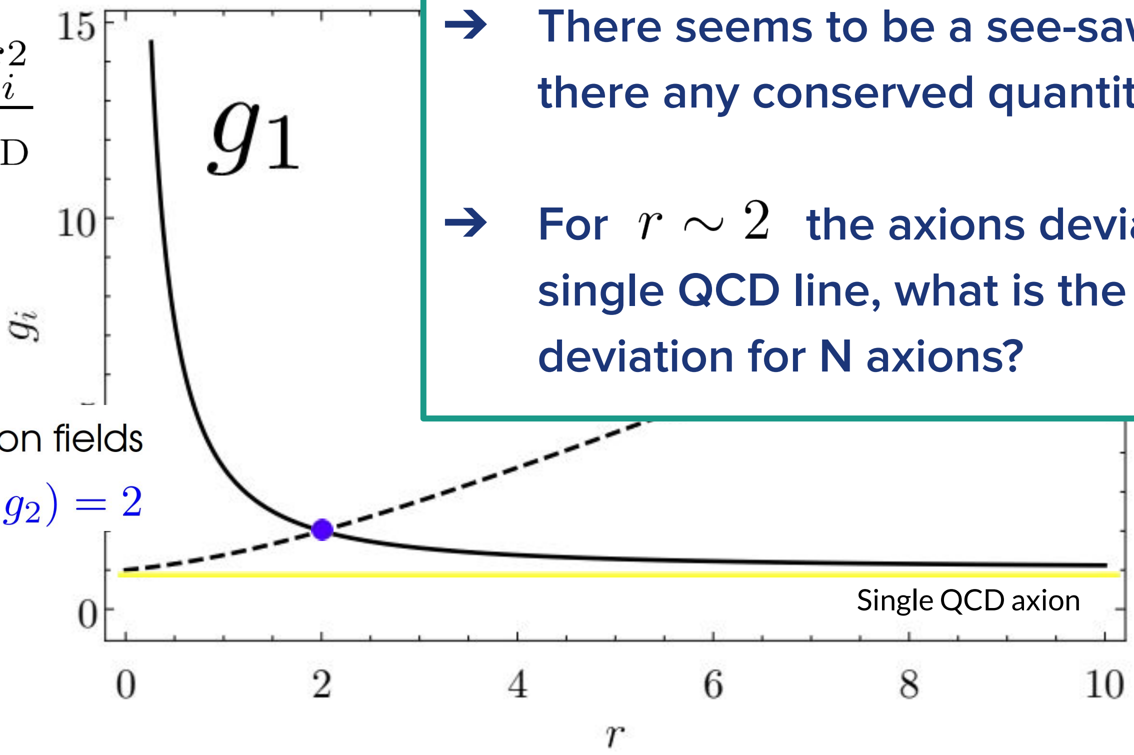


- There seems to be a see-saw like pattern, is there any conserved quantity or sum rule?
- For $r \sim 2$ the axions deviate from the single QCD line, what is the maximum deviation for N axions?

- 2 QCD maxion fields

$$\max \min\,(g_1,\ g_2) = 2$$

Based on **2305.15465** - Pablo Quílez

# Deviation from the QCD line: $g_i$-factor

$$\equiv \chi_{\rm QCD}$$

$$m_i^2 f_i^2 \equiv f_\pi^2 m_\pi^2 \frac{m_u m_d}{(m_u+m_d)^2} \times g_i$$

$$\mathcal{L} \supset \frac{\alpha_s}{8\pi}\frac{a_i}{f_i} G\widetilde{G}$$

$$\mathcal{L} \supset -\frac{1}{2} m_i^2 a_i^2$$