Contributions of CP Violating Operators to the Neutron/Proton EDM using Lattice QCD

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LANL EDM collaboration

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LANL Publications

- Bhattacharya et al, “Dimension-5 CP-odd operators: QCD mixing and renormalization”, PhysRevD.92.114026
- Bhattacharya et al, “Neutron Electric Dipole Moment and Tensor Charges from Lattice QCD”, PhysRevLett.115.212002
- Bhattacharya et al, “Isovector and isoscalar tensor charges of the nucleon from lattice QCD”, PhysRevD.92.094511
- Gupta et al, “Flavor diagonal tensor charges of the nucleon from $(2 + 1 + 1)$-flavor lattice QCD” PhysRevD.98.091501
- Bhattacharya et al, “Contribution of the QCD Θ-term to nucleon electric dipole moment”, Phys. Rev. D 103, 114507 (2021)
- Bhattacharya et al, “Quark Chromo-Electric Dipole Moment Operator on the Lattice”, Phys. Rev. D 108 (2023) 7, 074507

Outline

- Introduction
- Contribution of the quark EDM operator to nEDM
- Contribution of the Θ-term to nEDM
- Contribution of the quark chromo EDM operator to nEDM
- Contribution of the Weinberg 3-gluon operator to nEDM
- Future

The standard model does not explain

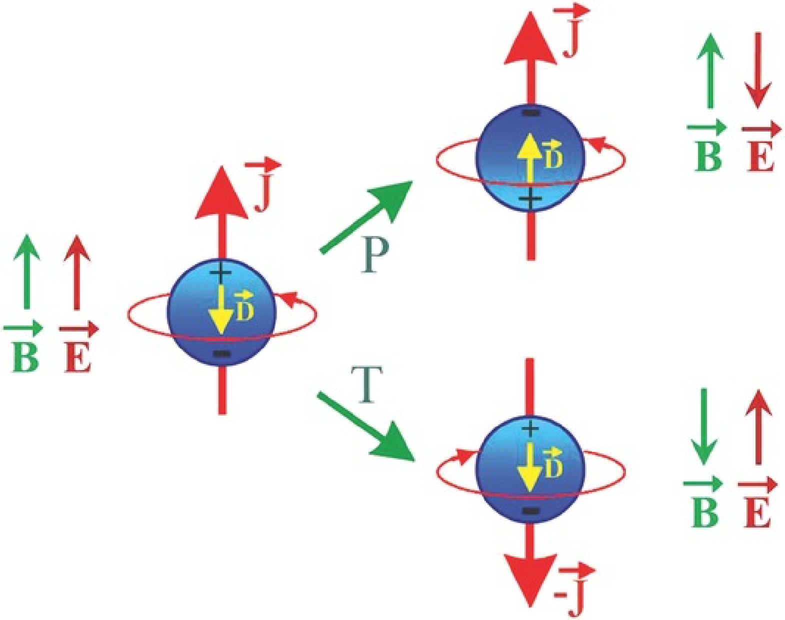
- The clumpiness of the universe cannot be explained by visible matter alone
  $\rightarrow$ Dark Matter
- accelerated expansion of the universe
  $\rightarrow$ Dark Energy
- Too much matter—the baryon asymmetry of the universe (BAU)
  $\rightarrow$ If created after inflation, then Sakharov's three conditions (baryon number violation, out of equilibrium evolution, and CP violation) have to be satisfied

Status

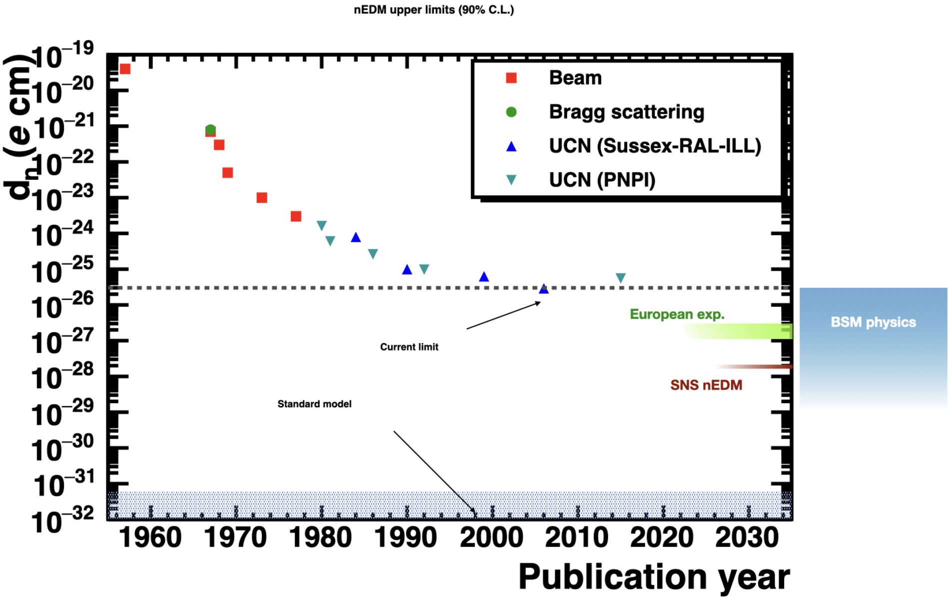
- The $\not{CP}$ in the CKM matrix is too small to explain BAU via baryogenesis
- The $\not{CP}$ coupling $\bar{\theta}$ of the Θ-term in QCD is constrained by the upper bound on neutron EDM. The current value, $\bar{\theta} \sim 10^{-10}$, is unnaturally small
- Most extensions of the standard model have additional sources of $\not{CP}$

Electric dipole moment in an elementary particle ($\propto$ Spin) $\Longleftrightarrow$ CP violating interactions

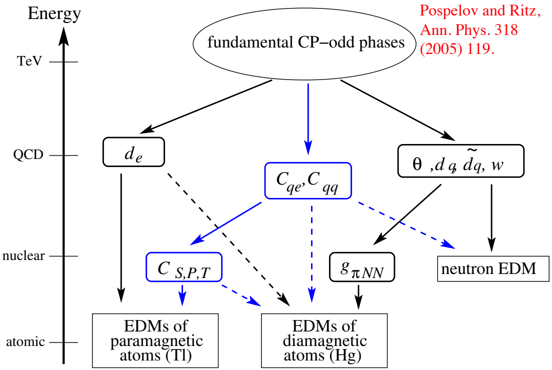
$\not{T}$ ($\not{CP}$ if CPT is a good symmetry) and EDMs



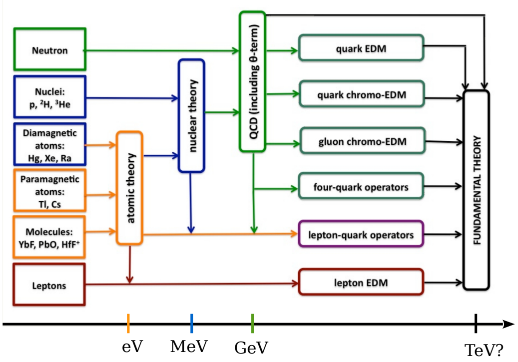
The race to measure EDMs is on: status of nEDM



Hierarchy of Scales: $\not{CP}$ in BSM $\rightarrow$ EDMs



Pospelov & Ritz



Jordy de Vries

$$\mathcal{L}_{\rm CPV} = \mathcal{L}_{\rm CKM} + \mathcal{L}_{\bar\theta} + \mathcal{L}_{\rm BSM} \quad \longrightarrow \quad \mathcal{L}^{\rm eff}_{\rm CPV}$$

Effective CPV Lagrangian at Hadronic Scale

$$
\begin{aligned}
\mathcal{L}_{\text{CPV}}^{d\leq 6} = & -\frac{g_s^2}{32\pi^2}\overline{\theta}G\tilde{G} && \text{dim=4 QCD } \theta\text{-term} \\
& - \frac{i}{2}\sum_{q=u,d,s,c} d_q \overline{q}(\sigma\cdot F)\gamma_5 q && \text{dim=5 Quark EDM (qEDM)} \\
& - \frac{i}{2}\sum_{q=u,d,s,c} \tilde{d}_q g_s \overline{q}(\sigma\cdot G)\gamma_5 q && \text{dim=5 Quark Chromo EDM (qcEDM)} \\
& + d_w \frac{g_s}{6} G\tilde{G}G && \text{dim=6 Weinberg's 3g operator} \\
& + \sum_i C_i^{(4q)} O_i^{(4q)} && \text{dim=6 Four-quark operators}
\end{aligned}
$$

- $\overline{\theta} \leq \mathcal{O}(10^{-8} - 10^{-11})$: Strong CP problem
- Dim=5 terms suppressed by $d_q \approx \langle \text{v} \rangle / \Lambda_{BSM}^2$; effectively dim=6
- Dim=6 terms suppressed by $d_W \approx \langle 1 \rangle / \Lambda_{BSM}^2$
- All terms up to $d = 6$ are leading order

Contributions to the Neutron EDM d_n

$$d_n = \overline{\theta} \cdot C^{\theta} + \sum_q d_q \cdot C_q^{\mathrm{qEDM}} + \sum_q \tilde{d}_q \cdot C_a^{\mathrm{qcEDM}} + \overline{\theta} \cdot C^{W} + \cdots$$

- $\not{C}\not{P}$ couplings in the SM and BSM
  $\longrightarrow$ Couplings in the low-energy effective CPV Lagrangian $(\overline{\theta}, d_q, \tilde{d}_q, \ldots)$

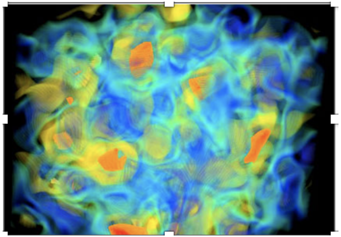
- Lattice QCD
  $\longrightarrow$ Nucleon matrix elements in presence of CPV interactions

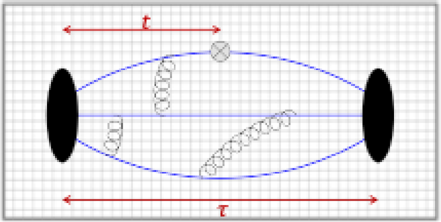
$$\begin{aligned} C_\theta &= \langle N | J^{\mathrm{EM}} | N \rangle |^{\theta} \\ C^{\mathrm{qEDM}} &= \langle N | J^{\mathrm{EM}} | N \rangle |^{\mathrm{qEDM}} \\ C^{\mathrm{qcEDM}} &= \langle N | J^{\mathrm{EM}} | N \rangle |^{\mathrm{qcEDM}} \\ C^{\mathrm{W}} &= \langle N | J^{\mathrm{EM}} | N \rangle |^{\mathrm{Weinberg}} \\ & \ldots \end{aligned}$$

Lattice QCD

- **Non-perturbative Vacuum of QCD**
  – Generate ensembles of gauge configurations distributed as e^{-L}
- **Calculate n-point correlation functions**
  – Quark line diagrams
- **Ensemble average $\Longrightarrow$ path integral:** $\int DUe^{-L}O \rightarrow 1/N \sum_i^N O_i$
- **Obtain matrix elements of operators composed of gauge and quark fields between hadronic states:** $\langle N(p_f)|\mathcal{O}(Q^2)|\overline{N}(p_i)\rangle$

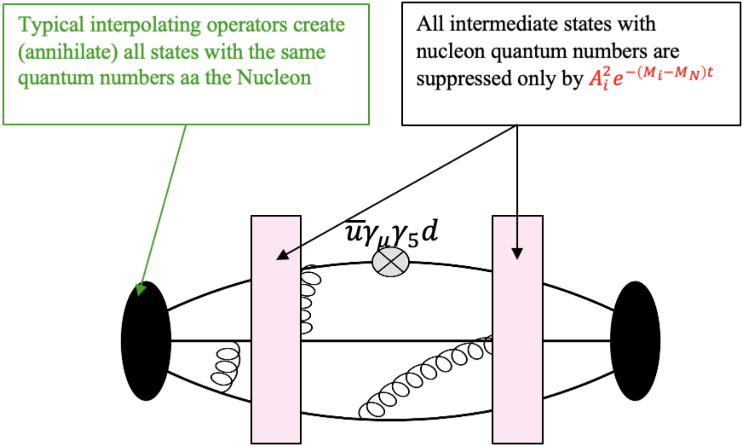
Numerical evaluation of the path integral $\Longrightarrow$ statistical errors





Removing excited-state contributions from Correlation Functions

- Lattice meson and nucleon interpolating operators also couple to excited states with the same quantum numbers
- The lowest excited states in $\Gamma^n_{\rm nucleon}$ are dense towers of $N\pi$, $N\pi\pi$, ... states!!



Lattice QCD $\Longrightarrow$ Physical Results

Extrapolate data at $\{a, M_\pi, M_\pi L\}$ to $a = 0$, $M_\pi = 135$ MeV, $M_\pi L \to \infty$

- **Renormalization: Lattice scheme $\longrightarrow$ continuum $\overline{\mathrm{MS}}$**
  - involves complicated/divergent mixing for qcEDM, Weinberg, 4-quark operators
- **Simulations at Physical Pion Mass!**
  - As $M_\pi \to 135$ MeV $\Longrightarrow$ computational cost increases $\Longrightarrow$ larger errors
- **Finite Lattice Spacing**
  - Extrapolate from finite lattice spacings $0.045 < a < 0.15$ fm
- **Finite Volume**
  - Finite lattice volume effects small in most EDM calculations for $M_\pi L > 4$

Neutron EDM from Quark EDM term

$$
\begin{aligned}
\mathcal{L}_{\text{CPV}}^{d\leq 6} = & -\frac{g_s^2}{32\pi^2}\overline{\theta}G\tilde{G} && \text{dim=4 QCD } \theta\text{-term} \\
& -\frac{i}{2}\sum_{q=u,d,s} d_q\overline{q}(\sigma\cdot F)\gamma_5 q && \text{dim=5 Quark EDM (qEDM)} \\
& -\frac{i}{2}\sum_{q=u,d,s} \tilde{d}_q g_s\overline{q}(\sigma\cdot G)\gamma_5 q && \text{dim=5 Quark Chromo EDM (qcEDM)} \\
& + d_w\frac{g_s}{6}G\tilde{G}G && \text{dim=6 Weinberg's 3g operator} \\
& + \sum_i C_i^{(4q)}O_i^{(4q)} && \text{dim=6 Four-quark operators}
\end{aligned}
$$

Contribution of $\mathcal{L}_{\rm qEDM}$ to nEDM

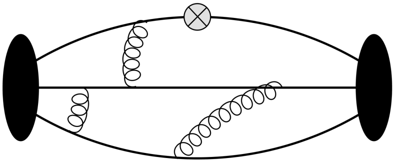
- $\mathcal{L}_{\rm CPV} = \mathcal{L}_{\rm CKM} + \mathcal{L}_{\rm qEDM}$ with $\mathcal{L}_{\rm qEDM} = -\frac{i}{2}\sum_{q=u,d,s,c} d_q \overline{q}(\sigma\cdot F)\gamma_5 q$
- Adding $\mathcal{L}_{\rm qEDM}$ to the theory adds $\overline{q}\sigma_{\mu\nu}q$ to the electromagnetic current.
- Its matrix elements $\langle N|\overline{q}\sigma_{\mu\nu}q|N\rangle = g_T^q \overline{u}_N\sigma_{\mu\nu}u_N$ where g_T are the tensor charges
- They give the leading contributions of qEDMs

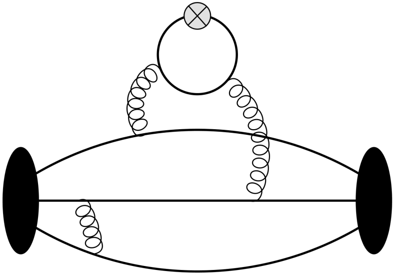
$$d_N = d_u g_T^u + d_d g_T^d + d_s g_T^s + d_c g_T^c \qquad (1)$$

- $d_q \propto m_q$ in many models $\Rightarrow$ Precision determination of $g_T^{\{s,c\}}$ is important

Calculating the Tensor Charges

Charges $g_T^{u,d,s,c}$ get contributions from “Connected” and “Disconnected” diagrams





- “Disconnected” diagram calculation is expensive. They are noisy & small
- Only the “Disconnected” diagram contributes to $g_T^{\{s,c\}}$
- Robust results with errors $\leq$5% have been obtaibned

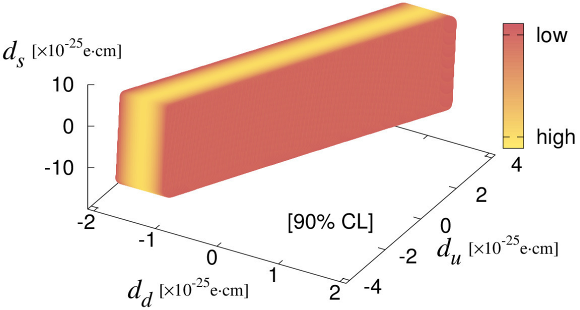
qEDM: FLAG2019, 2021 and Current Status

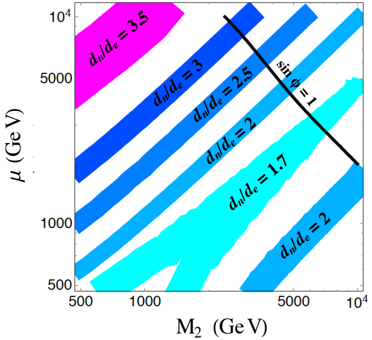
Collaboration	N_f	a	m_π	FV	Z	ESC	g_T^u	g_T^d
PNDME 20	2+1+1	★‡	★	★	★	○	0.783(27)(10)	−0.205(10)(10)
ETM 19	2+1+1	■	○	★	★	○	0.729(22)	−0.2075(75)
PNDME 18B	2+1+1	★‡	★	★	★	○	0.784(28)(10)#	−0.204(11)(10)#
PNDME 16	2+1+1	○‡	★	★	★	○	0.792(42)#&	−0.194(14)#&
Mainz 19	2+1	★	○	★	★	○	0.77(4)(6)	−0.19(4)(6)
JLQCD 18	2+1	■	○	○	★	○	0.85(3)(2)(7)	−0.24(2)(0)(2)
ETM 17	2	■	○	○	★	○	0.782(16)(2)(13)	−0.219(10)(2)(13)

Collaboration	N_f	a	m_π	FV	Z	ESC	g_T^s
PNDME 20	2+1+1	★‡	★	★	★	○	−0.0022(12)
ETM 19	2+1+1	■	○	★	★	○	−0.00268(58)
PNDME 18B	2+1+1	★‡	★	★	★	○	−0.0027(16)#
Mainz 19	2+1	★	○	★	★	○	−0.0026(73)(42)
JLQCD 18	2+1	■	○	○	★	○	−0.012(16)(8)
ETM 17	2	■	○	○	★	○	−0.00319(69)(2)(22)

Constraints on BSM Assuming qEDM is the only $\not{CP}$ Interaction

$$d_N = d_u g_T^u + d_d g_T^d + d_s g_T^s + d_c g_T^c$$





[Bhattacharya, *et al.* (2015), Gupta, *et al.* (2018)]

Status:

- $g_T^{u,d,s}$ results from multiple collaborations with control over $a \to 0$ extrapolation
- Single result from ETM 19 $g_T^c = -0.00024(16)$

Neutron EDM from QCD θ-term

$$
\begin{aligned}
\mathcal{L}_{\text{CPV}}^{d\leq 6} = & -\frac{g_s^2}{32\pi^2}\bar{\theta}G\tilde{G} && \text{dim=4 QCD } \theta\text{-term} \\
& -\frac{i}{2}\sum_{q=u,d,s} d_q \bar{q}(\sigma\cdot F)\gamma_5 q && \text{dim=5 Quark EDM (qEDM)} \\
& -\frac{i}{2}\sum_{q=u,d,s} \tilde{d}_q g_s \bar{q}(\sigma\cdot G)\gamma_5 q && \text{dim=5 Quark Chromo EDM (qcEDM)} \\
& + d_w \frac{g_s}{6} G\tilde{G}G && \text{dim=6 Weinberg's 3g operator} \\
& + \sum_i C_i^{(4q)} O_i^{(4q)} && \text{dim=6 Four-quark operators}
\end{aligned}
$$

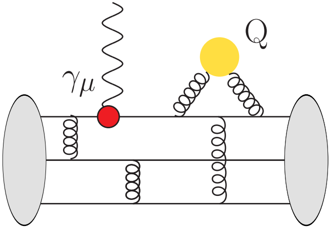
QCD θ-term

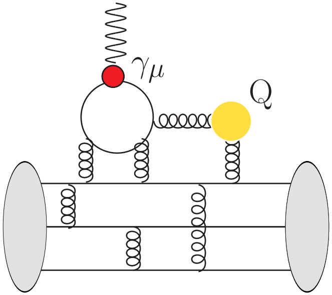
$$S = S_{QCD} + i\theta Q, \qquad Q = \int d^4x \frac{G\tilde{G}}{32\pi^2}$$

At the leading order, the correlation functions calculated are

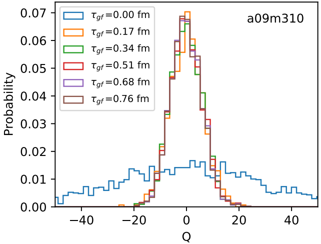
$$\langle N \mid J_\mu^{\mathrm{EM}} \mid N \rangle\Big|^{\overline{\Theta}} \approx \langle N \mid J_\mu^{\mathrm{EM}} \mid N \rangle\Big|^{\overline{\Theta}=0} - i\overline{\Theta} \left\langle N \left| J_\mu^{\mathrm{EM}} \int d^4x \frac{G^a_{\mu\nu}\widetilde{G}^a_{\mu\nu}}{32\pi^2} \right| N \right\rangle ,$$

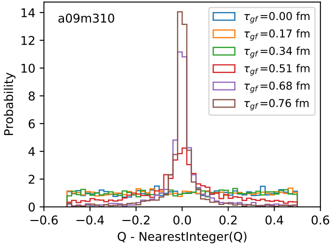
$\Longrightarrow$ on each configuration weight $\langle N \mid J_\mu^{\mathrm{EM}} \mid N \rangle$ by Q





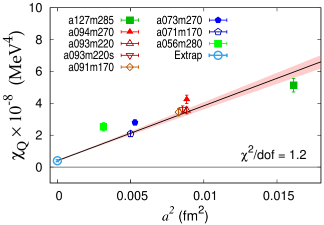
Topological charge $Q = \int d^4x \frac{G\tilde{G}}{32\pi^2}$, Susceptibility $\chi = \langle (Q - \bar{Q})^2 \rangle$

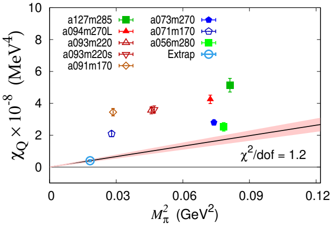




Reduce lattice artifacts by calculating Q under gradient flow (GF)

Distribution of Q stabilizes much faster than convergence to integer under GF





Estimate of χ consistent with chiral perturbation theory (PRD103, 114507 (2021)

d_n from the form factor F_3: $d_n = \lim_{q^2\to 0} F_3(q^2)/2M_N$

In simulations with *'imaginary θ'* and *'expansion in θ'* we extract the CPV form factor $F_3(0)$ from the the most general decomposition of the ground-state matrix element:

$$
\begin{aligned}
\langle N(p',s') \mid J_\mu^{\mathrm{EM}} \mid N(p,s)\rangle^{\overline{\Theta}}_{\not{C}\mathrm{P}} \;=\; & \overline{u}_N(p',s')\Big[\gamma_\mu F_1(q^2) \\
& + \frac{1}{2M_N}\sigma_{\mu\nu}q_\nu\Big(F_2(q^2) - iF_3(q^2)\gamma_5\Big) \\
& + \frac{F_A(q^2)}{M_N^2}(\not{q}q_\mu - q^2\gamma_\mu)\gamma_5\Big]u_N(p,s)\,,
\end{aligned}
$$

- Form Factors: Dirac F_1, Pauli F_2, Electric dipole F_3, Anapole F_A
- Electric $G_E = F_1 - \tau F_2$, Magnetic = $F_1 + F_2$

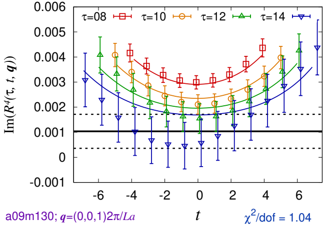
Removing ESC from n-point correlation functions Γ^n

$$\Gamma^{2\text{pt}}(\tau) \equiv \langle\Omega|N(\tau)\overline{N}(0)|\Omega\rangle = \sum_{i=0}^{\infty} |\langle N_i|\overline{N}|\Omega\rangle|^2 e^{-E_i\tau} \tag{2}$$

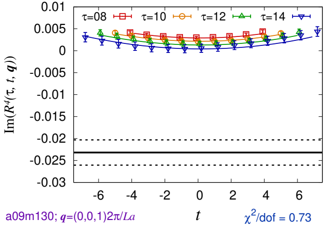
$$\Gamma^{3\text{pt}}_O(\tau,t) \equiv \langle\Omega|N(\tau)O(t)\overline{N}(0)|\Omega\rangle = \sum_{i,j=0}^{\infty} \langle\Omega|N|N_j\rangle^* \langle N_i|\overline{N}|\Omega\rangle \langle N_j|O|N_i\rangle e^{-E_i t} e^{-E_j(\tau-t)}$$

- ESC removed by making fits to the spectral decomposition of correlation functions
- Which excited states contribute significantly to Γ_n is not known $a\ priori$!
- $N\pi$ state with neutron quantum numbers has smallest mass gap
- Fits without/with the $N\pi$ state give a very different value in many Γ_n

$M_1 \sim 1.8\,\text{GeV}$



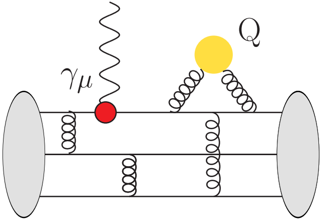
$M_1 \sim 1.1\,\text{GeV} + M_\pi$

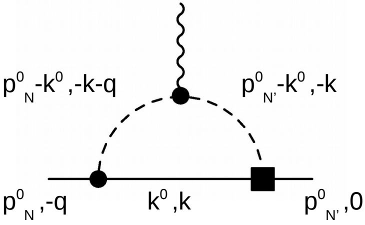


Do $N\pi$ excited states contribute to the 4-point function Γ_V^{Θ}, ?

A χPT analysis by Bhattacharya *et al.* (2021) shows :

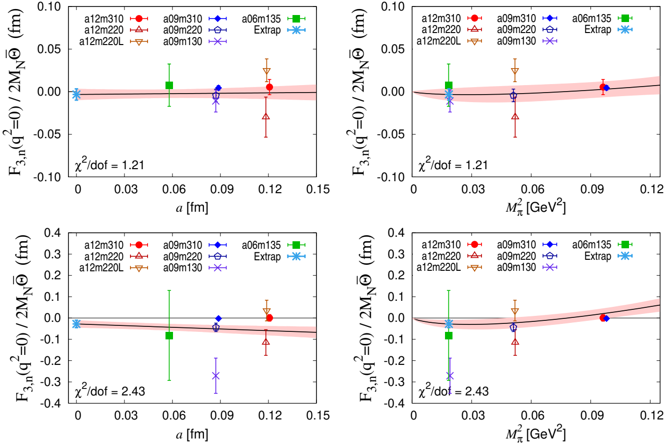
$\Longrightarrow$ Contribution of low energy $N\pi$ excited-state should grow as $M_\pi \to 135$ MeV





Large $\pi N N$ ~~CP~~ coupling $\bar{g}_0$

Results without and with including $N\pi$ state when removing ESC



Without (top) With (bottom) $N\pi$-state Bhattacharya, *et al.*, PRD 103, 114507 (2021)

- The two fits–with/without the $N\pi$ excited state–are not distinguished by the χ^2
- Resolving ESC is a major systematic

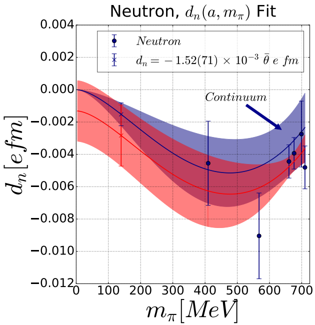
Status from Bhattacharya *et al.* (Updated 2023)

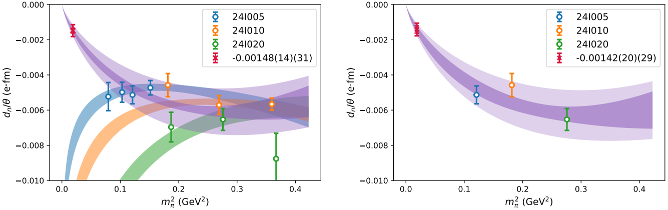
	Neutron $\overline{\Theta}$ e · fm	Proton $\overline{\Theta}$ e · fm
Bhattacharya 2021	$d_n = -0.003(7)(20)$	$d_p = 0.024(10)(30)$
Bhattacharya 2021 with $N\pi$	$d_n = -0.028(18)(54)$	$d_p = 0.068(25)(120)$
ETMC 2020	$\|d_n\| = 0.0009(24)$	–
Dragos 2019	$d_n = -0.00152(71)$	$d_p = 0.0011(10)$
Syritsyn 2019	$d_n \approx 0.001$	–
χQCD 2023	$d_n = 0.00148(14)(31)$	$d_p = 0.0038(11)(8)$

Table: Lattice results for the contribution of the Θ-term to the neutron and proton EDM.

- Including $N\pi$ excited state in the ESC analysis gives a much larger result
- Reliable estimate of the contribution of the Θ-term to nEDM requires higher statistical precision and control over ESC

Is the chiral fit robust in recent calculations of the Θ-term



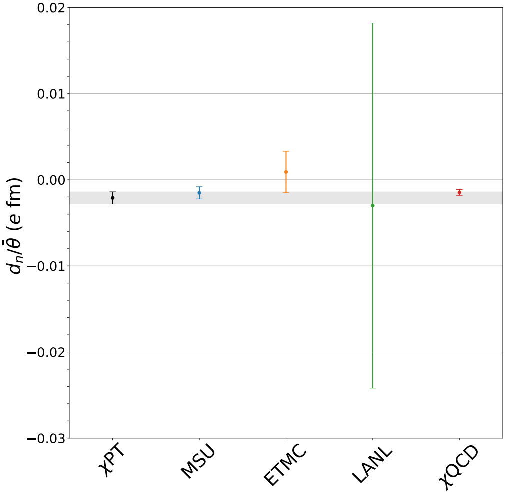


MSU: Dragos, *et al.* (2019) $\qquad$ χQCD, Jian Liang *et al.*, arXiv:2301.04331 (2023)

MSU: $d_n = -1.52(71) \times 10^{-3}\ \bar{\theta}\ e \cdot fm$ $\qquad$ χQCD, $d_n = 0.00148(14)(31)\ \bar{\theta}\ e \cdot fm$

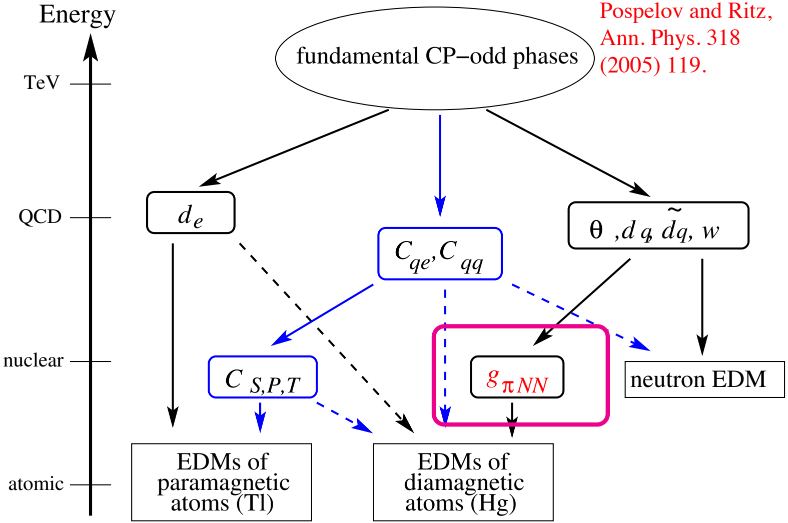
- multiple a but large pion mass $m_\pi > 400\text{MeV}$
- Inflection point occurs near smallest M_π to satisfy $d_n = 0$ at $M_\pi = 0$

LATTICE QCD Results for θ-term (Circa 2024)



- Small error points fixed $d_n = 0$ at $M_\pi = 0$ in fits to data at $M_\pi > 300$ MeV

Lattice Calculations for $g_{\pi NN}$



$g_{\pi NN}$: Current Status and Future Prospects

$$\mathcal{L}_{\pi NN}^{CPV} = -\frac{\bar{g}_0}{2F_\pi}\bar{N}\boldsymbol{\tau}\cdot\boldsymbol{\pi}N - \frac{\bar{g}_1}{2F_\pi}\pi_0\bar{N}N - \frac{\bar{g}_2}{2F_\pi}\pi_0\bar{N}\tau^3 N + \cdots$$

- Chiral symmetry relations + nucleon σ-term & mass splittings $\longrightarrow g_{\pi NN}$
  [Vries, Mereghetti, Seng, and Walker-Loud (2017)]
- No direct lattice calculation of $g_{\pi NN}$ published yet

Can be calculated from $\langle N|A_\mu(q)|N\rangle_{\rm CPV}$ following the same methodology used for neutron EDM, i.e.,
$\langle N|V_\mu(q)|N\rangle_{\rm CPV} \rightarrow \langle N|A_\mu(q)|N\rangle_{\rm CPV}$

Conclusions

- Significant progress.
- Issues of signal (statistics), $N\pi$ contribution, and renormalization remain
- Gradient flow scheme is, so far, best for renormalization
- quark-EDM: Lattice QCD has provided results with $\lesssim 5\%$ uncertainty
- Θ-term: Significant Progress. No reliable estimates yet
  - Statistics
  - Does $N\pi$ provide leading excited-state contamination?
- quark chromo-EDM: Signal in the 3 methods being used
  - Renormalization and mixing (Working on gradient flow scheme)
  - Does $N\pi$ provide leading excited-state contamination?
- Weinberg $G\widetilde{G}G$ Operator has signal
  - Address the mixing with Θ-term in gradient flow scheme
- Four-quark operators: No calculations yet

Need 10-100 X Larger Computational Resources