Search for an interaction mediated by axion-like particles with ultracold neutrons

Pin-Jung Chiu
On behalf of the nEDM collaboration at PSI

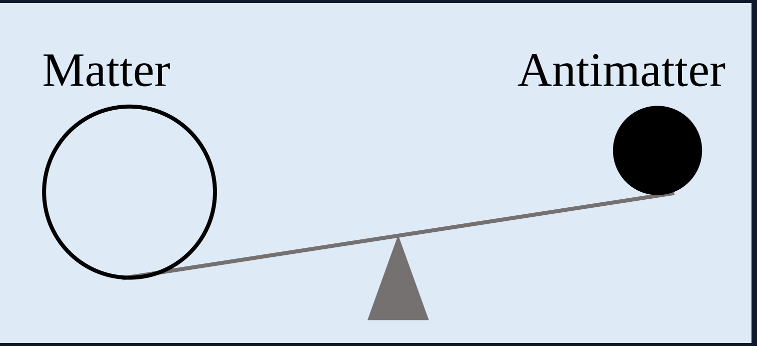
University of Zurich
pin-jung.chiu@physik.uzh.ch

August 7th, 2024
The Axion Quest – 20th Rencontres du Vietnam

Matter vs. antimatter

- Baryon asymmetry of the Universe → CP violation (CPV)
- $\bar{\theta}_{\mathrm{QCD}} \lesssim 10^{-10}$ in $\mathcal{L}_{\mathrm{QCD}}$ (bounded by nEDM, $\bar{\theta} \times 10^{-16}\ e \cdot \mathrm{cm}$)

  → strong CP problem



- Peccei-Quinn theory: $\mathrm{U(1)_{PQ}}$

  CP-violating angle $\bar{\theta}_{\mathrm{QCD}}$ → CP-conserving dynamical field

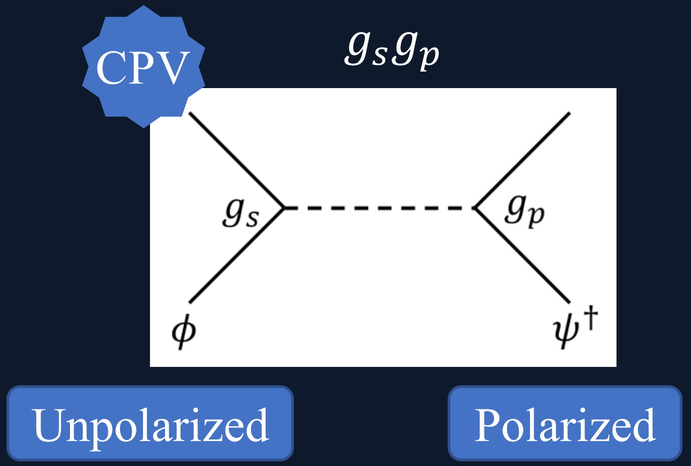
- Spontaneously broken $\mathrm{U(1)_{PQ}}$ → Nambu-Goldstone boson: axion

Ref.: R.D. Peccei and H. R. Quinn, Phys. Rev. Lett 38 (1977), 1440-1443.

See Mon. Axion Review from Y. Semertzidis

A new short-range spin-dependent interaction?

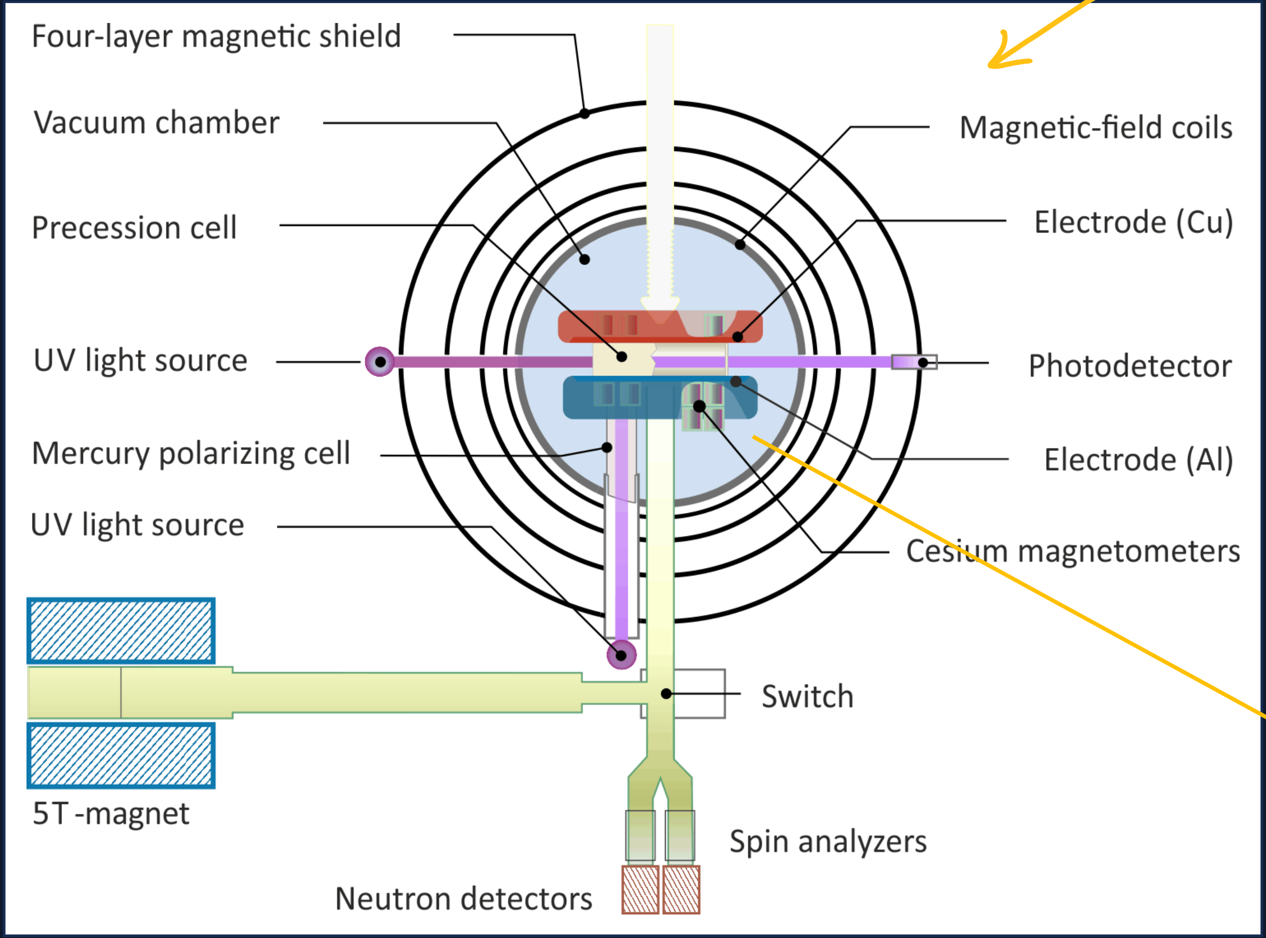
- Moody and Wilczek:
  A new short-range, spin-dependent macroscopic force?
- Allowed couplings for spin-0 bosons : scalar (g_s) and pseudoscalar (g_p) vertices
- Mediator particles:
  - Axions
    - Fixed relation between its mass and its coupling to other SM particles
    - 10^{-13} eV $< m_a < 10^{-2}$ eV → axion window
    - $\bar{\theta}_{\mathrm{QCD}} \lesssim 10^{-10}$ → bounded by nEDM constraint
  - Axionlike particles (ALPs)
    - Beyond standard model theories
    - Unrelated to $\bar{\theta}_{\mathrm{QCD}}$ → not constrained by nEDM limit
    - Dark matter candidates



Refs.:
1. J. E. Moody and F. Wilczek, Phys. Rev. D 30 (1984), 130-138.
2. S. Mantry, M. Pitschmann and M. J. Ramsey-Musolf, Phys. Rev. D 90, 054016 (2014).

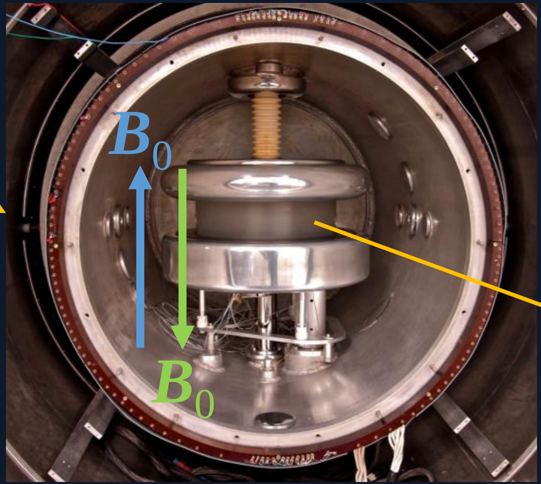
The nEDM spectrometer

- Located at the Paul Scherrer Institute, Villigen, Switzerland



- Set the currently most stringent nEDM upper limit
- Now commissioning n2EDM

(see Mon. nEDM from G. Ban & G. Pignol)

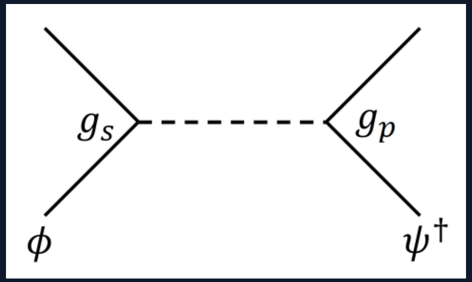


Measure spin-precession frequency of stored ultracold neutrons (UCN) under a constant magnetic field B_0

$$f_n = \gamma_n B_0 / 2\pi$$

Measurement principle

- Pseudomagnetic field



Nucleon UCN

$$b^*_{\mathrm{ALP}}(z) = g_s g_p^\dagger \frac{\hbar\lambda}{2\gamma^\dagger m^\dagger}\left(1 - e^{-a/\lambda}\right)\left[N_{\mathrm{bot}} e^{-(z+H/2)/\lambda} - N_{\mathrm{top}} e^{-(H/2-z)/\lambda}\right]$$

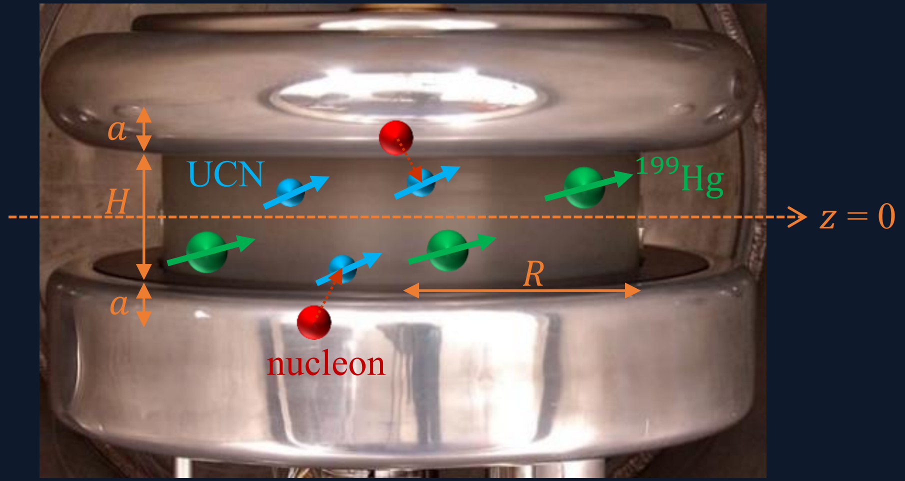
$$b^*_{\mathrm{UCN}} = \int_{-H/2}^{+H/2} b^*_{\mathrm{ALP}}(z)\,\rho_n(z)\,\mathrm{d}z$$

$$= g_s g_p^\dagger \frac{\hbar\lambda^2\left[H\left(N_{\mathrm{bot}} - N_{\mathrm{top}}\right) - 6\langle z\rangle\left(N_{\mathrm{bot}} + N_{\mathrm{top}}\right)\right]}{2\gamma^\dagger m^\dagger H^2}\left(1 - e^{-a/\lambda}\right)\left(1 - e^{-H/\lambda}\right)$$

γ: Gyromagnetic ratio
m: Mass
λ: Interaction length

$$\rho_{\mathrm{n}}(z) = \frac{1}{H}\left(1 + \frac{12\langle z\rangle}{H^2} z\right)$$

$N_{\mathrm{bot}} = N_{\mathrm{Al}} = 1.6 \times 10^{30}\ \mathrm{m}^{-3}$
$N_{\mathrm{top}} = N_{\mathrm{Cu}} = 5.4 \times 10^{30}\ \mathrm{m}^{-3}$
$H = 12$ cm
$a = 2.5$ cm
$R = 23.5$ cm



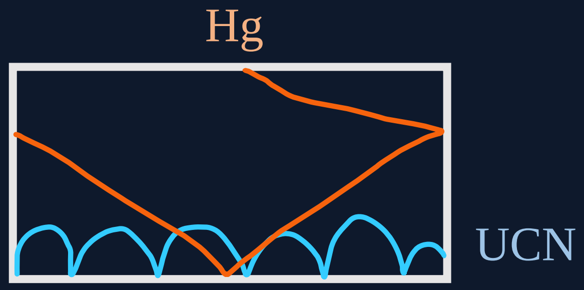
Observable: precession-frequency ratio $\mathscr{R}$

$$\mathcal{R}^{\uparrow/\downarrow} = \left(\frac{f_\mathrm{n}}{f_\mathrm{Hg}}\right)^{\uparrow/\downarrow} = \left|\frac{\gamma_\mathrm{n}}{\gamma_\mathrm{Hg}}\right|\left(1 \pm \frac{b^*_\mathrm{UCN}}{|\boldsymbol{B}_\mathbf{0}|} \pm \frac{G_\mathrm{grav}\langle z\rangle}{|\boldsymbol{B}_\mathbf{0}|} + \delta_\mathrm{else}\right) \longrightarrow b^*_\mathrm{UCN} = \frac{\mathcal{R}^\uparrow - \mathcal{R}^\downarrow}{\mathcal{R}^\uparrow + \mathcal{R}^\downarrow}|\boldsymbol{B}_\mathbf{0}| \longrightarrow g_s g_p^\dagger$$

Strategy:
observe $\mathscr{R}$ under different $G_{grav} = \frac{dB_z}{dz}$

$\langle B_n \rangle \neq \langle B_{Hg} \rangle$

$\langle z \rangle < 0$: center-of-mass offset of UCN

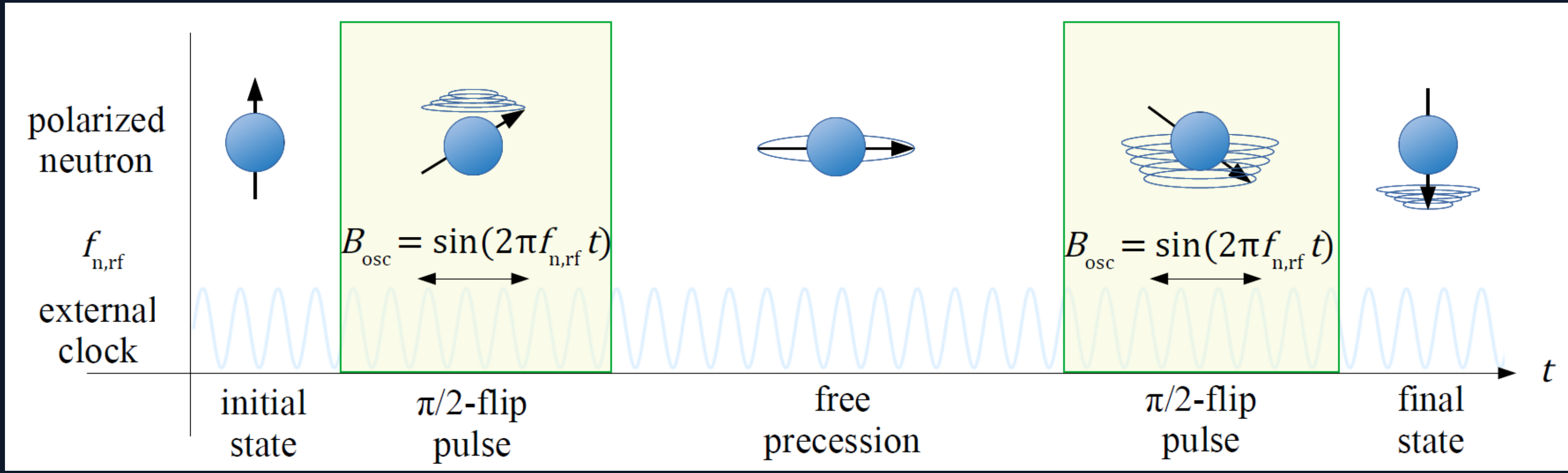


Systematic effects: $\delta_\mathrm{else} = \delta_\mathrm{T} + \delta_\mathrm{Earth} + \delta_\mathrm{light} + \delta_\mathrm{inc} + \delta_\mathrm{JNN}$

Ramsey's method of separated oscillatory fields

One measurement "cycle"

Ref.: N. F. Ramsey, Phys. Rev. 78 (1950), 695-699.

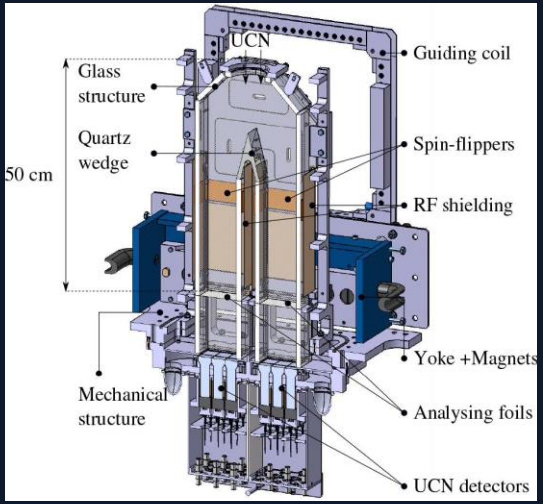


$\tau = 1.995$ s　　$\mathcal{T} = 180$ s　　$\tau = 1.995$ s

In phase

Asymmetry

$$\mathcal{A} = \frac{N^{\uparrow} - N^{\downarrow}}{N^{\uparrow} + N^{\downarrow}}$$

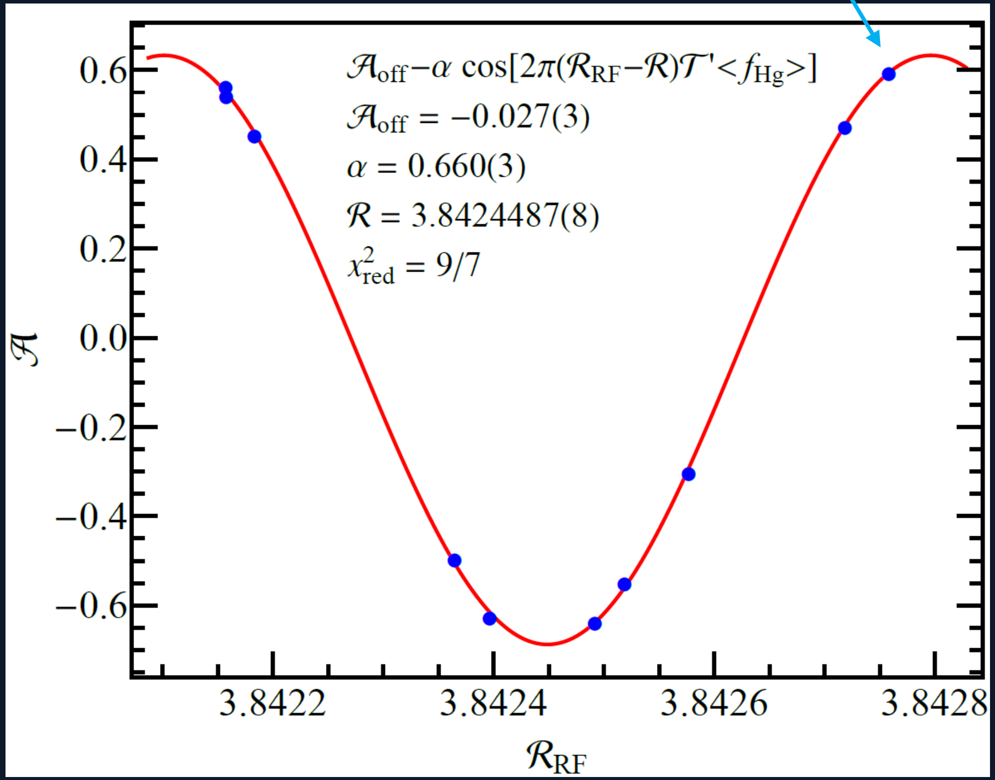


See Tue. BeamEDM from I. Schulthess

Ramsey pattern

One measurement "run" ~ 10 cycles

Constant G_{grav}
Different f_{rf}



Offset

$$\mathcal{A} = \frac{N^{\uparrow} - N^{\downarrow}}{N^{\uparrow} + N^{\downarrow}} \approx \mathcal{A}_{\mathrm{off}} - \alpha \cos\left[2\pi\left(\mathcal{R}_{\mathrm{RF}} - \mathcal{R}\right)\mathcal{T}'\left\langle f_{\mathrm{Hg}}\right\rangle\right]$$

Visibility (Ramsey contrast)

Resonant $\mathscr{R}$

$\mathcal{T}' = \mathcal{T} + 4\tau/\pi$ (effective time)

$\langle f_{Hg} \rangle$ average f_{Hg} of all cycles

Determination of G_{grav}

$$\mathcal{R}^{\uparrow/\downarrow} = \left(\frac{f_{\text{n}}}{f_{\text{Hg}}}\right)^{\uparrow/\downarrow} = \left|\frac{\gamma_{\text{n}}}{\gamma_{\text{Hg}}}\right| \left(1 \pm \frac{b^*_{\text{UCN}}}{|\boldsymbol{B_0}|} \pm \frac{G_{\text{grav}}\langle z\rangle}{|\boldsymbol{B_0}|} + \delta_{\text{else}}\right)$$

Polynomial expansion (Cartesian coordinates)

$$\boldsymbol{B}(\boldsymbol{r}) = \sum_l \sum_{m=-(l+1)}^{+(l+1)} G_{l,m} \begin{pmatrix} \pi_{x,l,m}(\boldsymbol{r}) \\ \pi_{y,l,m}(\boldsymbol{r}) \\ \pi_{z,l,m}(\boldsymbol{r}) \end{pmatrix}$$

$\boldsymbol{\pi}_{l,m}$: harmonic polynomials in *x, y,* and *z* directions of degree *l*, order *m*.

Expansion coefficients → Gradients

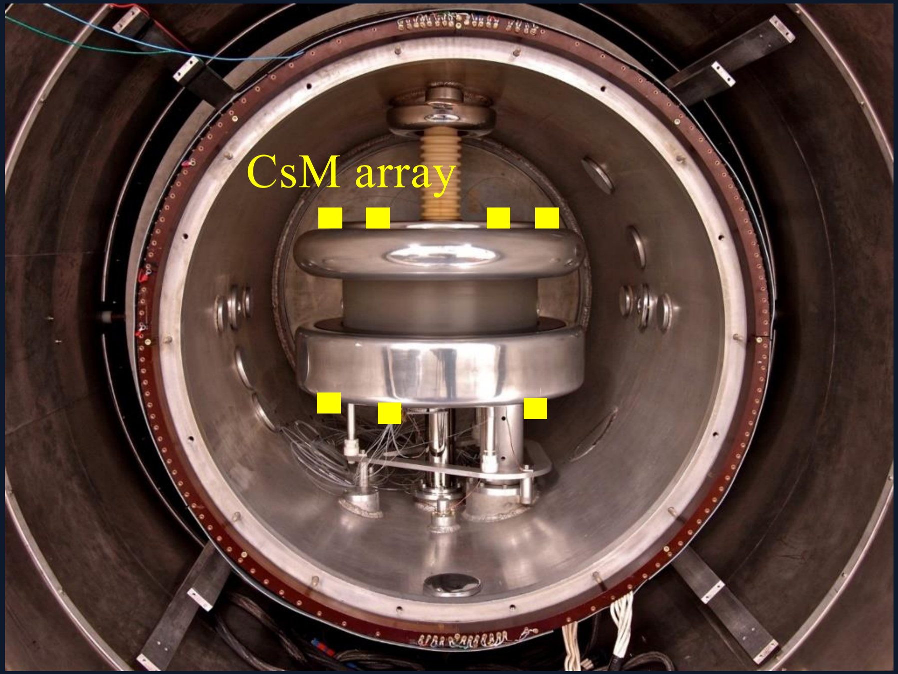
$$G_{\text{grav}} = G_{1,0} + G_{3,0}\left(\frac{3H^2}{20} - \frac{3R^2}{4}\right) + G_{5,0}\left(\frac{5R^4}{8} - \frac{3R^2H^2}{8} + \frac{3H^4}{112}\right)$$

$H = 12$ cm
$R = 23.5$ cm

Combining online and offline measurements

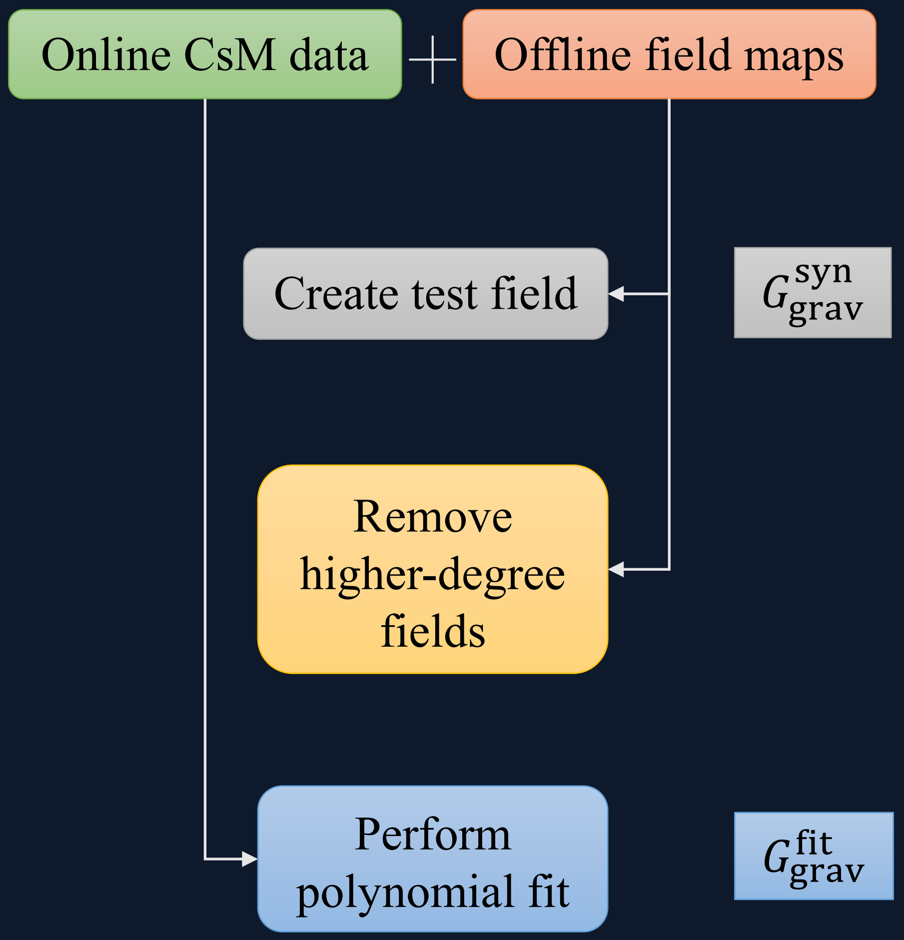
Online CsM data + **Offline field maps**

- 15 CsM during data taking
- Scalar magnetometer (z)



- Mapping campaign
- Three-axis fluxgate magnetometer (r, φ, z)

Synthesized data



random errors with standard deviations $\sigma_{l,m}^{\mathrm{map}}$

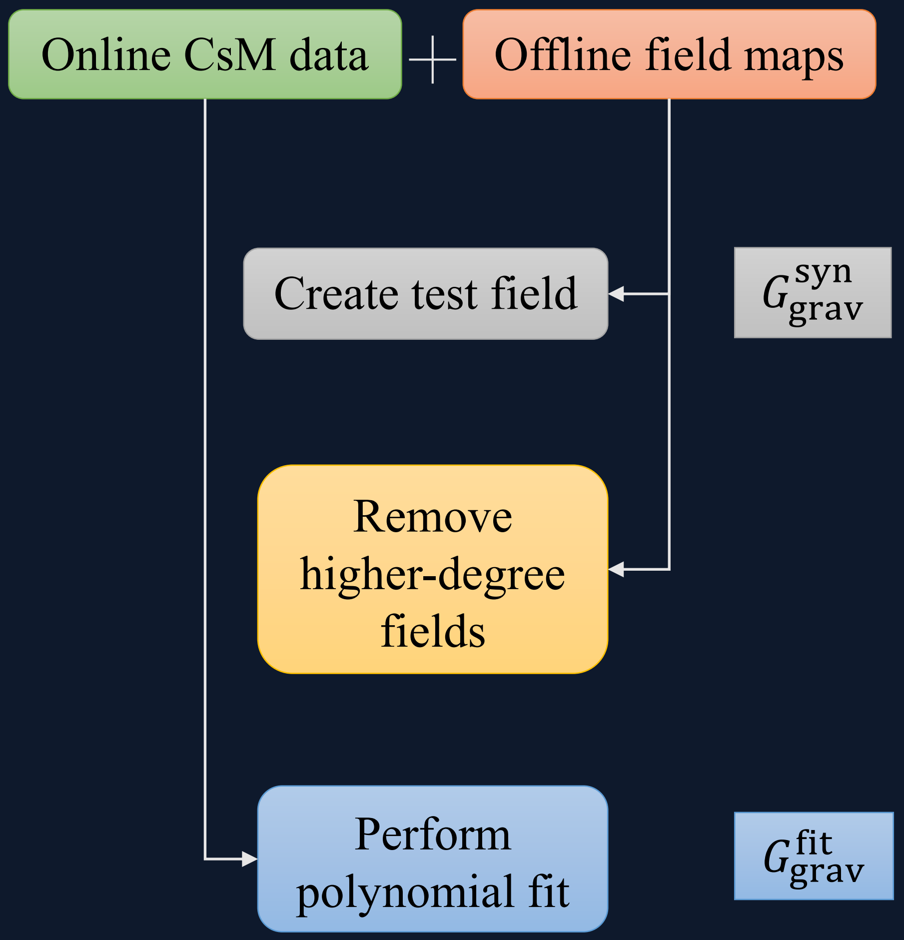
$$\boldsymbol{B}_{\mathrm{test}}^{i}(\boldsymbol{r}^{i}) = \sum_{l=0}^{6} \sum_{m=-(l+1)}^{+(l+1)} \left(G_{l,m}^{\mathrm{map}} + \delta_{l,m}^{\mathrm{map}}\right) \boldsymbol{\pi}_{l,m}(\boldsymbol{r}^{i}), \forall\, i \in [1,15]$$

$$B_{\mathrm{fit}}^{i}(\boldsymbol{r}^{i}) = \pm\left|\boldsymbol{B}_{\mathrm{test}}^{i}(\boldsymbol{r}^{i})\right| - \sum_{l,m} G_{l,m}^{\mathrm{map}}\, \pi_{z,l,m}(\boldsymbol{r}^{i})$$

$$B_{\mathrm{fit}}^{i}(\boldsymbol{r}^{i}) \longrightarrow \sum_{l,m} G_{l,m}^{\mathrm{fit}}\, \pi_{z,l,m}(\boldsymbol{r}^{i})$$

Ref.: P.-J. Chiu, Doctoral thesis, No. 27760, ETH Zurich (2021).

Synthesized data



- Two indicators:
  - $\Delta G_{grav} = G_{grav}^{syn} - G_{grav}^{fit}$ $\quad (G_{l,m}^{syn} = G_{l,m}^{map} + \delta_{G_{l,m}}^{map})$
  - $\sigma_{G_{grav}} =$

$$\sqrt{\left(\sigma_{G_{1,0}^{fit}}\right)^2 + \left(\sigma_{G_{3,0}^{fit}}\right)^2 \left(\frac{3H^2}{20} - \frac{3R^2}{4}\right)^2 + \left(\sigma_{G_{5,0}^{fit}}\right)^2 \left(\frac{5R^4}{8} - \frac{3R^2H^2}{8} + \frac{3H^4}{112}\right)^2}$$

- Optimal method: $\quad G_{3,0}^{map} \quad G_{5,0}^{map} \quad\quad G_{1,0}^{fit}$

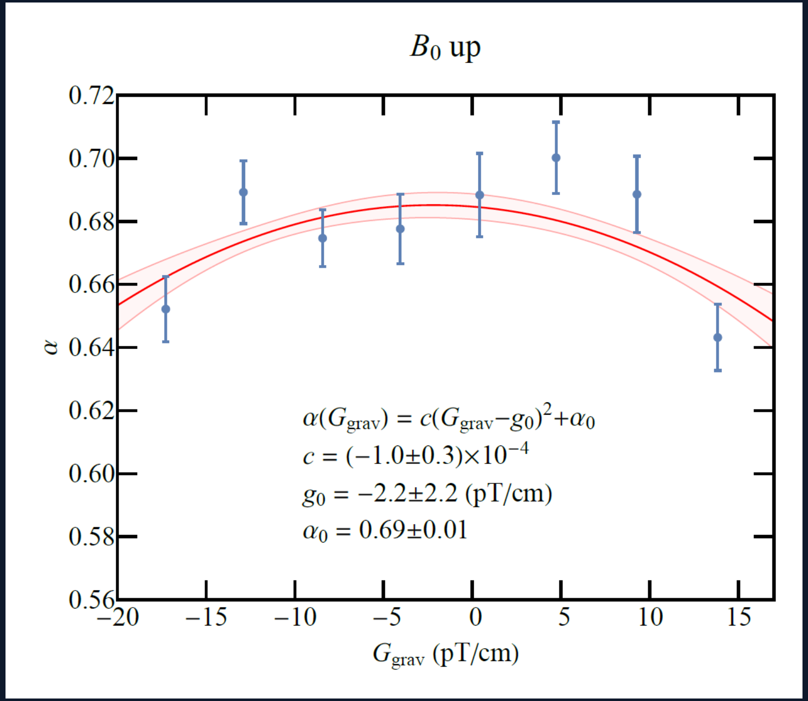
$$\pm\left|\boldsymbol{B}_{\mathrm{CsM}}^{i}(\boldsymbol{r}^{i})\right| - \sum_{l=3}^{6} G_{l,m}^{map}\, \pi_{z,l,m}(\boldsymbol{r}^{i}) \rightarrow \sum_{l=0}^{2} G_{l,m}^{fit}\, \pi_{z,l,m}(\boldsymbol{r}^{i})$$

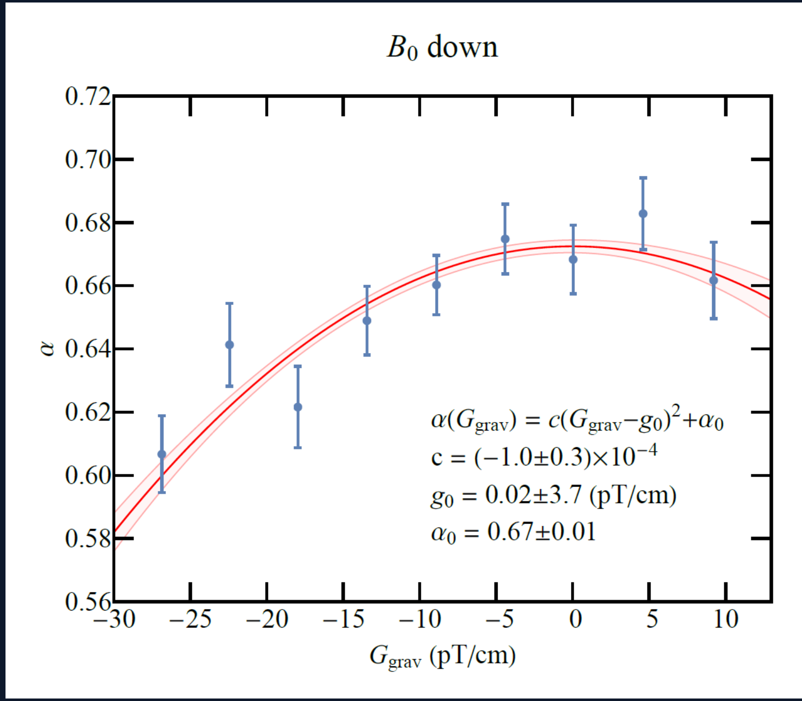
  - $|\Delta G_{grav}| \sim 2 - 3$ pT/cm
  - $\sigma_{G_{grav}} < 3.8$ pT/cm ($\boldsymbol{B}_0$ up) and $\sigma_{G_{grav}} < 4.5$ pT/cm ($\boldsymbol{B}_0$ down)

Ref.: P.-J. Chiu, Doctoral thesis, No. 27760, ETH Zurich (2021).

Visibility parabola

$$\mathcal{A} = \frac{N^{\uparrow} - N^{\downarrow}}{N^{\uparrow} + N^{\downarrow}} \approx \mathcal{A}_{\text{off}} - \boxed{\alpha} \cos\left[2\pi\left(\mathcal{R}_{\text{RF}} - \mathcal{R}\right)\mathcal{T}'\left\langle f_{\text{Hg}}\right\rangle\right]$$





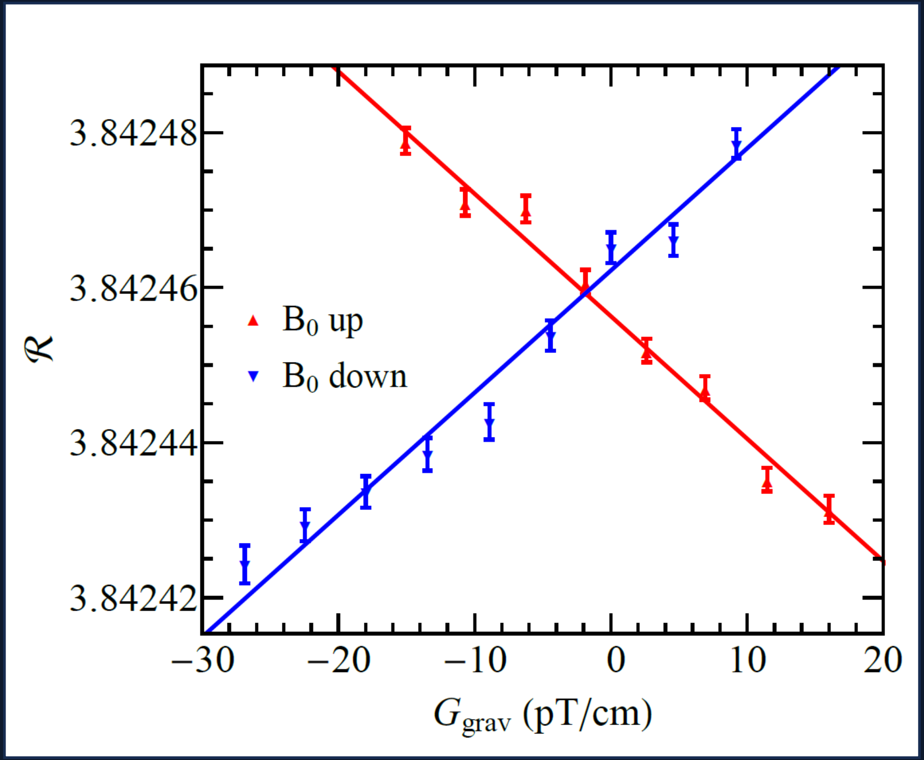
Correct the effective G_{grav} with a g_0 shift: -2.2 pT/cm ($\boldsymbol{B}_0$ up) or 0.02 pT/cm ($\boldsymbol{B}_0$ down)

Unprecedented precision on G_{grav} of 4.05 pT/cm

Crossing-point analysis

$$\mathcal{R}^{\uparrow/\downarrow} = \left(\frac{f_n}{f_{Hg}}\right)^{\uparrow/\downarrow} = \left|\frac{\gamma_n}{\gamma_{Hg}}\right| \left(1 \pm \frac{b^*_{UCN}}{|\boldsymbol{B_0}|} \pm \underbrace{\frac{G_{grav}\langle z \rangle}{|\boldsymbol{B_0}|}}_{\text{Corrected for the shift}} + \delta_{else}\right)$$

$$\delta_{else} = \underbrace{\delta_T + \delta_{Earth}}_{\text{Corrected for the shifts}} + \underbrace{\delta_{light} + \delta_{inc} + \delta_{JNN}}_{\text{No shift}}$$



$$\mathcal{R}^{\uparrow} = 3.8424563(08)_{stat}(36)_{sys}$$

$$\mathcal{R}^{\downarrow} = 3.8424622(12)_{stat}(59)_{sys}$$

$$\langle z \rangle = -0.43(2)\ \text{cm}$$

$$\gamma_n/\gamma_{Hg} = 3.8424574(30)$$

$$G_{\times} = -1.9(5)\ \text{pT/cm}$$

Refs.:
1. C. Abel et al., Phys. Rev. Lett. 124, 081803 (2020).
2. C. Abel et al., Phys. Rev. A 99, 042112 (2019).

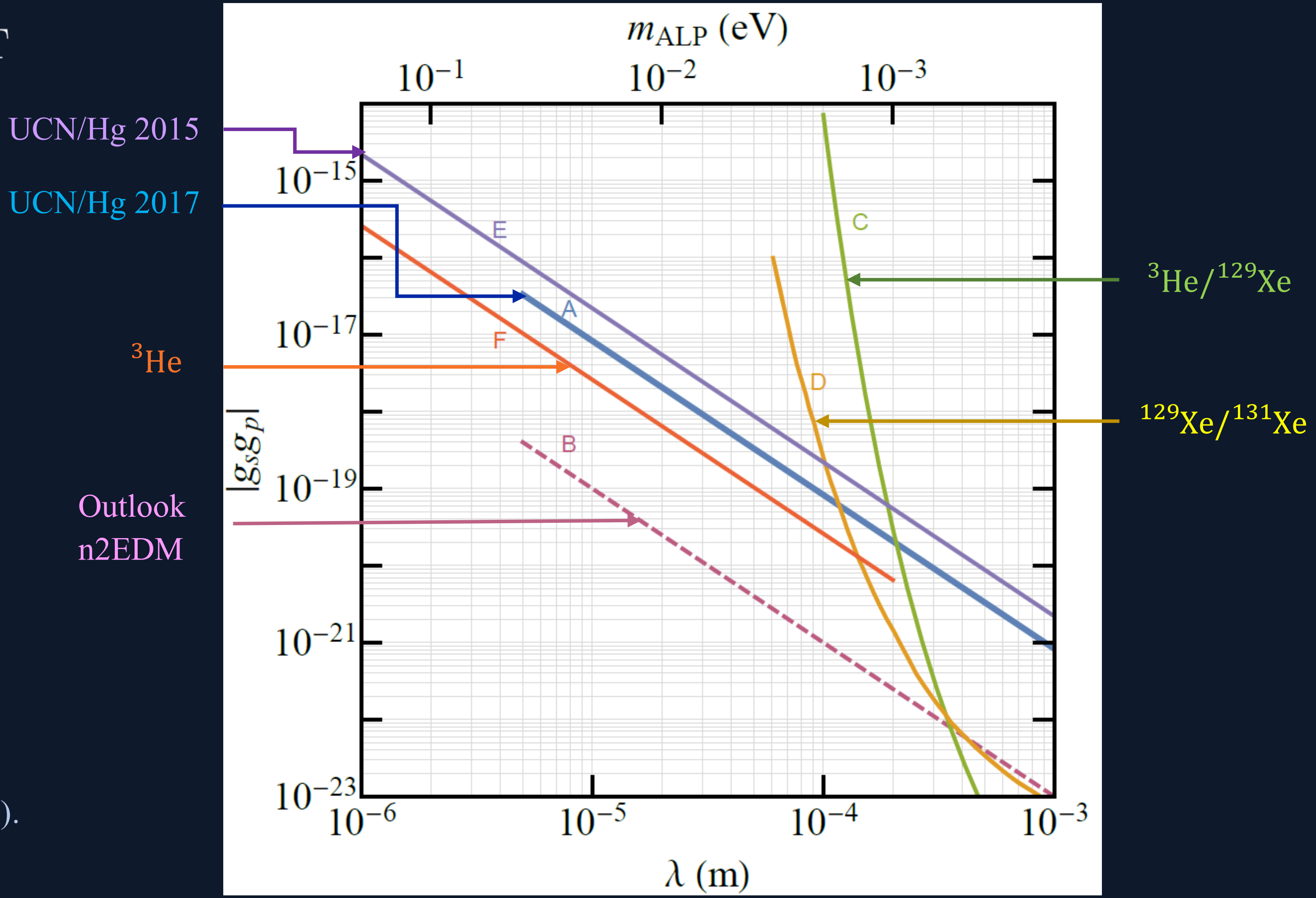
New exclusion limit

$$b^{*}_{\mathrm{UCN}} = \frac{\mathcal{R}^{\uparrow} - \mathcal{R}^{\downarrow}}{\mathcal{R}^{\uparrow} + \mathcal{R}^{\downarrow}} |\boldsymbol{B_0}| = (-0.80 \pm 0.96)\ \ \mathrm{pT}$$

→ $g_s g_p^{\dagger} \lambda^2 < 8.3 \times 10^{-28}\ \mathrm{m}^2$

Improves our previous limit by 2.7
Best limit obtained with free neutrons

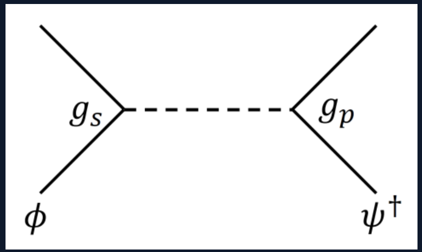
New J. Phys. 25 (2023) 103012
(DOI: 10.1088/1367-2630/acfdc3)



Refs.:
1. K. Tullney et al., Phys. Rev. Lett. 111, 100801 (2013).
2. M. Bulatowicz et al., Phys. Rev. Lett. 111, 102001 (2013).
3. S. Afach et al., Phys. Lett. B 745, 58 (2015).
4. M. Guigue et al., Phys. Rev. D 92, 114001 (2015).

Conclusion

- Axionlike particles (ALPs)
  - are promising candidates for dark matter
  - couple to fermions via scalar and pseudoscalar vertices

Nucleon  UCN

- A new short-range spin-dependent interaction
  - manifests as a pseudomagnetic field b^*_{UCN}
  - influence f_n

- Measure $\mathcal{R} = f_\mathrm{n}/f_\mathrm{Hg}$ under different G_grav for opposite B_0 polarities
- G_grav: online CsM measurements + offline field maps → unprecedented estimation on G_grav
- $b^*_{\mathrm{UCN}} \rightarrow g_\mathrm{s} g_\mathrm{p} \lambda^2 < 8.3 \times 10^{-28} \mathrm{m}^2$ (95% C. L.) in an interaction range of $5\ \mu\mathrm{m} < \lambda < 25$ mm
  → Best result obtained with free neutrons

Thank you for your attention!

Backup slides

Statistical

- Four effects were considered as stochastic uncertainties:
  - Neutron counting statistics
  - Uncertainty of the estimated HgM frequency
  - Magnetic-field-gradient G_{grav} drift between cycles
  - Ramsey-Bloch-Siegert shift induced by the $\pi/2$ pulse of the HgM onto the UCN spin
- Average uncertainties of each effect for all measurement cycles

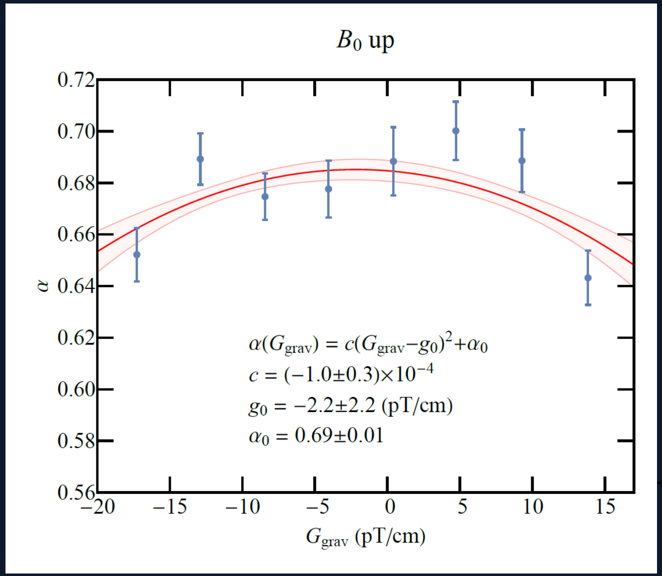
Effect / 1×10^{-7}	B_0 up	B_0 down
Neutron counts	1.84	2.26
HgM frequency	0.75	0.69
Gradient drift	0.02	0.02
^{199}Hg spin-flip pulse	0.07	0.23
Total stochastic effects	2.02	2.41

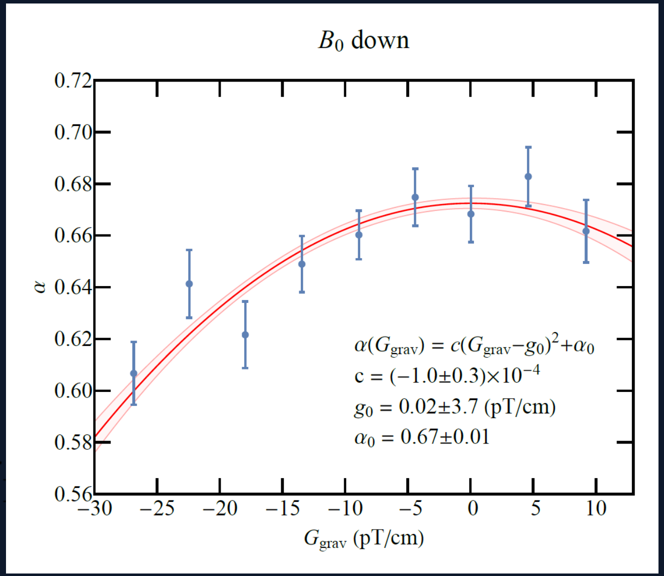
The gravitational shift δ_{grav}

Vertical magnetic field gradient

$$\mathcal{R}^{\uparrow/\downarrow} = \left(\frac{f_{\text{n}}}{f_{\text{Hg}}}\right)^{\uparrow/\downarrow} = \left|\frac{\gamma_{\text{n}}}{\gamma_{\text{Hg}}}\right| \left(1 \pm \frac{b^*_{\text{UCN}}}{|\boldsymbol{B_0}|} \pm \frac{G_{\text{grav}} \langle z \rangle}{|\boldsymbol{B_0}|} + \delta_{\text{else}}\right) \qquad \delta_{\text{grav}} = \frac{\langle B_z \rangle_{\text{n}}}{\langle B_z \rangle_{\text{Hg}}} - 1 = \pm \frac{G_{\text{grav}} \langle z \rangle}{|\boldsymbol{B}_0|},$$

$$G_{\text{grav}} = G_{1,0} + G_{3,0}\left(\frac{3H^2}{20} - \frac{3R^2}{4}\right) + G_{5,0}\left(\frac{5R^4}{8} - \frac{3R^2H^2}{8} + \frac{3H^4}{112}\right)$$





Error budget on $\mathcal{R}$

$$\sigma^{\uparrow}_{R_{\text{grav}}} = 35 \times 10^{-7}$$

$$\sigma^{\downarrow}_{R_{\text{grav}}} = 59 \times 10^{-7}$$

The transverse shift δ_{T}

Residual transverse field

$$\mathcal{R}^{\uparrow/\downarrow} = \left(\frac{f_{\mathrm{n}}}{f_{\mathrm{Hg}}}\right)^{\uparrow/\downarrow} = \left|\frac{\gamma_{\mathrm{n}}}{\gamma_{\mathrm{Hg}}}\right| \left(1 \pm \frac{b^*_{\mathrm{UCN}}}{|\boldsymbol{B_0}|} \pm \frac{G_{\mathrm{grav}}\langle z\rangle}{|\boldsymbol{B_0}|} + \delta_{\mathrm{else}}\right) \qquad \delta_{\mathrm{else}} = \boxed{\delta_{\mathrm{T}}} + \delta_{\mathrm{Earth}} + \delta_{\mathrm{light}} + \delta_{\mathrm{inc}} + \delta_{\mathrm{JNN}}$$

$$\delta_{\mathrm{T}} = \frac{\langle {B_{\mathrm{T}}}^2\rangle}{2{\boldsymbol{B}_0}^2} \qquad \langle {B_{\mathrm{T}}}^2\rangle = \langle \Delta B_x^2 + \Delta B_y^2\rangle \qquad \begin{aligned}\Delta B_x &= B_x - \langle B_x\rangle \\ \Delta B_y &= B_y - \langle B_y\rangle\end{aligned}$$

Systematic shift on $\mathcal{R}$

For each run, the resonant $\mathcal{R}$ deduced from Ramsey fit was corrected for δ_{T}

Error budget on $\mathcal{R}$

Reproducibility $\sigma_{\langle B_{\mathrm{T}}^2\rangle}$ of the field maps

$$\mathcal{R}_{\mathrm{T}}^{\uparrow} = (7.3 \pm 4.7) \times 10^{-7}$$
$$\mathcal{R}_{\mathrm{T}}^{\downarrow} = (6.4 \pm 4.1) \times 10^{-7}$$

Ref.: C. Abel et al., Phys. Rev. A 106, 032808 (2022).

Earth rotation δ_{Earth}

Rotating frame of reference

$$\mathcal{R}^{\uparrow/\downarrow} = \left(\frac{f_{\mathrm{n}}}{f_{\mathrm{Hg}}}\right)^{\uparrow/\downarrow} = \left|\frac{\gamma_{\mathrm{n}}}{\gamma_{\mathrm{Hg}}}\right| \left(1 \pm \frac{b^{*}_{\mathrm{UCN}}}{|\boldsymbol{B_0}|} \pm \frac{G_{\mathrm{grav}}\langle z\rangle}{|\boldsymbol{B_0}|} + \delta_{\mathrm{else}}\right) \qquad \delta_{\mathrm{else}} = \delta_{\mathrm{T}} + \boxed{\delta_{\mathrm{Earth}}} + \delta_{\mathrm{light}} + \delta_{\mathrm{inc}} + \delta_{\mathrm{JNN}}$$

$$\delta_{\mathrm{Earth}} = \mp\left(\frac{f_{\mathrm{Earth}}}{f_{\mathrm{n}}} + \frac{f_{\mathrm{Earth}}}{f_{\mathrm{Hg}}}\right)\cos\left(\theta_{\mathrm{PSI}}\right) = \mp 1.4 \times 10^{-6} \qquad \cos\left(\theta_{\mathrm{PSI}}\right) = 0.738$$

θ_{PSI} : Angle between the $\boldsymbol{B}_0$ direction and the rotational axis of the Earth

Systematic shift on $\mathcal{R}$

For each run, the resonant $\mathcal{R}$ deduced from Ramsey fit was corrected for δ_{Earth}

Hg light shift δ_{light}

Resonant UV laser beam traversing the precession chamber

$$\mathcal{R}^{\uparrow/\downarrow} = \left(\frac{f_{\text{n}}}{f_{\text{Hg}}}\right)^{\uparrow/\downarrow} = \left|\frac{\gamma_{\text{n}}}{\gamma_{\text{Hg}}}\right| \left(1 \pm \frac{b^*_{\text{UCN}}}{|\boldsymbol{B_0}|} \pm \frac{G_{\text{grav}}\langle z\rangle}{|\boldsymbol{B_0}|} + \delta_{\text{else}}\right) \quad \delta_{\text{else}} = \delta_{\text{T}} + \delta_{\text{Earth}} + \boxed{\delta_{\text{light}}} + \delta_{\text{inc}} + \delta_{\text{JNN}}$$

The vector light shift:

The projection of the magnetic field of the photons onto the $\boldsymbol{B}_0$ direction (depending on $\boldsymbol{B}_0$ direction)

$$\mathcal{R}^{\uparrow}_{\text{VL}} = (1.5 \pm 6.9) \times 10^{-7}$$
$$\mathcal{R}^{\downarrow}_{\text{VL}} = (1.2 \pm 5.4) \times 10^{-7}$$

The direct light shift:

The spin precesses at different frequencies; faster in the excited state (proportional to the light power)

$$\mathcal{R}^{\uparrow/\downarrow}_{\text{DL}} = (0.4 \pm 0.8) \times 10^{-7}$$

Error budget on $\mathcal{R}$

$$\mathcal{R}^{\uparrow}_{\text{light}} = (1.9 \pm 6.9) \times 10^{-7}$$
$$\mathcal{R}^{\downarrow}_{\text{light}} = (1.6 \pm 5.5) \times 10^{-7}$$

Refs.:

1. S. Afach et al., Phys. Lett. B 739, 128 (2014).
2. M. Fertl, Doctoral thesis, No. 21638, ETH Zurich (2013).
3. C. Abel et al., Phys. Rev. Lett. 124, 081803 (2020).

Incoherent scattering δ_{inc}

Spin-dependent nuclear interaction between UCN & Hg atoms

$$\mathcal{R}^{\uparrow/\downarrow} = \left(\frac{f_{\text{n}}}{f_{\text{Hg}}}\right)^{\uparrow/\downarrow} = \left|\frac{\gamma_{\text{n}}}{\gamma_{\text{Hg}}}\right| \left(1 \pm \frac{b^*_{\text{UCN}}}{|\boldsymbol{B_0}|} \pm \frac{G_{\text{grav}}\langle z \rangle}{|\boldsymbol{B_0}|} + \delta_{\text{else}}\right) \quad \delta_{\text{else}} = \delta_{\text{T}} + \delta_{\text{Earth}} + \delta_{\text{light}} + \boxed{\delta_{\text{inc}}} + \delta_{\text{JNN}}$$

Pseudomagnetic field

$$\boldsymbol{B}^* = -\frac{4\pi\hbar}{m_{\text{n}}\gamma_{\text{n}}} N_{\text{Hg}} b_{\text{inc}} \boldsymbol{P} \sqrt{\frac{I}{I+1}}$$

N_{Hg}: Hg number density
$b_{\text{inc}} = \pm 15.5$ fm: incoherent scattering length
$\boldsymbol{P}$: polarization vector
$I = ½$ Hg nuclear spin

In case of an imperfect Hg $\pi/2$ pulse

$$\left|\frac{\gamma_{\text{n}}}{\gamma_{\text{Hg}}}\right| \delta_{\text{inc}} = \pm \frac{\gamma_{\text{n}} |B^*_z|}{|\omega_{\text{Hg}}|} = \mp \frac{4\pi\hbar}{\gamma_{\text{Hg}} B_0 \sqrt{3} m_{\text{n}}} N_{\text{Hg}} b_{\text{inc}} P_z$$

Error budget on $\mathcal{R}$

$\sigma^{\uparrow/\downarrow}_{R_{\text{inc}}} \leq 5 \times 10^{-10}$ → Negligible

Refs.:

1. V. F. Sears, Neutron News 3, 26 (2006).
2. E. G. A. Chanel, Doctoral thesis, University of Bern (2021).

Magnetic Johnson-Nyquist noise δ_{JNN}

Magnetic-field fluctuation

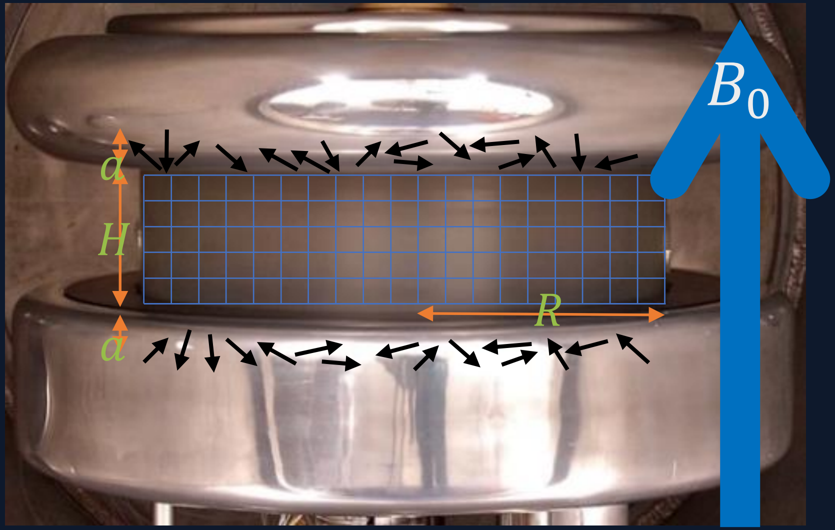
$$\mathcal{R}^{\uparrow/\downarrow} = \left(\frac{f_{\mathrm{n}}}{f_{\mathrm{Hg}}}\right)^{\uparrow/\downarrow} = \left|\frac{\gamma_{\mathrm{n}}}{\gamma_{\mathrm{Hg}}}\right|\left(1 \pm \frac{b^{*}_{\mathrm{UCN}}}{|\boldsymbol{B_0}|} \pm \frac{G_{\mathrm{grav}}\langle z\rangle}{|\boldsymbol{B_0}|} + \delta_{\mathrm{else}}\right) \quad \delta_{\mathrm{else}} = \delta_{\mathrm{T}} + \delta_{\mathrm{Earth}} + \delta_{\mathrm{light}} + \delta_{\mathrm{inc}} + \boxed{\delta_{\mathrm{JNN}}}$$

$\langle B_{\mathrm{n}}\rangle \neq \langle B_{\mathrm{Hg}}\rangle$

Finite-element method to estimate time-and-volume-averaged field

$$\langle B_{\mathrm{Hg}}\rangle = |\langle \boldsymbol{B}\rangle| = \sqrt{\langle B_x\rangle^2 + \langle B_y\rangle^2 + \langle B_0 + B_z\rangle^2}$$

$$\langle B_{\mathrm{UCN}}\rangle = \langle|\boldsymbol{B}|\rho_{\mathrm{UCN}}(z)\rangle = \left\langle\sqrt{(B_x)^2 + (B_y)^2 + (B_0 + B_z)^2}\,\rho_{\mathrm{n}}(z)\right\rangle$$



Discrete Biot-Savart law

$$\boldsymbol{B}(\boldsymbol{r},t) = \frac{\mu_0}{4\pi}\sum_{i=1}^{n_{\mathrm{dip}}}\sum_{\alpha=x,y,z}\frac{I_{\alpha,i}(\mathcal{T})\mathrm{d}\boldsymbol{l}\times(\boldsymbol{r}-\boldsymbol{r}'_i)}{|\boldsymbol{r}-\boldsymbol{r}'_i|^3}$$

Error budget on $\mathcal{R}$

$$\sigma^{\uparrow/\downarrow}_{R_{\mathrm{JNN}}} \leq 1\times10^{-9}$$ → Negligible

Ref.: N. J. Ayres et al., Phys. Rev. A 103, 062801 (2021).