"(-1)-form symmetry and its relation to Strong CP problem"
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Spontaneously Broken *(-1)-Form U(1)* Symmetries

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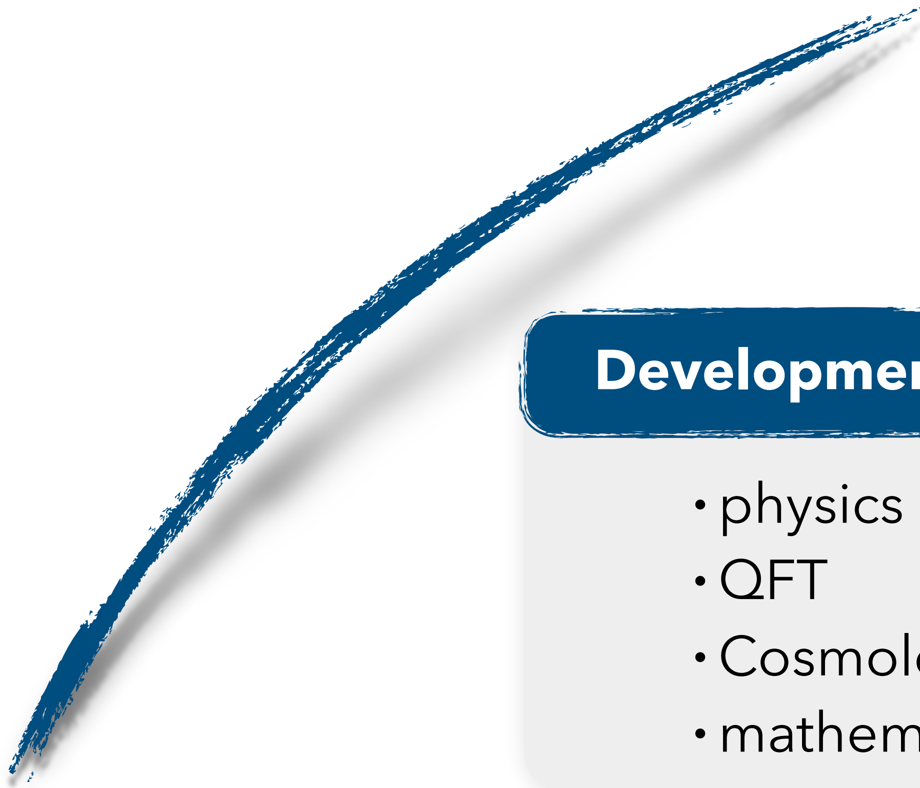
@The Axion Quest (2024)

My Challenge

High energy theory (unified theory)

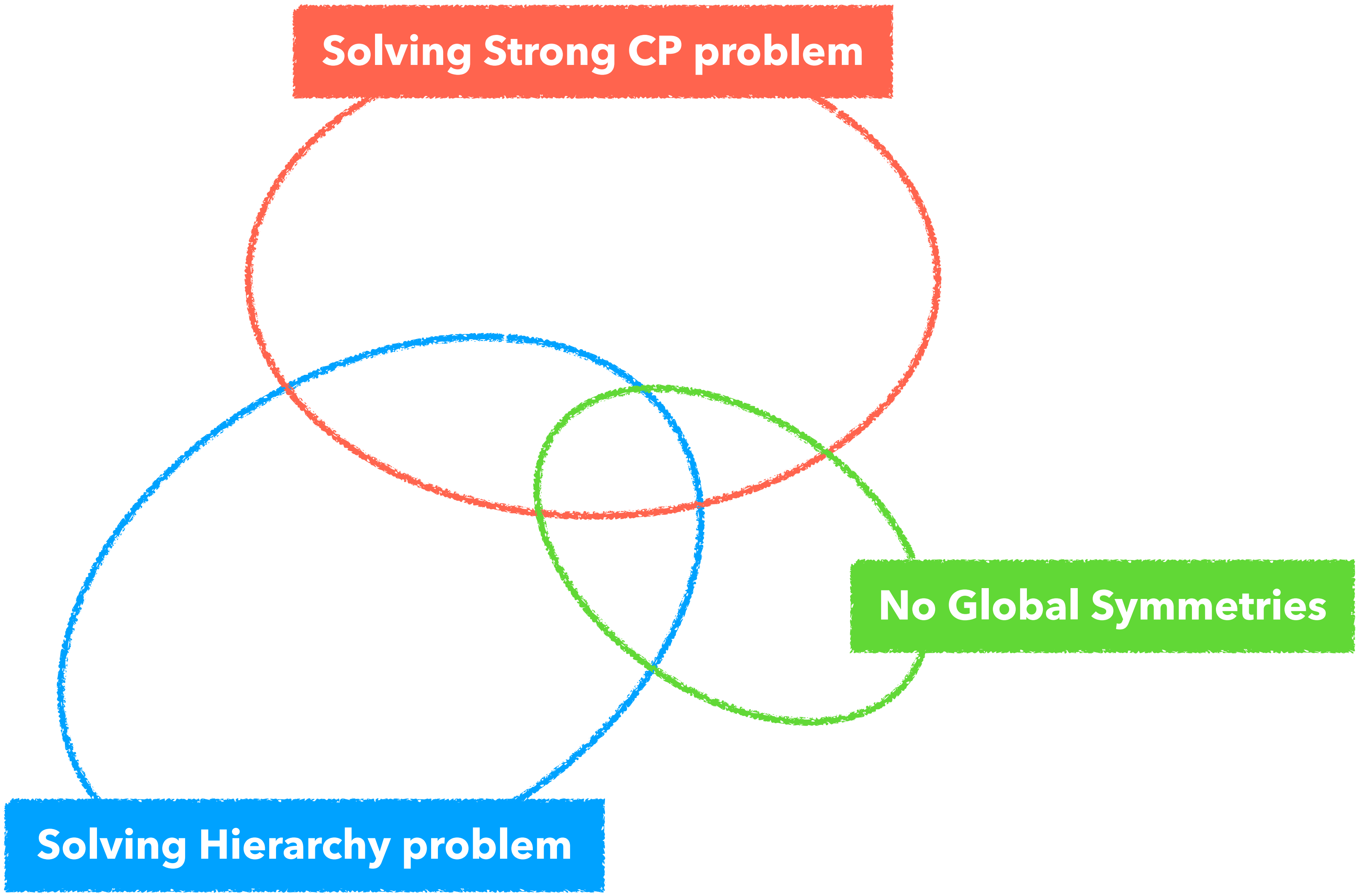


Developments in various fields

- physics including gravity
- QFT
- Cosmology
- mathematics

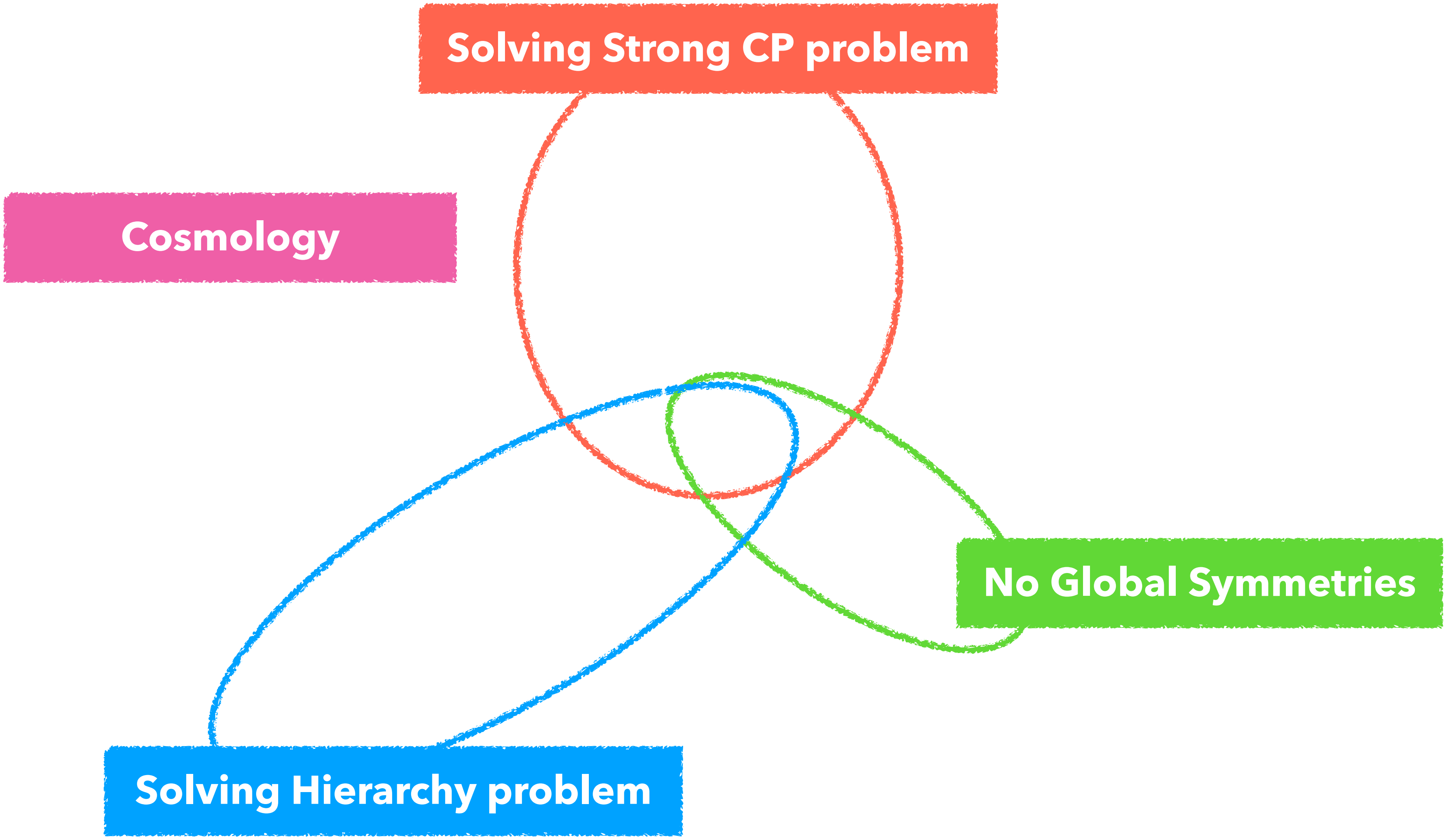
Low energy theory

My strategy



Solving Strong CP problem
No Global Symmetries
Solving Hierarchy problem

My strategy


Solving Strong CP problem
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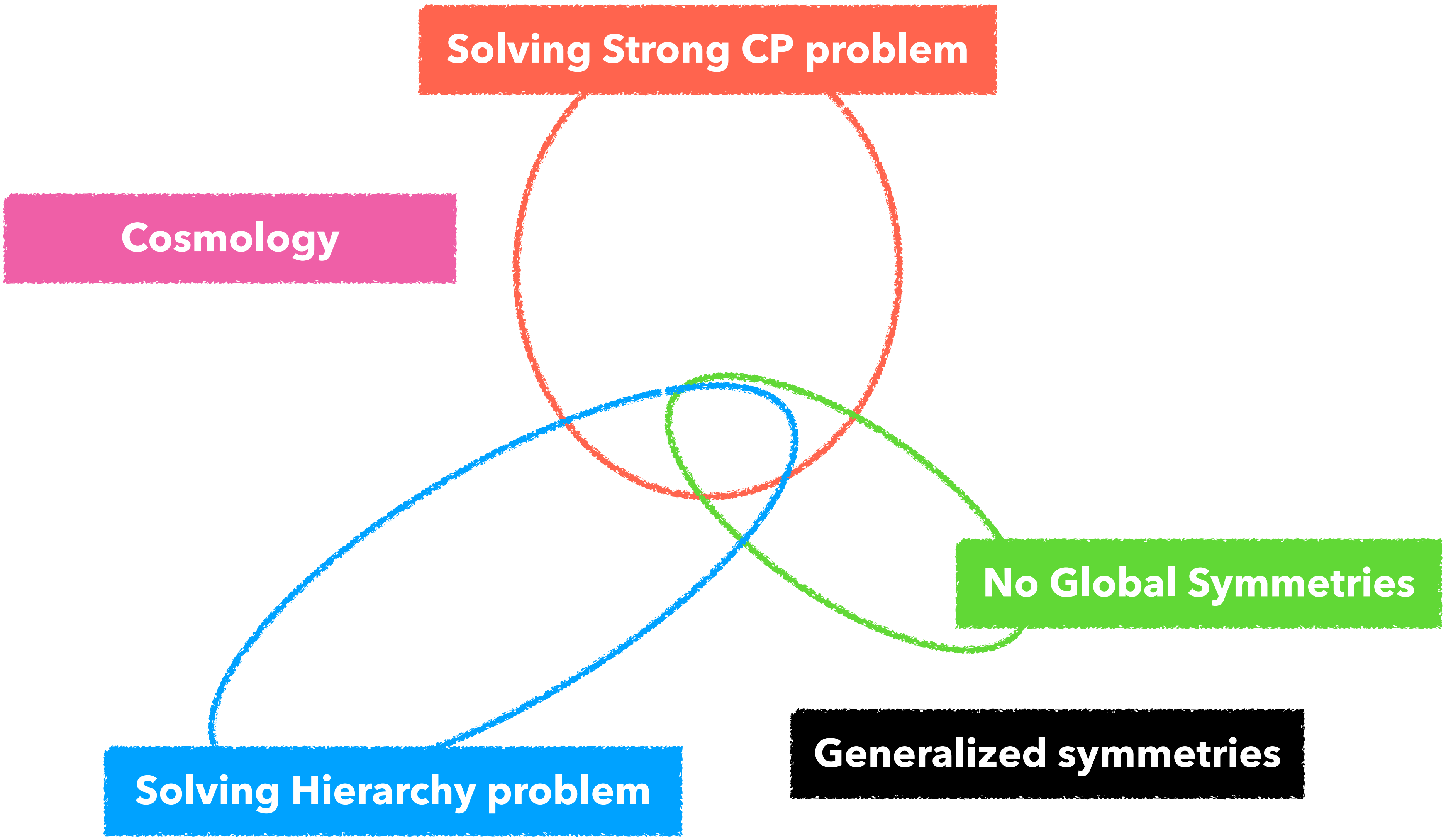
Solving Strong CP problem
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No Global Symmetries
Generalized symmetries
Solving Hierarchy problem

Strong CP problem

Strong CP problem

- QCD action:

$$S_{\rm QCD} = \int d^4x \left[\mathcal{L}_{quarks} - \theta \frac{g^2}{32\pi^2} G^{a\mu\nu} \tilde{G}^a_{\mu\nu} \right]$$

- QCD vacuum angle:

$$\bar{\theta} = \theta - \arg(\det M_u \cdot \det M_d)$$

- Experimental upper bound on the angle:

$$\bar{\theta} \lesssim 10^{-10}$$ '06 Baker *et.al*.

How to explain the smallness?

Strong CP problem

$$\bar{\theta} \lesssim 10^{-10}$$

- CP symmetry is maximally broken by CKM phase:

$$\delta_{\rm CKM} \sim 1$$

- The challenge for model building:

O(1) CP phase quark mass matrices

↓

$$|\arg(\det M_u \cdot \det M_d)| \sim 1$$

↓

$$\bar{\theta} \gg 10^{-10}$$ **Some mechanism needs!**

Solution to Strong CP problem

Axion: a dynamical solution to Strong CP problem

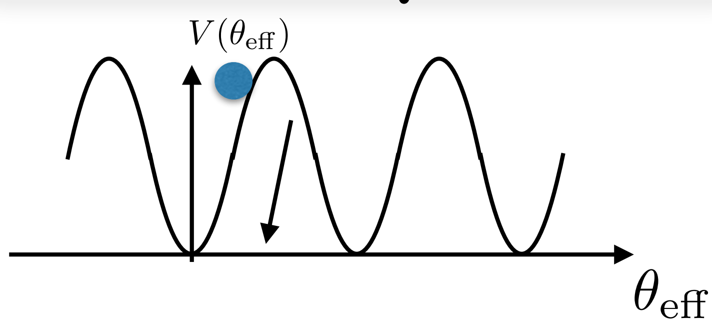
- θ-parameter becomes a dynamical field θeff

$$\theta \to \theta_{\rm eff}$$ (axion field)

$$\mathcal{L}_\theta = -\theta \frac{g^2}{32\pi^2} G^{a\mu\nu}\tilde{G}^a_{\mu\nu} \quad \to \quad \mathcal{L} = -\theta_{\rm eff} \frac{g^2}{32\pi^2} G^{a\mu\nu}\tilde{G}^a_{\mu\nu}$$

- Axion obtains potential by non-perturbative QCD effect

$$V(\theta_{\rm eff}) = -\Lambda^4_{\rm QCD}\cos(\theta_{\rm eff})$$



- At the potential minimum, Strong CP problem is solved!

$$\langle\bar{\theta}\rangle = 0$$

A concrete axion model

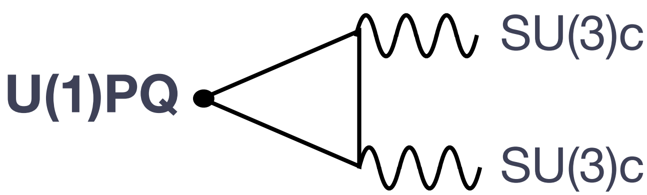
- KSVZ model ’79 Kim, ’80 Shifman, Vainshtein, Zakharov

$$\mathcal{L} \ni \Phi q_L \bar{q}_R$$ (extra quarks)

- PQ symmetry

$$\Phi \to e^{i\alpha}\Phi, \;\; q_L \to e^{-i\alpha} q_L, \;\; \bar{q}_R \to \bar{q}_R$$

- U(1)PQ - SU(3)c- SU(3)c triangle anomaly is nonzero



$$\neq 0 \;\leftrightarrow\; \partial j_{PQ} = \frac{g^2}{32\pi^2} G\tilde{G}$$

- SSB of U(1)PQ gives axion

$$\Phi \ni \frac{1}{\sqrt{2}} f_a e^{i\frac{a}{f_a}}$$

a: axion field

f_a: axion decay constant

$$\langle \Phi \rangle \neq 0 \to \frac{a}{f_a}\frac{g^2}{32\pi^2} G\tilde{G}(x)$$

Gravity may break PQ symmetry badly

- If physics at Planck scale **explicitly** breaks PQ symmetry

$$\mathcal{L} \ni \sum_{k=1,2,\dots} \left(\lambda_k \frac{\Phi^{k+4}}{M_{\rm pl}^k} + \tilde{\lambda}_k \frac{|\Phi^{2k}|\Phi}{M_{\rm pl}^k} + h.c. + \dots \right)$$

$\lambda_k, \ \tilde{\lambda}_k$: dimensionless couplings

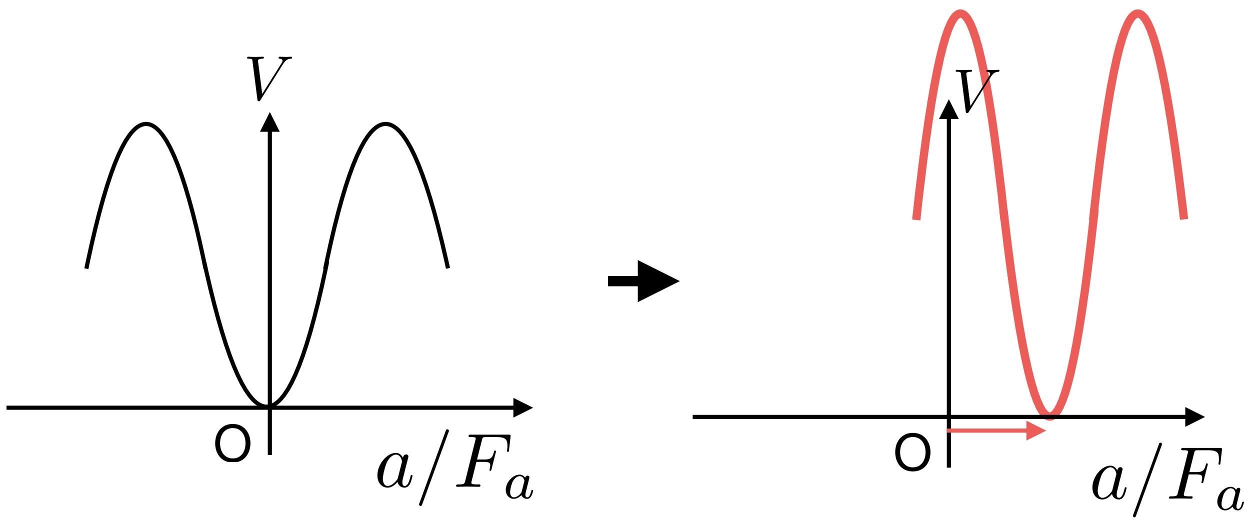
Axion potential is affected by Planck mass suppressed terms

$$V \sim -m_a^2 f_a^2 \cos\left(\frac{a}{f_a}\right) + \left(\lambda_1 \frac{(f_a/\sqrt{2})^5}{M_{\rm pl}} e^{i5a/f_a} + h.c. \right) + \dots$$

original axion potential

Gravity may break PQ symmetry badly

$$V(\theta_{\rm eff}) = -\Lambda_{\rm QCD}^4 \cos(a/F_a) + V_{\rm gravity}$$



$\langle\bar{\theta}\rangle \sim 1$ without very very tiny parameters

Quality problem in axion model

More quantitatively,
Shift of axion VEV should be smaller than 10^{-10}

$$V(a) \sim m_a^2 a^2 + |\lambda_1| \frac{f_a^5}{\sqrt{2}^5 M_{\rm pl}} \left(\left(\frac{5a}{f_a} \right)^2 + \frac{5a}{f_a}\delta \right) + \dots$$

$$\sim \quad \Lambda_{QCD}^4 \frac{a^2}{f_a^2} + |\lambda_1| \frac{f_a^4}{\sqrt{2}^5 M_{\rm pl}} \frac{5a}{f_a}\delta$$

$$\sim \quad \Lambda_{QCD}^4 \left(\frac{a}{f_a} + |\lambda_1| \frac{f_a^4}{\sqrt{2}^5 M_{\rm pl} \Lambda_{QCD}^4} \frac{5}{2}\delta \right)^2$$

$$|\Delta\theta| = | \left\langle \frac{a}{f_a} \right\rangle | = |\lambda_1| \frac{f_a^5}{\sqrt{2}^5 M_{\rm pl} \Lambda_{QCD}^4} \frac{5}{2}\delta < 10^{-10}$$

Couplings must be extremely tiny

$$|\lambda_1| < 10^{-56} \left(\frac{10^{12}\ GeV}{f_a} \right)^5 \left(\frac{\Lambda_{QCD}}{0.1\ GeV} \right)^4 \quad \text{for } \delta = O(1)$$

Axion Quality Problem !!

Addressing axion quality problem

- Mechanism only allows highly Planck mass suppressed terms

$$V \sim \frac{\Phi^{12}}{M_{\mathrm{PL}}^8} + \frac{\Phi^{13}}{M_{\mathrm{PL}}^9} + \dots$$

e.g. using (new) gauge symmetry

- Mechanism suppresses coefficients

$$V \sim \lambda \frac{\Phi^5}{M_{\mathrm{PL}}} + \dots \;, \lambda \ll 1$$

e.g. conformal dynamics

- Mechanism gives large axion mass around thetabar=0

$$V \sim \frac{1}{2} m_a^2 a^2 + \lambda \frac{\Phi^5}{M_{\mathrm{PL}}} \dots \;, m_a^2 \gg \frac{\Lambda_{\mathrm{QCD}}^4}{f_a^2}$$

e.g. visible axion model

- No axion (another quality problem?)

e.g. Nelson-Barr model

Our models to address quality problem

“Gauged” PQ symmetry

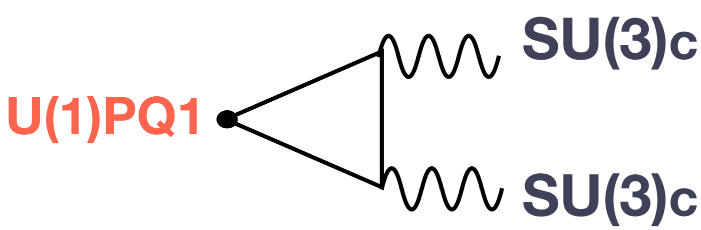
’17 H. Fukuda, M. Ibe, M.S., T. T. Yanagida

Solving quality problem: protecting axion by gauge symmetry

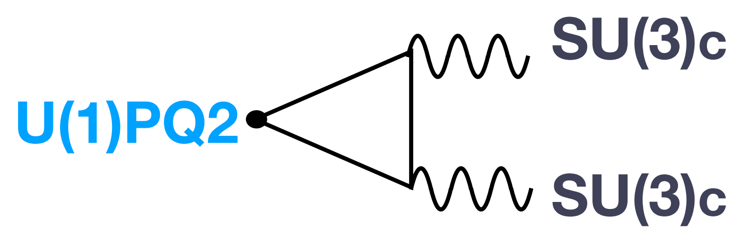
- Introducing two U(1)PQ sectors

U(1)PQ1 sector

U(1)PQ2 sector



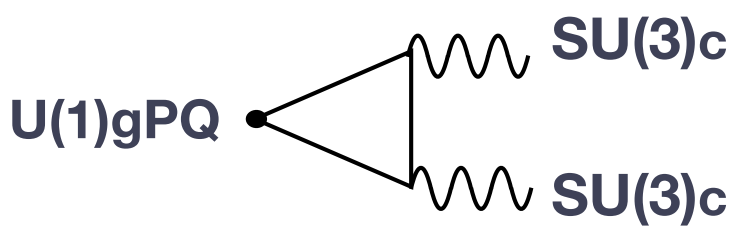
$\neq 0$



$\neq 0$

- A linear combination of two U(1)PQ’s cancel the anomaly and is gauged

$$U(1)_{gPQ} \equiv c_1 U(1)_{PQ_1} + c_2 U(1)_{PQ_2}$$



$= 0$

Cosmology of “Gauged” PQ symmetry

Main difference: SSBs of *two* PQ symmetries

We have two scenarios that may work

Pre-inflationary scenario

Time

- SSB of U(1)PQ1
- SSB of U(1)PQ2
- **End of inflation**
- Reheating
- T~ΛQCD, axion oscillation

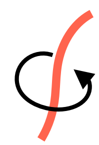
≈ standard pre-inflatinary scenario

Post-inflationary scenario

Time

- **End of inflation**
- Reheating
- SSB of U(1)PQ1

Global string1

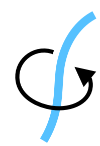


θ1

$\Phi_1 \sim v_1 e^{iq_1\theta_1}$

- SSB of U(1)PQ2

Global string2



θ2

$\Phi_2 \sim v_2 e^{iq_2\theta_2}$

- T~ΛQCD, axion oscillation

Domain wall problem

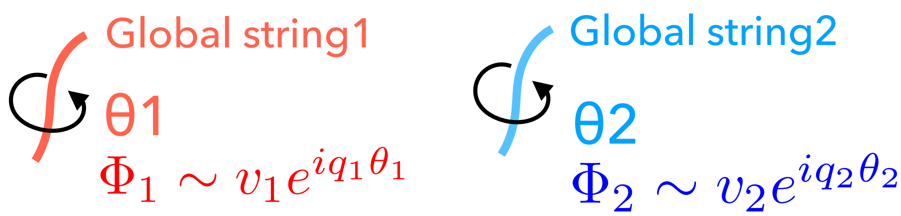
Solution to DW problem?

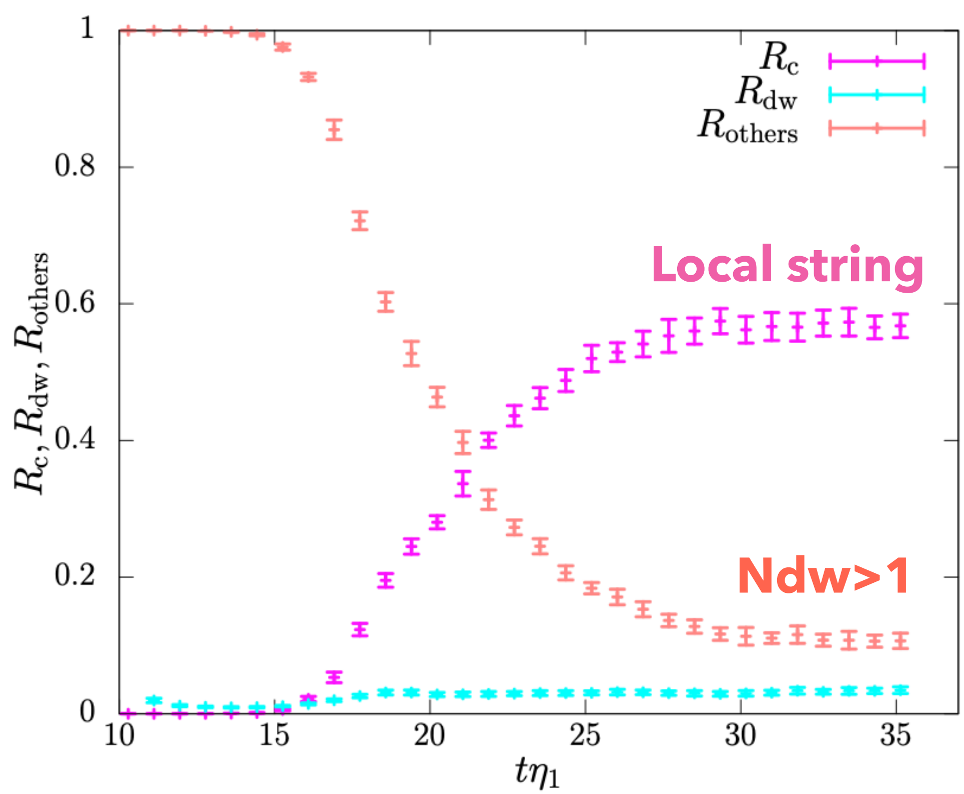
'19 '20 T. Hiramatsu, M. Ibe, M. Suzuki

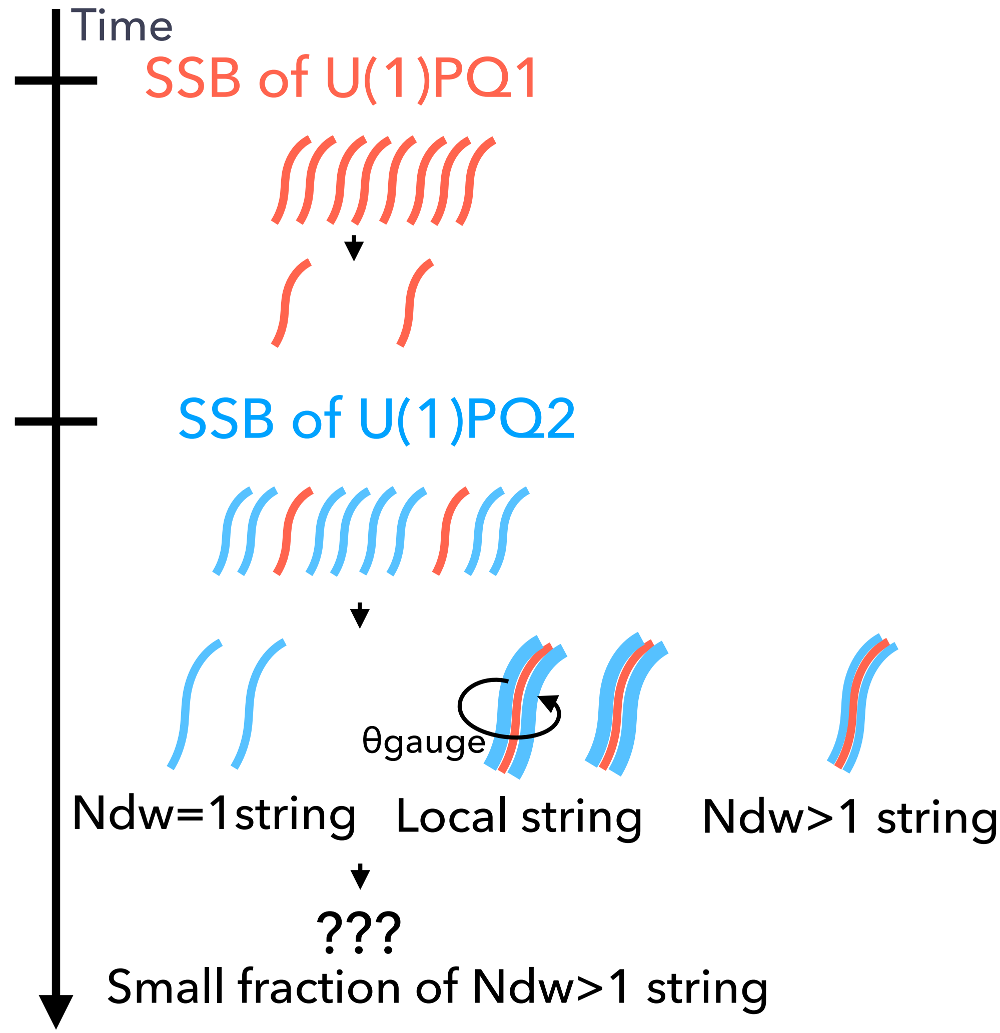
Special condition

$v_1/v_2 \gg 1$

$q_1 = 1,\ q_2 > 1$







Spontaneous CP violation model

'84 Nelson, '84 Barr

Energy scale

CP symmetric

$$\delta_{\rm CKM} = 0, \;\; \bar{\theta} = 0$$

SSB of CP symemtry

~~CP~~

$$\delta_{\rm CKM} \sim 1, \;\; \bar{\theta} \approx 0$$

✅ Tree level

✅ Loop levels

Nelson-Barr type model

- A simple model '91 L. Bento, G. C. Branco, and P. A. Parada

SM quark sector: $Q_f,\ \bar{u}_f,\ \bar{d}_f$

η_a, $\arg\langle\eta_a\rangle \neq 0$

vector-like quarks: $q,\ \bar{q}$

$$\mathcal{L} \supset + \sum_{f,f'} Y^d_{f,f'} H Q_f \bar{d}_{f'} + \sum_{a,f} a^d_{af} \eta_a q \bar{d}_f + \mu q \bar{q}$$

- The theta angle is zero at tree level:

$$\bar{\theta} = \theta - \arg(\det \hat{M}_u \cdot \det \hat{M}_d) = 0$$

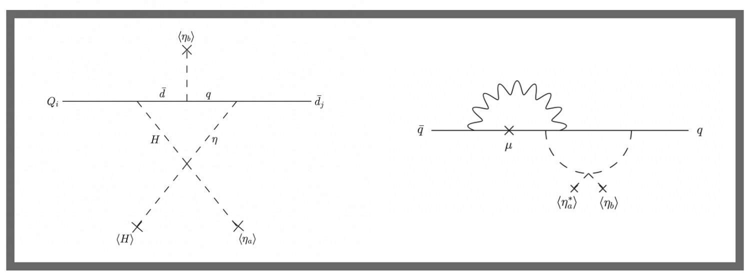
- Dangetous terms:

$$\sum_f H Q_f \bar{q} + \sum_a \eta_a q \bar{q} \qquad \frac{\eta_b^*}{\Lambda} \eta_a q \bar{q} + \frac{\eta_a^*}{\Lambda} H Q \bar{q}$$

$$\mathcal{L} \supset \gamma_{ab} \eta_a^* \eta_b H^\dagger H + \gamma_{abcd} \eta_a \eta_b \eta_c^* \eta_d^* + \text{h.c.}$$

+ flavor sym violating terms

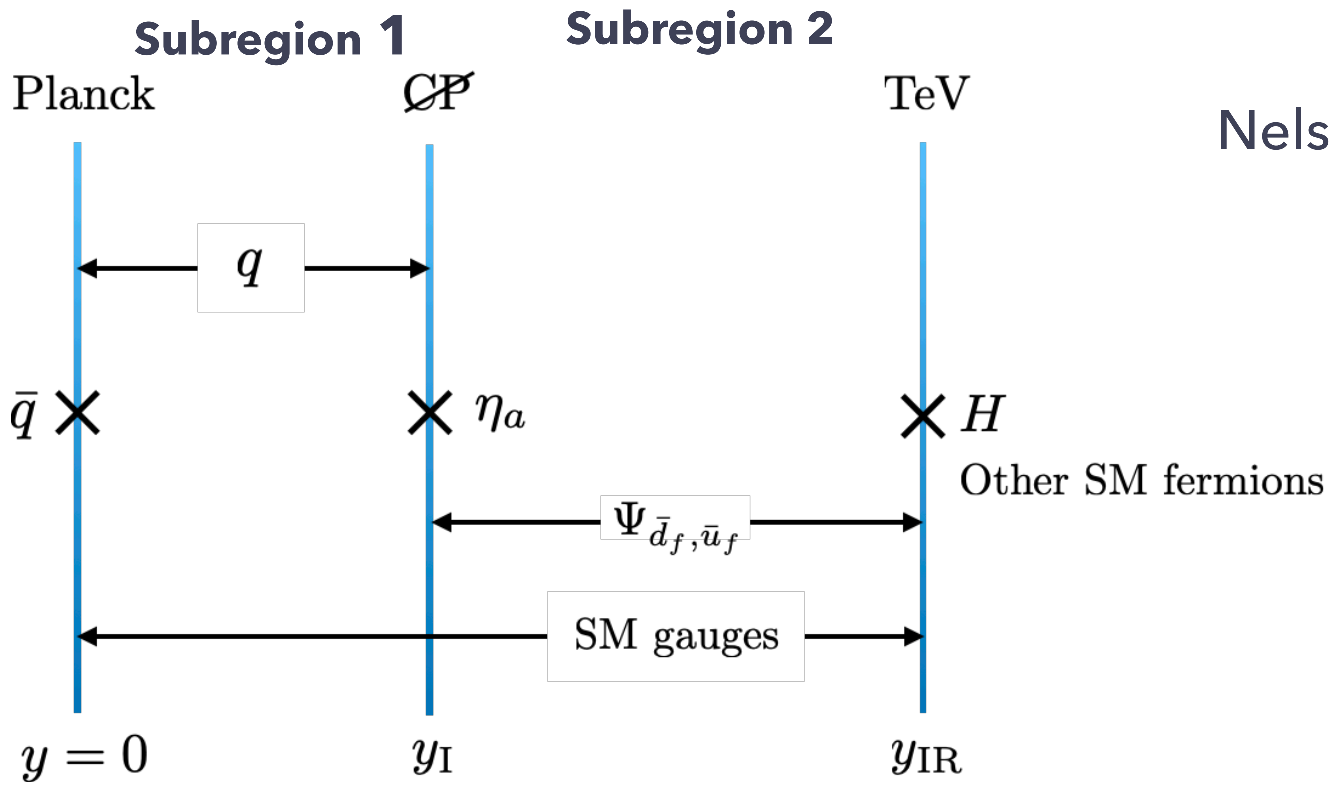
Radiative corrections



'15 Dine&Draper

A natural model

'22 S. Girmohanta, S. J. Lee, Y. Nakai, M.S.



Nelson-Barr, warped extra dim.

Three brane setup

Higgs (SM fields) : IR
η: intermediate
q: subregion 1
q: UV brane
$\overline{\rm u}$ and $\overline{\rm d}$: subregion 2

✅ 5d profiles → avoid dangerous term, small corrections

✅ Hierarchy problem

✅ Flavor structures

✅ radion stabilization in three branes

'21 S. J. Lee, Y. Nakai, M.S.

✅ cosmology, GWs production

'23 S. Girmohanta, S. J. Lee, Y. Nakai, M.S.

“Just a curiocity”

Another view of Strong CP problem or its solutions?

Generalized symmetries, (-1)-form symmetry

Understanding Strong CP problem in a different manner?

New way to solve Strong CP problem?

Replacing the Strong CP problem to another problem?

Review: higher-form symmetries

Generalized symmetries

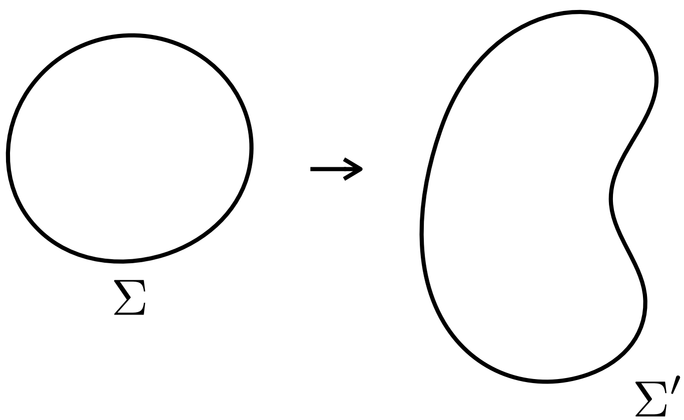
'14 A. Kupstin, N. Seiberg
'14 D. Gaiotto, A. Kapustin, N. Seiberg, and B. Willett

Generalization of notion of symmetry

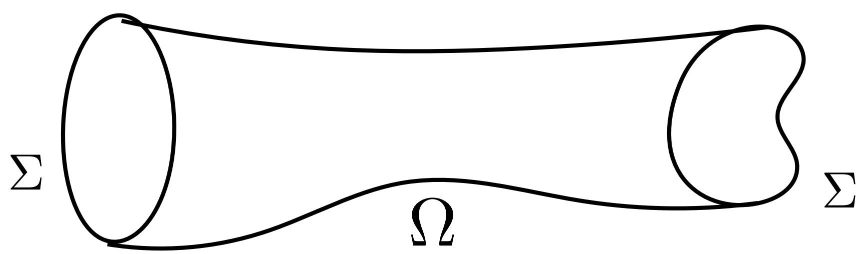
symmetry generator ~ **topological operator**

$$U = \exp(i\alpha \int_{\Sigma} \star j) \to \exp(i\alpha \int_{\Sigma'} \star j) = \exp(i\alpha \int_{\Sigma} \star j + i\alpha \int_{\Omega} d \star j)$$

$$= \exp(i\alpha \int_{\Sigma} \star j)$$



$$\Sigma' - \Sigma = \partial\Omega$$



Generalized symmetries

- 0-form symmetry (ordinary symmetry)

$$U(t) = \exp(i\alpha \int J^0 d^3x)$$

$$U(t) = U(t')$$

$$U(\Sigma_3(t)) = U(\Sigma_3(t'))$$

$$\Sigma_3(t') - \Sigma_3(t) = \partial \Sigma_4$$

0-form symmetry

Codimension 1, i.e. (4-1)-dim, topological operator

$$U_g(M^{(d-1)}) = \exp(i\alpha \int_{M^{(d-1)}} \star j)$$

Generalized symmetries

- p-form symmetry (p=0,1,2...)

p-form symmetry

Codimension (p+1), i.e. (4-(p+1))-dim, topological operator

$$U(\Sigma_{d-(p+1)}) = \exp(i\alpha \int_{\Sigma_{d-(p+1)}} \star j)$$

$$U(\Sigma_{d-(p+1)}) = U(\Sigma'_{d-(p+1)})$$

1-form symmetry in 4d Maxwell Theory

- d=4 Maxwell theory

$$S[a] = \int -\frac{1}{2e^2} F \wedge \star F = \int d^4x \, -\frac{1}{4e^2} F_{\mu\nu} F^{\mu\nu}$$

EOM: $d \star F = 0$

Bianchi identity: $dF = 0$

- Two conserved currents

$d \star F = 0 \rightarrow \star j_e = \star F$

$dF = 0 \rightarrow \star j_m = F$

- Two symmetry generators (topological operators)

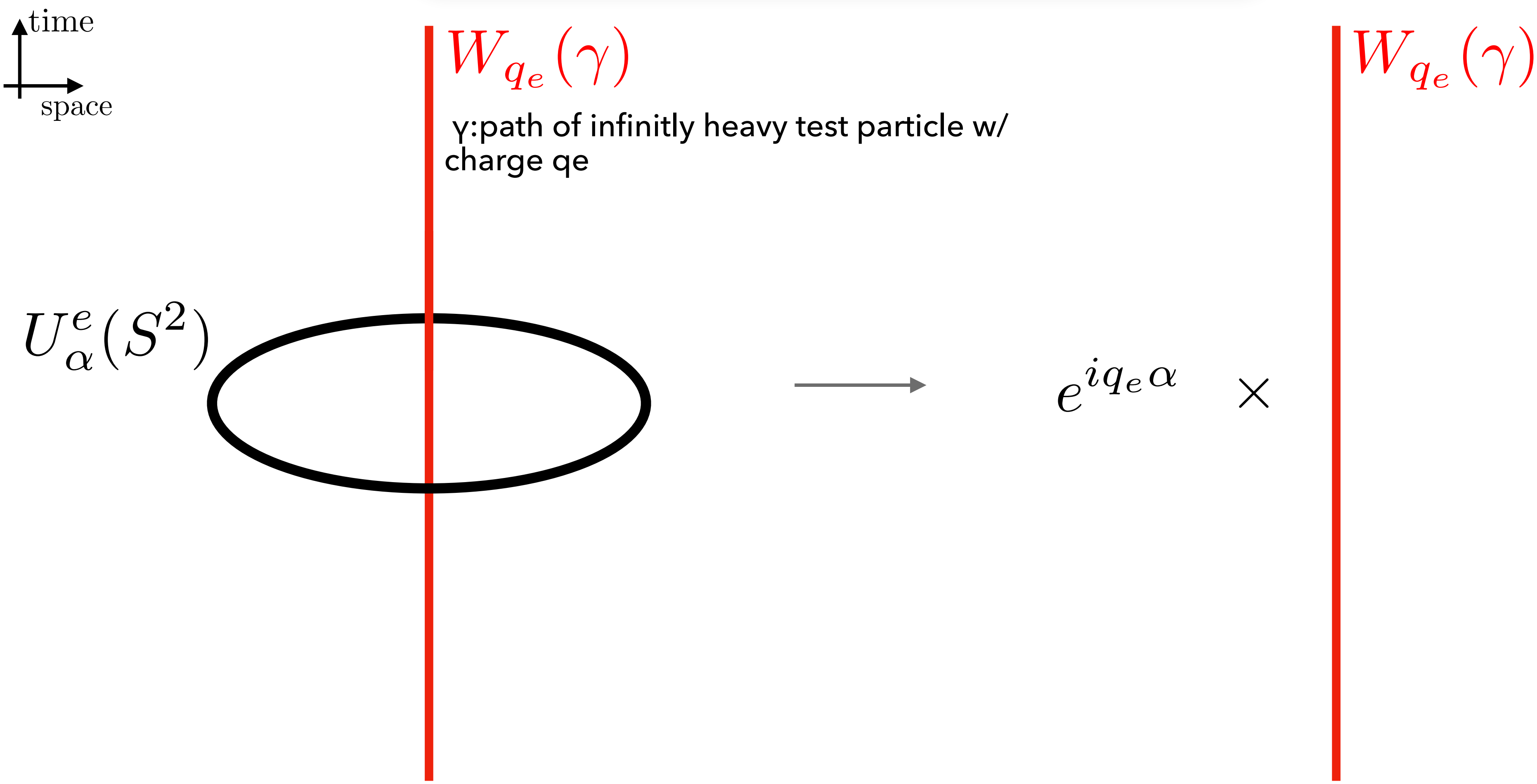
$$U_g^e(S^2) = \exp(i\alpha \int_{S^2} \star j_e) \qquad U_g^m(S^2) = \exp(i\alpha \int_{S^2} \star j_m)$$

1-form symmetry in 4d Maxwell Theory

- Charged operators are Wilson-line and 't Hooft line operators

$$U^e_\alpha(S^2)W_{q_e}(\gamma) = e^{iq_e\alpha}W_{q_e}(\gamma)$$

$$U^m_{\alpha'}(S^2)T_{q_m}(\gamma') = e^{iq_m\alpha'}T_q(\gamma')$$



1-form symmetry in 4d Maxwell Theory

- Gauging electric 1-form symmetry

$$A \to A + \Omega(x)$$
$$F \to F + d\Omega(x)$$
$$B_e \to B_e + d\Omega(x)$$

$$\Rightarrow \int (F - B_e) \wedge \star (F - B_e)$$

- SSB of electric 1-form symmetry

  Stuckelberg mass term, gauge fields obtain mass

(-1)-form symmetry

(-1)-form symmetry?

- p-form symmetry generator

$$U(\Sigma_{d-(p+1)}) = \exp(i\alpha \int_{\Sigma_{d-(p+1)}} \star j)$$

d=4, **p=-1** ↓

- (-1)-form symmetry generator??

$$U(\Sigma_4) = \exp(i\alpha \int_{\Sigma_4} \star j) \ ?$$

– not topological operator (whole spacetime integration)

– no (or we do not know) charged operators

Not the same to zero or higher-form symmetry

Another way to define (-1)-form symmetry?

– coupling with background gauge field

(-1)-form U(1) symmetry

Theory has **(-1)-form U(1) symmetry** when $\star j_0(x)$ can linearly couple to **periodic background field** $\theta(x)$

D. Aloni, E. García-Valdecasas, M. Reece, M. S.

$$e^{-S_E} \mapsto e^{-S_E} \exp(i \int_M \theta(x) \star j_0(x))$$

$$\theta(x) \cong \theta(x) + 2\pi$$

$$M : \text{spacetime manifold}$$

- $\star j_0(x)$ (-1)-form U(1) symmetry current
- $\int_M \star j_0(x) \in \mathbb{Z}$ (-1)-form symmetry charge

e.g. 4d

$$\star j_0 = \frac{1}{8\pi^2} \mathrm{tr}(F \wedge F)$$

– **instanton number** symmetry '19 D. Cordova, D. S. Freed, H. T. Lam, N. Seiberg

– coupling to **background axion field**

(-1)-form U(1) symmetry

- Gauging (-1)-form U(1) symmetry

Gauging (-1)-form U(1) symmetry
Background θ → Dynamical θ

– Analogous to usual gauging

$$A \wedge \star j^{(0)} \longleftrightarrow \theta(x)\frac{1}{8\pi^2}\mathrm{Tr}(F \wedge F)$$

– **Axion** field is the **gauge field** for **(-1)-form symmetry**!

Gauging (-1)-form symmetries are related to
any couplings are given by dynamical field VEVs

SSB of (-1)-form U(1) symmety

D. Aloni, E. García-Valdecasas, M. Reece, M. S. (2024)

SSB of (-1)-form U(1) symmetry

D. Aloni, E. García-Valdecasas, M. Reece, M. S. (2024)

We have various evidences of SSB of (-1)-form U(1) symmetry

SSB of 0-form U(1) symmetry	SSB of (-1)–form U(1) symmetry
Photon becomes massive	Axion obtains potential (mass)
NG boson	*NG field*
Stueckelberg mass term	Stueckelberg-like mass term
Dual: string bounded by monopole	Dual: domain wall bounded by string

SSB of (-1)-form U(1) symmetry

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e.g. axion obtains mass from non-zero topological susceptibility

$$\mathcal{X} = \lim_{q \to 0} i q^\mu q^\nu \int e^{iqx} \langle 0 | T K_\mu(x) K_\nu(0) | 0 \rangle d^4 x$$

$$\partial^\mu K_\mu \sim F\tilde{F}$$

SSB of (-1)-form U(1) symmetry

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$$\partial^\mu K_\mu \sim F\tilde{F}$$

Pole at q^2 =0 → NG field

$$q^\mu q^\nu \int e^{iqx} \langle 0|TK_\mu(x)K_\nu(0)|0\rangle d^4x \propto \frac{q_\mu q_\nu}{q^2}\frac{1}{q^2}$$

Order paramerer of SSB

We have various evidences of SSB of (-1)-form U(1) symmetry

SSB of 0-form U(1) symmetry	SSB of (-1)–form U(1) symmetry
Photon becomes massive	Axion obtains potential (mass)
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Order parameter of SSB of (-1)-form U(1) symmetry

Vacuum energy density E(θ) that depends on theta-angle

Strong CP problem & (-1)-form symmetry

The Strong CP problem

→ Partition function depends on θ

= Generating "functional" with θ-term as an external source term

→ Vacuum energy density depends on θ

→ SSB of the (-1)-form U(1) symmetry

A sufficient condition of Strong CP problem

If we encounter Strong CP problem,

there is SSB of global (-1)-form U(1) symmetry

Strong CP problem & (-1)-form symmetry

Equivalently,

A necessary condition to solve Strong CP problem

If SSB of global (-1)-form U(1) symmetry is avoided

Strong CP problem is solved

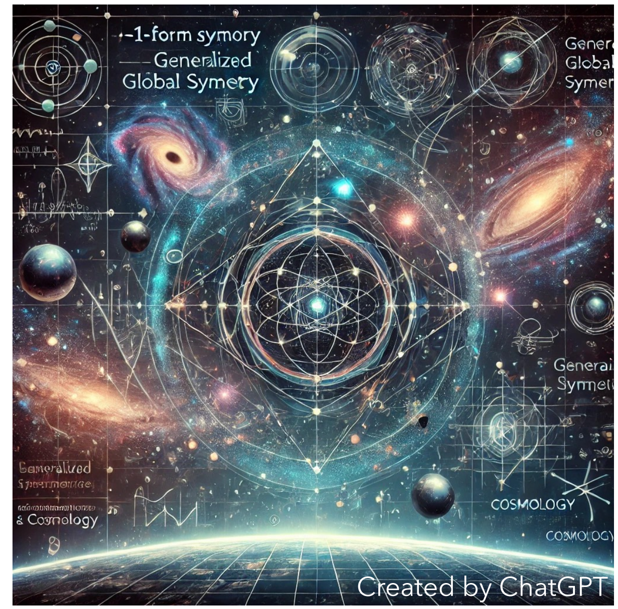
⬇

- ✅ Gauging the (-1)-form U(1) symmetry satisfies this condition
  i.e. axion, massless quark solutions

- ✅ Explicit breaking of the (-1)-form symmetry?
  E. García-Valdecasas, M. Reece, M. S. (2024)

Summary & Outlook

- Summary
  - (-1)-form U(1) sym ~ instanton number sym
  - Gauging ~ axion field
  - SSB ~ vacuum energy density E(theta)
  - Strong CP problem –> SSB of global (-1)-form
  - Still looking for new solutions



- Outlook
  - More on NG theorem for (-1)-form sym?
  - Hierarchy problem & (-1)-form sym?
  - More on explicit breaking of (-1)-form sym?
  - Axion quality problem & (-1)-form sym?

  etc.

Backup

Concrete model

Axion: pseudo-NG boson from SSB of U(1)PQ symmetry

- KSVZ model '79 Kim, '79 Shifman, Veinshtein, Zakharov

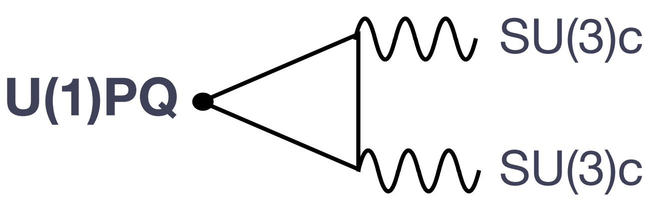
$$\mathcal{L} = -\phi Q \bar{Q}$$

- U(1)PQ symmetry

$$\phi \to e^{i2\alpha}\phi, \ Q \to e^{-i\alpha}Q, \ \bar{Q} \to e^{-i\alpha}\bar{Q}$$

φ: PQ field

- U(1)PQ - SU(3)c- SU(3)c triangle anomaly is nonzero



$$\neq 0 \ \leftrightarrow \ \partial j_{PQ} = \frac{g^2}{32\pi^2} G\tilde{G}$$

- SSB of U(1)PQ gives axion

$$\langle \phi \rangle \neq 0 \ \to \ \mathcal{L} = -\theta(x) \frac{g^2}{32\pi^2} G\tilde{G}(x)$$

$$\phi \sim v e^{i\theta(x)}$$

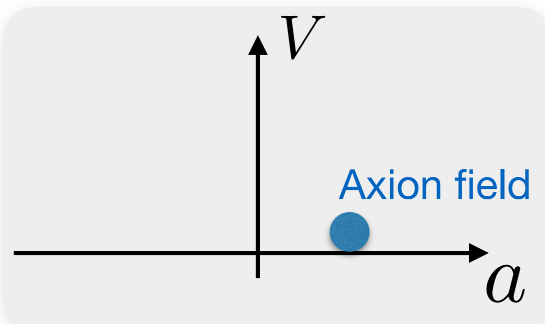
Axion Cosmology

Two standard scenarios

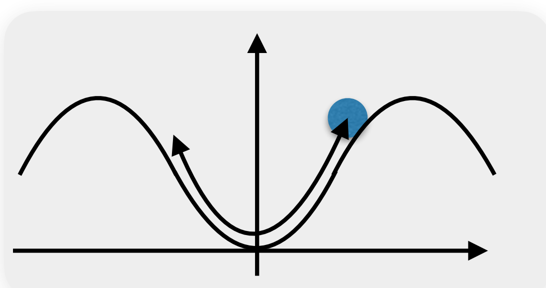
Pre-inflationary scenario

Time

- **SSB of U(1)PQ**
- **End of inflation**
- Reheating



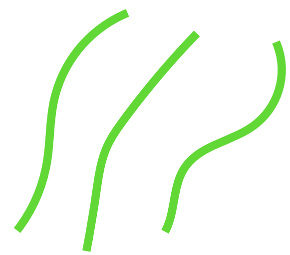
- **T~ΛQCD, axion oscillation**



Post-inflationary scenario

- **End of inflation**
- Reheating
- **SSB of U(1)PQ**

Global string formation



- **T~ΛQCD**

Domain wall formation