The axion quality problem & its possible solutions

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See, e.g., M. Redi, RS, 1602.05427, JHEP 05 (2016) 104

Relevant talks : Kiwoon Choi's talk on Tuesday,
Motoo Suzuki's talk on Wednesday,
and Sang Hui Im's talk on Friday

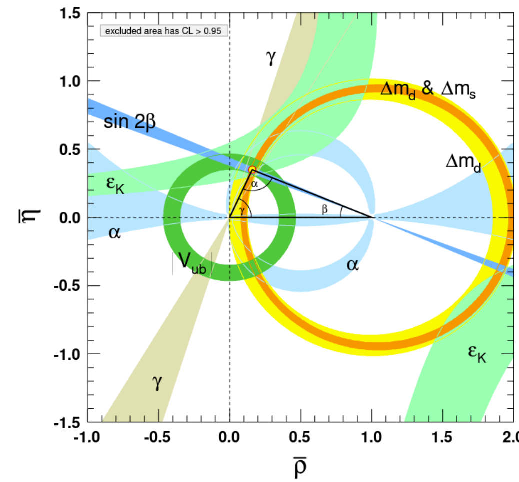
2024. 8. 8 @ The Axion Quest 2024, 20th Rencontres du Vietnam

CP violation in the quark sector

1. CP violation in flavor changing process

$K_L \to \pi\pi,\ B \to J/\psi K$, etc…

O(1) CP violation!



2. CP violation in flavor conserving process

Neutron EDM :

$|d_n| < 1.8 \times 10^{-26}$ e cm [Abel+ (2020)]

Neutron MDM :

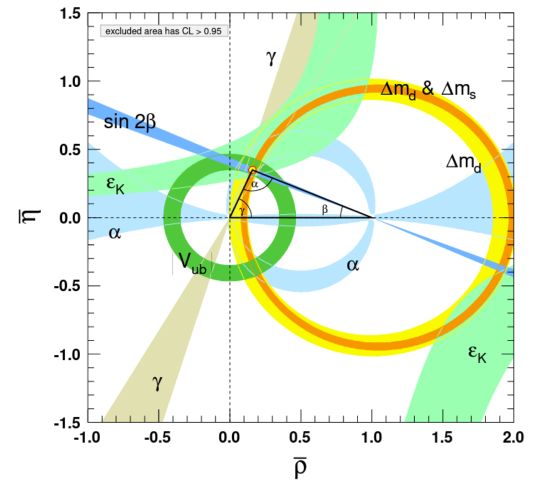
c.f.) $\mu_n \sim 2 \times 10^{-14}$ e cm

No observation yet. Only severe upperbound.

CP violation in the Standard Model

1. The phase of CKM matrix

$$\delta_{\mathrm{CKM}} = O(1) \qquad V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$



2. The phase of QCD vacuum

$$|\bar{\theta}| < 10^{-10}$$

The strong CP problem

Why $|\bar{\theta}| \ll \delta_{\mathrm{CKM}}$!?

[Jackiw, Rebbi (1976)]
[Callan, Dashen, Gross (1976)]
[Peccei, Quinn (1977)]

The QCD axion

[Peccei, Quinn, (1977); Weinberg (1978); Wilczek (1978)]

$\bar{\theta}$ -term in the Standard Model Lagrangian

$$L = \bar{\theta} \frac{g_s^2}{32\pi^2} G_{\mu\nu} \tilde{G}^{\mu\nu}$$

The QCD axion

[Peccei, Quinn, (1977); Weinberg (1978); Wilczek (1978)]

$\bar{\theta}$ -term + QCD axion

1. No potential (except for QCD effect)
2. $aG\tilde{G}$ coupling

$$L = \bar{\theta}\frac{g_s^2}{32\pi^2}G_{\mu\nu}\tilde{G}^{\mu\nu} + \frac{1}{2}\left(\partial_\mu a\right)^2 + \frac{a}{f}\frac{g_s^2}{32\pi^2}G_{\mu\nu}\tilde{G}^{\mu\nu}$$

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$$L = \frac{1}{2}\left(\partial_\mu a\right)^2 + \left(\bar{\theta} + \frac{a}{f}\right)\frac{g_s^2}{32\pi^2} G_{\mu\nu}\tilde{G}^{\mu\nu}$$

Axion can absorb $\bar{\theta}$

$\bar{\theta}$ is promoted to “dynamical scalar field”!

The QCD axion

[Peccei, Quinn, (1977); Weinberg (1978); Wilczek (1978)]

$\bar{\theta}$ -term + QCD axion

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2. $aG\tilde{G}$ coupling

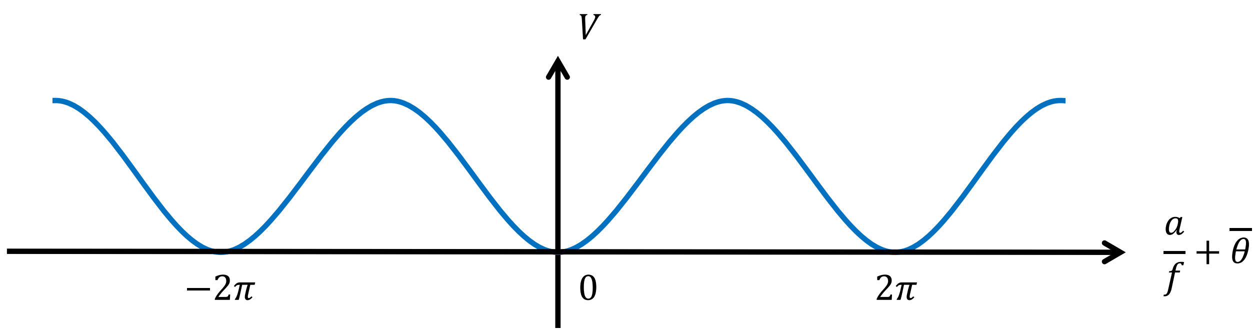
Low-energy effective description:

$$L_{\text{eff}} = \frac{1}{2}(\partial_\mu a)^2 - V\left(\bar{\theta} + \frac{a}{f}\right)$$

Non-perturbative effect from QCD

V has global minimum at $\frac{a}{f} + \bar{\theta} = 0$

[Vafa, Witten (1984)]



Where the QCD axion comes from?

[Peccei, Quinn, (1977); Weinberg (1978); Wilczek (1978)]

$\bar{\theta}$ -term + QCD axion

1. No potential ← Axion is NG boson from SSB of U(1)PQ sym.
2. $aG\tilde{G}$ coupling ← U(1)PQ sym. has chiral anomaly under QCD

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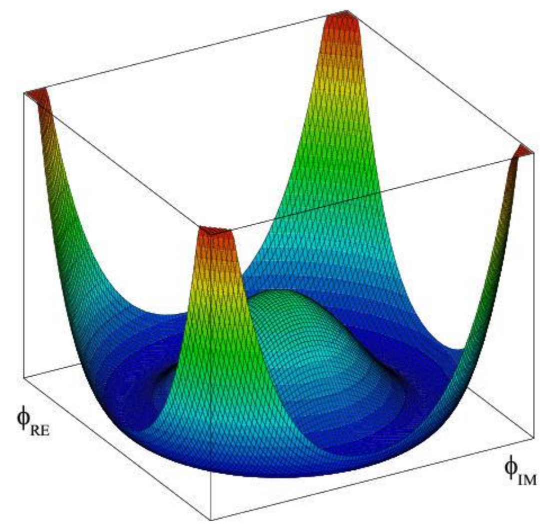
ex) KSVZ model [Kim (1979)] [Shifman, Vainshtein, Zakharov (1980)]

$$L = |\partial\Phi|^2 - \lambda\left(|\Phi|^2 - \frac{f_a^2}{2}\right)^2 - (y\Phi Q_L Q_R^c + h.c.)$$

$$\Phi = \frac{f_a}{\sqrt{2}}\exp\left(\frac{ia}{f_a}\right)$$

$$L_{eff} = \frac{1}{2}(\partial a)^2 + \frac{a}{f_a}\frac{g_s^2}{32\pi^2}G_{\mu\nu}\tilde{G}^{\mu\nu}$$



[en.wikipedia.org/wiki/Higgs_mechanism]

Does the QCD axion really works?

$\bar{\theta}$ -term + QCD axion

1. No potential $\leftarrow$ Axion is NG boson from SSB of U(1)PQ sym.
2. $aG\tilde{G}$ coupling $\leftarrow$ U(1)PQ sym. has chiral anomaly under QCD

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Are those two conditions compatible?

1. says "we should not break PQ symmetry explicitly"
2. says "we should break PQ symmetry explicitly (in a special way)"

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Are those two conditions compatible?

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2. says "we should break PQ symmetry explicitly (in a special way)"

In this regard the Peccei–Quinn symmetry is particularly puzzling because it is not a summetry of the full interacting theory but is broken by non-perturbative effects.

[Georgi, Hall, Wise (1981)]

The QCD axion (with small PQ violation)

$\bar{\theta}$ -term + QCD axion

1. No potential $\leftarrow$ Axion is NG boson from SSB of U(1)PQ sym.
2. $aG\tilde{G}$ coupling $\leftarrow$ U(1)PQ sym. has chiral anomaly under QCD

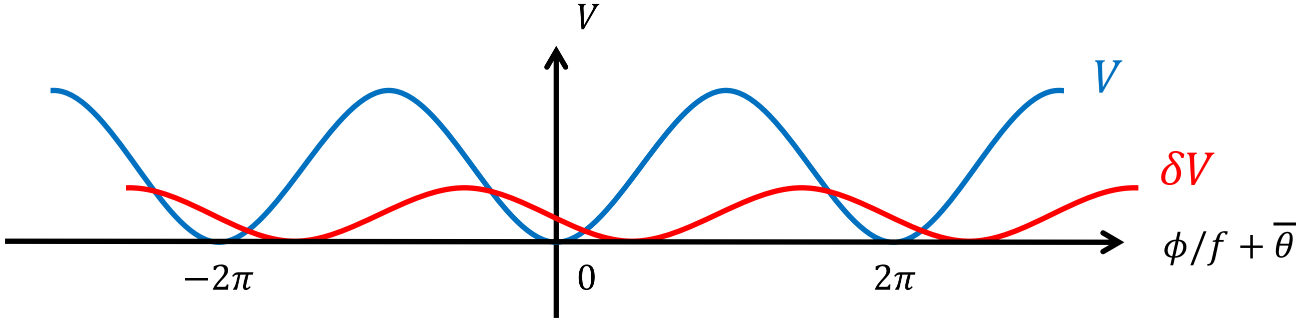
$$L = |\partial\Phi|^2 - \lambda\left(|\Phi|^2 - \frac{f_a^2}{2}\right)^2 + \left(\frac{c}{n!}\frac{1}{M_{pl}^{n-4}}\Phi^n + h.c.\right)$$

$$\Phi = \frac{f_a}{\sqrt{2}}\exp\left(\frac{ia}{f_a}\right)$$

$$L_{eff} = \frac{1}{2}(\partial a)^2 + \frac{|c|}{n!}\frac{f_a^n}{M_{pl}^{n-4}}\cos\left(\frac{a}{f} + \arg c\right)$$

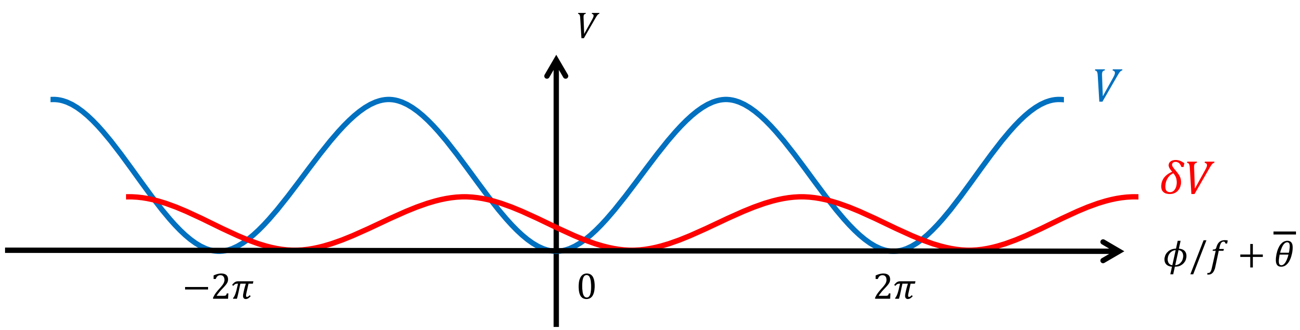
The QCD axion (with small PQ violation)

$$L_{eff} \sim \frac{1}{2}(\partial a)^2 \quad \underbrace{- \; V\left(\frac{a}{f} + \bar{\theta}\right) \qquad + \; \frac{|c|}{n!}\frac{f_a^n}{M_{pl}^{n-4}}\cos\left(\frac{a}{f} + \arg c\right)}_{V_{\text{eff}}(a)}$$



The QCD axion (with small PQ violation)

$$L_{eff} \sim \frac{1}{2}(\partial a)^2 \underbrace{- \frac{1}{2}\Lambda_{\rm QCD}^4\left(\frac{a}{f}+\bar{\theta}\right)^2 + \frac{|c|}{n!}\frac{f_a^n}{M_{pl}^{n-4}}\cos\left(\frac{a}{f}+\arg c\right)}_{V_{\rm eff}(a)}$$



$$\frac{dV_{\rm eff}(a)}{da} = 0 \quad \Rightarrow \quad \bar{\theta} \sim \frac{c}{n!}\frac{f_a^n}{M_{pl}^{n-4}\Lambda_{\rm QCD}^4}$$

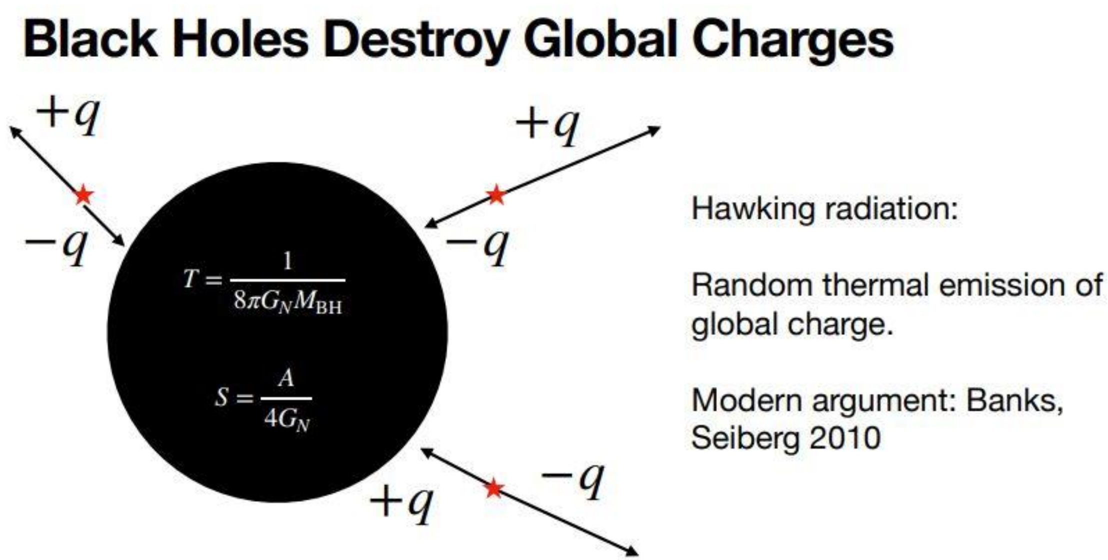
$\theta <\sim 10^{-10},\ f_a = 10^{12}\ \text{GeV} \rightarrow n \geq 11$

$\theta <\sim 10^{-10},\ f_a = 10^{9}\ \text{GeV} \rightarrow n \geq 8$

The axion quality problem

Even Planck-suppressed operator can spoil the QCD axion!

Quantum gravity breaks any global symmetry.



[taken from a slide by M. Reece @ AstroDark 2021]

Can we have PQ symmetry with good quality?
Can the QCD axion explain $|\bar{\theta}| < \sim 10^{-10}$?

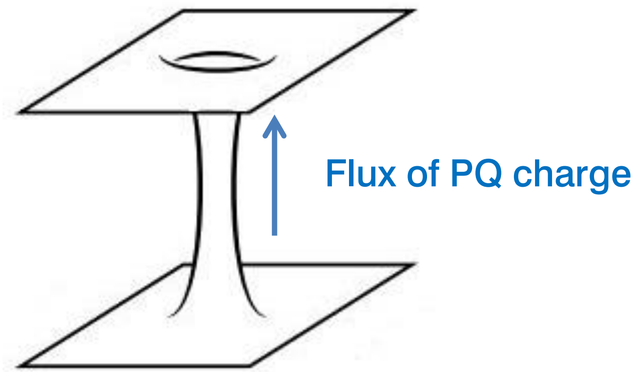
Quantum Gravity & PQ symmetry

How quantum gravity violates PQ symmetry? → Wormhole solution

[Giddings, Strominger (1988)] etc

$$S = \int d^4x \sqrt{g} \left(-\frac{1}{2} M_{pl}^2 R + \frac{1}{2} (\partial a)^2 \right)$$

⇒



[figures are taken from Hebecker, Mikhail, Soler (2018) & slide by P. Soler @ KEK-PH 2022]

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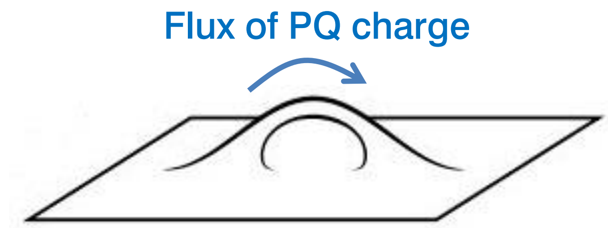

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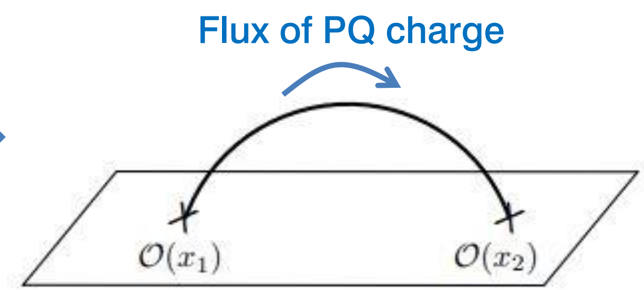

PQ charge appears/disappears at x_1 and x_2.

We need to sum topologies with wormholes.

+ ...

PQ charge appears/disappears everywhere
→ PQ symmetry is violated.

[figures are taken from Hebecker, Mikhail, Soler (2018) & slide by P. Soler @ KEK-PH 2022]

Quantum Gravity & PQ symmetry

Q. Is wormhole dangerous for the QCD axion?
A. This highly depends on UV physics.

$$\delta V \sim \Lambda^4 \exp(-S_{wormhole})$$

Particle Content	Gravitational Theory	Action Scaling	Quality Problem?	Refs.
Free Axion	Einstein Gravity	$M_{\rm pl}/f_a$	No	[32, 33]
Axion, Dynamical Radial Mode (incl. Arbitrary f Potential)	Einstein Gravity, *or* Kaluza-Klein/$f(\mathcal{R})$ Gravity	$\log M_{\rm pl}/f_a$	Yes	[30, 34]
Extended Global Symmetry	Einstein Gravity	$\sum_i \log M_{\rm pl}/f_{a,i}$	Yes	This Work
Axion, Dilaton	(Open*) String Theory	$(M_{\rm pl}/f_a)\cdot g_s^{-1}$	No	[35–40]
Axion, Dilaton, Dynamical Radial Mode	(Open*) String Theory	$\log(M_{\rm pl}/f_a)\cdot g_s^{-1}$	Yes	This Work

[Alvey, Escudero (2020)]

The axion quality problem?

Quantum gravity effect might be highly UV physics dependent.
Still do we have to care about the axion quality problem?

We are agnostic to UV physics.
Let us take a conservative attitude.
i.e., the most severe assumptions against the strong CP problem :

We include any gauge-invariant local operators in EFT.

ex) $$L_{\mathrm{eff}} = \frac{1}{M_{pl}}\Phi^5 + \frac{1}{M_{pl}^2}\Phi^6 + \cdots$$

What can we do if this is the case?

The axion quality problem

Incomplete list of references: (sorry if I miss your paper!)

- The axion quality problem

  [Georgi, Hall, Wise (1981)], [Dine, Seiberg (1986)]
  [Kamionkowski, March-Russell (1992)], [Dobrescu (1996)], …

- Solving the axion quality problem
  - Accidental Peccei Quinn symmetry

    [Randall (1992)], [RS, Redi (2016)], [Di Luzio, Nardi, Ubaldi (2017)]
    [Lillard, Tait (2017, 2018)], [Gavela, Ibe, Quilez, Yanagida (2019)], [Lee, Yin (2019)]
    [Nakai, Suzuki (2021)], [Contino, Podo, Revello (2021)], …

  - Higher dimensional setup

    [Cheng, Kaplan (2001)], [Izawa, Watari, Yanagida (2002, 2003)]
    [Fukunaga, Izawa (2003)], [Choi (2004)], [Grzadkowski, Wudka (2007)]

  - …

- Discussion on wormhole

  [Kallosh, Linde, Linde, Susskind (1995)], [Alosno, Urbano (2017)]
  [Hebecker, Mikhail, Soler (2018)], [Alvey, Escudero (2020)], …

How the quality problem arises

SSB of PQ symmetry means there exist operator O such that

- gauge-invariant
- Nonzero PQ charge
- $\langle O \rangle = f^n \neq 0$

$\Rightarrow$

$$O = \langle O \rangle \exp\left(\frac{iQ_O \phi}{f}\right)$$

Quantum gravity violates any global symmetry.

→ Any gauge invariant interaction can appear in EFT.

→ PQ is always violated by,

$$\delta L \sim \frac{1}{M_{pl}^{n\times d}} O^n \qquad n \geq 1, \qquad d = \dim(O)$$

Large $\dim(O)$ suppress PQ breaking effect

Accidental symmetry

c.f.) baryon number in the SM

Composite axion model

[Kim (1985); Choi, Kim (1985)]

- $SU(N)$ confines
- Dimension 3 operator gets VEV
- SSB : $SU(4)_L \times SU(4)_R \to SU(4)_V$

$$\longrightarrow \langle \psi_L \psi_R^c \rangle \sim \begin{pmatrix} \Lambda^3 & & & \\ & \Lambda^3 & & \\ & & \Lambda^3 & \\ & & & \Lambda^3 \end{pmatrix}$$

Gauge symmetry

		$SU(4)_L$	$SU(N)$	$SU(4)_R$	
ψ_L		4	N		
ψ_R^c			$\overline{N}$	$\overline{4}$	

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- $SU(N)$ confines
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- SSB : $SU(4)_L \times SU(4)_R \to SU(4)_V$
- $SU(3)_c$ is a subgroup of $SU(4)_V$
- 15 NG bosons = $8 + 3 + \bar{3} + 1$ in $SU(3)_c$

$$\langle \psi_L \psi_R^c \rangle \sim \begin{pmatrix} \Lambda^3 & & & \\ & \Lambda^3 & & \\ & & \Lambda^3 & \\ & & & \Lambda^3 \end{pmatrix}$$

axion

Gauge symmetry

QCD

	$SU(3)_c$	$U(1)_{PQ}$	$SU(4)_L$	$SU(N)$	$SU(4)_R$
Q	3	1	4	N	
S	1	-3			
$\bar{Q}$	$\bar{3}$	0		$\bar{N}$	$\bar{4}$
$\bar{S}$	1	0			

Accidental composite axion

[RS, Redi (2016)]

- $SU(N)_1$ and $SU(N)_2$ confines
- Dimension 6 operator gets VEV
- SSB : $SU(4)_L \times SU(4)_R \to SU(4)_V$
- $SU(3)_c$ is a subgroup of $SU(4)_V$
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axion

$$\langle \psi_L \chi_1 \chi_2 \psi_R^c \rangle \sim \begin{pmatrix} \Lambda^6 & & & \\ & \Lambda^6 & & \\ & & \Lambda^6 & \\ & & & \Lambda^6 \end{pmatrix}$$

Gauge symmetry

		$SU(4)_L$	$SU(N)_1$	$SU(4)_1$	$SU(N)_2$	$SU(4)_R$
ψ_L		4	N			
χ_1			$\overline{N}$	4		
χ_2				$\overline{4}$	N	
ψ_R^c					$\overline{N}$	$\overline{4}$

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axion

Gauge symmetry

QCD

	$SU(3)_c$	$U(1)_{PQ}$	$SU(4)_L$	$SU(N)_1$	$SU(4)_1$	$SU(N)_2$	$SU(4)_R$
Q	3	1	4	N			
S	1	-3					
χ_1				$\overline{N}$	4		
χ_2					$\overline{4}$	N	
$\overline{Q}$	$\overline{3}$	0				$\overline{N}$	$\overline{4}$
$\overline{S}$	1	0					

Accidental composite axion

[RS, Redi (2016)]

- $SU(N)_1$ and $SU(N)_2$ confines
- Dimension 6 operator gets VEV $\longrightarrow \langle \psi_L \chi_1 \chi_2 \psi_R^c \rangle \sim \begin{pmatrix} \Lambda^6 & & & \\ & \Lambda^6 & & \\ & & \Lambda^6 & \\ & & & \Lambda^6 \end{pmatrix}$
- SSB : $SU(4)_L \times SU(4)_R \to SU(4)_V$
- $SU(3)_c$ is a subgroup of $SU(4)_V$
- 15 NG bosons $= 8 + 3 + \bar{3} + 1$ in $SU(3)_c$

$O = Q\chi_1\chi_2\bar{Q},\ S\chi_1\chi_2\bar{S}$ are gauge invariant PQ-charged operators.

PQ violation suffer from $1/M_{pl}^2$ suppression $\quad L_{eff} = \frac{c}{M_{pl}^2} Q\chi_1\chi_2\bar{Q} + \frac{c'}{M_{pl}^2} S\chi_1\chi_2\bar{S}$

Straight forward extension to models with dim-9, 12, … operators.

Summary

- The axion should have

  1. Very flat potential = good PQ symmetry
  2. $aG\tilde{G}$ coupling = PQ is anomalous under QCD

- Even M_{pl} suppressed PQ violation is dangerous.

- Accidental PQ symmetry from composite axion is a possible way.

Backup

Topological susceptibility & CP viol.

$$\chi_t \sim \int d^4x \, \langle G\tilde{G}(x), G\tilde{G}(0)\rangle$$

$$\langle G\tilde{G}\rangle \sim \theta\chi_t$$ [Shifman, Vainshtein, Zakharov (1980)]

Large N_c analysis:

$$\chi_t \simeq \frac{m_*\langle q\bar{q}\rangle\chi_{t,\mathrm{pYM}}}{m_*\langle q\bar{q}\rangle + \chi_{t,\mathrm{pYM}}} \qquad m_* = \frac{m_u m_d}{m_u + m_d}$$

[Smilga, Leutwyler (1992)]
[Evans, Hsu, Schwetz (1996)]

$$m_{\eta'}^2 \simeq \frac{6\chi_{t,\mathrm{pYM}}}{f_\pi^2}$$

[Witten (1979)]
[Veneziano (1979)]

Heavy η' and no massless quark inevitably induces the strong CP problem