Lattice calculation of neutron electric dipole moment

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Outline

- Introduction : Current status of Lattice EDM calculations
- Our Results
- Summary

Nucleon EDM Experiments

Recent nEDM limits:

$$d_n < 2.9 \times 10^{-26} e\,.\,cm$$

C. A. Baker, Phys. Rev. Lett. 97(2006)

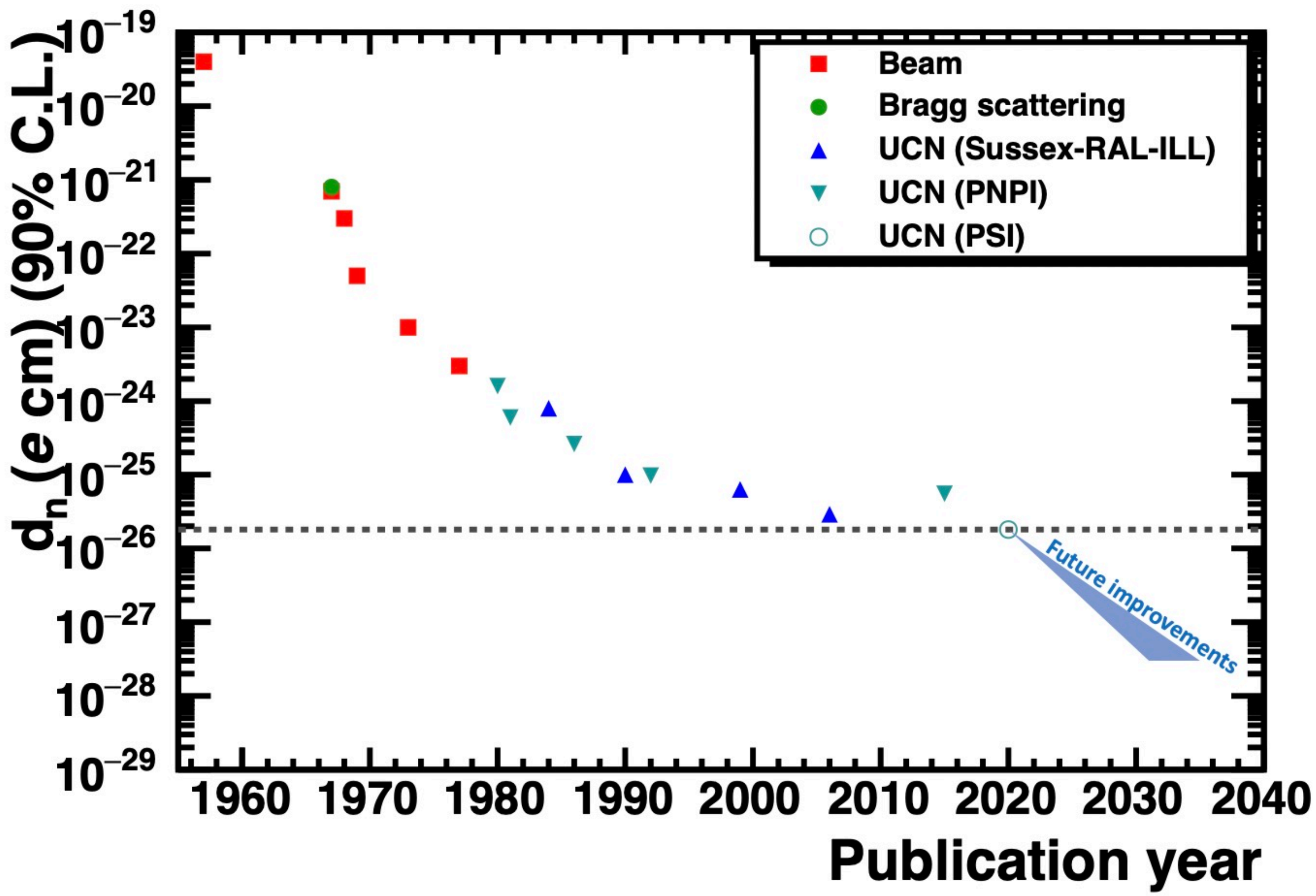
$$d_n < 1.6 \times 10^{-26} e\,.\,cm$$

B. Graner, Phys. Rev. Lett. 116(2016)

$$d_n = (0.0 \pm 1.1_{stat} \pm 0.2_{sys}) \times 10^{-26} e\,.\,cm$$

C. Abel et al, Phys. Rev. Lett. 124(2020)

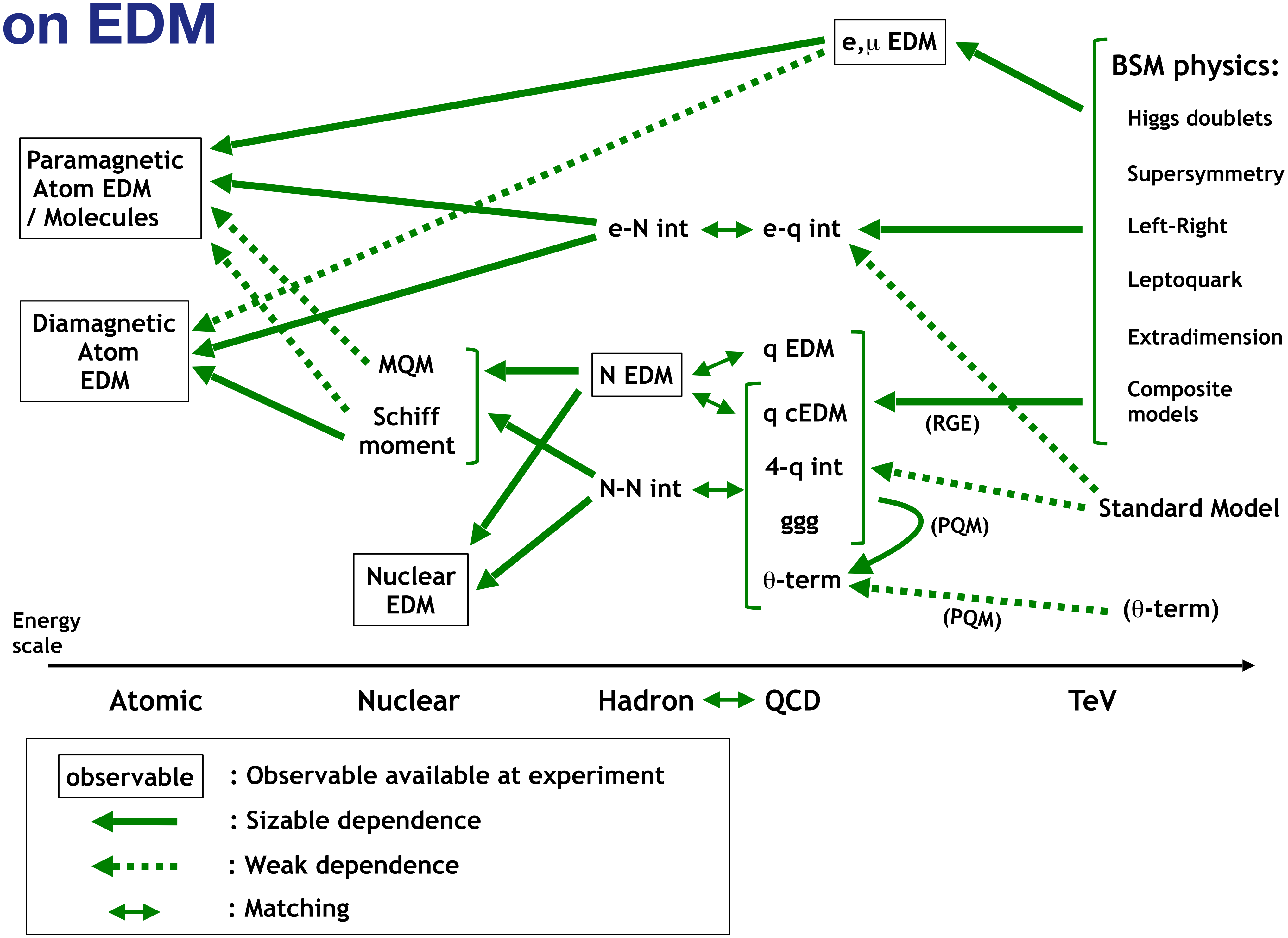
Snowmass 2021, 2203.08103



SM prediction

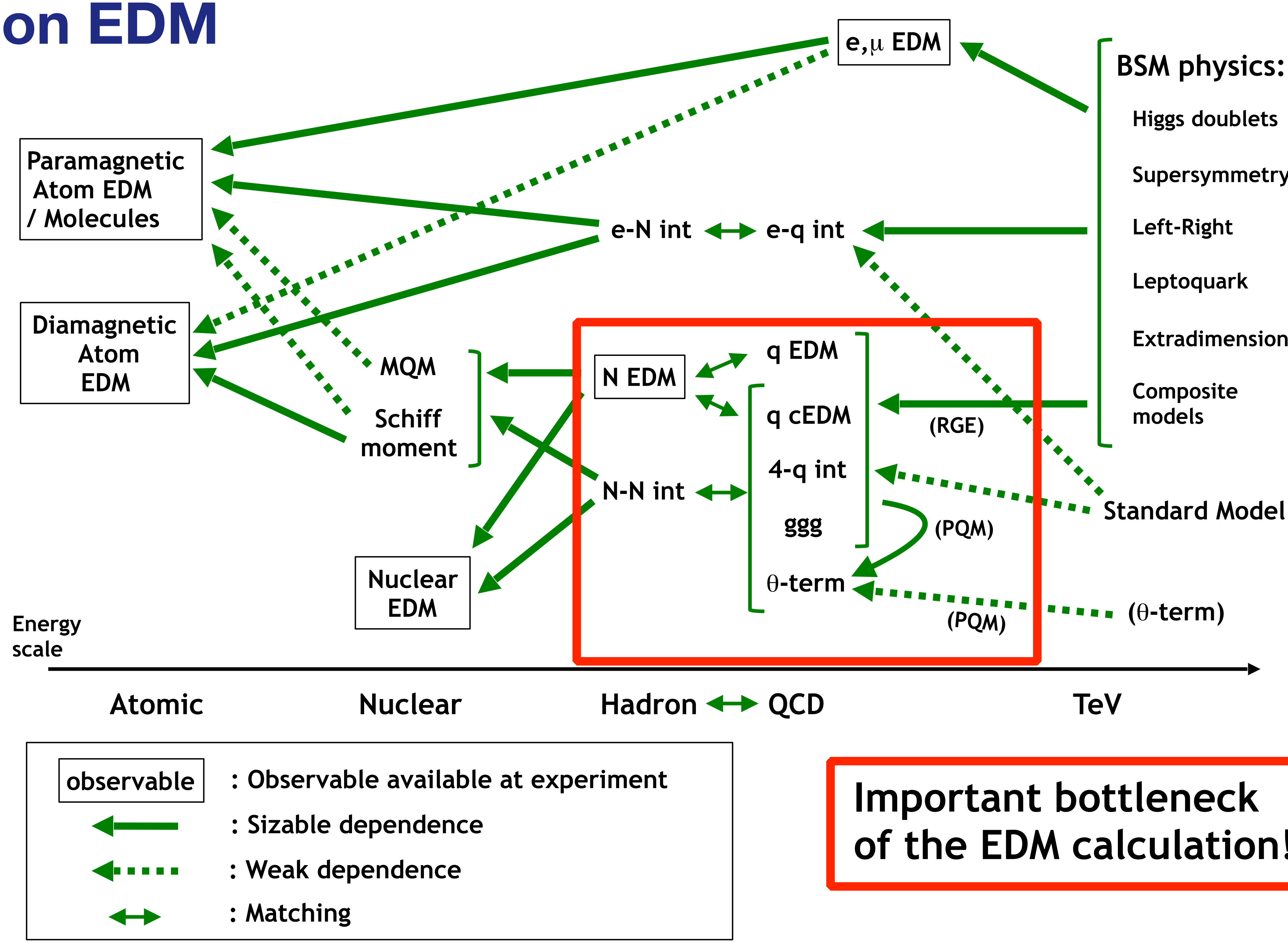
$|d_n| \sim 10^{-31}\ e{\cdot}cm.$

Nucleon EDM



[N. Yamanaka, et al. Eur. Phys. J. A53 (2017) 54, Ginges and Flambaum Phys. Rep. 397, 63, 2004]

Nucleon EDM



Important bottleneck of the EDM calculation!

[N. Yamanaka, et al. Eur. Phys. J. A53 (2017) 54, Ginges and Flambaum Phys. Rep. 397, 63, 2004]

Role of (lattice) QCD : connect quark/gluon-level (effective) operators to hadron/nuclei matrix elements and interactions (Form factor, dn)

Non-perturbative determination is important → Lattice QCD calculation

Effective CPV operators

$$\mathcal{L}_{eff}^{\not{CP}} = \frac{g_s^2}{32\pi^2}\bar{\theta} G_{\mu\nu}\tilde{G}^{\mu\nu}$$

dim=4, θ_{QCD}

$$- \frac{i}{2} \sum_{i=e,u,d,s} d_i \bar{\psi}_i F \cdot \sigma\gamma_5 \psi_i$$

dim=5, e, quark EDM

$$- \frac{i}{2} \sum_{i=u,d,s} \tilde{d}_i \bar{\psi}_i G \cdot \sigma\gamma_5 \psi_i$$

dim=5, chromo EDM

$$+ \omega f^{abc} G_{\mu\nu,a} G^{\mu\beta,b} G^{\nu,c}_{\ \ \beta}$$

dim=6, Weinberg three gluon

$$+ \sum C_i^{(4q)} \mathcal{O}_i^{(4q)}$$

dim=6, Four-quark operators

Effective CPV operators

$$\mathcal{L}_{eff}^{\not{CP}} = \frac{g_s^2}{32\pi^2}\bar{\theta}G_{\mu\nu}\tilde{G}^{\mu\nu}$$

dim=4, θ_{QCD}

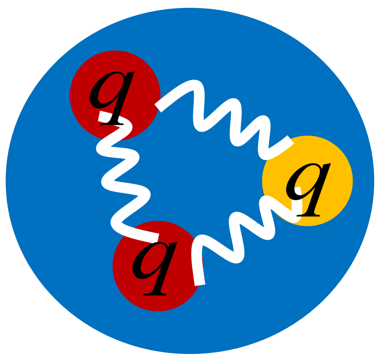
- Why $\bar{\theta} \leq \mathcal{O}(10^{-10})$? → Strong CP problem
- A dynamical solution : PQ symmetry (θ → 0 and Axion)
- Other dynamical solutions?

c.f. Conceptional discussions of "un-observability" of topological charges.

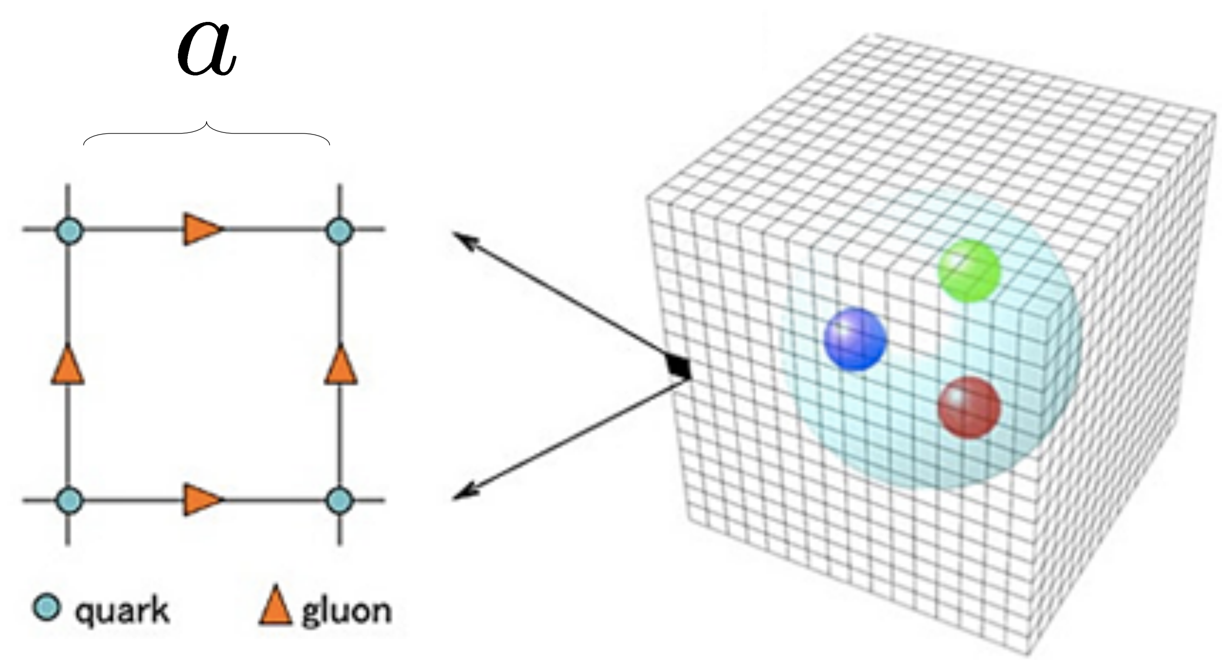
e.g., Topological charge decouples to hadron correlation functions in infinite volume limit？ [G. Schierholz, 2024].

→ Lattice QCD are important to confirm the problem and to constrain θ.

Lattice QCD : First principle calculation of QCD [Wilson, ’74]



Lattice regularization



Path integral of field theory

$$Z = \int \prod_{n,\mu} dU_{n,\mu} e^{-S}$$

1.Generate samples of vacuum, typically O(10)—O(1,000) samples of gauge configurations $U_{\mu}(n)$.

$$\{C^0\} \rightarrow \{C^1\} \rightarrow \cdots \rightarrow \{C^{i-1}\} \rightarrow \{C^i\} \rightarrow \cdots \rightarrow \{C^N\}$$

Thermalized configurations

2. Then measure physical observables on the vacuum ensemble (important sampling)

$$\langle \mathcal{O} \rangle = \frac{\int dU \mathcal{O}(U) e^{-S(U)}}{\int dU e^{-S(U)}} = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} \mathcal{O}(U_i)$$

Calculation of θ EDM on the lattice

Re-weighting method [S. Aoki et al. (2005); F. Berruto et al (2005), …]

$$e^{-S_{QCD}-i\theta Q} = e^{-S_{QCD}}\left[1 - i\theta Q + \mathcal{O}(\theta^2)\right]$$

$$\langle \mathcal{O} \rangle_{\not{CP}} = \langle \mathcal{O} \rangle_{CP-even} - i\theta \langle Q \cdot \mathcal{O} \rangle_{CP-even} + \mathcal{O}(\theta^2)$$

(CP-even) **(CP-odd)**

CPV operator : Q, cEDM, etc…, |θ| << 1

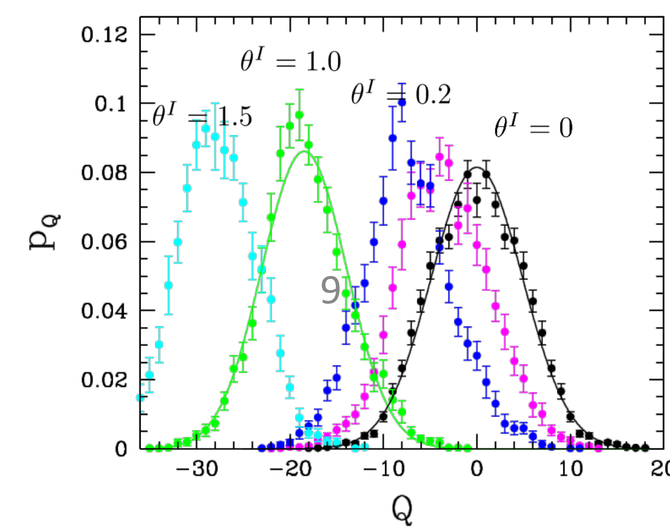
Original (CP-even) gauge configurations can be used.
No sign problem.

c.f. Dynamical simulation including CP-odd interactions

[R. Horsley et al. (2008); H. K. Guo, et al., 2015)]

$$\langle \mathcal{O} \rangle_\theta \sim \int \mathcal{D}U(\mathcal{O}) e^{-S_{QCD}-\theta_{imag}Q}$$

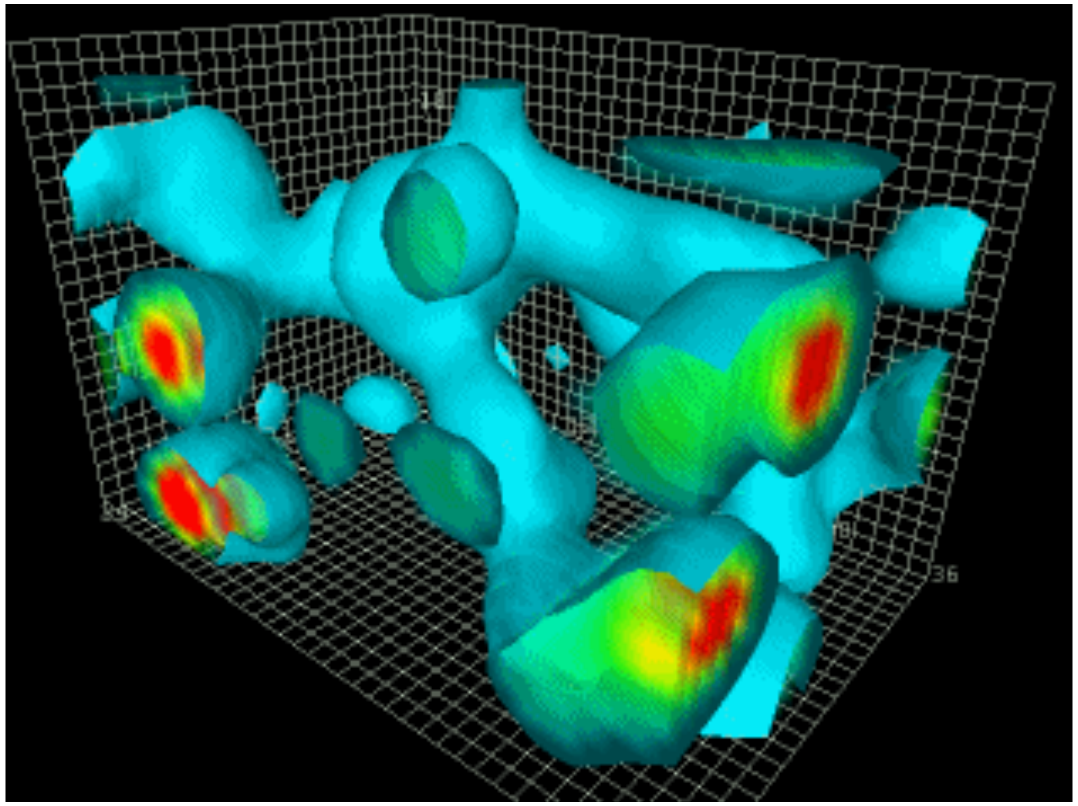
Need additional simulation for ensemble generations to get non-zero topological sector.
Better sampling of non-zero Q sector.
Check linearity of θ (ensemble generation for various imaginary θ)



[R. Horsley, et al (2008)]

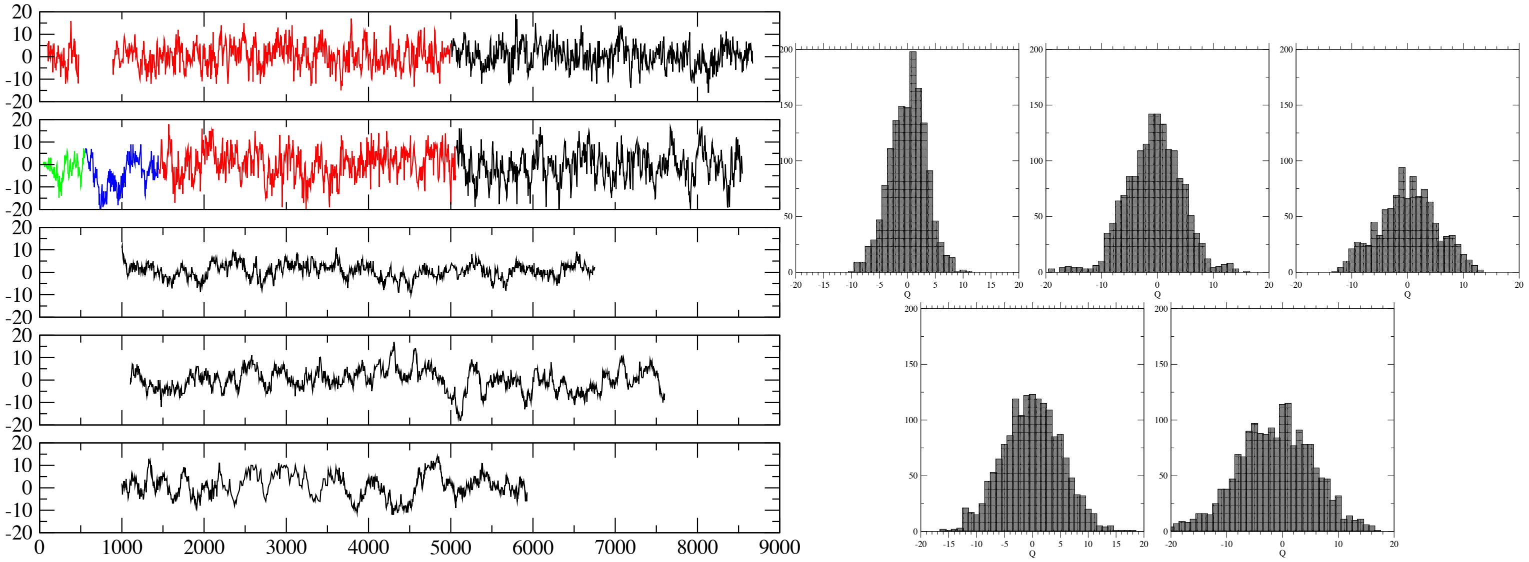
Example of Monte Carlo simulation of QCD vacuum

QCD non-trivial topological vacuum: source of CPV



(Courtesy of Derek Leinweber, CSSM, University of Adelaide)

Q (topological charge) in lattice Monte Carlo history (RBC /UKQCD collaboration)
Nf=2+1 DWF, mπ=290-420 MeV



Extraction of nucleon EDM

Form factor method

[Aoki et al (2005); Berruto et al (2005); Shindler et al (2015) ; Alexandrou et al (2015) ; Shintani et al (2016); Dragos et al(2019); Alexandrou et al(2020); Bhattacharya et al (2021) ;Liang et al (2023)]

Form factor is widely used to extract EDM,
one need to calculate the “3pt correlation function” with topological charge.

$$\langle N\,[\bar{q}\gamma^{\mu}q]\,\bar{N}\rangle_{\not{CP}} = \frac{1}{Z}\int \mathscr{D}U\,\mathscr{D}\bar{\psi}\mathscr{D}\psi N\,[\bar{q}\gamma^{\mu}q]\,\bar{N}e^{-S-iS_{\theta}}$$

$$S_{\theta} = \frac{\theta}{32\pi^2}\int d^4xTr[G_{\mu\nu}(x)\tilde{G}^{\mu\nu}(x)]$$

$$\langle p',\sigma'|J^{\mu}|p,\sigma\rangle = \bar{u}_{p',\sigma'}\left[F_1(Q^2)\gamma^{\mu} + F_2(Q^2)\frac{i\sigma^{\mu\nu}q_{\nu}}{2m_N} - F_3(Q^2)\frac{\gamma_5\sigma^{\mu\nu}q_{\nu}}{2m_N}\right]u_{p,\sigma}$$

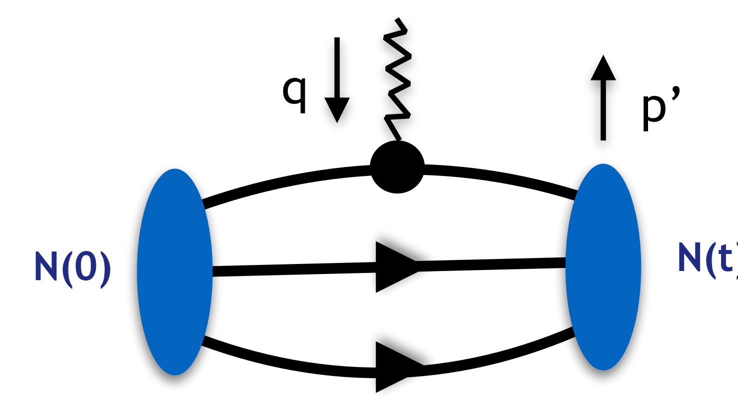
Dirac & Pauli Form Factor
(P, T even)

Electric Dipole Form Factor (EDFF)

Electric dipole moment ($Q^2 \to 0$ extrapolation)

$$d_n = \lim_{Q^2\to 0}\frac{F_3(Q^2)}{2m_N}$$

electric magnetic current



$$(q = p' - p, \quad Q^2 = -q^2)$$

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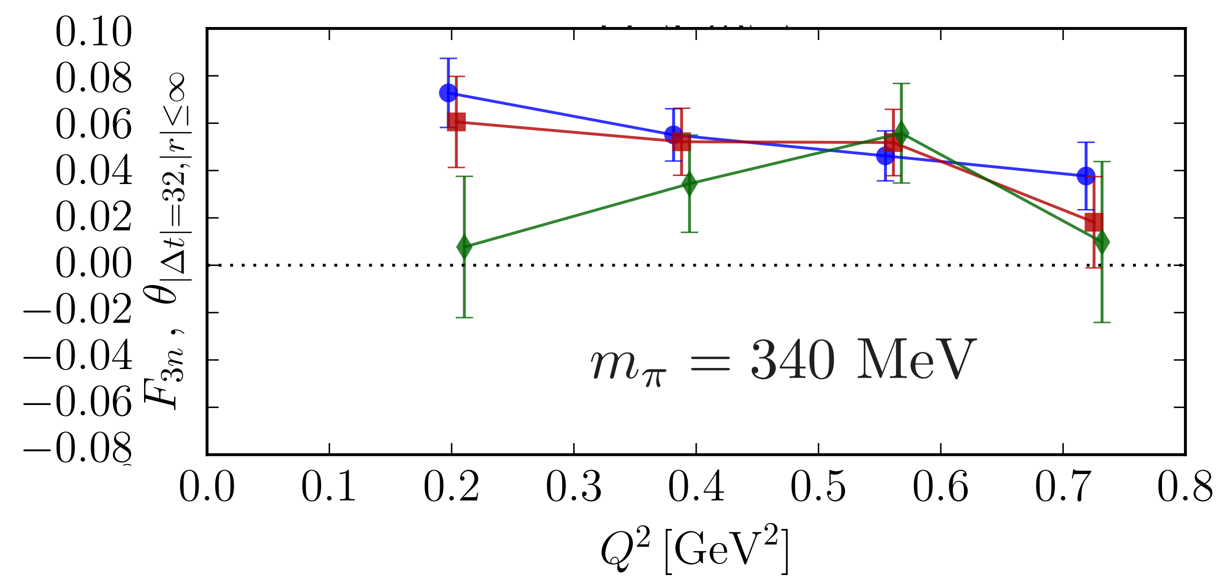

[S. Syritsyn et al. (2018)]

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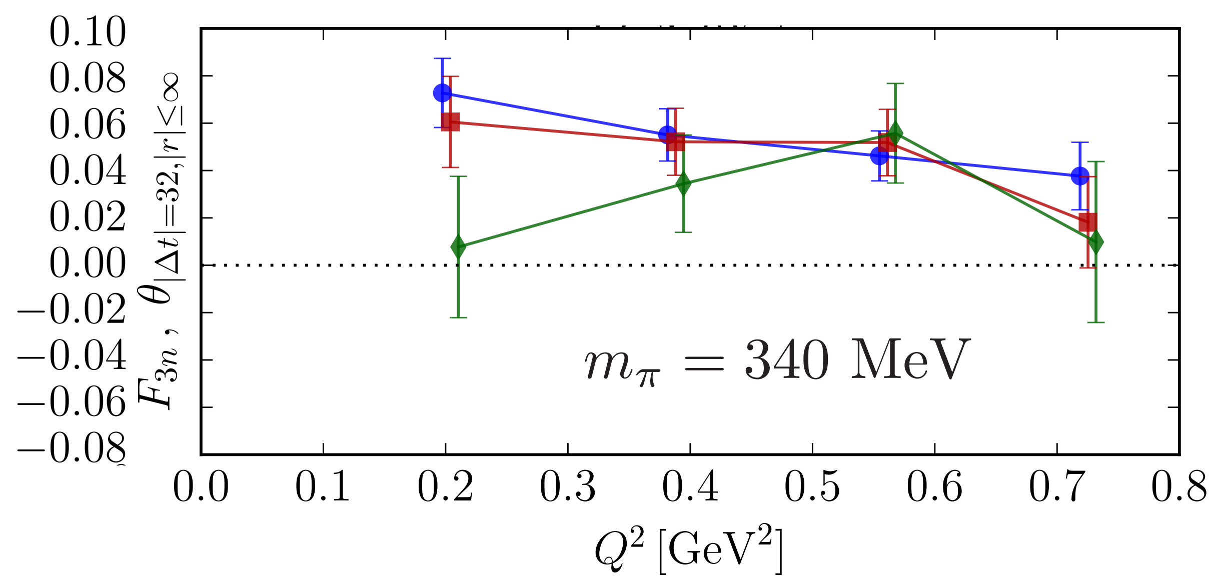

[S. Syritsyn et al. (2018)]

Problem: Prior to 2017, a spurious mixing between EDM and magnetic moments in all previous lattice computations of nucleon form factor.

Spurious mixing problem

[M. Abramczyk, et al. Phys.Rev.D 96 (2017)]

- CP violating interaction induces a chiral phase : $\langle 0|N|p,\sigma\rangle_{\not{CP}} = e^{i\alpha\gamma_5} u_{p,\sigma} = \tilde{u}_{p,\sigma}$

  $\tilde{u}_p$ is a solution spinor of the free Dirac equation in : $(\not{p} - m_N e^{-2i\alpha\gamma_5})\tilde{u}_p = 0$

  α is mixing angle (CP-violating mass correction)

- This mixing angle α has to be calculated, and rotated away to obtain "net" CP-violation effect.

$$\bar{\tilde{u}}_{p',\sigma'} \left[\tilde{F}_1 \gamma^\mu + (\tilde{F}_2 + i\tilde{F}_3\gamma_5) \frac{i\sigma^{\mu\nu}q_\nu}{2m_N} \right] \tilde{u}_{p,\sigma} \equiv \bar{u}_{p',\sigma'} \left[F_1\gamma^\mu + (F_2 + iF_3\gamma_5)\frac{i\sigma^{\mu\nu}q_\nu}{2m_N} \right] u_{p,\sigma}$$

[Previous "lattice" parametrization prior to 2017]

$$(F_2 + iF_3\gamma_5) = e^{2i\alpha\gamma_5}(\tilde{F}_2 + i\tilde{F}_3\gamma_5) \Rightarrow \begin{aligned} [F_2]_{\rm correct} &= \tilde{F}_2 + \mathcal{O}(\alpha^2) \\ [F_3]_{\rm correct} &= \tilde{F}_3 + 2\alpha F_2 \end{aligned}$$

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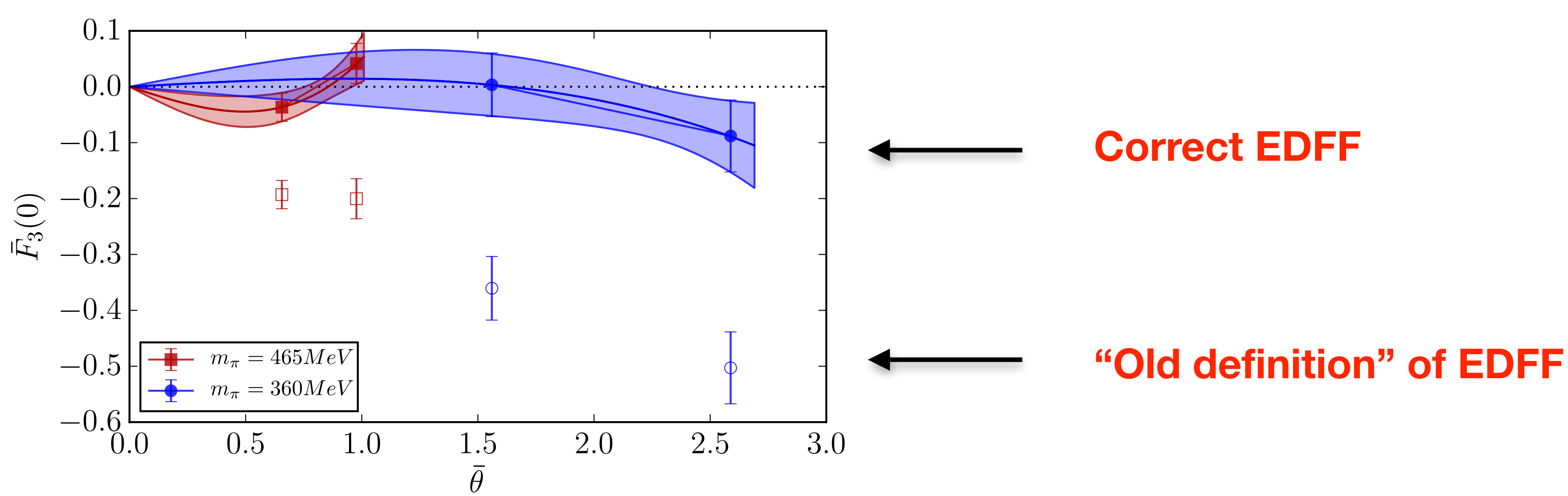


Form factor method with imaginary θ simulation [F. Guo et al., PRL 115, no.6, 062001 (2015)]

Recent lattice results for θ-induced nEDM (after 2017)

Lattice	Neutron EDM [e fm]	Proton EDM [e fm]
Dragos et al [2019]	$d_n/\theta = -0.00152(71)$	$d_p/\theta = 0.0011(10)$
Alexandrou et al [2020]	$\lvert d_n/\theta\rvert = 0.0009(24)$	–
Bhattacharya et al [2021]	$d_n/\theta = -0.003(7)(20)$	$d_p/\theta = 0.024(10)(30)$
Liang et al [2023]	$d_n/\theta = -0.00148(14)(31)$	$d_p/\theta = 0.0038(11)(8)$

Phenomenology	method	Neutron EDM [e fm]
Pospelov et al [2000]	ChPT/NDA	~ 0.002
Pospelov et al [1999]	QCD sum rules	0.0025(13)
Hisano et al [2012]	QCD sum rules	0.0004^{+3}_{-2}
Ema et al [2024]	QCD sum rules	$0.0005 \sim 0.0015$

↑

Re-analysis using “correct” nucleon operators from which unphysical chiral phase decouples.

Using the correct definition of F_3, the lattice results are more comparable with the phenomenological results but with huge errors.

→ Need to improve the signals.

Our results

- Nf=2+1 (Mobius) Domain wall fermion (chiral symmetry on the lattice)
- Iwasaki gauge action

Ensemble	Lattice size	Lattice spacing	Statistics	Pion mass
24I_005	24^3 x 64	0.1105fm	100cfgs	340MeV
24I_010	24^3 x 64	0.1105fm	1100cfgs	420MeV

- All-mode and low-mode averaging techniques for measurements

2pt with exact solver	1
2pt with sloppy solver	64
low mode all to all 2pt	Volume

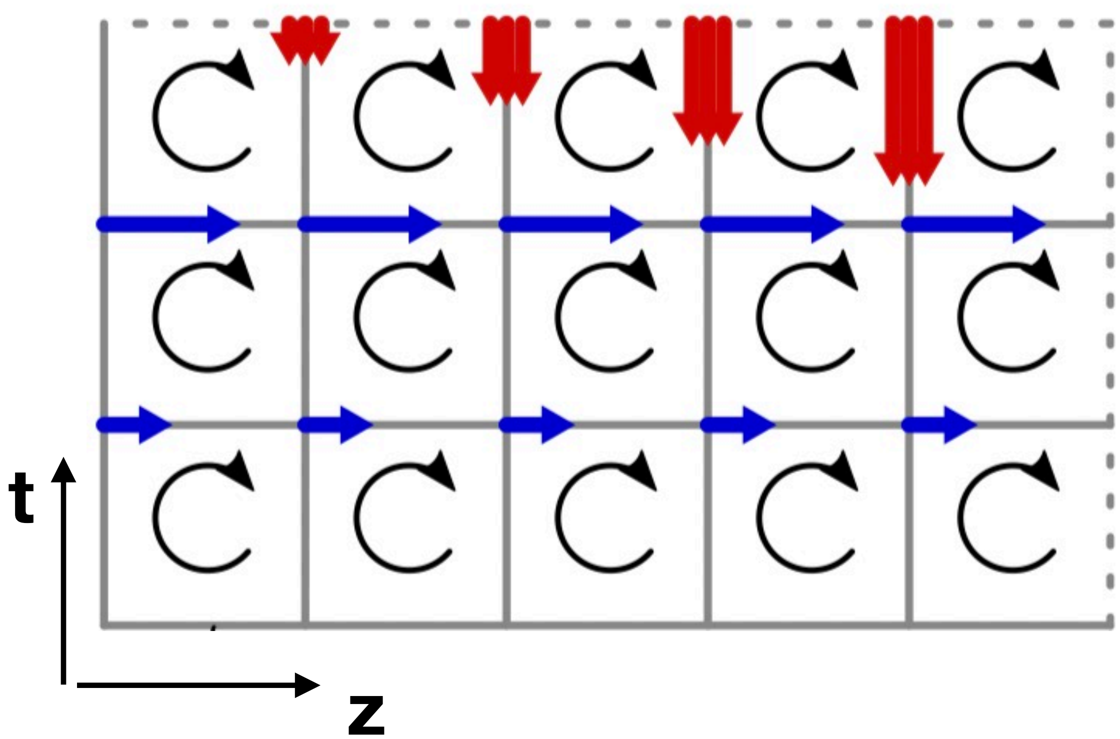
Background electric field method

W. Detmold, B. Tiburzi and A. Walker-Loud, Phys.Rev.D81(2010)

•Uniform electric field preserving translational invariance and periodic boundary conditions on a lattice (Euclidean imaginary electric field)

•No sign problem: analytic continuation of CP-odd interaction

•Neutron energy shift in background electric field $\Delta E = d_n \vec{S} \cdot \vec{\epsilon}$



The setup of U(1) gauge link

$$U_\mu \to e^{iqA_\mu} U_\mu$$

$$A_z(z,t) = -\epsilon_z t$$

$$A_t(z, L_t - 1) = \epsilon_z z \times L_t$$

ϵ_z **: Strength of background field**

Quantization condition $\epsilon_z = \dfrac{6\pi}{L_t L_x} n \qquad n = \pm 1, \pm 2, \ldots$

Lattice EDM with b.g. electric field

- The EDM can be extracted from the energy shift of 2pt in the background electric field

$$C^{2\text{pt},\vec{E}}_{\not{CP}}(\vec{0},t) = \langle N(t)\bar{N}(0)e^{i\theta Q}\rangle_E = C_{2\text{pt},\vec{E}}(\vec{0},t) + C^{Q}_{2\text{pt},\vec{E}}(\vec{0},t) \quad = |Z_N|^2 \sum_{s=\pm} \tilde{u}_{E_z,s}\bar{\tilde{u}}_{E_z,s} \frac{e^{-(m_N+s*\delta_E)t}}{2m_N}$$

$$= |Z_N|^2 \left(\frac{1+\gamma_4}{2} - i\frac{\kappa}{4m^2}\gamma_3\gamma_4\epsilon_z\right) e^{-m_N t} + |Z_N|^2 \left(i\alpha\gamma_5 - \frac{1+\gamma_4}{2}\Sigma_Z\delta E t - i\alpha\frac{\kappa}{2m^2}\Sigma_Z\epsilon_z + i\alpha\Sigma_Z\gamma_5\delta E t\right) e^{-m_N t}$$

α **: Chiral rotation angle** $\quad \tilde{u}_{p,\sigma} = e^{i\alpha\gamma_5}u_{p,\sigma}$

κ **: anomalous magnetic moment**

$$\delta E = d_n \epsilon_z$$
$$\Sigma_Z : -i\gamma_x\gamma_y$$

- 2pt nucleon correlation functions with topological charge

$$C^{Q}_{2\text{pt},\vec{E}}(0,t) = \sum_{\vec{y}} \langle N(\vec{y},t) \left(\sum_{\tau_q=0}^{T}\sum_{\vec{x}}[q(\vec{x},\tau_q)]\right) \bar{N}(\vec{0},0)\rangle_{\vec{E}}$$

- The extraction of EDM $\quad d_n \propto \frac{\mathrm{Tr}[\Sigma_Z C^{Q}_{2pt,\vec{E}}(0,t_f)]}{\mathrm{Tr}[C_{2pt,\vec{E}}(0,t_f)]} - \frac{\mathrm{Tr}[\Sigma_Z C^{Q}_{2pt,\vec{E}}(0,t_f-1)]}{\mathrm{Tr}[C_{2pt,\vec{E}}(0,t_f-1)]}$

Feynman-Hellman (FH) theorem

C. Bouchard, et al., PRD96(2017)

- The matrix elements can be related to the energy shift through FH theorem

2pt correlation function $$C_\lambda(t) = \langle \lambda | \mathcal{O}(t)\mathcal{O}^\dagger(0) | \lambda \rangle = \frac{1}{\mathcal{Z}_\lambda} \int D\Phi e^{-S-S_\lambda} \mathcal{O}(t)\mathcal{O}^\dagger(0)$$

FH theorem $$\left.\frac{dE(\lambda)}{d\lambda}\right|_{\lambda=0} = \left\langle N \left| \frac{d\hat{H}(\lambda)}{d\lambda} \right| N \right\rangle$$

- EDM using FH theorem

Nucleon Energy shift $$m^{\text{eff}} = m + \theta d_n^\theta \vec{\sigma} \cdot \vec{\epsilon}$$

EDM can be obtained from the matrix element $$d_n^\theta \epsilon_z = \langle N \uparrow | \sum_{\vec{x}} q(\vec{x}) | N \uparrow \rangle_E$$

d_n from global topological charge → (FH theorem) → d_n from local topological charge density operator

$$Q \sim \int_{V_4} G\tilde{G}, \quad \langle Q^2 \rangle \sim V_4$$ → (Statistical error)^2 $\propto$ V_4

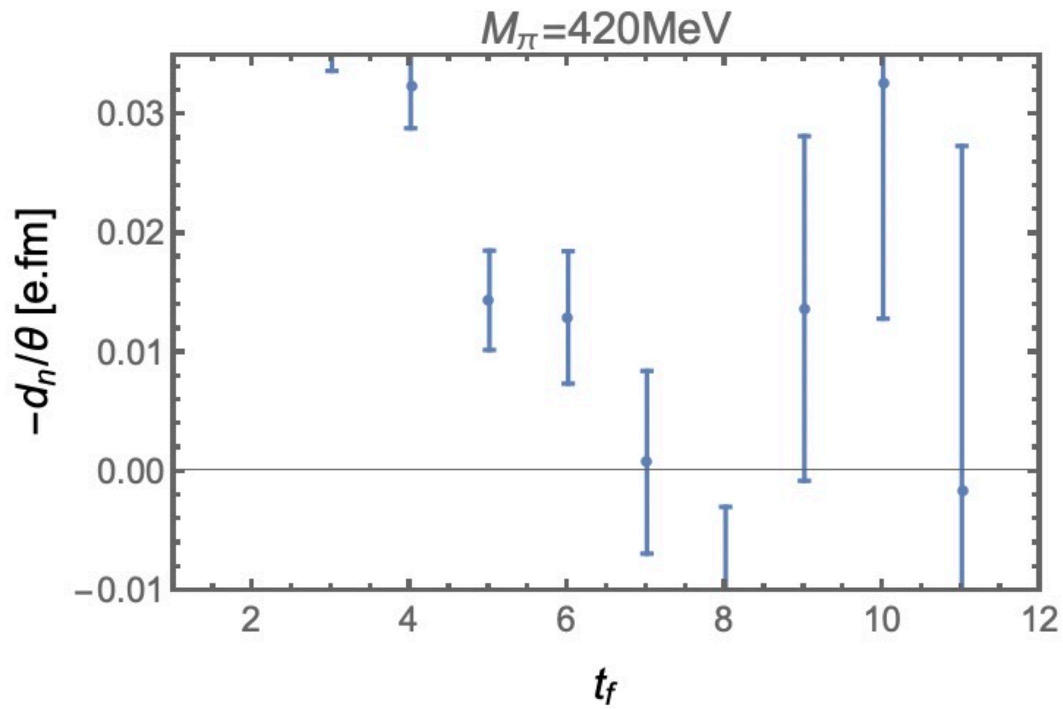
Signal can be improved.

Results of nEDM

- Comparison of results obtained using local and global topological charge

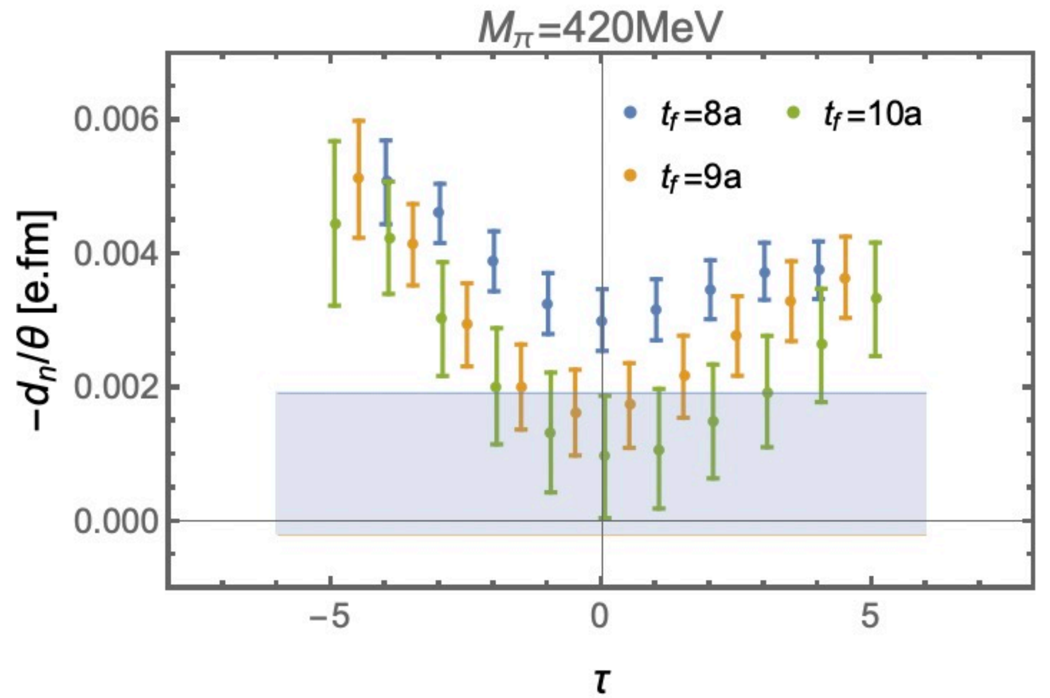
EDM using Global top. charge

$$d_n(t_f)\epsilon_z t_f \sim \langle N(t_f) \sum_{\tau=0}^{T} \sum_{\vec{x}} q(\vec{x},\tau) N(0) \rangle_E$$



EDM using local top. charge

$$d_n(t_f,\tau)\epsilon_z \sim \langle N(t_f) \sum_{\vec{x}} q(\vec{x},\tau) N(0) \rangle_E$$



$$\frac{-d_n}{\theta} = 0.0009(10)[e\,.fm]$$

The results obtained using the local operator have much better signal

Topological charge from quark loop

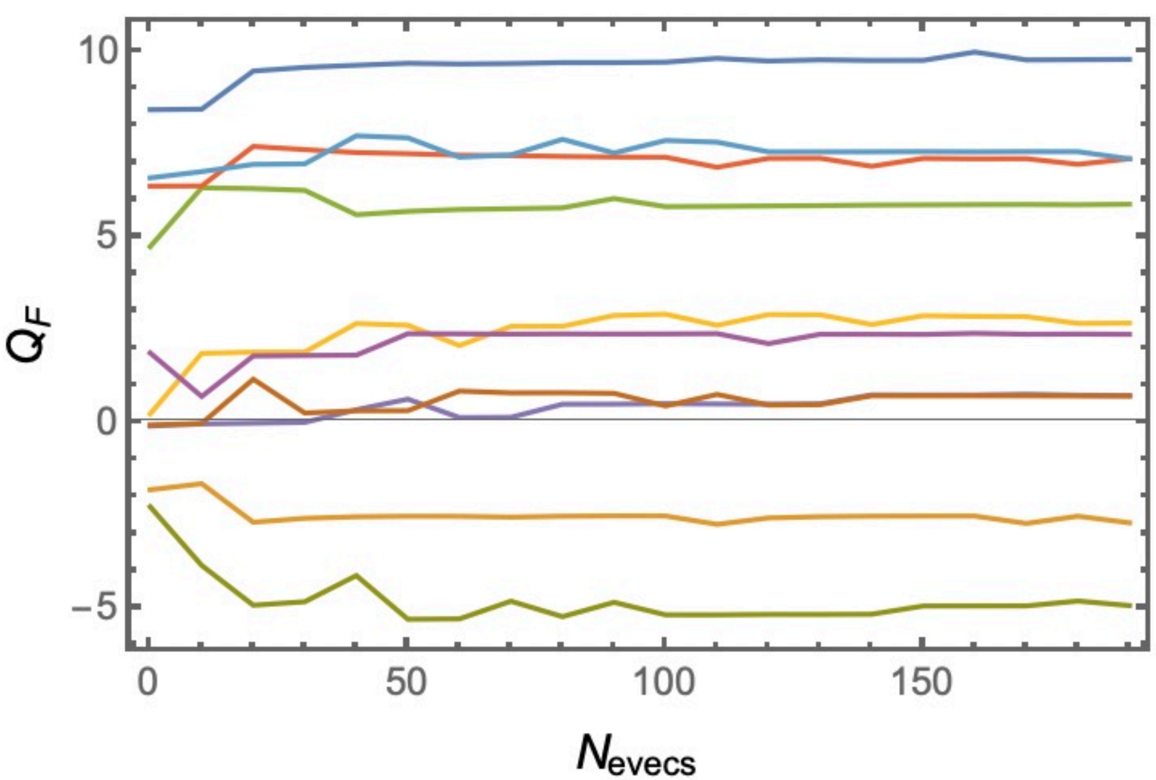
Gluonic definition: $$\rho_G(x) = \frac{1}{16\pi^2}\mathrm{Tr}\left[G_{\mu\nu}\tilde{G}^{\mu\nu}\right]_x$$

Fermionic definition: $$\rho_F(x) = -m_q J_A(x) = -m_q \mathrm{Tr}\left[\gamma_5 D^{-1}(x,x)\right]$$

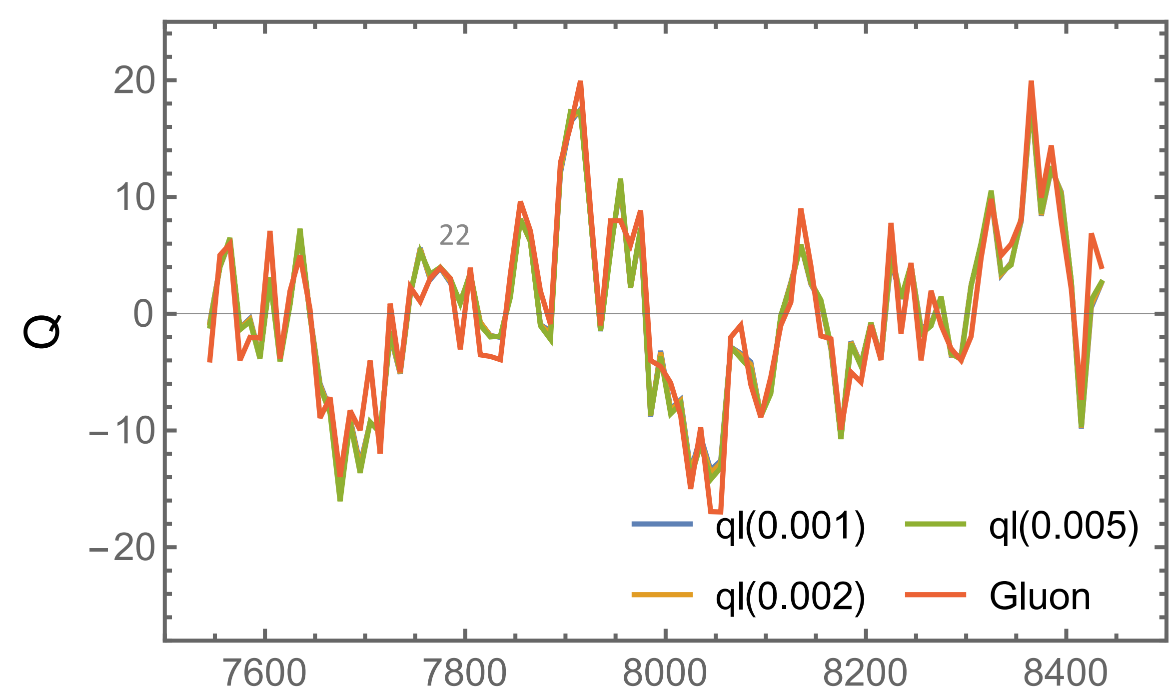
Index Theorem for Dirac operator: $Q_{\mathrm{top}} \equiv Q_G = Q_F$ Thanks to lattice chiral symmetry

$$Q_G = \sum_x \rho_Q(x),\ Q_F = \sum_x \rho_F(x)$$

The dependence of Q_F on #(Dirac eigenvectors)



Comparison of Q

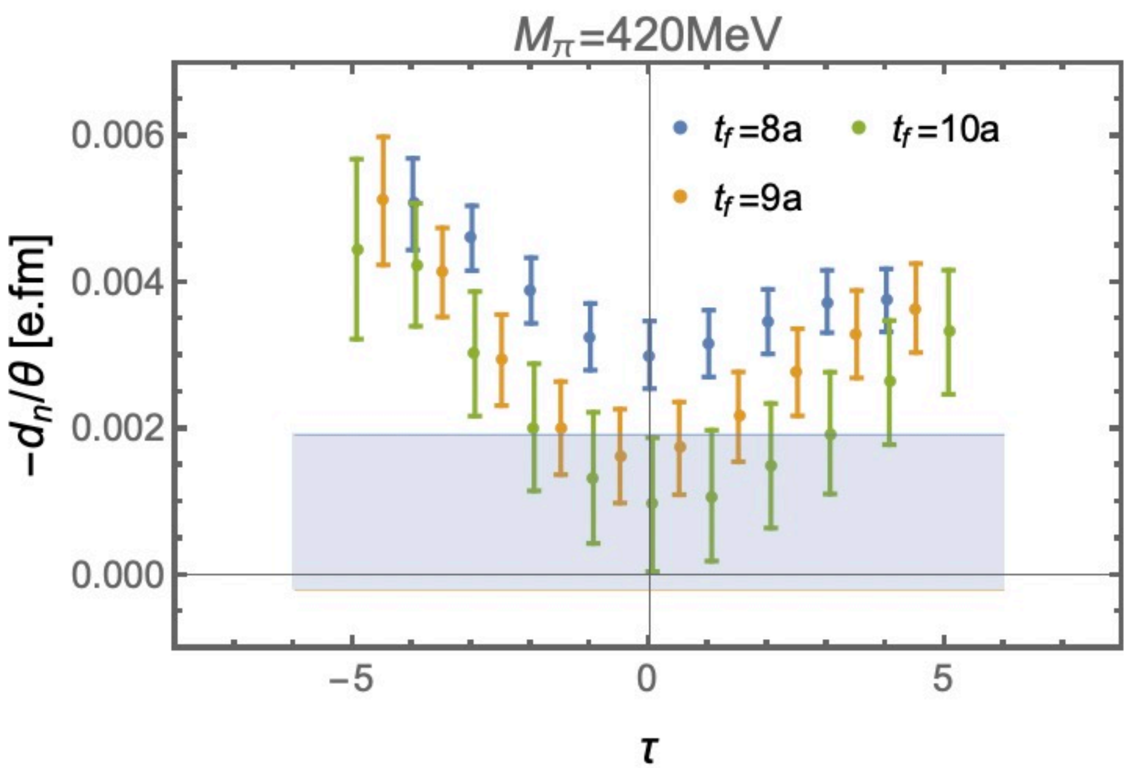


The top charge Q_F with different m_f is consistent.

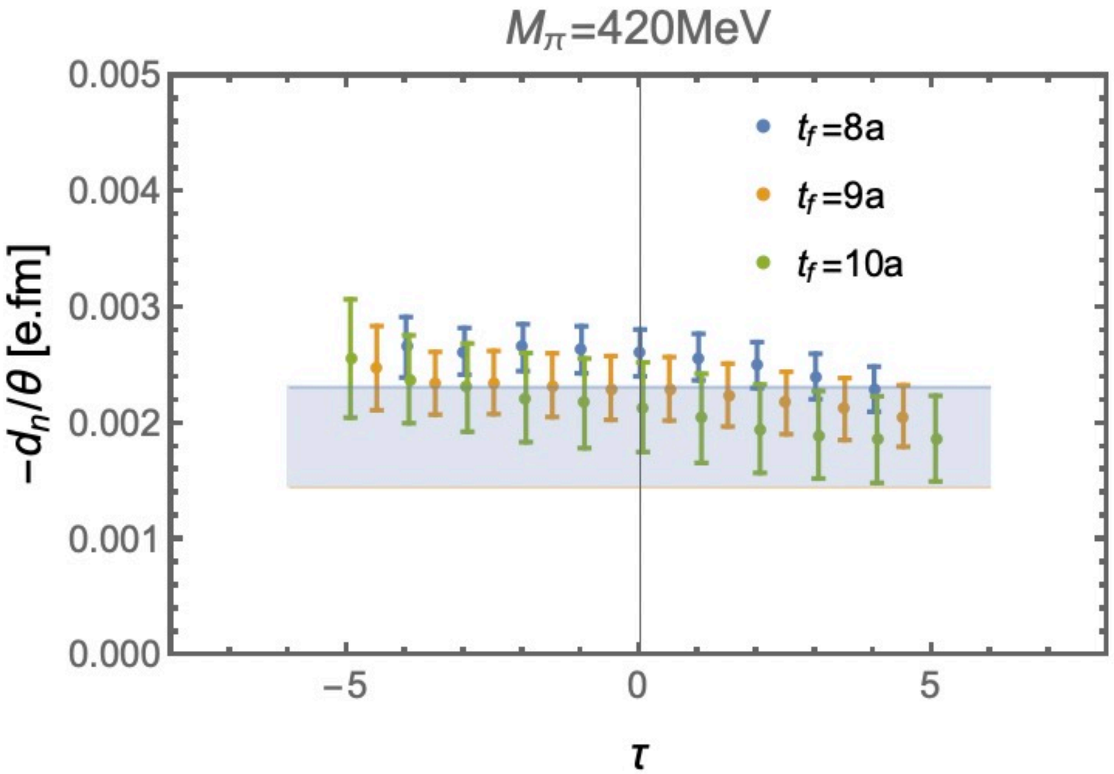
Results of nEDM

- The comparison of EDMs obtained using topological charge defined by gluon field and quark loop

Gluonic definition



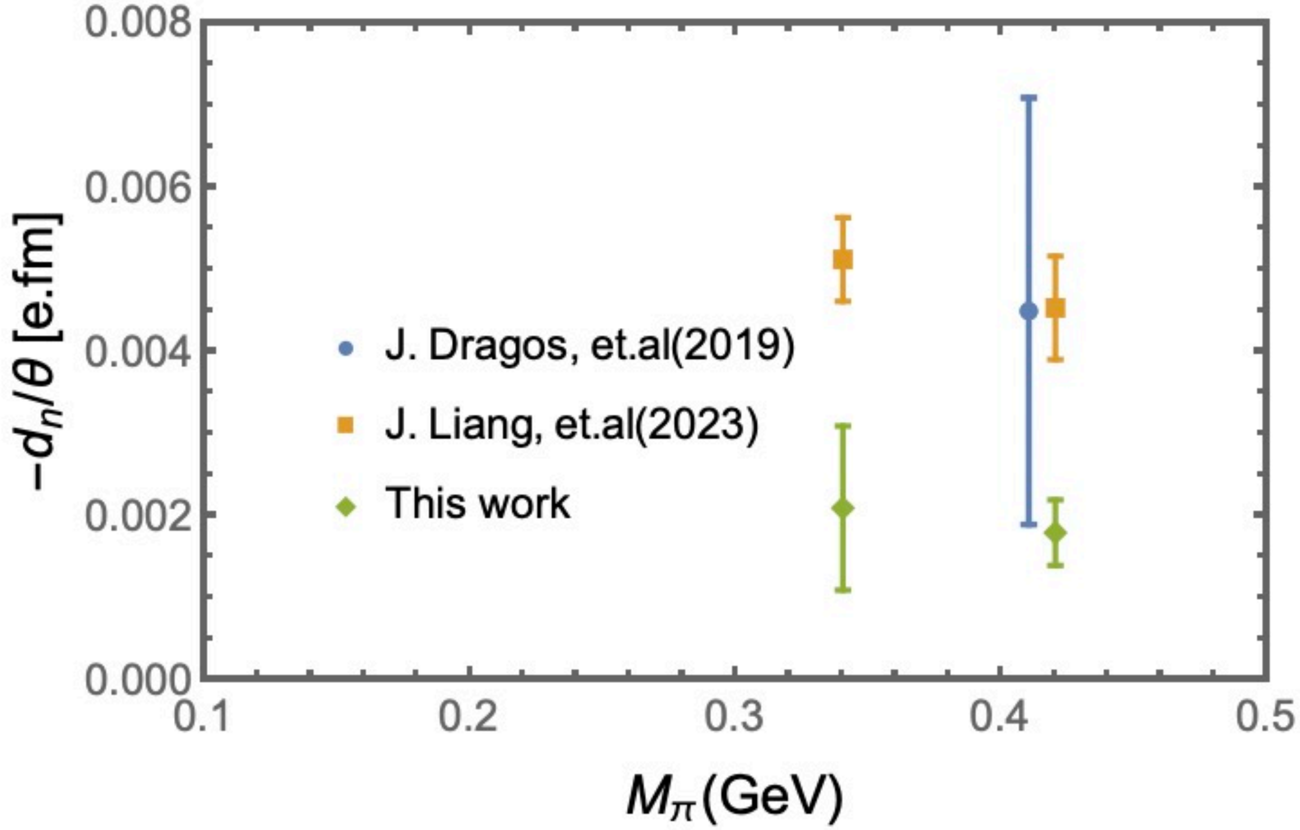
Fermionic definition



$$-\frac{d_n}{\theta} = 0.0018(4)$$

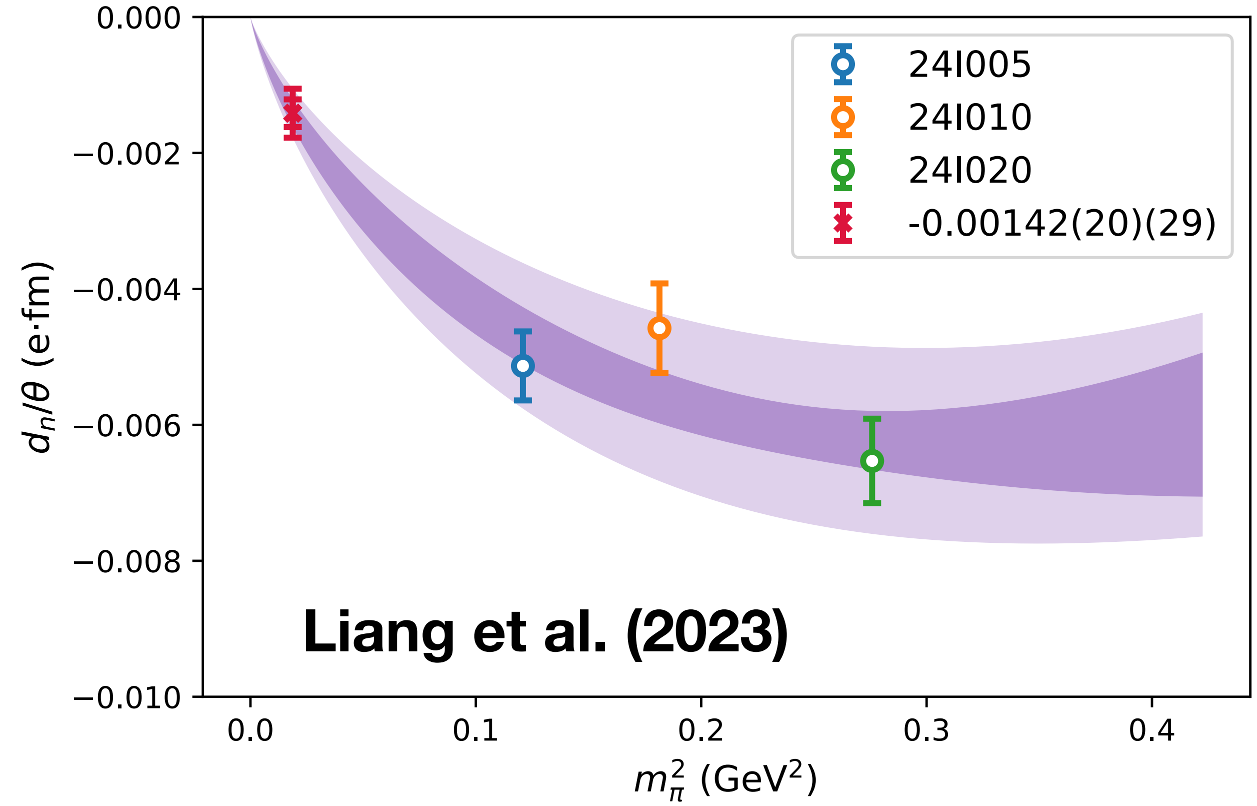
The results using the topological charge defined by quark loop have better signals.

Comparison of different groups' results

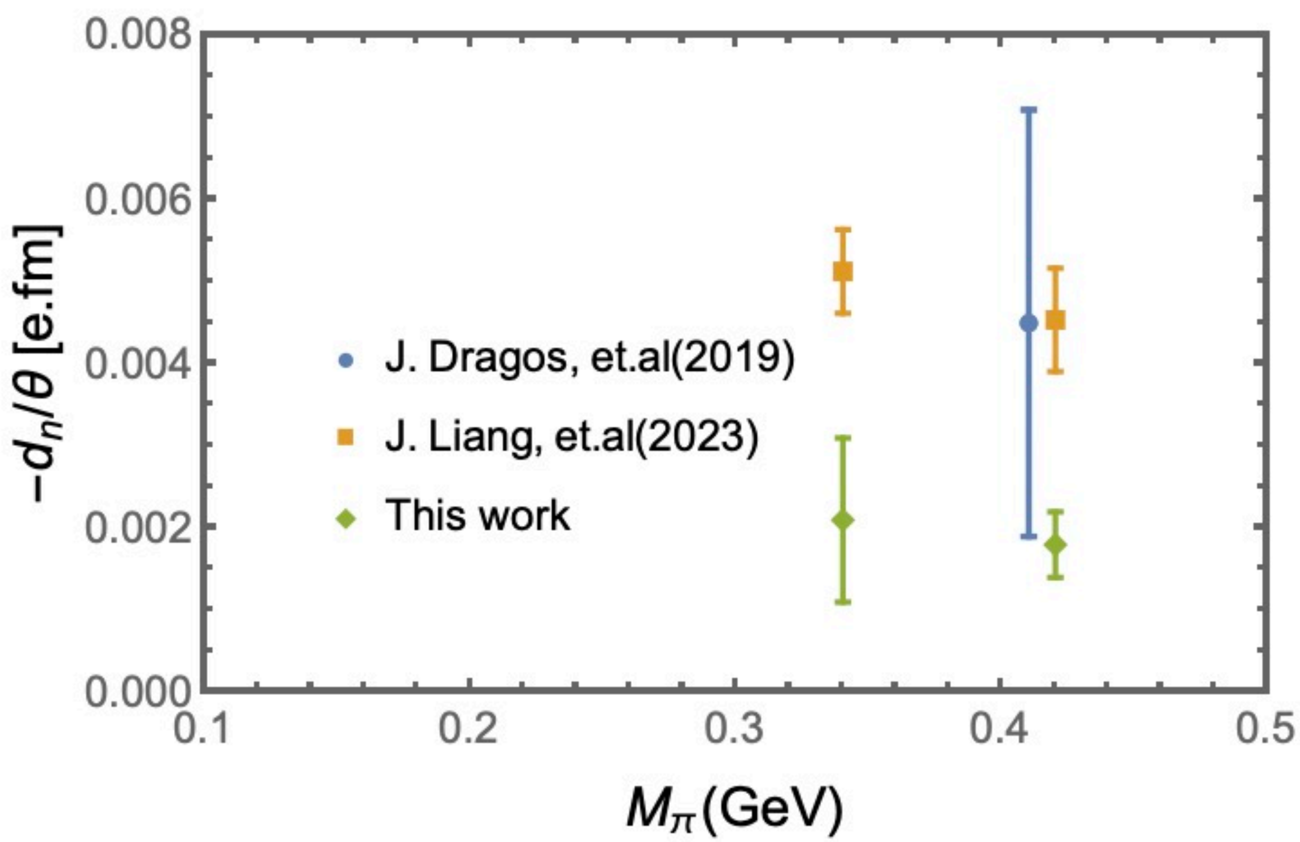


ChPT extrapolation

$$d_n = c_1 m_\pi^2 + c_2 m_\pi^2 log(m_\pi^2)$$

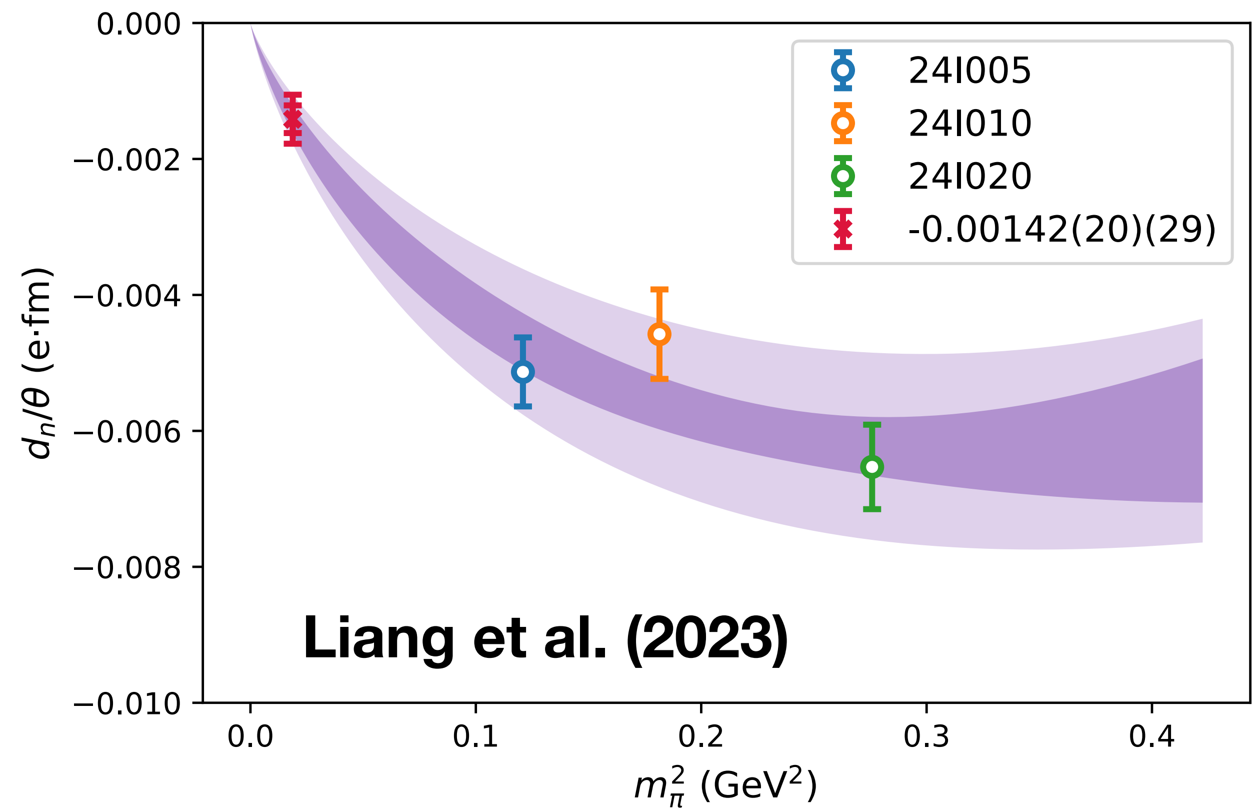


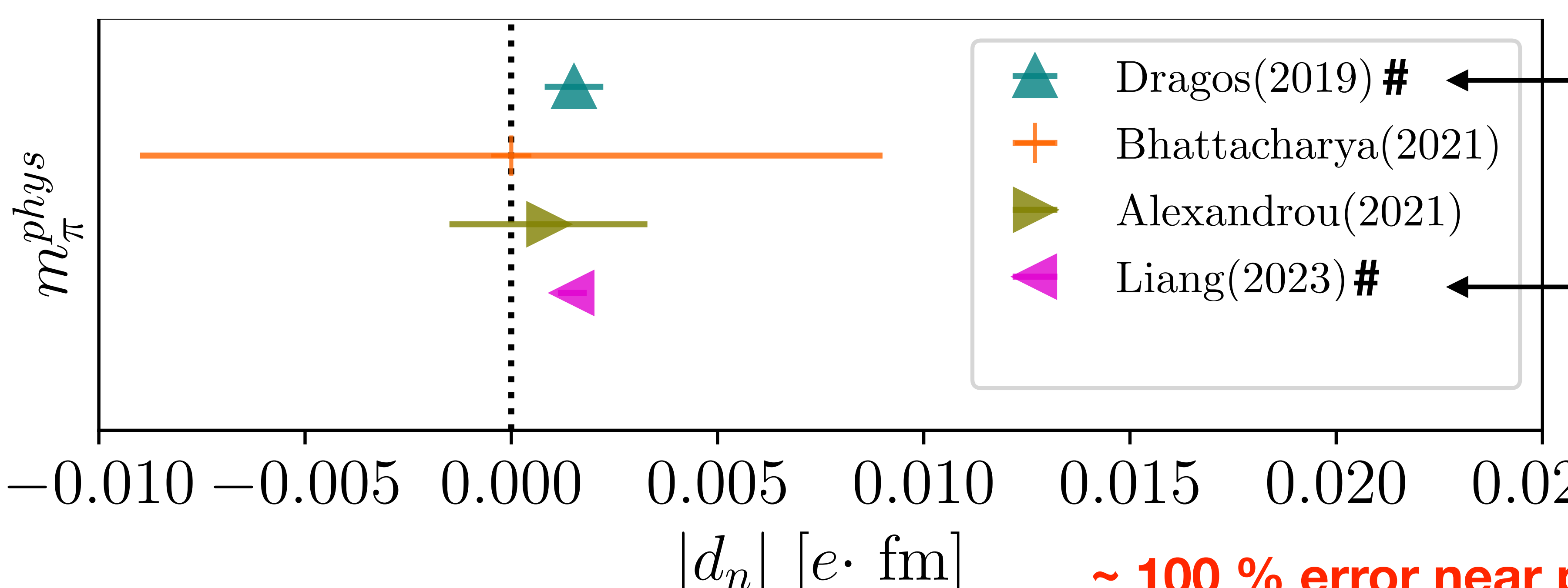
Comparison of different groups' results



ChPT extrapolation

$$d_n = c_1 m_\pi^2 + c_2 m_\pi^2 log(m_\pi^2)$$





$m_\pi \geq 410$ [MeV]

$m_\pi \geq 339$ [MeV]

 # ChPT extrapolation

~ 100 % error near physical point

Summary

- We test a new method: matrix elements with b.g. electric field, and fermionic definition of topological charge, has better signals and more stable plateau.
- So far the lattice results for neutron θ EDMs are consistent with model analyses.
- Constrain θ-nEDM at physical point is challenging, which will require order of magnitude accumulation of statistics (huge computational effort), and/or alternative noise reduction techniques (chiral basis dependent effect? c.f. [Ema et al. (2024)])

→ Constraint relaxation on θ parameter or No strong CP problem?

Thank you

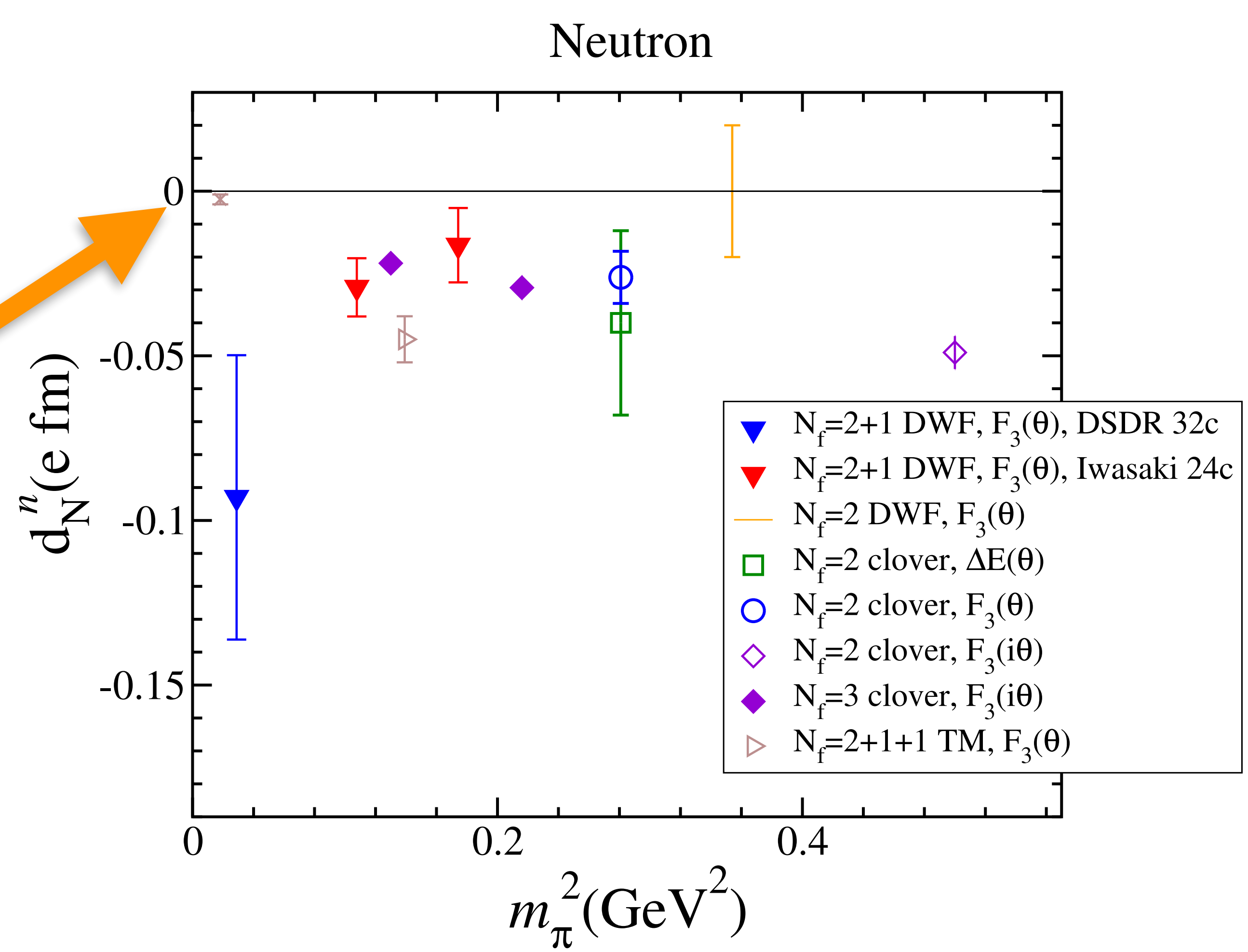
θ_{QCD} induced Nucleon EDMs

Phenomenological estimates

method	value
ChPT/NDA	~ 0.002 e fm
QCD sum rules [1,2]	0.0025 ± 0.0013 e fm
QCD sum rules [3]	$0.0004^{+0.0003}_{-0.0002}$ e fm

[1] M. Pospelov, A. Ritz, Nuclear Phys. B 573 (2000) 177,
[2] M. Pospelov, A. Ritz, Phys. Rev. Lett. 83 (1999) 2526,
[3] J. Hisano, J.Y. Lee, N. Nagata, Y. Shimizu, Phys. Rev. D 85 (2012) 114044.

Lattice calculations



[E. Shintani, T. Blum, T. Izubuchi, A. Soni, PRD93, 094503(2015)]

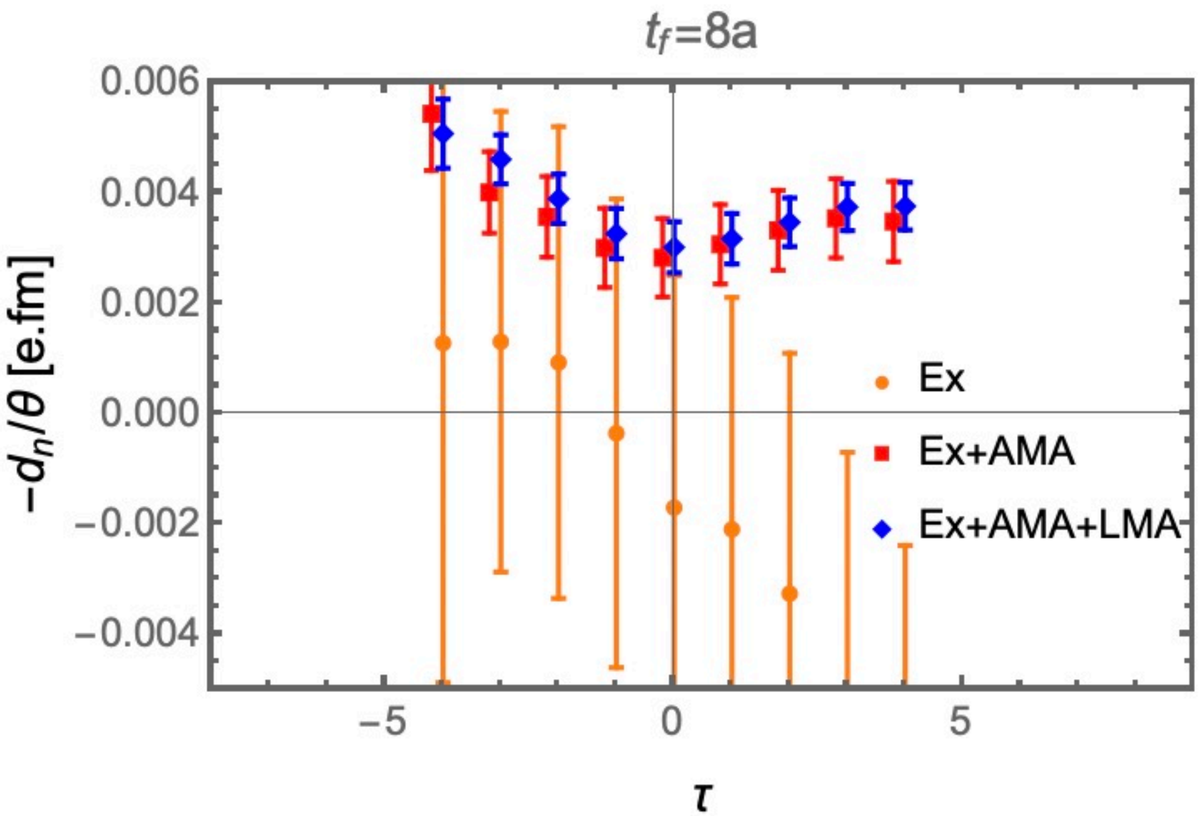
Phenomenology: |dn| ~ θ_{QCD} 10^{-3} e fm -> |θ_{QCD}| < 10^{-10}
Lattice : |dn| ~ θ_{QCD} 10^{-2} e fm -> severer constraint on |θ_{QCD}|

All mode average (AMA) and low mode average (LMA)

[T. Blum, T. Izubuchi and E. Shintani, *Phys.Rev.D* 88 (2013)]

Gauge ensembles	24I_010
2pt with exact solver	1
2pt with sloppy solver	64
low mode all to all 2pt	Volume

$$\langle \mathcal{O}_{\mathrm{AMA}} \rangle = \frac{1}{N_{\mathrm{ex}}} \langle \mathcal{O}_{\mathrm{ex}} \rangle - \frac{1}{N_{\mathrm{ex}}} \langle \mathcal{O}_{\mathrm{sl}} \rangle + \frac{1}{N_{\mathrm{sl}}} \langle \mathcal{O}_{\mathrm{sl}} \rangle \qquad (N_{sl} \gg N_{ex})$$



The signal can be significantly enhanced after using AMA and LMA.

Reanalysis of "lattice" θ induced EDM

Correction is simple: $$[F_3]_{\rm correct} = \tilde{F}_3 + 2\alpha F_2$$

Correction made by ourselves

		m_π [MeV]	m_N [GeV]	F_2	α	$\tilde{F}_3$	F_3
Ref[1]	n	373	1.216(4)	−1.50(16)	−0.217(18)	−0.555(74)	0.094(74)
Ref[2]	n	530	1.334(8)	−0.560(40)	−0.247(17)	−0.325(68)	−0.048(68)
	p	530	1.334(8)	0.399(37)	−0.247(17)	0.284(81)	0.087(81)
Ref[3]	n	690	1.575(9)	−1.715(46)	−0.070(20)	−1.39(1.52)	−1.15(1.52)
	n	605	1.470(9)	−1.698(68)	−0.160(20)	0.60(2.98)	1.14(2.98)
Ref[4]	n	465	1.246(7)	−1.491(22)	−0.079(27)	−0.375(48)	−0.130(76)
	n	360	1.138(13)	−1.473(37)	−0.092(14)	−0.248(29)	0.020(58)

Ref[1] : C. Alexandrou et al., Phys. Rev. D93, 074503 (2016),
Ref[2] : E. Shintani et al., Phys.Rev. D72, 014504 (2005).
Ref[3] : F. Berruto, T. Blum, K. Orginos, and A. Soni, Phys.Rev. D73, 054509 (2006)
Ref[4] : F. K. Guo et al., Phys. Rev. Lett. 115, 062001 (2015).

The lattice results are consistent with phenomenological estimates.

After removing spurious contributions, no signal of EDM.

How to improve the signal?

The extraction of gradient flow diffusion effect

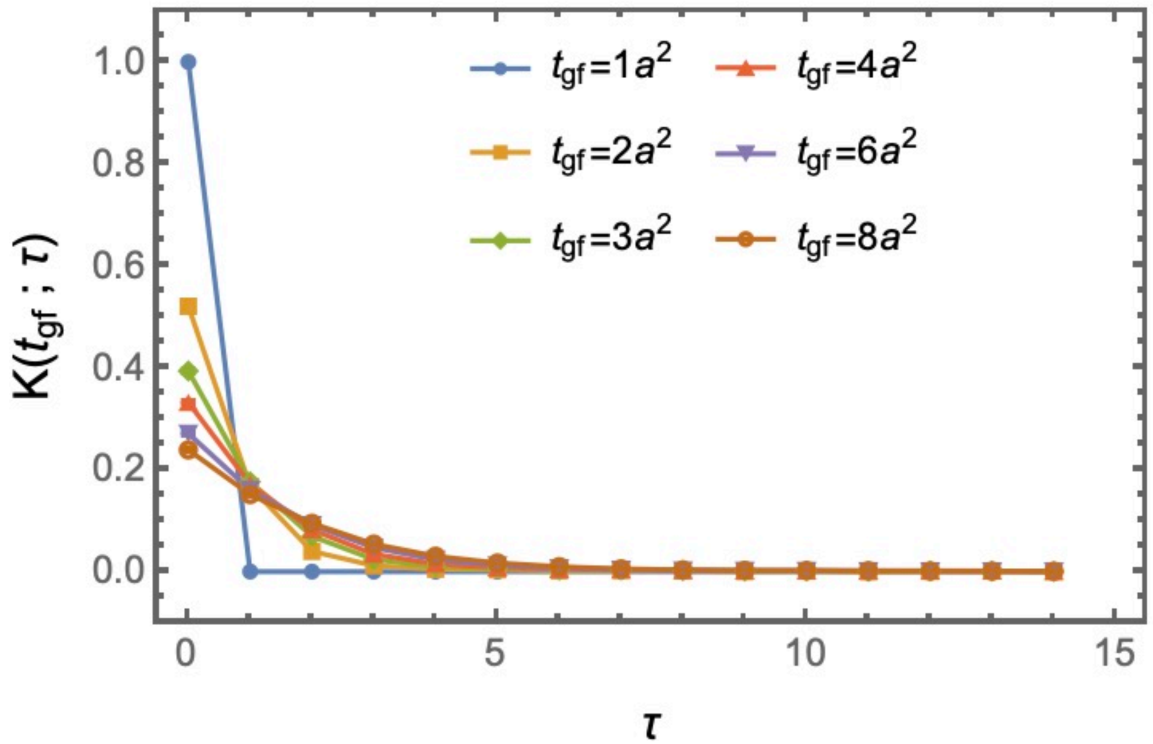
- **The diffusion effect in the gradient flow**

$$\tilde{q}(t_2^{gf};\tau) = \int dt' K(t_2^{gf} - t_1^{gf};|\tau-\tau'|)\tilde{q}(t_1^{gf};\tau') \xrightarrow{\text{Fourier transformation}} \tilde{q}(t_2^{gf};\omega) = K(t_2^{gf} - t_1^{gf};\omega)\tilde{q}(t_1^{gf};\omega)$$

The diffusion kernel can be extracted through

$$K(t_2^{gf} - t_1^{gf};\tau) = \widetilde{\mathrm{FT}}_{\omega\to t}\left[\sqrt{\frac{\mathrm{FT}_{\tau_2\to\omega}[\langle\tilde{q}(t_2^{gf};0)\tilde{q}(t_2^{gf};\tau_2)\rangle]}{\mathrm{FT}_{\tau_1\to\omega}[\langle\tilde{q}(t_1^{gf};0)\tilde{q}(t_1^{gf};\tau_1)\rangle]}}\right]$$

Diffusion kernel under gradient flow



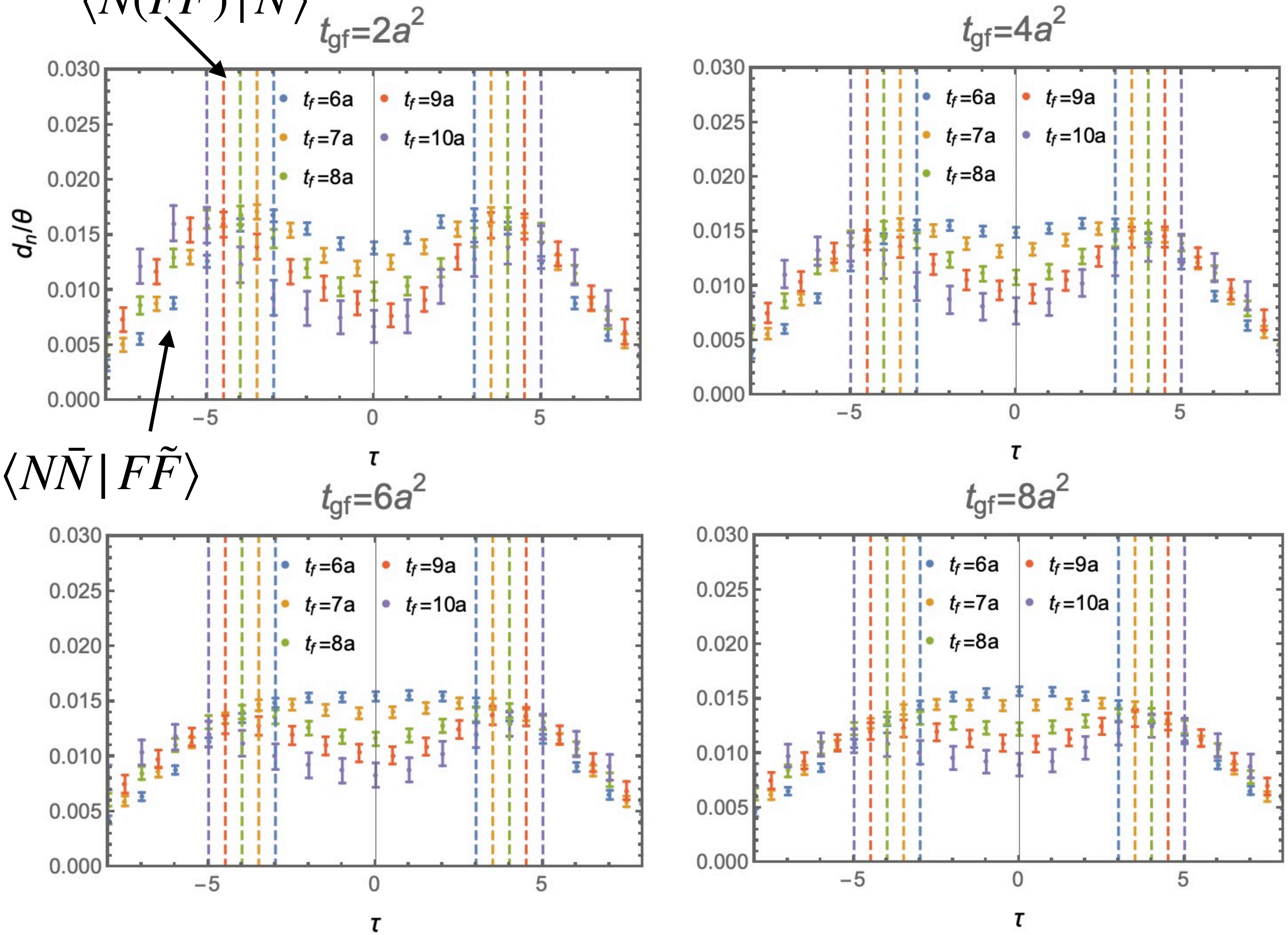
Normalization $\sum_{\tau} K(t_{gf};\tau) = 1$

The correlation length become larger with increasing t_{gf}

The correlation will be zero when $\tau > 6$

Gradient flow dependence

Contact term $\langle N(F\tilde{F})|N\rangle$



- **The noise is suppressed at larger gradient flow time.**
- **The plateau will be shifted due to the diffusion.**

Gradient flow diffusion

$$\langle \tilde{q}(\tau, t_{gf})\tilde{q}(0,t_{gf})\rangle \propto e^{-C\frac{\tau^2}{t_{gf}}}$$

$$C_3(t_2^{gf};\tau,t_{sep}) = \underline{K(t_2^{gf}-t_1^{gf};|\tau-\tau'|)} \overset{\tau'}{\otimes} C_3(t_1^{gf};\tau',t_{sep})$$

Diffusion kernel

3pt function with topological charge density in the presence of background electric field

- **Consider 3-pt functions of topological charge density**

$$\Delta C_{3pt}(\tau,\vec{\mathcal{E}}) = \langle \hat{N}(T)\bar{Q}(\tau)\hat{\bar{N}}(0)\rangle_{\vec{\mathcal{E}}}, \quad (0<\tau<T)$$

- **Performing the spectral decomposition**

$$\begin{aligned}\Delta C_{3pt}(\tau,\vec{\mathcal{E}}) &= \langle \hat{N}(T)\bar{Q}(\tau)\hat{\bar{N}}(0)\rangle_{\vec{\mathcal{E}}} \sim \sum_{n,m} e^{-E_n(T-\tau)-E_m\tau}\langle 0|\hat{N}|n,\mathcal{E}\rangle\langle n,\mathcal{E}|\bar{Q}|m,\mathcal{E}\rangle\langle m,\mathcal{E}|\hat{\bar{N}}|0\rangle \\ &= |Z_N|^2 e^{-m_N T}\boxed{\langle N_+,\mathcal{E}|\bar{Q}|N_+,\mathcal{E}\rangle} + (\text{excited states})\end{aligned}$$

$|N_+,\mathcal{E}\rangle$ **: ground state nucleon in the presence of b.g. electric field**

This matrix element can be non-zero due to non-zero electric field, which corresponds to the energy shift (δE)

$$\boxed{\langle N_+,\mathcal{E}|\bar{Q}|N_+,\mathcal{E}\rangle = \delta E = d_n \times \vec{\Sigma}\cdot\vec{\mathcal{E}}}$$

c.f. 1st order energy correction in the perturbation theory of quantum mechanics

$$\hat{H} = \hat{H}_0 + \delta\hat{H}, \qquad \delta E_n = \langle n|(\delta\hat{H})|n\rangle$$

More details on Perturbative analysis with background electric field

[G. Baym, and H. Beck, PNAS 7438, 113, 27, 2016]

- **State mixing due to electric field (without CP-odd operator)**

$$|N_+, \mathcal{E}\rangle = |N_+, 0\rangle + \vec{\mathcal{E}} \cdot \vec{D} |N_-, 0\rangle + \cdots$$

$$\vec{D} = \frac{e}{2m_{N-}(m_{N-} - m_{N+})} \int dx^3 \vec{x} \langle N^-, 0 | \rho_{EM}(x) | N^+, 0\rangle$$

(expectation value of the dipole operator)

$$\rho_{EM} = \frac{2}{3}\bar{u}\gamma_0 u(x) - \frac{1}{3}\bar{d}\gamma_0 d(x)$$

(c.f. 1st order state mixing in quantum mechanics)

Ground state nucleon (originally P-even) can mix with the negative parity nucleon (N-)

$$\langle N_+, \mathcal{E} | \bar{Q} | N_+, \mathcal{E}\rangle = \langle N_+, 0 | \bar{Q} | N_+, 0\rangle$$

← zero due to P Sym.

$$+ \vec{\mathcal{E}} \cdot \vec{D} \langle N_+, 0 | \bar{Q} | N_-, 0\rangle + (c.c.) \rightarrow d_n \vec{\mathcal{E}} \cdot \vec{\Sigma}$$

CP-even quantity → Non-zero VEV

(Same as mixing angle α)

EDM : interplay of both electric field (P-odd state mixing) and CP-odd matrix element (energy splitting) in 1-st order perturbation