Spectroscopy Theory:
progress and open questions from a lattice point of view

focus on exotic hadrons

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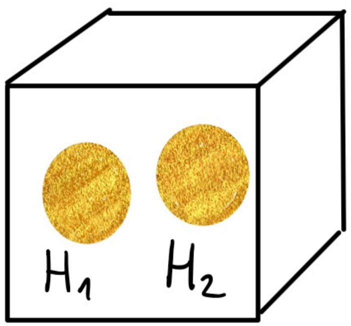
Spectroscopy in decays & in femtoscopic correlations

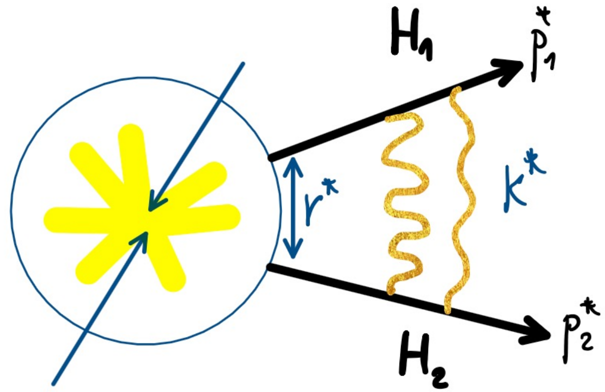
16th December 24, Orsay, France



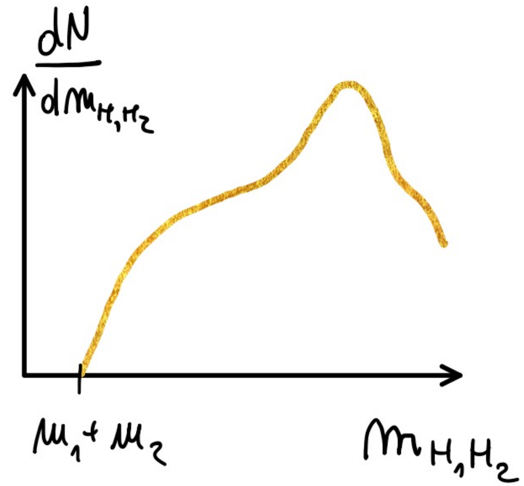
lattice QCD:

easier to explore E

relatively near thr.



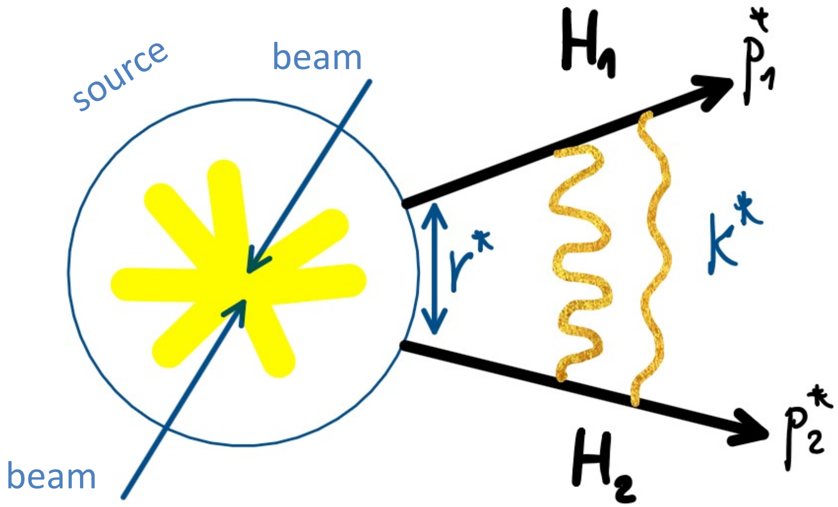
femtoscopy



invariant mass distribution

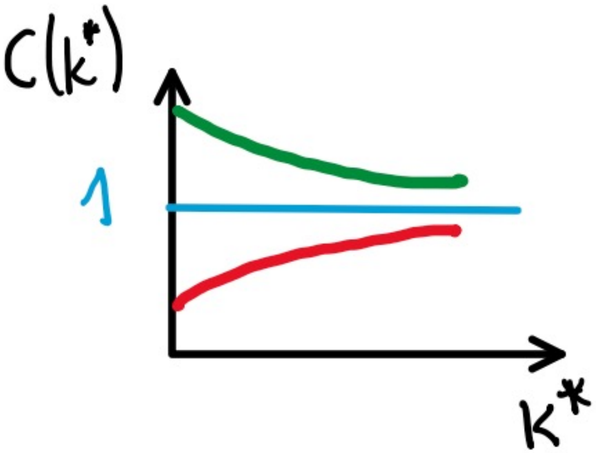
My limited knowledge on femtoscopy

Disclaimer: I have not done the homework to know what femtoscopy can and has measured already, but I am keen to learn. All followup thoughts concerning femtoscopy are very naive



$$k^* = \frac{1}{2}|\vec{p_1^*} - \vec{p_2^*}|$$

$$C(k^*) = \int \mathbf{d}\boldsymbol{r}^* \, S(\boldsymbol{r}^*)|\Psi(\boldsymbol{r}^*, \boldsymbol{k}^*)|^2$$



- attractive
- repulsive

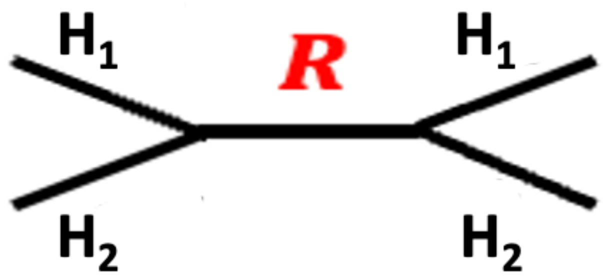
S: source function

r*: relative distance of particles at production

Ψ: two-particle wave function

I expect that femtoscopy is most valuable to explore interactions near threshold. That is where lattice QCD is most efficient and provides most of the results.

One of aims to study of hadron-hadron interactions:
intermediate exotic hadrons that form



$\bar{q}_1 \bar{q}_2 q_3 q_4$

J^P

Questions for a lattice theorist:

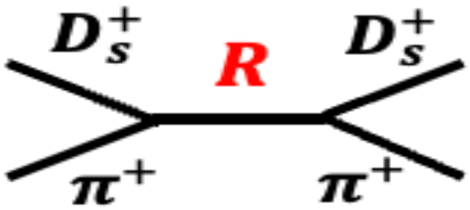
Does it exist ?

Mass ?

Width?

Binding mechanism ?   explore dependence on m_q

Aim: study of interactions/scattering of two hadrons

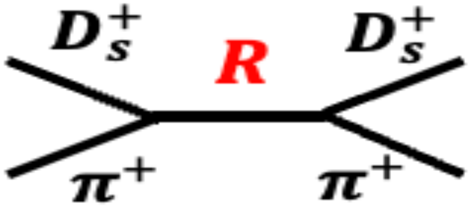


Initial particles are most often decaying electro-weakly in Nature

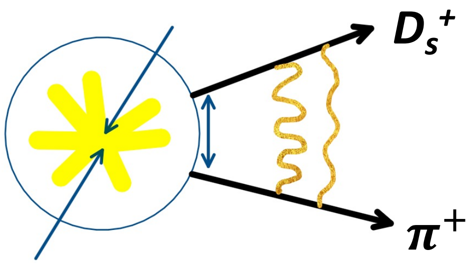
Different production mechanisms :

theory: with only QCD

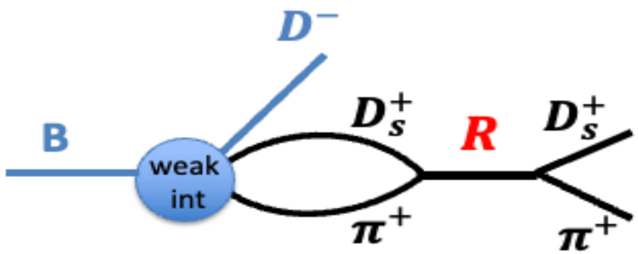
~~electro-weak~~



experiment: femtoscopy



experiment: production via weak int.



Observables depend on the same scattering amplitude

How difficult is it to study a given hadron in lattice QCD?
Depends whether it is strongly decaying or not

spectroscopic studies throught this lecture:

- strong int (QCD)
- el~~ectro-weak~~ int

$\bar{u}u$	$\bar{s}u$	$\bar{c}u$
$\pi^{\pm}$	$K^{\pm}$	$D^{\pm}$
π^{0}	K^{0}	D^{0}
η	K^0_S	$D^*(2007)^0$
$f_0(500)$	K^0_L	$D^*(2010)^{\pm}$
aka σ; was	$K_0^*(700)$	$D_0^*(2300)$
$f_0(600)$,	aka κ; was	was $D_0^*(2400)$
$f_0(400-1200)$	$K_0^*(800)$	$D_1(2420)$
$\rho(770)$	$K^*(892)$	$D_1(2430)^0$
$\omega(782)$	$K_1(1270)$	$D_2^*(2460)$
$\eta'(958)$	$K_1(1400)$	$D_0(2550)^0$
$f_0(980)$	$K^*(1410)$	$D_1^*(2600)^0$
$a_0(980)$	$K_0^*(1430)$	was $D_J^*(2600)$
$\phi(1020)$	$K_2^*(1430)$	$D^*(2640)^{\pm}$
$h_1(1170)$	$K(1460)$	$D_2(2740)^0$
$b_1(1235)$	$K_2(1580)$	was $D(2740)^0$
$a_1(1260)$	$K(1630)$	$D_3^*(2750)$
$f_2(1270)$	$K_1(1650)$	
$f_1(1285)$	$K^*(1680)$	

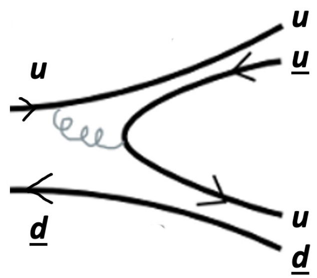
- ❑ strongly stable states

$$\pi^- \to \mu^- \nu_\mu$$

eigenstate of H_{QCD}

- ❑ strongly decaying resonances

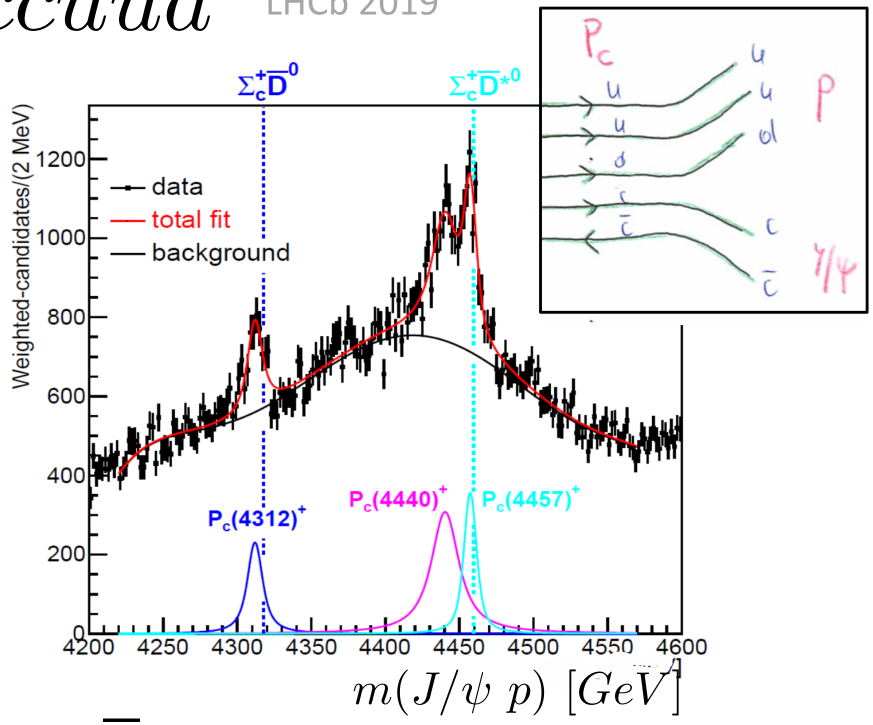
$$\rho \to \pi\pi$$



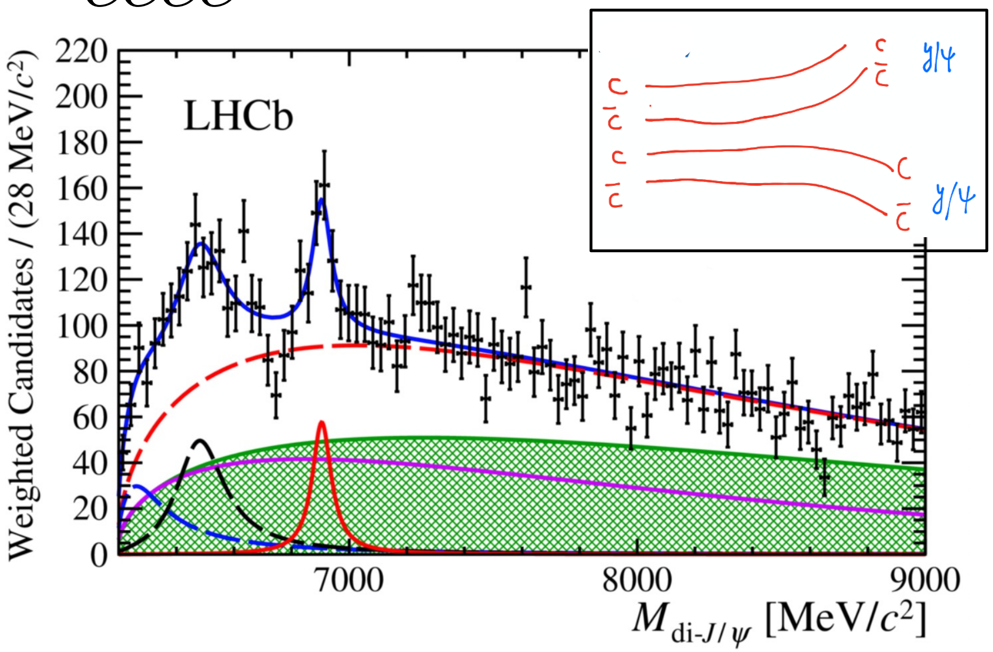
not eigenstate of H_{QCD}

All experimentally discovered exotic hadrons strongly decay

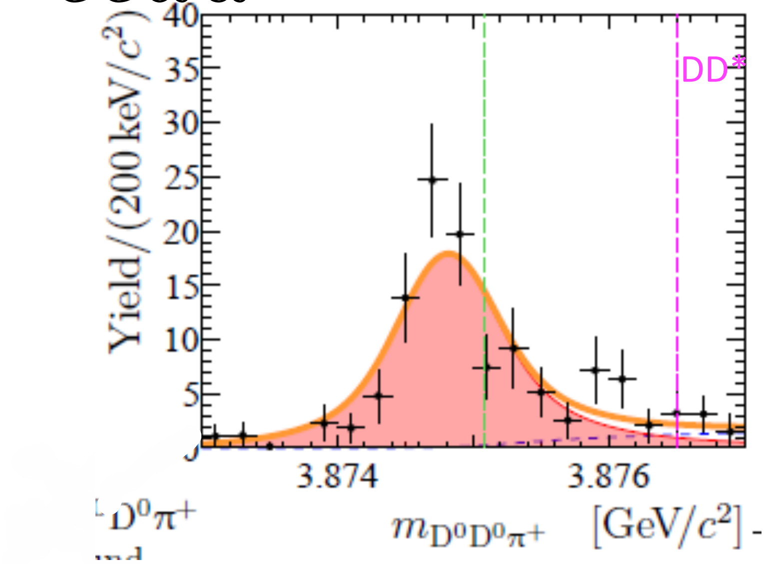
$\bar{c}cuud$ LHCb 2019



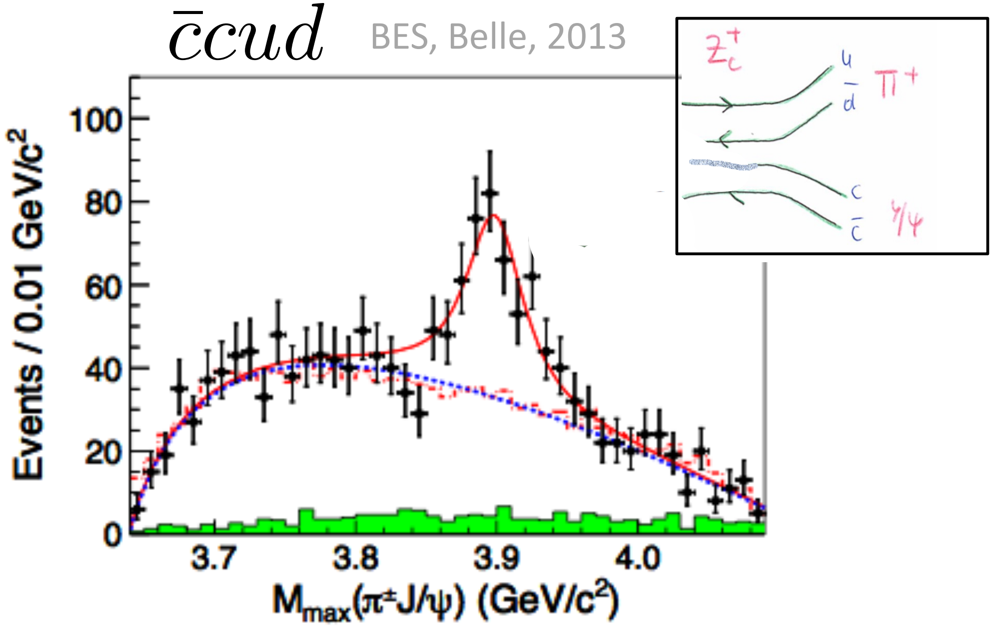
$\bar{c}c\bar{c}c$ LHCb 2021



$cc\bar{d}\bar{u}$ LHCb, 29th Jul 2021



$\bar{c}cu\bar{d}$ BES, Belle, 2013

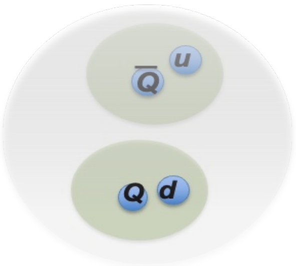


Exotic hadrons

Simplistic argument: for a given V

(that does not significantly depend on mQ)

heavier particles are easier to bind

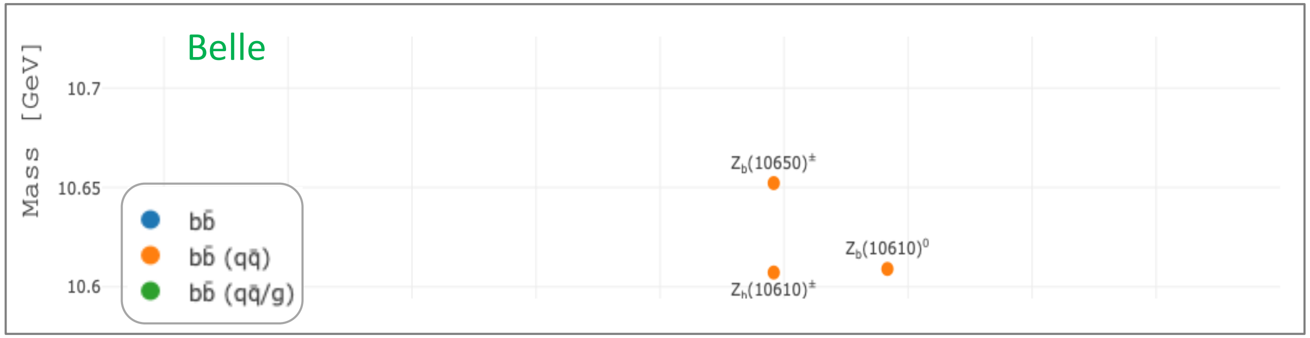


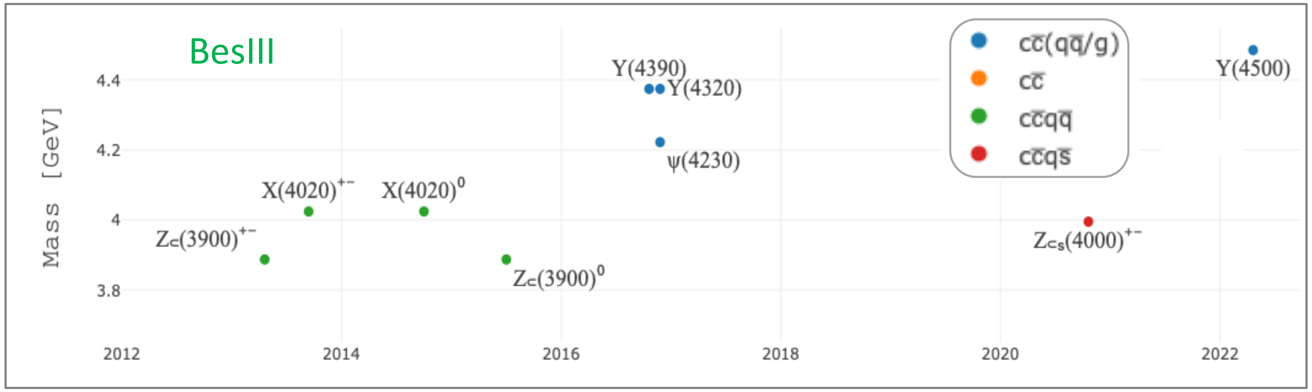
$$\hat{H} = \frac{\hat{p}^2}{2m_r} + V$$

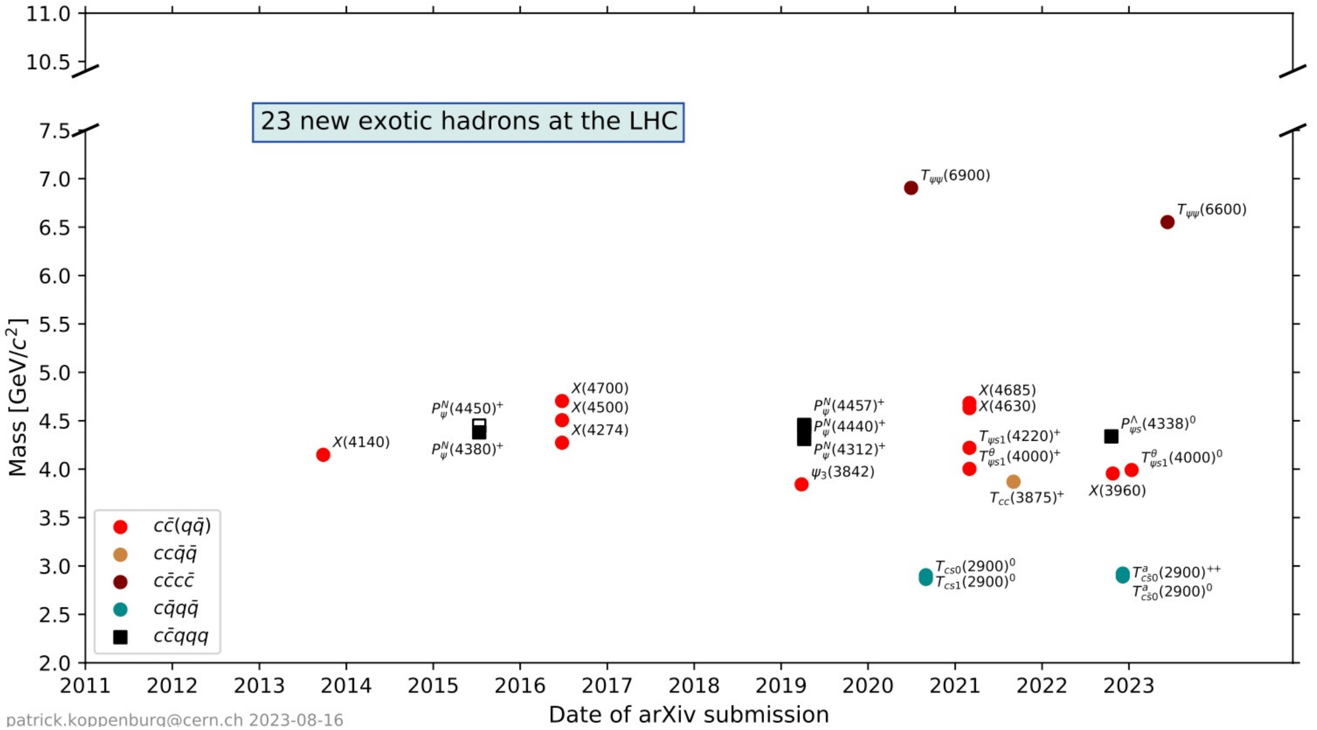
https://www.nikhef.nl/~pkoppenb/particles.html

https://qwg.ph.nat.tum.de/exoticshub/

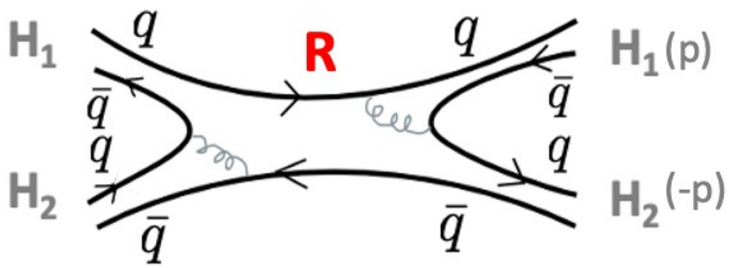
All experimentally discovered exotic hadrons strongly decay

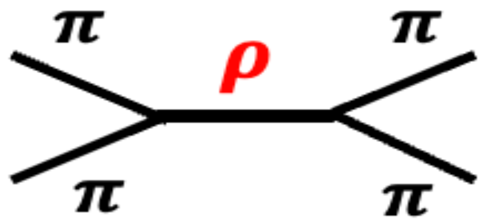






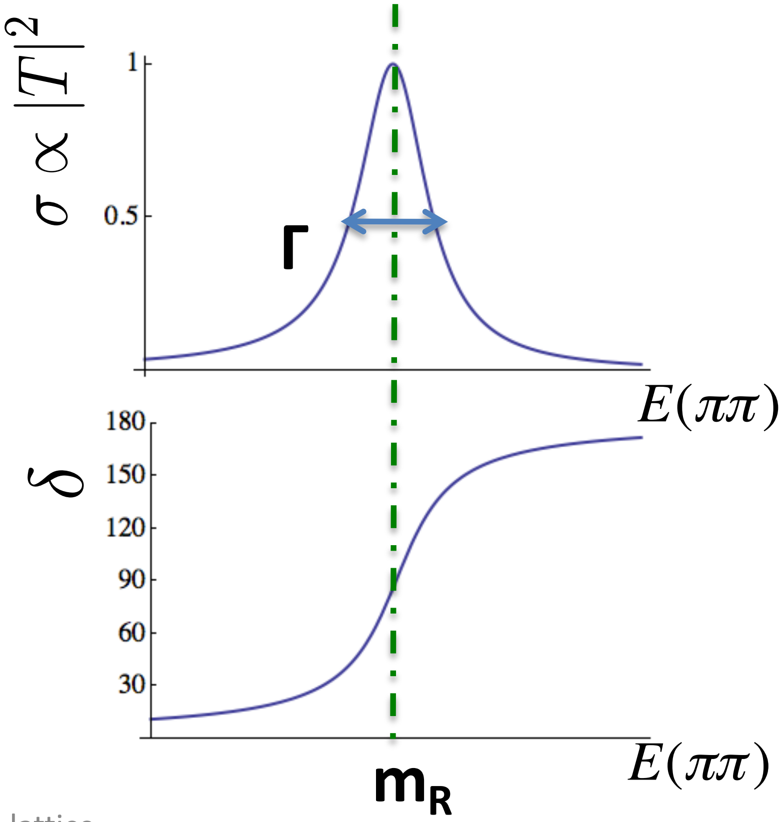
Strongly decaying hadrons (resonance)





Task for lattice QCD:

determine scattering amplitude T



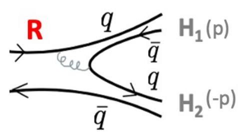
How difficult it is to study a given hadron with lattice QCD?

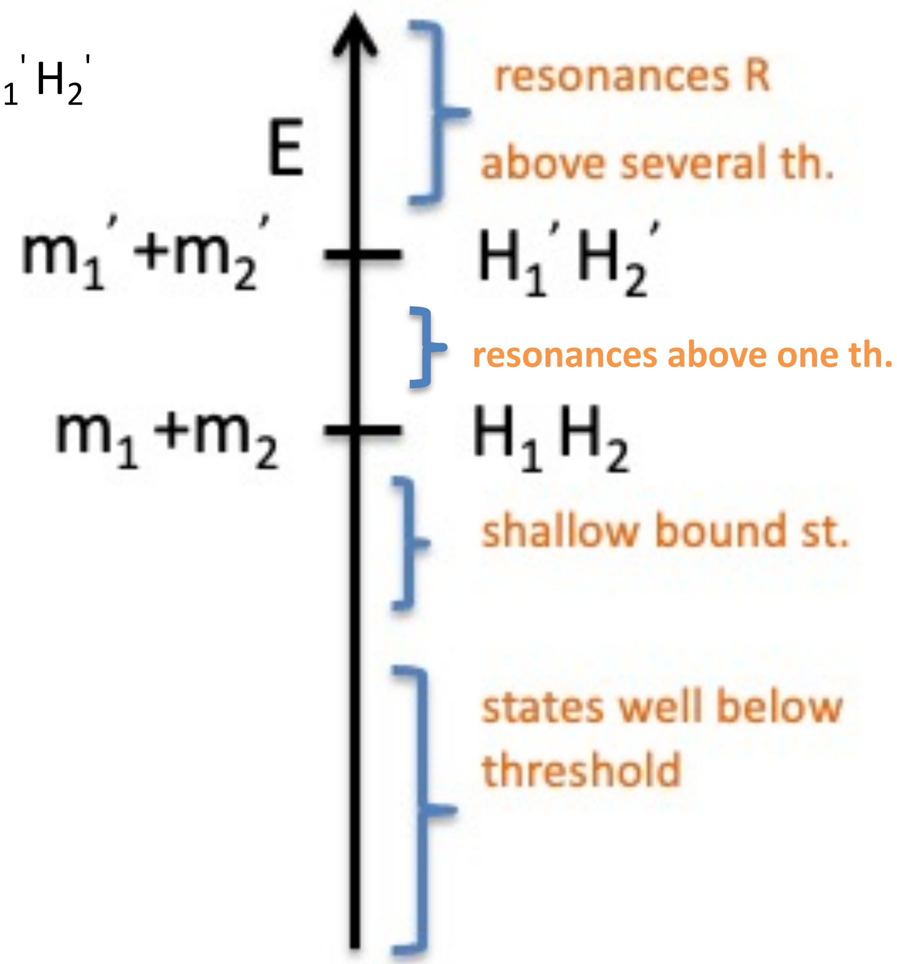
strong, ~~EW~~

Outline

$R \to H_1 H_2 , H_1' H_2'$

$R \to H_1 H_2$





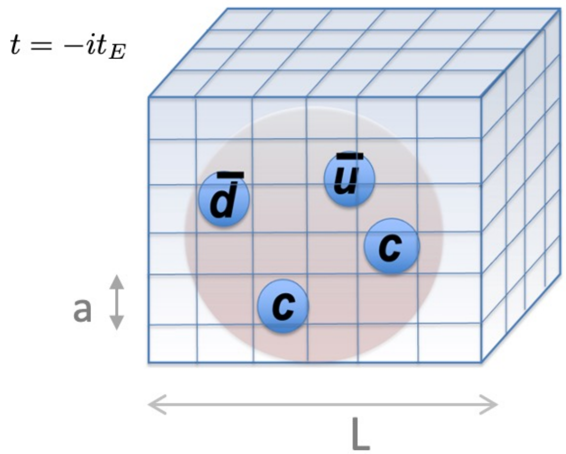
- ➢ hadrons from coupled-channel scat.
- ➢ hadrons from one-channel scattering
- ➢ hadrons well below threshold

- all from lattice eigen-energies E_n

$$\hat{H}_{QCD}|n\rangle = E_n|n\rangle$$

- examples presented
- this is NOT a review of all existing results !

Quantum ChromoDynamics on lattice



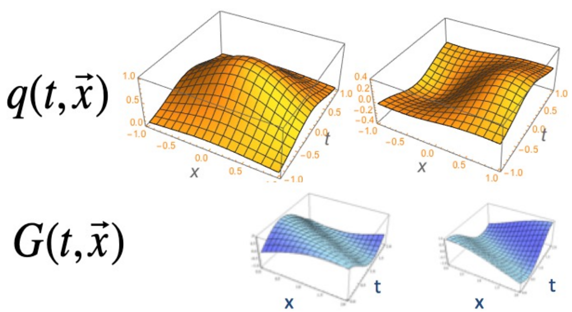
- numerical evaluation of path integral
  in discretized finite Eucledian space-time
  $t_M = -it$

- typical : $a \approx 0.05$ fm , L = 40-100 a
  a $\to 0$, $L \to \infty$

- input: g_s , m_q

$$\mathcal{L}_{QCD} = \frac{1}{4} G_a^{\mu\nu} G_a^{\mu\nu} + \bar{q} i \gamma_\mu (\partial^\mu + i g_s G_a^\mu T^a) q - m_q \bar{q} q$$

$$S_{QCD} = \int d^4x \; \mathcal{L}_{QCD}$$

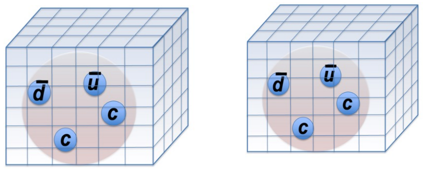


$$\langle C \rangle \propto \int \mathcal{D}G \mathcal{D}q \mathcal{D}\bar{q} \; C \; e^{-S_E/\hbar}$$

Main quantity extracted: E_n

$$\hat{H}_{QCD}|n\rangle = E_n|n\rangle$$

C: different correlation than in case of femtoscopy



$$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t)\ \mathcal{O}_j^{+}(0) | 0 \rangle$$

$$\sum_n |n\rangle\langle n|$$

$$\mathcal{O}(t_M) = e^{iHt_M}\mathcal{O}(0)e^{-iHt_M}$$

$$= \sum_n \langle 0|\ e^{iHt_M}\mathcal{O}_i(0)e^{-iHt_M}\ |n\rangle\langle n|\mathcal{O}_j^{\dagger}(0)|0\rangle$$

$$t_M = -it$$

$$= \sum_n \langle 0|\ e^{Ht}\mathcal{O}_i(0)e^{-Ht}\ |n\rangle\langle n|\mathcal{O}_j^{\dagger}(0)|0\rangle$$

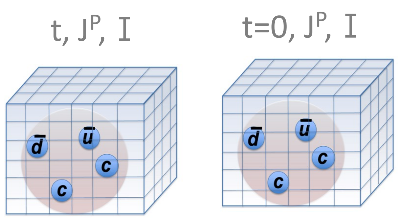
$$= \sum_n \langle 0|\mathcal{O}_i|n\rangle\ e^{-E_n t}\ \langle n|\mathcal{O}_j^{+}|0\rangle$$

$$Z_i^n = \langle 0|\mathcal{O}_i|n\rangle$$ overlap

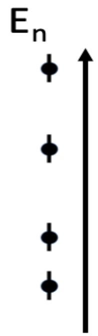
$$= \sum_n Z_i^n\ Z_j^{n*}\ e^{-E_n t}$$

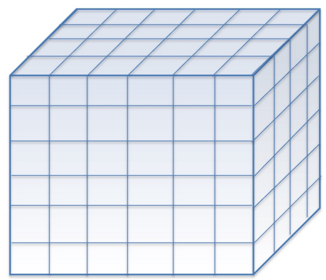
All results in this talk will be based on E_n

- for strongly stable state well below threshold : $E_n(P=0) = m$
- resonances (Luscher's relation) $E_n^{cm} \to T(E_n^{cm})$ } often "non-precision" studies: single a, $m_{u/d} > m_{u/d}^{phy}$, $m_\pi > 140$ MeV
- static potentials: $E_n \to V(r)$



$\mathcal{O} = \mathcal{O}(q, G)$



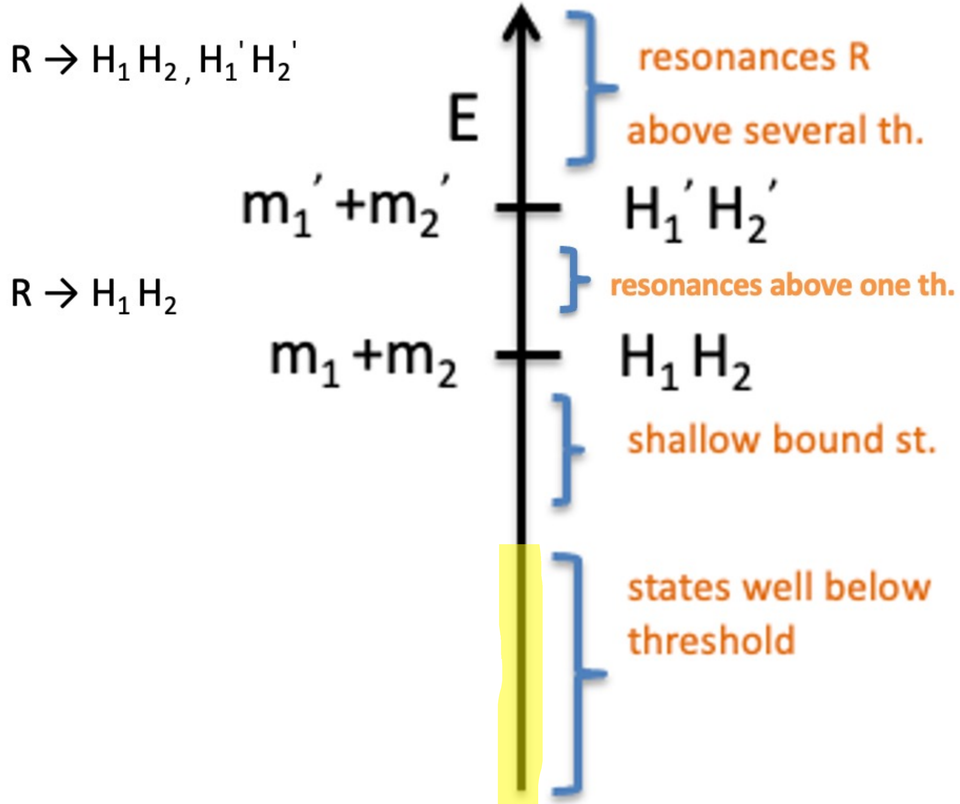


$$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t)\ \mathcal{O}_j^+(0) | 0 \rangle = \sum_n \langle 0 | \mathcal{O}_i | n \rangle\ e^{-E_n t}\ \langle n | \mathcal{O}_j^+ | 0 \rangle$$

All E_n with given quantum numbers must be extracted:

$s\bar{u}$ $K^*(890)$: $E > m_K + m_\pi \simeq 640$ MeV **OK**

$c\bar{c}u\bar{d}$ $Z_c(4430)$: $E > m_{J/\psi} + m_\pi \simeq 3240$ MeV ✖



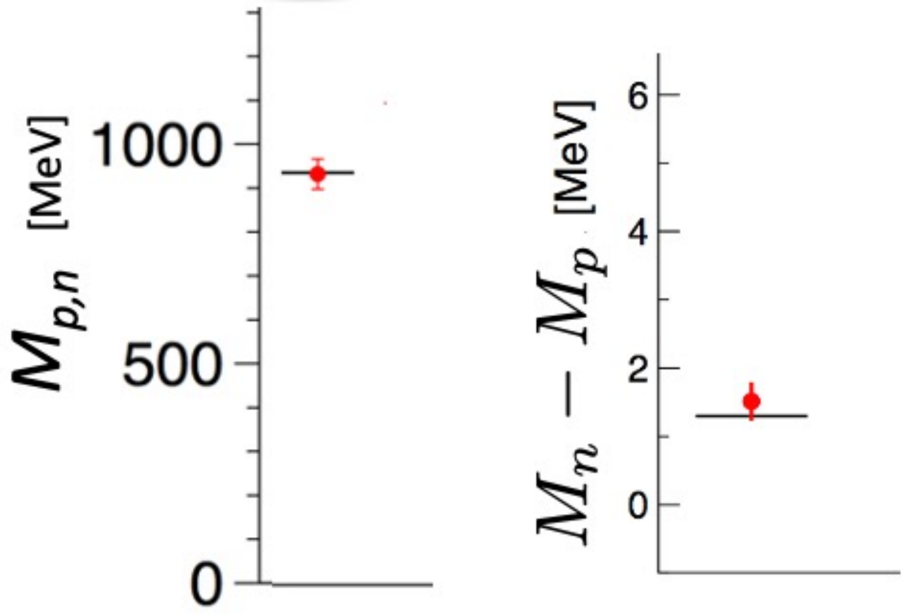
$$E^2 = m^2c^4 + P^2c^2$$

$$E_{(P=0)} = mc^2$$ $a \to 0,\ L \to \infty,\ \ m_q \to m_q^{phy}$

Hadrons well below threshold

(or studied as if located well below threshold)

Proton and neutron mass



— exp

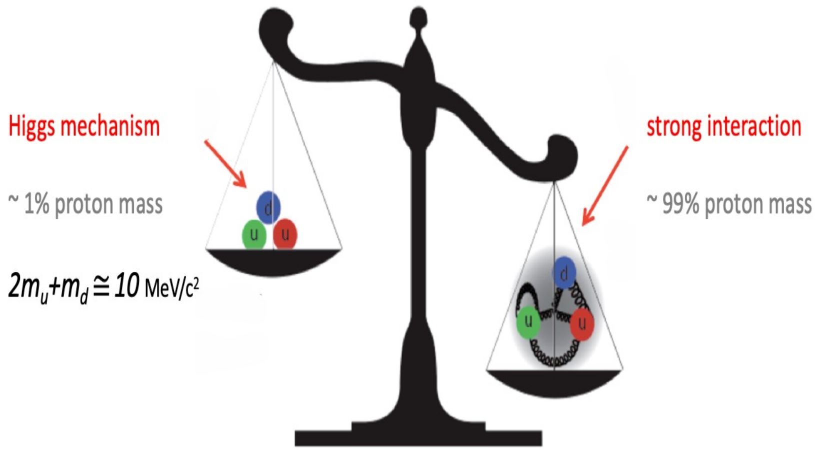
● lattice QCD
$m_u = m_d$

BMW collaboration
Science 322, 2008

— exp

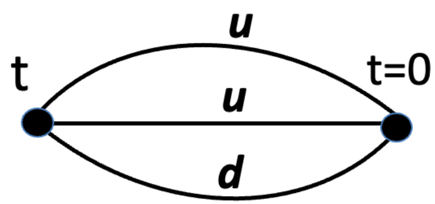
● lattice QCD+QED
$m_u \neq m_d$

BMW collaboration
Science 347,2015



$$m_p \simeq 938\ \mathrm{MeV/c^2}$$

$$\mathcal{O}_p = \epsilon_{ijk}[u_i^T C \gamma_5 d_j] u_k \simeq uud$$

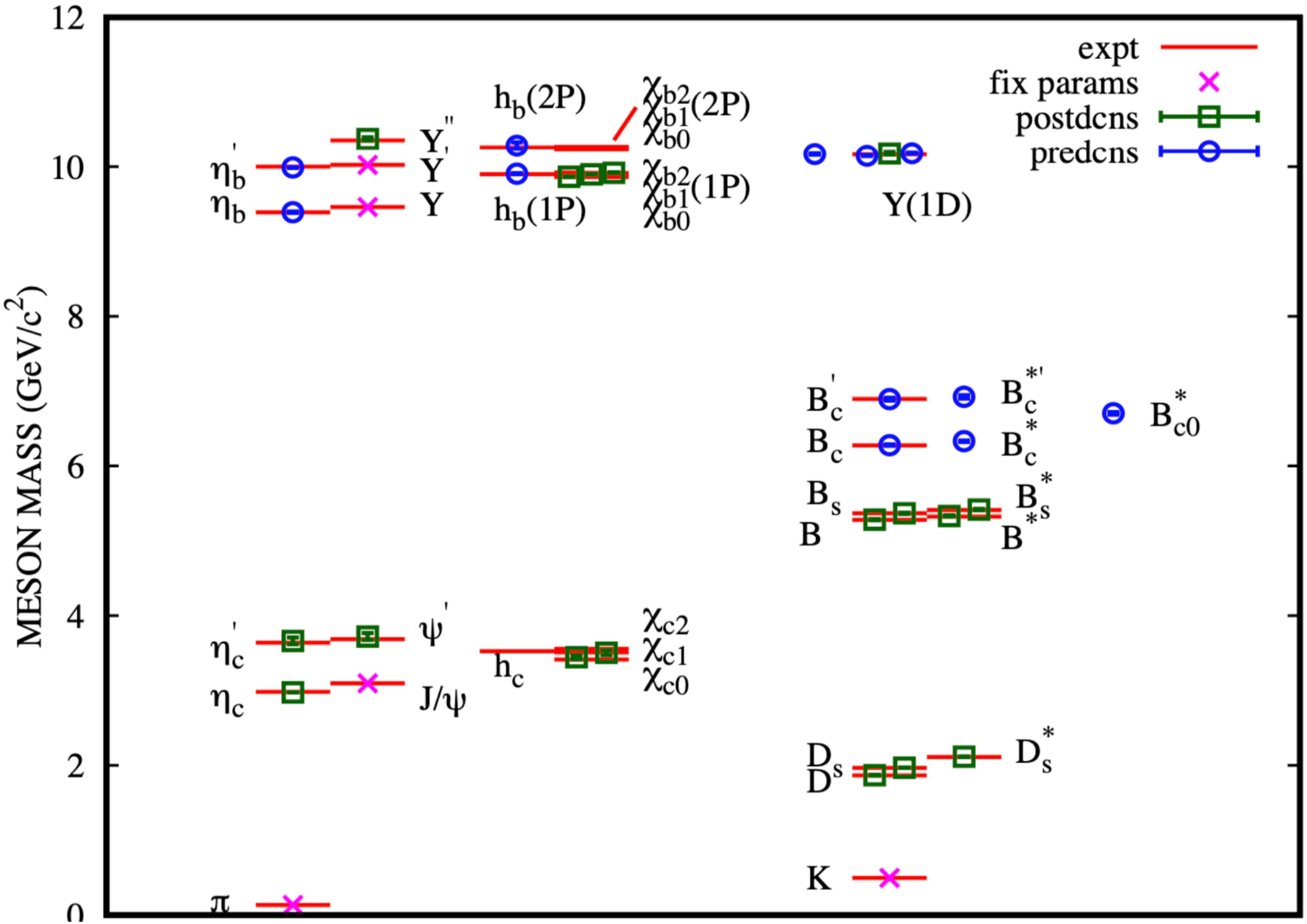


$$C = \langle 0 | \mathcal{O}_p(t)\ \mathcal{O}_p^\dagger(0) | 0 \rangle$$

$$C(t) = \sum_n A_n e^{-E_n t} \to A_1 e^{-E_1 t}$$

$$E_1 = m_p c^2$$

Strongly stable hadrons (HPQCD coll)

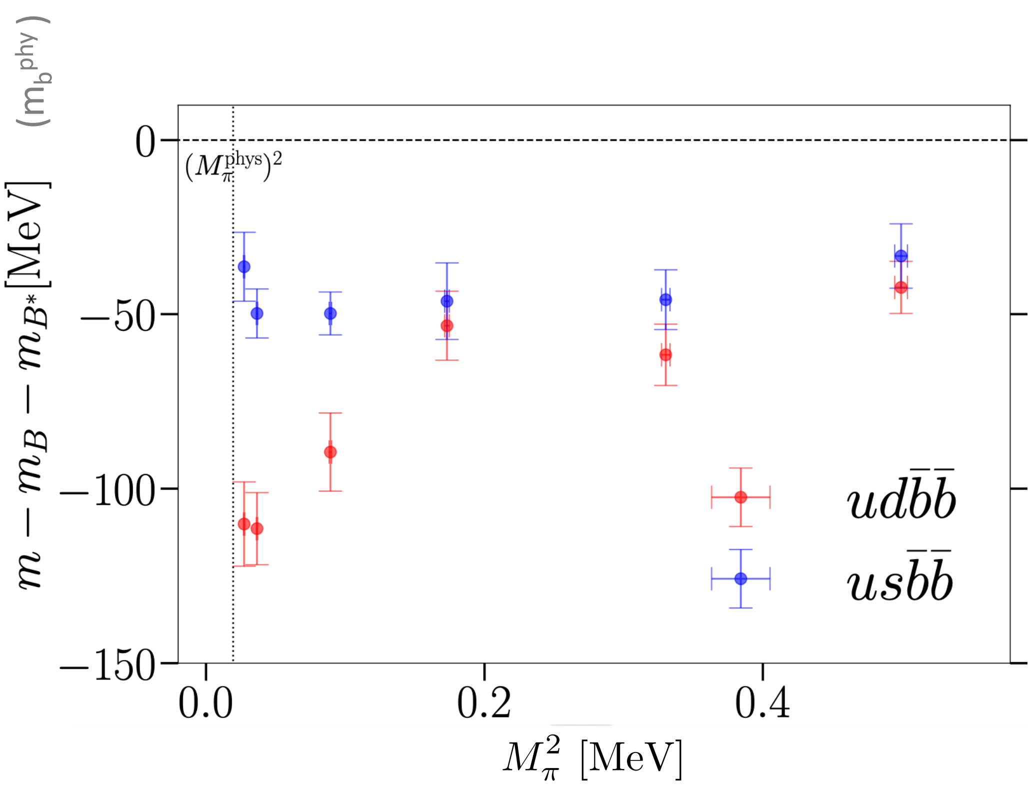


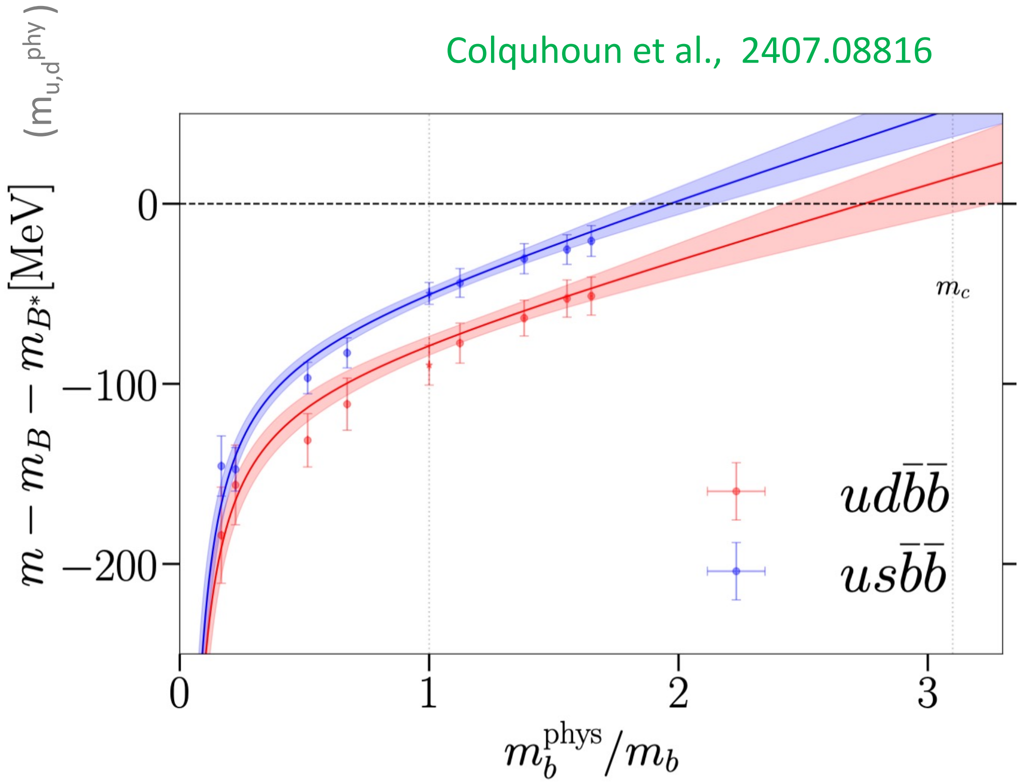
HPQCD, update of 1207.5149 (thanks to Christine Davies)

Doubly bottom tetraquarks $bb\bar{d}\bar{u}$ $bb\bar{s}\bar{u}$ $I=0,\ J^P=1^+$

lattice: dependence on m_b and $m_{u,d}$

Colquhoun et al., 2407.08816





Other doubly heavy tetraquarks: $QQ'\bar{q}\bar{q}'$

Theoretically expected near or above threshold

States near or above threshold have to be identified from scattering T(E): next Section

Di-baryons with heavy quarks

... being aware those are not best suited for exp

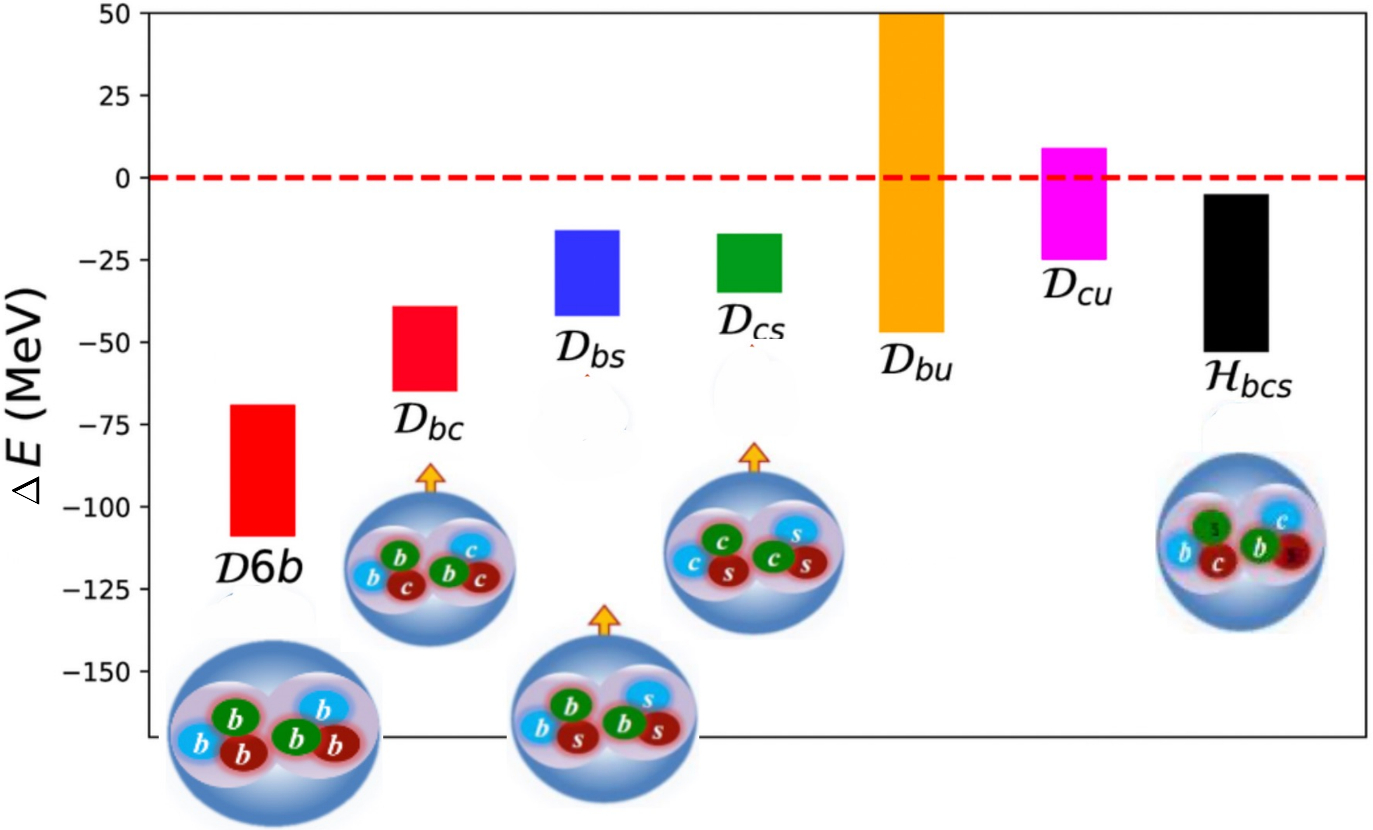
$$O = qqq\ qqq$$

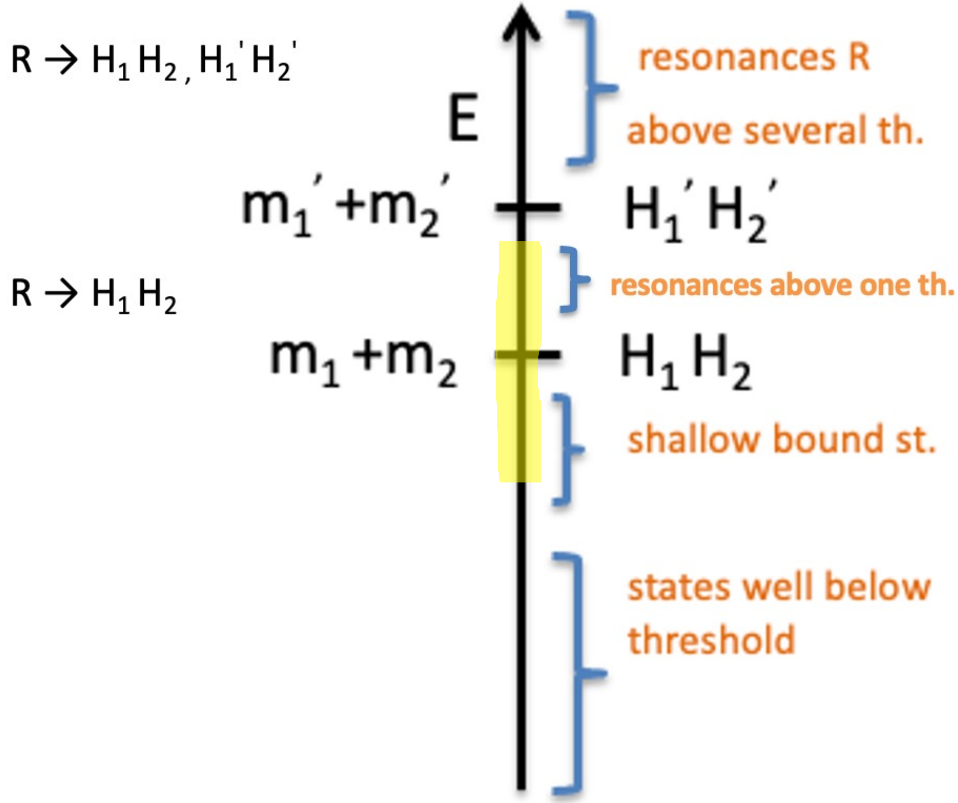
binding energy $\Delta E = m - m_{B1} - m_{B2}$

Junnarkar Mathur
1906.06054, PRL

Mathur, Padmanath, Chakraborty
2205.02862

Junnarkar, Mathur, 2206.02942, PRL

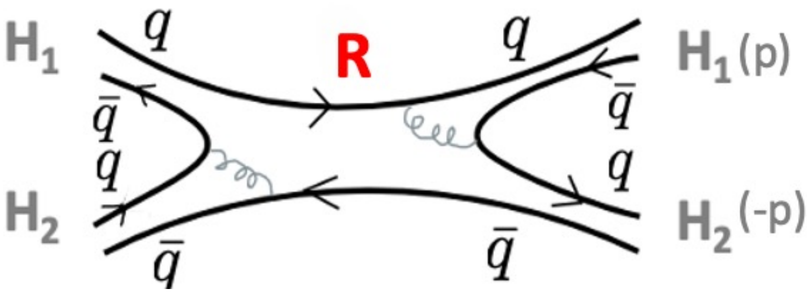




often "non-precision" studies:

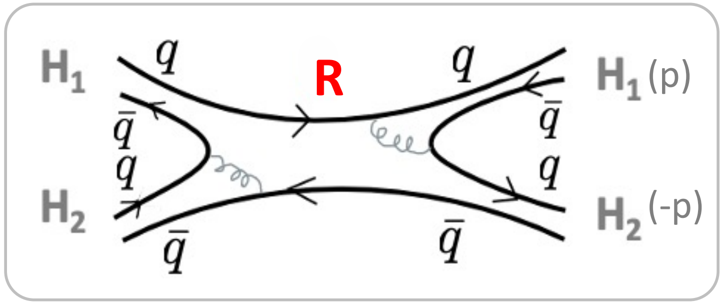
single a, $m_{u/d} > m_{u/d}^{phy}$, $m_\pi > 140$ MeV

Hadrons from one-channel scattering



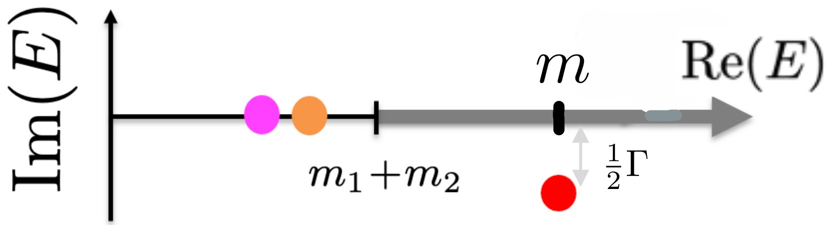
States from one-channel scattering

scattering amplitude T(E) for given partial wave l



$$S(E)=e^{2i\delta(E)}=1+i\,\frac{p}{4\pi E}\,T(E)\;\rightarrow\;\boxed{T(E)=\frac{8\pi E}{p\cot\delta-ip}}$$

$$E=\sqrt{m_1^2+p^2}+\sqrt{m_2^2+p^2}$$

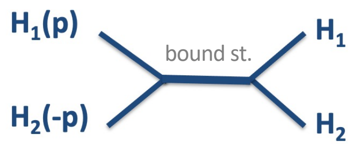


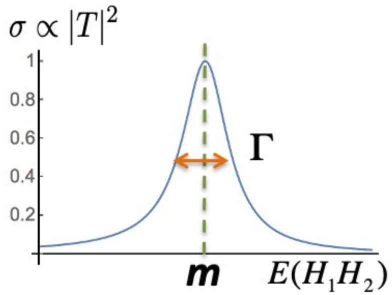
Virtual bound st.	Bound st.	Resonance
p = - i |p|, sheet II	p = i |p|, sheet I	sheet II

$p^2<0$

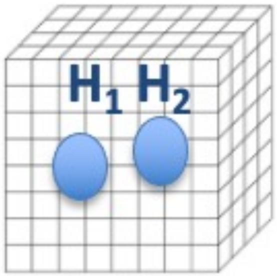
$$T(E)\propto\frac{1}{E^2-m^2}$$

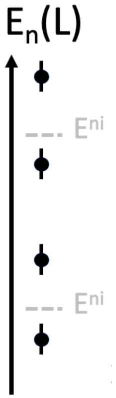
$$T(E)\propto\frac{1}{E^2-m^2+iE\Gamma}$$





Scattering amplitude T(E) from lattice QCD





$$\sigma(E)\propto|T(E)|^2$$

for real E — for complex E

$$E\;\rightarrow\;T(E)\;\rightarrow\;T(E^c)$$

analytic relation:
Luscher 1991
generalizations by many authors

analytic contin.
to complex E

Relation between E and δ(E) , T(E): 1D quantum mechanics

derivation of relation

$\psi(x) = A\cos(p|x| + \delta)$

- this form already ensures $\psi(L/2) = \psi(-L/2)$
- the other BC: $\psi'(L/2) = \psi'(-L/2)$

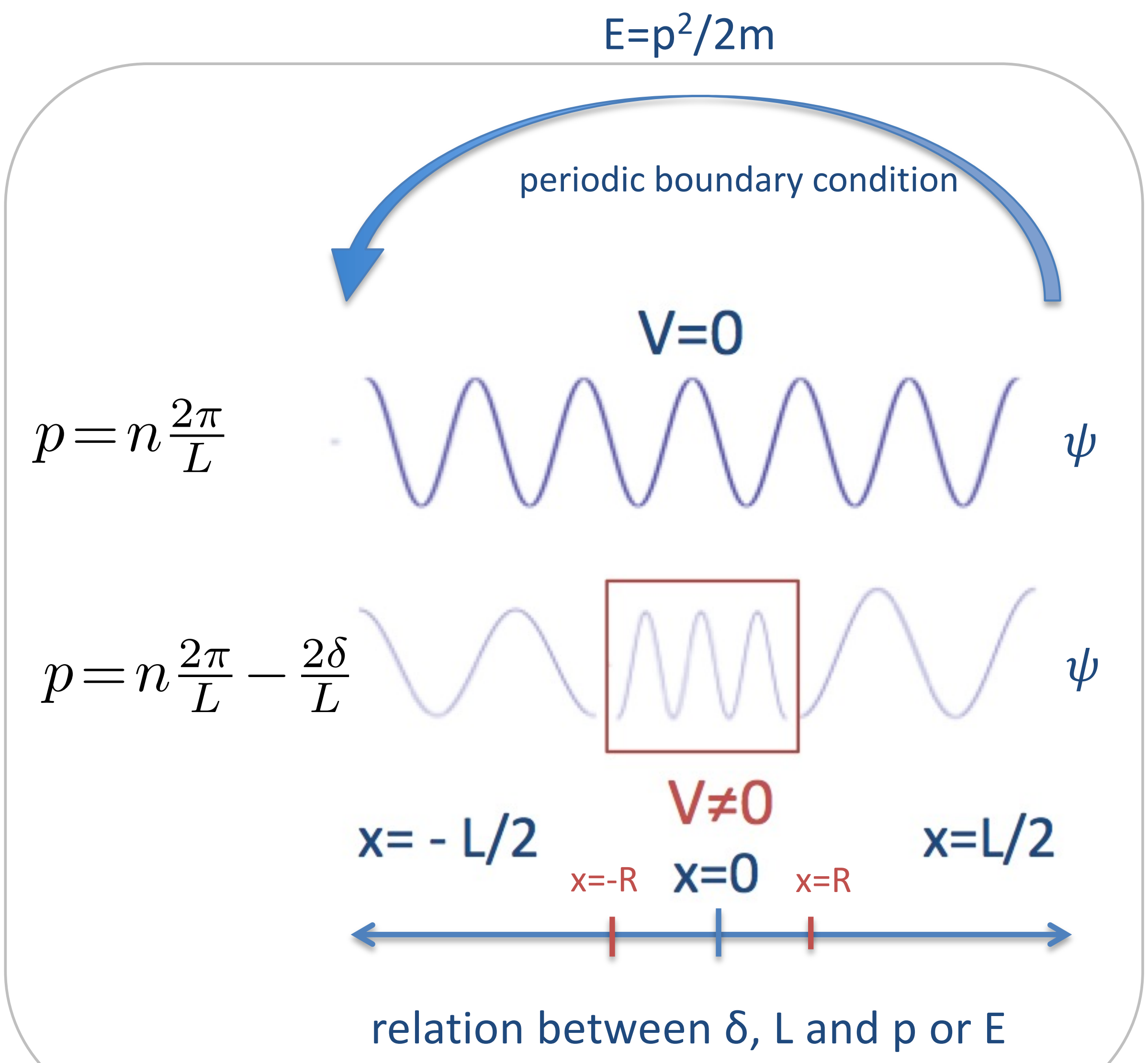
  this requires

  $Ap\sin(p(\frac{L}{2})+\delta) = -Ap\sin(-p(-\frac{L}{2})+\delta)$

  $\rightarrow \psi'(L/2)=0\ ,\ \sin(p\frac{L}{2}+\delta)=0$

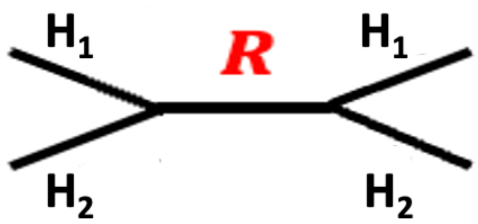
  $p\frac{L}{2}+\delta = n\pi \qquad \boxed{p = n\frac{2\pi}{L} - \frac{2}{L}\delta}$

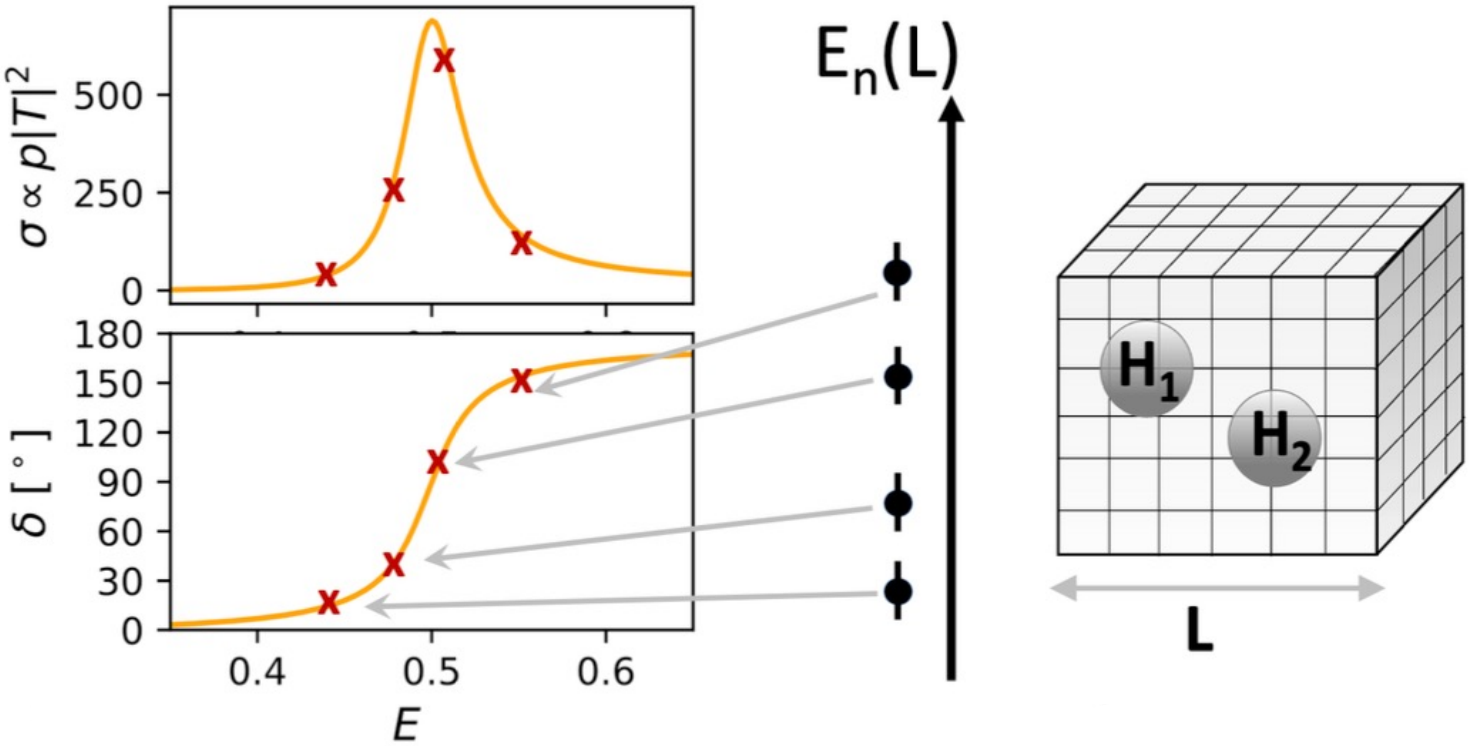
  relation between δ, L



Relation between E and δ(E), T(E)

$$T(E) \propto \frac{1}{p\cot\delta - ip}$$





at infinite volume δ(E), T(E) ← Luscher 1991 — E(L) energies from lattice with spatial extent L

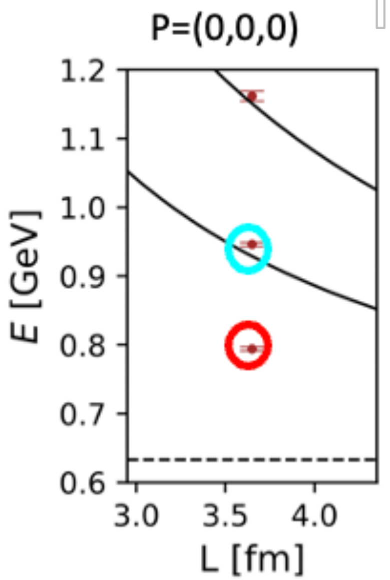
Verifying formalism on conventional mesons

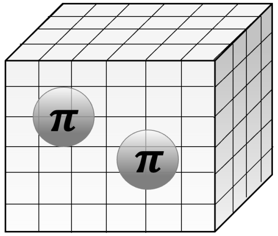
Alexandrou et al, 1704.05439

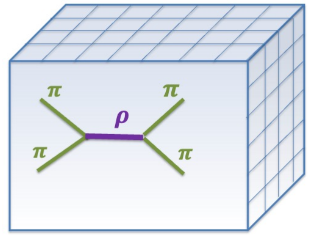
m_π=320 MeV, Nf=2+1, L~3.6 fm

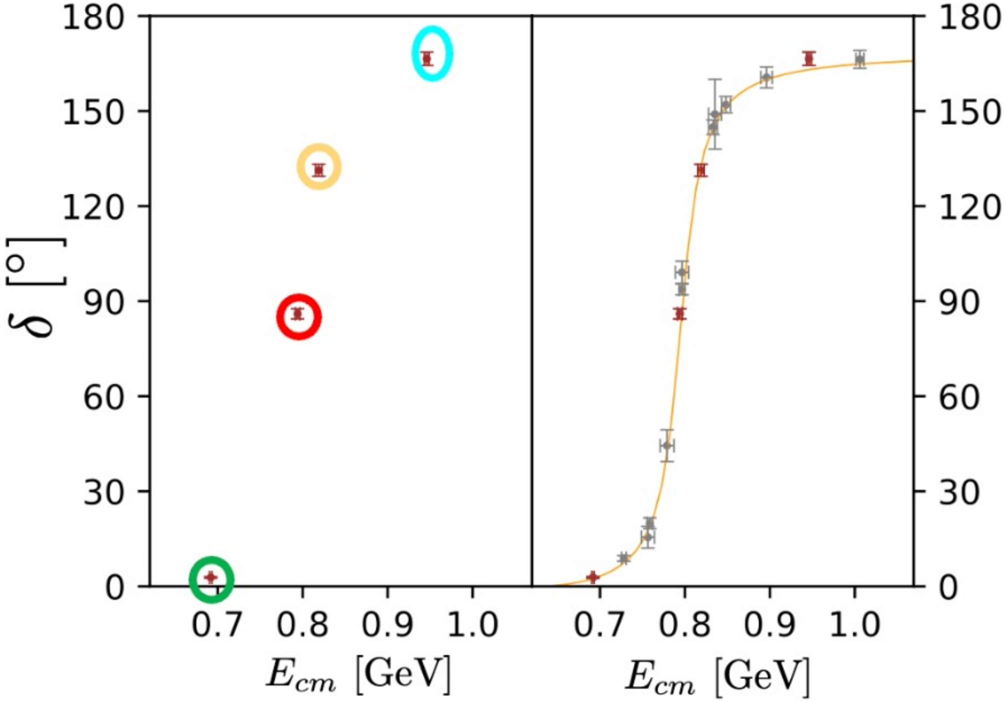
ρ-resonance in $\pi\ \pi$ scattering

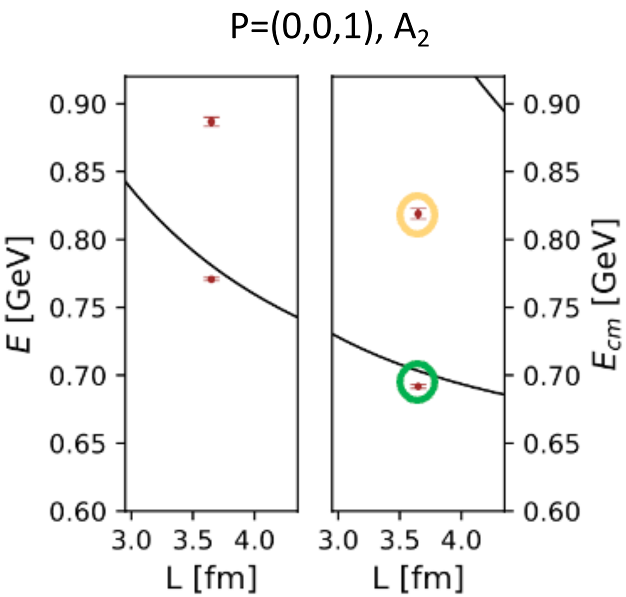
Luscher's relation: E-> δ(E), T(E)











$$\frac{p}{8\pi E}\ T(s) = \frac{\sqrt{s}\,\Gamma(s)}{m_R^2 - s - i\sqrt{s}\,\Gamma(s)}$$

$$m_\pi = 797(5)\ \mathrm{MeV} \qquad g = 5.7(2)$$

$$\Gamma(s) = g^2 \frac{p^3}{6\pi s}$$

Verifying formalism on conventional mesons

$\pi\,\pi$ and K π scattering at almost physical quark masses

Boyle et al. 2406.19194

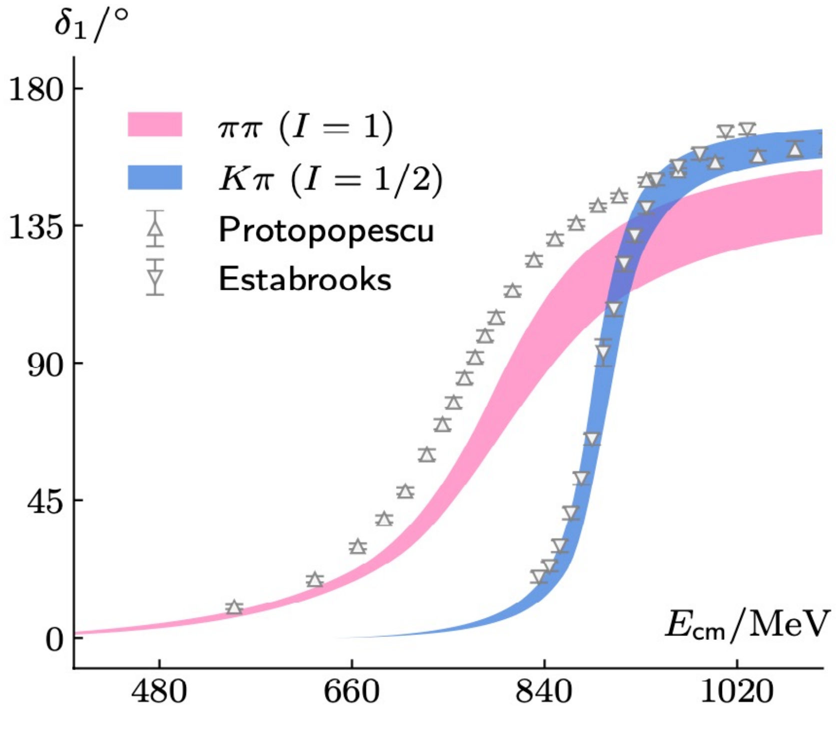
RBC/UKQCD ensemble

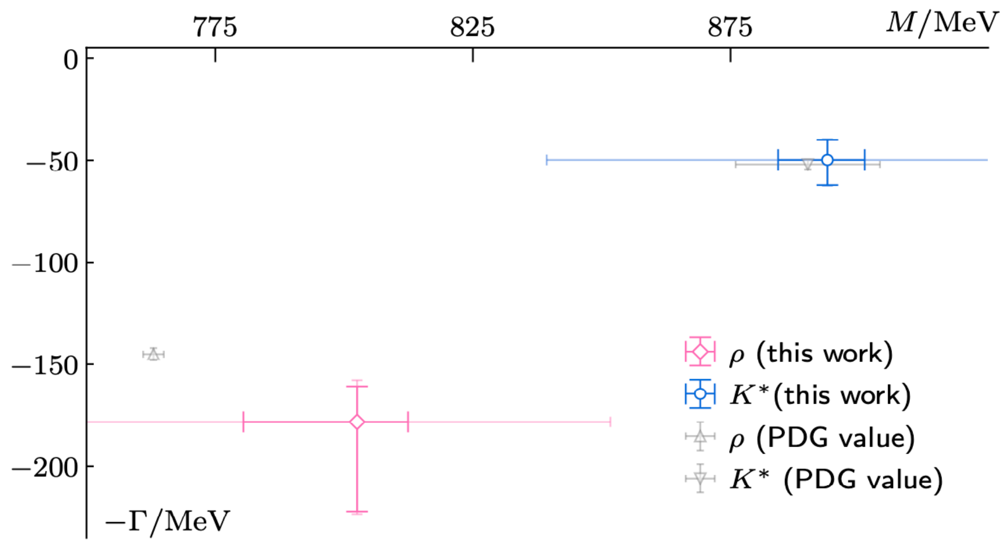
$m_\pi = 138.5(2)$ MeV

$(L/a)^3 \times (T/a) = 48^3 \times 96$

$a \simeq 0.11$ fm

$\vec{P} = \vec{0}$





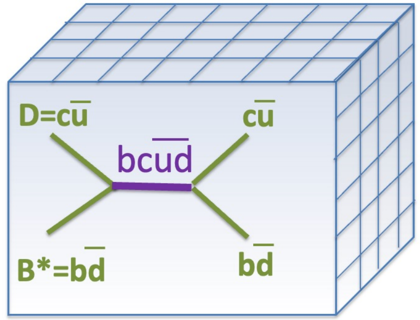
Alexandrou et al, 2312.02925 PRL

thanks to S. Meinel for figures!

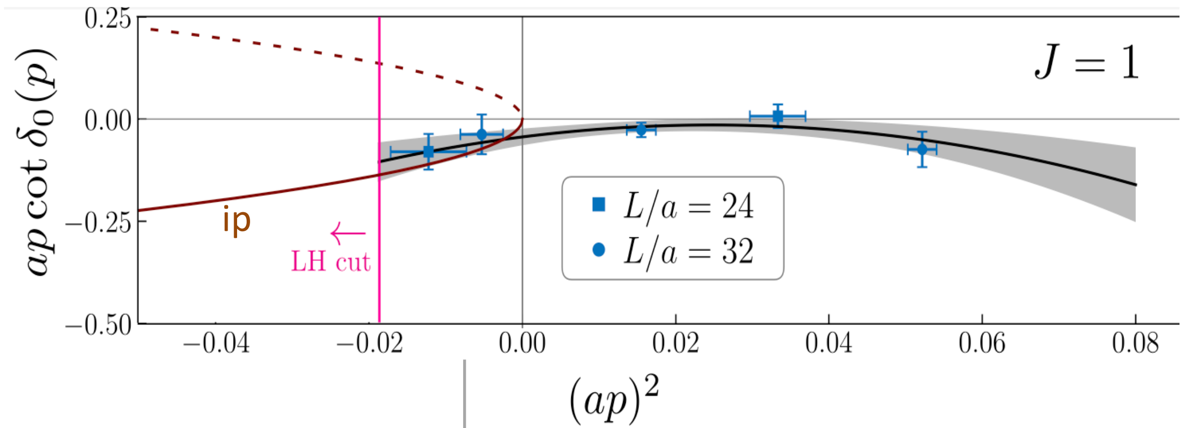
$m_\pi \approx 220 MeV$

T_{bc} $\quad bc\bar{u}\bar{d} \quad I=0$

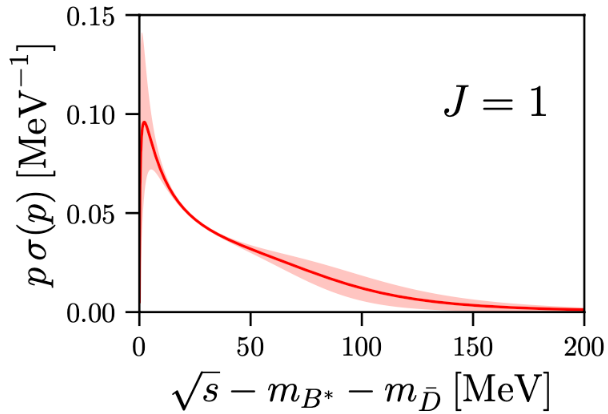
$B^*D \quad J^P=1^+$



$$T_0 \propto \frac{1}{p\cot\delta_0 - ip}$$



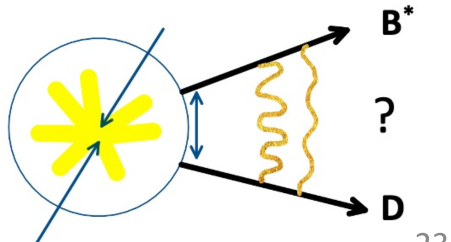
$p_b = i|p_b|$



$$m_{T_{bc}} = \sqrt{m_1^2 + p_b^2} + \sqrt{m_2^2 + p_b^2}$$

$$m_{T_{bc}} - m_{B^*} - m_D = -2.4^{+2.0}_{-0.7} \text{ MeV}$$ bound state

$$m_R - m_{B^*} - m_D = \quad 67 \pm 24 \text{ MeV} \quad \Gamma_R = 132 \pm 32 \text{ MeV}$$ resonance



see also Padmanath, Padmanath, Mathur, 2307.14128, PRL

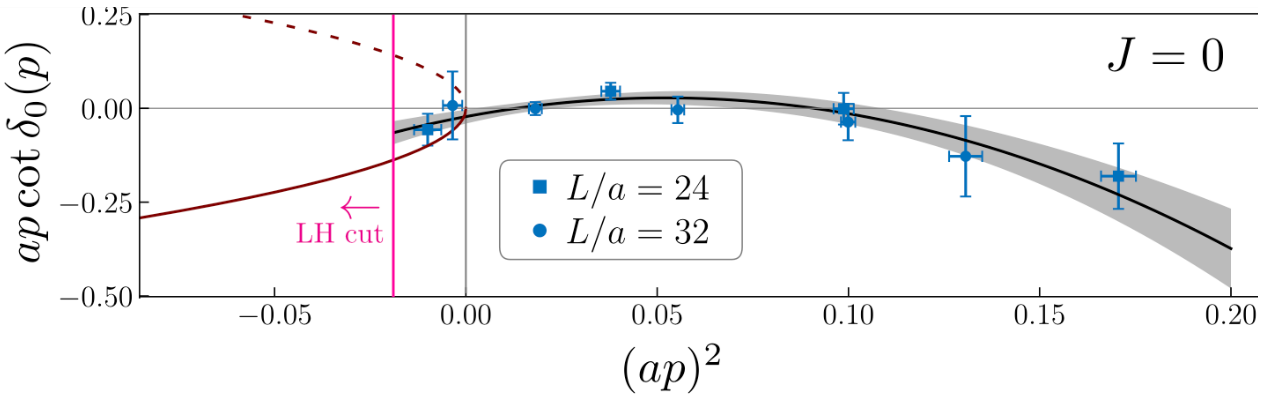
T_{bc} $\quad bc\bar{u}\bar{d} \quad I=0$

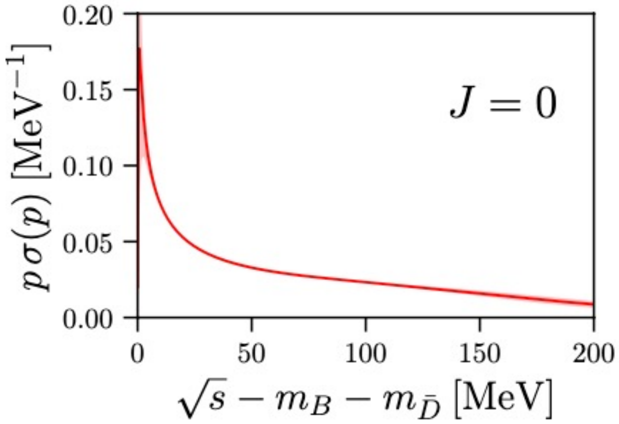
Alexandrou et al, 2312.02925 PRL

thanks to S. Meinel for figures!

$m_\pi \approx 220 MeV$

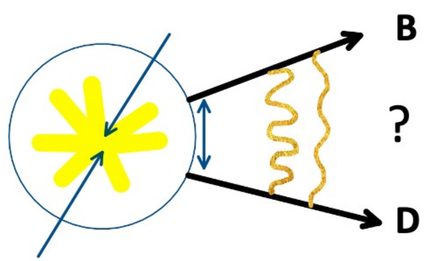
BD $\quad J^P=0^+$





$$m_{T_{bc}} - m_B - m_D = -0.5^{+0.4}_{-1.5}\ \text{MeV}$$ **bound state**

$$m_R - m_{B^*} - m_D = \ \ 138 \pm 13\ \text{MeV} \quad \Gamma_R = 229 \pm 35\ \text{MeV}$$ **resonance**

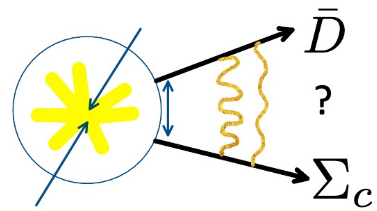


see also Radhakrishanan, Padmanath, Mathur, 2404.08109, PRD

P_c

$\bar{c}cuud$ $\rightarrow (\bar{c}u)(cud), \ldots$

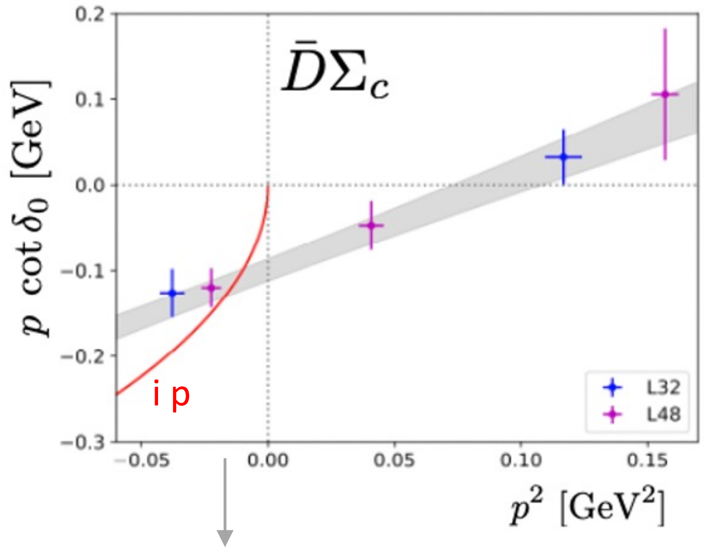
$\not\rightarrow (\bar{c}c)(uud)$



caution: coupling to charmonium+proton omitted

H. Xiang et al., 2210.08555 $m_\pi \approx 294$ MeV

$\bar{D}\Sigma_c$ in s-wave $J^P=1/2^-$

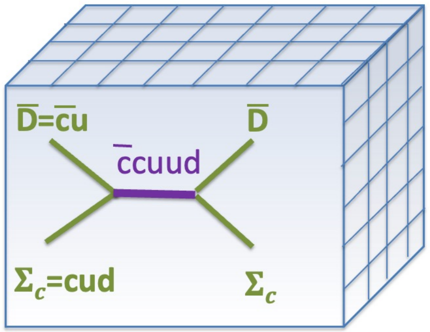


$p_b = i|p_b|$

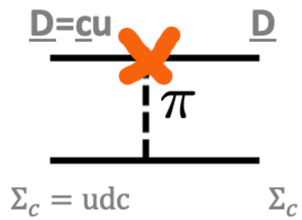
i p = i (i |p|)=-|p|

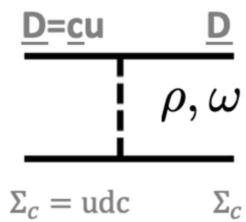
$$m_{P_c} = \sqrt{m_1^2 + p_b^2} + \sqrt{m_2^2 + p_b^2}$$

$$m_{P_c} - (m_D + m_{\Sigma_c}) = -6 \pm 3 \text{ MeV}$$ bound state

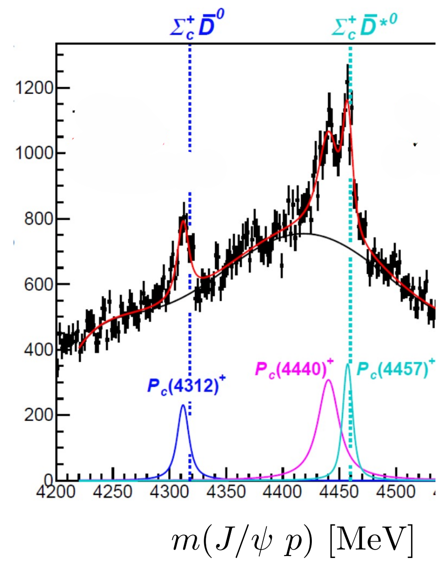


Wu, Molina, Oset, Zou, PRL 2010





LHCb 2019



The only previous study did not find significant p-J/ψ interaction at Pc energies assuming one-channel p-J/ψ scattering, Skerbis, SP [1811.02285, PRD]

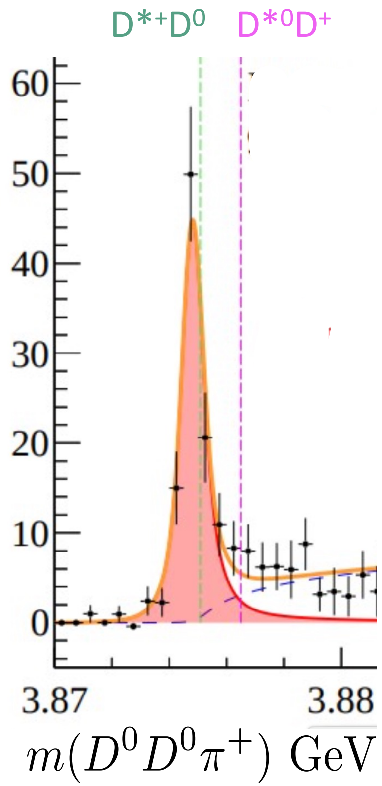
T_{cc} $cc\bar{d}\bar{u}$ I=0, $J^P=1^+$

experiment

$$D^* \to D\pi$$

$$m_{\pi^0} \simeq 135 \text{ MeV}$$
$$m_{D^{*+}} - m_{D^+} \simeq 140 \text{ MeV}$$

$$\delta m_{pole} = -0.36 \pm 0.04 \text{ MeV}$$

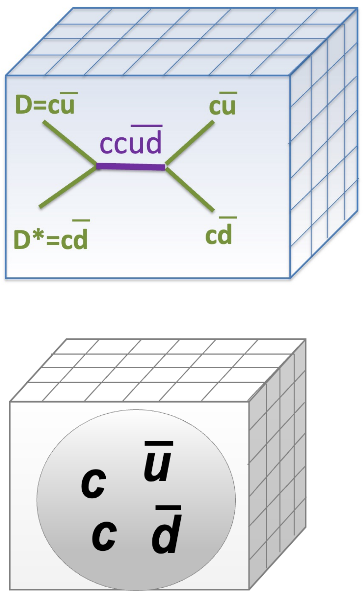


LHCb 2021 Nature Physics

lattice

$$D^* \not\to D\pi$$

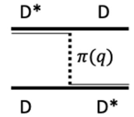
$$m_u = m_d > m_{u,d}^{ph}$$

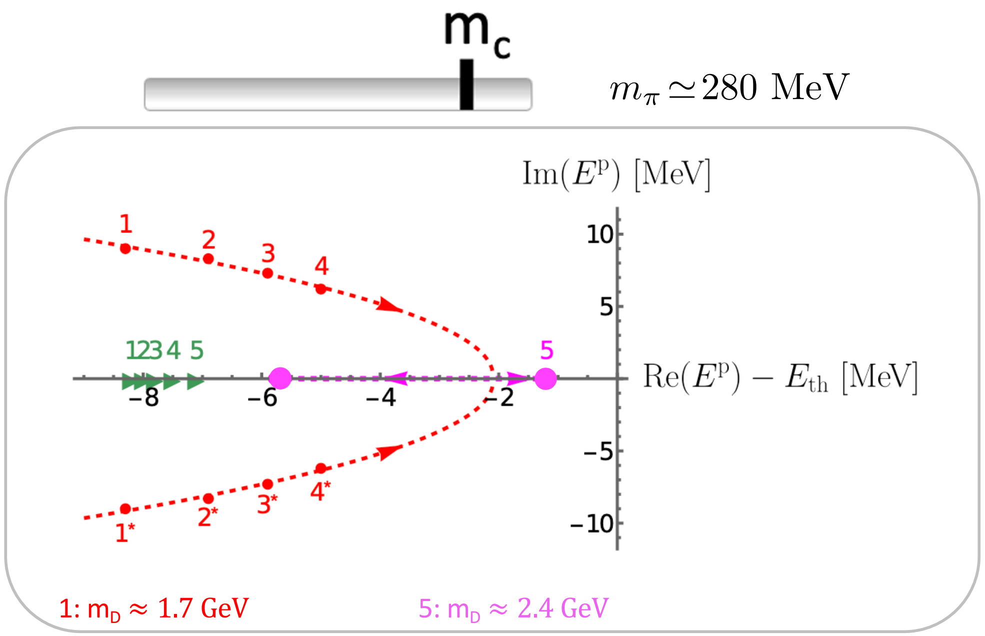


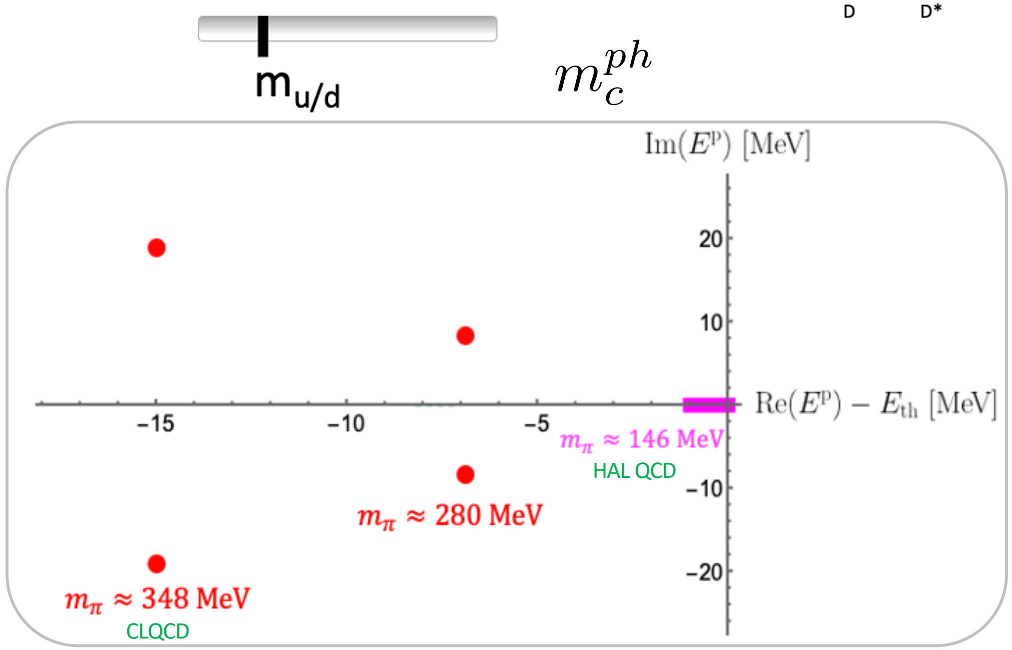
T_{cc} : scattering amplitude and pole trajectory

Collins, Nefediev, Padmanath , SP, 2402.14715, PRD

analysis incorporates analytic properties related to





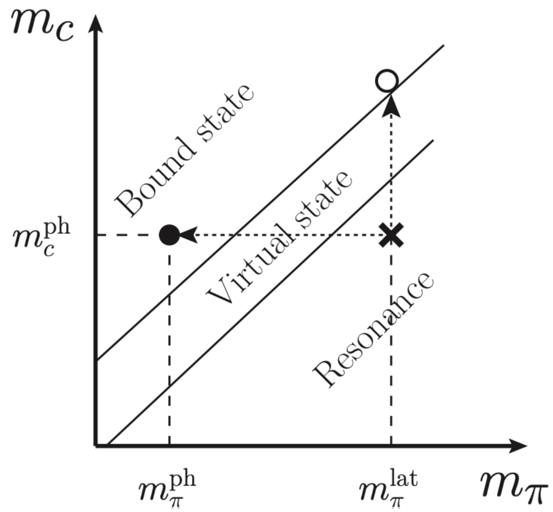


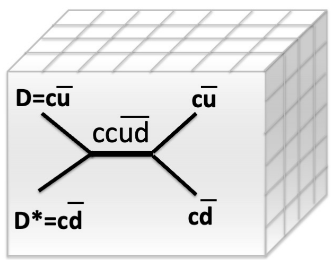
arrow: increasing mc

resonance pole

virtual state pole

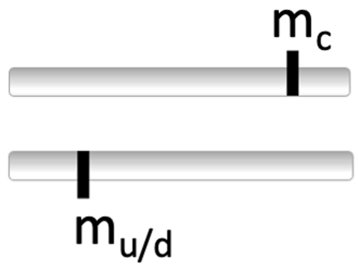
left hand cut

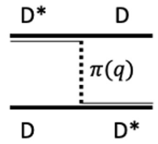




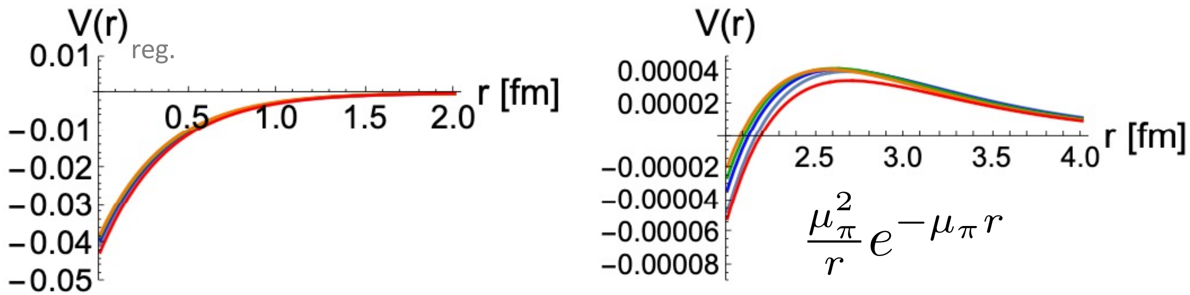
T_{cc} : interpretation

Collins, Nefediev, Padmanath , SP, 2402.14715, PRD

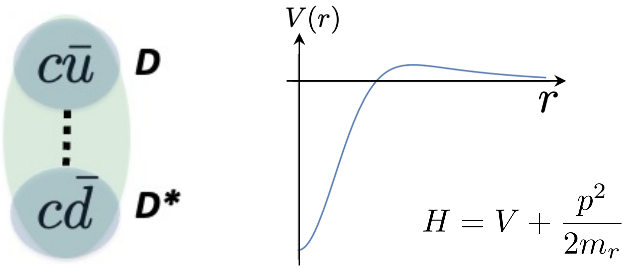




Potential for five different m_c

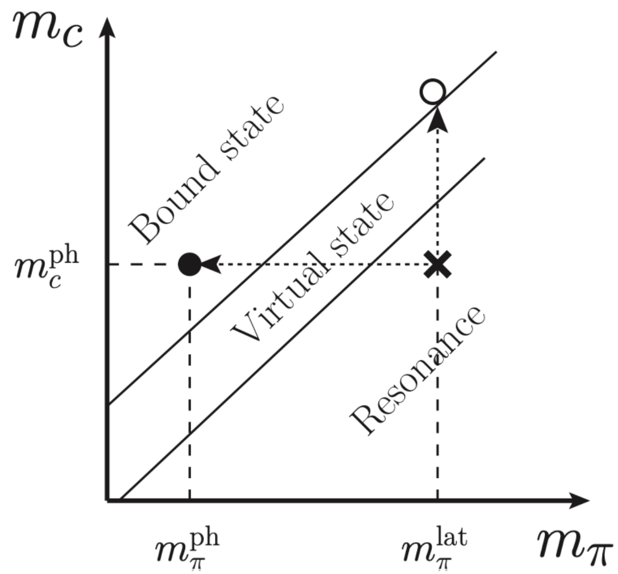


$$\mu_\pi^2 = m_\pi^2 - (m_{D^*} - m_D)^2 > 0$$



V almost independent on m_c

attraction increases with decreasing $m_{u/d}$



T_{cc}: I=0 vs. I=1 ($J^P=1^+$)

Meng, Pacheco, Baru, Epelbaum, Padmanath, SP, 2411.06266

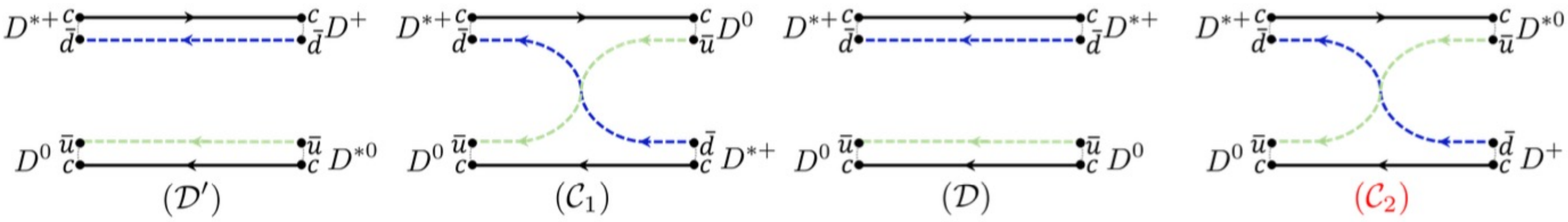
CLQCD, 2206.06185

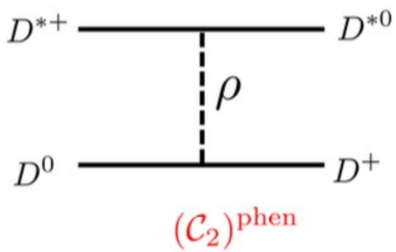
$cc\bar{d}\bar{u}$ $\quad \bar{u}\bar{d}-\bar{d}\bar{u}$ $\quad \bar{u}\bar{d}+\bar{d}\bar{u}$

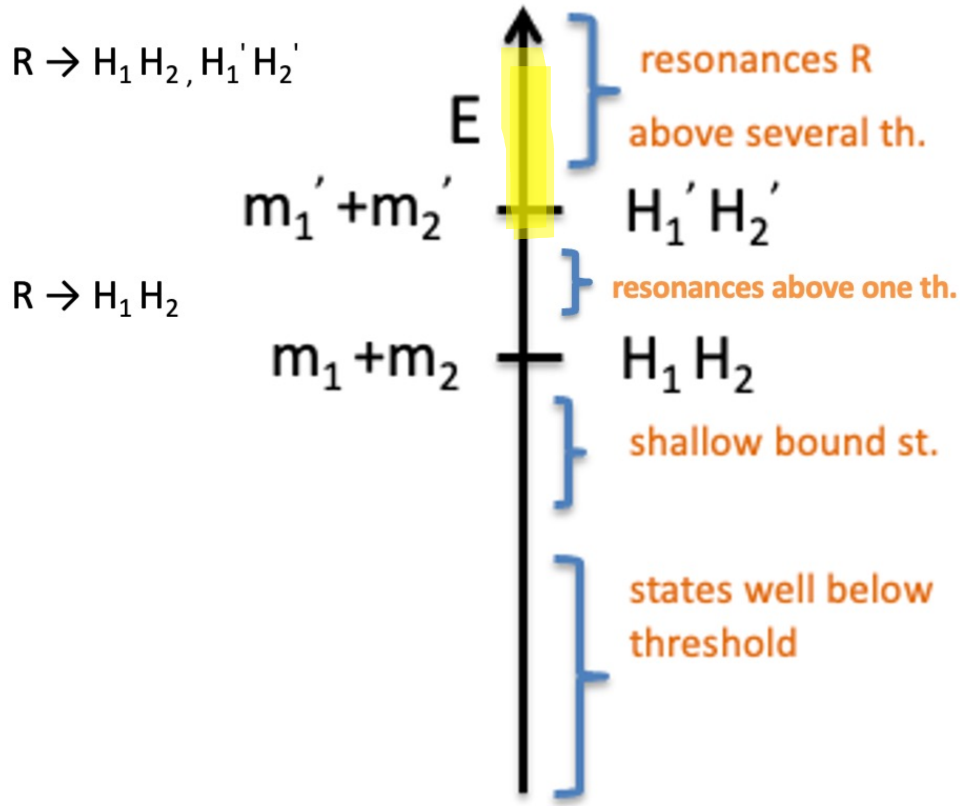
	I=0	I=1
	$a_0>0$	$a_0<0$ and small
	attractive	repulsive (on average)
	virtual state or resonance	no pole

$a_0 \equiv p\cot\delta|_{p=0}$

dominantly responsible for difference between I=0,1:







Hadrons from coupled-channel scattering

Coupled-channel scattering

most of hadronic resonances decay strongly to several final states

$R \to H_1 H_2\,,\; H_1' H_2'$

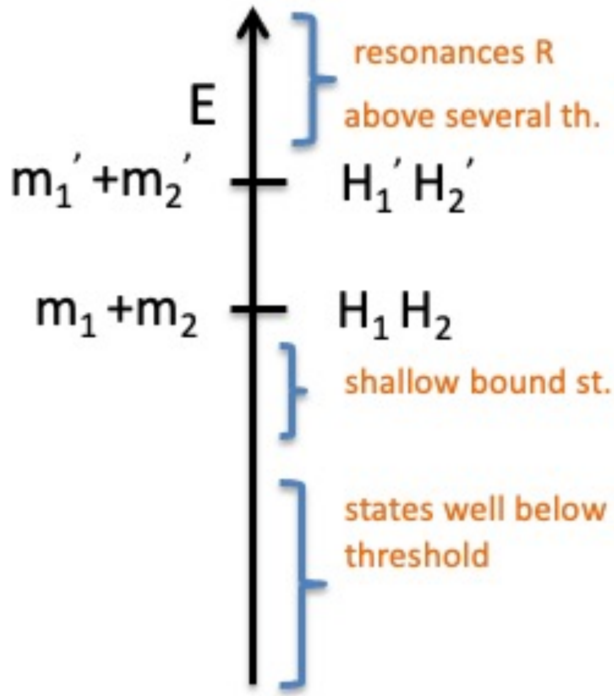
$f_0(980) \to \pi\pi,\; K\bar{K}$

$a_0(980) \to \pi\eta,\; K\bar{K}$

$a_1(1260) \to \rho\pi,\; \sigma\pi,\; \ldots$

$K_0^*(1430) \to K\pi,\; K\eta,\; K\eta'$

$D_3^*(2750) \to D\pi,\; D^*\pi$



almost all exotic hadrons decay stronly to several final states

$\bar{c}cu\bar{d}:\; Z_c \to J/\psi\,\pi,\; D\bar{D}^*,\; \eta_c\rho,\; \ldots$

$\bar{b}bu\bar{d}:\; Z_b \to \Upsilon(1S)\pi,\; h_b(1P)\pi,\; B\bar{B}^*,\; \ldots$

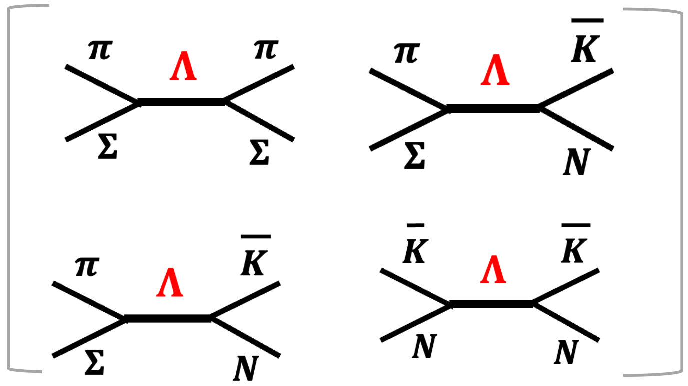
$\bar{c}cuud:\; P_c \to J/\psi\, p,\; \Sigma_c D,\; \ldots$

$\bar{c}c\bar{c}c:\; X(6900) \to J/\psi\, J/\psi,\; \eta_c\eta_c,\; \ldots$

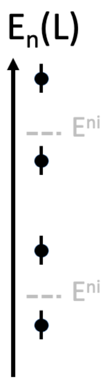
Coupled-channel scattering matrix

one-channel scattering	two-channel scattering
1x1 1x1 1x1	2x2 2x2 2x2
$S = I + i\frac{p}{4\pi E}T$	$S = I + i\frac{p}{4\pi E}T$

$$T(E) = \begin{pmatrix} T_{aa}(E) & T_{ab}(E) \\ T_{ab}(E) & T_{bb}(E) \end{pmatrix} =$$

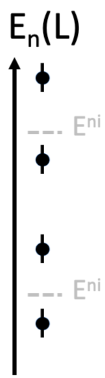


$$f(T(E)) = 0$$



$$E \rightarrow T(E)$$

$$f\big(T_{aa}(E), T_{bb}(E), T_{ab}(E)\big) = 0$$



$$E \rightarrow T_{ij}(E)$$

strategy:

- parametrize energy dependence of K matrix
- perform global fit to all eigen-energies

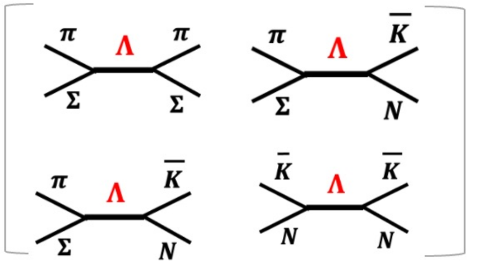
$$T_{ij}(E, \vec{\kappa})$$

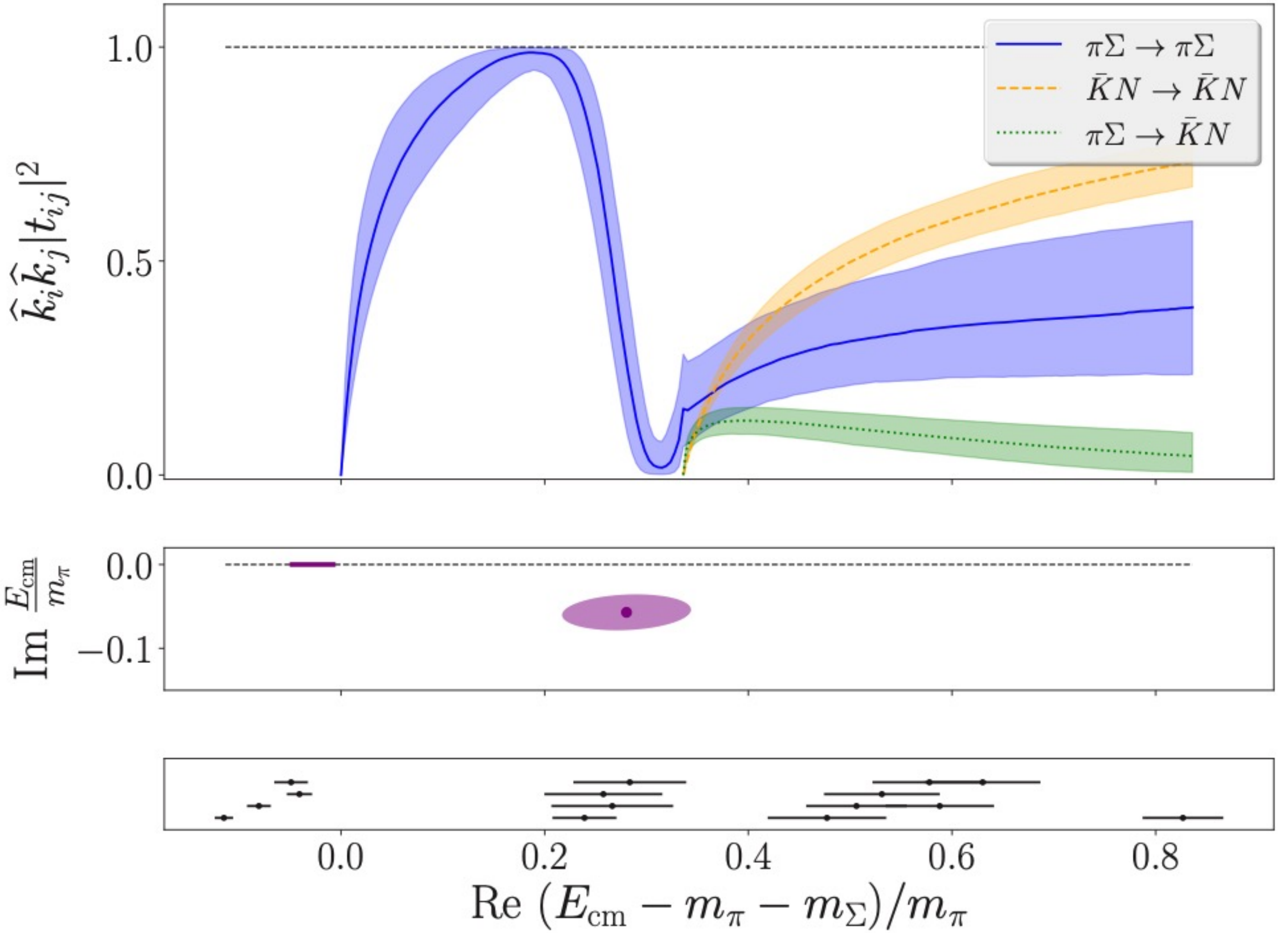
- applied for many meson resonances by HadSpec, mostly those composed of u,d,s

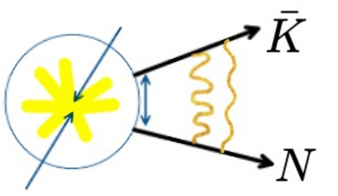
Two-pole structure of baryon $\Lambda(1405)$

Bulava et al, 2307.10413, PRL

$m_\pi \simeq 200\ \mathrm{MeV}$ $\quad I(J^P) = 0(\frac{1}{2}^-)$ $\quad$ S= -1







ALICE (2205.15176)
also finds two poles

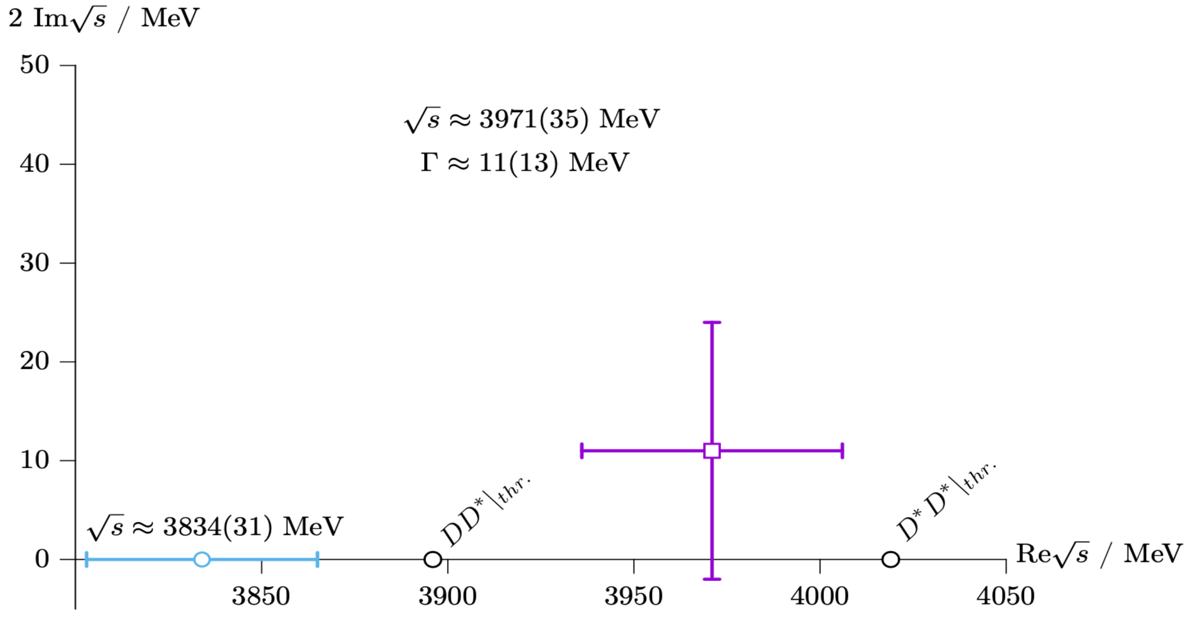
Coupled-channel *DD*-D*D** scattering

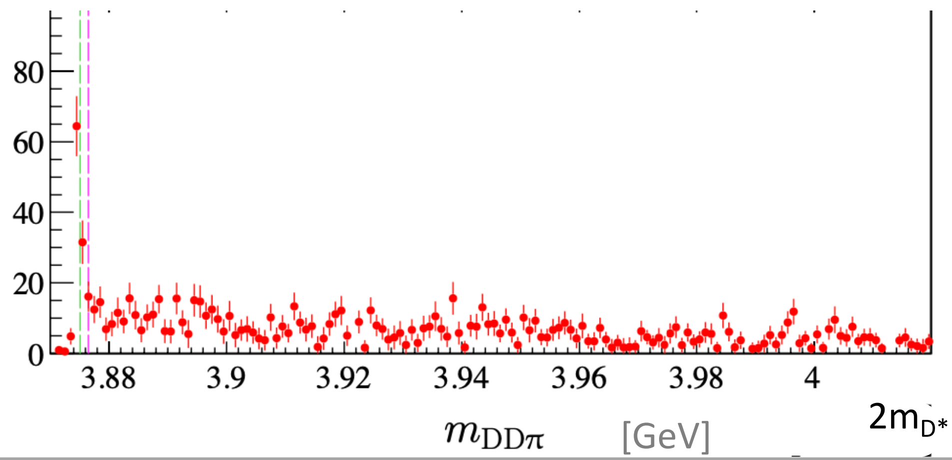
T_{cc} virtual state below DD* threshold (effects from left-hand cut not incorporated)

T_{cc}' resonance below D*D* threshold : look for it in experiment !

Hadspec 2405.15741

$m_\pi \simeq 391$ MeV





LHCb, 2109.01056

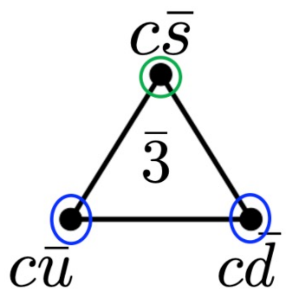
is there a particular reason to stop at D*D* threshold?

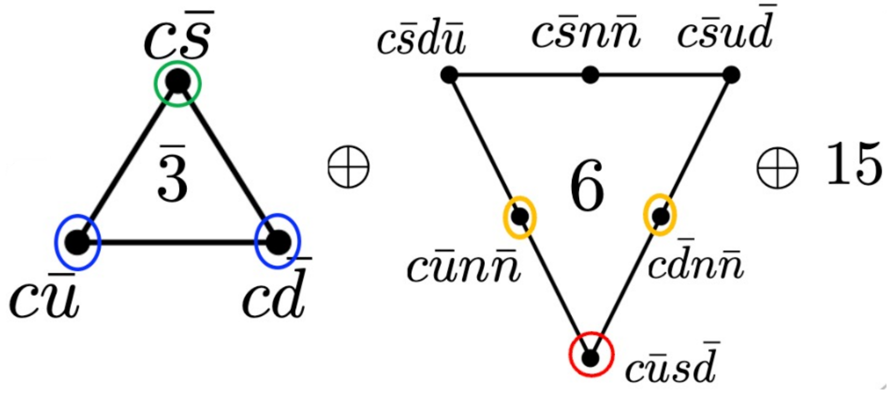
Scalar heavy-light mesons $J^P = 0^+$

Conventional quark model

$c\bar{q}$

$q=u,d,s$





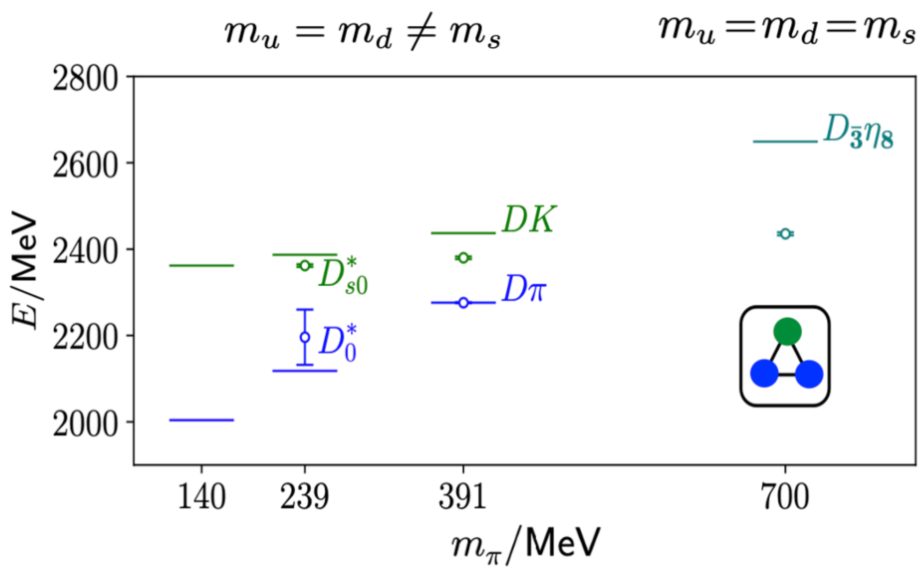
New paradigm

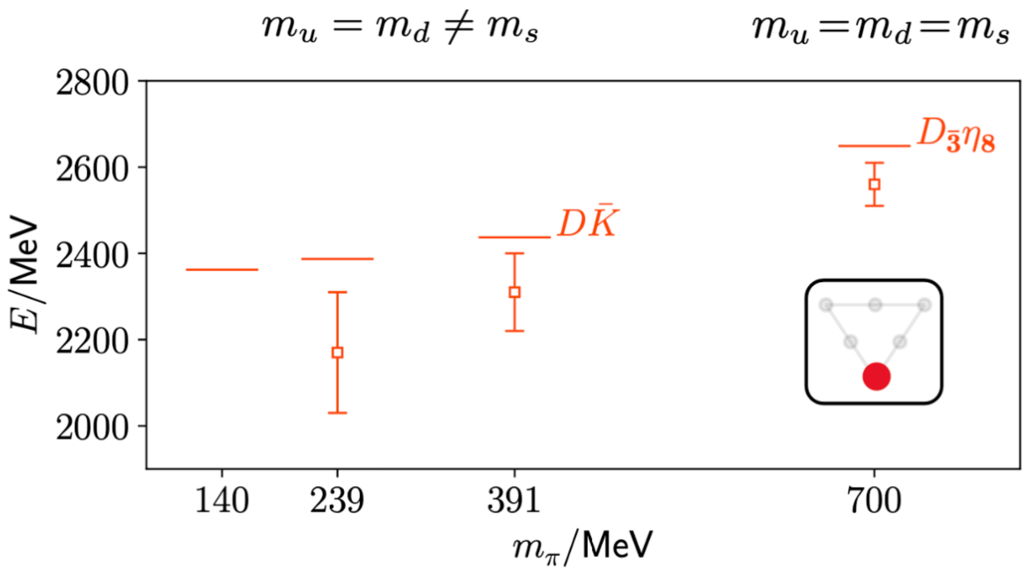
Lutz et al, 2003 PLB, 2209.10601 ; Du et al, 1712.07957, PRD
earlier lattice work: Mohler, SP, Lang, Leskovec, Woloshyn (several papers)
recent D-pi study: CLQCD 2404.13479
Lattice results below: HadSpec (several papers)

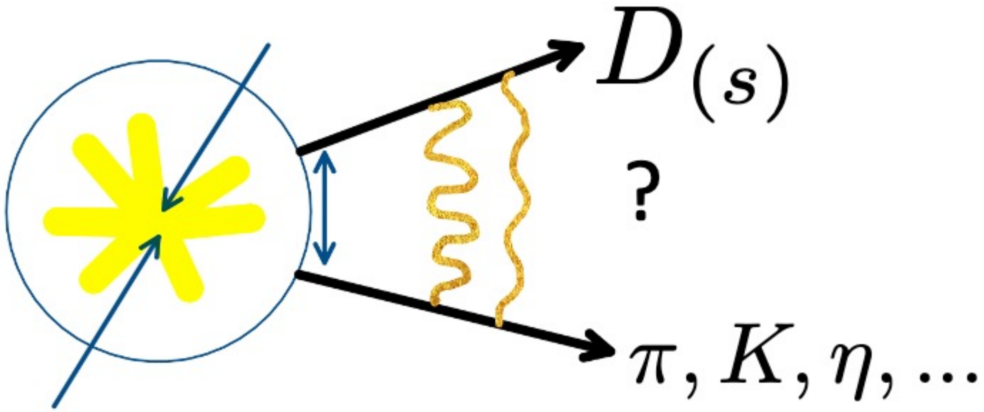
$c\bar{q}$ + $c\bar{q}\ q\bar{q}$ $q=u,d,s$ $n=u,d$

$\underline{3} \otimes 8 = \underline{3} \oplus 6 \oplus 15$ $SU(3)_F$

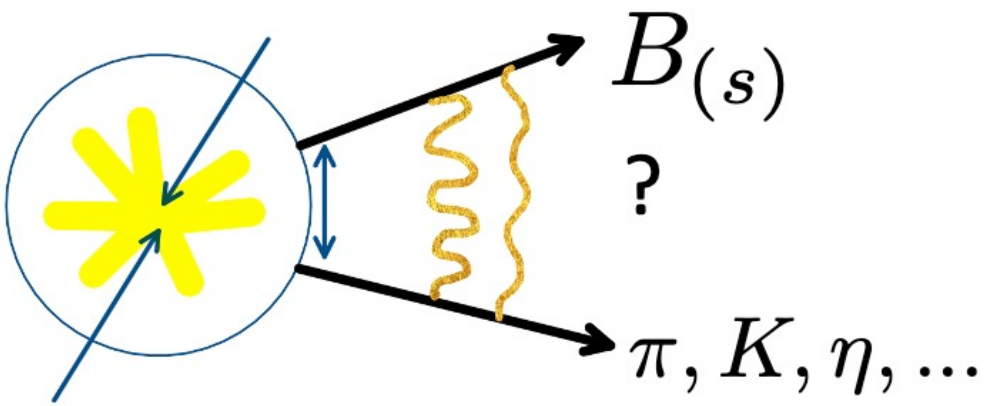
most attractive, attractive, repulsive







talk by Daniel Battistini
at Erice 2023

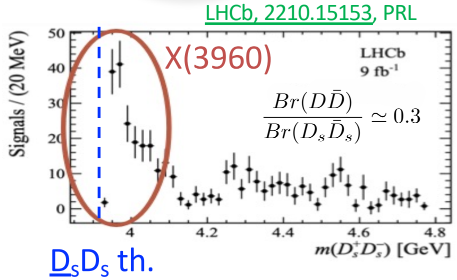


Charmonium(like) states from $D\bar{D} - D_s\bar{D}_s$ $\quad \bar{c}c$, $\bar{c}q\bar{q}c$ $\quad$ q=u,d,s $\quad I=0$

$\bar{D}_s D_s$ $J^P=0^+$ state

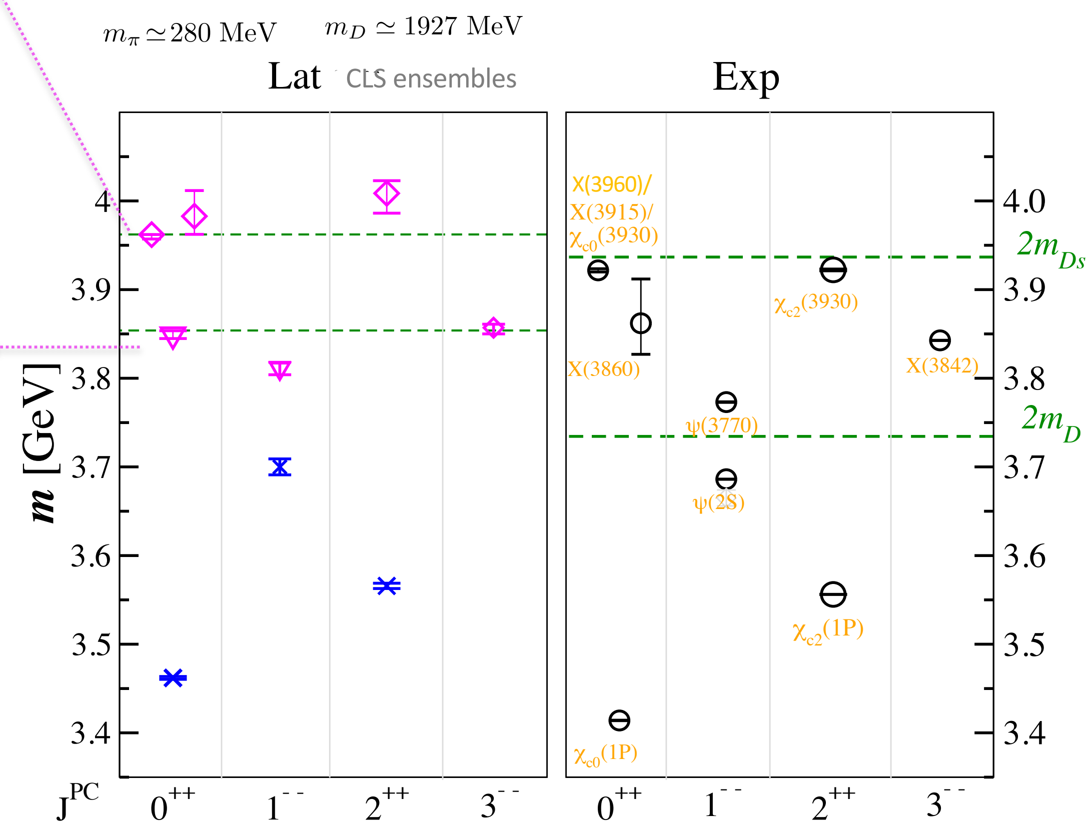
$$T_{ij}(E_{cm}) \sim \frac{c_i\ c_j}{E_{cm}^2 - m^2}$$

$$\frac{|c^2_{D\bar{D}}|}{|c^2_{D_s\bar{D}_s}|} \overset{\text{lat}}{=} 0.02\ ^{+0.02}_{-0.01}$$

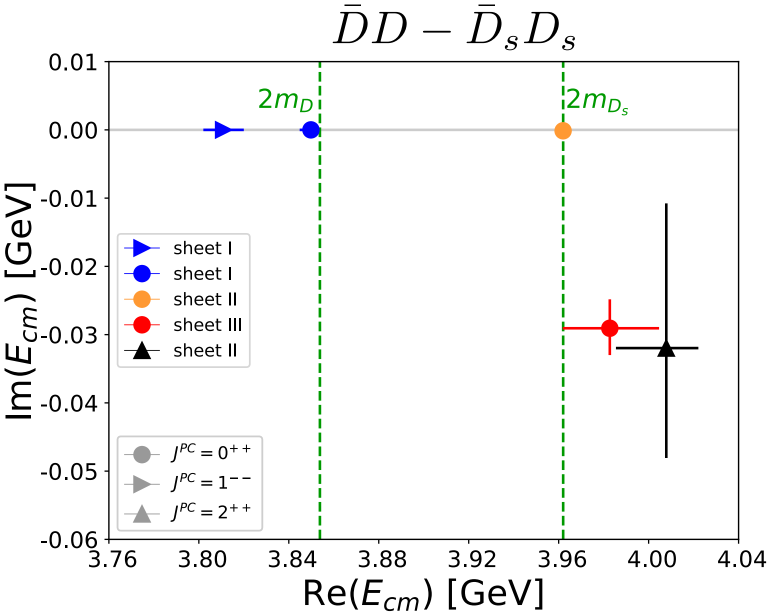


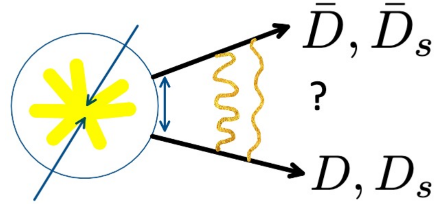
$\bar{D}D$ $J^P=0^+$ state

+ expected conventional charmonia



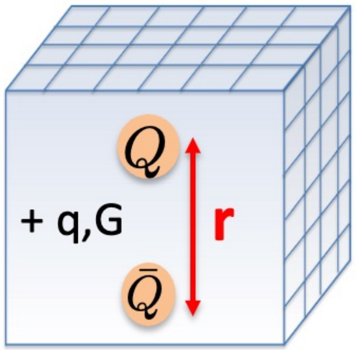
S.P. , Collins, Padmanath, Mohler, Piemonte
2011.02541 JHEP, 1905.03506 PRD





Exotic hadrons from static potentials

Static potentials from Born-Oppenheimer approximation



System with

- two heavy partices QQ or QQ or ...
- light degrees of freedom q=u,d, G

$$E = m_Q + m_{\bar{Q}} + W^Q_{kin} + W(q,G)$$

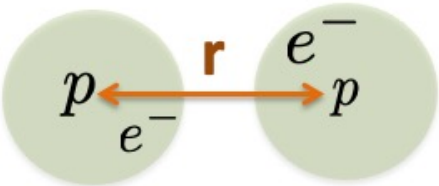
$$E = m_Q + m_{\bar{Q}} + W^Q_{kin} + V(r)$$

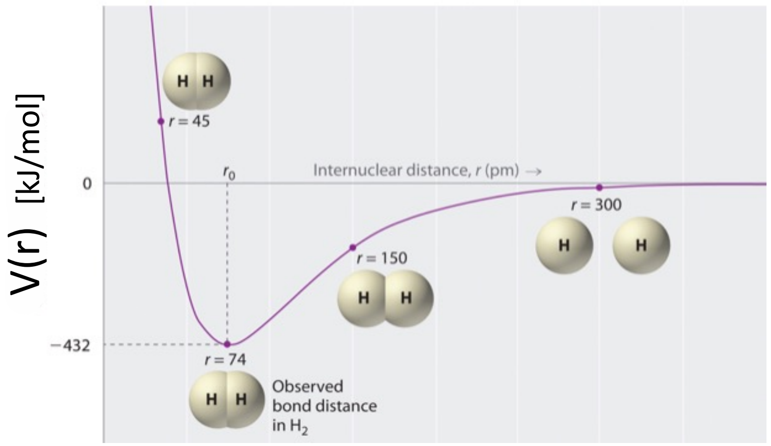
seminal recent work aimed at exotica:

Brambilla et al (TUM): 2408.04719

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t) = -\frac{\hbar^2}{2m}\nabla^2\Psi(\mathbf{r},t) + V(\mathbf{r})\Psi(\mathbf{r},t)$$

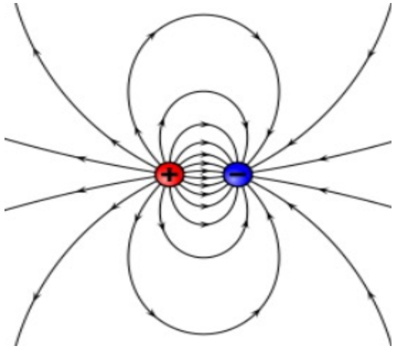
H_2

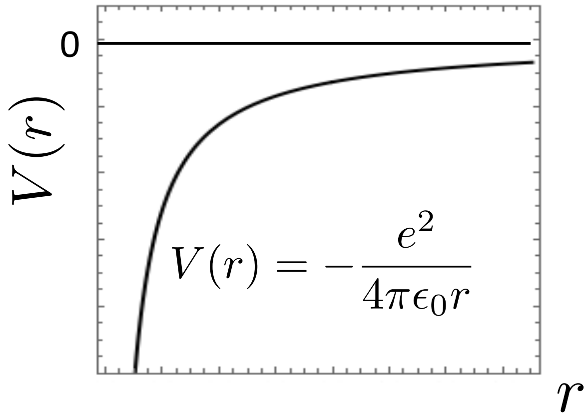




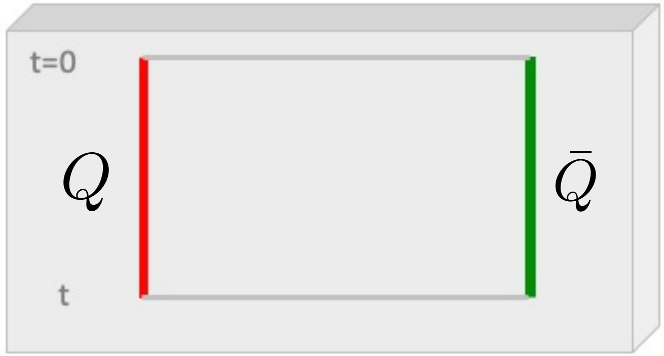
Potential and confinement

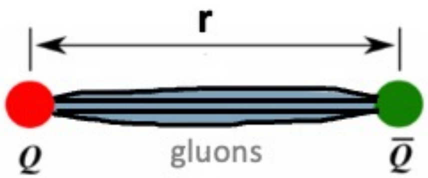
EM interactions

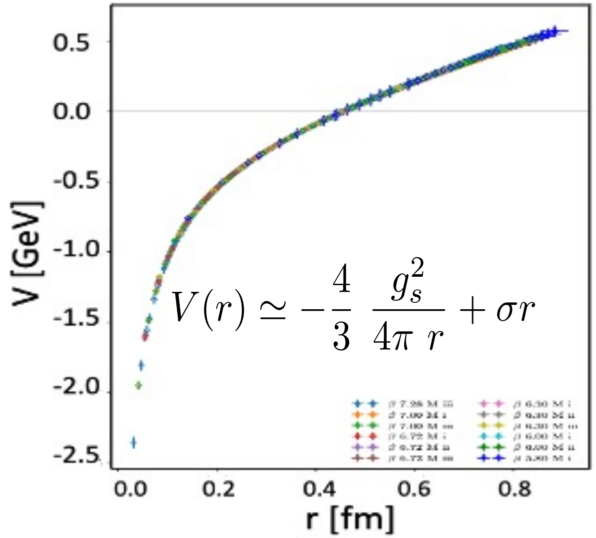




strong interactions (QCD without dynamical quarks)







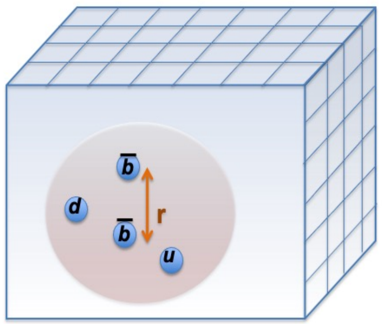
TUMQCD, 2206.03156, PRD

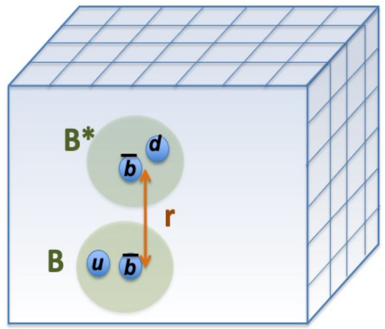
Deeply bound doubly bottom tetraquark

$I=0,\ J^P=1^+$

$\bar{b}\bar{b}ud$

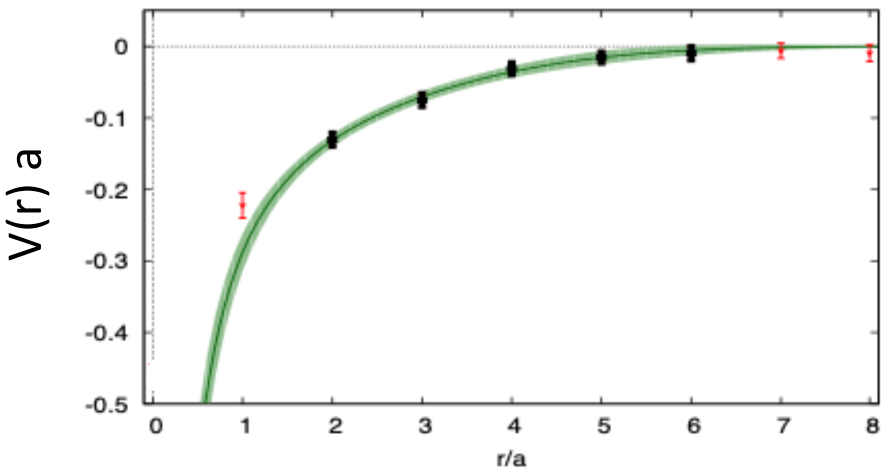
$$E_n(r) \ \rightarrow \ V(r) \ \rightarrow \ m$$





$$-\frac{\hbar^2}{2m_r}\nabla^2\Psi(\mathbf{r},t)+V(\mathbf{r})\Psi(\mathbf{r},t) \ = E^{nr}\Psi(\vec{r},t)$$

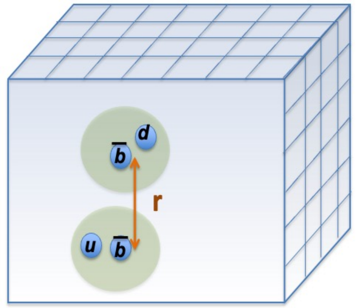
$E^{nr} = m\text{-}m_B\text{-}m_{B*} = \text{-}\ 38\ (18)$ MeV



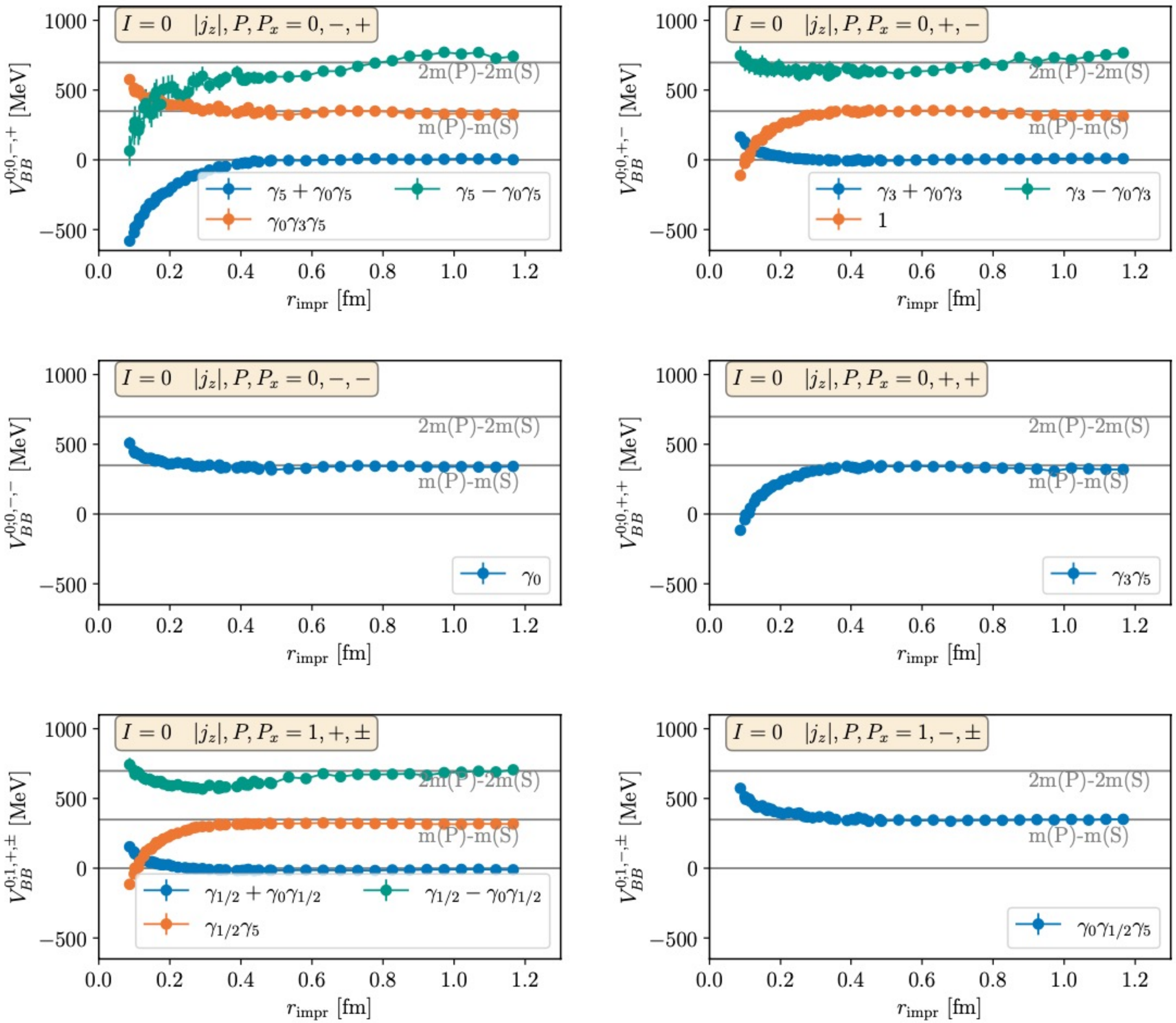
Bicudo, Wagner, Peters, Cichy (1209.6274)

More doubly-bottom tetraquark potentials

$\bar{b}\bar{b}ud$



quantum numbers in the static limit ⇔ can be related to various J^P channels



$B_0 B_0$

$B\ B_0$

$B\ B$ (also BB*, B*B)

$m_b \to \infty: B = B^*,\ B_0 = B_1$

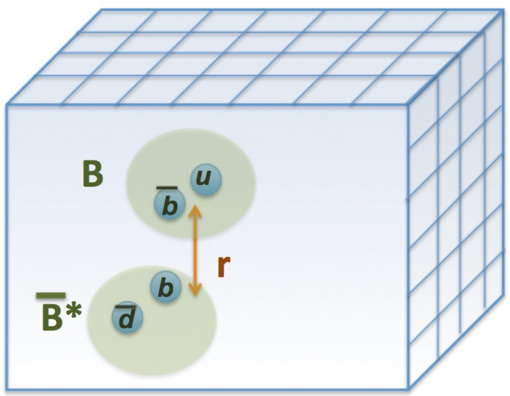
Bicudo, Marinkovic, Muller, Wagner, 2409.10796

Z_b tetraquark

$\bar{b}b\bar{d}u \quad J^P = 1^+$

S.P., Bahtiyar, Petkovic, PLB 2019

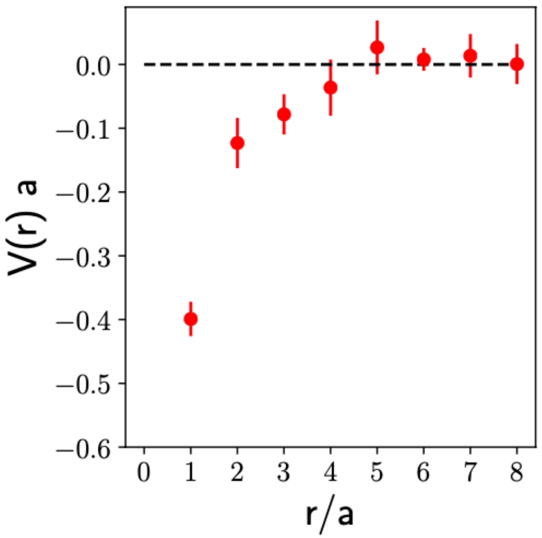
Sadl, S.P., PRD 2021



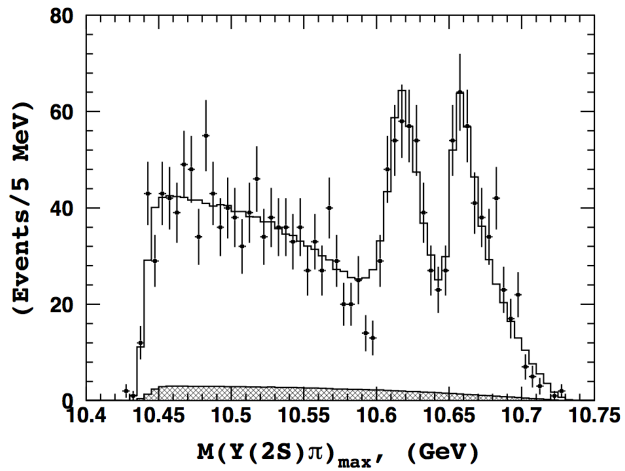
challenge

$\bar{b}b\bar{d}u \rightarrow \bar{b}b + \bar{d}u$ Υ(nS) π , Υ(2S) π, Υ(3S) π, $h_b(1S)$ π, $h_b(2S)$ π

$\rightarrow \bar{b}u + \bar{d}b$ B $\underline{B}$* , B* $\underline{B}$*



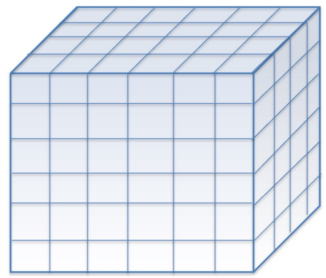
Belle 2011 PRL 2011



All presented results are extracted from E_n

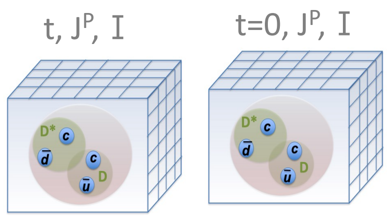
$$\langle C \rangle = \int DG\, Dq\, D\bar{q}\, C\, e^{-S_{QCD}/\hbar}$$

$$C_{ij}(t) = \langle 0 | O_i(t)\, O_j^{+}(0) | 0 \rangle = \sum_n \langle 0 | O_i | n \rangle\, e^{-E_n t} \langle n | O_j^{+} | 0 \rangle$$

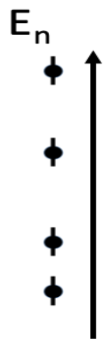


often "non-precision" studies:

single a, $m_{u/d} > m_{u/d}^{phy}$, $m_\pi > 140$ MeV

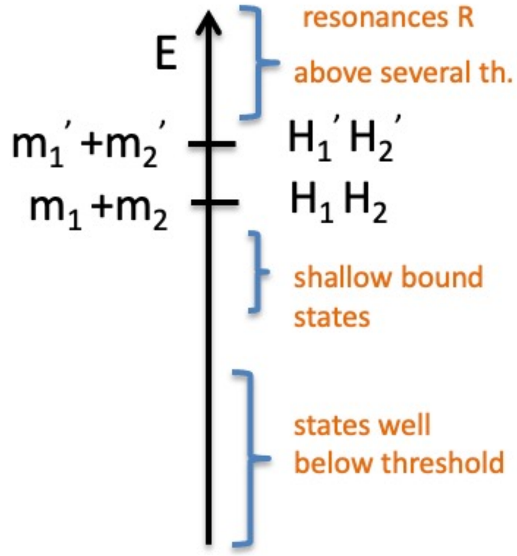


$$\mathcal{O} = \mathcal{O}(q, G)$$



- for strongly stable state well below threshold : $E_n(P=0) = m$
- resonances (Luscher's relation) $$E_n^{cm} \to T(E_n^{cm})$$
- static potentials: $$E_n \to V(r)$$

Conclusions



Status on exotic hadrons from Lattice :

- exotic hadrons that are not resolved (yet)

  strongly decay via many decay channels: $Z_c(4430)$, X(6900),...

- available: valuable results on exotic (and conventional) hadrons

  strongly stable ; strongly decaying to 1,2,3 channels $H_1 H_2$

- significant progress on three-hadron scattering and R-> $H_1 H_2 H_3$ (not discussed here)

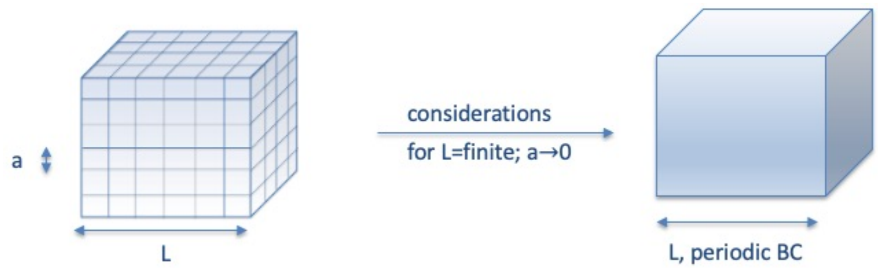
- HALQCD method to extract scattering amplitude (not discussed here)

- looking forward to learn what femtoscopy can do or has accomplished

Reviews:
N. Brambilla et al. 1907.07583, Phys. Rept
M. Mai, U. Meissner, C. Urbach, 2206.01477
N. Brambilla, 2111.10788
P. Bicudo, 2212.07793
...
S. Prelovsek, Lattice QCD calculations of hadron spectroscopy, Encyclopedia of Particle Physics, Elsevier (on the way)

Backup

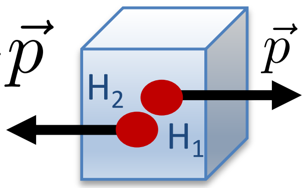
Towards relation between E and T(E) in finite-volume QCD



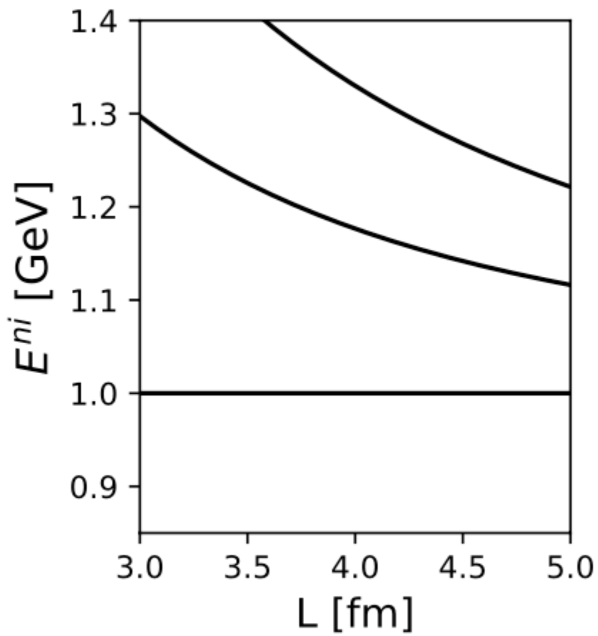
E for non-interacting H_1 H_2 (P=0)

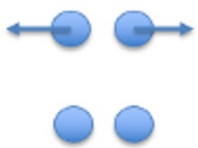
$$E^{n.i.} = \sqrt{m_1^2 + \vec{p}^2} + \sqrt{m_2^2 + (-\vec{p})^2}$$

$-\vec{p}$



$\vec{p} = \vec{n}\frac{2\pi}{L}$





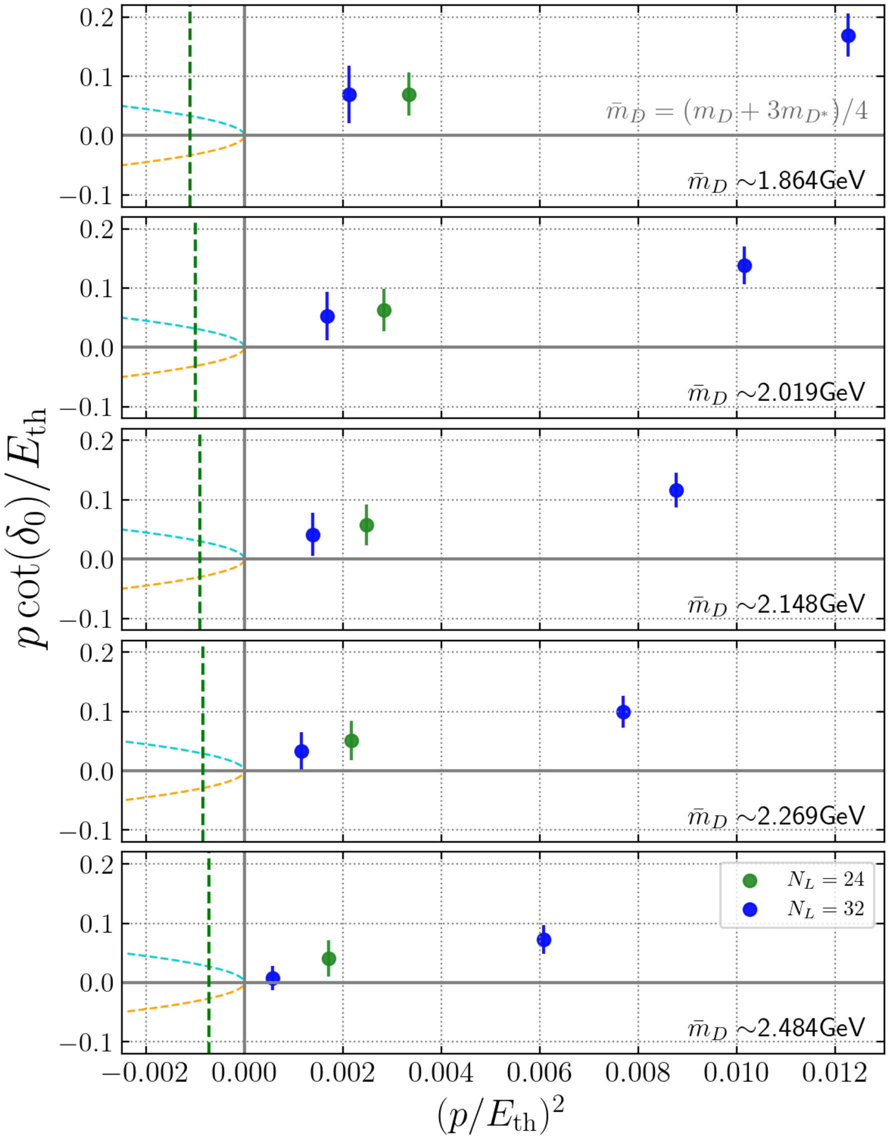
$$p = \frac{2\pi}{L}(1,1,0)$$

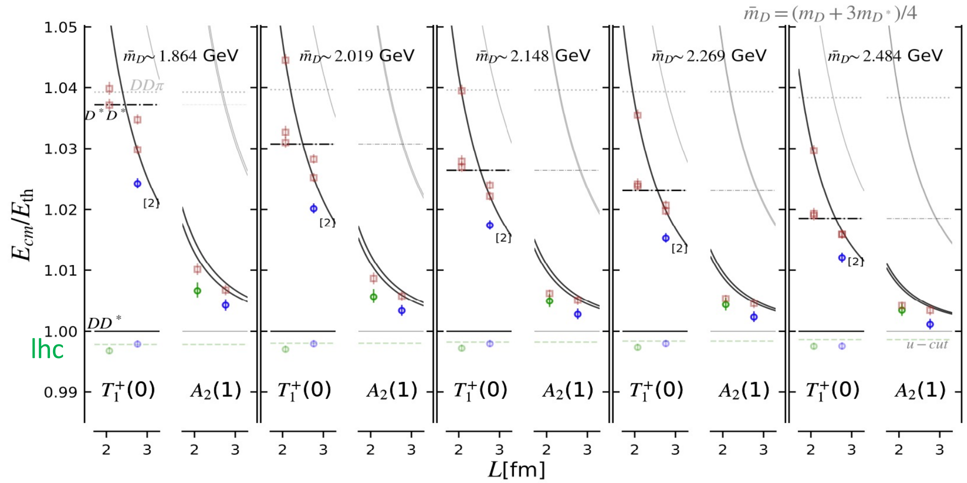
$$p = \frac{2\pi}{L}(1,0,0)$$

$$p = 0$$

example: m=0.5, L=3.6 fm

T_{cc} : scattering amplitude





δ(E) E